# Modeling Corner Solutions with Panel Data : *Application to the Industrial Energy Demand in France.*

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23rd September 2003

### Abstract

This paper provides an empirical application of Lee and Pitt's (1986) approach to the problem of corner solutions in the case of panel data. This model deals with corner solutions in a manner consistent with the firm behavior theory while controlling for unobserved heterogeneity. In this model, energy demand at industrial plant level is the result of a discrete choice of the type of the energy to be consumed and a continuous choice that defines the level of demand. The econometric model is, essentially, an endogenous switching regime model which requires the evaluation of multivariate probability integrals. We estimate the random effect model by maximum likelihood using a panel of industrial French plants from the paper and pulp industry. We calculate empirical price elasticities of energy demand from the model. We also study the effects on energy demand of an environmental policy aimed at reducing $CO_2$ emissions.

JEL Classification: C33, C34, D21, Q41.

Key words: Energy demand system, Zero expenditures, Panel data, Random effects model.

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# 1 Introduction

The oil crisis of the mid 1970's increased the interest of economists and policy makers in the study of energy demand. Interesting questions which arose included: whether supplies of non-renewable energies were running out, whether alternative forms of energy would arise to alleviate the problem, and whether national economies could adjust to a changing price structure of energy and many empirical studies arose from these concerns. More recently, energy demand models became of particular interest from an environmental point of view. The consumption of different types of energy is associated with different levels of pollution emissions ($CO_2$, $SO_2$,) and the combustion of fossil fuels (coal, gas, oil,) is the primary source of greenhouse gases emissions. Therefore, if different forms of energy are substitutes, it may be possible to reduce pollution by taxing the most polluting energies.

Studies on energy demand usually examine either substitution possibilities between energy forms (Fuss, 1977) or substitution between energy (as an aggregate) and other inputs such as labor and capital (Berndt and Wood, 1975). Most of these studies use aggregated data. Typically these data are averages of variables over various industries. The use of aggregate data leads to two particular difficulties. First, the aggregation prevents the use of firm or plant level data directly in models that are designed to be applied at the microeconomic level. Second, prices used are average prices while the economic theory of cost minimization recognizes that the appropriate price to be used as an explanatory variable is the marginal price for energy forms.

The use of disaggregated data alleviates these two problems but raises other modeling issues, in particular the existence of zero expenditure. When a significant proportion of observations in which expenditure on one or more goods is zero, the econometric model should allow for zero expenditure to occur with positive probability. Standard estimation methods for these models[1] do not take into account zero expenditure and consequently yield inconsistent estimates of parameters. If observations containing zero expenditure are excluded for the purpose of the estimation, this will reduce significantly the sample size and estimators would be biased and inconsistent. In addition, the application of Tobit estimation will, for systems with more than two goods, result in biased estimates since they fail to consider that consumers' response to price depends on the set of goods it consumes at corners[2].

Wales and Woodland (1983) and Lee and Pitt (1986) derived models which offer an economic interpretation of zero expenditure as well as a direct and appropriate method to specify the econometric model. The Wales and Woodland (1983) approach is based upon Kuhn-Tucker conditions associated with the maximization of a utility function subject to the budget constraint and the non-negativity constraints on goods demands. Zero expenditures are obtained when non-negativity constraints are binding, leading to a corner solution of the conventional utility maximization or cost minimization problem. The approach proposed by Lee and Pitt(1986) is based on the notion of virtual price. Their method consists of deriving consumer demand systems from indirect cost or utility functions including popular flexible functional forms such as the translog. They derive notional demand functions

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[1] Such as SUR (Seemingly unrelated Regressions) or maximum likelihood estimator

[2] Lee and Pitt (1987)

which are defined over the full real line, effective demands are rationed to be non-negative. Virtual prices are such that notional and observed demands coincide.

The Kuhn-Tucker approach (Wales and Woodland (1983)) and the virtual prices approach (Lee and Pitt(1986)) lead to equivalent regime conditions. Lee and Pitt's approach has the advantage of allowing the use of flexible-form indirect cost function such as the translog. Lee and Pitt (1987) apply the virtual price approach to estimate interfuel substitution between electricity, fuel oil and other fuels from a cross-section of Indonesian firms in the Weaving and Spinning sector and in the Metal product sector. Bousquet and Ivaldi (1998) propose a combined and coherent treatment of both the zero expenditures and missing data (prices)[3]. In this case, price equations are added to the demand system. Bjørner & Jensen (2000) focus on substitution between three different energy inputs. Estimations are carried out conditioning on the observed energy pattern. This means that the choice of the type of energy to consume is exogenously defined for firms.

The objective of this paper is to estimate an energy demand system, using the approach of Lee & Pitt (1987) to treat the zero expenditure problem and taking into account the panel form of the data. Panel data control for individual heterogeneity by identifying and measuring effects that are not detectable in pure cross-section or pure time-series data. This paper gives efficient estimates of a structural system of censored variables with random effects on panel data.

The present paper is organized as follows: section 2 describes the model of energy demand. The data and estimation results are presented in section 3. Section 4 is devoted to the exploration of empirical results. We present a complete simulation model based on our estimates. We study the effects of energy price variations and analyze the effects on energy demand of a CO2 tax.

# 2 The model

## 2.1 Energy demand specification

The production structure used here in deriving energy demand relationships are the same as in Fuss (1977). First, we assume that the production function is homothetic weakly separable in energy inputs. Thus the cost-minimizing mix of energy inputs is independent of the mix of the other factors and the cost minimization is a two stages procedure. In the first stage, the optimal amount of aggregate energy demand is determined and in the second stage relative prices of energy are used to determine the market share of each form of energy. In this paper we consider this second stage to determine the demand for energy input components.

The unit cost of energy is thus described by the translog cost function,

$$\ln P_{EN} = \alpha_0 + \sum_{m=1}^{3} \alpha_m \ln p_m + \frac{1}{2} \sum_{m=1}^{3} \sum_{k=1}^{3} \beta_{mk} \ln p_m \ln p_k,$$

where index 1 denotes natural gas, 2 denotes oil product and 3 denotes electricity.

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[3] This occur because most surveys generally report prices only for the subset of goods purchased.

The system of energy shares obtained from the translog cost function is defined as:

$$S_m = \frac{\partial \ln P_{EN}}{\partial \ln p_m} = \alpha_m + \sum_{k=1}^{3} \beta_{mk} \ln p_k, \quad m = 1, 2, 3.$$

The properties of neo-classical production theory require the following parameter restrictions:

$$\begin{aligned} \beta_{mk} &= \beta_{km}, \quad \forall\, m, k = 1, 2, 3, \\ \sum_{m=1}^{3} \alpha_m &= 1 \quad \text{and} \quad \sum_{k=1}^{3} \beta_{mk} = 0, \quad \forall\, m = 1, 2, 3. \end{aligned}$$

## 2.2 Errors specification

Our model extends the approach of Lee & Pitt (1986) to the case of panel data. This type of data helps control for individual heterogeneity. Time series and cross-section studies not controlling for this heterogeneity run the risk of obtaining biased coefficient estimates.

We add disturbances $w_{mit}$ to the system of energy shares described above:

$$S_{mit} = \alpha_m + \sum_{k=1}^{3} \beta_{mk} \ln p_{kit} + w_{mit}, \quad m = 1, 2, 3 \quad i = 1, \ldots N \quad \text{and} \quad t = 1, \ldots T \tag{1}$$

$m$ denotes index for type of energy, $i$ is the plant index and $t$ is the date index.

Following the standard procedure in panel data analysis, we decompose the error term as:

$$w_{mit} = \mu_{mi} + \varepsilon_{mit}, \quad m = 1, 2, 3 \quad i = 1, \ldots N \quad \text{and} \quad t = 1, \ldots T \tag{2}$$

where $\mu_{mi}$ is the individual effect of firm $i$ and energy $m$ and $\varepsilon_{mit}$ is an iid error term for equation $m$. We suppose that:

$$\mu_{mi} \sim N(0, \sigma^2_{\mu_m}), \quad \varepsilon_{mit} \sim N(0, \sigma^2_{\varepsilon}), \quad E(\mu_{mi}\varepsilon_{mit}) = 0, \quad \forall\, m = 1, 2, 3, \tag{3}$$

with:

$$E(\mu_{mi}\mu_{kj}) = \begin{cases} \sigma^2_{\mu_{mk}} & \text{if} \quad i = j \\ 0 & \text{otherwise} \end{cases} \quad \forall\, m, k = 1, 2, 3,$$

$$E(\varepsilon_{mit}\varepsilon_{kjs}) = \begin{cases} \sigma^2_{\varepsilon_{mk}} & \text{if} \quad i = j \quad \text{and} \quad t = s \\ 0 & \text{otherwise} \end{cases} \quad \forall\, m, k = 1, 2, 3.$$

Using notations in Schmidt (1990), one can write the system of $M = 3$ notional share equations where the $m$-th equation is of the form:

$$S_m = \alpha_m + \beta_m^{'} \ln p + w_m, \qquad m = 1, 2, 3 \tag{4}$$

where $S_m = (S_{m11,}...S_{m1T,}...S_{mN1,}...S_{mNT})'$, $\alpha_m = (\alpha_{m1}, \alpha_{m2}, \alpha_{m3})'$, $\beta_m = (\beta_{m1}, \beta_{m2}, \beta_{m3})'$, $\ln p = (\ln p_1, \ln p_2, \ln p_3)'$, $\ln p_m = (\ln p_{m11,}..., \ln p_{m1N}, \ln p_{mN1,}... \ln p_{mNT})'$, $\mu'_m = (\mu_{m1}, ..., \mu_{mN}) \otimes e'_T$ and $\varepsilon_m = (\varepsilon_{m11}, ...\varepsilon_{m1T}, ...\varepsilon_{mN1}, ...\varepsilon_{mNT})$

## 2.3 Energy regimes description

In this study three forms of energy are identified: electricity E, oil products O and natural gas G. To derive the likelihood function for this model, we need to distinguish different regimes. For three-energy models there are seven regimes $(2^3 - 1)$ in total. We assume that it is not feasible to produce without using any form of energy. In our sample electricity is always employed, therefore the choice set is reduced to $2^{3-1} = 4$ cases.

Denoting $S_m^*$ the observed shares for $m = 1, 2, 3$ the regime conditions[4] are summarized as in table (1).

Table 1: Presentation of energy regimes

| Regimes definitions | Observed share equations | Regime conditions |
|---|---|---|
| Regime $XXX$<br>$S_1^* > 0,\ S_2^* > 0,$<br>$1 - S_1^* - S_2^* > 0$ | <br>$S_1^* = S_1$<br>$S_2^* = S_2$ | <br>$0 < S_1, 0 < S_2$<br>$0 < S_1 + S_2 < 1$ |
| Regime $OXX$<br>$S_1^* = 0,$<br>$0 < S_2^* < 1$ | <br>$S_1^* = 0$<br>$S_2^* = S_2 - \frac{\beta_{12}}{\beta_{11}} S_1$ | <br>$S_1 \leqslant 0$<br>$0 < S_2 - \frac{\beta_{12}}{\beta_{11}} S_1 < 1$ |
| Regime $XOX$<br>$0 < S_1^* < 1,$<br>$S_2^* = 0$ | <br>$S_1^* = S_1 - \frac{\beta_{12}}{\beta_{22}} S_2$<br>$S_2^* = 0$ | <br>$0 < S_1 - \frac{\beta_{12}}{\beta_{22}} S_2 < 1$<br>$S_2 \leqslant 0$ |
| Regime $OOX$<br>$S_1^* = 0,$<br>$S_2^* = 0$ | <br>$S_1^* = 0$<br>$S_2^* = 0$ | <br>$S_1 - \frac{\beta_{12}}{\beta_{22}} S_2 \leqslant 0$<br>$S_2 - \frac{\beta_{12}}{\beta_{11}} S_1 \leqslant 0$ |

## 2.4 The likelihood function

For panel data, the presence of individual effects complicates matters significantly. In this case $\mu_{mi}$ are unknown parameters and for fixed $T$ the number of parameters $\mu_{mi}$ increases with $N$. This means that $\mu_{mi}$ cannot be consistently estimated for a fixed $T$. However if $T \to \infty$, then the maximum likelihood estimators (MLE) of $\mu_{mi}$ and the other parameters are consistent. For the linear regression model, when $T$ is fixed, the parameters of the

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[4] The formal expressions for the likelihood function associated with each regime of demand are available upon request from the authors.

model can be estimated consistently by removing $\mu_{mi}$, using the *within* transformation. This is not, however, the case for a non-linear model, as demonstrated by Chamberlain (1980).

The two common statistical model specifications that are used to analyze pooled cross-section and time-series data are the fixed effects model and the random effects model[5]. Heckman and Macurdy (1980) consider a fixed effects Tobit model to estimate a life-cycle model of female labor supply. They argue that the individual effects have a specific meaning in a life-cycle model and therefore cannot be assumed independent of the explicative variables. Therefore, a fixed effects, rather than a random effects specification is estimated, using a two-steps iterative method.

The random effects model is more appropriate when the sample is randomly drawn from a large population. In this case, $N$ is usually large and a fixed effects model would lead to an enormous loss of degrees of freedom. To our knowledge efficient estimates of a structural system of limited dependent variables with random effects have not been developed in the literature.

We assume that $\mu_{1i}$ and $\mu_{2i}$ are independent of exogenous variables and are random sampling from a bivariate normal distribution indexed by a finite number of parameters $\sigma_{\mu_1}, \sigma_{\mu_2}, \rho_{\mu_1\mu_2}$:

$$\begin{pmatrix} \mu_{1i} \\ \mu_{2i} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{\mu_1} & \rho_{\mu_1\mu_2}\sigma_{\mu_1}\sigma_{\mu_2} \\ \rho_{\mu_1\mu_2}\sigma_{\mu_1}\sigma_{\mu_2} & \sigma^2_{\mu_2} \end{pmatrix}\right)$$

Conditional on given values of $\mu_{1i}$ and $\mu_{2i}$, the marginal likelihood for plant $i$ is:

$$l_i = \int_{\mu_1}\int_{\mu_2} \Pi_{t=1}^{T}\Pi_{r\in R}\left[(L_r)^{I_{itr}}\right] dF(\mu_{1i}, \mu_{2i}|\sigma_{\mu_1}, \sigma_{\mu_2}, \rho_{\mu_1\mu_2}) \tag{5}$$

$L_r$ is the likelihood function of regime $r \in R = \{XXX, OXX, XOX, OOX\}$ and $I_{itr}$ is such that,

$$I_{itr} = \begin{cases} 1, & \text{if the observation } (i,t) \text{ belongs to regime r} \\ 0, & \text{otherwise.} \end{cases}$$

The sample log-likelihood function is:

$$L = \sum_{i=1}^{N} \log(l_i) \tag{6}$$

### 2.5 Coherency conditions of the model

The econometric model is correctly specified only if certain coherency conditions are satisfied. In the case of an incoherent likelihood function, the sum of the probabilities for all demand regimes is not equal to one[6]. Ransom (1987), Van Soest and Kooreman (1990) and Van Soest et al. (1993) have shown that the model will be coherent if the matrix of parameters is negative semi-definite. Imposing this condition also ensures the global concavity of translog cost function ( Diewert and Wales, 1987). In this model, coherency conditions and global concavity conditions are equivalent.

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[5] See Hsiao 1986 and Baltagi 1996

[6] see Gourieroux and al. (1980).

In order to impose the coherency conditions (and the concavity restrictions) on the translog functional form, we impose negative semidefiniteness on the matrix of parameters $B = [\beta_{ij}]_{i,j=1,2,3}$. We use the following technique due to Wiley, Schmidt and Bramble (1973): imposing negative semidefiniteness on the matrix $B$ is equivalent to writing:

$$B = -AA'$$

where A is a matrix such that: $A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ -a_{11} - a_{12} & -a_{22} & 0 \end{bmatrix}$

so:

$$B = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{12} & \beta_{22} & \beta_{23} \\ \beta_{13} & \beta_{23} & \beta_{33} \end{bmatrix} = \begin{bmatrix} -a_{11}^2 & -a_{11}a_{12} & a_{11}\left(a_{11}+a_{12}\right) \\ -a_{11}a_{12} & -a_{12}^2 - a_{22}^2 & a_{12}\left(a_{11}+a_{12}\right) + a_{22}^2 \\ a_{11}\left(a_{11}+a_{12}\right) & a_{12}\left(a_{11}+a_{12}\right) + a_{22}^2 & -\left(a_{11}+a_{12}\right)^2 - a_{22}^2 \end{bmatrix}$$

The new parameters satisfy both homogeneity and symmetry constraints imposed. The likelihood function is rewritten with the new parameters $a_{11}, a_{12}, a_{22}$.

# 3 Application to energy demand

## 3.1 The data

In this paper, zero expenditure is supposed to be the result of cost minimization of the firms leading to corner solutions. An energy source will not be used if it is too expensive: if its market price is greater than its marginal productivity in value. This assumption can be justified by the fact that a significant proportion of firms use multi-energy systems in order to be able to take advantage of the uncertainty of price variations, by shifting from one energy regime to another, at almost no cost. According to the MECS (Manufacturing Energy Consumption Survey), 20% of American industrial plants have the technology which enables them to substitute energies within an extremely short period of time and without disturbing production or incurring additional costs for modifying their equipment. The frequent observation of firms which abandon and then re-adopt a form of energy each year, during the 14 years of observation, suggests either that there exist many real capacities of substitution or that the costs of adaptation of the energy techniques are relatively low, or both.

The sample used for estimations is drawn from a yearly survey on energy consumption conducted by the *Service des Statistiques Industrielles* (SESSI) of the French Ministry of Industry. The sample contains 324 plants from Pulp and Paper sector observed over the period $(1983 - 1996)$. The energy survey includes information about expenditures as well as the consumption in physical units of different forms of energy.

Table 2: Descriptive statistics according to energy regimes

| Variable | **E** | | **E-G** | | **E-O** | | **E-G-O** | |
|---|---|---|---|---|---|---|---|---|
| Number of observations | 343 | | 396 | | 2946 | | 851 | |
| | Mean | s.d | Mean | s.d | Mean | s.d | Mean | s.d |
| Labor (number of employees) | 70 | 63 | 167 | 113 | 115 | 102 | 211 | 171 |
| Electricity power (KW) | 540 | 609 | 1777 | 4894 | 1459 | 6267 | 5726 | 14928 |
| **Energy consumption (toe)** | | | | | | | | |
| Total | 355 | 497 | 3069 | 7713 | 2299 | 9522 | 9771 | 17295 |
| Electricity | 355 | 497 | 1484 | 3460 | 1406 | 8304 | 5011 | 9286 |
| Gas | 0 | 0 | 1585 | 4377 | 0 | 0 | 3725 | 7727 |
| Oil | 0 | 0 | 0 | 0 | 803 | 2273 | 827 | 2753 |
| **Expenditure ($10^3$ Euro)** | | | | | | | | |
| Electricity | 106 | 128 | 269 | 415 | 263 | 1101 | 808 | 1239 |
| Gas | 0 | 0 | 225 | 542 | 0 | 0 | 568 | 1197 |
| Oil | 0 | 0 | 0 | 0 | 148 | 445 | 135 | 474 |
| **Quantity shares (%)** | | | | | | | | |
| Electricity | 100 | 0 | 64 | 16 | 72 | 21 | 57 | 17 |
| Gas | 0 | 0 | 36 | 16 | 0 | 0 | 31 | 21 |
| Oil | 0 | 0 | 0 | 0 | 28 | 21 | 11 | 16 |
| **Cost shares (%)** | | | | | | | | |
| Electricity | 100 | 0 | 69 | 16 | 74 | 19 | 62 | 16 |
| Gas | 0 | 0 | 31 | 16 | 0 | 0 | 28 | 19 |
| Oil | 0 | 0 | 0 | 0 | 26 | 19 | 11 | 13 |
| **Average prices (Euro/tep)** | | | | | | | | |
| Electricity | 356 | 94 | 310 | 77 | 321 | 88 | 273 | 77 |
| Gas | 329 | 55 | 240 | 74 | 334 | 57 | 226 | 92 |
| Oil | 455 | 10 | 455 | 19 | 303 | 115 | 307 | 139 |

A description of the plants' characteristics according to their energy regimes is presented in table (1). All plants use electricity, so only four energy regimes are possible for the three types of energy modelled: electricity, natural gas and oil products (which is a composite of heavy fuel, oil fuel and butane-propane). We note that:

- 19% of plants use all three types of energy, these plants are generally very large (the average number of employees is 211 and the subscribed electricity power is 5726 $KW$). These large plants are also energy intensive as they consume about 63% of the overall energy in the sample.
- 65% of plants use electricity and oil products. In terms of number of plants, this is the most significant regime. These plants are generally small (the average number of employees being 115).
- In the same way, 9% of plants use electricity and natural gas. These plants are larger than those using electricity and oil products in terms of number of employees and subscribed electricity power.
- Finally a small percent of the plants use electricity only. These are relatively small plants.

Figure 1: Distribution of the number of energy regime transitions

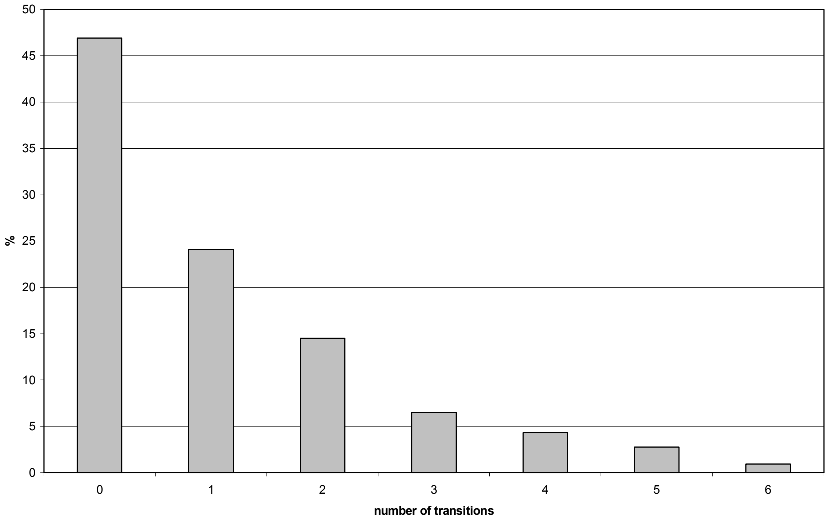

Figure (1) shows the distribution of the number of transitions between energy regimes over the 14 years of observation. Only 48% of the plants did not change their energy regime at all during this period. The plants which change their energy regime once (24%) switch for a large majority (54%) from the regime $E-O$ to $E-G-O$. 15% and 10% of these plants switched respectively from $E-O$ to $E$ and $E-G-O$ to $E-G$.

As explained previously, for each observation, factor prices are only observed conditionally on the realization of strictly positive demand. The most common procedure consists of replacing missing prices by average prices as in Lee and Pitt (1987). However since the observed price distribution is truncated and so observed average prices are lower than the true average. This is not appropriate in our case since we cannot associate a price to a zero expenditure at which other producers have positive optimal demand. In our model we substitute the empirical maxima of price distributions for missing prices for each type of energy. This method was originally applied by Flinn and Heckman (1982) to replace non observed wages for non-participants in the labor market.

### 3.2 Estimation results

The loglikelihood function of the model is maximized, to obtain MLE of the parameters $a_{11}, a_{12}, a_{22}, a_1, a_2$, $\sigma_{\varepsilon_1}$, $\sigma_{\varepsilon_2}$, $\rho_{\varepsilon_1\varepsilon_2}$, $\sigma_{\mu_1}$, $\sigma_{\mu_2}$ and $\rho_{\mu_1\mu_2}$.

Table (3) displays estimates of the translog cost parameters, obtained under global curvature condition and homogeneity of the cost function. Most of parameters are significantly different from zero.

Table 3: Estimation results

| | 1 | | 2 | | 3 | |
|---|---|---|---|---|---|---|
| | Natural Gas | | Oil | | Electricity | |
| | Estimation | s.e | Estimation | s.e | Estimation | s.e |
| $a_.$ | -0.1723 | 0.0113 | 0.3924 | 0.1411 | 0.7799 | 0.0247 |
| $b_{1.}$ | -0.7965 | 0.0269 | 0.5776 | 0.0183 | 0.2189 | 0.0179 |
| $b_{2.}$ | 0.5776 | 0.0183 | -0.4785 | 0.0168 | -0.0991 | 0.0105 |
| $b_{3.}$ | 0.2189 | 0.0179 | -0.0991 | 0.0105 | -0.1198 | 0.0135 |
| $\sigma^2_{\varepsilon.}$ | 0.3548 | 0.0098 | 0.2611 | 0.0067 | 0.2507 | 0.0040 |
| $\sigma^2_{\mu.}$ | 0.0021 | 0.0129 | 0.0093 | 0.0481 | 0.0095 | 0.0159 |
| $\rho_{\varepsilon_{1.}}$ | - | - | -0.708 | 0.169 | -0.677 | 0.0024 |
| $\rho_{\varepsilon_{2.}}$ | -0.708 | 0.169 | - | - | -0.039 | 0.0364 |
| $\rho_{\mu_{1.}}$ | - | - | -0.2435 | 0.0345 | -0.217 | 0.0215 |
| $\rho_{\mu_{2.}}$ | -0.2435 | 0.0345 | - | - | -0.975 | 0.0175 |

In order to test the relevance of estimating the random effect model, we compare our model to a model without random individual effect using the Breusch-Pagan (1980) test. This is a Lagrange-multiplier (LM) test for random effects model. The result of the test is to reject the null hypothesis in favor of the random effect model. We conclude that the classical regression model with one single constant term is inappropriate for these data.

## 4 Simulations

Studies using aggregate data have the drawback of smoothing the reaction of demand to variation in prices. In contrast, our microeconometric analysis allows us to distinguish among individual behaviors. In fact the model decomposes the choice of energy mix in two combined decisions, a qualitative and a quantitative one. Substitution effects between energies can be separated in a direct effect on changes in quantities (the distribution of regimes being unchanged), and in an indirect effect through the probabilities of choosing a particular regime. Note also that the existence of continuous (positive demand) and discrete (zero demand) components of demand complicates the definition of elasticities because these depend on conditional probabilities of consumption. Hence, we propose a simulation approach to measure elasticities.

In order to study the effects of price variations on energy demand, we compute the expected relative cost shares which depend on the probability of each regime and the associated vector of expected shares. Details of computation are available from the authors upon request. They are based on general results on the moments of truncated multivariate normal distribution[7].

The expected relative share of energy $m$ for firm $i$ is computed according to the following formula[8]:

$$E(S_{mi}) = \sum_{r=1}^{7} \Pr(i \in r) \times E\left[S_{mi}/i \in r\right], \quad m = E, O, G, \quad i = 1, ..., N.$$

Consequently, the impacts of price changes on expected shares should be decomposed into two effects: the first one on the probabilities of regimes and the second one on the expected conditional shares. Marginal effect of price $p_{ki}$ variation on expected relative share of energy $m$ is given by:

$$\frac{\partial E(S_{mi})}{\partial p_{ki}} = \sum_{r=1}^{7} \left[ \frac{\partial \Pr(i \in r)}{\partial p_{ki}} \times E\left[S_{mi}/i \in r\right] + \frac{\partial E\left[S_{mi}/i \in r\right]}{\partial p_{ki}} \times \Pr(i \in r) \right], \quad m, k = E, O, G$$

Then the variation of the price of one energy affects both the probability of choosing each regime and the conditional cost share of the energy.

## 4.1 Simulation of energy price variation

The simulation of price variations is obtained by shifting exogenously prices of the three forms of energy. The price effects may be expressed more conveniently in terms of price elasticities of demands. These elasticities defined by equation (7) take into account the effects of prices variation on the probabilities of regimes and on the conditional demand for regimes.

$$\begin{cases} \eta_{mk} = \frac{\partial \ln(E(S_m))}{\partial \ln(p_k)} + E(S_k), & \forall\, m, k = E, O, G \text{ and } k \neq m, \\ \eta_{kk} = \frac{\partial \ln(E(S_k))}{\partial \ln(p_k)} + E(S_k) - 1, & \forall\, m, k = E, O, G. \end{cases} \tag{7}$$

The own price elasticities of the three forms of energy are as follows: gas (-1.792), oil (-1.705) and electricity (-0.801). Thus, oil and gas have elastic own-price demands. Not surprisingly, the own-price elasticity of electricity is quite small: electricity is much more expensive than the other fuels, and should therefore be used only when there is little possibility of using an alternative fuel.

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[7] See Johnson and Kotz (1973).

[8] Note that we calculate regime probabilities and conditional expected shares even for regimes that are not observed. As we have three forms of energy, the number of energy regimes is equal to $2^3 - 1 = 7$.

Cross-price elasticities show that oil and gas are substitutes. However, oil is more sensitive to the variation of the price of gas. That is, a 1% increase in the price of gas implies a 0.87% increase in the oil demand, while a 1% increase in the price of oil implies only 0.65% increase in the demand of gas.

The elasticity of demand for electricity with respect to the price of oil is 0.024. The positive sign indicates that electricity and oil are substitutes. Nevertheless, the elasticity estimate is fairly small. This means that electricity demand is not very sensitive to oil price variations. However a 1% increase in electricity price leads to 0.32% increase in oil demand.

Finally, the demand for electricity reacts differently to oil and gas price changes. If the price of electricity increases by 1%, firms in the pulp and paper sector will increase more their gas demand.

We compared our results with two other studies that analyze inter-fuel substitution possibilities with firm level data: Lee and Pitt (1987) and Bjørner & Jensen (2000).

Lee and Pitt (1987) estimate interfuel substitution between electricity, fuel oil and other fuels from a cross-section of Indonesian firms in two industrial sectors. To take into account the observed zero demand, they estimate a discrete/continuous translog system. The authors found that the three forms of energy are substitutes and very elastic to their own price. Own-price elasticities were found to be: oil (-3.51), electricity (-1.96) and fuel (-2.91). Price elasticities were slightly larger than most of those reported in the literature. Lee and Pitt explain this by the fact that, almost all of the existing estimates of these types of energy price elasticities are for the industrialized countries

Bjørner & Jensen (2000) focus on substitution between three different energy forms, electricity, district heating and other fuels using micro panel data for Danish industrial companies. In their work, estimations were made conditioning on the observed energy pattern which means that the choice of the type of energy was assumed to be exogenously defined for firms. They found very small price elasticities. For example, for the translog model with fixed effects the own-price elasticities were between -0.26 and -0.06 for electricity, between -0.55 and -0.29 for district heating and between -0.24 and -0.04 for fuels.

In fact, cross-section and pooled data models are believed to have more long-run nature than fixed effects models. The random effect model estimated in this paper is a combination of the *within* and the *between* methods, which leads to intermediate term elasticities.

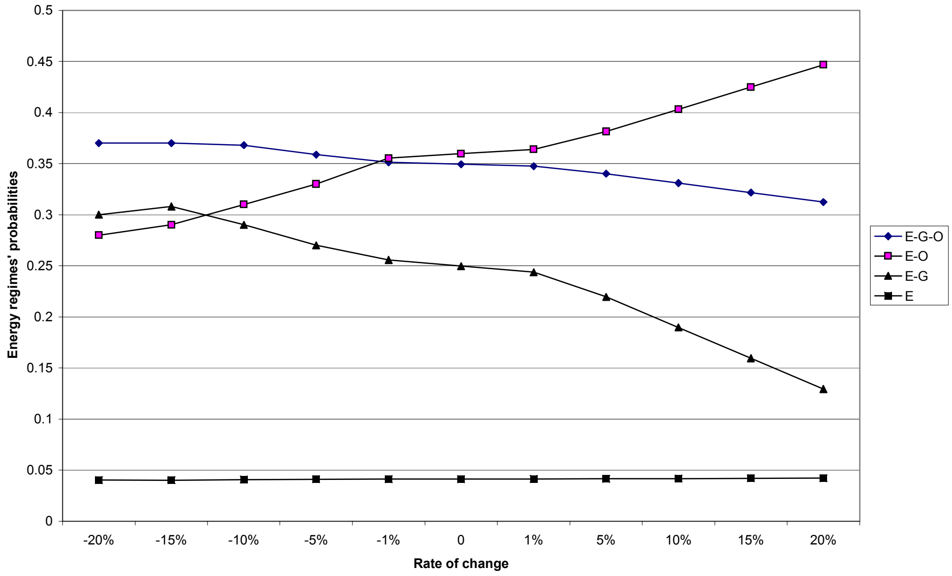
Note also that elasticities calculated in this paper take into account qualitative and quantitative effects of price variations, which in not the case in Lee and Pitt (1987) and Bjørner & Jensen (2000). This makes comparisons of elasticities less obvious.

### 4.1.1 Effects of prices changes on the probabilities of energy regimes

We consider now the effects of prices changes on the regimes' probabilities. Figures (2) and (3) show how probabilities of energy regimes are modified when the prices of gas and oil products change respectively.

From these figures, one can see that the probability of the regimes O-E and G-E are the most sensitive to price variations among observed regimes. A 10% change in the price of gas will decrease the probability of the

Figure 2: Effects of changes in gas price on probabilities of energy demand



regime G-E from 0.25 to 0.19 and increase the probability of the regime O-E from 0.36 to 0.40. Hence, when the prices of gas or oil change, firms switch between the two regimes O-E and G-E.

### 4.1.2 Effects of prices changes on conditional expected cost shares

In our model, we can also calculate the effect of price variations on conditional demand for regimes. To study this effect we define $\epsilon_{mk}$, (for $m, k = E, G, O$) as the percentage change in conditional demand of energy type $m$ when the price of energy type $k$ increases by 1%.

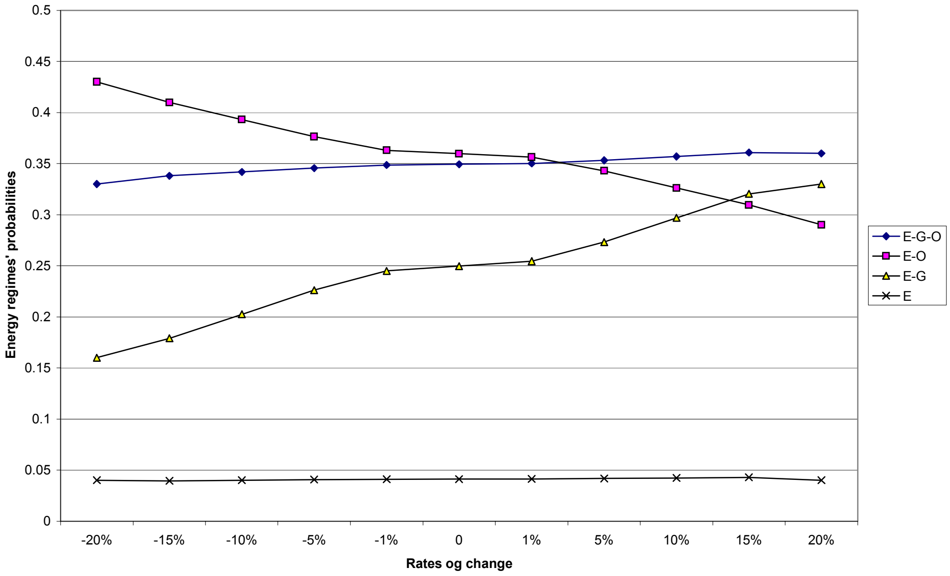
$$\epsilon_{mk} = \frac{\partial \ln(E(x_m / regime\ r))}{\partial \ln(p_k)},\ m, k = E, G, O.$$

The elasticity estimates by energy regime are reported in table (4). It appears that in general, elasticities are not exactly the same across the different energy regimes. For example an increase of 1% in the electricity price will increase the oil demand by 0.376% in the regime E-O and by only 0.223% in the regime E-G-O. Note also that own price elasticities are greater (in absolute value) in the regime E-G-O than in the other regimes. And that cross-price elasticities are almost smaller in the regime E-G-O than in the other regimes.

## 4.2 Simulation of a $CO_2$ tax

In France, the industrial sector generates 23% of the total national greenhouse emissions. $CO_2$ is the most significant gas emitted, representing 76% of the total industrial greenhouse emissions. Ecological tax reform presents an opportunity to meet the target of reducing emissions of greenhouse gases set out in the Kyoto Protocol.

Figure 3: Effects of changes in oil products prices on probabilities of energy demand



Carbon and energy taxes have been frequently advocated by economists and international organizations as a policy instrument for reducing carbon dioxide emissions. An increasing number of Western European countries (Sweden, Norway, The Netherlands, Denmark,...) have implemented taxes based on the carbon content of the energy products. In France, the government had proposed a reform of environmental taxation by extending the TGAP[9] to the field of energy starting in 2001. However, this new energy/$CO_2$ tax on industrial companies was cancelled in December 2000 during the political debate.

### 4.2.1 How big a tax should be to stabilize the CO$_2$ level in the pulp and paper industry?

Before examining the effects of a CO$_2$ tax on energy demand, we need to define the "optimal " tax rate to be implemented. This tax rate will depend on the objectives of policy makers. To meet its commitments to the Kyoto protocol, France will need to stabilize CO$_2$ emissions in 2010 at their level of 1990. We calculate the tax rate that would be necessary in the pulp and paper sector in order to stabilize the CO$_2$ emissions over the periods 1990-2010 and 1990-2040. We assume that energy consumption will continue to grow at the average growth rate observed in the data, the average rate of growth of total energy demand over the observation period 1983-1996 is equal to +3.6% per year. We also assume that price elasticity of demand for the energy aggregate is equal to -1.[10] We finally assume that there is no substitution between forms of energy and thus the reduction of CO$_2$ emissions is only possible by reducing the demand of total energy.

We found that a tax of 17 Euro/tCO$_2$ is necessary in order to keep the same level of energy consumption

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[9] Taxe Générale sur les Activités Polluantes (general tax on polluting activities).

[10] See Atkinson and Manning (1995) for a survey of international energy demand elasticities.

Table 4: Conditional price elasticities

| | Energy Regimes | | |
|---|---|---|---|
| | E-G-O | E-G | E-O |
| $\epsilon_{GG}$ | -1.466 | -1.115 | - |
| $\epsilon_{OG}$ | 0.621 | - | 0.203 |
| $\epsilon_{EG}$ | 0.171 | 0.317 | -0.101 |
| $\epsilon_{GO}$ | 0.445 | 0.119 | - |
| $\epsilon_{OO}$ | -1.378 | - | -1.197 |
| $\epsilon_{EO}$ | 0.124 | -0.078 | 0.173 |
| $\epsilon_{GE}$ | 0.396 | 0.461 | - |
| $\epsilon_{OE}$ | 0.223 | - | 0.376 |
| $\epsilon_{EE}$ | -0.857 | -0.802 | -0.774 |

during the period 1990-2010, which ensures stable emissions $CO_2$ emissions. Similarly, we obtain a tax level of 32 Euro/t$CO_2$ when the objective is to stabilize energy consumption, and then $CO_2$ emissions, during the period 1990-2040.

Now we will illustrate, using our estimates, the real impact of these taxes on $CO_2$ emissions when we take into account the interfuel substitution possibilities. We will show how the $CO_2$ tax affect the optimal energy mix and so the level of $CO_2$ emissions.

### 4.2.2 Simulation results

We simulate a $CO_2$ tax with two rates 17 Euro/t$CO_2$ and 32 Euro/t$CO_2$. These two tax rates imply that the average price of gas increases by 14% and 28% respectively, and the average price of oil by 15% and 29% respectively. Most of the electricity produced in France is nuclear or hydraulic. We assume that the electricity used in the industry, and in particular in the pulp and paper sector, is base load electricity and thus is not $CO_2$ polluting.

Simulation results show that a $CO_2$ tax will lead to substitution from fossil fuels to electricity. According to these results a 17 Euro/t$CO_2$ tax will increase the electricity demand by 7% while demand of oil and gas will decrease by 22% and 27% respectively. Consequently, we found that $CO_2$ emissions in the pulp and paper sector will decrease by 25%.

If the objective of policy makers is to stabilize $CO_2$ emissions in this industry over 50 years (1990-2040), a tax of 32 Euro/t$CO_2$ will be necessary, as shown earlier. In this case the changes in the energy mix induced by the tax leads to the reduction of $CO_2$ emissions by 40%. Electricity demand will increase in this case by 11% and gas and oil demand will decrease by 43% and 33% respectively.

These simulation results show the great importance of interfuel substitution possibilities. It shows clearly that the effects of a $CO_2$ tax could change a lot depending on substitution possibilities between different forms of energy. Such exercise should motivate further studies and empirical analysis on interfuel substitution. It is then very important to take into account and to properly estimate interfuel substitution for energy policies and the derived environmental constraints.

## 5 Conclusion

This paper presents an application of Lee and Pitt's (1986) approach to panel data. This approach deals with corner solutions in a consistent manner with behavioral theory. We define an indirect translog cost function. Optimal input demands are derived by Shephard lemma. The virtual price concept allows us to characterize zero expenditure. Null demands are considered as the result of endogenous rationing and are explained by price excess. Firms do not consume inputs for which prices on the market exceed the virtual prices. The econometric model is essentially an endogenous switching regime model. We apply the model to estimate energy demand in the pulp and paper sector in France. We use panel data which helps us to control for individual heterogeneity.

Our empirical findings are that: (1) electricity, gas and oil products are substitutes, (2) oil and gas have elastic own-price demands which is not the case for electricity, (3) comparing to price elasticities found in two other micro interfuel substitution studies, our results seem to be of intermediate run, (4) price elasticities conditional to regimes show that the three forms of energy are more sensitive to their own prices in the regime E-G-O than in the other regimes. (5) when the prices of gas or oil change, most of firms switch between the two regimes O-E and G-E (6) $CO_2$ tax simulations show that there are real interfuel substitution possibilities in the pulp and paper sector and then a carbon tax will be less costly for firms in this industry.

For further work, it will be interesting to disaggregate the oil products and to estimate the model for the case of more than three energy forms. The difficulty of estimating such models motivated the search for a method of evaluation of high-dimensional integrals that keep a reasonable balance between accuracy and computational costs. We propose to estimate the model of Lee and Pitt (1987) for the case of more than three goods using the Maximum Simulated Likelihood Estimator.

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