Second Best Analysis in a General Equilibrium Climate Change Model

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Abstract

The paper considers a general equilibrium climate change model with two endogenous R&D sectors. First, we characterize the set of decentralized equilibria: to each vector of public tools – a carbon tax and a subsidy to each R&D sector – is associated a particular equilibrium. Second, we compute the optimal tools. Third, we perform various second-best analyses. The main results of the paper are the following: i) the effect of the green research subsidy on resource extraction, and thus on carbon emissions, can override the carbon tax one; ii) R&D subsidies have a very large impact on the total social welfare, as compared with the carbon tax; iii) those subsidies allow to spare the earlier generations who are, on the other hand, strongly penalized by a carbon tax.

JEL classification: H23, O32, Q43, Q54, Q55.

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1 Introduction

The strand of literature on economic growth and climate change contains mostly optimization models (see for instance Bosetti et al., 2006; Edenhofer et al., 2005, 2006; Gerlagh and Van Der Zwaan 2006; Popp, 2006a, 2006b). In these papers, the problem consists in determining the temporal trajectories which maximize a social welfare function. Generally, the constraints of the problem are both technological and climatic: in this case, one gets the first-best optimum. Sometimes, additional constraints are added. For instance, Popp (2006a) presents the results of a simulated optimal carbon tax without research subsidy. However, to our knowledge, the basic problem of a policy-maker facing the agent behaviors in a decentralized economy is generally nor formalized neither analyzed.

There exists several reasons to think that it is impossible to reach the first-best optimum in the real world. Some of them are standard in the literature, as the existence of ex-ante distortionnary taxes in the system (Sandmo, 1975), or the restriction to linear taxes. For instance, Cremer et al. (2001) study how second-best considerations change the level of the optimal tax on a polluting good, but in a static model. In our paper, we assume that, because of budgetary, socioeconomic or political constraints, it is difficult to enforce the first-best policies. For example, consider a policy-maker who is constrained on several policy tools among the vector of all the instruments he can spare, e.g. the environmental tax and/or some research subsidies are set below their first-best levels. In these cases, the policy-maker can only play with the remaining unconstrained tools in order to maximize the social welfare. The basic point is that the structure of the decentralized economy becomes an additional constraint for him and then, he can only reach a second-best optimum.

Before conducting a second-best analysis, it is thus necessary to characterize the set of equilibria: a particular equilibrium is associated to each vector of economic policy tools. Hence, if some of these tools are constrained, the policy-maker determines the other(s) in order to maximize the welfare in the remaining sub-set of equilibria.

Beyond the fact that it allows to perform second-best analysis, the general equilibrium approach has several other advantages. First, it allows to analyze the dissociated impacts of various policy tools on the time pace of all the variables, prices and quantities. For instance, one can study the impact of a change in the carbon tax, the other tools being given. Second, it allows to understand the role of prices, that are the channels by which policy tools act on the economy. Third, it permits to avoid the inaccuracies inherent in any partial equilibrium analysis, as for instance the ones implied by the use of the standard cost-benefit approach when the policy (or project) choices lead to more than marginal perturbations (see Dietz et al., 2008, for the special case of climate change mitigation policies).

The objective of this paper is to propose a methodological framework in order to perform second-best analysis in a general equilibrium climate change model with dedicated endogenous R&D. More precisely, we focus on the study of the set of equilibria in the decentralized economy. The main difficulty of this approach lies in the way the research activity is modeled, in particular the type of innovation goods which are developed as well as their pricing. In the standard endogenous growth theory (Aghion and Howitt, 1998; Romer, 1990...), when an innovation is produced, it is associated with a particular intermediate good. However, the more often, embodying knowledge into intermediate goods becomes inextricable in more general computable endogenous growth models with pollution and/or natural resources. In addition, those technical difficulties are emphasized when dealing with several research sectors, i.e. when there are several types of specific knowledge, each of them being dedicated to a particular input (resource, labor, capital, backstop...), as it is proposed in Acemoglu (2002). To circumvent those obstacles, we assume that the pieces of knowledge are directly priced (see for instance Grimaud and Rougé, 2008). We compute the social and the market values of an innovation and we suppose that the policy-maker can reduce the gap between these two values owing to dedicated R&D subsidies.

We develop an endogenous growth model in which energy services can be produced from a polluting non-renewable resource as well as a clean backstop. We introduce two R&D sectors, the first one improving the efficiency of energy production, the second one, the efficiency of the backstop. With this respect, we have to consider two types of market failures: the pollution from fossil resource use and the research spillovers in each R&D sector. That is why, in the decentralized equilibrium, we introduce two kinds of economic policy instruments in accordance: an environmental tax on the carbon emissions and a research subsidy for the energy and backstop sectors. There is an equilibrium associated to each vector of instruments, which allows to study the impact of one or several policy changes on the equilibrium trajectories. Clearly, when public instruments are optimally set, the equilibrium of the decentralized economy coincides with the first best optimum.

Next, we calibrate the model to fit the world 2005 data. We obtain results that highlight the role of the research grants, in particular the backstop ones. First, in a second-best world, the effect of the green research subsidy on resource extraction, and thus on carbon emissions, can override the carbon tax one. Second, R&D subsidies have a very large impact on the total social welfare, as compared with the carbon tax. Third, those subsidies allow to spare the earlier generations who are, on the other hand, strongly penalized by a carbon tax.

The article is organized as follows. Section 2 presents the decentralized economy and solves the equilibrium. In section 3, we characterize the first-best optimal solutions and we compute the optimal policy tools that implement it. In section 4, we analyze a selection of second-best cases and we illustrate numerically our main results. We conclude in section 5.

2 The decentralized economy

We consider a worldwide decentralized economy containing four production sectors: final output, energy services and two primary energy inputs, namely a fossil fuel and a carbon-free backstop. The fossil fuel (e.g. refining industry in the case of oil) is obtained from a polluting non-renewable resource whose combustion yields carbon emissions which are accumulated in the atmosphere, implying an increase of the mean atmospheric temperature and then, some economic penalties. Following on Nordhaus (2007b), we assume here that these penalties take the form of a damage function affecting the level of final output, instead of the consumer's utility. The production of final energy services and backstop requires some specific knowledges provided by two specific R&D sectors. We assume that all sectors, except R&D sectors, are perfectly competitive. The population grows exogenously and is equal to the labor supply. Finally, in order to correct the two types of distortions involved by the model (pollution and research spillovers in each R&D sector), we introduce two types of policy tools: an environmental tax on the fossil fuel use and a subsidy for each R&D sector. The model is sketched in Figure 1 and is detailed sector by sector in the following subsection.

2.1 Behavior of agents

2.1.1 The final good sector

We assume that global warming affects the economy through the final output such that, when the average temperature increase is T_t, the instantaneous penalty rate is $D(T_t) =$

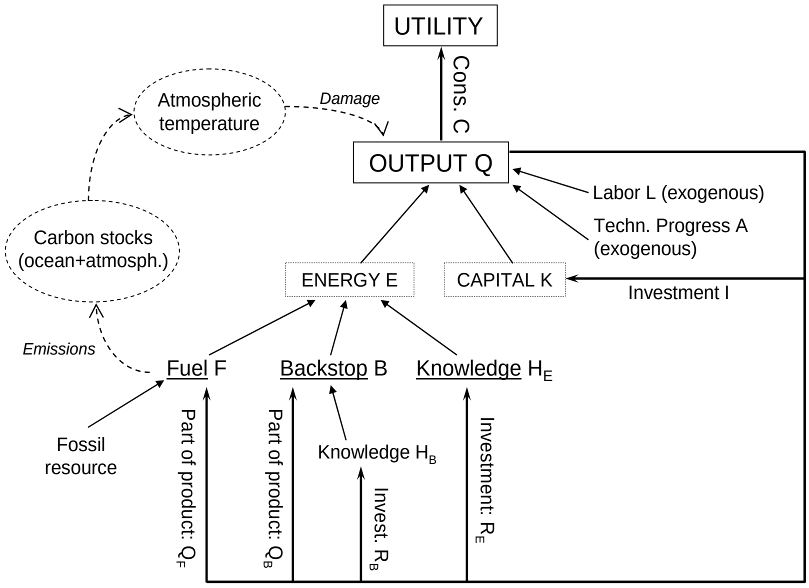

Figure 1: Description of the model

$1/(1+\alpha_T T_t^2)$, $\alpha_T > 0$. At each time t, the production of final output is $D(T_t)Q_t$, where Q_t is given by:

$$Q_t = Q(K_t, E_t, L_t, A_t) \;=\; A_t K_t^{\gamma} E_t^{\beta} L_t^{1-\gamma-\beta}, \quad \beta, \gamma \in (0,1), \tag{1}$$

in which K_t, E_t and L_t, $L_t \equiv L_0 e^{\int_0^t g_{L,s} ds}$, denote the capital, the flow of energy services and the labor employed at time t, respectively. A_t, $A_t \equiv A_0 e^{\int_0^t g_{A,s} ds}$, is an efficiency index that measures the total productivity of factors. Growth rates $g_{L,t}$ and $g_{A,t}$ are exogenously given: $g_{j,t} = g_{j0} e^{-d_j t}$, with $d_j > 0$, $\forall j = \{A, L\}$.

Denoting respectively by $p_{E,t}$, w_t, r_t and δ the price of energy services, the real wage, the interest rate and the depreciation rate of capital, and normalizing the output price to one, the instantaneous profit of the final output producer writes[1]: $\Pi_t^Q = D(T_t)Q_t - p_{E,t}E_t - w_t L_t - (r_t+\delta)K_t$. At each time t, the program of the final output producer consists in choosing K_t, E_t and L_t that maximizes Π_t^Q, subject to (1). The first order conditions

[1] We assume here that the representative household holds the capital and rents it to the firm at a rental price R_t. Standard arbitrage conditions imply $R_t = r_t + \delta$.

are:

$$\begin{aligned} D(T_t)Q_K - (r_t + \delta) &= 0 && (2)\\ D(T_t)Q_E - p_{E,t} &= 0 && (3)\\ D(T_t)Q_L - w_t &= 0, && (4) \end{aligned}$$

where J_X stands for the partial derivative of function $J(.)$ with respect to X.

2.1.2 The energy sector

The instantaneous production of a flow of energy services, E_t, requires a bundle of imperfect substitute primary energies and some knowledge (see Popp, 2006a):

$$E_t = E(F_t, B_t, H_{E,t}) = \left[(F_t^{\rho_B} + B_t^{\rho_B})^{\frac{\rho_H}{\rho_B}} + \alpha_H H_{E,t}^{\rho_H}\right]^{\frac{1}{\rho_H}}, \quad \alpha_H, \rho_H, \rho_B \in (0,1), \qquad (5)$$

where F_t is the fossil fuel use, B_t is a backstop energy source and $H_{E,t}$ represents a stock of specific technological knowledge dedicated to energy efficiency. Denoting by $p_{F,t}$ and $p_{B,t}$ the prices of fossil fuel and backstop and by τ_t the carbon tax, assumed here to be additive, the energy producer must chooses F_t and B_t at each time t that maximizes $\Pi_t^E = p_{E,t}E_t - (p_{F,t} + \tau_t)F_t - p_{B,t}B_t$, subject to (5). Note that, because of the carbon tax, the fuel price paid by the firm, i.e. $p_{F,t} + \tau_t$, is larger than the selling price $p_{F,t}$, i.e. the price which is received by the resource-holder. The first order conditions write:

$$\begin{aligned} p_{E,t}E_F - p_{F,t} - \tau_t &= 0 && (6)\\ p_{E,t}E_B - p_{B,t} &= 0. && (7) \end{aligned}$$

2.1.3 The fossil fuel sector

The fossil fuel is obtained from some carbon-based non-renewable resource and some specific productive investment (see Grimaud et al., 2007):

$$F_t = F(Q_{F,t}, Z_t) = \frac{Q_{F,t}}{c_F + \alpha_F (Z_t/\bar{Z})^{\eta_F}}, \quad c_F, \alpha_F, \eta_F > 0, \qquad (8)$$

where $Q_{F,t}$ is the amount of final product devoted to the production of fossil fuel and Z_t, $Z_t \equiv \int_0^t F_s ds$, is the cumulative extraction of the exhaustible resource from the initial date up to t, with $\bar{Z}$: $Z_t \leq \bar{Z}$, $\forall t \geq 0$. Then, the fuel supply is constrained by the resource scarcity. The instantaneous profit of the fuel producer is: $\Pi_t^F = p_{F,t}F_t - Q_{F,t}$ and its program consists in choosing $\{Q_{F,t}\}_0^\infty$ that maximizes $\int_0^\infty \Pi_t^F e^{-\int_0^t r_s ds} dt$, subject to $Z_t = \int_0^t F_s ds$ and (8). Denoting by η_t the multiplier associated with the state equation, static and dynamic first order conditions are:

$$\begin{aligned}(p_{F,t}F_{Q_F} - 1)e^{-\int_0^t r_s ds} + \eta_t F_{Q_F} &= 0 \quad (9)\\ p_{F,t}F_Z e^{-\int_0^t r_s ds} + \eta_t F_Z &= -\dot{\eta}_t, \quad (10)\end{aligned}$$

together with the transversality condition $\lim_{t\to\infty} \eta_t Z_t = 0$. Integrating (10) and using (9), it comes:

$$p_{F,t} = \frac{1}{F_{Q_F}} - \int_t^\infty \frac{F_Z}{F_{Q_F}} e^{-\int_t^s r_x dx} ds, \quad (11)$$

which reads as a specific version of the standard Hotelling rule in the case of an extraction technology given by function (8).

2.1.4 The backstop sector

The backstop resource technology is given by:

$$B_t = B(Q_{B,t}, H_{B,t}) = \alpha_B Q_{B,t} H_{B,t}^{\eta_B}, \quad \alpha_B, \eta_B > 0, \quad (12)$$

where $Q_{B,t}$ is the amount of final product that is devoted to the backstop production sector and $H_{B,t}$ is the stock of knowledge pertaining to the backstop. At each time t, the backstop producer maximizes its profit $\Pi_t^B = [p_{B,t}B_t - Q_{B,t}]$, subject to technological constraint (12), which implies the following first order condition:

$$p_{B,t}B_{Q_B} - 1 = 0. \quad (13)$$

2.1.5 The R&D sectors

There are two stocks of knowledge, H_E and H_B, each associated with a specific R&D sector (i.e. the energy and the backstop ones). We consider that each innovation is a non-rival, indivisible and infinitely durable piece of knowledge (for instance, a scientific report, a data base, a software algorithm...) which is simultaneously used by the sector which produces the good i and the R&D sector i, $i = \{B, E\}$.

Here, an innovation is not directly embodied into tangible intermediate goods and thus, it cannot be financed by the sale of these goods. However, in order to fully describe the equilibrium, we need to find a way to assess the price received by the inventor for each piece of knowledge. We proceed as follows: i) In each research sector, we determine the *social* value of an innovation. Since an innovation is a public good, this social value is the sum of marginal profitabilities of this innovation in all sectors which use it. If the inventor was able to extract the willingness to pay of each user, he would receive this social value and the first best optimum would be implemented. ii) In reality, there are some distortions that constrain the inventor to extract only a part of this social value[2]. This implies that the *market* value (without subsidy) is lower than the social one. iii) The research sectors are eventually subsidized in order to reduce the gap between the social and the market values of innovations.

Let us apply this three-steps procedure to the R&D sector i, $i = \{B, E\}$. Each innovation produced by this sector is used by the R&D sector i itself as well as by the production technology of good i. Thus, at each date t, the instantaneous social value of this innovation is $\bar{v}_{H_i,t} = \bar{v}^i_{H_i,t} + \bar{v}^{H_i}_{H_i,t}$, where $\bar{v}^i_{H_i,t}$ and $\bar{v}^{H_i}_{H_i,t}$ are the marginal profitabilities of this innovation in the production and R&D sectors i, respectively. The social value of this innovation at t is $\bar{V}_{H_i,t} = \int_t^\infty \bar{v}_{H_i,s} e^{-\int_t^s r_x dx} ds$. We assume that, without any public intervention, only a share γ_i of the social value is paid to the innovator, with $0 < \gamma_i < 1$. However, the government can decide to grant this R&D sector by applying a non-negative subsidy rate $\sigma_{i,t}$. Note that if $\sigma_{i,t} = 1 - \gamma_i$, the market value matches the social one. The instantaneous market value (including subsidy) is:

$$v_{H_i,t} = (\gamma_i + \sigma_{i,t})\bar{v}_{H_i,t}, \tag{14}$$

and the market value at date t is:

$$V_{H_i,t} = \int_t^\infty v_{H_i,s} e^{-\int_t^s r_x dx} ds. \tag{15}$$

Note that differentiating (15) with respect to time leads to the usual arbitrage relation:

$$r_t = \frac{\dot{V}_{H_i,t}}{V_{H_i,t}} + \frac{v_{H_i,t}}{V_{H_i,t}}, \quad \forall i = \{B, E\}, \tag{16}$$

which reads as the equality between the rate of return on the financial market and the rate of return on the R&D sector i.

We can now analyze the behaviors of the R&D sectors. The dynamics of the stock of knowledge in sector i is governed by the following innovation function $H^i(.)$:

$$\dot{H}_{i,t} = H^i(R_{i,t}, H_{i,t}) \; = \; a_i R_{i,t}^{b_i} H_{i,t}^{\phi_i}, \tag{17}$$

where $a_i > 0$, and $b_i, \phi_i \in [0, 1]$, $\forall i = \{B, E\}$. $R_{i,t}$ is the R&D investment into sector i, i.e. the amount of final output that is devoted to R&D sector i. At each time t, each

[2]For instance, Jones and Williams, 1998, estimate that actual investment in research are at least four times below what would be socially optimal; on this point, see also Popp, 2006a.

sector i, $i = \{B, E\}$, supplies the flow of innovations $\dot{H}_{i,t}$ at price $V_{H_i,t}$ and demands some specific investment $R_{i,t}$ at price 1, so that the profit function to be maximized is $\Pi_t^{H_i} = V_{H_i,t} H^i(R_{i,t}, H_{i,t}) - R_{i,t}$. The first order condition implies:

$$\frac{\partial \Pi_t^{H_i}}{\partial R_{i,t}} = 0 \quad \Rightarrow V_{H_i,t} = \frac{1}{H_{R_i}^i}. \tag{18}$$

The marginal profitability for specific knowledge of R&D sector i is:

$$\bar{v}_{H_i,t}^{H_i} = \frac{\partial \Pi_t^{H_i}}{\partial H_{i,t}} = V_{H_i,t} H_{H_i}^i = \frac{H_{H_i}^i}{H_{R_i}^i}. \tag{19}$$

Finally, in order to determine the social and the market values of an innovation in all research sectors, we need to know the marginal profitabilities of innovations in the backstop and the energy production sectors. From the expressions of Π_t^B and Π_t^E, those values are given respectively by $\bar{v}_{H_B,t}^B = \partial \Pi_t^B / \partial H_{B,t} = B_{H_B}/B_{Q_B}$ and $\bar{v}_{H_E,t}^E = \partial \Pi_t^E / \partial H_{E,t} = E_{H_E}/E_B B_{Q_B}$. Therefore, the instantaneous market values (including subsidies) of innovations are:

$$v_{H_B,t} = (\gamma_B + \sigma_{B,t}) \left(\frac{B_{H_B}}{B_{Q_B}} + \frac{H_{H_B}^B}{H_{R_B}^B} \right) \tag{20}$$

$$v_{H_E,t} = (\gamma_E + \sigma_{E,t}) \left(\frac{E_{H_E}}{E_B B_{Q_B}} + \frac{H_{H_E}^E}{H_{R_E}^E} \right). \tag{21}$$

2.1.6 The household and the government

The social welfare function is defined as:

$$W = \int_0^\infty U(C_t) e^{-\int_0^t \rho_s ds} dt \; = \; v_1 \int_0^\infty L_t \frac{(C_t/L_t)^{1-\epsilon}}{(1-\epsilon)} e^{-\int_0^t \rho_s ds} dt + v_2, \tag{22}$$

where ρ_t, $\rho_t \equiv \rho_0 e^{-g_\rho t}$, is the instantaneous social rate of time preferences, g_ρ, $g_\rho > 0$, is the (constant) declining rate of ρ_t, $U(C_t)$ is the instantaneous utility function, ϵ, $\epsilon > 0$, is the elasticity of marginal utility, and $v_1, v_2 > 0$ are scaling parameters. The households maximize W subject to the following dynamic budget constraint:

$$\dot{K}_t = r K_t + w_t L_t + \Pi_t - C_t - T_t^a, \tag{23}$$

where Π_t is the total profits gained in the economy and T_t^a is a lump-sum tax (subsidy-free) that allows to balance the budget constraint of the government. This maximization leads to the following condition:

$$\rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} = r_t \Rightarrow U'(C_t) = U'(C_0) e^{\int_0^t (\rho_s - r_s) ds}. \tag{24}$$

Assuming that the government's budget constraint holds at each time t (i.e. sum of the various taxes equal R&D subsidies), then it writes:

$$T_t^a + \tau_t F_t = \sum_i \frac{\sigma_i}{(\gamma_i + \sigma_i)} V_{H_i,t} \dot{H}_{i,t}, \quad i = \{B, E\}. \tag{25}$$

Finally, remark that expanding $\Pi_t = \Pi_t^Q + \Pi_t^E + \Pi_t^B + \Pi_t^F + \Pi_t^{H_B} + \Pi_t^{H_E}$ into (23) and replacing T_t^a by its value coming from (25), we obtain:

$$D(T_t)Q_t = C_t + Q_{F,t} + Q_{B,t} + I_t + R_{E,t} + R_{B,t}, \tag{26}$$

where I_t is the instantaneous investment in capital defined by:

$$I_t = \dot{K}_t + \delta K_t. \tag{27}$$

Hence, we verify that the final output is devoted to the aggregated consumption, the fossil fuel production, the backstop production, the investment in capital, and in the two R&D sectors.

2.2 The environment

Pollution is generated by fossil fuel burning. Let ξ, $\xi > 0$, be the unitary carbon content of fossil fuel, G_0 the stock of carbon in the atmosphere at the beginning of the planning period, G_t the stock at time t and ζ, $\zeta > 0$, the natural rate of decay. As in the DICE-07 model (Nordhaus, 2007b), the atmospheric carbon concentration does not directly enter the damage function. In fact, the increase in carbon concentration drives the global mean temperature away from a given state – here the 1900 level – and the difference between this state and the present global mean temperature is taken as an index of climate change. Let T_t denote this difference. Then, the climatic dynamic system is captured by the following two state equations:

$$\dot{G}_t = \xi F_t - \zeta G_t \tag{28}$$

$$\dot{T}_t = \Phi(G_t) - mT_t = \alpha_G \log G_t - mT_t, \quad \alpha_G, m > 0. \tag{29}$$

Function $\Phi(.)$, which links the atmospheric carbon concentration to the dynamics of temperature, is in fact the reduced form of a more complex function that takes into account the inertia of the climate dynamics (i.e. the radiative forcing, see Nordhaus 2007b)[3].

[3]In the analytical treatment of the model, we assume, for the sake of clarity, that the carbon cycle through atmosphere and oceans as well as the dynamic interactions between atmospheric and oceanic

2.3 Characterization of the decentralized equilibrium

From the previous analysis of individual behaviors, we can now characterize an equilibrium in the decentralized economy, which is done by the following proposition:

Proposition 1 *For a given triplet of policies* $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$*, the equilibrium conditions can be summed up as follows:*

$$\left[D(T_t)Q_E E_F - \tau_t - \frac{1}{F_{Q_F}}\right] U'(C_t) e^{-\int_0^t \rho_s ds} + \int_t^{\infty} \frac{F_Z}{F_{Q_F}} U'(C_s) e^{-\int_0^s \rho_x dx} ds = 0 \tag{30}$$

$$\begin{aligned}
D(T_t)Q_E E_B B_{Q_B} &= 1 &(31)\\
D(T_t)Q_K - \delta &= \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} &(32)\\
-\frac{\dot{H}^B_{R_B}}{H^B_{R_B}} + (\gamma_B + \sigma_{B,t})\left(\frac{B_{H_B} H^B_{R_B}}{B_{Q_B}} + H^B_{H_B}\right) &= \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)} &(33)\\
-\frac{\dot{H}^E_{R_E}}{H^E_{R_E}} + (\gamma_E + \sigma_{E,t})\left(\frac{E_{H_E} H^E_{R_E}}{E_B B_{Q_B}} + H^E_{H_E}\right) &= \rho_t - \frac{\dot{U}'(C_t)}{U'(C_t)}. &(34)
\end{aligned}$$

The corresponding system of prices is:

$$\begin{aligned}
r_t^* &= D(T_t)Q_K - \delta &(35)\\
w_t^* &= D(T_t)Q_L &(36)\\
p_{F,t}^* &= \frac{1}{F_{Q_F}} - \int_t^{\infty} \frac{F_Z}{F_{Q_F}} e^{-\int_t^s r_x dx} ds &(37)\\
p_{B,t}^* &= \frac{1}{B_{Q_B}} &(38)\\
p_{E,t}^* &= \frac{p_{B,t}^*}{E_B} = D(T_t)Q_E &(39)\\
V_{H_i,t}^* &= \frac{1}{H^i_{R_i}}, \ \forall i = \{B, E\}. &(40)
\end{aligned}$$

Proof. See Appendix A1.

A particular equilibrium is associated with a given triplet of policies $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$ and the set of equations given by Proposition 1 allows to compute quantities and prices for this equilibrium. If the triplet of policy tools is optimal, this set of equations characterizes

temperatures, are captured by the reduced form (28) and (29). Goulder and Mathai (2000), or Kriegler and Bruckner (2004), have recourse to such simplified dynamics. From the DICE-99 model, the formers estimate parameters ξ and ζ that take into account the inertia of the climatic system. They state that only 64% of current emissions actually contribute to the augmentation of atmospheric CO_2 and that the portion of current CO_2 concentration in excess is removed naturally at a rate of 0.8% per year. However, in the numerical simulations, we adopt the full characterization of the climate dynamics from the 2007 version of DICE (see http://nordhaus.econ.yale.edu/).

the first-best optimum, together with the system of prices that implement it. Note that we will get the same kind of conditions than the ones of Proposition 1 to characterize the first-best optimum (cf. Proposition 2 below), so that we defer their interpretations to the next section.

3 Implementation of the first-best optimum

The social planner problem consists in choosing $\{C_t, Q_{B,t}, Q_{F,t}, R_{B,t}, R_{E,t}\}_{t=0}^{\infty}$ that maximizes W, as defined by (22), subject to the output allocation constraint (26), the technological constraints (1), (5), (8) and (12), the environmental constraints (28) and (29), and, finally, the stock accumulation constraints (17), (27) and $\dot{Z}_t = F_t$. After eliminating the co-state variables, the first order conditions reduce to the five characteristic conditions of Proposition 2 below, which hold at each time t (we drop time subscripts for notational convenience).

Proposition 2 *At each time t, an optimal solution is characterized by the following five conditions:*

$$\begin{aligned}\left[D(T)Q_E E_F - \frac{1}{F_{Q_F}}\right] U'(C)e^{-\int_0^t \rho ds} + \int_t^\infty \frac{F_Z}{F_{Q_F}} U'(C)e^{-\int_0^s \rho dx} ds & \\ +\xi \int_t^\infty \left[\int_s^\infty D'(T)QU'(C)e^{-\int_0^x \rho dy - m(x-s)}dx\right] \Phi'(G)e^{-\zeta(s-t)}ds &= 0 \qquad (41)\end{aligned}$$

$$D(T_t)Q_E E_B B_{Q_B} = 1 \qquad (42)$$

$$D(T)Q_K - \delta = \rho - \frac{\dot{U}'(C)}{U'(C)} \qquad (43)$$

$$H^B_{H_B} + \frac{H^B_{R_B} B_{H_B}}{B_{Q_B}} - \frac{\dot{H}^B_{R_B}}{H^B_{R_B}} = \rho - \frac{\dot{U}'(C)}{U'(C)} \qquad (44)$$

$$H^E_{H_E} + \frac{H^E_{R_E} E_{H_E}}{E_B B_{Q_B}} - \frac{\dot{H}^E_{R_E}}{H^E_{R_E}} = \rho - \frac{\dot{U}'(C)}{U'(C)}. \qquad (45)$$

Proof. See Appendix A2.

Equation (41) reads as a particular version of the Hotelling rule in this model, which takes into account the carbon accumulation in the atmosphere, the dynamics of temperatures and their effects on output. Equation (42) tells that the marginal productivity of specific input $Q_{B,t}$ equals its marginal cost. The three last equations are Keynes-Ramsey conditions. Equation (43) characterizes the optimal trade-off between capital K_t and consumption C_t, as in more standard growth models. Equation (44) (resp. (45)) characterizes the same kind of optimal trade-off between specific investment into backstop R&D sector, $R_{B,t}$ (resp. energy R&D sector, $R_{E,t}$) and consumption.

Recall that for a given set of public policies, a particular equilibrium is characterized by conditions (30)-(34) of Proposition 1. This equilibrium will be said to be optimal if it satisfies the optimum characterizing conditions (41)-(45) of Proposition 2. By analogy between these two sets of conditions, we can show that there exists a single triplet $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$ that implements the optimum.

First, by comparing conditions (30) and (41), the optimal pollution tax can be identified as:

$$\tau_t^o = -\frac{\xi}{U'(C)} \left\{ \int_t^{\infty} \left[\int_s^{\infty} D'(T)QU'(C)e^{-m(x-s)-\int_t^x \rho dy} dx \right] \Phi'(G)e^{-\zeta(s-t)} ds \right\}. \qquad (46)$$

This expression reads as the ratio between the marginal social cost of climate change – the marginal damage in terms of utility coming from the consumption of an additional unit of fossil resource – and the marginal utility of consumption. In other words, it is the environmental cost of one unit of fossil resource in terms of final good.

Next, the correspondence between the equilibrium characterizing condition (33) (resp. (34)) and the optimum characterizing condition (44) (resp. (45)) is achieved if and only if $\sigma_{i,t}$ is equal to $1-\gamma_i$, $i = \{B, E\}$, i.e. if the two sectors are fully subsidized. The remaining conditions of the two sets are equivalent. These findings are summarized in Proposition 3 below.

Proposition 3 *The equilibrium defined in Proposition 1 is optimal if and only if the triplet of policies* $\{\sigma_{B,t}, \sigma_{E,t}, \tau_t\}_{t=0}^{\infty}$ *is such that* $\sigma_{B,t} = 1 - \gamma_B$, $\sigma_{E,t} = 1 - \gamma_E$ *and* $\tau_t = \tau_t^o$, *for all* $t \geq 0$.

4 Second-best policies

4.1 Methodology

The characteristic conditions of Proposition 1 yield the intertemporal equilibrium profiles of quantities $\{C_t^e, T_t^e, F_t^e, ...\}_0^{\infty}$ and prices $\left\{p_{F,t}^e, p_{B,t}^e, ...\right\}_0^{\infty}$ associated with any profile of policy tools $\{\tau_t, \sigma_{B,t}, \sigma_{B,t}\}_0^{\infty}$. For each equilibrium solution, one can compute the associated welfare value as a function of those public tools: $W\left(\{\tau_t, \sigma_{B,t}, \sigma_{B,t}\}_0^{\infty}\right)$. When W is maximized simultaneously with respect to the three tools, one gets the first-best optimum as described by Proposition 3.

Assume now that the social planner faces some constraints on her choices. For instance, she cannot subsidy research, or she cannot implement the first-best carbon tax. In this case, she only uses the remaining unconstrained tool(s) to maximize the social welfare in the remaining sub-set of equilibria. Among the infinity of possible second-best problems, we focus on the particular cases described in Table 1.

Case	τ_t	σ_E	σ_B	Comment
FB	τ_t^o	σ_E^o	σ_B^o	First-best optimum
LF	0	0	0	Laisser-faire
$SB1$	$\tau_t^{sb_1}$	0	0	Second-best, no R&D subs.
$SB2$	$\tau_t^{sb_2}$	σ_E^o	0	Second-best, no green R&D subs.
$SB3$	$\tau_t^{sb_3}$	0	σ_B^o	Second-best, no energy R&D subs.
$SB4$	0	σ_E^{sb}	σ_B^{sb}	Second-best, no carbon tax

Table 1: Summary of the various cases

In table 1, polar cases "FB" and "LF" refer to the first-best and the laisser-faire, respectively. All the other cases are second-best analysis. "SB1" is the case where neither energy nor backstop R&D can be subsidized and it gives the associated second-best carbon tax τ_t^{sb1}. "SB2" (resp. "SB3") is the case where the green (resp. energy) research cannot be granted, the other subsidy been set at its first-best optimal level; the associated second-best tax is denoted by τ_t^{sb2} (resp. τ_t^{sb3}). The first and second-best carbon taxes are depicted in Figure 2(a). Finally, "SB4" is the case where the fossil resource is not taxed at all. It gives the associated second-best R&D subsidies, given the additional constraints that those subsidies are equal and constant over time. Under these simplifying assumptions, we find $\sigma_i^{sb} = 1.04 \times \sigma_i^o$, $i = \{B, E\}$.

4.2 Main results

As shown in figure 2(a), when the social planner is not able to grant research at all, then she must impose a higher carbon tax than the first-best one: $\tau_t^o < \tau_t^{sb1}$. In order to identify the relevant research sector to explain this result, we must look at "SB2" and "SB3". It appears that only green R&D matters. Then, under an economic policy point of view, an insufficient σ_B can be partially balanced by an increase in τ, but not an insufficient σ_E. To sum up, one gets:

$$\tau^{lf} = \tau^{sb4} = 0 < \tau_t^{o} \approx \tau_t^{sb3} < \tau_t^{sb1} \approx \tau_t^{sb2},\ \forall t \geq 0.$$

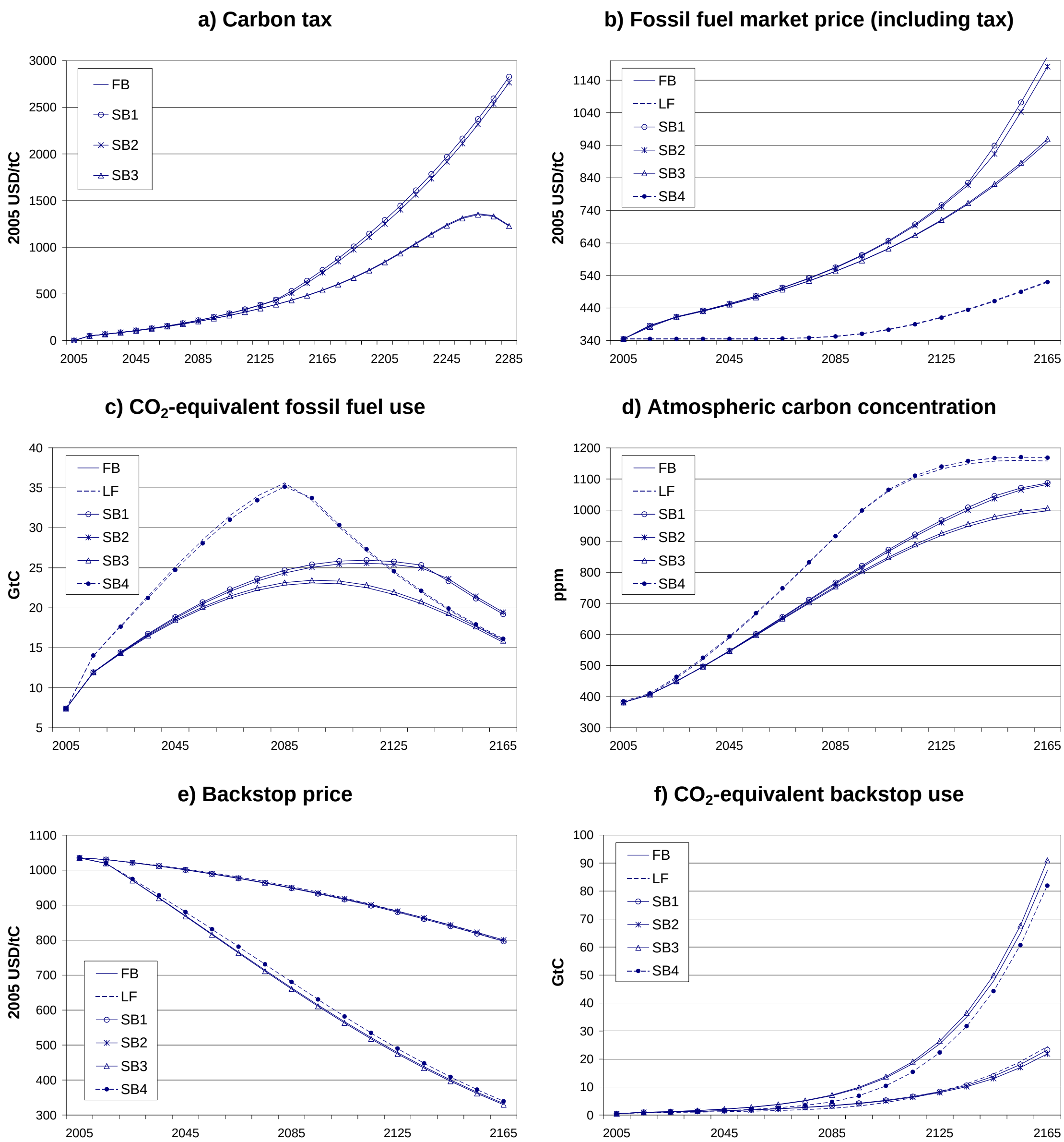

Figure 2: Results in resources and pollution

This ranking of the various taxes is transfered to the fossil fuel market prices, i.e. the selling prices including tax, as shown in Figure 2(b):

$$p_{F,t}^{lf} \approx p_{F,t}^{sb4} < p_{F,t}^{o} \approx p_{F,t}^{sb3} < p_{F,t}^{sb1} \approx p_{F,t}^{sb2},\ \forall t \geq 0.$$

We could expect that this ranking of taxes and fossil prices would lead to a corresponding inverted ranking of the extraction trajectories. However, we can see in Figure 2(c) that this not the case. Indeed, we have (at least until the end of this century):

$$F_t^{lf} \approx F_t^{sb4} > F_t^{sb1} \approx F_t^{sb2} > F_t^{o} \approx F_t^{sb3}, \forall t \geq 0.$$

The first inequality is the expected one: an increase in τ causes F to decrease. On the other hand, more surprising is the second one. As compared to "FB", the carbon tax increases in "SB1" and "SB2", but the fossil fuel extraction flow also increases. The reason is that the green R&D subsidy, σ_B, decreases: the effect of the green research subsidy overrides the carbon tax one. The same results prevail when we look at the carbon accumulation in the atmosphere (Figure 2(d)) and the variations of temperatures (not shown). Those results illustrates the role that green research subsidies can play in climate change mitigation policies.

From Figures 2(e) and 2(f), we observe that the carbon tax has only very weak effect on the backstop price and production, and on the green R&D (not shown). The basic relevant policy tool on these markets is the specific subsidy σ_B.

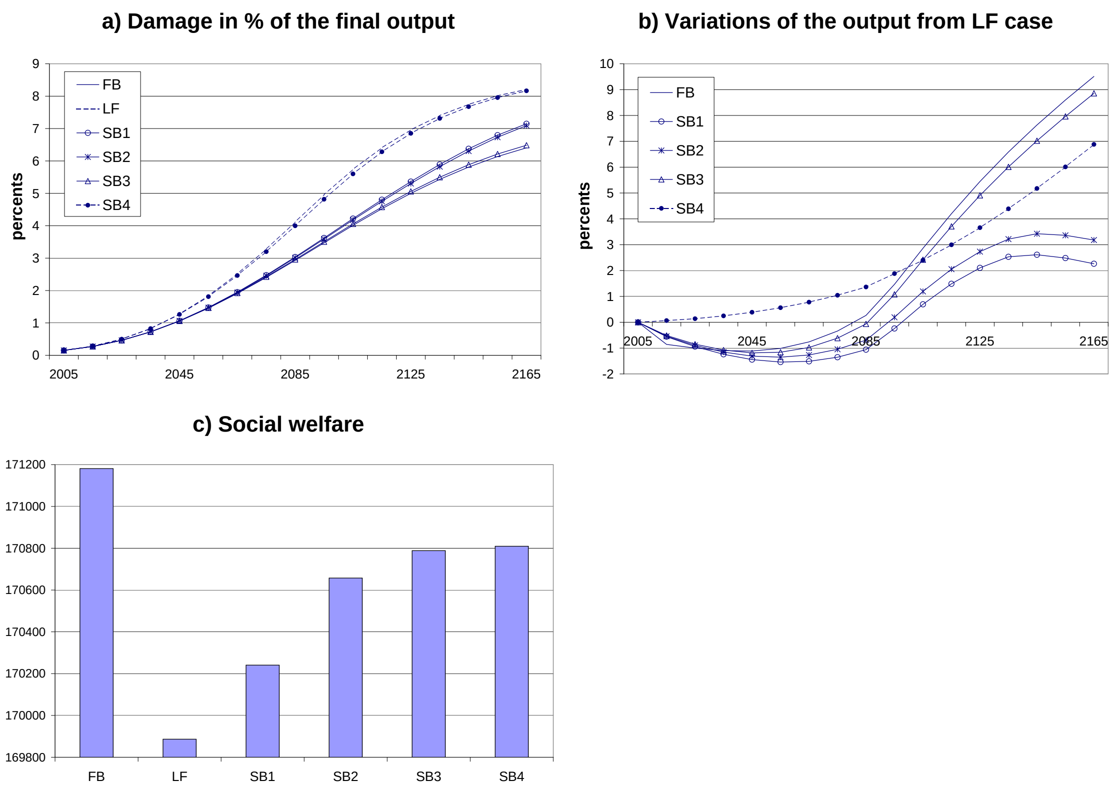

Figure 3: Macroeconomic impacts

Figure 3 focuses on more general macroeconomic effects of the various scenarios. Figure 3 (a) depicts the variations in percents of the final output, formally $(1 - D)/D$. Unsurprisingly, the results directly follow the variations of carbon accumulation and thus of temperatures analyzed above.

In Figure 3(b), we analyze the losses and gains in terms of final output, and thus in terms of instantaneous utility, implied by the various public interventions, as compared with the laisser-faire case. First, whatever the case in which a carbon tax is set up, we can observe a loss for the earlier generations. Second, the larger the carbon tax is, the stronger this loss. Third, one can attenuate the loss caused by the carbon tax and reach earlier the date at which gains will occur again, by increasing simultaneously the green research subsidy. Finally, the intergenerational effort can be smoothed if the planner uses less the tax and more the subsidy. However, in this case, the long run gain reveals to be less important than the one implied by the use of the carbon tax alone.

Last, Figure 3(c) gives some results on the relative impacts of both the carbon tax and the R&D subsidies on the social welfare (i.e. the present value of the flows of instantaneous utility). For instance, the gap between "FB" and "LF" highlights the social cost of doing nothing. Similarly, the gap between "FB" and "SB1" (resp. "SB4") measures the social cost when research sectors are not subsidized at all (resp. when the fossil resource is not taxed). It is then notable to observe that R&D subsidies have a very large impact on the total social welfare, as compared with the carbon tax. Moreover, as shown in Figure 3(b) (cf. "SB4" curve), those subsidies allow to spare the earlier generations who are, on the other hand, strongly penalized by a carbon tax.

5 Conclusion

We have conducted various second-best analysis in a general equilibrium climate change model with endogenous and dedicated R&D. To do that, we have characterized the set of equilibria in the decentralized economy, and we have imposed some institutional constraints on the policy tool(s): i) the impossibility to implement the first-best carbon tax; ii) the impossibility to subsidize one or two R&D sectors. In each case, we have computed the second-best level of the remaining unconstrained tool(s). The second-best results have been compared with, on the upper side, the first-best trajectories and, on the lower side, the laisser-faire ones. Those comparisons have allowed to appreciate the effects of each policy tool on the trajectories of the main following variables: fossil fuel extraction and price, backstop use and price, atmospheric carbon concentration, instantaneous damage, final output. We have also illustrated the assessment of each tool in terms of social welfare gain with respect to the laisser-faire benchmark case.

The main results have highlighted the role of the research grants, in particular the backstop ones. First, in a second-best world, the effect of the green research subsidy on resource extraction, and thus on the flow of pollution, has proved to counter-balance the carbon tax one. Second, R&D subsidies have a very large impact on the total social welfare, as compared with the carbon tax. Third, those subsidies allow to spare the earlier generations who are, on the other hand, strongly penalized by a carbon tax.

References

[1] Acemoglu, D., 2002. Directed technical change. Review of Economic Studies 69(4), 781-809.

[2] Aghion, P., Howitt, P., 1998. Endogenous growth theory. Cambridge, MA: MIT Press.

[3] Bosetti, V., Carraro, C., Graleotti, M., 2006. The dynamics of carbon and energy intensity in a model of endogenous technical change. The Energy Journal, Special issue, 191-206.

[4] Cremer, H., Gahvari, F., Ladoux, N., 2001. Second-best pollution taxes and the structure of preferences. Southern Economic Journal, 68(2), 258-280.

[5] Dietz, S., Hepburn, C., Hope, C., 2008. Discounting and climate change: a non-marginal policy choice. Mimeo, London School of Economics.

[6] Edenhofer, O., Bauer, N., Kriegler, E., 2005. The impact of technological change on climate protection and welfare: Insights from the model MIND. Ecological Economics 54, 277-292.

[7] Edenhofer, O., Lessman, K., Bauer, N., 2006. Mitigation strategies ans costs of climate protection: The effects of ETC in the hybrid model MIND. The Energy Journal, Special issue, 207-222.

[8] Gerlagh, R., Lise, W., 2005. Carbon taxes: A drop in the ocean, or a drop that erodes the stone? The effect of carbon taxes on technological change. Ecological Economics 54, 241-260.

[9] Gerlagh, R., van der Zwaan, B.C.C., 2006. Options and instruments for a deep cut in CO_2 emissions: Carbon capture or renewables, taxes or subsidies? The Energy Journal 27, 25-48.

[10] Goulder, L.H., Mathai, K. 2000. Optimal CO_2 abatement in the presence of induced technological change. Journal of Environmental Economics and Management 39, 1-38.

[11] Grimaud, A., Lafforgue, G., Magné, B., 2007. Decentralized equilibrium analysis in a growth model with directed technical progress and climate change mitigation, mimeo LERNA, Toulouse School of Economics.

[12] Grimaud, A., Rougé, L., 2008. Environment, directed technical change and economic policy. Forthcoming in Environmental and Resource Economics.

[13] IEA, 2007. Key World Energy Statistics 2007. Available at: http://www.iea.org/Textbase/stats/index.asp.

[14] Jones, C.I., Williams, J.C., 1998. Measuring the social return to R&D. The Quarterly Journal of Economics 113(4), 119-1135.

[15] Jones, C.I, Williams, J.C., 2000. Too much of a good thing? The economics of investment in R&D. Journal of Economic Growth 5(1), 65-85.

[16] Kriegler, E., Bruckner, T., 2004. Sensitivity analysis of emissions corridors for the 21^{st} century. Climatic change 66, 345-387.

[17] Nordhaus, W.D., 2006. After Kyoto: Alternative mechanisms to control global warming. American Economic Review 96(2), 31-4.

[18] Nordhaus, W.D., 2007a. A Review of the Stern Review on the Economics of Global Warming. Journal of Economic Literature, forthcoming.

[19] Nordhaus, W.D., 2007b. The challenge of global warming: Economic models and environmental policy. Available at http://nordhaus.econ.yale.edu/.

[20] Popp, D., 2006a. ENTICE-BR: The effects of backstop technology R&D on climate policy models. Energy Economics 28, 188-222.

[21] Popp, D., 2006b. Comparison of climate policies in the ENTICE-BR model. The Energy Journal, Special issue, 163-174.

[22] Popp, D., 2006c. R&D subsidies and climate policy: Is there a "free lunch"? Climatic Change 77, 311-341.

[23] Romer, P.M., 1990. Endogenous technical change. Journal of Political Economy 98(5), 71-102.

[24] Sandmo, A., 1975. Optimal taxation in the presence of externalities. Swedish journal of Economics, 77, 86-98.

[25] Stern, N., 2006. The economics of climate change. The Stern Review. Cambridge University Press.

Appendix

A1. Proof of Proposition 1

The first characterizing condition (30) is obtained by replacing η into (9) by its value $\eta_0 - \int_0^t \left[F_Z/F_{Q_F} \exp\left(-\int_0^s rdu\right)\right] ds$ and by noting that $p_F = p_E E_F - \tau$ from (6), where $p_E = D(T)Q_E$ from (3) and $\exp(-\int_0^t rds) = U'(C)\exp(-\int_0^t \rho ds)$ from (24). Combining (3), (7) and (13) leads to condition (31). Next, using (2) and (24), we directly get condition (32). Finally, the differentiation of (18) with respect to time leads to:

$$\frac{\dot{V}_{H_i}}{V_{H_i}} = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}}, \quad i = \{B, E\}.$$

Substituting this expression into (16) and using (14), (18) and (19), it comes:

$$r = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}} + (\sigma_i + \gamma_i)H^i_{R_i}\left(\bar{v}^i_{H_i} + \frac{H^i_{H_i}}{H^i_{R_i}}\right), \quad \forall i = \{B, E, S\}.$$

We obtain the two last characterizing equilibrium conditions (33) and (34) by replacing into this last equation $\bar{v}^B_{H_B}$ and $\bar{v}^E_{H_E}$ by their expressions.

A2. Proof of Proposition 2

Let H be the discounted value of the Hamiltonian of the optimal program:

$$\begin{aligned} H &= U(C)e^{-\int_0^t \rho ds} + \lambda D(T)Q\left\{K, E\left[F(Q_F, Z), B(Q_B, H_B), H_E\right]\right\} \\ &\quad -\lambda\left(C + Q_F + Q_B + \delta K + \sum_i R_i\right) + \sum_i \nu_i H^i(R_i, H_i) \\ &\quad +\mu_G\left[\xi F(Q_F, Z) - \zeta G\right] + \mu_T[\Phi(G) - mT] + \eta F(Q_F, Z). \end{aligned}$$

The associated first order conditions are:

$$\begin{aligned}
\frac{\partial H}{\partial C} &= U'(C)e^{-\int_0^t \rho ds} - \lambda = 0 && (47)\\
\frac{\partial H}{\partial Q_F} &= \lambda[D(T)Q_E E_F F_{Q_F} - 1] + \xi \mu_G F_{Q_F} + \eta F_{Q_F} = 0 && (48)\\
\frac{\partial H}{\partial Q_B} &= \lambda[D(T)Q_E E_B B_{Q_B} - 1] = 0 && (49)\\
\frac{\partial H}{\partial R_i} &= -\lambda + \nu_i H^i_{R_i} = 0, \quad i = \{B, E\} && (50)\\
\frac{\partial H}{\partial K} &= \lambda[D(T)Q_K - \delta] = -\dot{\lambda} && (51)\\
\frac{\partial H}{\partial H_B} &= \lambda D(T)Q_E E_B B_{H_B} + \nu_B H^B_{H_B} = -\dot{\nu}_B && (52)\\
\frac{\partial H}{\partial H_E} &= \lambda D(T)Q_E E_{H_E} + \nu_E H^E_{H_E} = -\dot{\nu}_E && (53)\\
\frac{\partial H}{\partial G} &= -\zeta \mu_G + \mu_T \Phi'(G) = -\dot{\mu}_G && (54)\\
\frac{\partial H}{\partial T} &= \lambda D'(T)Q - m\mu_T = -\dot{\mu}_T && (55)\\
\frac{\partial H}{\partial Z} &= \lambda D(T)Q_E E_F F_Z + \xi \mu_G F_Z + \eta F_Z = -\dot{\eta} && (56)
\end{aligned}$$

The transversality conditions are:

$$\begin{aligned}
\lim_{t\to\infty} \lambda K &= 0 && (57)\\
\lim_{t\to\infty} \nu_i H_i &= 0, \quad i = \{B, E\} && (58)\\
\lim_{t\to\infty} \mu_G G &= 0 && (59)\\
\lim_{t\to\infty} \mu_T T &= 0 && (60)\\
\lim_{t\to\infty} \eta Z &= 0 && (61)
\end{aligned}$$

First, from (47), (48) and (56), we can write the following differential equation:

$$\dot{\eta} = -\frac{F_Z}{F_{Q_F}} U'(C)e^{-\int_0^t \rho ds}.$$

Integrating this expression and using transversality condition (61), we obtain:

$$\eta = \int_t^\infty \frac{F_Z}{F_{Q_F}} U'(C)e^{-\int_0^s \rho du} ds. \tag{62}$$

From (47) and (55), we have:

$$\dot{\mu}_T = m\mu_T - D'(T)QU'(C)e^{-\int_0^t \rho ds}.$$

Using (60), the solution of such a differential equation can be computed as:

$$\mu_T = \int_t^\infty D'(T)QU'(C)e^{-\left[m(s-t)+\int_0^s \rho dx\right]} ds. \tag{63}$$

Equations (54) and (59) imply:

$$\mu_G = \int_t^{\infty} \mu_T \Phi'(G) e^{-\zeta(s-t)} ds. \tag{64}$$

Replacing into (48) λ, η, μ_T and μ_G by their expressions coming from (47), (62), (63) and (64), respectively, gives us the equation (41) of Proposition 1.

Second, equation (43) directly comes from condition (49). Next, log-differentiating (47) and (50) with respect to time yields:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{U}'(C)}{U'(C)} - \rho \tag{65}$$

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\nu}_i}{\nu_i} + \frac{\dot{H}^i_{R_i}}{H^i_{R_i}}. \tag{66}$$

Combining (65) and (51) yields condition (43). Condition (44) comes from (50), (52), (65) and (66), and from (49) by using $D(T)Q_E E_B = 1/B_{Q_B}$. Similarly, conditions (45) is obtained from the equations (50), (53), (65) and (66).

A3. Calibration of the model

Param.	Value	Description	Source
γ	0.3	Capital elasticity in output prod.	Nordhaus (2007b)
β	0.07029	Energy elasticity in output prod.	Nordhaus (2007b)
α_T	0.0028388	Scaling param. on damage	Nordhaus (2007b)
ρ_B		Elasticity of subs. for backstop	Calibrated
ρ_E	0.38	Elasticity of subs. for energy	Popp (2006a)
α_H	0.336	Scaling param. of H_E on energy	Popp (2006a)
F_0	7.401	2005 fossil fuel use in GtC	IEA (2007)
c_F	345	2005 fossil fuel price in USD	Computed from IEA (2007)
α_F	700	Scaling param. on fossil fuel cost	Popp (2006a)
η_F	4	Exponent in fossil fuel prod.	Popp (2006a)
B_0	0.55	2005 backstop use in GtC	IEA (2007)
α_B	1035	2005 backstop price in USD	Nordhaus (2007b)
η_B		Exponent in backstop prod.	Calibrated
a_B	0.0122	Scaling param. in backstop innovation	Popp (2006a)
a_E	0.0264	Scaling param. in energy innovation	Popp (2006a)
b_B	0.3	Rate of return of backstop R&D	Popp (2006a)
b_E	0.2	Rate of return of energy R&D	Popp (2006a)
Φ_i	0.54	Elasticity of knowledge in innovation	Popp (2006a)
ϵ	2	Elasticity of intertemporal subst.	Nordhaus (2007b)
A_t		Total factor productivity trend	Nordhaus (2007b)
L_t		World population trend	Nordhaus (2007b)
ρ_t		Time preference rate	Nordhaus (2007b)
		Other param. and initial values	Calibrated

Table 2: Calibration of parameters