Existence and specific characters of rentiers. A savers-spenders theory approach*

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Abstract: Using the savers-spenders theory developed by Mankiw (2000, AER), we propose microfoundations to the existence of rentiers in macroeconomic growth models. From an OLG model which acknowledges the great heterogeneity of consumer behavior apparent in the data, we capture the dynamic considerations of potential rentiers as a natural consequence of intertemporal utility maximization and we analyze realistic characteristics (proportion, wealth, propensity to save) of rentiers.

Keywords: Growth models, Heterogeneity of preferences, Endogenous labor supply.

JEL codes: E13, D64, J22.

*This paper is adapted from the fourth chapter of my Ph.D Thesis. Then, I thank Alain VENDITTI, my Ph.D Supervisor and Antoine d'AUTUME and Pierre PESTIEAU my Ph.D Referees. Earlier version of this paper were presented at the International conference: New perspectives on (un)stability: the role of heterogeneity at Marseille, June 7-9, 2001 and at the X-th Spring School of the Associated European Laboratory (CNRS-FNRS/CORE-GREC-GREQAM) at Aix, 2001. I thank conference participants, in particular Olivier CHARLOT, Christian GHIGLINO, Kiminori MATSUYAMA and Philippe MICHEL for suggestions, helpful comments and discussions.

1 Introduction

The macrodynamic models of equilibrium are usually divided in two streams depending on whether households are life cycler (finite horizon) or altruistic, leaving positive bequests (infinite horizon). Recently, Gregory Mankiw pointed at numerous empirical facts to persuade economists that neither the Barro-Ramsey model of intergenerational altruism nor the Diamond-Samuelson model of overlapping generations are consistent with the empirical findings[1].

Instead of these two canonical models, Mankiw (2000a,b) proposes a macroeconomic framework which seems to be best-suited to the large heterogeneity in consumer behavior apparent in the data. Some altruistic people (namely, the savers) have long time horizons, which is consistent with the great concentration of wealth and the importance of bequests in aggregate capital accumulation. Others (namely, the spenders) have short time horizons, as evidenced by the failure of consumption smoothing and the prevalence of households with near zero net worth. This model which combines agents *à la Diamond* and agents *à la Barro* yields some new and surprising conclusions about fiscal policy[2] and for Mankiw (2000b, p. 124):

> " *The savers-spenders theory sketched here takes a small step toward including this microeconomic heterogeneity in macroeconomic theory.* "

The purpose of this paper is to study other important implications of this new sort of heterogeneity. Indeed, we shall show that the savers-spenders theory is adequate to give microfoundations to the macroeconomic models with rentiers. Indeed we shall study a savers-spenders model which allows us to show that the emergence

[1] Mankiw (2000a, p. 1 to 5) discusses empirical regularities which led him to reject both Diamond's (1965) model and Barro's (1974) model.

[2] Indeed, fiscal policy such as public debt is found to be neutral at the aggregate level of capital accumulation but redistributes resources from nonaltruistic to altruistic households (see also Michel and Pestieau (1998) or Smetters (1999)).

of rentiers is a natural consequence of intertemporal utility maximization. The emergence of rentiers is then a consequence of microeconomic heterogeneity and of the labor-leisure choice introduced in Mankiw's (2000a,b) model.

In the growth literature there are broadly two kinds of macroeconomic models with rentiers. The most famous are issued from the Kaldorian tradition of two-class growth models (capital owners and workers). In these models (see, for instance, Kaldor (1956) or Pasinetti (1962)), it is assumed the existence in the society of an exogenous proportion of workers and rentiers (i.e., capital owners who do not work). Then, the long run equilibrium is also determined by the exclusive savings of rentiers (see Britto (1972)). The most recent macroeconomic models with rentiers are used to study stationary sunspots or/and endogenous business cycle (see, for instance, Grandmont, Pintus and de Vilder (1998) or Barinci (2001)) and are based on the finance constrained economy developed by Woodford (1986). Briefly outlined, these models consider two assets - money and capital - and feature two classes of infinite lived agents labelled workers and rentiers. It is assumed that workers supply a variable quantity of labor time units, depending on their per period liquidity constraint. Such a constraint is meant to reflect the difficulty they have in borrowing against labor income, whereas rentiers do not work.[3]

To give realistic microfoundations to models with an exogenous proportion of rentiers, we focus on an OLG model in which the population consists of agents with heterogenous degrees of altruism toward their offspring whose labor supply is endogenous. This model allows to encompass most of the OLG models in which there is at least one agent *à la Barro*[4].

[3]Under some mild assumptions, along equilibria close enough to the steady state, workers save their end-of-period wage earnings only in the form of money while rentiers never wish to accumulate money.

[4]Our approach embodies a wide class of OLG models with exogenous labor supply: those where agents are exclusively altruists (see, for instance, Weil (1987) or Thibault (2000)), those

Our framework, in which a finite number of dynasties can be distinguished by their sole degree of altruism, is more general that Mankiw's model of two types of agents (altruists and selfish). It integrates recent empirical evidence on saving and bequest motives (see Arrondel, Masson and Pestieau (1997)): the majority of life-cyclers individuals are not selfish but constrained altruists. Hence, our micro-foundations can be considered quite realistic, and one does not necessarily need to assume the existence of egoistic agents, i.e. parents who do not care about the welfare of their children.

Our model bears some similarities to Becker (1980) with heterogenous infinite lived agents. It may be useful to give a brief outline of the similarities and denote the differences. There are at least four reasons why our model differs from Ramsey (1928) or Becker (1980). First, in our context the individuals' labor supply is endogenous. Second, contrary to a widespread opinion, even if bequests are positive, the OLG model with dynastic altruism à la Barro (1974) is not equivalent to the infinite horizon representative agent model[5]. Third, contrary to Becker (1980), our model can possess life cyclers, and, therefore, consists of a mix of agents with finite and infinite horizons.[6] Fourth, the market structure of Becker (1980) requires capital assets to be nonnegative at each moment of time and that agents without capital have no access to the loan market; for high discount rate consumers with zero initial capital stocks, this implies that the wage income is consumed at each

where population is a mix of agents *à la Diamond* and agents *à la Barro* (see, for instance, Michel and Pestieau (1998) or Nourry and Venditti (2001)) or those with heterogenous dynasties (see, for instance, Smetters (1999)). We also extend Michel and Pestieau's (1999) analysis of a successive generations model to an overlapping economy.

[5]As pointed by Michel, Thibault and Vidal (2003), there are at least four differences between these two canonical models. Note that these differences are so important that they can lead to surprising fiscal policy results (see Michel and Thibault (2003)).

[6]The differences between economies with finite or infinite lived consumers and those with both finite and infinite lived consumers are stressed by Muller and Woodford (1988).

time. Such an artificial constraint is naturally incorporated in our model, in which the so-called nonnegative bequest constraint prevents individuals from transferring resources away from their children.

In the long run, as in Mankiw (2000a,b), the society is divided in two classes: altruistic agents who make positive transfers (the savers) and agents who cannot afford making positive bequest (the spenders). Indeed, only the dynasty endowed with the highest degree of altruism, has the possibility to leave a bequest to their children. In such a situation, the steady state of the economy is the golden rule modified by the degree of altruism of the most altruistic agents; regardless their relative number.

If, as Mankiw (2000a,b), we interpret degree of altruism as a degree of patience or as a propensity to save,[7] this result is consistent with the intuition of Ramsey (1928) and the findings of Becker (1980). Indeed, considering (in an heuristic way) the case where different people discount future utility at different rates, Ramsey (1928, p. 559) concluded his seminal paper as follows:

> " *In such a case, therefore, equilibrium would be attained by a division of society into two classes, the thrifty enjoying bliss and the improvident at the subsistence level.* "

Since the most altruists (i.e., the savers) can inherit, they can behave as rentiers, i.e, as individuals who can choose not to work. The aim of this paper is to establish the conditions under which rentiers emerge and to analyze their characteristics (proportion, wealth, propensity to save).

First, we focus on the wealth of rentiers. We show that, *ceteris paribus*, there exists a level of wealth above which savers decide to be rentiers. Importantly, this

[7]Separating the two concepts, Falk and Stark (2001) analyzes the roles of altruism and impatience in the evolution consumption and bequests in an exogenous labor supply context. Drugeon (2000) reexamines the role of long-run endogenous impatience in homothetic growth path.

endogenous threshold depends on the proportion and the saving propensity of savers. To give rise to the emergence of rentiers in the society, we show that it has to be a sufficiently large proportion of spenders. In this case, spenders'saving are lower and a large share of capital belongs to the savers. Since production is provided by spenders, savers choose not to work. Hence, savers are rentiers if (and only if) their proportion in the economy is sufficiently low.

Note that the existence of rentiers is really a consequence of the microeconomic heterogeneity introduced in our macroeconomic model. Indeed, in a society exclusively composed of homogenous savers, individuals choose to work so that the production sector does not vanish.

After having studied their proportion, we focus on the propensity to save of rentiers. Indeed, we examine the impact of the saving propensity of savers on their labor supply. Then, we can distinguish two opposite effects when the previous propensity increases. A *wealth effect*: since agents accumulate more, they are urged to lower their labor supply because they are richer. But, since the wage increases and the interest factor decreases, savers are incited to work more (*wage/interest rate effect*).

In a theoretical Cobb-Douglas economy we show that the *wealth effect* begins to dominate the *wage/interest rate effect* and the labor supply of savers decreases. These drop can lead savers to stop working. However, from a high value of the propensity to save, the *wage/interest rate effect* is larger than the *wealth effect* and the labor supply of savers increases.

Consequently, we show that to be a rentier, the saving propensity of a saver must be sufficiently high to avoid that the saver is a spender, but not too high. Indeed when his propensity to save is too large, a saver wishes to work to accumulate more (and more) capital.

To conclude, we can make two remarks about the existence of a cut-off propensity to save above which savers always choose to work. First, this result proves that for a given saving propensity, there exists a level of wealth above which savers choose not to work. However, this wealth is not always attained. The same applies to the richer.

Finally, our results can also contribute to an alternative interpretation of observed variations in wealth inequality and can explain why rentiers seem to vanish during the twentieth century. We can imagine that the rise of individualism during the last century has generated a higher propensity to save and accumulate capital.

The remainder of this paper is organized as follows. Section 2 sets up the OLG model with heterogenous dynasties and endogenous labor supply. In section 3 we exhibit conditions under which savers and rentiers may appear in the society. We also examine the wealth of these rentiers. In section 4, we focus on the proportion and the propensity to save of rentiers. Section 5 sets out our conclusions. Proofs are gathered in appendix.

2 The model

Consider a perfectly competitive economy which extends over infinite discrete time. The economy consists of $N \geq 1$ families denoted with $h \in \{1, ..., N\}$. In each period t, the size of each family h is denoted with N_t^h and grows at rate n. We consider a population of size N_t which consists of a fraction p_t^h of each family h where the proportion p_t^h does not vary through time. Hence:

$$\forall t > 0 : \frac{N_t^h}{N_t} = p_t^h = p^h \quad \text{and} \quad \sum_{h=1}^{h=N} p^h = 1 \quad \text{and} \quad \frac{N_{t+1}}{N_t} = \frac{N_{t+1}^h}{N_t^h} = 1 + n$$

We assume that $p^h \in (0, 1]$ for $h \in \{1, ..., N\}$.

The consumers

Individuals of a family h are identical within as well as across generations and live for two periods. Hence, a family can be identified with a dynasty. For altruistic agents, we adopt Barro (1974)'s definition of altruism: parents care about their children welfare by weighting their children's utility in their own utility function and possibly leave them a bequest. When young, altruists of dynasty h born at time t receive a bequest x_t^h, work a portion l_t^h of their first period, receive the market wage $w_t l_t^h$, consume c_t^h and save s_t^h. When old, they consume part of the proceeds of their savings and bequeath the remainder $(1+n)x_{t+1}$ to their $(1+n)$ children. Agents perfectly foresee the interest factor R_{t+1}. Importantly, the bequest is restricted to be non-negative. We denote by V_t^h the utility of an altruist of dynasty h:

$$
\begin{aligned}
V_t^h(x_t^h) = \max_{c_t^h,\ell_t^h,s_t^h,d_{t+1}^h,x_{t+1}^h} \quad & U(c_t^h, \ell_t^h, d_{t+1}^h) + \beta^h \; V_{t+1}^h(x_{t+1}^h) \\
s.t \quad & w_t(1-\ell_t^h) + x_t^h = c_t^h + s_t^h && (1) \\
& R_{t+1} s_t^h = d_{t+1}^h + (1+n)x_{t+1}^h && (2) \\
& x_{t+1}^h \geq 0 && (3) \\
& \ell_t^h \in [0,1] && (4)
\end{aligned}
$$

where $V_{t+1}^h(x_{t+1}^h)$ denotes the utility of a representative descendant who inherits x_{t+1}^h, $U(c^h, \ell^h, d^h)$ his life cycle utility which depends on consumptions (c^h, d^h) and leisure $\ell^h = 1 - l^h$ and β^h the intergenerational degree of altruism of the dynasty h.

We assume that $\beta^N \in (0,1)$ and (if $N > 1$) $\beta^h \in [0, \beta^N)$ for $h \in \{0, ..., N-1\}$. Therefore, N is the most altruistic dynasty. Moreover, agents of all dynasties have the same life cycle utility $U(c^h, \ell^h, d^h)$ satisfying Assumption 1.

Assumption 1 *$U(c^h, \ell^h, d^h)$ is strictly concave, twice continuously differentiable over $\mathbb{R}_+^\star \times (0,1) \times \mathbb{R}_+^\star$ and $U_c(c^h, \ell^h, d^h) > 0$, $U_\ell(c^h, \mathcal{L}^h, d^h) > 0$, $U_d(c^h, \ell^h, d^h) > 0$, and $\lim_{\varrho \to 0} U_c(\varrho, \ell^h, d^h) = +\infty$, $\lim_{\varrho \to 0} U_\ell(c^h, \varrho, d^h) = +\infty$, $\lim_{\varrho \to 0} U_d(c^h, \ell^h, \varrho) = +\infty$. The Hessian of U is negative definite. Moreover, c_t^h, ℓ_t^h and d_{t+1}^h are normal goods.*

Solving $V_t(x_t)$ gives the following optimality conditions:

$$U_c(c_t^h, \ell_t^h, d_{t+1}^h) = R_{t+1} U_d(c_t^h, \ell_t^h, d_{t+1}^h) \tag{5}$$

$$w_t U_c(c_t^h, \ell_t^h, d_{t+1}^h) - U_\ell(c_t^h, \ell_t^h, d_{t+1}^h) \quad \begin{cases} = 0 & \text{if } l_t^h > 0 \qquad (6a) \\ \leq 0 & \text{if } l_t^h = 0 \qquad (6b) \end{cases}$$

$$-(1+n) U_d(c_t^h, \ell_t^h, d_{t+1}^h) + \beta U_c(c_{t+1}^h, \ell_{t+1}^h, d_{t+2}^h) \leq 0 \quad (= \text{if } x_{t+1} > 0) \tag{7}$$

and the transversality condition (see Michel (1990)): $\lim\limits_{t \to +\infty} \beta^{t+1} U_d(c_{t-1}^h, \ell_{t-1}^h, d_t^h) x_t = 0.$

Contrary to models with exogenous labor supply, the optimization problem of altruistic consumers possesses two inequality constraints ($x_{t+1}^h \geq 0$ and $l_t^h \geq 0$).

In Appendix, we prove that the solution s_t^h of (5) can be expressed by a differentiable function $\breve{s}^h(l_t^h, w_t, R_{t+1}, x_t^h, x_{t+1}^h)$. After substitution of c_t^h and d_{t+1}^h in (5) and (6a), the solutions l_t^h and s_t^h of these equations can be expressed by differentiable functions $s^h(.)$ and $l^h(.)$ of w_t, R_{t+1}, x_t^h and x_{t+1}^h. Since c and d are normal goods, $s^h(.)$ is increasing with respect to (w.r.t.) x_t^h and x_{t+1}^h and $l^h(.)$ is decreasing w.r.t. x_t^h, increasing w.r.t. x_{t+1}^h. The higher is the inheritance, the higher are savings and leisure. The more an altruist wants to leave a bequest, the more he works and saves. An increase in w_t can induce two opposite effects: it can increase labor supply because a higher wage incites to work more but it may decrease labor supply because, to keep his income constant, an agent can work less. Hence, $s^h(.)$ is not necessarily increasing w.r.t. its first argument. Concerning the second argument, things are more complex and the sign of s_2^h and l_2^h are indeterminate. Taking into account the constraint $l_t^h \geq 0$, the labor supply and the saving levels of an altruist who inherits x_t^h and wants to bequeath x_{t+1}^h to each of his children may be locally expressed by some continuous functions $\tilde{l}^h(.)$ and $\tilde{s}^h(.)$ of $(w_t, R_{t+1}, x_t^h, x_{t+1}^h)$:[8]

$$l_t^h = \tilde{l}^h(w_t, R_{t+1}, x_t^h, x_{t+1}^h) \equiv \max[0, l^h(w_t, R_{t+1}, x_t^h, x_{t+1}^h)]$$

[8]All the details of these tedious computations are gathered in Appendix 1.

$$s_t^h = \tilde{s}^h(w_t, R_{t+1}, x_t^h, x_{t+1}^h) \equiv \breve{s}^h(\tilde{l}^h(w_t, R_{t+1}, x_t^h, x_{t+1}^h), w_t, R_{t+1}, x_t^h, x_{t+1}^h)$$

These functions allow to characterize the bequest and labor supply of an agent of dynasty h. Remark that if an altruist chooses to work, his savings function $\tilde{s}^h(.)$ corresponds to the function $s^h(.)$. Since $l^h(.)$ is increasing with respect to x_t^h, the higher is the inheritance of an altruist, the lower is his labor supply.

It is also important to note that when x_t^h and x_{t+1}^h are zero, then the functions $s^h(.)$ and $l^h(.)$ give the savings and the labor supply of each selfish agents as functions of the wage rate and the interest rate. As the life cycle utility function U is identical for the N dynasties, there exist some differentiable functions $s^{De}(.)$ and $l^{De}(.)$ such[9] that for all h: $s^{De}(w_t, R_t) \equiv s^h(w_t, R_t, 0, 0)$ and $l^{De}(w_t, R_t) \equiv l^h(w_t, R_t, 0, 0)$. From these function, we also define the function $\vartheta : \mathbb{R}_+ \to \mathbb{R}_+$ such that:

$$\vartheta(z) = \frac{s^{De}(f(z) - zf'(z), f'(z))}{(1+n)l^{De}(f(z) - zf'(z), f'(z))}$$

The firms

Production occurs according to a constant returns to scale technology $F(.)$ using two inputs, capital K_t and labor L_t.

Assumption 2 *F(K,L) is twice continuously differentiable, homogeneous of degree* 1 *with respect to capital and labor over the set* $(0, +\infty) \times (0, +\infty)$ *and satisfies:* $\forall L > 0 \quad F_K(., L) > 0 \quad F_{KK}(., L) < 0$ *and* $\lim_{L \to 0} F(K, L) = 0$.

Homogeneity of degree one allows us to write output per young as a function of the capital/labor ratio per young $f(z_t) = F(z_t, 1)$ where $z_t = K_t/L_t$.

[9]In this paper we denote with the upper-script "De" the variables which correspond to those of the Diamond model with endogenous labor supply in which the population consists of life-cyclers. Nourry (2001) contains a general dynamic study of this Diamond model.

Markets are perfectly competitive. Each factor is paid its marginal product. Assuming that capital fully depreciates after one period we obtain:

$$w_t = F_L(z_t, 1) = f(z_t) - z_t f'(z_t) \quad \text{and} \quad R_t = F_K(z_t, 1) = f'(z_t) \tag{8}$$

In each period, the labor market clears, i.e., $L_t = N_t l_t$ with $l_t = 1 - \sum_{h=1}^{h=N} p^h \ell_t^h$. The capital stock at time $t+1$ is financed by the savings of the young generation born in t. Hence, we have: $K_{t+1} = N_t s_t$ with $s_t = \sum_{h=1}^{h=N} p^h s_t^h$. Therefore, in intensive form:

$$k_{t+1} = \frac{s_t}{1+n} \quad \text{with} \quad k_{t+1} = \frac{K_t}{N_t} \tag{9}$$

3 Emergence of rentiers

We now confine our analysis to steady states. According to equations (5) and (7), the long-term behavior of each dynasty h ($h \in \{1, ..., N\}$) must satisfy:

$$\beta^h \leq \frac{1+n}{R} \quad (= \text{if } x^h > 0) \tag{10}$$

Hence, only agents of dynasty N i.e., the dynasty endowed with the highest degree of altruism, have the possibility to leave a bequest to their children. Indeed, if there exists $y \in \{1, ..., N-1\}$ such that $x^y > 0$ then equation (10) is not satisfied for dynasties j where $j \in \{y+1, ..., N\}$. Remark that it is sufficient to have some unconstrained altruistic agents to reach the modified golden rule, and this result holds true regardless of the proportion p^N. Indeed, when x^N is positive, according to (8) and (10) the steady state capital/labor ratio z is equal to:

$$z = f'^{-1}(\frac{1+n}{\beta^N}) \equiv \hat{z}$$

Whatever their size, as well-known since Becker (1980), the most patients (or altruists) impose their view on the long-run capital accumulation. This result does not imply that savings and bequests both vanish for the less altruistic agents: their bequests are nil but their savings are not nil. However, they are only constituted by their *life-cycle* savings. Indeed, we can distinguish two types of savings: *life-cycle* and *intertemporal* savings (i.e., the bequests). As in Mankiw (2000a,b), all the agents have *life-cycle* savings (denoted s_t). But only one type of individuals leaves bequests, and consequently has *intertemporal* savings (denoted x_t). Using Mankiw (2000a,b)'s terminology, these agents are labelled as savers. Only altruists of dynasty N can behave as savers because they are the only ones who can leave positive bequests. Members of other dynasties (constrained altruists) are labelled as spenders.

Existence of savers

Using Mankiw (2000a,b)'s terminology and according to the previous result, only altruists of dynasty N can behave as savers. We now focus on the conditions under which the most altruists are savers.

First, even though the capital/labor ratio of the modified golden rule[10] is independent of the sequence of $\{p^h\}_{h=1}^{h=N}$, we show that the level of bequest x^N is not. Interestingly, when bequests of dynasty N are positive they depend on the proportion p^N but not on the other proportions $\{p^h\}_{h=1}^{h=N-1}$. This set of results follows from the next lemma.

Lemma 1 DIFFERENT TRANSFERS DESIRED BY THE PARENT

(i) There exists a differentiable function $\breve{x}$ *of* p^N *and* β^N *such that* $\breve{x}(p^N, \beta^N)$ *is the unique solution of:*

$$\psi(x, p^N, \beta^N) = (1+n)f'^{-1}(\hat{R}) - \frac{p^N s^N(\hat{w}, \hat{R}, x, x) + (1-p^N)s^{De}(\hat{w}, \hat{R})}{p^N l^N(\hat{w}, \hat{R}, x, x) + (1-p^N)l^{De}(\hat{w}, \hat{R})} = 0$$

(ii) There exists a differentiable function $\vec{x}$ *of* p^N *and* β^N *such that* $\vec{x}(p^N, \beta^N)$

[10] In this paper we denote with the upperscript "$\wedge$" the variables evaluated at the modified golden rule $f'^{-1}((1+n)/\beta^N)$. Hence, $\hat{w} = f(\hat{z}) - \hat{z}f'(\hat{z})$ and $\hat{R} = f'(\hat{z}) = (1+n)/\beta^N$.

is the unique solution of:

$$\zeta(x, p^N, \beta^N) = (1+n)f'^{-1}(\hat{R}) - \frac{p^N \breve{s}^N(0, \hat{w}, \hat{R}, x, x) + (1-p^N)s^{De}(\hat{w}, \hat{R})}{(1-p^N)l^{De}(\hat{w}, \hat{R})} = 0$$

The variable $\breve{x}(p^N, \beta^N)$ (positive or negative) is the transfer which is desired by the parent. Indeed, it is obtained by maximization of the utility with respect to x^N and ℓ^N when ignoring inequality constraints $x^N \geq 0$ and $\ell^N \leq 1$. The solution $\vec{x}(p^N, \beta^N)$, is the transfer which is desired by the parent when he does not work (i.e., assuming that $\ell^N = 1$). According to Appendix 2, transfers $\breve{x}(p^N, \beta^N)$ and $\vec{x}(p^N, \beta^N)$ are increasing with respect to p^N. The smaller the proportion of altruists of dynasty N in the society, the larger the optimal amount of bequests. Intuitively, bequests are going to offset the lack of savings by the spenders.

We can also remark that the function $\psi(0, p^N, \beta^N)$ does not depend on the proportion p^N since, when x is equal to zero, the function ψ is equivalent to $(1+n)f'^{-1}(\hat{R}) - \vartheta(\hat{z})$. Using methodology developed in Thibault (2000, 2002), the function ϑ is a convenient tool for deriving a general condition under which savers exist. A simple extension of Thibault (2000) or of the theorem 1 (step 2) of Thibault (2002) allow us to establish the next proposition.[11]

Proposition 1 EXISTENCE OF SAVERS

The economy experiences savers if and only if $\vartheta(\hat{z})$ is lower than the modified golden rule capital/labor ratio $\hat{z}$.

Contrary to the optimal level of bequest, the condition to obtain positive bequests does not depend on the proportion of agents of dynasty N. It is the same as that in a society consisting only of altruists. A strength of this condition is that it is valid whatever the form of the economy without bequests motive. Indeed, this condition

[11] Details of proof are contained in a preliminary version of this paper available on the web: `http://durandal.cnrs-mrs.fr/GREQAM/dt/wp-pdf00/00A32.pdf`

holds whatever the number and stability properties of the equilibria of the Diamond model with endogenous labor supply. Note that if we assume that the Diamond economy has a unique and stable (non-trivial) steady state, we can enlighten a threshold value of the degree of altruism above which altruists of dynasty N leaves an inheritance.[12]

Emergence of rentiers

Importantly, when bequests are positive, agents of dynasty N are savers. Then, the economy is at the modified golden rule steady state which depends on the saving propensity β^N of savers, but not on their proportion. Note that this result is similar to those of Kaldorian models (see Britto (1972)) but it is obtained in an endogenous way.

Interestingly, savers may not work and capital owners can endogenously emerge. Indeed, since the interest factor is equal to $(1+n)/\beta^N$, investing $\beta^N x^N$ is sufficient to leave $(1+n)x^N$ to one's children. The difference, $x^N - \beta^N x^N$, between the bequest received and the actualized value of bequest handing down is defined as the rent[13] (or patrimony return) of the saver and is denoted by ρ.

When a saver chooses not to work his labor income is zero and his wealth only consists of his bequest x^N. Hence, this patrimony corresponds to the wealth of a rentier. Then, to study conditions under which such rentiers can emerge in the society, we first focus on the desired wealth by the savers to be rentiers. Indeed, according to the next proposition, we can exhibit a wealth value $\bar{x}$ above which savers do not work.

[12]This result extends the well known result obtained by Weil (1987) in a theoretical setup where altruists are homogenous and have an exogenous labor supply.

[13]From (1) and (2): $w(1-\ell^N) + (1-(1+n)/R)x^N = c^N + d^N/R$. The life cycle income of a saver is composed of his labor income $w(1-\ell^N)$ and the return $(1-(1+n)/R)x^N$ of his patrimony. Since $Rx^N = ((1+n)/\beta^N)x^N$ this return is equal to $(1-\beta^N)x^N$.

Proposition 2 Emergence and wealth of rentiers

There exists a (unique) wealth $\bar{x}$ (depending on β^N) such that savers behave as rentiers if and only if their wealth is larger than $\bar{x}$.

Given their propensity to save β^N, the wealth level $\bar{x}$ is the minimal wealth of rentiers. Hence, intuitively, $\bar{\rho} = (1-\beta)\bar{x}$ corresponds at the minimum patrimony return level which incites a saver to behave as a capital owners. Importantly, $\bar{x}$ only depends on the propensity to save β^N, but, according to lemma 1, wealth of rentiers $\vec{x}$ depends both on p^N and β^N.

4 Specific characters of rentiers

Since endogenous wealths $\bar{x}$ and x^N depend on exogenous parameters p^N and β^N, we now focus on the correlation between these two specific characters of savers and the existence of rentiers.

Proportion of rentiers

Although the existence of savers is independent of their proportion, existence of rentiers is based on the relative weight of savers in our economy. Indeed, according to their proportion, savers are rentiers or not. More precisely we can show:

Proposition 3 Existence and proportion of rentiers

There exists a (unique) proportion $p^\star$ (depending on β^N) such that savers behave as rentiers if and only if their proportion is lower than $p^\star$.

According to lemma 1, bequests are a decreasing function of p. Hence, below the critical value $p^\star$, the size of wealth x is larger than $\bar{x}$ and incites savers not to work. The proportion $p^\star \in [0,1)$ only depends on β^N. And, when p^N is lower than $p^\star$, contrary to the Kaldorian tradition of two-class growth models, rentiers emerge endogenously.

Intuitively, to have rentiers in a society, it is necessary that spenders are in a large proportion so that savers choose not to work. Savings of spenders are lower and a large share of capital belongs to a few savers. Since production is provided by spenders, savers choose not to work.

We can also remark that savers are obliged to work when the society consists exclusively of savers. Intuitively, they choose to work so that the production sector does not vanish. Indeed, there are no other workers when $p^N = 1$. Hence, the assumption $F(K, 0) = 0$ implies $p^\star < 1$ and the existence of rentiers is really a consequence of the microeconomic heterogeneity introduced in our model.

Propensity to save of rentiers

Both thresholds $\bar{x}$ and $p^\star$ only depends on β^N. In the light of this fact, we now study the relation existing between the propensity to save β^N and the existence of rentiers. According to proposition 2, this relation depends on the correlation between x^N and β^N. Through a simple example in an exogenous labor supply framework, Thibault (2001) shows that an increase in β^N can result in a decrease in x^N even if the Diamond model has a unique and stable steady state.[14] To avoid any complications that may arise from multiple locally stable steady states or exotic production function, we consider a Cobb-Douglas economy to study the propensity of save of rentiers.

Then we assume the life cycle utility function of each dynasty is: $U(c_t^h, \ell_t^h, d_{t+1}^h)$ $= \mu \ln c_t^h + \xi \ln \ell_t^h + \gamma \ln d_{t+1}^h$ with $(\mu, \xi, \gamma) \in \mathbb{R}_+^{3\star}$ and $\mu + \xi + \gamma = 1$. Moreover, we assume that the production function is $f(z_t) = A z_t^\alpha$ with $A > 0$ and $\alpha \in (0, 1)$.

With these specifications, we can use proposition 1 and 3 to determine under

[14]To rule out this counterintuitive case, an assumption on the curvature of the production function is necessary (see Thibault (2001)).

which conditions savers exist and what is their labor supply.[15]

(i) There exist savers if and only if the propensity to save β^N is sufficiently strong, i.e., larger than $\beta^\star$ with:

$$\beta^\star = \frac{\gamma(\alpha^{-1} - 1)}{\mu + \gamma}$$

Since savers must exist to eventually obtain rentiers, we assume that $\beta^\star$ is lower than one or, equivalently, μ/γ greater than $\alpha^{-1} - 2$.

(ii) Savers are rentiers if and only if their proportion p^N in the society is lower than $p^\star$ with:

$$p^\star = \frac{(1-\beta^N)\xi((\mu+\gamma)\beta^N - (\alpha^{-1}-1)\gamma)}{(\gamma+\mu\beta^N)(\alpha^{-1}-1) + (1-\beta^N)\xi((\mu+\gamma)\beta^N - (\alpha^{-1}-1)\gamma)}$$

It is necessary that the proportion of spenders is large enough so that a spender chooses not to work. Hence, below the critical value p^N, their wealth incites savers not to work.

To be rentier, we can remark that a saver must have a sufficiently large propensity to save because bequest must be positive. However, this propensity to save must not be too high. Indeed, a saver can wish work to accumulate more (and more) capital. In such a case, he may accumulate so much that he would not benefit from a patrimony return. For instance, if the saving propensity of savers is full ($\beta^N = 1$), they choose to work as spenders to accumulate maximum of wealth.[16]

More precisely, we are going to show (proposition 4) that a saver does not work if and only if his propensity to save is greater than $\beta^\star$ and satisfies $\mathcal{A}\beta^{N2} + \mathcal{B}\beta^N + \mathcal{C} \leq 0$

[15] In long-run, bequests of agents of dynasty i ($i \in \{0, ..., N-1\}$) are nil and their labor supply are constant and equal to $\mu + \gamma$.

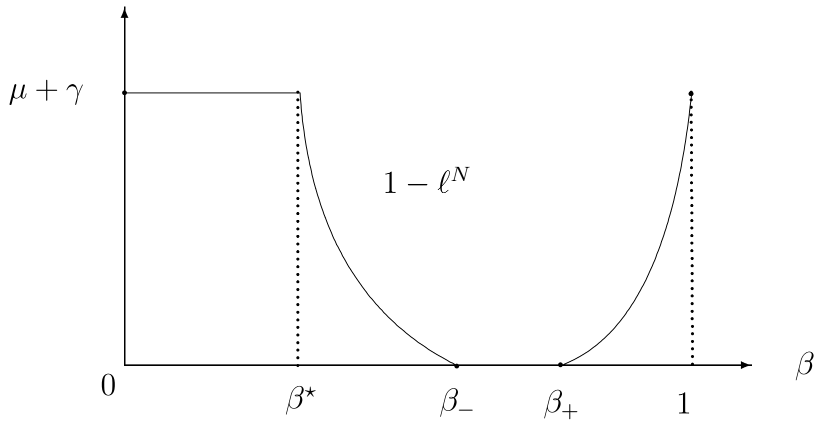
[16] Reinterpreting β^N as a degree of altruism the intuition is the following. Agents of dynasty N can be too altruistic to benefit from a rent. For instance, if they are fully altruistic (i.e. $\beta^N = 1$), they transmit the received bequest in its entirety ($\rho = p^\star = 0$).

with $\mathcal{A} = (1 - p^N)\xi(\mu + \gamma)$, $\mathcal{B} = p^N\mu(\alpha^{-1} - 1) - (1 - p^N)\xi(\mu + \gamma\alpha^{-1})$ and $\mathcal{C} = (\alpha^{-1} - 1)(p^N\gamma + (1 - p^N)\xi\gamma)$. We also show that the discriminant $\Delta \equiv \mathcal{B}^2 - 4\mathcal{AC}$ of this polynomial is a polynomial of p which experiences two roots included in $(0, 1)$. These roots only depend on the elasticity of life cycle utility and production function. We assume than p^N is lower than $\hat{p}$; the smallest of the two previous roots. From $\beta_{\pm} = (-\mathcal{B} \pm \sqrt{\Delta})/(2\mathcal{A})$, we now can determine the propensity to save of rentiers.

Proposition 4 EXISTENCE AND PROPENSITY TO SAVE OF RENTIERS

Savers are rentiers if and only if their propensity to save is greater than β_- and lower than β_+.

To give an illustration of this proposition, we represent the labor supply $1 - \ell^N$ of savers according to their propensity to save.



Then, we can distinguish two effects.[17] When agents of dynasty N becomes savers a *wealth effect* implies a fall in the labor supply $1 - \ell^N$. Intuitively, when a dynasty begins to accumulate capital, its members are incited to lower their labor

[17]Indeed, after computations we have: $l^N(\hat{w}, \hat{R}, x) = \mu + \gamma - \frac{\xi}{\hat{w}}(1 - (1 + n)/\hat{R})x$. According to this equation we obtain, since $\partial l^N/\partial\hat{w} \geq 0$, $\partial l^N/\partial\hat{R} \leq 0$, $\partial l^N/\partial x \geq 0$, $\partial\hat{w}/\partial\beta > 0$ and $\partial\hat{R}/\partial\beta < 0$, our two opposite effects.

supply because they are richer. When spenders are numerous ($1 - p^N > 1 - \hat{p}$), this drop is so important that savers choose not to work if their propensity to save is contained between β_- and β_+. However, when β^N is greater than β_+, a *wage/interest rate effect* incites savers to work. Intuitively, When β^N increases, the wage $\hat{w}$ and the interest rate $\hat{R}$ are more attractive. For higher propensities to save, the *wage/interest rate effect* dominates the *wealth effect.* Hence, a saver wishes to work to accumulate more and more capital.

In contrast to an accepted idea, a rentier is not a rich with a very large propensity to save. Intuitively, he gives up his labor income in order to enjoy leisure. Interestingly, contrary to the main assumption of kaldorian models, the propensity to save β^w of the agents who decide to work is not always lower than that of capital owners, β^c. Indeed, a two class society in which the propensity to save of workers is greater than that of capital owners endogenously emerge as soon as $\beta_- < \beta^c < \beta_+ < \beta^w$.

5 Conclusion

The goal of this paper was to analyze the characteristics of rentiers. Precisely, we have focused on the proportion, the wealth and the propensity to save of rentiers.

Using Mankiw's (2000a,b) savers-spenders theory, we have developed a theoretical framework to investigate the microfoundations of the macroeconomic models with rentiers. Indeed, we have analyzed a growth model in which the emergence of rentiers is a natural consequence of the intertemporal utility maximization of altruists individuals.

Our main findings can be summarized as follows: rentiers in a society are necessarily in a low proportion,[18] and have a sufficiently large wealth. Interestingly,

[18]Even if this result seems intuitive, its importance comes from the analysis of the threshold proportion under which savers are rentiers. Indeed, for some parameter sets this threshold value can be (relatively) large and can explain why rentiers are localized in particular countries.

their propensity to save must be sufficient to reach a high level of wealth but not too large. In such a case, savers wish to work to accumulate more capital.

Many economists have constructed macroeconomic models to investigate the wealth distribution, inequality and/or potential forces behind the rise and fall of class societies (see, for instance, Piketty (1997), Matsuyama (2000,2002)). In these studies, all individuals are altruists and the source of heterogeneity across households is not their preferences but their wealth.

Contrary to these strands of literature, our framework allows us to explain the emergence of rentiers and can also contribute to an alternative interpretation for understanding the mechanisms behind the formation of class society.

Finally, our analysis takes a first step toward studying the observed variations in wealth inequality. As noted by Ghiglino and Olszak-Duquenne (2001, p. 2): "a natural way to treat this issue (wealth inequality) is provided by fully competitive, dynamic general equilibrium models with heterogenous agents". Then, the framework developed in this paper seems relevant to treat this issue which is on our research agenda.

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Appendix

Appendix 1

Let $V^h(s_t^h, l_t^h) = U(x_t^h + w_t l_t^h - s_t^h, 1 - l_t^h, R_{t+1}s_t^h - (1+n)x_{t+1}^h)$.

Then $V_{11}^h = U_{11}^h - 2U_{13}^h R_{t+1} + U_{33}^h R_{t+1}^2$, $V_{22}^h = U_{11}^h w_t^2 - 2U_{12}^h w_t + U_{22}^h$ and $Hess\ V^h = \begin{pmatrix} V_{11}^h & D \\ D & V_{22}^h \end{pmatrix}$ where $D = -U_{11}^h w_t + U_{13}^h R_{t+1} w_t + U_{12}^h - U_{23}^h R_{t+1}$ and $U_{ij}^h = U_{ij}(c_t^h, \ell_t^h, d_{t+1}^h)$. Since $Hess\ V^h = A \times Hess\ U^h \times A^T$ where $A = \begin{pmatrix} -1 & 0 & R_{t+1} \\ w_t & -1 & 0 \end{pmatrix}$ is a matrix of rank 2, $Hess\ U^h$ negative definiteness implies $Hess\ V^h$ negative definiteness. Hence V_{11}^h, V_{22}^h are negative and the determinant of $Hess\ V$ denoted ζ is positive. V^h is concave with respect to s_t^h and l_t^h and, when the conditions (5) is equivalent to $V_1^h(s_t^h, l_t^h, w_t, R_{t+1}, x_t^h, x_{t+1}^h) = 0$. Since $V_{11}^h \neq 0$, there exist[19] a differentiable function $\breve{s}^h$ such that $s_t^h = \breve{s}^h(l_t^h, w_t, R_{t+1}, x_t, x_{t+1})$ satisfies equations (5).

We can also remark that the two conditions (5) and (6a) are equivalent to $V_1^h(s_t^h, l_t^h, w_t, R_{t+1}, x_t^h, x_{t+1}^h) = 0$ and $V_2^h(s_t^h, l_t^h, w_t, R_{t+1}, x_t^h, x_{t+1}^h) = 0$. Since $\zeta \neq 0$, there exist[20] two differentiable functions s^h and l^h such that $s_t^h = s^h(w_t, R_{t+1}, x_t, x_{t+1})$ and $l_t^h = l^h(w_t, R_{t+1}, x_t, x_{t+1})$ satisfy equations (5) and (6a).

[19] Given l_t^h, w_t, R_{t+1}, x_t and x_{t+1}, the set of constraints under which we maximize the concave function V^h is a compact set. This ensures the existence of $\breve{s}^h(.)$.

[20] Given w_t, R_{t+1}, x_t and x_{t+1}, the set of constraints under which we maximize the concave function V^h is a compact set. This ensures the existence of $s^h(.)$ and $l^h(.)$.

The differentiability follows from the implicit function theorem:

$$\begin{bmatrix} \dfrac{\partial s^h}{\partial w_t} & \dfrac{\partial l^h}{\partial w_t} \\ \dfrac{\partial s^h}{\partial R_{t+1}} & \dfrac{\partial l^h}{\partial R_{t+1}} \\ \dfrac{\partial s^h}{\partial x_t^h} & \dfrac{\partial l^h}{\partial x_t^h} \\ \dfrac{\partial s^h}{\partial x_{t+1}^h} & \dfrac{\partial l^h}{\partial x_{t+1}^h} \end{bmatrix} = \frac{1}{\zeta} \begin{bmatrix} DU_1^h + l_t^h \gamma_1^h & -U_1^h V_{11}^h + l_t^h R_{t+1} \gamma_3^h \\ -V_{22}^h U_3^h + s_t^h \gamma_2^h & DU_3^h + s_t^h \gamma_3^h \\ \gamma_1^h & R_{t+1} \gamma_3^h \\ -(1+n)\gamma_2^h & -(1+n)\gamma_3^h \end{bmatrix}$$

where $\gamma_1^h = R_{t+1}(\phi_3^h + \dfrac{\phi_6^h}{R_{t+1}} - w_t \phi_5^h)$, $\gamma_2^h = R_{t+1}[w_t(\phi_2^h + \dfrac{\phi_5^h}{R_{t+1}} - w_t\phi_4^h) - (\phi_1^h + \dfrac{\phi_3^h}{R_{t+1}} - w_t\phi_2^h)]$ and $\gamma_3^h = R_{t+1}(\phi_2^h + \dfrac{\phi_5^h}{R_{t+1}} - w_t\phi_4^h)$.

The ϕ_i^h's parameter is defined by: $\phi_1^h = U_{22}^h U_{33}^h - {U_{23}^h}^2$, $\phi_2^h = U_{12}^h U_{33}^h - U_{13}^h U_{23}^h$, $\phi_3^h = U_{12}^h U_{23}^h - U_{13}^h U_{22}^h$, $\phi_4^h = U_{11}^h U_{33}^h - {U_{13}^h}^2$, $\phi_5^h = U_{11}^h U_{23}^h - U_{12}^h U_{13}^h$ and $\phi_6^h = U_{11}^h U_{22}^h - {U_{12}^h}^2$.

Let Ω^h the life cycle income of an agent of dynasty h. With this income he can buy c^h at price 1, leisure ℓ^h at price w and d^h at $1/R$. Hence, consider the static program $\max_{c^h,\ell^h,d^h} U(c^h, \ell^h, d^h)$ subject to the constraint $\Omega^h = c^h + w\ell^h + d^h/R$. This program is equivalent to $\max_{c^h,\ell^h} \mathcal{A}(c^h, \ell^h, \Omega^h)$ or $\max_{d^h,\ell^h} \mathcal{B}(d^h, \ell^h, \Omega^h)$ where $\mathcal{A}(c^h, \ell^h, \Omega^h) = U(c^h, \ell^h, R(\Omega^h - w\ell^h - c^h))$ and $\mathcal{B}(d^h, \ell^h, \Omega^h) = U(\Omega^h - w\ell^h - d^h/R, \ell^h, d^h)$. Using implicit-function theorem we can show that: $\partial c^h/\partial\Omega^h$ has the sign of $\mathcal{A}_{12}\mathcal{A}_{23} - \mathcal{A}_{13}\mathcal{A}_{22}$, $\partial\ell^h/\partial\Omega^h$ the sign of $\mathcal{B}_{12}\mathcal{B}_{13} - \mathcal{B}_{11}\mathcal{B}_{23}$ and $\partial d^h/\partial\Omega^h$ the sign of $\mathcal{B}_{12}\mathcal{B}_{23} - \mathcal{B}_{13}\mathcal{B}_{22}$. After computations, $\partial c^h/\partial\Omega^h$ has the sign of $\phi_1^h + \dfrac{\phi_3^h}{R} - w\phi_2^h$, $\partial\ell^h/\partial\Omega^h$ has the sign of $-\phi_2^h - \dfrac{\phi_5^h}{R} + w\phi_4^h$, and $\partial d^h/\partial\Omega^h$ has the sign of $\phi_3^h + \dfrac{\phi_6^h}{R} - w\phi_5^h$. By definition of the normality, c^h, ℓ^h, and d^h are normal goods if and only if $\partial c^h/\partial\Omega^h$, $\partial\ell^h/\partial\Omega^h$ and $\partial d^h/\partial\Omega^h$ are positive. Therefore, under Assumption 2 we have: $\gamma_1^h > 0$, $\gamma_2^h < 0$ and $\gamma_3^h < 0$. Hence $s_3^h > 0$, $s_4^h > 0$, $l_3^h < 0$, and $l_4^h > 0$. **QED**

Appendix 2: Proof of lemma 1

(i) Using the function $\psi : (-\infty, x^{\star}) \times (0,1] \times (0,1) \to \mathbb{R}$ we[21] have:

$$\psi'_x(x, p^N, \beta^N) = -\frac{p^N}{u^2} \times [s'^N_x(\hat{w}, \hat{R}, x, x)u - l'^N_x(\hat{w}, \hat{R}, x, x)v]$$

where u and v are positive.

According to appendix 1, $s^N(w, R, x, x)$ is an increasing function of x and $l'^N_x(\hat{w}, \hat{R}, x, x)$ has the sign of $(\hat{R} - (1+n))\gamma_3^N$. Since γ_3^N is negative and $\hat{R} = (1+n)/\beta^N > 1+n$, $l'^N_x(\hat{w}, \hat{R}, x, x)$ is negative. Intuitively, since $\hat{R} > 1+n$, agents work less because invest x at rate $\hat{R}$ is sufficient to leave $(1+n)x$ to their children.

Hence, $s'^N_x(\hat{w}, \hat{R}, x, x)u - l'^N_x(\hat{w}, \hat{R}, \breve{x}^N, x, x)v$ is positive. Since $\psi'_x(x, p^N, \beta^N) \neq 0$, there exist a differentiable function $\breve{x}$ of $(p^N$,$\beta^N)$ such that: $x = \breve{x}(p^N, \beta^N)$ verify $\psi(x, p^N, \beta^N) = 0$. Moreover, since $s'^N_x(\hat{w}, \hat{R}, x, x)$ is positive and $l'^N_x(\hat{w}, \hat{R}, x, x)$ is negative, we have $s^N(\hat{w}, \hat{R}, \breve{x}, \breve{x}) > s^{De}(\hat{w}, \hat{R})$ and $l^N(\hat{w}, \hat{R}, \breve{x}, \breve{x}) < l^{De}(\hat{w}, \hat{R})$ when $\breve{x}$ is positive. Hence, $\psi'_{p^N}(\breve{x}, p^N, \beta^N)$ is negative when $\breve{x}$ is positive. Since $\psi'_x(\breve{x}, p^N, \beta^N)$ is negative, $\partial\breve{x}(p^N, \beta^N)/\partial p^N$ is negative when $\breve{x}(p^N, \beta^N)$ is positive.

(ii) Using the function $\zeta : \mathbb{R} \times (0,1] \times (0,1) \to \mathbb{R}$ we have:

$$\zeta'_x(x, p^N, \beta^N) = -\frac{p^N}{1-p^N} \times \frac{\breve{s}'^N_x(0, \hat{w}, \hat{R}, x, x)}{(1+n)l^{De}(\hat{w}, \hat{R})}$$

Since $\breve{s}^N(0, w, R, x, x)$ is an increasing function of x, $\zeta'_x(x, p^N, \beta^N)$ is negative. Since $\zeta'_x(x, p^N, \beta^N) \neq 0$, there exist a differentiable function $\vec{x}$ of $(p^N$,$\beta^N)$ such that: $x = \vec{x}(p^N, \beta^N)$ verify $\zeta(x, p^N, \beta^N) = 0$. Moreover, $\zeta'_{p^N}(\breve{x}, p^N, \beta^N) < 0$. Hence, since $\zeta'_x(\breve{x}, p^N, \beta^N) < 0$, $\partial\vec{x}(p^N, \beta^N)/\partial p^N < 0$. **QED**

Appendix 3: Proof of Proposition 2

According to appendix 2, $l'^N_x(\hat{w}, \hat{R}, x, x)$ is negative. Hence, since $l^N(\hat{w}, \hat{R}, 0, 0) = l^{De}(\hat{w}, \hat{R})$ is positive, there exists a unique $\bar{x}$ ($\bar{x} \in (0, +\infty)$ or $\bar{x} = +\infty$) such that $l^N(\hat{w}, \hat{R}, x, x)$ is positive if and only if $x \in [0, \bar{x})$.

[21]The threshold $x^{\star}$ is defined in the proof of proposition 3.

When an agent of dynasty N choose to leave a positive bequest, is at the modified golden rule equilibrium. Hence, according to appendix 1 and lemma 1, the stationary labor supply of an agent of dynasty N is $\bar{l}^N = Max[0, l^N(\hat{w}, \hat{R}, \breve{x}, \breve{x})]$.

Hence, according to proposition 1 and lemma 1, our model has a steady state in which agents of dynasty N do not work if and only if $\vartheta(\hat{z}) < \hat{z}$ and $l^N(\hat{w}, \hat{R}, \breve{x}, \breve{x}) \leq 0$, i.e., if and only if $l^N(\hat{w}, \hat{R}, \breve{x}, \breve{x})$ is not positive while $\breve{x}$ is positive. According to the first point of this appendix, this condition is equivalent to $\breve{x} \geq \bar{x}$. **QED**

Appendix 4: Proof of Proposition 3

(i) According to appendix 2, the sign of $\breve{x}(p^N, \beta^N)$ is independent of p^N. Indeed it is entirely determined by the sign of $\hat{z} - \vartheta(\hat{z})$. Therefore, we can distinguish two cases according to the sign of $\breve{x}$.

- If $\breve{x}(p^N, \beta^N)$ is not positive, $\bar{x}(\beta^N) - \breve{x}(p^N, \beta^N) > 0$ for all p^N. Hence $p^\star = 0$.

- If $\breve{x}(p^N, \beta^N)$ is positive, we have shown in the proof of lemma 4 that $\breve{x}(p, \beta^N)$ is decreasing with respect to p. Moreover it is possible to show that for $p = 1$ we have $\bar{x}(\beta^N) > \breve{x}(1, \beta^N)$, i.e., $l^N(\hat{w}, \hat{R}, \breve{x}(1, \beta^N), \breve{x}(1, \beta^N))$ is positive. Indeed the realistic assumption $\lim_{L \to 0} F(K, L) = 0$ implies[22] that $\ell_t^N < 1$. Intuitively, agents choose to work so that the production sector does not vanish. Indeed, they are no other workers when $p^N = 1$. Hence, since $\breve{x}'_p(p, \beta^N) < 0$ and $\breve{x}(1, \beta^N) < \bar{x}(\beta^N) >$, there exists a unique $p^\star \in [0, 1)$ such that $\bar{x}(\beta^N) - \breve{x}(p^N, \beta^N) > 0$ if and only if $p^N < p^\star$.

(ii) According to the previous appendix, our model has a steady state in which agents of dynasty N do not work if and only if $\vartheta(\hat{z}) < \hat{z}$ and $l^N(\hat{w}, \hat{R}, \breve{x}, \breve{x}) \leq 0$, i.e., if and only if $l^N(\hat{w}, \hat{R}, \breve{x}, \breve{x})$ is not positive while $\breve{x}$ is positive. According to part (i) of this appendix, this condition is equivalent to $p^N < p^\star$. **QED**

[22] $\lim_{L \to 0} F(K, L) = 0$ implies $\lim_{L \to 0} K F_K(K, L) = 0$. Hence, if $l_t^N = 0$ then $R_{t+1} s_t^N = 0$. According to (1) that implies $d_{t+1}^N = 0$. Impossible because $\lim_{\varrho \to 0} U_d(c^N, \ell^N, \varrho) = +\infty$.

Appendix 5: Proof of Proposition 4

Inequation $p \leq p^\star$ is equivalent to $P(\beta^N) = \mathcal{A}\beta^{N2} + \mathcal{B}\beta^N + \mathcal{C} \leq 0$. According to proposition 6, $\bar{l}^N = 0$ if and only if $P(\beta^N) \leq 0$.

Assume that $\beta^\star < 1$ [23]. Sine $P(\beta^\star) = p^N(\alpha^{-1} - 1)(\mu\beta^\star + \gamma)$ and $P(1) = p^N(\alpha^{-1} - 1)(\mu + \gamma)$ are positive and since $\lim_{\varrho \to \pm\infty} P(\varrho) = +\infty$, if P has two roots denoted β_- and β_+, these roots are either both lower than $\beta^\star$ or both in $(\beta^\star, 1)$ or both greater than one.

Since $\mathcal{A}$ is positive, the only degrees of altruism for which $\bar{l}^N = 0$ are the β^N such that:$\beta^\star < \beta_- < \beta^N < \beta_+ < 1$. For the existence of β_- and β_+ it is necessary and sufficient that $\Delta \geq 0$. Moreover, $(\beta_-, \beta_+) \in (\beta^\star, 1)^2$ if and only if $\mathcal{C} < \mathcal{A} < -\mathcal{B}/(2\beta^\star)$. Indeed, if $\mathcal{C} < \mathcal{A}$ we have $\beta_-\beta_+ < 1$. Hence β_- and β_+ are not both greater than one. Similarly, if $-\mathcal{B}/\mathcal{A} > 2\beta^\star$, we have $\beta_- + \beta_+ > 2\beta^\star$. Hence β_- and β_+ are not both lower than $\beta^\star$.

Hence, agents of dynasty N are rentiers if and only if:

$$(i)\ \Delta \geq 0,\ (ii)\ \mathcal{C} < \mathcal{A} < -\mathcal{B}/(2\beta^\star),\ (iii)\ \beta^N \in (\beta_-, \beta_+),\ (iv)\ \beta^\star < 1 \tag{11}$$

We have $\Delta = Q(p) = ap^2 + bp + c$ with $a = f + g + h$, $b = -(2g + h)$ and $c = g$ where $f = \mu^2(\alpha^{-1} - 1)^2$, $g = [\xi(\mu + \gamma(2 - \alpha^{-1})]^2$ and $h = 2(\alpha^{-1} - 1)\xi[\mu(\mu + \gamma\alpha^{-1}) + 2(\mu + \gamma)\gamma]$. Q increases as long as $p < p_0$ and decreases if $p > p_0$ with $p_0 = -b/2a \in (0, 1)$. Moreover, $Q(p_0)$ is negative (sign of $-2[\gamma^2 + \mu\gamma\alpha^{-1}]$). Since a, c and f are positive, we have $Q(0) > 0$, $Q(1) > 0$ and $\lim_{p \to \infty} Q(p) = +\infty$. Hence, Q has two roots $\hat{p}$ and $\tilde{p}$ which are both in $(0, 1)$. Thus, $\Delta > 0 \Leftrightarrow p \in (0, \hat{p}) \cup (\tilde{p}, 1)$.

Let $\mathcal{U} = \xi'[\mu + \gamma(2 - \alpha^{-1})]$. With this notation we have:

$[\mathcal{C} < \mathcal{A}$ and $\mu/\gamma > \alpha^{-1} - 2] \Leftrightarrow p < p_1 = \mathcal{U}/(\mathcal{U} + (\alpha^{-1} - 1)\gamma)$ and

$[\mathcal{A} < -\mathcal{B}/(2\beta^\star)$ and $\mu/\gamma > \alpha^{-1} - 2] \Leftrightarrow p < p_2 = \mathcal{U}/(\mathcal{U} + (\alpha^{-1} - 1)\mu)$.

$Q(p_2)$ has the sign of $-2[\gamma^2 + \mu\gamma\alpha^{-1}]$. Hence, $Q(p_2) < 0$.

[23] If $\beta^\star \geq 1$, altruists of dynasty N work for all $\beta^N \in (0, 1)$.

Assume now that $p_1 < p_2$[24] i.e. $\gamma > \mu$.
Since $\mu^2 + \gamma\mu(2-\alpha^{-1}) - 2\gamma\mu - 2\gamma^2\alpha^{-1} \leq \gamma(-\mu - 2\gamma(\alpha^{-1}-1)) < 0$ we have $Q(p_1) < 0$. Since $Q(p_1)$ and $Q(p_2)$ are negative we have $\hat{p} < p_1 < p_2 < \tilde{p}$. Consequently, $[\beta^\star < 1$, $\Delta \geq 0$ and $\mathcal{C} < \mathcal{A} < -\mathcal{B}/(2\beta^\star)] \Leftrightarrow [p \leq \hat{p}$ and $\beta^\star < 1]$.

Then, according to (11), our model experiences a steady state in which agents of dynasty N do not work if and only if $\beta^\star < 1$, $p \leq \hat{p}$ and $\beta^N \in (\beta_-, \beta_+)$. **QED**

[24] If $p_2 < p_1$ then $p < p_2$ implies $p < p_1$. Therefore $Q(p_2) < 0$ is sufficient to obtain our result.