Weightless Economy, Knowledge Goods and Schumpeterian Growth *

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Abstract

The main purpose of the paper is to extend the Schumpeterian growth theory to the case of a weightless economy. Basically, we consider a formalization of knowledge accumulation which is slightly different from the Aghion-Howitt's one. First, in the line of these authors, we use this formalization in a standard model, i.e. a model of tangible economy in which intermediate goods embody knowledge. Second, we use it in a model of weightless economy, i.e. a model in which knowledge (or information) goods replace intermediate goods. In this framework, we consider an equilibrium with Cournot competition to overcome the problems caused by the non-convexity of technology. Then, we can extend the Schumpeterian growth analysis; in particular, we study the creative destruction mechanism with knowledge goods. Finally, we analyse the case in which patents directly protect new knowledge.

Keywords: Weightless Economy, Knowledge Goods, Schumpeterian Growth

JEL Classification: O31, O34.

1 Introduction

Two paradigms in innovation-based growth models are today available. First, the models with an expanding variety of products (Romer [20], Grossman and Helpman [10]). Second, the models with quality improving innovations, and in particular the Schumpeterian growth theory developed by P. Aghion and P. Howitt [1, 2]. A common feature of these models is that new knowledge (i.e. new ideas) is embodied in private intermediate goods, and that all authors consider an equilibrium in which these goods are monopolized. In the present paper, we call *tangible economy* the case in which these intermediate goods exist.

However, in the new technology sectors, these intermediate goods generally disappear because new ideas are embodied in knowledge (or information) goods which have the same essential property than knowledge: they are non rival [1]. Here we call *weightless economy* the case where intermediate goods are replaced by knowledge goods in which ideas are embodied.

The main purpose of this paper is to extend the Schumpeterian paradigm to the case of a weightless economy [2]. The basic difficulty is that we have to propose a new concept of equilibrium. Indeed, since intermediate goods have disappeared, we cannot continue to assume that an innovator obtains a flow of monopoly profits on this type of good. On the contrary, we must assume that knowledge goods (or, possibly, knowledge) are directly priced. Because of the public good nature of knowledge goods, new questions arise. In particular, since knowledge is used as an input in production processes,

[1] For instance, S. Scotchmer [25] writes: "By information goods, we usually mean computer software and entertainment products stored in digital form, such as music. Information goods have a feature that sets them apart from ordinary private goods. They are public goods in the technical sense meant by economists: use by one person does not preclude use by any other person and does not cost additional resources, except the small cost of distributing them. That is, the use of such a good is non rival". On this point, see also Quah [18, 19].

[2] One can find several arguments in favor of the Schumpeterian paradigm. See for instance Aghion and Howitt [3].

technologies are non convex, and a competitive equilibrium does not exist. Thus we have to construct an equilibrium in which firms are able to buy knowledge, that is to say in which their profits remain strictly positive once private inputs (labor, capital, ...) are paid.

Our first stage is to present a formalization of knowledge accumulation which is slightly different from the Aghion-Howitt's one. Basically, this analysis is in the line of these authors. As Aghion and Howitt, we assume a Poisson arrival rate of new ideas. However, we propose a different formalization of the intersectoral spillovers: we assume that when a new idea occurs in a sector, the increase in knowledge is a function of the total stock of knowledge accumulated in the economy. This leads to very simple expressions for the law of motion of knowledge. We use this formalization of knowledge accumulation to study a tangible economy. In a first stage, we analyze a standard equilibrium in which intermediate goods are monopolized, and we find again the basic results of Aghion and Howitt, especially the fact the laissez-faire growth may be greater or lower than the optimal one. In a second stage, we determine the system of prices which supports the firs-best optimum. To do it, we assume that knowledge is directly priced (independently of the intermediate goods in which it is embodied) and we compute the social value of each unit of knowledge. This methodological step gives us the basic tools which allow to analyze later the economy with knowledge goods.

Afterwards, we extend the model to the case of a weightless economy. Then, to solve the problems raised by the non-convexity of technology, we assume that firms compete "à la Cournot" on the markets of consumption goods. In this case, they get positive profits which are used to buy the knowledge goods. At the same time, we introduce a market for knowledge in which we assume imperfect exclusion. Then, we can in particular examine what becomes the creative destruction mechanism in a weightless economy context: this case occurs when innovators have a monopoly on knowledge goods. It can be compared to the situation in which property rights are directly put on ideas.

The paper is organized as follows. In section 2, we present the formalisation of knowledge accumulation, we distinguish tangible economy and weightless economy, we analyse the standard ("à la Aghion-Howitt") equilibrium in the first one, and we calculate the expression of the social value of one unit of knowledge. In section 3, we study the equilibrium with Cournot competition and free entry in the weightless economy. We conclude in section 4.

2 From intermediate goods ("tangible economy") to knowledge goods ("weightless economy")

In this section, we present a model which can describe two types of economies, that we call tangible ("standard") and weightless ("new"). The common feature of these economies is that they have the same research sector which produces new ideas, that are non-rival goods. The basic difference is that in the standard economy ideas are embodied in intermediate goods (i.e. private goods), whereas in the new economy they are embodied in knowledge goods that are also non-rival, and thus that can be assimilated to ideas themselves.

2.1 The model

2.1.1 The research sector

In both economies, a fixed flow of labor (L) has two competing uses at each date t : it can be used to produce the final good (Y), and it can be used in research. That is :

$$L = L_t^Y + L_t^A, \tag{1}$$

where L_t^Y is the amount of labor used in the final good sector and L_t^A is the amount of labor used in research.

There is a continuum of sectors, each sector being denoted by $j, j \in [0, 1]$. Each sector j is characterized by a level of knowledge A_{jt}, and it has its own research activity. This activity produces *news ideas,* that are successive increases in knowledge. As P. Aghion and P. Howitt [2], we want to formalize the intersectoral spillovers, that it to say the fact that new technologies "diffuse gradually, through a process in which one sector gets ideas from the experience of others". As in their model, we assume that "the innovations themselves draw on the same pool of shared technological knowledge". But we depart from their assumption along with the state of this knowledge is represented by the "leading-edge" technology whose the productivity parameter is $A_t^{\max}$. Here we assume that the common pool is represented by the total stock of knowledge in the economy, $A_t = \int_0^1 A_{jt} dj$. Formally, we make two assumptions.

First, as Aghion and Howitt, we assume that the Poisson arrival rate of new ideas in each sector j is $\lambda L_{jt}^A, \lambda > 0$, where L_{jt}^A is the amount of labor devoted to research in this sector.

Second, we make a very simple assumption to express the increase in knowledge when a new idea occurs. We denote by $\tau(j)$ the index of ideas in sector $j(\tau(j) = 1, 2, 3, \ldots)$ and by $A_{t(\tau(j))}$ the level of total knowledge when the level of knowledge is $A_{\tau(j)}$ in sector j. Our basic assumption is

$$A_{\tau+1(j)} - A_{\tau(j)} = \sigma A_{t(\tau(j))}, \qquad \sigma > 0. \tag{2}$$

It says that, when a new idea occurs in sector j, the increase in knowledge is a linear function of the total knowledge in the economy[3].

These two assumptions allow to obtain very simple laws of motion of the average knowledge in each sector j, and in the whole economy. One gets :

$$\dot{A}_{jt} = \lambda \sigma L_{jt}^A A_t \quad \text{and} \quad \dot{A}_t = \lambda \sigma L_t^A A_t. \tag{3}$$

The proof is the following. Consider a time interval $(t, t + \Delta t)$. If knowledge in sector j is A_{jt} at date t, its value at $(t+\Delta t)$ is a random variable $A_{jt+\Delta t}$

[3]This assumption is a simple formalization of the intersectoral spillovers. It will allow to highlight the working of the market for knowledge. That is why we use this type of model rather than the ones of Grossman and Helpman [9], or O'Donoghue and Zweimüller [6] for instance.

which can take two values : $A_{jt+\Delta t} = A_{jt} + \sigma A_t$ with probability $\lambda L_{jt}^A \Delta t$ (one innovation during the time interval) and A_{jt} with probability $1 - \lambda L_{jt}^A \Delta t$ (no innovation). One gets $E(A_{jt+\Delta t}) = (A_{jt} + \sigma A_t)\lambda L_{jt}^A \Delta t + A_{jt}(1 - \lambda L_{jt}^A \Delta t) = A_{jt} + \lambda \sigma L_{jt}^A A_t \Delta t$. When Δt tends to zero, we have $E(\dot{A}_{jt}) = \lambda \sigma L_{jt}^A A_t$. Summing on j, one gets in average the law of motion of A_t given by (3).

Let us note that the technology described by (3) displays constant returns to scale with respect to the private input, i.e labor, and increasing ones with respect to all inputs, i.e labor and knowledge, taken together.

Remark : more generally, one can assume that the Poisson arrival rate of new ideas in sector j is $\phi(L_{jt}^A)$, $\phi' > 0$, and that $A_{\tau+1(j)} - A_{\tau(j)} = \psi[A_{t(\tau(j))}]$, $\psi' > 0$. Thus the laws of motion of knowledge are $\dot{A}_{jt} = \psi(A_t)\phi(L_{jt}^A)$ in sector j, and $\dot{A}_t = \psi(A_t) \int_0^1 \phi(L_{jt}^A) dj$ in the whole economy. Several cases can be envisaged. Let us consider some of them. If $\phi(L_{jt}^A) = \lambda L_{jt}^A$ and $\psi(A_t) = \sigma A_t$, we recover the previous case. If $\phi(L_{jt}^A) = \lambda L_{jt}^A / L_t$ (now, L_t can be growing) and $\psi(A_t) = \sigma A_t$, one gets $\dot{A}_t = \lambda \sigma A_t L_t^A / L_t$, that leads to an endogenous growth model without scale effects. Assume now $\phi(L_{jt}^A) = \zeta_t L_{jt}^A$, $\psi(A_t) = \sigma A_t^\gamma, \gamma < 1$, and $\zeta_t = \lambda (L_t^A)^{\delta - 1}$, $\lambda > 0$, $\delta \in (0,1)$, where ζ_t is a productivity factor which is external to each sector. One gets $\dot{A}_{jt} = \lambda \sigma (L_t^A)^{\delta-1} L_{jt}^A (A_t)^\gamma$ and $\dot{A}_t = \lambda \sigma (L_t^A)^\delta (A_t)^\gamma$, that leads to a semi-endogenous growth model: see for instance Jones and Williams [11].

In the following text, we use the simple specification given in (3) and we provide complementary results in remarks.

2.1.2 Tangible ("standard") economy versus weightless ("new") economy

Now we can distinguish the two types of economies.

In the *tangible economy,* ideas are embodied in intermediate goods. In this case, we can keep the formalization of Aghion and Howitt. The final good is produced according to

$$Y_t = (L_t^Y)^{1-\alpha} \int_0^1 A_{jt}(x_{jt})^{\alpha} dj \quad , \quad 0 < \alpha < 1 \tag{4}$$

where x_{jt} is the quantity of intermediate good j used at date t.

Each intermediate good j is produced from final output according to

$$x_{jt} = \frac{y_{jt}}{A_{jt}} \quad , \quad j \in [0, 1], \tag{5}$$

where y_{jt} is the quantity of final output used to produce x_{jt}.

Finally we have

$$Y_t \;\; = \;\; Lc_t + \int_0^1 y_{jt} dj, \tag{6}$$

where c_t is the consumption of the representative household.

In the *weightless economy,* there are no intermediate goods with significant positive cost and ideas are embodied in knowledge goods which can be produced at zero cost. In this case, we assume that the technology in the final good sector is

$$Y_t = L_t^Y \int_0^1 A_{jt} dj = A_t L_t^Y. \tag{7}$$

Note that, in accordance with the replication argument, we assume that (4) and (7) display constant returns to scale with respect to the private (i.e. rival) input(s): labor and intermediate goods in the tangible economy, only labor in the weightless economy. On the other hand, in both cases, these technologies display increasing returns to scale with respect to rival and non rival inputs taken jointly. Since there are no intermediate goods, the whole final good is used for consumption:

$$Y_t = Lc_t. \tag{8}$$

Finally, in the tangible economy and in the weightless one, preferences are given by

$$U = \int_0^{\infty} \frac{c_t^{1-\varepsilon} - 1}{1 - \varepsilon} e^{-\rho t} dt, \varepsilon > 0, \varepsilon \neq 0, \rho > 0. \tag{9}$$

2.2 Economy with intermediate goods ("tangible economy")

Here we analyse two types of equilibria. The first one, that we call the standard equilibrium, is a direct extension of the analysis of Aghion and Howitt. The second one, that we call the first-best, allows to exhibit the system of prices which supports the first-best optimum. It allows to present several elements that will be useful when we analyze the weightless economy.

2.2.1 Standard equilibrium

The price of good Y is normalized to one, and we denote by w_t, r_t and q_{jt}, the wage, the interest rate and the price of the intermediate j. The financial market, the labor market and the market of good Y are perfectly competitive. As Aghion and Howitt, we assume that "the firm that succeeds in innovating can monopolize the intermediate sector until replaced by the next innovator [2]". Formally, a standard equilibrium ("*à la Aghion-Howitt*") is such that:

- in the final sector, firms maximize the profit $\pi_t^Y = (L_t^Y)^{1-\alpha} \int_0^1 A_{jt} x_{jt}^{\alpha} dj - w_t L_t^Y - \int_0^1 q_{jt} x_{jt} dj$,
- in each intermediate sector j, the monopoly maximizes the profit $\pi_{jt} = q_{jt} x_{jt} - y_{jt}$,
- in the research activity, the free entry condition is $w_t = \lambda \Pi_{jt}$, where $\Pi_{jt} = \int_t^{\infty} \pi_{js} e^{-\int_t^s (r_u + \lambda L_{ju}^A) du} ds$ is the sum of the present values of the expected profits in sector j at date t.

The results are summarized in the following proposition.

Proposition 1 *In the standard ("à la Aghion-Howitt") equilibrium, rates of* *growth, quantities and prices are the following:*

$$
\begin{aligned}
g_A &= g_Y = \frac{\lambda\alpha L - \rho}{\varepsilon - 1 + (1+\alpha)/\sigma} \\
L^A &= \frac{g_Y}{\lambda\sigma}, \quad L^Y = L - L^A, \quad x_j = x = \alpha^{\frac{1}{1-\alpha}} L^Y, \ \forall j \\
w_t &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t \ , \ q_{jt} = q_t = \frac{A_t}{\alpha} \ , \ \forall j
\end{aligned}
$$

where g_z is the rate of growth of any variable z.

Proof: see appendix A.

It can be shown (see Proposition 2 below) that, at optimum, we have $g_Y = (\lambda\sigma L - \rho)/\varepsilon$ and $x = \alpha^{1/(1-\alpha)} L^Y$. As usual, the standard equilibrium is not optimal. In particular, as in the basic formulation of Aghion and Howitt, the laissez-faire growth may be greater or lower than the optimal one.

Remark : let us consider the case in which the Poisson arrival rate of new ideas in sector j is $\phi(L^A_{jt}) = \lambda A_{jt}/L_t$, where population L_t grows at the exogenous rate n. Moreover, we continue to assume that $A_{\tau+1(j)} - A_\tau(j) = \sigma A_{t(\tau(j))}$ (see (2)). Thus we know that in average the law of motion of knowledge is $\dot{A}_{jt} = \lambda\sigma A_t L^A_{jt}/L_t$ in sector j and $\dot{A}_t = \lambda\sigma A_t L^A_t/L_t$ in the whole economy (see the final remark in 2.1.1 above). In this case, one gets $g_c = g_A = (\lambda\alpha - \rho)/[\varepsilon - 1 + (1-\alpha)/\sigma]$, $g_Y = g_c + n, L^A_t/L_t = g_A/\lambda\sigma, x_{jt} = x_t = \alpha^{\frac{2}{1-\alpha}} L^Y_t, w_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t$, and $q_{jt} = q_t = A_t/\alpha$. Basically, there is now no scale effects in this "à la Aghion-Howitt" equilibrium.

2.2.2 Welfare, social value of innovations and first-best prices

Now we depart from the usual analysis made in growth theory. The main purpose of this sub-section is to exhibit the system of prices which supports the first-best optimum of the standard economy. The basic point is to compute the optimal unit price of knowledge, that is to say to provide a detailed expression of the social value of one unit of knowledge. We see later that this analysis will be useful to study the weightless economy. We introduce two modifications with respect to the standard equilibrium.

First, we complete the markets by assuming that ideas are priced. Second, having in mind the first theorem of welfare, we assume that the markets of private goods (final good, labor, *but also intermediate goods*) are perfectly competitive, and that ideas are priced at their optimal level.

As previously, we normalise to one the price of good Y, and w_t, r_t and q_{jt} are the wage, the interest rate and the price of the intermediate j. We know that each unit of knowledge is simultaneously used (ideas are non-rival goods) by three types of firms, which belong to the three sectors of the economy: the final good sector (see (4)), the research sector (see (3)) and the intermediate goods sector (see(5): these goods embody the knowledge). We denote respectively by v_{jt}^Y, v_{jt}^i and v_{jt}^x, the marginal profitability of one unit of knowledge A_j at date t in the final sector, the sector i, and the intermediate goods sector. Then, $v_{jt} = v_{jt}^Y + \int_0^1 v_{jt}^i di + v_{jt}^x$ is the total marginal profitability of one unit of knowledge A_j: it is the *instantaneous social marginal value of this unit.* Indeed, since each unit of knowledge is simultaneously used by three sectors, this social value is the sum of the marginal profitabilities of this unit in these sectors.

Since ideas are infinitely-lived, we can define $V_{jt} = \int_t^\infty v_{js} e^{-\int_t^s r_u du} ds$ as the *social value of one unit of knowledge* A_j at date t. Differentiating the expression of V_{jt} with respect to time gives the usual condition $r_t = v_{jt}/V_{jt} + g_{V_{jt}}$.

The results are summarized in the following proposition.

Proposition 2 *In the tangible economy, the first-best rates of growth, quan-*

tities, and prices are (upper-script ^{o} is used for optimum):

$$\begin{aligned} g_A^o &= g_Y^o = \frac{\lambda\sigma L - \rho}{\varepsilon} \\ (L^A)^o &= \frac{g_Y^o}{\lambda\sigma},\ (L^Y)^o = L - (L^A)^o,\ x^o = \alpha^{\frac{1}{1-\alpha}}(L^Y)^o \\ w_t^o &= (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_t,\ q_t^o = A_t,\ v^o = L(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}},\ V^o = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\lambda\sigma} \end{aligned}$$

Proof: see appendix B.

These results confirm that in the standard equilibrium studied in 2.1.1, there are two market failures with respect to the first-best one: first, intermediate goods are sold by monopolies; second, markets are incomplete, since knowledge has no price. We have now the main tools to construct an equilibrium in the weightless economy.

Remark : let us again consider the case in which $\dot{A}_{jt} = \lambda\sigma A_t L_{jt}^A/L_t$, $\dot{A}_t = \lambda\sigma A_t L_t^A/L_t$, and $\dot{L}_t/L_t = n$. Then one gets $g_c^o = g_A^o = (\lambda\sigma - \rho)/\varepsilon, g_Y^o = g_c^o + n, (L_t^A)^o/L_t = g_A^o/\lambda\sigma, x_t^o = \alpha^{\frac{1}{1-\alpha}}(L_t^Y)^o, w_t^o = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A_t, q_t^o = A_t, v_t^o = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}L_t, V_t^o = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}L_t/\lambda\sigma$. There are two differencies with respect to the results given in proposition 2: scale effects disappear and the social value of one unit of knowledge increases at the same rate than population.

2.3 Economy with knowledge goods: first-best and basic problems

Let us now consider the weightless economy. In order to characterize the optimal path, one maximizes (9) under (1)-(3)-(7)-(8). Clearly the solutions are the same as in the tangible economy, except that intermediate goods disappear. Thus one gets $g_Y = g_c = g_A = (\lambda\sigma L - \rho)/\varepsilon, L^A = g_Y/\lambda\sigma$, and $L^Y = L - L^A$.

Using the methodology presented in 2.2.2, we can easily obtain the first-best prices.

In the final sector, the profit is $\pi_t^Y = A_t L_t^Y - w_t L_t^Y$. One gets $w_t = A_t$, and the marginal profitability of one unit of knowledge is $v_t^Y = \partial \pi_t^Y / \partial A_t = L_t^Y$. In any research sector j, the profit is $\pi_{jt}^A = V_{jt} \lambda \sigma A_t L_{jt}^A - w_t L_{jt}^A$. The free entry condition is $w_t = V_{jt} \lambda \sigma A_t$. Since $w_t = A_t$, one gets $V_{jt} = V_t = 1/\lambda\sigma$, that implies $g_V = 0$. The marginal profitability of one unit of knowledge is $v_{jt}^A = \partial \pi_{jt}^A / \partial A_t = V_t \lambda \sigma L_{jt}^A$.

From the representative household's behavior and the arbitrage condition between financial market and research, one has at steady-state $r = \rho + \varepsilon g_Y = v^Y/V + v^A/V$. Since $v^Y/V = \lambda\sigma L^Y$ and $v^A/V = \lambda\sigma L^A$, one gets $g_Y = (\lambda\sigma L - \rho)/\varepsilon$, which is the optimal rate of growth.

We can now come back to the two expressions of the social value of knowledge, that is to say to the optimal price of knowledge in both economies. In the tangible economy, it is the sum of the marginal profitabilities of knowledge in three sectors: the final good sector, the research sector, and the intermediate goods sector. In the weightless economy, intermediate goods disappear. They are replaced by knowledge goods which are non-rival, i.e. which are consubstantial with knowledge itself. That is why the social value of knowledge is now the sum of only two terms: the marginal profitabilities of knowledge in the final good sector and in the research sector.

The results are summarized in the following proposition.

Proposition 3 *In the weightless economy, the first-best rates of growth, quantities and prices are:*

$$\begin{aligned} g_A^o &= g_Y^o = \frac{\lambda\sigma L - \rho}{\varepsilon} \\ (L^A)^o &= \frac{g_Y^o}{\lambda\sigma} \ , \ (L^Y)^o = L - (L^A)^o \\ w_t^o &= A_t \ , \ v^o = L \ , \ V^o = \frac{1}{\lambda\sigma}. \end{aligned}$$

This equilibrium is formally very simple, but it is not realistic. Assume for example that patents are given to the inventors of new ideas [4]. First, in order to implement the first-best optimum, we must assume that these inventors are able to identify who uses an idea, and to exclude those who do not pay.

Second, in a competitive economy, firms which use ideas are unable to pay for them, because the payment of private factors exhaust their revenue. Thus this equilibrium exists only if research is totally publicly funded.

These two types of difficulties come from the fact that ideas and knowledge goods are non rival, and generally only partially excludable. In particular, the non-rivalness property explains why the technologies of production of final good and of innovations display constant returns to scale with respect to private inputs and increasing ones with respect to all inputs (privates ones and knowledge) taken together: see (3), (4) and (7).

Remark : if $\dot{A}_{jt} = \lambda\sigma A_t L^A_{jt}/L_t, \dot{A}_t = \lambda\sigma A_t L^A_t/L_t$, and $\dot{L}_t/L_t = n$, one gets $g^o_c = g^o_A = (\lambda\sigma - \rho)/\varepsilon, g^o_Y = g^o_c + n, (L^A_t)^o/L_t = g^o_A/\lambda\sigma, w^o_t = A_t, v^o_t = L_t$, and $V^o_t = L_t/\lambda\sigma$. Again, scale effects disappear and the social value of one unit of knowledge increases at the same rate than population.

3 Equilibrium in the weightless economy

In the previous section, we have analyzed two types of equilibria: the standard ("à la Aghion-Howitt") one in which there are monopolies on intermediate goods, and the first-best one in which knowledge is directly priced at its optimal level. Unfortunately, when one considers the weightless economy, none of them can be used: the standard equilibrium is unusable because intermediate goods have disappeared, and the first-best one is not realistic. Thus we have to propose a new concept of equilibrium.

We have seen that the main difficulties come from the public good nature

[4] An alternative case would be to give patents directly to new knowledge goods. Both cases will be considered in sections 3.2 and 3.3.

of knowledge which rises two types of questions. First, we have to construct an equilibrium in which the firms which use knowledge as an input get strictly positive profits on the sales of consumption goods in order to buy knowledge. The simple idea of the model analyzed below is to assume that these profits are earned on the markets of consumption goods, which are imperfectly competitive. Second, we have to explain how is organized the market for knowledge.

We consider a disaggregated version of the weightless economy presented in 2.1.2. There is now a continuum of consumption goods, and each good is produced by an endogenous number of firms. Each firm engages simultaneously in research, and we assume that firms compete "à la Cournot" on the markets of consumption goods. Since the price of these goods is higher than the unit cost, each firm obtains a positive profit which is used to buy the knowledge goods (or ideas themselves). Then we study two polar cases. If innovators have a monopoly on knowledge goods, we get a "destructive creation" equilibrium, which can be compared with the equilibrium studied by Aghion and Howitt in the tangible economy. If property rights are directly put on new ideas, one gets an equilibrium which is noticeably different from the previous one. In all cases, we determine the number of firms in each consumption goods sector which results from the free entry condition[5].

3.1 Disaggregated model and agents behavior

3.1.1 Model

As previously, there is a continuum of sectors. Now each sector $j, j \in [0, 1]$, produces a specific consumption good. We choose this differentiated goods structure in order to exhibit a demand function on each market (see (11) below) and to introduce imperfect competition. Each individual is endowed

[5] A possible extension of this analysis would be to link this model of weightless economy with the patent-design literature, as it is done by O'Donoghue and Zweimüller [6] for a model of tangible economy.

with one unit of labor and preferences are given by

$$U = \int_0^\infty \frac{[\int_0^1 (c_{jt})^\theta]^{(1-\varepsilon)/\theta} - 1}{1-\varepsilon} e^{-\rho t} dt, \varepsilon > 0, \rho > 0, 0 < \theta < 1 \tag{10}$$

where c_{jt} is the consumption of differentiated good j.

In each sector j, there are $N_j (n_j = 1, \ldots, N_j)$ firms. Each firm n_j engages simultaneously in research and in production. We denote by $A_{n_j t}$ the stock of knowledge produced by firm n_j until date t, by $A_{jt} = \sum_{n_j=1}^{N_j} A_{n_j t}$, the stock of knowledge produced in sector j, and by $A_t = \int_0^1 A_{jt} dj$ the total stock of knowledge.

Inside each firm n_j, new ideas occur under the two assumptions presented in 2.1.1. First, the Poisson arrival rate of these ideas is $\lambda L^A_{n_j t}$, $\lambda > 0$, where $L^A_{n_j t}$ the amount of labor devoted to research in this firm. Second, when a new idea occurs in firm n_j, the increase in knowledge inside this firm is $\sigma A_t^{n_j}$, $\sigma > 0$, where $A_t^{n_j}$ is the stock of knowledge used by the firm (we have $A_t^{n_j} \leq A_t$). Thus, as explained in 2.1.1, one gets in average the law of motion $\dot{A}_{n_j t} = \lambda \sigma L^A_{n_j t} A_t^{n_j}$, for all n_j. The technology of production of good j by firm n_j is $Y_{n_j t} = A_t^{n_j} L^Y_{n_j t}$, where $L^Y_{n_j t}$ is the amount of labor devoted to production.

If $A_t^{n_j} = A_t$ (i.e. all firms use the whole stock of knowledge, that will be the case at equilibrium : see Proposition 4), one gets $\dot{A}_{jt} = \sum_{n_j=1}^{N_j} \dot{A}_{n_j t} = \lambda \sigma L^A_{jt} A_t$ in each sector j, and $\dot{A}_t = \int_0^1 \dot{A}_{jt} dj = \lambda \sigma L^A_t A_t$ in the whole economy. In the same way, the total production of good j is $Y_{jt} = \sum_{n_j=1}^{N_j} Y_{n_j t} = A L^Y_{jt}$.

Finally, one has $Y_{jt} = L c_{jt}$ and $\int_0^1 [\sum_{n_j=1}^{N_j} (L^Y_{n_j t} + L^A_{n_j t})] dj = L^Y_t + L^A_t = L$.

Let us remark that in the symmetric case ($Y_j = Y, \forall j$) one finds again the aggregated model presented in section 2.1. Thus, the optimal path is the same: see section 2.3.

3.1.2 Basic assumptions and market for knowledge

We normalize the price of labor to one, and we denote by p_{jt} and r_t the price of consumption good j and the interest rate. We know that each firm

n_j has two activities. First, it sells a quantity Y_{n_jt} of good j. Since there is imperfect competition on this market, the firm gets a strictly positive profit, $p_{jt}Y_{n_jt} - L^Y_{n_jt}$, on this activity. Second, it gets an additional profit on its research activity. The expected value of this profit is $\tilde{V}_t \lambda \sigma A_t L^A_{n_jt} - L^A_{n_jt}$, where $\tilde{V}_t$ is the market price of one unit of knowledge. We analyze later two cases in which $\tilde{V}_t$ take two different values. Either firms sell knowledge goods which have a finite life in average, as intermediate goods in the standard Aghion and Howitt's model. Either property rights can be directly given to ideas which are infinitely-lived.

Now we have to explain how the market for knowledge works. To simplify we assume that firms directly exchange units of knowledge. These goods are non rival and they are also differentiated and perfect substitutes, as it can be seen in (3) and (7) where units of knowledge can be permuted. We make three assumptions.

First, we assume that each firms n_j is characterized by an instantaneous marginal willingness to pay for *knowledge,* $v_{n_j}(A^{n_j}_t)$, which is a decreasing function of $A^{n_j}_t$ (see equation (15) below for the analytical expression of this function). Second, we assume for a while (see remark below) that there is perfect exclusion : non-buyers cannot copy the knowledge and buyers cannot resell it. Thus each firm can be considered as an independent market for the sellers of knowledge, and different firms generally pay different prices. In this model, there are $\int_0^1 N_j dj$ independent micro-markets. Third, since units of knowledge are perfect substitutes, we assume that each micro-market works as the standard competitive market for an homogenous good, where the total supply is A_t. Thus, on the micro-market of firm n_j, the unit price of knowledge, v_{n_jt}, adjusts so that the firm uses the total stock of knowledge : one gets $A^{n_j}_t = A_t$ and $v_{n_jt} = v_{n_j}(A_t)$, for all n_j. Since knowledge is non-rival, the price received by any seller for one unit of knowledge is the sum of the prices paid by firms which use this unit, that is $v_t = \int_0^1 \sum_{n_j=1}^{N_j} v_{n_j}(A_t)dj$. In other words, different firms generally pay different unit prices, but the price received by the sellers is the same for all units of knowledge.

An important point here is that, under the assumption of perfect exclusion, the market for non-rival (but substitute) units of knowledge leads to the optimal price. Indeed, v_t is the social value of one unit of knowledge, i.e. the sum of the user's marginal profits from this unit.

Remark : partial exclusion
We have now to take into account the problem of exclusion, which is at the heart of the weightless economy. To introduce partial exclusion, we relax the second assumption above, and we use the following simple formalization. We assume that, for any firm n_j, each unit of knowledge used by this firm has a probability ϕ , $0 \leq \phi \leq 1$, to be paid to the seller. Thus, in average, the seller perceives $\phi v_{n_j}(A_t)$ from this firm, and the average payment received from all firms for one unit of knowledge is $\tilde{v}_t = \int_0^1 \sum_{n_j=1}^{N_j} \phi v_{n_j}(A_t)dj = \phi \int_0^1 \sum_{n_j=1}^{N_j} v_{n_j}(A_t)dj = \phi v_t$.[6]

These results can be summarized in the following proposition.

Proposition 4 *Each firm n_j uses the whole available knowledge, and the instantaneous market value of one unit of knowledge ($\tilde{v}_t$) is a given percentage, ϕ, of its social value (v_t). That is*

$$A_t^{n_j} = A_t \quad \textit{for all} \quad n_j$$

$$\textit{and} \qquad \tilde{v}_t = \phi v_t \ , \ \textit{with}\ 0 \leq \phi \leq 1 \ \textit{and}\ v_t = \int_0^1 \sum_{n_{j=1}}^{N_j} v_{n_j}(A_t)dj$$

In section 3.2 and 3.3 below, we will consider two differents organizations of the market for knowledge. In each case, we will obtain a specific expression for the market value of one unit of knowledge at date t , $\tilde{V}_t$.

[6] In the present model, innovators appropriate only a part of the social value they create, that leads to under-investment in research : see below. This is consistent to empirical studies. For instance, Jones and Williams [11] estimate that actual investments are at least four times what would be socially optimal.

3.1.3 Behaviors and basic results

In this section, we examine the behavior of agents, and we compute some equilibrium prices.

The representative individual maximizes (10) subject to the budget constraint $\dot{b}_t = r_t b_t + w_t - \int_0^1 p_{jt} c_{jt} dt$ (b_t is the per-capita wealth, and we know that $w_t = 1$). One gets the following standard results. The inverse total demand function for consumption good j is

$$p_{jt} = (LE_t)^{1-\theta}(C_{jt})^{\theta-1}, \tag{11}$$

where $C_{jt} = Lc_{jt}$, and $E_t = (\int_0^1 p_{kt} c_{kt} dk)/(\int_0^1 (p_{kt})^{\theta/(\theta-1)} dk)$.

The Ramsey-Keynes rule is

$$r_t = (1-\theta) g_{c_{jt}} + [1 - \frac{1-\varepsilon}{\theta}] g_{\Omega_t} + \rho + g_{p_{jt}}, \tag{12}$$

where $\Omega_t = \int_0^1 (c_{jt})^\theta dj$.

The profit of firm n_j (without the payment of knowledge) is the sum of the profit on the production activity, $p_{jt} Y_{n_j t} - w_t L^Y_{n_j t}$, and the profit on the research activity, $\tilde{V}_t \lambda \sigma A_t L^A_{n_j t} - w_t L^A_{n_j t}$. Since $Y_{n_j t} = A_t L^Y_{n_j t}$ and using (11), this profit can be written:

$$\tilde{\pi}_{n_j t} = Y_{n_j t}[(LE_t)^{1-\theta}(\sum_{n_{j=1}}^{N_j} Y_{n_j t})^{\theta-1} - 1/A_t] + \tilde{V}_t \lambda \sigma A_t L^A_{n_j t} - L^A_{n_j t}.$$

The first-order condition with respect to $Y_{n_j t}$ gives

$$[(LE_t)^{1-\theta}(Y_{jt})^{\theta-1} - 1/A_t] + (\theta-1)(LE_t)^{1-\theta}(Y_{jt})^{\theta-2} Y_{n_j t} = 0 \tag{13}$$

which implicitly yields the best response of firm $n_j, Y_{n_j t}$, to the choice of production of the other firms in sector j.

The first-order condition with respect to $L^A_{n_jt}$ yields

$$\tilde{V}_t = 1/\lambda\sigma A_t \tag{14}$$

saying that the selling price of one unit of knowledge, $\tilde{V}_t$, is equal to its marginal cost, $1/\lambda\sigma A_t$.

The marginal profitability of one unit of knowledge, that is to say the instantaneous willingness to pay for this unit is $v_{n_jt} = \partial\tilde{\pi}_{n_jt}/\partial A_t = Y_{n_jt}/(A_t)^2 + \tilde{V}_t\lambda\sigma L^A_{n_jt}$. Since $Y_{n_jt} = A_t L^Y_{n_jt}$ and $\tilde{V}_t\lambda\sigma = 1/A_t$, one gets

$$v_{n_jt} = (L^Y_{n_jt} + L^A_{n_jt})/A_t. \tag{15}$$

Therefore, the instantaneous willingness to pay by all firms in the economy for one unit of knowledge, that is the *instantaneous social value of one unit of knowledge* is $v_t = \int_0^1 (\sum_{n_j=1}^{N_j} v_{n_jt})dj = (L^Y_t + L^A_t)/A_t$, that gives

$$v_t = L/A_t \tag{16}$$

Using (11), in which $C_{jt} = Y_{jt}$, (13) becomes $(p_{jt} - 1/A_t) + (\theta-1)p_{jt}Y_{n_jt}/Y_{jt} = 0$. Assume that all firms in sector j are identical, that implies $Y_{n_jt}/Y_{jt} = 1/N_j$. Then, one gets $p_{jt} = 1/A_t[1 + (\theta - 1)/N_j]$.

Since $0 < \theta < 1, p_{jt}$ is a decreasing function of N_j, and $\lim\limits_{N_j \to +\infty} p_{jt} = 1/A_t$ (i.e the marginal cost). If $N_j = 1$, we have $p_{jt} = 1/\theta A_t$, which is the standard result in the monopoly case. We see also that p_{jt} is a decreasing function of θ: the mark-up on the marginal cost decreases when the price-elasticity of the consumption goods demand increases.

Now let us assume that all sectors are symmetric, i.e $N_j = N$ for all j. Then we have

$$p_{jt} = p_t = \frac{1}{A_t(1 + \frac{\theta-1}{N})}, \ \forall j \in [0,1]. \tag{17}$$

The results concerning prices are summarized in the following proposition.

Proposition 5 *In the weightless economy with Cournot competition in all consumption goods sectors, and partial exclusion on the knowledge market, prices are*

$$\frac{\tilde{v}_t}{p_t} = \phi L(1 + \frac{\theta - 1}{N}), \frac{\tilde{V}_t}{p_t} = \frac{1}{\lambda\sigma}(1 + \frac{\theta - 1}{N}), \frac{w_t}{p_t} = A_t(1 + \frac{\theta - 1}{N}).$$

Since we have normalized the wage w_t to one, we have divided all prices by p_t to compare the results to the first-best ones given in proposition 3 (in which we have $p_t = 1$). One gets $\tilde{v}_t/p_t < (v_t/p_t)^0 = L, \tilde{V}_t/p_t < (V_t/p_t)^0 = 1/\lambda\sigma$, and $w_t/p_t < (w_t/p_t)^0 = A_t$. On account of Cournot competition and partial exclusion, the prices of knowledge and of labor are lower than the first-best ones, with convergence when the number of firms in each sector increases. However, as it is studied below, the number of firms is endogenous and finite because the payment of knowledge by firms appears as a fix cost, and because their profits must be non-negative.

3.2 Destructive creation with knowledge goods

In section 2.2.1, we have analyzed the standard (*"à la Aghion-Howitt"*) equilibrium in the tangible economy. In this equilibrium, the monopoly on an intermediate good disappears when this good is replaced by a new one: it is the destructive creation mechanism. The purpose of this sub-section is to extend this analysis to the case of the weightless economy.

Now, in each sector, the firm which has obtained the last innovation embodies this innovation in a knowledge (or information) good. As Aghion and Howitt, we assume that it has a monopoly on this good until an innovation occurs in this sector. Thus the value at t of one unit of knowledge is $\tilde{V}_t = \int_t^\infty \phi v_s e^{-\int_t^s (r_u + \lambda L_u^A) du} ds$, that gives after differentiation $r_t + \lambda L_t^A = g_{\tilde{V}_t} + \phi v_t/\tilde{V}_t$ (note that we consider the symmetric case, where $L_j^A = L^A, \forall j$).

Let us now calculate the rate of growth of the output (we continue to assume that all sectors are symmetric).

From (14), $\tilde{V}_t = 1/\lambda\sigma A_t$, we have $g_{\tilde{V}_t} = -g_{A_t}$, that gives at steady state $r + \lambda L^A = -g_A + \phi(L/A_t)\lambda\sigma A_t = -g_A + \phi\lambda\sigma L$, in which we know that $\lambda L^A = g_Y/\sigma$.

In the symmetric case, (12) becomes $r = \rho + \varepsilon g_Y + g_p$. From (17), we have $g_p = -g_A$, that gives $r = \rho + \varepsilon g_Y - g_A$. Plugging this expression of r in the previous equation gives $\rho + \varepsilon g_Y - g_A + g_Y/\sigma = -g_A + \phi\lambda\sigma L$, and thus

$$g_Y = \frac{\phi\lambda\sigma L - \rho}{\varepsilon + 1/\sigma}. \tag{18}$$

We see that g_Y is an increasing function of ϕ : the more firms can extract on each unit of knowledge, the more they put in research, that spurs growth.

However, even if $\phi = 1$ (perfect exclusion and thus total extraction of surplus) the equilibrium growth rate is lower than the optimal one, given by $g_Y^0 = (\lambda\sigma L - \rho)/\varepsilon$ (see proposition 3). This result is different from the one obtained by Aghion and Howitt in the tangible economy (see also proposition 1) along with the laissez-faire growth may be greater or lower than the optimal one. In fact, in the tangible economy, the innovator extracts a surplus on the demand of the intermediate good in which its idea is embodied: this surplus can be higher or lower than the value of the idea. On the contrary, in the weightless economy, it extracts directly a part of this social value. We know that the length of life of knowledge goods is finite in average, when ideas are infinitely-lived. That is why, even if $\phi = 1$, the rate of growth is sub-optimal.

We can now calculate the number of firms in each consumption good sector. The profit of a firm before the payment of knowledge is $\tilde{\pi} = (pY - L^Y + \tilde{V}\lambda\sigma AL^A - L^A)/N$. Since $\tilde{V}\lambda\sigma A = 1$ from (14), we have $\tilde{\pi} = (pY - L^Y)/N$. The price paid for each firm for one unit of knowledge is $\tilde{V}/N$. Thus, the firm pays $\tilde{V}\lambda\sigma AL^A/N = L^A/N$ to buy knowledge. Then, its total profit is $\pi = \tilde{\pi} - L^A/N = (pY - L^Y - L^A)/N = (pY - L)/N$. If there is free entry, one gets $pY - L = 0$. Using (7), (17) and (18), one obtains

$$N = \frac{(\varepsilon + 1/\sigma)(1-\theta)}{\phi - \rho/\lambda\sigma L}. \tag{19}$$

N is a decreasing function of ϕ. An increase in ϕ means that innovators can appropriate a greater amount of the willingness to pay for knowledge. We have seen that growth is stimulated (see (18)), but simultaneously the cost supported by firms to buy knowledge increases. That is why their number decreases. We see also that an increase in θ, i.e. an increase in the price-elasticity of consumption goods demand, leads to a decrease in N. Indeed, when θ increases, the profit which allows to fund knowledge decreases. That is why the number of firms decreases in the free entry equilibrium.

Remark : if $\dot{A}_t = \lambda\sigma A_t L_t^A / L_t$ and $\dot{L}_t/L_t = n$, one gets $g_c^o = g_A^o = (\phi\lambda\sigma - \rho)/(1+1/\sigma)$, $g_Y^o = g_c^o + n$ and $N = (1-\theta)(\varepsilon + 1/\sigma)/(\phi - \rho/\lambda\sigma)$. Scale effects disappear and the number of firms at equilibrium does not depend on the size of the population.

3.3 Infinite patents on knowledge

Instead of patents on knowledge goods (which disappear when the good is replaced), we now examine an equilibrium in which patents would be directly given to the knowledge embodied in this good. The basic difference is that these patents are infinitely-lived: the innovator receives a payment for an idea, even when the associated knowledge good has disappeared. Then, the value at t of one unit of knowledge is $\tilde{V}_t = \int_t^\infty \phi v_s e^{-\int_t^s r_u du} ds$, that gives after differentiation $r_t = g_{\tilde{V}_t} + \phi v_t / \tilde{V}_t$. Since $v_t = L/A_t$ (see (16)), and $\tilde{V}_t = 1/\lambda\sigma A_t$, (see (14)), one gets at steady-state $r = -g_A + \phi\lambda\sigma L$. From (12), we have at steady-state $r = \rho + \varepsilon g_Y + g_p$, where $g_p = -g_A$ (from (17)). Finally, one gets the equilibrium rate of growth

$$g_Y = \frac{\phi\lambda\sigma L - \rho}{\varepsilon} \tag{20}$$

which is lower than the optimal one, $g_Y^0 = (\lambda\sigma L - \rho)/\varepsilon$ (see proposition 3).

Note that if $\phi = 1$, that is to say if there is total extraction of the willingness to pay by each innovator (i.e. perfect exclusion), the equilibrium is optimal despite the Cournot competition in consumption goods markets. Basically, on account of Cournot competition, the real wage is $A_t(1 + (\theta - 1)/N)$ (see proposition 4), which is lower than the optimal one, $w_t^0 = A_t$ (see proposition 3). However, since the supply of labor is assumed exogenous, there is no distorsion on the labor market. Clearly, in a model with an elastic labor supply, the equilibrium would not be optimal.

As previously, we can also calculate the number of firms in each consumption good sector. Here also, the free entry condition leads to $pY - L = 0$, where $p = 1/A[1 + (\theta - 1)N]$ (see (17)) and $Y = AL^Y$ (see (7)). From (20), we have $g_Y = g_A = \lambda\sigma L^A = (\phi\lambda\sigma L - \rho)/\varepsilon$ that allows to compute L^A, and thus $L^Y = L - L^A = L - (\phi\lambda\sigma L - \rho)/\varepsilon$. Plugging this result in the free entry condition, one gets

$$N = \frac{\varepsilon(1-\theta)}{\phi - \rho/\lambda\sigma L}. \tag{21}$$

Note that, here also, an increase in ϕ or in θ leads to a decrease in N : the interpretation is the same than in the previous case.

Remark : if $\dot{A}_t = \lambda\sigma A_t L_t^A/L_t$, and $\dot{L}_t/L_t = n$, one gets $g_c^o = g_A^o = (\phi\lambda\sigma - \rho)/\varepsilon$ and $N = \varepsilon(1-\theta)/(\phi - \rho/\lambda\sigma)$. Here also, scale effects disappear ant the number of firms does not depend on the population size.

4 Conclusion

The rise in importance of new technology industries leads to a progressive dematerialization of the economy. Using the terminology of this paper, one progressively moves from a tangible economy to a weightless one. The implication for economic growth models is that ideas (i.e. new knowledge) are embodied in knowledge goods, which are non rival, rather that in private intermediate goods.

The purpose of this paper was to extend the Schumpeterian growth theory to the case of a weightless economy. In the latter, since tangible intermediate goods have made way for weightless knowledge goods, one cannot use the standard equilibrium concept of growth theory; on the contrary, the challenge is to propose a new concept, in which knowledge goods (or, eventually, knowledge) are directly priced.

First, we have proposed a formalisation of knowledge accumulation which is slightly different from the Aghion-Howitt's one, and we have shown that it can be easily used to analyse the case of a tangible economy.

Second, in the weightless economy, we have analyzed an equilibrium with Cournot competition and free entry in consumption goods markets, so that firms can buy knowledge (or knowledge goods) despite the non-convexity of technology. In this equilibrium, the real wage and the price of knowledge are lower than the optimal ones, because the price is higher than the marginal cost on consumption goods markets. Then, we have studied the case of "destructive creation", that is to say the case in which the monopoly on a knowledge good disappears when this good is replaced by a new one: we have shown that the output growth rate is always lower than the optimal one, contrary to the result obtained by Aghion-Howitt in a tangible economy. We have also studied the case where infinitely-lived patents directly protect ideas (rather than knowledge goods which embody them). In this case, the equilibrium is optimal if firms are able to extract the total willingness to pay for each unit of knowledge, in spite of the prices distorsions caused by the Cournot competition.

Appendix A : Standard (*"à la Aghion-Howitt"*) equilibrium

In the final good sector, the maximisation of the profit $\pi_t^Y = (L_t^Y)^{1-\alpha} \int_0^1 A_{jt} x_{jt}^{\alpha} dj - w_t L_t^Y - \int_0^1 q_{jt} x_{jt} dj$ leads to

$$(1-\alpha) Y_t / L_t^Y - w_t = 0 \tag{A.1}$$

$$\text{and} \qquad \alpha (L_t^Y)^{1-\alpha} A_{jt} x_{jt}^{\alpha - 1} - q_{jt} = 0. \tag{A.2}$$

In each intermediate good sector j, the profit is $\pi_{jt} = q_{jt} x_{jt} - y_{jt}$, where q_{jt} and y_{jt} are respectively given by (A.2) and (5). After maximisation, one gets

$$x_{jt} = \alpha^{\frac{2}{1-\alpha}} L_t^Y, q_{jt} = \frac{A_{jt}}{\alpha}, \text{ and } \pi_{jt} = \alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha) L_t^Y A_{jt}. \tag{A.3}$$

The sum of the present values of the expected profits in sector j at date t is $\Pi_{jt} = \int_t^{\infty} \pi_{js} e^{-\int_t^s (r_u + \lambda L_{ju}^A) du} ds$. Differentiating with respect to time gives

$$r_t + \lambda L_{jt}^A = \frac{\dot{\Pi}_{jt}}{\Pi_{jt}} + \frac{\pi_{jt}}{\Pi_{jt}}. \tag{A.4}$$

Now we use the usual free entry condition, $w_t = \lambda \Pi_{jt}$, and we consider the symmetric case in which $x_{jt} = x_t, A_{jt} = A_t, L_{jt}^A = L_t^A, \pi_{jt} = \pi_t, \Pi_{jt} = \Pi_t$. Using (A.1) and the free entry condition, we obtain $\Pi_t = \frac{(1-\alpha)\alpha^{2\alpha/(1-\alpha)}}{\lambda} A_t$, that gives $g_{\Pi} = g_A$. Using this result and (A.3), we get $\pi_t / \Pi_t = \lambda \alpha L_t^Y$.

Let us now consider the steady-state, in which L^A (and thus also L^Y) are constant. Using (3) and (4), we have $g_{\Pi} = g_A = g_Y = \lambda \sigma L^A$. Thus (A.4) writes $r + \lambda L^A = g_Y + \lambda \alpha (L - L^A)$, that gives $r = \lambda \alpha L + g_Y (1 - (1+\alpha)/\sigma)$. From the maximisation of utility, we have $r = \rho + \varepsilon g_c$, where $g_c = g_Y$. One gets finally

$$g_Y = \frac{\lambda \alpha L - \rho}{\varepsilon - 1 + (1+\alpha)/\sigma},$$

and all the results given in proposition 1.

Appendix B : Welfare analysis in the tangible economy

The Hamiltonian of the social planner's program is:

$$\begin{aligned} H &= \frac{c^{1-\varepsilon}-1}{1-\varepsilon}e^{-\rho t} + \mu\left[(L^Y)^{1-\alpha}\int_0^1 A_j(x_j)^\alpha dj - Lc - \int_0^1 A_j x_j d_j\right] \\ &+ \int_0^1 \eta_j\left(\lambda\sigma L_j^A \int_0^1 A_h dh\right)dj + \nu\left(L - L^Y - \int_0^1 L_j^A dj\right). \end{aligned}$$

The first-order conditions $\partial H/\partial c = 0,\ \partial H/\partial L^Y = 0,\ \partial H/\partial L_j^A = 0$, and $\partial H/\partial x_j = 0$ yield

$$c^{-\varepsilon}e^{-\rho t} - \mu L = 0 \tag{B.1}$$

$$\mu(1-\alpha)Y/L^Y - \nu = 0 \tag{B.2}$$

$$\eta_j\lambda\sigma A - \nu = 0 \tag{B.3}$$

$$\alpha(L^Y)^{1-\alpha}x_j^{\alpha-1} - 1 = 0 \tag{B.4}$$

Moreover, $\partial H/\partial A_j = -\dot{\eta}_j$ yields

$$\mu\left[(L^Y)^{1-\alpha}x_j^\alpha - xj\right] + \int_0^1 \eta_j\lambda\sigma L_j^A dj = -\dot{\eta}_j \tag{B.5}$$

From (B.4), we have $x_j = x = \alpha^{1/(1-\alpha)}L^Y,\ \forall j$, that implies $Y = A(L_Y)^{1-\alpha}x^\alpha$, and thus $g_Y = g_A$ at steady-state.
Log-differentiating (B.1) and (B.2) with respect to time gives $-g_\mu = \varepsilon g_c + \rho = g_A - g_\nu$.
From (B.3), one gets $\eta_j = \eta = \nu/\lambda\sigma A,\ \forall j$, and $g_\eta = g_\nu - g_A$.
From (B.5), we have $(\mu/\eta)[(L^Y)^{1-\alpha}x^\alpha - x] + \lambda\sigma L^A = -g_\eta$.

Using (B.2) and (B.3)to compute μ/η, the first term on the left hand side is equal to $\lambda\sigma L^Y$. Moreover, we know that $-g_\eta = -g_\mu = \varepsilon g_c + \rho$. Thus, one gets

$$g_c = g_Y = \frac{\lambda\sigma L - \rho}{\varepsilon}$$

Now, in order to compute the prices which sustain the optimum, we examine the behavior of the different sectors.

In the final good sector, the profit is $\pi_t^Y = (L_t^Y)^{1-\alpha} \int_0^1 A_{jt} x_{jt}^{\alpha} dj - w_t L_t^Y - \int_0^1 q_{jt} x_{jt} dj$. The maximisation of π_t^Y with respect to L_t^Y and x_{jt} gives

$$(1-\alpha) Y_t / L_t^Y - w_t = 0 \tag{B.6}$$

and $$\alpha (L_t^Y)^{1-\alpha} A_{jt} x_{jt}^{\alpha-1} - q_{jt} = 0. \tag{B.7}$$

Moreover, the marginal profitability of one unit of knowledge A_{jt} is

$$v_{jt}^Y = \frac{\partial \pi_{jt}^Y}{\partial A_{jt}} = (L_t^Y)^{1-\alpha} x_{jt}^{\alpha}. \tag{B.8}$$

In any sector i, the expected profit in the research activity is $\pi_{it}^A = V_{it} \lambda \sigma A_t L_{it}^A - w_t L_{it}^A$. Indeed, if labor L_{it}^A is engaged in research on interval $(t, t+\Delta t)$, the probability to get an innovation is $\lambda L_{it}^A \Delta t$ and, in this case, the increase in knowledge is σA_t. The free entry assumption gives $V_{it} \lambda \sigma A_t - w_t = 0$, that implies[7]

$$V_{it} = V_t = \frac{w_t}{\lambda \sigma A_t} \text{ , } \forall i. \tag{B.9}$$

The marginal profitability of one unit of knowledge A_j is $v_{jt}^i = \partial \pi_{it}^A / \partial A_{jt} = V_{it} \lambda \sigma L_{it}^A = w_t L_{it}^A / A_t$. Thus, the total marginal profitability of one unit of knowledge A_j in the whole research sector, $v_{jt}^A = \int_0^1 v_{jt}^i di$, is

$$v_{jt}^A = v_t^A = V_t \lambda \sigma L_t^A = \frac{w_t}{A_t} L_t^A \text{ , } \forall j. \tag{B.10}$$

Finally, *in the intermediate sector j*, the profit is $\pi_{jt}^x = x_{jt}(q_{jt} - A_{jt})$. Since there is now perfect competition, the price is equal to the marginal cost:

$$q_{jt} = A_{jt}. \tag{B.11}$$

[7]In fact, condition (B.9) is the classic optimality condition for a public good first derived by Samuelson [21, 22]: see for instance Mas-Colell, Whinston and Green [14]. Indeed, V_{it} is the sum of the present values of users' marginal benefits from one unit of knowledge, and $w_t / \lambda \sigma A_t = w_t L_{it}^A / \dot{A}_{it}$ is the cost of this unit.

The marginal profitability of one unit of knowledge A_j in this sector is

$$v_{jt}^x = \frac{\partial \pi_{jt}^x}{\partial A_{jt}} = -x_{jt}. \tag{B.12}$$

This marginal profitability is negative because an increase in A_j leads to an increase in the cost of production of the intermediate good x_j (see (5)).

It is now easy to compute the equilibrium solutions.

From (B.7) and (B.1), one gets $x_{jt} = x_t = \alpha^{1/(1-\alpha)} L_t^Y$, for all j, that is the optimal production of intermediate goods. Thus (4) becomes $Y_t = (L_t^Y)^{1-\alpha} A_t x_t^\alpha = \alpha^{\alpha/(1-\alpha)} A_t L_t^Y$, and (B.6) gives $w_t = (1-\alpha)\alpha^{\alpha/(1-\alpha)} A_t$.

From (B.9), one gets $V_t = (1-\alpha)\alpha^{\alpha/(1-\alpha)}/\lambda\sigma$, that implies $g_V = 0$. From (B.8) and (B.12), we have $v_{jt}^Y = v_t^Y = \alpha^{\alpha/(1-\alpha)} L_t^Y$ and $v_{jt}^x = v_t^x = -\alpha^{1/(1-\alpha)} L_t^Y$.

Then, at steady-state, the basic arbitrage condition $r = (v^Y + v^A + v^x)/V$ becomes $r = \frac{\lambda\sigma L^Y}{1-\alpha} + \lambda\sigma L^A - \frac{\alpha\lambda\sigma L^Y}{1-\alpha} = \lambda\sigma L$. Since $r = \rho + \varepsilon g_Y$ from the household behavior, one gets $g_Y = (\lambda\sigma L - \rho)/\varepsilon$, that is the optimal rate of growth.

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