Carbon Storage and Climate Policy in a Growth Model with Innovation

André Grimaud[1], Bertrand Magné[2] and Luc Rouge[3]

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[1]Toulouse School of Economics (IDEI and LERNA), Manufacture des Tabacs, 21 Allée de Brienne, 31000 Toulouse, France, and Toulouse Business School. E-mail: grimaud@cict.fr

[2]Paul Scherrer Institute (LEA), 5232 Villigen PSI, Switzerland. E-mail: bertrand.magne@psi.ch

[3]Corresponding author. Toulouse Business School, 20 Bd Lascrosses, 31068 Toulouse Cedex 7, France. E-mail: l.rouge@esc–toulouse.fr Tel: +33 5 61 29 48 20 Fax: +33 5 61 29 49 94

Abstract

We study the implications of the carbon capture and storage (CCS) technology availability on the optimal use of polluting exhaustible resources and on optimal climate policies. We develop an endogenous growth model in which the accumulated stock of greenhouse gas emissions harms social welfare. Since CCS technology allows reducing the effective pollution for each unit of resource use, extraction and pollution are partially disconnected. CCS accelerates the optimal extraction pace, though it may foster CO2 emissions for the early generations. Moreover, it is detrimental to output growth. Next, we study the implementation of a unit tax on pollution. Contrary to previous results of the literature, its level here matters, as it provides the right incentives to CCS effort. The optimal growth rate of this carbon tax is positive, though we indicate that this climate policy instrument can be interpreted ex-post as a decreasing ad-valorem tax on the resource.

Keywords: carbon storage, endogenous growth, polluting non-renewable resources.

JEL classification: O32, O41, Q20, Q32

1 Introduction

The exploitation of fossil resources raises two concerns: the first one is scarcity, because fossil resources are exhaustible by nature, the second one is related to greenhouse gases (GHG) emission associated to their combustion.

Numerous models deal with this double issue. Some of them are placed in the context of partial equilibrium (e.g. Sinclair [17, 18]), Withagen [23], Ulph and Ulph [22], Tahvonen [21]) whereas some others tackle this issue in a general equilibrium growth frameworks (Schou [16, 17], Grimaud and Rouge [5, 6], Groth and Schou [7]). Two main questions are addressed: the socially optimal outcome on the one hand, and, on the other hand, its implementation in a decentralized economy along with the impacts of environmental policies. It is generally shown that postponing the resource extraction, and thus the polluting emissions, is optimal. In addition, model recommendations in terms of environmental policy are less unanimous. For instance, Sinclair [18] advocates a decreasing ad valorem tax on resource use, whereas Ulph and Ulph [22], among others, show that such a tax may not always be optimal, especially when the pollution stock partially decays. Considering the sole endogenous growth models with polluting exhaustible resources, with the exception of Schou [16, 17] for whom no environmental policy is required, results generally exhibit a decreasing optimal carbon tax (see Grimaud and Rouge [5, 6] or Groth and Schou [7]). Moreover, as in Sinclair [18], a change of the tax level only has redistributive effects and does not alter the model dynamics, e.g. neither the extraction nor the pollution emission time-paths.

A common feature of those papers lies in the systematic link between resource extraction and pollution emission, in the form of a simple functional relation, generally linear. It is therefore equivalent to tax either the pollution stream or the resource use itself. Nevertheless, the emergence of a technological option such as carbon sequestration, more precisely CO_2 capture and storage (hereafter CCS), in order to tackle climate change, partially breaks this link[1]. Indeed, the possibility to sequester a fraction of the CO2 emission inherent to fossil resource combustion disconnects the resource use from the effective pollution. This article aims at considering the availability of such a technology in the context of an endogenous growth model with a polluting exhaustible resource and at assessing how the main literature results recalled above, namely in terms of optimal policy, are modified in such a framework[2].

We develop an endogenous growth model in which the production of consumption goods requires the input of an extracted resource, whose stock is available in limited quantities. Furthermore, this resource use generates polluting emission, interpreted as GHG emission, whose flow in turn damages the environment, whose quality index is here considered as a stock. Notice that the environment features partial natural regeneration capacity. Finally, the index of environmental quality enters the utility function as an argument and thus allows gauging how the pollution accumulation affects the welfare. But the main novelty of the model lies in the consideration of the CCS availability, which, via some effort, allows for the partial storage of CO_2 release. Then, we distinguish between the total potential CO_2 emission associated to one unit of fossil resource (referred to as total carbon content per unit of resource in the remainder) and the effective emission, i.e. the remaining pollution fraction left after CO_2 removal. The implication in terms of climate change policy is then straightforward: the first best outcome can only be restored by taxing the pollution but not by taxing the resource itself[3].

Our main results can be summarized as follows. The availability of CCS speeds up the

[1] The possibility of capturing and sequestering some fraction of the carbon dioxide arising from fossil fuel combustion has recently caught a lot of attention, reinforced by its recent demonstrated viability (for an overview see IPCC special report [11]).

[2] Numerous studies have addressed the effect of pollution abatement in models with environmental concerns and growth (e.g. Smulders and Gradus [20]). With respect to this literature, one can consider CCS technology to be an important abatement possibility. This question has been addressed in several empirical studies on climate change (e.g. Gerlagh and van der Zwaan [3]) but to our knowledge, it has never been examined in theoretical models with endogenous growth.

[3] Here we assume that the regulator is able to fully measure the greenhouse gases emissions. This may not be systematically the case: While emission data is fairly reliable in industrialized countries, collecting accurate data on industrial activities from developing regions and deducting the emissions may prove more difficult.

optimal pace of resource extraction while relaxing the environmental constraint. Additionally, it modifies the emissions time-path of GHG. In the long term, the pollution level decreases without ambiguity. But, if the preference for environmental quality is not manifest enough, the pollution level may increase in the short term; in this case, the following counter-intuitive result emerges: the introduction of a carbon sequestration technology leads to an increase of CO_2 emissions from the early generations. Lastly, the availability of such a technology reveals detrimental for the output growth because of acceleration in resource extraction combined with a negative effect on R&D effort.

In our framework, as mentioned earlier, a tax on pollution is not equivalent to a tax on resource use anymore; emissions are the ones to be taxed in order to obtain first best results. Besides, contrary to results obtained in a context without CCS, as in Sinclair [18] or Grimaud and Rouge [5, 6] for instance, the tax level here matters and especially allows for setting the optimal CCS effort level. We also show that an increase in this tax leads the economy to postpone the extraction (which falls back in the standard literature discussed above), and also modifies the pollution quantity emitted per unit of resource used.

We finally derive the optimal tax trajectory which exhibits a positive optimal growth rate (that stems from the decreasing marginal utility of consumption). We also show that this tax can be expressed ex-post as a decreasing ad-valorem tax on the resource.

The remainder of the paper is organized as follows. We give some additional elements on carbon capture and sequestration in section 2. We present the model as well as the social optimum in section 3 and we portray the decentralized equilibrium in section 4. In section 5, we compare both market and optimal outcomes. We then analyze the effects of climate change policy and the incentives to carry out R&D. Lastly, we characterize the optimal policies. In section 6, we conduct a numerical illustration to examine the effect of technical change in CCS technology. Conclusive remarks are given in section 7.

2 Background - The carbon capture and sequestration technology

As formulated by Hoffert et al. [9], the decarbonization, i.e. the reduction of the carbon content of each fossil fuel unit, i.e. the amount of carbon emitted per unit of primary energy, is intimately linked to sequestration. Carbon capture, sometimes referred to as emissions control (see Kolstad and Toman [13]), is the way of achieving this decarbonization. This process consists in separating the carbon dioxide from other flux gases during the process of energy production. It is particularly adapted to large-scale centralized power stations but may also indirectly apply to non electric energy supply[4]. Once captured, the gases are then being disposed into various reservoirs. The sequestration reservoirs include depleted oil and gas fields, depleted coal mines, deep saline aquifers, oceans, trees and soils. Those various deposits differ in their respective capacities, their costs of access or their effectiveness in storing the carbon permanently.

Despite the numerous uncertainties still surrounding the sizable deployment of carbon capture technologies, especially with regard to the ecological consequences of massive carbon injection, this technological option has become promising for the fossil energy extractive industry. The estimated cost of carbon capture ranges from 40 to 90USD per ton of CO2 captured and stored (IEA [10]). This would translate into an increase of the electricity cost by 25 to 45%, depending on the technologies. According to IEA forecasts, the use of carbon capture and storage technologies will account for 20 to 28% of the CO2 emission reductions in 2050, i.e. from 6500 to 7500 million tons of CO2 could be avoided, 60% in the sole power sector.Coal use will then be 13% to 32% higher than today's level.

In what follows, since the focus of this paper is the impact of the carbon storage option on optimal and equilibrium paths, as well as the design of climate and R&D policies, we take the

[4] The hydrogen obtained without carbon emission from fossil fuels and CO2 removal devices, could then supply the transportation energy needs owing to fuel cells.

following assumptions. Without loss of generality we do not distinguish the capture phase from the injection one. We may also neglect the carbon sinks per se, and we implicitly assume that their capacity is of infinite size[5]. More importantly, we assume for the sake of simplicity that carbon capture can be applied to any consumed fossil fuel unit whatever its use, i.e. we do not distinguish among the various fossil fuel uses as long as the extraction is dedicated to energy production matters.

3 Model and Optimal Paths

3.1 Disaggregated Model

There is a continuum of consumption goods, indexed on the unit interval. Each good j, $j \in [0;1]$, is produced by N_j firms. Each firm n_j $(n_j = 1, ..., N_j)$ simultaneously produces good j, performs research and stores carbon. For firm n_j, production function of good j is

$$Y_{n_j t} = A_t^{\nu} L_{Y n_j t}^{\alpha} R_{n_j t}^{1-\alpha},\ 0 < \alpha < 1 \text{ and } \nu > 0. \tag{1}$$

A_t is the stock of existing knowledge at time t, $L_{Y n_j t}$ is the amount of labour devoted to consumption goods, and $R_{n_j t}$ is the flow of non-renewable resource.

Technology for production of knowledge is

$$\dot{A}_{n_j t} = \delta L_{A n_j t} A_t,\ \delta > 0, \tag{2}$$

where $L_{A n_j t}$ is the amount of labour devoted to research and $A_{n_j t}$ is the stock of knowledge produced by firm n_j; we have $A_t = \int_0^1 (\sum_{n_j}^{N_j} A_{n_j t}) dj$.

[5] The level of aggregation of our model makes this assumption reasonable: one can consider that deep saline aquifers and ocean carbon sinks are sufficiently large with regard to the ultimate amount of CO2 needed to be sequestered.

Pollution is generated by the use of the non-renewable natural resource within the production process. In case of no carbon storage, pollution flow would be a linear function of resource use: $\gamma R_{n_j t}$, where $\gamma > 0$. In this way, $\gamma R_{n_j t}$ can be seen as the carbon content of resource extraction by firm n_j or, equivalently, as maximum potential pollution by firm n_j. Nevertheless, firm n_j can store part of this carbon so that the actual emitted flow of pollution is

$$P_{n_j t} = \gamma R_{n_j t} - Q_{n_j t}, \tag{3}$$

where $Q_{n_j t}$ is stored carbon. We assume that $Q_{n_j t}$ is produced from two inputs, the pollution content $\gamma R_{n_j t}$ via the amount of extracted resource $R_{n_j t}$ and dedicated labour $L_{Q n_i t}$ according to the following Cobb-Douglas CCS technology:

$$Q_{n_j t} = (\gamma R_{n_j t})^{\eta} L_{Q n_i t}^{1-\eta},\ 0 < \eta < 1, \text{ if } L_{Q n_i t} < \gamma R_{n_j t} \tag{4}$$

and

$$Q_{n_j t} = \gamma R_{n_j t}, \text{ if } L_{Q n_i t} \geq \gamma R_{n_j t}.$$

For any given $\gamma R_{n_j t}$, the total cost of labour, $L_{Q n_i t} = Q_{n_j t}^{1/(1-\eta)} (\gamma R_{n_j t})^{-\eta/(1-\eta)}$, is an increasing and convex function of $Q_{n_j t}$. The marginal and average labour costs, respectively $\partial L_{Q n_i t} / \partial Q_{n_j t} = [1/(1-\eta)]\, Q_{n_j t}^{\eta/(1-\eta)} (\gamma R_{n_j t})^{-\eta/(1-\eta)}$ and $L_{Q n_i t} / Q_{n_j t} = Q_{n_j t}^{\eta/(1-\eta)} (\gamma R_{n_j t})^{-\eta/(1-\eta)}$, are also increasing functions of $Q_{n_j t}$. The Cobb-Douglas form allows simple analytical developments. Let us briefly discuss this CCS technology. Given any quantity of potentially emitted carbon $\gamma R_{n_j t}$, it is the effort in terms of labour only that enables carbon capture. Of course, one could also consider physical capital for instance. However, this would yield further computational complexity as it would add another state variable. Our CCS technology is such that the fraction of stored carbon, $Q_{n_j t} / \gamma R_{n_j t}$, is comprised between 0 and 1. The pollution flow is fully stored as soon as $L_{Qn_it} \geq \gamma R_{n_jt}$.

We denote by $L_{Yt} = \int_0^1 (\sum_{n_j}^{N_j} L_{Yn_jt})dj$, $L_{At} = \int_0^1 (\sum_{n_j}^{N_j} L_{An_jt})dj$ and $L_{Qt} = \int_0^1 (\sum_{n_j}^{N_j} L_{Qn_jt})dj$ the total amount of labour in production, $R\&D$ and carbon storage. Similarly, total extracted resource is $R_t = \int_0^1 (\sum_{n_j}^{N_j} R_{n_jt})dj$, total stored carbon is $Q_t = \int_0^1 (\sum_{n_j}^{N_j} Q_{n_jt})dj$ and total flow of pollution is $P_t = \int_0^1 (\sum_{n_j}^{N_j} P_{n_jt})dj = \gamma R_t - Q_t$.

The non-renewable resource is extracted from an initial finite stock S_0. There are no extraction costs. At each date t, a flow $-\dot{S}_t$ of non renewable resource is extracted,

$$\dot{S}_t = -R_t. \tag{5}$$

The flow of pollution (P_t) affects negatively the stock of environment (E_t). We assume $E_t = E_0 - \int_0^t P_s e^{\theta(s-t)} d_s$, with $E_0 > 0$, and θ is the (supposed constant) positive rate of regeneration. This gives the following law of motion

$$\dot{E}_t = \theta(E_0 - E_t) - P_t. \tag{6}$$

Population is assumed constant, normalized to one, and each individual is endowed with one unit of labour. Thus we have:

$$1 = L_{Yt} + L_{At} + L_{Qt}. \tag{7}$$

The household's instantaneous utility function depends on both consumption c_{jt}, $j \in [0;1]$, and the stock of environment E_t[6]. The intertemporal utility function is:

$$U = \int_0^{+\infty} \left[\ln\left(\int_0^1 c_{jt}^{\varepsilon} dj\right)^{1/\varepsilon} + \omega E_t \right] e^{-\rho t} dt,\ 0 < \varepsilon < 1,\ \rho > 0 \text{ and } \omega \geq 0, \tag{8}$$

[6]It would be equivalent to assume that utility is a decreasing function of the pollution stock $X_t = X_0 + \int_0^t P_s e^{\theta(s-t)} d_s$. From this expression, one gets the law of motion $\dot{X}_t = \theta(X_0 - X_t) + P_t$ and we have the following correspondence: $X_t - X_0 = E_0 - E_t$.

where $c_{jt} = Y_{jt} = \sum_{n_j}^{N_j} Y_{n_j t}$, that is, the whole production of good j is consumed by the representative household. Note that, contrary to Aghion and Howitt [1] for instance, the instantaneous marginal utility of the stock of environment, ω, is constant. In the case of strong damages to the environment, it may be more realistic to consider that this marginal utility is increasing (think of catastrophic events). Nevertheless, this assumption allows for simple computations in a general equilibrium model.

3.2 Welfare

3.2.1 Characterization of optimal paths

Now we characterize the optimum in the symmetric case, in which $N_j = N$, $Y_{n_j} = Y/N$, $R_{n_j} = R/N$, $L_{Yn_j} = L_Y/N$, $L_{An_j} = L_A/N$, $L_{Qn_j} = L_Q/N$ and $Q_{n_j} = Q/N$. The results are given in Appendix 1, where we fully characterize the optimal transition time-paths of the economy. The main results are summarized in the following Proposition 1. We drop time subscripts for notational convenience (upper-script ^{o} stands for optimum and g_X is the rate of growth of any variable X).

Proposition 1 *(i) In the case of strictly positive environmental preference ($\omega > 0$), due to the presence of the environmental stock E, the economy is always in transition and asymptotically converges towards the case where pollution does not matter ($\omega = 0$).*

(ii) The extraction flow, R^o, decreases over time (i.e. $g_R^o < 0$); moreover, strictly positive environmental preference slows down the process. As the optimal flows of sequestration (Q^o) and of pollution (P^o) are proportional to R^o, they also decrease over time.

(iii) Labor in production, L_Y^o, is constant over time. Labor in sequestration, L_Q^o, is proprotional to the flow of extraction, R^o, and thus follows the same dynamics (i.e. $g_{L_Y}^o = g_R^o$). Therefore, labor in research, L_A^o, increases over time and converges to $1 - L_Y^o$ as time goes to infinity.

All optimal levels and growth rates are given in Appendix 1.

Proof. See Appendix 1. ■

3.2.2 General comments

Let us give some comments on formulas (30)-(38) and let us first consider the case where $\omega = 0$, i.e., the environmental quality does not affect the households's utility. Here, the econony immediately jumps to its steady-state. From (30), (31), (32) and (34), we can see that $L^o_{Qt} = 0$, $Q^o_t = 0$ and $L^o_{At} = 1 - \alpha\rho/\delta\nu$: no sequestration is undertaken, and the efforts dedicated to production and $R\&D$ are constant. Moreover, B becomes nil and $g^o_{Rt} = -\rho$ from (37). Since we are in a no-CCS case, $P^o_t = \gamma R^o_t$ (from (35)): this means that the total carbon content of each unit of extracted resource is emitted. Hence, the growth rate of pollution is constant, as the growth rate of extraction.

Finally, one also easily obtains from (38) that the growth rate of output, g^o_{Yt}, is equal to $\nu\delta - \rho$, as in more general endogenous growth models with non-polluting non-renewable resources (see for example Grimaud and Rouge [4]). In addition, it will be shown later that the optimal outcome of this economy when $\omega = 0$ is identical to the decentralized outcome of an economy where no climate policy is implemented but where research is optimally funded.

We now turn to the case where $\omega > 0$. Contrary to the preceding case, the economy is now always in transition. From (33), R^o_t also decreases over time but g^o_{Rt} is now greater than $-\rho$. In other words, when the environmental quality affects the households's utility, the social planner postpones the resource extraction (see Withagen [23] for a similar result in a partial equilibrium context). As L^o_{Qt}, Q^o_t and P^o_t are linear function of R^o_t, they exhibit similar dynamics: they decrease over time and so do their growth rates. Evidently, this also implies that the fraction of captured emissions, i.e. Q^o_t/P^o_t, remains constant over time.

Note that L^o_Y is also constant over time (see (30)). Hence, the remaining flow of labour is split between carbon storage and research. As L^o_{Qt} decreases over time, L^o_{At} increases: as the effort in carbon storage gets lower and lower, $R\&D$ investment is rising.

As t tends to infinity, $g^o_{Rt} = g^o_{L_{Qt}} = g^o_{Q_t} = g^o_{P_t}$ tends to $-\rho$. At the same time, L^o_{Qt} decreases down to 0, L^o_{At} tends to $1-\alpha\rho/\delta\nu$ and g^o_{Yt} tends to $\nu\delta-\rho$. Those asymptotic values are identical to the ones from the steady state obtained above where $\omega = 0$. The resource is asymptotically exhausted and thus the pollution flow tends to zero. That is the reason why, at infinity, the socially optimal time-path converges to the one of an economy where pollution does not matter anymore.

For the sake of illustration, we conduct numerical simulations of the model owing to the following parameters values: $E_0 = 0, S_0 = 250, A_0 = 1, \alpha = 0.667, \rho = 0.025, \gamma = 0.5, \eta = 0.8, \omega = 0.01, \delta = 0.02, \theta = 0.05, \nu = 1.5$. The trajectories of resource extraction, pollution, environmental quality and final good production are depicted in figures 1, 2, 3 and 4 respectively.

3.2.3 Impact of carbon storage on optimal paths

We denote by $X^{o\varnothing}_t$ the optimal level of any variable X_t when no technology of carbon storage is available (which is the case in most standard growth models). We give the optimal levels and growth rates in Appendix 2.

Comparing the social optimum in this case with the optimum presented above leads to the following proposition.

Proposition 2 *Introducing carbon storage alters the optimum results as follows:*

(i) Resource extraction is faster (i.e. $g^o_{Rt} < g^{o\varnothing}_{Rt}$): more resource is extracted in the early stages, and less in the future.

(ii) The short and long run effects on pollution may differ. In the short run, the increase in resource extraction (see (i) above) favors pollution augmentation whereas carbon storage activity leads to the opposite outcome: the overall effect is ambiguous. In the long run, since resource

extraction diminishes (see (i) above) and part of the emissions is stored, the pollution flow decreases without ambiguity.

(iii) Economic growth is lower (i.e. $g^{o}_{Yt} < g^{o\varnothing}_{Yt}$*).*

The speed up of resource extraction ($g^{o}_{Rt} < g^{o\varnothing}_{Rt}$) is depicted in Figure 1. Standard models with non-renewable resources show that the optimal extraction is less fast when pollution is taken into account. Here, we can see that CCS allows to partially relax this environmental constraint. As formulated in the above proposition, the impact of carbon storage on the optimal pollution paths is less obvious. Let us first consider the early generations. Two opposite effects drive the pollution path: an extraction effect and a CCS effect. Since resource extraction increases, pollution tends to increase as well; however CCS activity diminishes pollution emissions. The question is then: which effect dominates? In fact, this depends on the parameters of the model featured in the terms between brackets in formula (35). In particular, one can check that for high values of ω the CCS effect tends to be the strongest. This means that when households value environment a lot, carbon storage is intensive, and thus pollution diminishes despite the increase in resource use. In this case, carbon storage diminishes optimal pollution for the first generations (as illustrated in Figure 2). If ω is low, i.e., households are less sensitive to environmental quality, the extraction effect dominates the CCS one: pollution increases since carbon storage activity is low. We thus have the counter-intuitive case in which carbon storage leads to a simultaneous increase in resource extraction and pollution for the first generations. In the long-term, carbon storage unambiguously induces lower pollution for the future generations. Indeed, we have shown that extraction decreases; thus, whatever the amount of carbon stored, pollution decreases.

Let us now turn to the effect of CCS on optimal growth. First, $L^{o\varnothing}_{Qt}$ and $Q^{o\varnothing}_{t}$ are obviously nil. This implies $L^{o}_{At} < L^{o\varnothing}_{At}$: the amount of labour devoted to $R\&D$ is higher in the "no-storage case" as there is no need to use labour for storage. So there is a first research effect which is detrimental for growth. In addition, the aforementioned extraction effect also holds growth back. In other words, the first two inequalities presented in Proposition 2 immediately yield the following one: $g^o_{Yt} = \nu\delta L^o_{At} + (1-\alpha)g^o_{Rt} < g^{o\varnothing}_{Yt} = \nu\delta L^{o\varnothing}_{At} + (1-\alpha)g^{o\varnothing}_{Rt}$, that is, carbon storage is detrimental for economic growth. The technology of carbon storage allows to relax the environmental constraint. Hence, in an economy with carbon storage technology, early generations extract more resource and consume more at the optimum. In other words, their "sacrifice" is reduced (see Figure 4).

4 Decentralized Economy

Let us now give some words about the decentralized economy, and, in particular, the way we model innovation activities.

In contrast with the standard endogenous growth literature, in our model, new pieces of knowledge are not embodied in intermediate goods. They are directly used by firms and protected by infinitely-lived patents (that is, directly priced). As knowledge is a public good, there are two main difficulties for funding it. First, it is difficult to extract the whole willingness to pay of agents that use knowledge (see for instance Popp [15]); for Jones and Williams [12], investments in R&D in the US are at least two to four times lower than their optimal level. We therefore introduce one exogenous parameter ψ which represents the gap between the willingness to pay and the price perceived by sellers of innovations in the research sector (this parameter will be interpreted as a subsidy to R&D later in the text). A second difficulty arises from the non-convexity of technologies of firms using knowledge as a productive factor. In a perfectly competitive environment, profits for these firms would be negative and a general competitive equilibrium would not exist. We therefore assume an imperfect competition (*à la Cournot*) in markets for consumption goods. By selling these goods at a price which is higher than the marginal cost, firms get resources that allow them to buy knowledge.

Thus, this model features three basic distortions with respect to the optimum. First, the flow of pollution, P_t, which damages the stock of environment; second, the distortion on innovations markets mentionned above; finally, the Cournot competition in the markets for consumption goods. This latter distortion will be shown not to prevent equilibrium variables from being optimal. Hence we introduce two economic tools: a tax on pollution, and a subsidy to research.

Note that our climate policy consists in a tax on pollution, and not on the polluting resource, as in Grimaud and Rouge [5, 6] or Groth and Schou [7]. Indeed, the basic externality is carbon emissions and, as technology for carbon storage is available, a tax on these emissions and a tax on the polluting resource are no more equivalent (contrary to what happens in the papers mentioned above).

As will be shown below, this tax on carbon emissions has two main effects: it leads to postponing extraction (as in the models without carbon storage possibility). It also yields incentives to produce optimal efforts in carbon storage at each time t.

4.1 Agents' behaviour

Wage is normalized to one: $w_t = 1$, and p_{jt} ($j \in [0;1]$), p_{Rt}, r_t and V_t are, respectively, the price of consumption good j, the price of the non-renewable resource, the interest rate on a perfect financial market and the unit price of knowledge (in terms of labour). We drop time subscripts for notational convenience.

Household

The representative household maximizes (8) subject to her budget constraint $\dot{b} = rb + w + \pi - \int_0^1 p_j c_j dj + T$, where b is her total wealth, π represents total profits in the economy and T is a lump-sum subsidy (or tax). Recall that we normalized w to 1. One gets the two following standard results. Total demand for good j is

$$c_j = p_j^{1/(\varepsilon-1)}\Omega, \tag{9}$$

where $\Omega = (\int_0^1 p_k c_k dk)/(\int_0^1 p_k^{\varepsilon/(\varepsilon-1)} dk)$, and Ramsey-Keynes condition is

$$r = \rho + (1-\varepsilon) g_{c_j} + g_\Gamma + g_{p_j}, \text{ with } j \in [0;1], \tag{10}$$

where $\Gamma = \int_0^1 c_j^\varepsilon dj$.

Non-renewable resource sector

On the competitive natural resource market, the maximization of the profit function $\int_t^{+\infty} p_{Rs} R_s e^{-\int_t^s r_u du} ds$, subject to $\dot{S}_s = -R_s$, $S_s \geq 0$, $R_s \geq 0$, $s \geq t$, yields the standard equilibrium "Hotelling rule":

$$\frac{\dot{p}_R}{p_R} = r. \tag{11}$$

As usual, the transversality condition is $\lim_{t \to +\infty} S_t = 0$.

Firms

Recall that firms have three activities. First, each one produces and sells a differentiated good on an imperfect market. Second, it produces and sells innovations which we assume traded using bilateral contracts between inventors and users. Thirdly, firm stores part of emitted carbon.

V_t is the price of one innovation at date t in the research sector. Let us denote by $\tilde{\pi}_{n_j t}$ the profit of firm n_j without payment of knowledge. At each moment, firm n_j maximizes $\tilde{\pi}_{n_j t} = p_{jt} Y_{n_j t} - L_{Y n_j t} - L_{Q n_j t} - p_{Rt} R_{n_j t} - \tau_t \gamma \left[R_{n_j t} - \gamma^{\eta-1} R_{n_j t}^{\eta} L_{Q n_j t}^{1-\eta} \right] + V_t \dot{A}_{n_j t} - L_{A n_j t}$, subject to (1), (2), and (9), where τ_t is a unit tax on pollution (note that it also corresponds to the price of permits on a competitive market in the case of tradeable permits). After substitutions, one gets the following program:

$$\max \tilde{\pi}_{n_j} = Y_{n_j}[\Omega^{1-\varepsilon}(\sum_{n_k=1}^{N_j} Y_{n_k})^{\varepsilon-1}]L_{Yn_j} - L_{Qn_j} - p_R R_{n_j} - \tau\gamma\left[R_{n_j} - \gamma^{\eta-1}R_{n_j}^{\eta}L_{Qn_j}^{1-\eta}\right]$$
$$+V\delta A L_{An_j} - L_{An_j}$$
$$\text{subject to } Y_{n_j} = A^{\nu}L_{Yn_j}^{\alpha}R_{n_j}^{1-\alpha},$$

The first order conditions with respect to Y_{n_j}, R_{n_j}, L_{Yn_j}, L_{Qn_j}, and L_{An_j} are respectively (λ is the Lagrange multiplier):

$$\Omega^{1-\varepsilon}(\sum_{n_k=1}^{N_j} Y_{n_k})^{\varepsilon-1} + (\varepsilon-1)Y_{n_j}\Omega^{1-\varepsilon}(\sum_{n_k=1}^{N_j} Y_{n_k})^{\varepsilon-2-}\lambda = 0. \tag{12}$$

This equation implicitely yields the best response of firm n_j to the choice of production of the other firms on the market of consumption good j.

$$-p_R + \lambda\frac{(1-\alpha)Y_{n_j}}{R_{n_j}} - \tau\gamma\left[1 - \eta\gamma^{\eta-1}R_{n_j}^{\eta-1}L_{Qn_j}^{1-\eta}\right] = 0, \tag{13}$$

$$-1 + \lambda\frac{\alpha Y_{n_j}}{L_{Yn_j}} = 0, \tag{14}$$

$$-1 + \tau\gamma^{\eta}(1-\eta)R_{n_j}^{\eta}L_{Qn_j}^{-\eta} = 0, \tag{15}$$

$$V\delta A - 1 = 0. \tag{16}$$

The willingnesses to pay for pieces of knowledges A at each date t is

$$v_{n_j} = \partial\tilde{\pi}_{n_j}/\partial A = V\delta L_{An_j} + \lambda\nu Y_{n_j}/A. \tag{17}$$

Each piece of knowledge being simultaneously used by research and production activities, $V\delta L_{An_j}$ is the willingness to pay relative to research activity and $\lambda\nu Y_{n_j}/A$ is the willingness to pay relative to production -the public good nature of knowledge inside the firm is here manifest.

4.2 Equilibrium

Here, an equilibrium is a set of profiles of quantities and prices such that: the representative household maximizes utiliy and firms maximize profits; labour, resource and financial markets are perfectly competitive; on each consumption good market, there is Cournot competition; pieces of knowledge are traded using bilateral contracts. We focus on a symmetric equilibrium.

Let us first express both the social value and the market value of one unit of knowledge. In (17), λ can be replaced by $L_{Yn_j}/\alpha Y_{n_j}$ (see formula (14)). Summing on n_j and j, one gets $v = V\delta L_A + \nu L_Y/\alpha A$: this corresponds to the instantaneous *social* value of one piece of knowledge.

From now on, we assume that firms are unable to extract the whole willingnesses to pay for knowledge due to information and excludability problems. In fact, they only extract a fraction $\bar{v}$. The extracted value for one unit of knowledge is: $\bar{v} = V\delta L_A + \psi\nu L_Y/\alpha A$, where $\psi \in [0;1]$. This is the instantaneous *market* value of one piece of knowledge. This formulation allows for simple computations and can be interpreted as follows: innovators are able to fully observe the social value of innovations in the research activity, but not in the production activity. Furthermore, in the following, we will interpret an increase in ψ as an economic policy aiming at fostering research. Finally, unit price paid for knowledge is $V_t = \int_t^{+\infty} \bar{v}_s e^{-\int_t^s r_u du} ds$. Differentiating with respect to time, one gets the standard following formula:

$$r_t = \frac{\dot{V}_t}{V_t} + \frac{\bar{v}_t}{V_t}, \tag{18}$$

which states that the rate of return is the same in both the financial market and the research sector.

We now turn to the derivation of the consumption goods price. Since we are in the symmetric case (in particular we have $Y_{n_j} = Y_j/N = Y/N$ and $p_j = p$), equations (12) and (14) lead to

$$p\left[1 + (\varepsilon - 1)/N\right] = L_Y/\alpha Y \tag{19}$$

where $L_Y/\alpha Y$ is the marginal cost. Since $\varepsilon < 1$, this equation means that the price of any consumption good is higher than its marginal cost. The discrepancy between price and marginal cost allows firms to buy knowledge despite the non-convexity of technology. Observe that, if $N = 1$ (monopolistic case), (19) becomes $p =$ (marginal cost)$/\varepsilon$, which is the standard result.

The main findings concerning the equilibrium are summarized in the following Proposition. We drop time subscripts for notational convenience (upper-script ^{e} stands for equilibrium).

Proposition 3 *At the equilibrium in the decentralized economy with a strictly positive carbon tax (i.e. $\tau > 0$) at each date:*

(i) The economy is always in transition.

(ii) The flow of resource extraction, R^e, as well as the flows of sequestration, Q^e, and of pollution, P^e, decrease over time.

(iii) Labour in final good production, L_Y^e, is constant over time. Labour devoted to storage activities, L_Q^e, is proportional to the flow of resource extraction, R^e, and thus follows the same dynamics: $g_{L_Qt}^e = g_{Rt}^e < 0$. Therefore, labour devoted to research, L_A^e, increases over time and converges to the constant level $1 - L_Y^e$ as time goes to infinity.

All equilibrum levels and growth rates are given in Appendix 3.

Proof. See Appendix 3. ■

Let us now consider that there is no climate policy (i.e. $\tau = 0$ at each date). Here, the economy immediatly jumps to its steady-state, where the amount of labour devoted to carbon storage is nil (see formula (51)): $L^e_Q = 0$, which means that no carbon is stored ($Q^e = 0$). This, in turn, implies that the total potential emission is released in the atmosphere, i.e. $P^e = \gamma R^e$. Moreover, labor used in the production of the final good, L^e_Y, is constant, and thus labor devoted to the research sector, $L^e_A = 1 - L^e_Y$ is also constant. The flow of extraction at date t is $R^e_t = \rho S_0 e^{-\rho t}$, and $p_{R0} = (1-\alpha)/\psi\nu\delta S_0$. This implies $g^e_R = -\rho$ for all t when no tax on pollution is levied. This latter case corresponds to the optimum without pollution (and no carbon storage).

We now compare the equilibrium growth rate of resource extraction (g^e_R) in the absence of climate policy to its optimal level. Combining the previous results with those given in Proposition 1, we obtain the following inequalities:

$$g^e_R = -\rho < g^o_{Rt} < g^{o\varnothing}_{Rt}.$$

Recall that $g^{o\varnothing}_{Rt}$ is the optimal growth rate of extraction in the case of no available technology for carbon storage (defined in section 3.2.3). First, $g^e_R < g^{o\varnothing}_{Rt}$ means that, in an economy in which no technology for carbon storage is available, resource extraction in the laissez-faire economy is too fast, compared to the optimal path. For a similar result in a partial equilibrium context, see Withagen [23]. Nevertheless, introducing carbon storage into the analysis leads to two complementary results. The inequality $g^e_R = -\rho < g^o_{Rt}$ is an extension of the previous result: even if carbon storage is possible, it is optimal to postpone extraction, relative to what is done in the decentralized laissez-faire equilibrium. However, the inequality $g^o_{Rt} < g^{o\varnothing}_{Rt}$ states that in the case of carbon storage, the optimal extraction paths is less restrictive than in the absence of such technology. In other words, carbon storage partially relaxes the environmental constraint. As we stated earlier, the sacrifice of earlier generations is reduced.

Figures 1, 2, 3, 4, obtained through numerical simulations (see section 3.2.2), illustrate the preceding results.

5 Impact of Economic Policies

5.1 Impact of climate and $R\&D$ policies

Let us first study the impacts of climate policy (a carbon tax on carbon emissions P^e) on the equilibrium paths of this economy.

Proposition 4 *An increase in the carbon tax τ has the following effects:*

(i) Resource extraction and carbon emissions decrease at a lower pace, and so does the effort in CCS, as well as sequestration activity itself (i.e.: g_R^e, g_P^e, $g_{L_Q}^e$ and g_Q^e increase).

(ii) The intensity of effort in CCS (L_{Qt}^e/Q_t^e), the effort by unit of carbon content ($L_{Qt}^e/\gamma R_t^e$), as well as the instantaneous rate of carbon storage ($Q_t^e/\gamma R_t^e$), all increase.

(iii) Effective pollution by unit of carbon content ($P_t^e/\gamma R_t^e$) decreases.

(iv) The effort in production (L_Y^e) remains unchanged.

Assume $0 \leq \tau \leq 1/(1-\eta)$. An increase in tax τ has two basic effects: first, pollution gets more costly, which leads the economy to postpone extraction (g_{Rt}^e increases). A second effect is that carbon storage becomes more profitable; hence the amount of labour by unit of carbon content ($L_{Qt}^e/\gamma R_t^e$) increases. Therefore, $Q_t^e/\gamma R_t^e$, that is, the instantaneous rate of carbon storage, also increases. Simultaneously, effective pollution by unit of carbon content ($P_t^e/\gamma R_t^e$) decreases. As carbon sequestration gets more profitable, the intensity of labour in this activity (L_{Qt}^e/Q_t^e) increases.

Let us now give some elements on the-short term effects of this climate policy on output's level and growth. First, as g_{Rt}^e increases, early generations extract less resource; since labour devoted to output is unchanged, output level diminishes for these generations. Second, since $g_{L_{Qt}}^e$ increases, L_Q^e, the effort in carbon storage, decreases in the short-run[7] (see Figure 5). Then, as L_Y^e is unchanged, L_A^e and thus g_A^e increase. Finally, output growth ($g_Y^e = \nu g_A^e + (1-\alpha)g_R^e$)

[7] Using (51) and (53), one can show that $\partial L_{Qt}^e/\partial t \leq 0$ if t is low enough.

is fostered for early generations.

Now we analyse the effects of the *R&D* policy. The impact of an increase in the subsidy to *R&D*, ψ, on the price of an innovation is illustrated in 6.

Proposition 5 *An increase in the subsidy to R&D, ψ, has the following effects:*

(i) The effort in production (L_Y^e) diminishes.

(ii) Resource extraction and carbon emissions decrease at a lower pace, as well as the effort in CCS and sequestration activity itself (i.e.: g_R^e, g_P^e, $g_{L_Q}^e$ and g_Q^e increase).

(iii) The intensity of effort in CCS (L_Q^e/Q^e), the effort by unit of carbon content ($L_Q^e/\gamma R^e$), the instantaneous rate of carbon storage ($Q^e/\gamma R^e$), and effective pollution by unit of carbon content ($P^e/\gamma R^e$) remain unchanged.

The impact of such a policy on output is similar to what is obtained in standard endogenous growth models. Since g_{Rt}^e increases, R^e decreases in the short-run. As L_Y^e decreases (less efforts in output production), early levels of output Y^e diminish. Moreover, since $g_{L_{Qt}}^e$ increases, L_Q^e decreases in the short-run. Thus we have a simultaneous decrease in L_Y^e and L_Q^e, which yields an increase in L_A^e (recall that total amount of labour is constant). For that reason, the accumulation of knowledge is faster in the early generations: g_A^e increases. As we also have an increase in g_{Rt}^e, output growth is unambiguously fostered in the short run.

5.2 Optimal policy

Comparing values in propositions 1 and 2, we obtain the following result which gives the design of optimal policy instruments.

Proposition 6 $\psi^o = 1$ *(optimal financing of research) and* $\tau^o = \frac{\rho\omega}{\delta\nu(\rho+\theta)}$ *are the levels of* ψ *and* τ *for which the equilibrium path is optimal.*

First, note that this optimal tax level is expressed in terms of labour. Dividing this level by price p (given in (19)), in which L_Y is at its equilibrium level (see (50)), and $\psi = 1$, we obtain:

$$\tau^o / p_t = \frac{\omega \left[1 + (\varepsilon - 1)/N\right]}{\rho + \theta} Y_t. \tag{20}$$

This corresponds to the optimal tax level in terms of consumption good. This tax grows at the same rate as output, as depicted in Figure 7 (left panel).

As we commented earlier, the tax level here matters, contrary to standard results of the literature (see Sinclair [18], Grimaud and Rouge [5, 6], Groth and Schou [7] for instance). Indeed, when CCS technology is available, the social planner has to give the right signal in terms of social costs of pollution to firms, so as to induce their optimal effort in sequestration. We now elaborate on this issue.

First, let us show that the optimal tax level (that we will refer to as the optimal price of carbon) is equal to the sum of discounted marginal social costs for all (present and future) generations, expressed in terms of good Y. For each generation, ω is the social cost of one unit of carbon, given in terms of utility. Thus, at date t, $\int_t^{+\infty} \omega e^{-(\rho+\theta)(s-t)} ds = \omega/(\rho + \theta)$ is the sum for all generations of this cost's present values (taking regeneration into account). Moreover, the marginal utility of any good j is $1/c_j$ (see formula (8)). Taking into account the mark-up on each consumption good's market due to Cournot competition, and the fact that we consider a symmetric equilibrium, we obtain the expression given in (20). Observe that, when it is expressed in terms of utility, this optimal tax is constant over time. However, it is an increasing function of time when given in terms of consumption good. Indeed, economic growth being positive, the marginal utility of consumption decreases over time.

Second, this optimal tax leads to the optimal arbitrage between production and pollution, given the availability of carbon storage technology. Let us assume a labour transfer from the carbon storage sector towards production, resulting in the emission of one additional unit of pollution. The optimal tax corresponds to the subsequent marginal increase in good Y.

Proof. Taking mark-ups into account as we did above, one gets:

$\tau^o/p_t = [(\partial Y_t/\partial L_{Yt})/(\partial Q_t/\partial L_{Qt})]\,[1+(\varepsilon-1)/N]$.

This comes from the fact that $\partial Y_t/\partial L_{Yt} = \alpha Y_t/L_Y = 1/p\,[1+(\varepsilon-1)/N]$ (from (1) and (19)) and $\partial Q_t/\partial L_{Qt} = (1-\eta)Q_t/L_{Qt} = 1/\tau$ (see formulas (4), (51) and (54)). ∎

Finally, the optimal tax on pollution, which in particular leads to postponing resource extraction, can be interpreted as a decreasing ad valorem tax on the resource (see Figure 7 (right panel)). This allows to make a link with standard literature in the case of no carbon storage (see Sinclair [18], Grimaud and Rouge [5, 6] or Groth and Schou [7]). When the optimal tax is implemented, the "total" (i.e., including the price of the resource and the carbon tax) unit price paid by users for the resource increases less fast than the unit price perceived by owners of the resource (whose growth rate is the interest rate). That is why extraction is postponed. Ex-post, this has the same effect as a decreasing ad valorem tax.

Proof. "Total" price paid by firms is:

$p_R R + \tau^o\gamma(R - \gamma^{\eta-1}R^\eta L_Q^{1-\eta}) = p_R R\left[1+(\tau^o/p_R)(1-(L_Q/\gamma R)^{1-\eta})\right].$

Using (51) and $\tau^o = \rho\omega/\delta\nu(\rho+\theta)$ (see proposition 5), this price is given by

$$p_R R\left[1+\left(1-\left(\frac{\omega\rho(1-\eta)}{\delta\nu(\rho+\theta)}\right)^{(1-\eta)/\eta}\right)\frac{\omega\gamma}{(\rho+\theta)p_R}\right],$$

that is, $p_R R(1+\sigma)$ where σ can be interpreted as an ad valorem tax on the resource, which is decreasing since p_R is an increasing function of time (recall that $g_{p_R} = r$). ∎

6 The effect of technical progress in CCS: A numerical illustration

The model solutions have been obtained so far for a CCS technology without technical progress. However, progressive improvements in this technology are likely to occur due to learning in the early stages of its development (IPCC [11]). In order to grasp insights on how specific technical progress to sequestration technology would alter our results, we introduce some exogenous trend in the productivity of pollution mitigation activities. In this section, we restrict ourselves to the analysis of the optimal paths that we illustrate owing to a sole numerical simulation. We thus modify the carbon storage technology given by (4) using the following functional specification that corresponds to labour-augmenting technical progress:

$$\begin{aligned} Q_{n_jt} &= (\gamma R_{n_jt})^{\eta}(\kappa_t.L_{Qn_it})^{1-\eta},\ 0<\eta<1,\ \text{if } L_{Qn_it}<\gamma R_{n_jt} \\ \text{with } \kappa_t &= \kappa_0-(\kappa_0-1)*e^{-\overline{\kappa}.t} \end{aligned}$$

The numerical values for κ_0 and $\overline{\kappa}$ are chosen to be 20 and 0.02, so that the function is strictly increasing and concave. Technical progress is quickly enhancing the carbon storage productivity in the early periods, due to the strong concavity of our specification. Thus the amount of labour dedicated to carbon removal, as well as the effective amount of carbon sequestered (depicted in Figure 8, Left panel), is increasing as compared to the no technical progress case developed in section 3. The cumulative amount of carbon that is ultimately sequestered, corresponding to the surface below the time-path of Q_t^o, is increasing significantly when dedicated technical progress in introduced. As a result, the environmental quality E_t^o is degrading less rapidly and is returning to its initial state in shorter time owing to natural regeneration (see Figure 8, Right panel).

It is beyond the scope of the current section to test the robustness of our results to the specification for technical progress. Still, the qualitative effects of technical progress shall be preserved with alternative specifications: Technical progress makes this climate change mitigation option more effective by reinforcing the capture of CO_2 emissions throughout the entire horizon. It alleviates the burden of pollution emission on environmental quality while hindering economic growth to a lower extent.

7 Conclusion

We proposed a model of endogenous growth (à la Romer) in which output is produced from knowledge, labour and a polluting non-renewable resource. The aim of the paper was to study how previous results of the literature on growth and polluting non-renewable resources are modified when carbon storage technology is available. Here, part of the carbon flow that is emitted when the resource is used within the production process can be stored. This implies that, contrary to standard literature, pollution is dissociated from resource extraction. The remaining flow of carbon damages the state of the environment, which is harmful for household's utility.

We fully characterized the optimal trajectories. We showed how the CCS option speeds up the optimal resource extraction and thus helps to partially relax the environmental constraint, which reduces the sacrifice of early generations. Moreover, the path of GHG emissions is modified. In the long-run, emissions unambiguously decrease, but we proved that pollution may increase for the early generations if environmental preferences are low. Finally, we showed that the availability of CCS technology is detrimental for growth.

Then we studied the impact of climate and R&D policies on the main relevant variables in the decentralized economy. The climate policy consists in a tax on pollution (which is not equivalent to a tax on resource, which would only yield second best outcome, contrary to models without carbon storage). First, we showed that the level of the tax matters, as it provides the right incentives for an optimal effort in storage activity. In addition, the optimal carbon tax is proved to be equal to the sum of discounted marginal social costs for all (present and future) generations (taking regeneration into account). Second, the optimal carbon tax is an increasing function of time and leads to postponing extraction. Moreover, it can be interpreted (ex-post) as a decreasing ad valorem tax on the resource: climate policy reduces the growth rate of the "total" resource price (i.e., the resource price including carbon tax). Finally, we briefly studied the case of dedicated (exogenous) technical progress in carbon storage through a numerical simulation.

The decarbonization of the economy and the switch to renewable or non fossil fuel-based energy remains necessary (Gerlagh [2]). In order to keep the model tractable, the availability of a clean and renewable energy source has not been introduced. This so-called backstop would not drastically alter the qualitative properties of our results. Nevertheless, it would be interesting to study the impact of the CCS option on the adoption timing of these alternative sources of energy. We can infer that the possibility to store the carbon underground would delay the introduction of renewable energy. Indeed, the availability and use of carbon sequestration technologies may notably encourage a shift of electricity generation from natural gas to coal-based power plants thus favoring a coal renaissance (Newell et al. [14]) over the next decades, while decreasing reliance on renewable energy sources.

Appendix

Appendix 1: Welfare

Let us consider the symmetric case in which $N_j = N$, $Y_{n_j} = Y/N$, $R_{n_j} = R/N$, $L_{Yn_j} = L_Y/N$, $L_{An_j} = L_A/N$, $L_{Qn_j} = L_Q/N$ and $Q_{n_j} = Q/N$. Then functions (1), (2), (3) and (4) become $Y = A^{\nu} L_Y^{\alpha} R^{1-\alpha}$, $\dot{A} = \delta L_A A$, $P = \gamma R - Q$ and $Q = (\gamma R)^{\eta} L_Q^{1-\eta}$. Utility is now $U = \int_0^{+\infty} (\ln c_t + \omega E_t) e^{-\rho t} dt$. The social planner maximizes U subject to the modified versions of (1), (2), (3), (4), (5), (6) and (7). The current value Hamiltonian of the program is

$$H = \nu \ln A + \alpha \ln(1 - L_A - L_Q) + (1-\alpha) \ln R + \omega E + \mu \delta L_A A - \varphi R + \zeta \left[\theta(E_0 - E) - \gamma(R - \gamma^{\eta-1} R^{\eta} L_Q^{1-\eta}) \right],$$

where μ, φ and ζ are the co-state variables. The first order conditions $\partial H/\partial L_A = 0$, $\partial H/\partial R = 0$ and $\partial H/\partial L_Q = 0$ yield

$$-\alpha/(1 - L_A - L_Q) + \mu \delta A = 0, \tag{21}$$

$$(1-\alpha)/R - \varphi - \zeta\gamma(1 - \eta\gamma^{\eta-1} R^{\eta-1} L_Q^{1-\eta}) = 0, \tag{22}$$

$$\text{and } -\alpha/(1 - L_A - L_Q) + \zeta\gamma^{\eta}(1-\eta) R^{\eta} L_Q^{-\eta} = 0. \tag{23}$$

Moreover, $\partial H/\partial A = \rho\mu - \dot{\mu}$, $\partial H/\partial S = \rho\varphi - \dot{\varphi}$,and $\partial H/\partial E = \rho\zeta - \dot{\zeta}$ yield

$$\rho\mu - \dot{\mu} = \nu/A + \mu\delta L_A, \tag{24}$$

$$\rho\varphi - \dot{\varphi} = 0, \tag{25}$$

$$\text{and } \rho\zeta - \dot{\zeta} = \omega - \zeta\theta. \tag{26}$$

i) Computation of L_Y.

Log-differentiating (21) with respect to time and using (24), one gets the following Ricatti differential equation: $\dot{L}_Y = (\delta\nu/\alpha)L_Y^2 - \rho L_Y$, whose solution is $L_Y = \alpha\rho/\left[\delta\nu + (\alpha\rho/L_{Y0} - \delta\nu)e^{\rho t}\right]$. Using transversality condition $\lim\limits_{t \longrightarrow +\infty} \mu A e^{-\rho t} = 0$, we show that L_Y immediately jumps to its steady-state level:

$$L_Y = \alpha\rho/\delta\nu. \tag{27}$$

Indeed, using (21) it turns out that the transversality condition is only satisfied when $L_Y = L_{Y0} = \alpha\rho/\delta\nu$.

ii) Computation of ζ.

The solution for equation (26) is $\zeta = e^{(\rho+\theta)t}\left[(\omega/(\rho+\theta)(e^{-(\rho+\theta)t} - 1) + \zeta_0\right]$. Moreover, the transversality condition associated to E writes

$\lim\limits_{t \longrightarrow +\infty} \zeta E e^{-\rho t} = \lim\limits_{t \longrightarrow +\infty} e^{\theta t}\left[\omega/(\rho+\theta)(e^{-(\rho+\theta)t} - 1) + \zeta_0\right]\left[E_0 e^{\theta t} - \int_0^t P_s e^{\theta s} ds\right] = 0.$

Normalizing E_0 such that the second term between brackets is not nil, we obtain

$$\zeta = \zeta_0 = \omega/(\rho+\theta). \tag{28}$$

iii) Computation of L_Q.

Replacing the value of L_Y in (27) in (23), we get

$$L_Q = \left[\frac{\rho\omega(1-\eta)}{\delta\nu(\rho+\theta)}\right]^{1/\eta}\gamma R. \tag{29}$$

iv) Computation of R.

Using (22), (28) and (29) we obtain $R = \frac{1-\alpha}{\varphi_0 e^{\rho t} + B}$ in which $B = \frac{\omega\gamma}{\rho+\theta}\left[1 - \eta\left(\frac{\rho\omega(1-\eta)}{\delta\nu(\rho+\theta)}\right)^{(1-\eta)/\eta}\right]$.

Using the constraint $\int_0^{+\infty} R_t dt = S_0$, after some calculations we obtain $\varphi_0 = B/(e^{\frac{B\rho S_0}{1-\alpha}} - 1)$.

v) Computation of Q and P.

Plugging (29) into $Q = (\gamma R)^\eta L_Q^{1-\eta}$, one gets $Q = \left(\frac{\rho\omega(1-\eta)}{\delta\nu(\rho+\theta)}\right)^{(1-\eta)/\eta}\gamma R$.

Then, using $P = \gamma R - Q$, we have $P = \left[1 - \left(\frac{\rho\omega(1-\eta)}{\delta\nu(\rho+\theta)}\right)^{(1-\eta)/\eta}\right]\gamma R.$

vi) Computation of growth rates.

The growth rates directly follow from the log-differenciation of the preceding results.

In summary, one gets:

$$L_Y^o = \frac{\alpha\rho}{\delta\nu}, \tag{30}$$

$$L_{Qt}^o = \left[\frac{\rho\omega(1-\eta)}{\delta\nu(\rho+\theta)}\right]^{(1/\eta)} \gamma R_t^o, \tag{31}$$

$$L_{At}^o = 1 - L_Y^o - L_{Qt}^o, \tag{32}$$

$$R_t^o = \frac{1-\alpha}{\varphi_0 e^{\rho t} + B}, \tag{33}$$

where $\varphi_0 = \frac{B}{e^{\frac{B\rho S_0}{1-\alpha}} - 1}$ and $B = \frac{\omega\gamma}{\rho+\theta}\left[1 - \eta\left[\frac{\rho\omega(1-\eta)}{\delta\nu(\rho+\theta)}\right]^{\frac{1-\eta}{\eta}}\right]$,

$$Q_t^o = \left(\frac{\rho\omega(1-\eta)}{\delta\nu(\rho+\theta)}\right)^{(1-\eta)/\eta} \gamma R_t^o, \tag{34}$$

$$P_t^o = \left[1 - \left(\frac{\rho\omega(1-\eta)}{\delta\nu(\rho+\theta)}\right)^{(1-\eta)/\eta}\right] \gamma R_t^o, \tag{35}$$

$$g_{At}^o = \delta L_{At}^o, \tag{36}$$

$$g_{Rt}^o = g_{L_{Qt}}^o = g_{Q_t}^o = g_{P_t}^o = \frac{-\rho}{1 + (e^{\frac{B\rho S_0}{1-\alpha}} - 1)e^{-\rho t}}, \tag{37}$$

$$g_{Yt}^o = \nu g_{At}^o + (1-\alpha) g_{Rt}^o. \tag{38}$$

Appendix 2: Welfare in the no-storage case

When no storage technology is available, maximizing welfare leads to the following results (recall that we denote by $X_t^{o\varnothing}$ the optimal level of any variable X_t in this case):

$L_Y^{o\varnothing} = \alpha\rho/\delta\nu,\ L_A^{o\varnothing} = 1-(\alpha\rho/\delta\nu),\ R_t^{o\varnothing} = \frac{1-\alpha}{\varphi_0^{\varnothing} e^{\rho t}+B^{\varnothing}},\ g_R^{o\varnothing} = \frac{-\rho}{1+B^{\varnothing}/\varphi_0^{\varnothing} e^{\rho t}},\ g_A^{o\varnothing} = \delta L_A^{o\varnothing}$, $g_Y^{o\varnothing} = \nu\delta L_A^{o\varnothing} + (1-\alpha)g_R^{o\varnothing}$, where $\varphi_0^{\varnothing} = \frac{B^{\varnothing}}{e^{(B^{\varnothing}\rho S_0/(1-\alpha))}-1}$ and $B^{\varnothing} = \omega\gamma/(\rho+\theta)$.

Appendix 3: Equilibrium

Here also, we consider the symmetric case, in which we also have $p_j = p$ and $c_j = c = Y$ for all j. Then it can be easily verified that $\Omega = p^{1/(1-\varepsilon)}Y$ and $\Gamma = Y^{\varepsilon}$. This implies $g_\Gamma = \varepsilon g_Y$.

In this case, formulas (10)-(16) become:

$$r = \rho + g_Y + g_p \text{ (Ramsey-Keynes)}, \tag{39}$$

$$r = \dot{p}_R/p_R \text{ (Hotelling rule)}, \tag{40}$$

$$p\left[1+(\varepsilon-1)/N\right] = \lambda, \tag{41}$$

$$-p_R + \lambda(1-\alpha)Y/R - \tau\gamma\left[1-\eta(L_Q/\gamma R)^{1-\eta}\right] = 0, \tag{42}$$

$$\lambda = L_Y/\alpha Y, \tag{43}$$

$$-1 + \tau(1-\eta)(L_Q/\gamma R)^{-\eta} = 0, \tag{44}$$

$$V\delta A = 1. \tag{45}$$

Moreover, remember that the instantaneous market value of one unit of knowledge is

$$\bar{v} = V\delta L_A + \psi\nu L_Y/\alpha A, \text{ where } 0 \le \psi \le 1, \tag{46}$$

and that the standard arbitrage equation writes

$$r = \dot{V}/V + \bar{v}/V. \tag{47}$$

i) Computation of L_Y.

From (41) and (43), we have $p(1-(\varepsilon-1)/N) = L_Y/\alpha Y$. Hence we get $g_p = g_{L_Y} - g_Y$. Plugging this into (39), we obtain $r = \rho + g_{L_Y}$, which, together with (47) yield $g_V + \bar{v}_t/V_t = \rho + g_{L_Y}$.

Since (45) implies $g_V = -g_A = -\delta L_A$ and $\bar{v}_t/V_t = \delta L_A + \psi\nu\delta L_Y/\alpha$, we have the following Ricatti differential equation $-\rho L_Y^2 + \psi\nu\delta L_Y/\alpha = \dot{L}_Y$. Using the transversality condition of the household program, we can show that L_Y immediately jumps towards its steady-state level, as we did in Appendix 1. Thus one gets

$$L_Y = \alpha\rho/\psi\nu\delta. \tag{48}$$

Note that, since g_{L_Y} is nil, $r = \rho$ (remember that we normalized wage to 1).

ii) Computation of L_Q.

Using (44), we immediately get

$$L_Q = [(1-\eta)\tau]^{1/\eta}\gamma R. \tag{49}$$

iii) Computation of R.

First, note that formula (40) implies $p_R = p_{R0}e^{rt} = p_{R0}e^{\rho t}$. Then (43) and (42) together with (48) and (49) yield

$$R_t^e = \frac{(1-\alpha)\rho/\psi\nu\delta}{p_{R0}e^{\rho t} + G}, \text{ with } G = \tau\gamma\left[1 - \eta\left[(1-\eta)\tau\right]^{(1-\eta)/\eta}\right] \text{ when } \tau \neq 0.$$

Using $\int_0^{+\infty} R_t dt = S_0$ we obtain $p_{R0} = G/(e^{\psi\nu\delta S_0 G/(1-\alpha)} - 1)$.

iv) Computation of Q and P.

Plugging (49) into $Q = (\gamma R)^\eta L_Q^{1-\eta}$, one gets $Q = [(1-\eta)\tau]^{(1-\eta)/\eta}\gamma R$.

Then, using $P = \gamma R - Q$, we have $P = \left[1 - ((1-\eta)\tau)^{(1-\eta)/\eta}\right] \gamma R$.

v) Computation of growth rates.

The growth rates directly follow from the log-differenciation of the preceding results. In summary, at the equilibrium, quantities and rates of growth take the following values:

$$L_Y^e = \frac{\alpha\rho}{\psi\nu\delta}, \tag{50}$$

$$L_{Qt}^e = \left[(1-\eta)\tau\right]^{1/\eta} \gamma R_t^e, \tag{51}$$

$$L_{At}^e = 1 - L_{Yt}^e - L_{Qt}^e, \tag{52}$$

$$R_t^e = \frac{(1-\alpha)\rho/\psi\nu\delta}{p_{R0}e^{\rho t} + G}, \tag{53}$$

with $p_{R0} = \frac{G}{e^{\frac{\psi\nu\delta S_0 G}{1-\alpha}} - 1}$, and $G = \tau\gamma\left[1 - \eta\left[(1-\eta)\tau\right]^{(1-\eta)/\eta}\right]$ when $\tau \neq 0$.

$$Q_t^e = \left[(1-\eta)\tau\right]^{(1-\eta)/\eta} \gamma R_t^e, \tag{54}$$

$$P_t^e = \left[1 - ((1-\eta)\tau)^{(1-\eta)/\eta}\right] \gamma R_t^e, \tag{55}$$

$$g_{At}^e = \delta L_{At}^e, \tag{56}$$

$$g_{Rt}^e = g_{L_{Qt}}^e = g_{Qt}^e = g_{Pt}^e = \frac{-\rho}{1 + (e^{\frac{\psi\nu\delta S_0 G}{1-\alpha}} - 1)e^{-\rho t}}, \tag{57}$$

$$g_{Yt}^e = \nu g_{At}^e + (1-\alpha) g_{Rt}^e. \tag{58}$$

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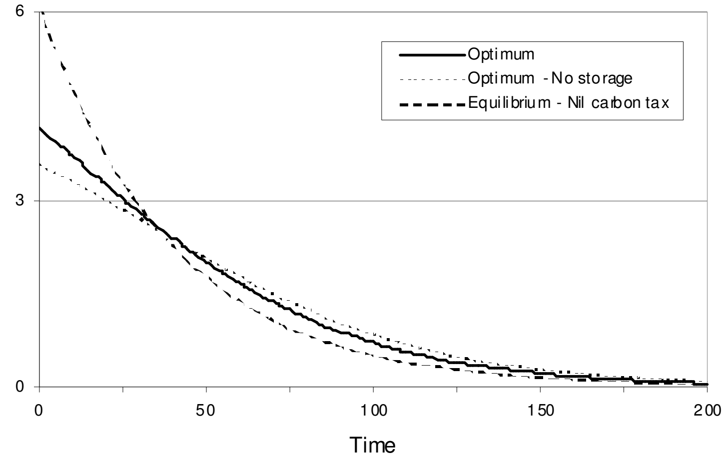

Figure 1: Resource extraction R_t

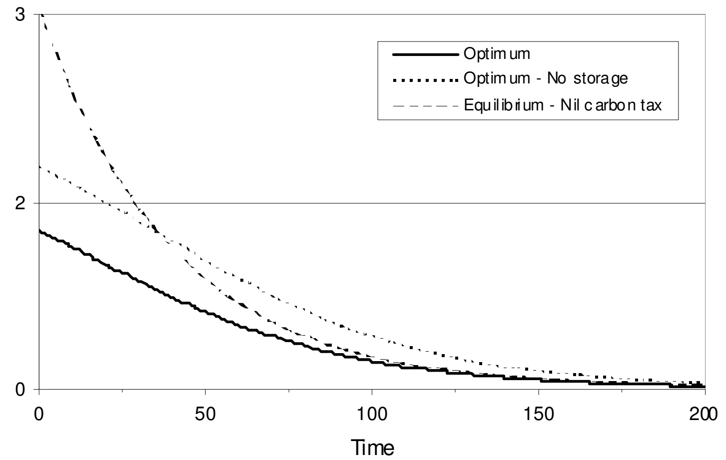

Figure 2: Pollution P_t

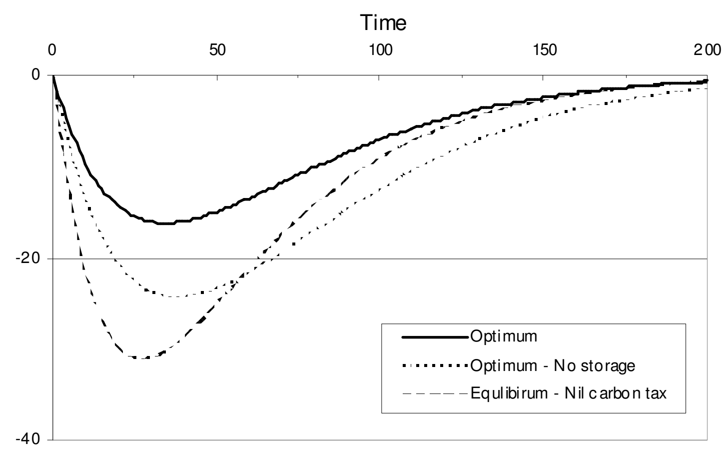

Figure 3: Stock of environment E_t

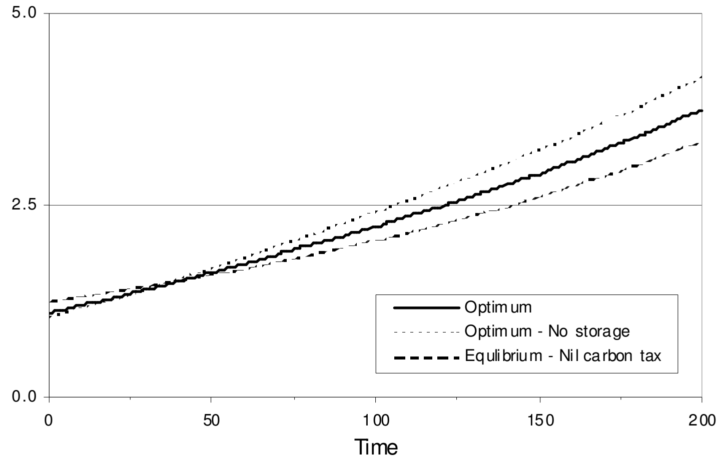

Figure 4: Final good production Y_t

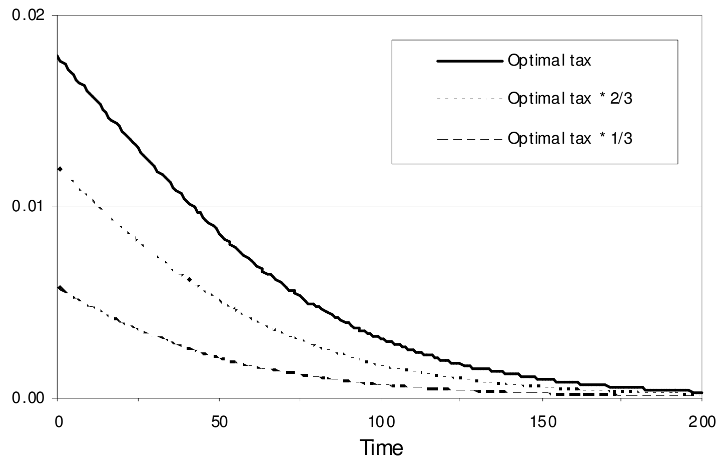

Figure 5: Sensitivity of labour dedicated to carbon sequestration L_{Q_t} to carbon tax

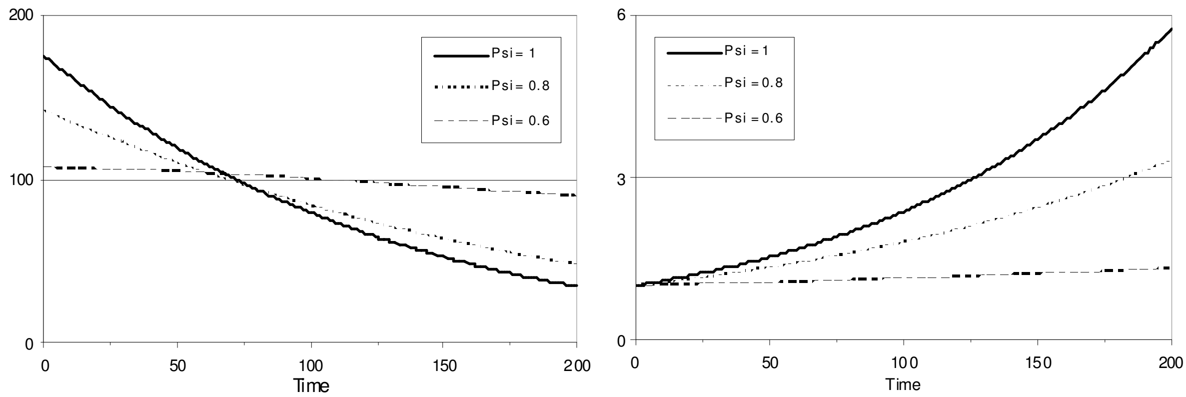

Figure 6: Sensitivity to parameter ψ of the innovation price expressed in terms of good (Left panel) and sensitivity of the stock of knowledge A_t (Right panel)

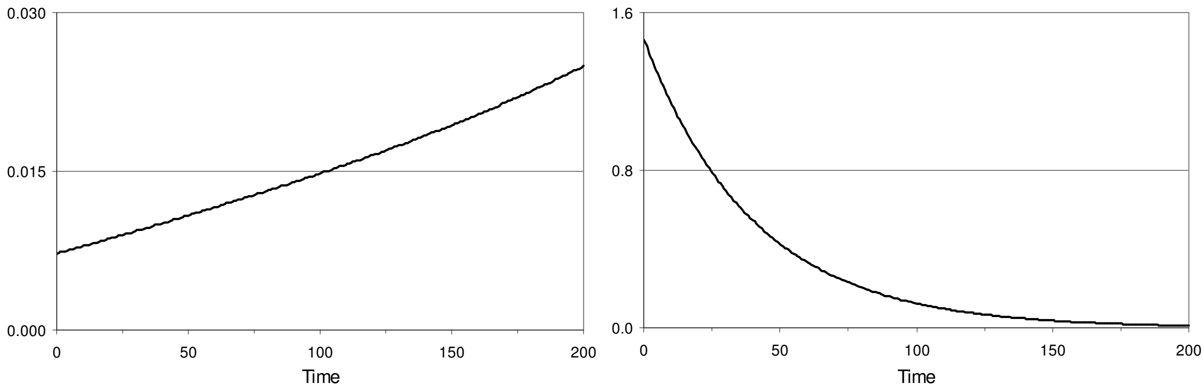

Figure 7: Optimal carbon tax in terms of consumption good τ^o/p_t (Left panel) and equivalent optimal "ad valorem" tax on resource extraction (Right panel)

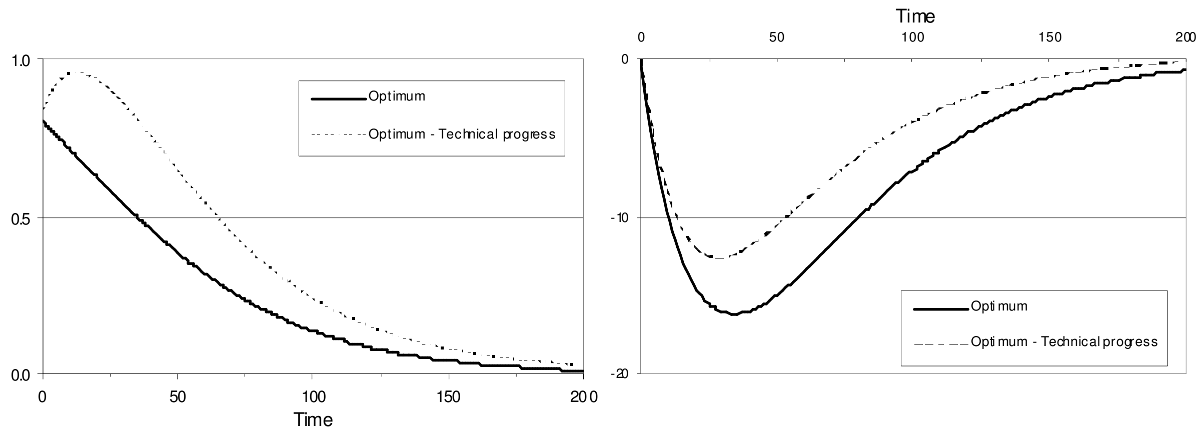

Figure 8: Effect of exogenous technical progress on the sequestred carbon Q_t (Left panel) and on the environmental quality E_t (Right panel)