Non-Renewable Resources and Growth with Vertical Innovations : Optimum, Equilibrium and Economic Policies *

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Abstract

We consider a Schumpeterian model of endogenous growth with creative destruction in which we introduce a non-renewable natural resource. We characterize the optimum and the equilibrium paths, and we derive the precise levels of economic policy instruments that allow the implementation of the optimum. Moreover, we study the effects of these policies on the relevant steady-state variables, in particular the rate of extraction of the resource.

Keywords: Non-Renewable Resources, Endogenous Growth, Vertical Innovations, Optimum, Equilibrium, Economics Policies, $R\&D$, Knowledge.

1 Introduction

It may seem paradoxical to ask whether positive infinite growth is possible despite the fact that the production process uses non-renewable natural resources. For several decades, this question has given birth to an important economic literature, most notably in growth theory. This literature has established that under some properties of the resource and some technological characteristics, positive long-run growth is possible even if the stock of the natural resource is finite.

In fact, many questions can be addressed and the following ones seem especially relevant to us:

- Is continuous growth compatible with a finite stock of natural resources ?
- What is the optimal path, and what are its properties ? In particular, even if positive growth is possible, is it optimal ?
- What are the properties of the equilibrium path ? Is it optimal ? If not, are there economic policies that allow the implementation of the optimum ? More generally, what are the effects of these policies ?

In the 1970s, Dasgupta and Heal (1974), Solow (1974), Stiglitz (1974), and Garg and Sweeney (1978), among others, analyzed the problem in "standard" growth models ("à la Ramsey"). They showed that under certain technological conditions, positive long-run growth is possible in the presence of non-renewable natural resources. Moreover, they studied the optimal and the equilibrium paths. More recently, this analysis was relaunched within the context of endogenous growth models. In this new framework, the first-order conditions that characterize the optimum are, in some cases, not fulfilled at equilibrium, essentially because of the intertemporal externalities arising from the fact that knowledge is a public good. Indeed, if Barbier (1999) and Aghion and Howitt (1998) focus mainly on optimality aspects, and Scholz and Ziemes (1999) on equilibrium, Schou (1996) and Grimaud (2000) make use of a model of horizontal innovations to show that while positive optimal long-run growth is possible, the equilibrium path is not optimal.

In this paper, we use a Schumpeterian model of endogenous growth (i.e., with vertical innovations) "à la Aghion-Howitt (1992)" to tackle this problem which has generally been done with "à la Romer (1990)" models and raise the same questions as above. In fact, our results partly resemble those obtained by authors working with "standard" growth models (e.g., Stiglitz

(1974), and Garg and Sweeney (1978)), but we also find noticeable differences that raise new questions that we investigate. Moreover, we employ a very simple framework (in particular, we assume that there is a single intermediate good) so as to avoid computational complexity and to highlight the relevant phenomena.

In our model, the natural resource is necessary but non-essential (as defined by Dasgupta and Heal (1979)), and a positive long-run growth is always possible if the $R\&D$ sector is productive enough. However, we find that this positive long-run growth may be non-optimal, because the optimum could also be characterized by a negative growth of output. As in Schou's (1996) paper, we show that, at equilibrium, growth (which can be positive or negative) is not optimal. However, contrary to Schou who finds that growth is under-optimal, we show that it may be either under or over-optimal. We then demonstrate that there exist economic policy tools that allow the implementation of the optimum and we compute the precise levels of these tools that equate both paths. We also perform some comparative statics exercises to analyze how the relevant variables of the model, in particular, the rate of extraction of the resource, are affected by these policy tools. Throughout the paper, we focus on optimum and equilibrium along the balanced growth paths only, i.e., on paths along which the growth rate of any variable is constant.

The remainder of the paper is organized in five sections. In section 2, we present the model. We characterize the optimum in section 3, and the equilibrium in section 4. In the latter section, we also compare the optimum and the equilibrium and we analyze the impact of the economic policy tools on the relevant variables. Section 5 is devoted to the implementation of the optimum by means of these tools. A summary and some concluding remarks are given in section 6.

2 The model

There are four goods in the economy: an homogeneous good (Y) used only for consumption (c), an intermediate good (x), labor (L) and a non-renewable resource (R).

At each date t, the final output is produced by a competitive sector according to

$$Y_t = A_t x_t^{\alpha} R_t^{1-\alpha} \qquad 0 < \alpha < 1, \tag{1}$$

where x_t and R_t are the amounts of intermediate good and resource used to produce Y_t, and A_t is the level of technology at time t : see (3) below.

Concerning the intermediate good sector, we use the Aghion-Howitt approach (see Aghion-Howitt (1998), chapter 2). We assume that the labor supply is fixed and has two competing uses. First, it can produce the intermediate good, one for one. Second, it can be used for research. Normalizing the total flow of labor to one ($L = 1$), we have, at each time t :

$$1 \;=\; x_t + n_t, \tag{2}$$

where x_t is the amount of labor used in manufacturing (recall that, as in Aghion and Howitt, due to the one for one technology, x_t is also the amount of intermediate good) and n_t is the amount of labor used for research.

If one unit of labor is used for research, innovations arrive randomly with a Poisson arrival rate $\lambda > 0$. Each innovation τ replaces the old one $\tau - 1$ (τ is an index for innovations), and is such that

$$A_\tau = \gamma A_{\tau-1}, \quad \gamma > 1, \qquad \text{for all } \tau. \tag{3}$$

An innovation consists in a new technology that is embodied inside a new kind of intermediate good. This new intermediate good will then be produced by a monopoly and sold to the final sector until replaced by a new good (when the next innovation occurs). Following Aghion and Howitt, we assume that the amount of labor used for research is determined by an arbitrage condition which states that the wage (i.e., the cost of one unit of labor) is equal to the expected value of this unit used for research.

If we denote by S_0 the initial stock of resource, the stock at t is given by

$$S_t \;=\; S_0 - \int_0^t R_\nu d_\nu, \tag{4}$$

and we assume that there are no extraction costs.

Starting from equation (1), we have $g_Y = g_A + (1-\alpha) g_R$ (where $g_z = \frac{\dot{z}}{z}$ is the growth rate of any variable z). Assume that we are at steady-state ; the resource constraint $\int_0^{+\infty} R_t dt \leq S_0$ can be rewritten $\int_0^{+\infty} R_0 e^{(g_Y - g_A)t/(1-\alpha)} \leq S_0$. This integral converges if and only if $g_A > g_Y$, in other words, if and only if knowledge grows faster than output.

The utility function of the infinitely lived representative agent is

$$\int_0^\infty \frac{c_t^{1-\varepsilon}}{1-\varepsilon} e^{-\rho t} dt, \tag{5}$$

where ρ is a positive rate of time preference and $1/\varepsilon$ is the intertemporal elasticity of substitution.

3 Welfare analysis

This section essentially tackles two objectives. First, we set out to obtain a characterization of balanced optimal growth path. Second, we study the impact of parameter variations on this path, and we compare our results with those of more standard "exogenous growth models" (e.g. Stiglitz (1976) and Garg and Sweeney (1978)).

In a first step, it is useful to observe that, *on average*, the law of motion of A_t is

$$\dot{A}_t = (\gamma - 1)\lambda n_t A_t, \quad \text{for all} \quad t. \tag{6}$$

Indeed, if n_t is the quantity of labor devoted to research at t, then the expected level of the random variable A at $t + \Delta t$ is

$$E(A_{t+\Delta t}) = \lambda n_t \Delta t \gamma A_t + (1 - \lambda n_t \Delta t)A_t = A_t + \lambda(\gamma - 1)n_t A_t \Delta t$$

that yields (6) when Δt tends to zero.

We are now ready to characterize the optimal path.

3.1 Existence and characterization of the steady-state optimum

The program of the social planner is to maximize the utility

$$\begin{array}{ll} \int_0^\infty \frac{1}{1-\varepsilon}(A_t(1-n_t)^\alpha R_t^{1-\alpha})^{1-\varepsilon} e^{-\rho t} dt & \\ \text{subject to} & \\ \dot{A}_t = \lambda(\gamma - 1)n_t A_t & (\mu_t) \\ \dot{S}_t = -R_t & (\nu_t). \end{array}$$

Proposition 1 *A balanced optimal growth path is a set of quantities and growth rates that take the following values*

$$n^o = \frac{\alpha}{\varepsilon}\left(1 - \frac{\rho}{\lambda(\gamma - 1)}\right) + 1 - \alpha \tag{7}$$

$$x^o = 1 - n^o \tag{8}$$

$$g_A^o = \frac{-\alpha\rho + \lambda(\gamma - 1)(\varepsilon + \alpha - \alpha\varepsilon)}{\varepsilon} \tag{9}$$

$$g_Y^o = g_c^o = \frac{\lambda(\gamma - 1) - \rho}{\varepsilon} \tag{10}$$

$$g_R^o = g_S^o = \frac{\lambda(\gamma - 1)(1 - \varepsilon) - \rho}{\varepsilon}. \tag{11}$$

The (unique) transversality condition is

$$(1-\varepsilon)\lambda(\gamma-1) < \rho. \tag{12}$$

This condition ensures that $n^o < 1$. *In order to have* $n^o \geq 0$, *it is furthermore necessary that* $\varepsilon > \frac{\alpha}{1-\alpha}\left(\frac{\rho}{\lambda(\gamma-1)} - 1\right)$.

First of all, observe that, since ε is positive, the transversality condition is equivalent to $g_R^o = g_S^o < 0$.

A second remark is that when the tranversality condtion holds, the integral in the stock constraint (4) is convergent. Then, since the resource stock is exhausted along an optimal path, we obtain

$$R_0 = -g_A S_0 = (\rho - \lambda(\gamma-1)(1-\varepsilon))S_0/\varepsilon.$$

Thirdly, since (1) yields $g_Y^o = g_A^o + (1-\alpha)g_R^o$, we have $g_Y^o < g_A^o$ (because $g_R^o < 0$). That is, along the optimal path, the level of technological knowledge in the economy grows faster than output.

Fourth, observe from (10) that growth is positive if and only if $\lambda(\gamma-1) - \rho \geq 0$, and thus $1 - \rho/\lambda(\gamma-1) \geq 0$: along an optimal path, output grows if and only if the effectiveness of the $R\&D$ sector is greater than the psychological discount rate. Hence, an optimal negative growth of output is possible : we confirm, in an endogenous growth framework, the opinion of Solow (1974-a) who said : "even when the technology and the resource base could permit a plateau level of consumption per head, or even a rising standard of living, positive social time preference might in effect lead society to prefer eventual extinction, given the drag exercised by exhaustible resources".

We can gather these remarks as follows.

If $\rho < \lambda(\gamma-1)$, we have $\frac{\alpha}{1-\alpha}\left(\frac{\rho}{\lambda(\gamma-1)} - 1\right) < 0 < 1 - \frac{\rho}{\lambda(\gamma-1)}$. In this case, we know that $g_Y^o > 0$. The transversality condition is satisfied only if $\varepsilon > 1 - \rho/\lambda(\gamma-1)$. Then, we have $n^o < 1$. Since $\frac{\alpha}{1-\alpha}\left(\frac{\rho}{\lambda(\gamma-1)} - 1\right) < 0$ and $\varepsilon > 0$, we have $\varepsilon > \frac{\alpha}{1-\alpha}\left(\frac{\rho}{\lambda(\gamma-1)} - 1\right)$: therefore $n^o > 0$.

If $\rho > \lambda(\gamma-1)$, we have $1 - \frac{\rho}{\lambda(\gamma-1)} < 0 < \frac{\alpha}{1-\alpha}\left(\frac{\rho}{\lambda(\gamma-1)} - 1\right)$. Then we know that $g_Y^o < 0$. Since ε is positive, we have $\varepsilon > 1 - \frac{\rho}{\lambda(\gamma-1)}$, and thus $(1-\varepsilon)\lambda(\gamma-1) > \rho$: in other words, the transversality condition is always satisfied and $n^o < 1$. Finally, n^o is positive only if $\varepsilon > \frac{\alpha}{1-\alpha}\left(\frac{\rho}{\lambda(\gamma-1)} - 1\right)$. In this case, the optimal rate of growth is negative because the effectiveness of the $R\&D$ sector ($\lambda(\gamma-1)$) is lower than the psychological discount rate (ρ).

These results are summarized in Figure 1.

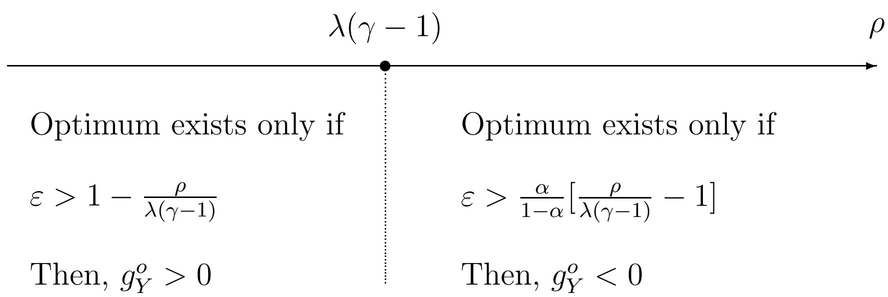

Figure 1: Existence of interior optimum

Remark : if $\rho > \lambda(\gamma-1)$, and ε tends to $\frac{\alpha}{1-\alpha}[\frac{\rho}{\lambda(\gamma-1)} - 1]$, then n^o tends to zero (no $R\&D$). Thus, g_A^o also tends to zero and g_Y^o tends to $(1-\alpha)g_R^o$. This is the case where we have the quickest decay.

Proof 1 The current value Hamiltonian of the program presented above is

$$H = \frac{1}{1-\varepsilon} A_t^{1-\varepsilon}(1-n_t)^{\alpha(1-\varepsilon)} R_t^{(1-\alpha)(1-\varepsilon)} + \mu_t \lambda(\gamma-1) n_t A_t - \nu_t R_t.$$

where μ_t and ν_t are the costate variables.

The first order conditions $\partial H / \partial n_t = 0$ and $\partial H / \partial R_t = 0$ yield

$$\mu_t = \frac{\alpha A_t^{-\varepsilon}(1-n_t)^{\alpha(1-\varepsilon)-1} R_t^{(1-\alpha)(1-\varepsilon)}}{\lambda(\gamma-1)} \tag{13}$$

$$\text{and} \quad \nu_t = (1-\alpha) A_t^{1-\varepsilon}(1-n_t)^{\alpha(1-\varepsilon)} R_t^{(1-\alpha)(1-\varepsilon)-1}. \tag{14}$$

Moreover, $\partial H / \partial A_t = \rho \mu_t - \dot{\mu}_t$ and $\partial H / \partial S_t = \rho \nu_t - \dot{\nu}_t$ yield

$$g_\mu = \rho - \frac{\lambda(\gamma-1)}{\alpha} + \frac{\lambda(\gamma-1)(1-\alpha)}{\alpha} n_t \tag{15}$$

$$\text{and} \quad g_\nu = \rho. \tag{16}$$

At steady state, all variables grow at constant rates. Thus $g_A^o = \lambda(\gamma-1)n_t^o$ is constant, and then n_t^o and $x_t^o = 1 - n_t^o$ are constant. From now on, we thus can drop the time subscripts.

From (13), we obtain $g_\mu = -\varepsilon g_A^o + (1-\alpha)(1-\varepsilon)g_R^o = -\varepsilon\lambda(\gamma-1)n^o + (1-\alpha)(1-\varepsilon)g_R^o$.

Then, using (15), we have

$$\rho - \frac{\lambda(\gamma-1)}{\alpha} + \frac{\lambda(\gamma-1)(1-\alpha+\varepsilon\alpha)n^o}{\alpha} \quad = \quad (1-\alpha)(1-\varepsilon)g_R^o. \qquad (17)$$

From (14), we obtain $g_\nu = (1-\varepsilon)g_A^o + ((1-\alpha)(1-\varepsilon)-1)g_R^o = (1-\varepsilon)\lambda(\gamma-1)n^o - (\varepsilon+\alpha-\alpha\varepsilon)g_R^o$. Using (16), we have

$$\rho - (1-\varepsilon)\lambda(\gamma-1)n^o \quad = \quad (\alpha\varepsilon-\varepsilon-\alpha)g_R^o. \qquad (18)$$

Eliminating g_R^o between (17) and (18) gives, after some calculations,

$$n^o \quad = \quad \frac{\alpha}{\varepsilon}\left(1 - \frac{\rho}{\lambda(\gamma-1)}\right) + (1-\alpha),$$

that is, the value given by (7).

From this result, all growth rates (9) - (10) - (11) can be easily computed.

Using (13), (15) and the fact that $g_A^o = \lambda(\gamma-1)n^o$, the first transversality condition,

$$\lim_{t\to+\infty} \mu_t A_t^o e^{-\rho t} = 0,$$

implies

$$\rho - \frac{\lambda(\gamma-1)}{\alpha} + \frac{\lambda(\gamma-1)(1-\alpha)}{\alpha}n^o + \lambda(\gamma-1)n^o - \rho < 0.$$

This inequality comes from $n^o < 1$ and thus, using (7), $(1-\varepsilon)\lambda(\gamma-1) < \rho$ (see (12)).

Using (11) and (16) the second transversality condition,

$$\lim_{t\to+\infty} \nu_t S_t^o e^{-\rho t} = 0,$$

implies $g_S^o < 0$, and thus $(1-\varepsilon)\lambda(\gamma-1) < \rho$.

Hence, there is a unique transversality condition given by (12). ■

3.2 Properties of the steady-state optimal path

Our objective is now to present some properties of the optimal path and to compare them with (more standard) properties that have been obtained in "exogenous growth models". For instance, Stiglitz (1974) uses the Cobb-Douglas technology $Y = K^{\alpha_1}L^{\alpha_2}R^{\alpha_3}e^{\eta t}$, where η is the rate of technological progress (Garg and Sweeney (1978) use the same technology).

The main results are stated in Table 1: there, we present the signs of the partial derivatives of n^o, g_A^o, g_Y^o and g_R^o with respect to the corresponding parameters of the model in columns. Letters S and GS indicate that similar results have been obtained by Stiglitz and Garg-Sweeney, respectively.

	$\xi = \lambda$	$\xi = \gamma$	$\xi = \rho$	$\xi = \varepsilon$
$\frac{\partial n^o}{\partial \xi}$	> 0	> 0	< 0	< 0 if $g_Y^o > 0$
$\frac{\partial g_A^o}{\partial \xi}$	> 0	> 0	< 0	< 0 if $g_Y^o > 0$
$\frac{\partial g_Y^o}{\partial \xi}$	S GS > 0	S GS > 0	S GS < 0	S < 0 if $g_Y^o > 0$
$\frac{\partial g_R^o}{\partial \xi}$	S < 0 if $\varepsilon > 1$	S < 0 if $\varepsilon > 1$	< 0	S < 0 if $g_Y^o > 0$

Table 1: Properties of the optimal path

- First of all, as noted above, growth is positive if and only if $\lambda(\gamma - 1) > \rho$, while in Stiglitz (1974) it is positive if and only if $\eta/\alpha_3 > \rho$ (see formula (31)). Both results are very close; indeed, in our Schumpeterian model, the term $\lambda(\gamma - 1)$, which characterizes the effectiveness of the $R\&D$ sector, can be seen as the *exogenous* part of technological progress (that is, η in Stiglitz).

  Let us now comment the effects of an *increase in* λ *or* γ on the relevant variables. When λ or γ increases, it becomes socially more efficient to invest in the $R\&D$ sector (relative to the production of the intermediate good), thus n^o increases (and x^o decreases). More investment in this sector means a higher growth of technological knowledge, that is, g_A^o increases (see (6)). The social planner will thus choose a higher growth of output; yet, if the elasticity of marginal utility, ε, is higher than one (which means that consumers derive relatively more utility from a uniform path of consumption and thus of output), then a smaller growth rate of the resource extraction will soften the increase of g_Y^o, because $g_Y^o = g_A^o + (1-\alpha)g_R^o$. This explains our result in the last line of Table 1 : $\partial g_R^o/\partial\lambda < 0$ if $\varepsilon > 1$.

In Stiglitz (1974) (see p.134), it is shown that the sensitivity of this rate with respect to the rate of technological progress η depends on the inverse of the intertemporal elasticity of substitution ε : $\partial(R^o/S^o)/\partial\eta$ is positive (resp. negative) if ε is higher (resp. lower) than one. The present model confirms these results.

- An *increase in* ρ means that households obtain more utility from current consumption relative to future consumption. Then, investment in $R\&D$, which implies a sacrifice today for the sake of future gains, does not interest them. As a result, n^o must decrease, and by consequence g_A^o will do so, too. Moreover, a higher ρ means a lower growth of consumption, because consumers prefer present consumption, and thus lower output growth. Then, g_R^o will also decrease, because the social planner, in order to produce more today, will extract the resource with less care for its future exhaustion.

- An *increase in* ε means that the elasticity of marginal utility increases : households will derive more utility from a uniform consumption path, ceteris paribus. If $g_Y^o > 0$, then a social planner will not invest in $R\&D$ (investing would imply a higher consumption tomorrow) and thus n^o and g_A^o will decrease. At the same time, he will choose a lower consumption growth rate (i.e., output growth rate) to achieve a flatter path, and therefore a lower growth rate of the resource extraction. If $g_Y^o < 0$, the opposite results stem from the same fact : the planner tries to flatten the consumption path, and thus lowers g_A^o and g_R^o.

  In Stiglitz (1974), we have $\partial(R^o/S^o)/\partial\varepsilon \geq 0$ (resp. ≤ 0) if $\eta/\alpha_3 \geq \rho$ (resp. $\leq \rho$) ; our result is similar. Recall that, in both models, conditions $\eta/\alpha_3 \geq \rho$ and $\lambda(\gamma-1) \geq \rho$ are necessary and sufficient for the economy to grow at a positive rate.

4 Equilibrium

4.1 Existence and characterization of the steady-state equilibrium

4.1.1 Basic assumptions and behavior of agents

The price of good Y is normalized to one and w_t, p_t, p_t^R and r_t are, respectively, the wage, the price of the intermediate good, the price of the resource and the interest rate on a perfect financial market. In order to eliminate the two market failures arising from the monopolistic character of the intermediate-good sector and from the intertemporal spillover, we use two public tools : a research subsidy (σ) and a demand subsidy for the intermediate good (θ). We now examine the behavior of the different agents.

a) At each time t, the profit in the final sector is

$$\pi_t^Y = A_t x_t^{\alpha} R_t^{1-\alpha} - p_t(1-\theta_t)x_t - p_t^R R_t.$$

Differentiating with respect to x_t and R_t and equating to zero gives the two following first order conditions :

$$x_t = \left(\frac{\alpha A_t}{p_t(1-\theta_t)}\right)^{\frac{1}{1-\alpha}} R_t \tag{19}$$

$$p_t^R = (1-\alpha)A_t\left(\frac{x_t}{R_t}\right)^{\alpha}. \tag{20}$$

b) In the intermediate good sector, the monopoly at time t maximizes the profit $\pi_t^m = p_t x_t - w_t x_t$ where the demand function is given by (19). This maximization yields

$$x_t = \left(\frac{\alpha^2 A_t}{(1-\theta_t)w_t}\right)^{\frac{1}{1-\alpha}} R_t, \tag{21}$$

or, equivalently,

$$p_t = \frac{w_t}{\alpha}. \tag{22}$$

Now, let us look at the $R\&D$ side.

Assume that an innovation occurs at t. Then the profit at $s(s > t)$ is a random variable $\tilde{\pi}_s$ that takes the value $\pi_s^m > 0$ with probability $exp(-\int_t^s \lambda n_u du)$ (the probability that there is no innovation between t and s), and 0 with probability $1 - exp(-\int_t^s \lambda n_u du)$. The expected value of $\tilde{\pi}_s$ is $E(\tilde{\pi}_s) = \pi_s^m exp(-\int_t^s \lambda n_u du)$ and its present value at t is $\pi_s^m exp(-\int_t^s (r_u + \lambda n_u) du)$. Thus, the value of an innovation at t, that is, the sum of the present values of expected profits, is

$$V_t = \int_t^\infty \pi_s^m e^{-\int_t^s (r_u + \lambda n_u) du} ds. \tag{23}$$

At t, the cost of one unit of labor per unit of time is $w_t(1 - \sigma_t)$. Simultaneously, the probability of an innovation is λ, that gives the expected pay-off λV_t. Hence, the arbitrage condition is (see for instance Aghion-Howitt (1998), chapter 2)

$$w_t(1 - \sigma_t) = \lambda V_t. \tag{24}$$

c) On the competitive natural resource market, the maximization of the profit function

$$\int_t^\infty p_s^R R_s e^{-\int_t^s r_u du} ds, \quad \text{for all} \quad t$$

subject to $\dot{S}_s = -R_s, S_s \geq 0, R_s \geq 0, s \geq t$, leads to the "Hotelling rule" :

$$\frac{\dot{p}_t^R}{p_t^R} = r_t, \quad \forall\, t. \tag{25}$$

As usual, the transversality condition of this problem is

$$\lim_{t \to +\infty} S_t = 0,$$

that is, an asymptotic exhaustion of the resource stock.

d) The government's budget constraint is

$$\int_0^\infty (\theta_t p_t x_t + \sigma_t w_t n_t - T_t) e^{-\int_0^t r_u du} dt = 0 \tag{26}$$

where T_t is a lump-sum tax used to finance the research subsidy $(\sigma_t w_t n_t)$ and the subsidy for the intermediate good $(\theta_t p_t x_t)$. θ_t and σ_t are chosen by the government in order to maximize welfare (see section 5 and proposition 6 below). The choice of any profile $(T_t)_{t=0}^{+\infty}$ simultaneously determines the profile of government borrowings, so that the government's budget constraint is satisfied at each time t.

e) Finally, the representative household maximizes the utility function

$$\int_0^\infty \frac{c_t^{1-\varepsilon}}{1-\varepsilon} e^{-\rho t} dt$$

subject to

$$\dot{B}_t = w_t + p_t^R R_t + \pi_t^m - T_t - c_t, \text{ for all } t,$$

where B_t is the stock of bonds at t.

At each time t, the research sector borrows $w_t(1-\sigma_t)n_t = \dot{B}_t$ from households on the financial market. Once one innovation has occured, the monopolist uses its profits π_t^m to make the interest payments. Thus, we have $\pi_t^m = r_t B_t + \hat{\pi}_t^m$, where $\hat{\pi}_t^m$ (which can be positive or negative) is the profit distributed to the household which owns the firm. Observe that, on the financial market, the households' total lendings are $w_t + p_t^R R_t + \pi_t^m - T_t - c_t$. Replacing π_t^m by $p_t x_t - w_t x_t$, T_t by $-\theta_t p_t x_t - \sigma_t w_t x_t$, and c_t by $y_t = p_t(1-\theta_t)x_t - p_t^R R_t$, we obtain exactly $w_t(1-\sigma_t)n_t$, that is, the research sector's borrowings.

The above maximization leads to the usual condition :

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\varepsilon}. \tag{27}$$

4.1.2 Computation of the equilibrium

Until now we used time subscripts for the two subsidies θ and σ ; indeed we do not see any reason why they would not be time dependent outside the steady-state. Henceforth, we drop time subscripts for these two variables since, as we said before, we only study optimum and equilibrium at steady-state and similar studies (for example Barro and Sala-i-Martin (1995)) show that these tools are constant in this case. In Proposition 2, we show that with θ and σ constant, we can characterize the steady-state equilibrium paths (we use the symbol e for equilibrium, except for prices, that do not appear at optimum).

Proposition 2 *A balanced equilibrium growth path is a set of quantities, prices and rates of growth that take the following values :*

Quantities :

$$n^e = \frac{\lambda\gamma(1-\alpha)(\alpha+\varepsilon(1-\alpha))-\alpha(1-\sigma)\rho}{(\lambda\alpha(1-\sigma)+\lambda\gamma(1-\alpha))(\alpha+\varepsilon(1-\alpha))+\alpha(1-\sigma)\lambda(\gamma-1)(\varepsilon-(1-\alpha)(1-\varepsilon))} \tag{28}$$

$$x^e = 1-n^e \tag{29}$$

$$A_t^e = A_0 e^{g_A^e t} \tag{30}$$

$$Y_t^e = c_t^e = A_t^e (x^e)^\alpha (R_t^e)^{1-\alpha} \tag{31}$$

$$R_t^e = R_0 e^{g_R^e t} \tag{32}$$

Prices :

$$r = \frac{\varepsilon\lambda(\gamma-1)n^e+\alpha\rho}{\alpha+\varepsilon(1-\alpha)} \tag{33}$$

$$w_t = \frac{\alpha^2 A_t^e}{(1-\theta)(x_t^e)^{1-\alpha}}(R_t^e)^{1-\alpha} \tag{34}$$

$$p_t = \frac{w_t}{\alpha} \tag{35}$$

$$p_t^R = (1-\alpha)A_t^e\left(\frac{x^e}{p_t}\right)^\alpha \tag{36}$$

Rates of growth :

$$g_n^e = g_x^e = g_r = 0 \tag{37}$$

$$g_Y^e = g_c^e = g_w = g_p = \frac{r-\rho}{\varepsilon} = \frac{\lambda(\gamma-1)n^e-\rho(1-\alpha)}{\alpha+\varepsilon(1-\alpha)} \tag{38}$$

$$g_A^e = (\gamma-1)\lambda n^e \tag{39}$$

$$g_S^e = g_R^e = g_Y^e - g_{p^R} = \frac{\lambda(\gamma-1)(1-\varepsilon)n^e-\rho}{\alpha+\varepsilon(1-\alpha)} \tag{40}$$

$$g_{p^R} = r \tag{41}$$

Proof 2 Let us observe that in proposition 2, several formulas (namely (29), (31), (34), (35), (36), (39) and (41)) come directly from the definition of the model and the agents' behavior. In order to obtain the others, we proceed in two stages. First, we obtain (33), (38) and (40), that is to say r, g_Y^e and g_R^e as functions of n^e. Second, we calculate n^e, the quantity of labor devoted to research, that is, the central variable of the model.

From (6), (20) and (25), we have $g_A^e = (\gamma-1)\lambda n^e$, $g_{p^R} = g_A^e - \alpha g_R^e$, and $g_{p^R} = r$. Thus we have $g_R^e = (\lambda(\gamma-1)n^e - r)/\alpha$. In the same way, from (1), (6) and (27), we have $g_Y^e = g_A^e + (1-\alpha)g_R^e$, $g_A^e = (\gamma-1)\lambda n^e$ and

$g_c^e = g_Y^e = (r-\rho)/\varepsilon$. Thus we obtain $g_R^e = ((r-\rho)/\varepsilon - (\gamma-1)\lambda n^e))/(1-\alpha)$. Simplifying g_R^e, we obtain $r = (\varepsilon\lambda(\gamma-1)n^e + \alpha\rho)/(\alpha+\varepsilon(1-\alpha))$, that is the formula (33) in proposition 1.

Replacing r by this value in $g_R^e = (\lambda(\gamma-1)n^e - r)/\alpha$ and $g_Y^e = (r-\rho)/\varepsilon$ gives easily $g_R^e = (\lambda(\gamma-1)(1-\varepsilon)n^e - \rho)/(\alpha+\varepsilon(1-\alpha))$ and $g_Y^e = (\lambda(\gamma-1)n^e - \rho(1-\alpha))/(\alpha+(1-\alpha))$, that is, formulas (40) and (38) in proposition 2.

Now our objective is to compute n^e. We start from the arbitrage condition (24) : $w_t(1-\sigma) = \lambda V_t$, in which V_t is given by (23). In order to calculate V_t, we first observe that r and n^e are constant at the steady-state. Second, we have to calculate the profit $\pi_t^m = p_t x_t - w_t x_t$ in the intermediate good sector. Since $p_t = w_t/\alpha$ (see (22)) and using the expression of x_t given by (21), we have

$$\pi_t^m \quad = \quad \frac{A_t R_t^{1-\alpha}}{(1-\theta)^{\frac{1}{1-\alpha}}}\tilde{\pi}_t^m, \tag{42}$$

where we have $\tilde{\pi}_t^m = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\tilde{w}_t^{\frac{\alpha}{\alpha-1}}$ and $\tilde{w}_t = w_t/A_t R_t^{1-\alpha}$. In fact, we introduce these two variables because they are constant at the steady-state. The free entry condition, $w_t(1-\sigma) = \lambda V_t$, can be written

$$\tilde{w} A_t R_t^{1-\alpha}(1-\sigma) \quad = \quad \lambda \int_t^{\infty} \frac{A_s R_s^{1-\alpha}}{(1-\theta)^{\frac{1}{1-\alpha}}}(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\tilde{w}^{\frac{\alpha}{\alpha-1}}e^{-(r+\lambda n^e)s}ds$$

where $A_s = \gamma A_t, \forall s > t$, because there is an innovation at time t (see(3)), and where $R_s = R_t e^{g_R^e s}$ at the steady state. After integration and simplification, we obtain

$$\tilde{w}^{\frac{1}{1-\alpha}}(1-\sigma) = \frac{\lambda\gamma(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}}{r+\lambda n^e - (1-\alpha)g_R^e}\,\frac{1}{(1-\theta)^{\frac{1}{1-\alpha}}}.$$

From (21), we have

$$x_t \quad = \quad \left(\frac{\alpha^2 A_t}{(1-\theta)w_t}\right)^{\frac{1}{1-\alpha}} R_t = \frac{\alpha^{\frac{2}{1-\alpha}}}{(1-\theta)^{\frac{1}{1-\alpha}}}\left(\frac{A_t R_t^{1-\alpha}}{w_t}\right)^{\frac{1}{1-\alpha}} = \frac{\alpha^{\frac{2}{1-\alpha}}}{(1-\theta)^{\frac{1}{1-\alpha}}}\frac{1}{\tilde{w}^{\frac{1}{1-\alpha}}},$$

that gives $\tilde{w}^{\frac{1}{1-\alpha}} = \alpha^{\frac{2}{1-\alpha}}/x_t(1-\theta)^{\frac{1}{1-\alpha}}$.

Replacing $\tilde{w}^{\frac{1}{1-\alpha}}$ by this expression in the aforementioned free entry condition gives

$$x^e \quad = \quad \frac{\alpha(1-\sigma)(r+\lambda n^e - (1-\alpha)g_R^e)}{\lambda\gamma(1-\alpha)}. \tag{43}$$

Now, using (33) for r and (40) for g_R^e, we obtain the expression of n^e given by (28). ■

Remark : observe that the rate of subsidy θ does not appear in quantities (in particular in n^e), in prices, nor in growth rates at steady-state (see proposition 2). In fact, we impose this subsidy in order to eliminate the distorsion due to the monopoly status of the intermediate firm. Nevertheless, we see that this subsidy does not modify the equilibrium values, because it affects p and w the same : both effects compensate each other. Thus, one instrument is enough to implement the optimum.

4.1.3 Existence of the steady-state equilibrium

In this section, we assume that there is no public intervention. Thus, we assume $\sigma = 0$ in proposition 2. We then obtain results that closely resemble those obtained at the optimum (see 3.1 and 3.2 above).

First, if $\sigma = 0$ in (28), we obtain, after some calculations,

$$n^e = \frac{\lambda\gamma(1-\alpha)(\alpha+\varepsilon(1-\alpha))-\alpha\rho}{\lambda(\alpha+\varepsilon(\gamma-\alpha))}. \tag{44}$$

Then, we can see that $n^e < 1$ is equivalent to $\varepsilon > 1-(\gamma+\rho/\lambda)/(\gamma-1+\gamma(1-\alpha))$. Similarly, we have $n^e > 0$ if and only if $\varepsilon > \alpha(\rho/(\lambda\gamma(1-\alpha))-1)/(1-\alpha)$.

Secondly, using (44) and (40), we calculate the rate of growth of the flow of extraction (which is equal to the rate of growth of the stock of resource at the steady-state) :

$$g_S^e = g_R^e = \frac{\lambda\gamma(\gamma-1)(1-\varepsilon)(1-\alpha)-\gamma\rho}{\alpha+\varepsilon(\gamma-\alpha)}. \tag{45}$$

From (45), we have $g_R^e < 0$ if and only if $\varepsilon > 1-\rho/(\lambda(\gamma-1)(1-\alpha))$.

Thirdly, using (44) and (33), we find the interest rate without intervention

$$r = \frac{\varepsilon\gamma(\gamma-1)\lambda(1-\alpha)+\alpha\rho}{\alpha+\varepsilon(\gamma-\alpha)}. \tag{46}$$

Now, using (46) and (38), we obtain the rate of growth without intervention

$$g_Y^e = \frac{\lambda\gamma(\gamma-1)(1-\alpha)-\rho(\gamma-\alpha)}{\alpha+\varepsilon(\gamma-\alpha)}. \tag{47}$$

Then, from (47), we observe that growth is positive if and only if $\rho < \lambda\gamma(\gamma-1)(1-\alpha)/(\gamma-\alpha)$.

Thus, we have two possible cases :

If $\rho < \lambda\gamma(\gamma-1)(1-\alpha)/(\gamma-\alpha)$, we can verify that we have $1-\rho/(\lambda(\gamma-1)(1-\alpha)) > 1-(\gamma+\rho/\lambda)/(\gamma-1+\gamma(1-\alpha)) > \alpha(\rho/\lambda\gamma(1-\alpha)-1)/(1-\alpha)$. In this case, equilibrium exists ($g_R^e < 0$) only if $\varepsilon > 1-\rho/(\lambda(\gamma-1)(1-\alpha))$, and we have $0 < n^e < 1$ and $g_Y^e > 0$.

If $\rho > \lambda\gamma(\gamma-1)(1-\alpha)/(\gamma-\alpha)$, we have $\alpha(\rho/\lambda\gamma(1-\alpha)-1)/(1-\alpha) > 1-(\gamma+\rho/\lambda)/(\gamma-1+\gamma(1-\alpha)) > 1-\rho/(\lambda(\gamma-1)(1-\alpha))$. Here, an interior equilibrium exists ($n^o > 0$) if $\varepsilon > \alpha(\rho/(\lambda\gamma(1-\alpha))-1)/(1-\alpha)$, and we have $g_Y^e < 0$.

These results are summarized in Figure 2, which can be likened to Figure 1 above.

$\frac{\lambda\gamma(\gamma-1)(1-\alpha)}{\gamma-\alpha}$ ρ

Equilibrium exists only if	Equilibrium exists only if
$\varepsilon > 1-\frac{\rho}{\lambda(\gamma-1)(1-\alpha)}$	$\varepsilon > \frac{\alpha}{1-\alpha}[\frac{\rho}{\lambda\gamma(1-\alpha)}-1]$
Then, $g_Y^e > 0$	Then, $g_Y^e < 0$

Figure 2: Existence of interior equilibrium

Remark : if $\rho > \frac{\lambda\gamma(\gamma-1)(1-\alpha)}{\gamma-\alpha}$, and ε tends to $\frac{\alpha}{1-\alpha}[\frac{\rho}{\lambda\gamma(1-\alpha)}-1]$, then n^e tends to zero (no *R&D*). Thus g_A^e also tends to zero and g_Y^e tends to $\frac{-\rho(1-\alpha)}{\alpha+\varepsilon(1-\alpha)}$ (see (38)). This is the case where we have the quickest decay at equilibrium.

4.2 Properties of the steady-state equilibrium path

As done for the optimum (3.2), the impact of variations of different parameters of the model on n^e, r and on the rates of growth g_A^e, g_Y^e and g_R^e is presented in Table 2.

	$\xi = \lambda$	$\xi = \gamma$	$\xi = \rho$	$\xi = \varepsilon$
$\dfrac{\partial n^e}{\partial \xi}$	> 0	> 0	< 0	< 0 if $g_Y^e > 0$
$\dfrac{\partial g_A^e}{\partial \xi}$	> 0	> 0	< 0	< 0 if $g_Y^e > 0$
$\dfrac{\partial g_Y^e}{\partial \xi}$	> 0	> 0	< 0	< 0 if $g_Y^e > 0$
$\dfrac{\partial g_R^e}{\partial \xi}$	< 0 if $\varepsilon > 1$	< 0 if $\varepsilon > 1$	< 0	< 0 if $g_Y^e > 0$
$\dfrac{\partial r}{\partial \xi}$	> 0	> 0	> 0	> 0 if $g_Y^e > 0$

Table 2: Properties of the equilibrium path

The results depicted in Table 2 closely resemble those obtained at the optimum: see Table 1. Recall that the latter table presented the effects of parameter variations arising from the social planner's decisions. In Table 2, the effects of the same parameter variations are shown, only this time they result from market mechanisms.

- An *increase in* λ *or* γ means that the *R&D* sector becomes more productive, inducing *R&D* firms to hire more workers. Because of the intertemporal externality on knowledge accumulation (see (39)), the rate of growth of knowledge, g_A^e, will also increase. In the meantime, more research will make the growth rate of output, g_Y^e, higher. Indeed, more research in the *R&D* sector implies that *R&D* firms will borrow more, and consequently, that the interest rate, r, will increase. For consumers, a higher interest rate means a higher growth rate of consumption, and thus a higher g_Y^e (see (38)).

  Moreover, we know (see (38) and (40)) that the resource extraction growth rate, g_R^e, is equal to the difference between the wage growth rate, g_w, and the resource price growth rate, g_{p^R}; that is, g_R^e is equal to the growth rate of the relative price between labor and resource (w/p_R). In fact, g_w is equal to $(r-\rho)/\varepsilon$ (see (38)), and g_{p^R} to r (see (41)) : this is the Hotelling rule. Thus, if the elasticity of marginal utility, ε, is higher than one, g_w will be less sensitive to an increase in r than g_{p^R}. Then, if $\varepsilon > 1$ (resp. < 1), and if λ or γ increases, g_w will increase less (resp. more) than g_{p^R}, which is why g_R^e will decrease (resp. increase). In other words, when the productivity of the $R\&D$ sector increases, the resource price increases relative to the wage.

- An *increase in* ρ means a higher taste for present consumption (relative to the future). In this case, the representative household will lend less because it prefers to consume today, and thus r will increase. Therefore, investment in $R\&D$, n^e, will decrease, and so will g_A^e. The increase in ρ will, however, dominate the increase in r in (38), and g_Y^e will decrease. Indeed, consumers derive more utility from present consumption, thus they have no interest in increasing $g_c^e (= g_Y^e)$.

  Finally, the decrease in g_w and the increase in g_{p^R} make the effect on g_R^e obvious (see (40)): g_R^e decreases.

- When $g_Y^e > 0$, the effects of an *increase in* ε closely resemble those obtained above for an increase in ρ. A higher ε means that consumers are more interested in a uniform path of consumption : thus, they will lend less (have less interest in future gains) ans r will increase. Then n^e and g_A^e will decrease.

  There are two opposite effects on g_Y^e (see (38)): an increase in r and an increase in ε, but it is the latter that dominates the former. Consumers prefer a more uniform consumption path : $g_c^e (= g_Y^e)$ decreases.

  At the end, the effect on g_R^e is unambiguous (as for the previous case): g_R^e decreases when ε grows because g_w will decrease while g_{p^R} will increase; that is, the resource price will increase faster than the wage if the elasticity of marginal utility takes higher values.

4.3 Comparison between optimum and equilibrium

Here, we continue to assume that there is no public intervention ($\sigma = 0$) at equilibrium. Before making a comparison between optimum and equilibrium, it is useful to define the range of parameters in which this comparison is possible. This is done in the following proposition :

Proposition 3 *Let us consider interior solutions only. An optimal path and an equilibrium path both exist in the following cases :*

Case a : $\rho < \dfrac{\lambda\gamma(\gamma-1)(1-\alpha)}{\gamma-\alpha}$, *if* $\varepsilon > 1 - \dfrac{\rho}{\lambda(\gamma-1)}$

Case b : $\dfrac{\lambda\gamma(\gamma-1)(1-\alpha)}{\gamma-1} < \rho < \lambda(\gamma-1)$, *if* $\gamma < \dfrac{1}{\alpha}$ *and* $\varepsilon > 1 - \dfrac{\rho}{\lambda(\gamma-1)}$,

or if $\gamma > \dfrac{1}{\alpha}$ *and* $\varepsilon > \dfrac{\alpha}{1-\alpha}[\dfrac{\rho-\lambda\gamma(1-\alpha)}{\lambda\gamma(1-\alpha)}]$

Case c : $\rho > \lambda(\gamma-1)$, *if* $\gamma < \dfrac{1}{\alpha}$ *and* $\varepsilon > \dfrac{\alpha}{1-\alpha}[\dfrac{\rho}{\lambda(\gamma-1)} - 1]$,

or if $\gamma > \dfrac{1}{\alpha}$ *and* $\varepsilon > \dfrac{\alpha}{1-\alpha}[\dfrac{\rho}{\lambda\gamma(1-\alpha)} - 1]$.

Proof 3 In order to prove the proposition, it suffices to recall Figures 1 and 2, and to observe that $\lambda\gamma(\gamma-1)(1-\alpha)/(\gamma-\alpha)$ is lower than $\lambda(\gamma-1)$.

Case a) An optimum path exists if $\varepsilon > 1-\rho/\lambda(\gamma-1)$ and an equilibrium path exists if $\varepsilon > 1-\rho/\lambda(\gamma-1)(1-\alpha)$. We have $1-\rho/\lambda(\gamma-1) > 1-\rho/\lambda(\gamma-1)(1-\alpha)$. Thus, the result follows.

Case b) An optimum path exists if $\varepsilon > 1-\rho/\lambda(\gamma-1)$ and an equilibrium exists if $\varepsilon > \dfrac{\alpha}{1-\alpha}[\dfrac{\rho}{\lambda\gamma(1-\alpha)} - 1]$. It can be verified that $\dfrac{\alpha}{1-\alpha}[\dfrac{\rho}{\lambda\gamma(1-\alpha)}] < 1 - \dfrac{\rho}{\lambda(\gamma-1)}$, except when $\gamma > \dfrac{1}{\alpha}$. The result follows.

Case c) An optimum path exists if $\varepsilon > \alpha(\rho/\gamma(\gamma-1)-1)/(1-\alpha)$ and an equilibrium path exists if $\varepsilon > \dfrac{\alpha}{1-\alpha}[\dfrac{\rho}{\lambda\gamma(1-\alpha)} - 1]$. It can be verified that $\dfrac{\alpha}{1-\alpha}[\dfrac{\rho}{\lambda\gamma(1-\alpha)} - 1] < \dfrac{\alpha}{1-\alpha}[\dfrac{\rho}{\lambda(\gamma-1)} - 1]$, except when $\gamma > \dfrac{1}{\alpha}$. The result follows. ■

Now, we consider the cases in which the two paths exist and we make a comparison in the following proposition.

Proposition 4 *We have* $n^o > n^e, g_Y^o > g_Y^e$ *and* $g_R^o > g_R^e$ *(resp.* $g_R^o < g_R^e$*) if* $\varepsilon < 1$ *(resp.* $\varepsilon > 1$*), except in the following case :*

$\rho > \lambda(\gamma-1), \gamma < 1/\alpha$ *and* $\varepsilon < \dfrac{\rho-\lambda(\gamma-1)}{\lambda(\gamma-1)^2}$,

where we have $n^o < n^e, g_Y^o < g_Y^e$ *and* $g_R^o < g_R^e$ *(resp.* $g_R^o > g_R^e$*) if* $\varepsilon < 1$ *(resp.* $\varepsilon > 1$*).*

Remark : in the last case, we have $g_Y^o < g_Y^e < 0$.

Proof 4 First, we compare n^o (see (7)) and n^e (see (44)). It can easily be demonstrated that $n^o > n^e$ if and only if $\varepsilon > (\rho - \lambda(\gamma-1))/\lambda(\gamma-1)^2$. Then we can distinguish two cases.

If $\rho < \lambda(\gamma-1)$, an optimum exists (and thus an equilibrium exists) only if $\varepsilon > 1-\rho/\lambda(\gamma-1)$ (see Fig 1). In this case we have $(\rho-\lambda(\gamma-1))/\lambda(\gamma-1)^2 < 0 < 1-\rho/\lambda(\gamma-1)$, and thus $n^o > n^e$.

Moreover, let us observe that $g_Y^o > 0$.

If $\rho > \lambda(\gamma-1)$, an optimum exists (and thus an equilibrium also exists) only if $\varepsilon > \alpha(\rho/(\lambda(\gamma-1))-1)/(1-\alpha)$. Here, we have $g_Y^o < 0$. Two sub-cases can be distinguished.

If $\gamma > 1/\alpha$, we have

$0 < (\rho-\lambda(\gamma-1))/\lambda(\gamma-1)^2 < \alpha(\rho/(\lambda(\gamma-1))-1)/(1-\alpha)$, and thus $n^o > n^e$.

If $\gamma < 1/\alpha$, we have

$0 < \alpha(\rho/(\lambda(\gamma-1))-1)/(1-\alpha) < (\rho-\lambda(\gamma-1))/\lambda(\gamma-1)^2$. Then if $\varepsilon > (\rho-\lambda(\gamma-1))/\lambda(\gamma-1)^2$, we have $n^o > n^e$. But if $\alpha(\rho/\lambda(\gamma-1)-1)/(1-\alpha) < \varepsilon < (\rho-\lambda(\gamma-1))/\lambda(\gamma-1)^2$, we have $n^o < n^e$.

Now, we compare g_Y^o (see (10)) and g_Y^e (see (47)). It can be demonstrated that $g_Y^o > g_Y^e$ if and only if $\varepsilon > (\rho-\lambda(\gamma-1))/\lambda(\gamma-1)^2$, which is exactly the condition that gives $n^o > n^e$. Then, we can proceed in exactly the same manner and obtain the same result : we have $g_Y^o > g_Y^e$, except in the case $(\rho > \lambda(\gamma-1); \gamma < 1/\alpha; \alpha(\rho/\lambda(\gamma-1)-1) < \varepsilon < (\rho-\lambda(\gamma-1))/\lambda(\gamma-1)^2)$, where we have $g_Y^o < g_Y^e < 0$.

Finally, we compare $g_R^o = (\lambda(\gamma-1)(1-\varepsilon)n^o-\rho)/(\alpha+\varepsilon(1-\alpha))$(see (18)) and $g_R^e = (\lambda(\gamma-1)(1-\varepsilon)n^e-\rho)/(\alpha+\varepsilon(1-\alpha))$ (see (40)). Clearly, if $n^o > n^e$, we have $g_R^o > g_R^e$ (resp. $g_R^o > g_R^e$) if $\varepsilon < 1$ (resp. $\varepsilon > 1$). If $n^o < n^e$, we have $g_R^o < g_R^e$ (resp. $g_R^o > g_R^e$) if $\varepsilon < 1$ (resp. $\varepsilon > 1$). ∎

Let us make some comments on proposition 4.

Clearly, the steady-state path is not optimal. In fact, we distinguish two cases : in the first one, we have $n^o > n^e$ and $g_Y^o > g_Y^e$ (in other words, there is not enough labor in research and development at equilibrium, and thus not enough growth). Moreover, if $\varepsilon > 1$ (resp. < 1), then $R^o/S^o = -g_R^o > R^e/S^e = -g_R^e$ (resp. $R^o/S^o < R^e/S^e$): see Fig 3. Observe that the same result has been obtained by Schou (1996) in an endogenous growth model with horizontal innovations. In the second case, we have $n^o < n^e, g_Y^o < g_Y^e$, and $R^o/S^o < R^e/S^e$ (resp. $R^o/S^o > R^e/S^e$) if $\varepsilon > 1$ (resp. $\varepsilon < 1$): see Fig 4. This result, obtained in a model with vertical innovations, does not appear in Schou. However, we could probably obtain it in a Schou/Scholz-Ziemes/Grimaud horizontal innovations framework, by slightly modifying the model, namely by changing the production function for the final good as suggested by Benassy (1998).

Remark : we have just seen that in the particular case where $\rho > \lambda(\gamma - 1), \gamma < 1/\alpha$ and $\varepsilon < \dfrac{\rho - \lambda(\gamma-1)}{\lambda(\gamma-1)^2}$, we had $n^o < n^e, g_Y^o < g_Y^e$ and $g_R^o < g_R^e$ (resp. $g_R^o > g_R^e$) if $\varepsilon < 1$ (resp. $\varepsilon > 1$). We can interpret this result by underlining that it occurs when the technology parameter γ is low, that is, when $\gamma < 1/\alpha$. Then, we can see that the combination of the "business stealing effect", and the positive externality due to $R\&D$ (both described by Aghion and Howitt (1998)) results in a domination of the former. Let us illustrate this by focusing on the extreme case when γ tends to 1, that is, when the size of the technological step due to a new invention is nil. In this case (see (6)), we clearly see that the positive external effect of $R\&D$ becomes null. On the contrary, at equilibrium, people will still work in the $R\&D$ sector, because V_t (the discounted pay off to the next innovation) stays positive at steady-state. Thus the "business stealing effect" remains positive in this case, and dominates the positive externality. That is why we have $n^e > n^o$.

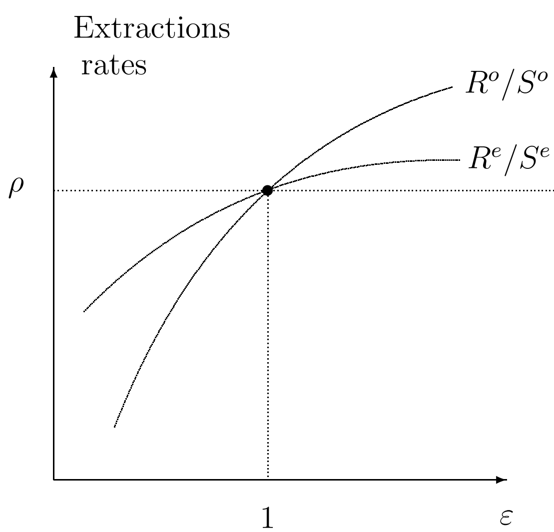

Fig 3 : extraction rates and elasticity of marginal utility (ε) when $n^o > n^e$.

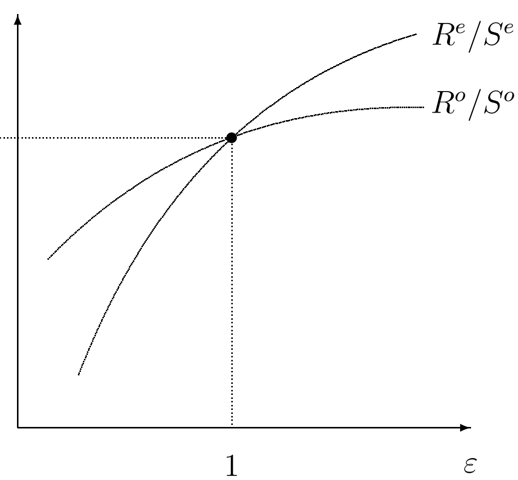

Fig 4 : extraction rates and elasticity of marginal utility (ε) when $n^o < n^e$.

4.4 Effects of public policies on equilibrium

In the two preceeding sections, 4.2 and 4.3, where we have assumed that there was no public intervention, we saw that, in general, an equilibrium is not optimal. We now study the effects of the rate of subsidy σ on the steady-state variables at equilibrium.

Proposition 5 *n^e, g_Y^e and r are increasing functions of σ. Moreover, we have $\partial g_R^e/\partial\sigma > 0$ (resp. $\partial g_R^e/\partial\sigma < 0$) if $\varepsilon < 1$ (resp. $\varepsilon > 1$).*

Proof 5 From (28) it can be proved after some calculations that $\partial n^e/\partial\sigma$ is positive if $\varepsilon > 1-(\gamma+\rho/\lambda)/(\gamma-1+\gamma(1-\alpha))$. Above we saw (see for instance Fig 2) that if $\rho > \lambda\gamma(\gamma-1)(1-\alpha)/(\gamma-\alpha)$, an interior equilibrium exists only if $\varepsilon > \alpha(\rho/(\lambda\gamma(1-\alpha))-1)/(1-\alpha)$, which is higher than $1-(\gamma+\rho/\lambda)/(\gamma-1+\gamma(1-\alpha))$ (see p. 15) : thus, in this case, we have $\partial n^e/\partial\sigma > 0$. If we consider the case $\rho < \lambda\gamma(\gamma-1)(1-\alpha)/(\gamma-\alpha)$, we know that an equilibrium exists if $\varepsilon > 1-\rho/\lambda(\gamma-1)(1-\alpha)$. But, it can be verified that in this case we have $1-\rho/(\lambda(\gamma-1)(1-\alpha)) > 1-(\gamma+\rho/\lambda)/(\gamma-1+\gamma(1-\alpha))$. Thus, we always have $\partial n^e/\partial\sigma > 0$.

Since $\partial n^e/\partial\sigma$ is positive, we immediately have $\partial r^e/\partial\sigma > 0$ from (33) and $\partial g_Y^e/\partial\sigma > 0$ from (38).

Finally, from (40), we have $\partial g_Y^e/\partial\sigma > 0$ if $\varepsilon < 1$, and $\partial g_R^e/\partial\sigma < 0$ if $\varepsilon > 1$. ■

The first part of proposition 5 (n^e, g_Y^e and r increasing functions of σ) is not surprising. Indeed, a higher σ stimulates research, and thus stimulates growth. The second part (effects of σ on g_R^e, hence on the extraction rate $R^e/S^e = -g_R^e$) can be understood as follows : from (40), we have $g_R^e = g_Y^e - g_{p^R} = (r-\rho)/\varepsilon - r$, since $g_Y^e = g_w = g_p = (r-\rho)/\varepsilon$ (see (38)) and $g_{p^R} = r$ (see (41)). The resource extraction rate is then $R^e/S^e = -g_R^e = \rho/\varepsilon - r(1-\varepsilon)/\varepsilon$. When σ increases, the growth rate $g_Y^e = g_w = g_p$ increases, but simultaneously g_{p^R} increases, and the total effects on R^e/S^e depend on ε.

If $\varepsilon = 1$, the two effects compensate each other and we always have $R^e/S^e = \rho$.

If $\varepsilon < 1$, the effect on g_Y^e is higher than the effect on g_{p^R}. Thus, the extraction rate R^e/S^e decreases. If $\varepsilon > 1$, the inverse result obtains : R^e/S^e increases. These results are summarized in Fig 5 and Fig 6, where we represent the trajectories of the pair $(R^e/S^e, g_Y^e)$ between point E (equilibrium without public intervention) and point O (optimum) when σ progressively increases from zero to its optimal value.

Let us observe that these results are alike the ones obtained by Stiglitz (1974) and that we have here found again at the *optimum* (see 3.2 above). At *equilibrium*, an increase in σ has the same effects as a positive shock on technological progress, equivalent to an increase in Stiglitz's η, or an increase in λ or γ in our endogenous growth model.

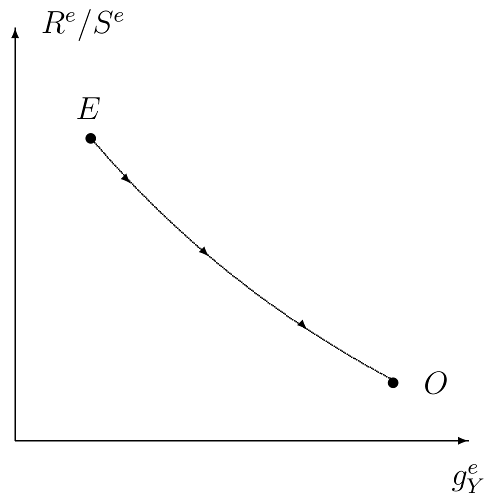

Fig 5 : effects of an increase of σ, when $\varepsilon < 1$

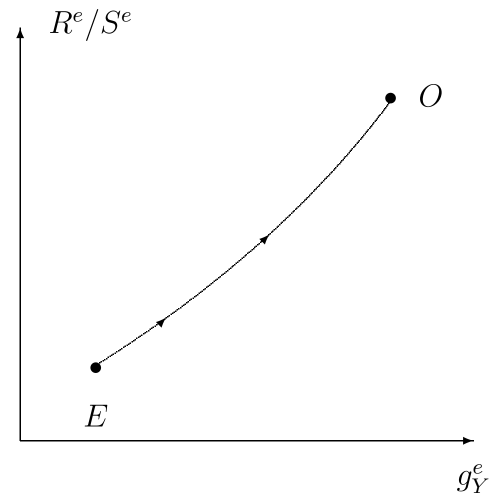

Fig 6 : effects of an increase of σ, when $\varepsilon > 1$

5 Implementation of optimum

Let us now calculate the optimal σ that leads to an optimal equilibrium path.

Proposition 6 *If*

$$\sigma = \frac{\lambda(\gamma-1)(\gamma\varepsilon+1)-\rho}{\lambda(\gamma-1)[\gamma(\varepsilon-(1-\alpha)(1-\varepsilon))+1-\varepsilon]+\rho(\gamma(1-\alpha)-1)},$$

then the equilibrium path is optimal.

Proof 6 We are searching σ such as n^o (see (7)) and n^e (see (28)) be equal. Simple computations allow us to find σ. ■

Remark : as we said above, θ does not intervene in a policy aimed at the implementation of the optimum.

The analysis of the sign of σ shows that $\sigma > 0$ when $n^o > n^e$, and that $\sigma < 0$ when $n^o < n^e$. These results were predictable. We have just seen (proposition 5) that n^e and g_Y^e are increasing functions of σ, and that $\partial g_R^e/\partial\sigma > 0$ (resp. < 0) for $\varepsilon < 1$ (resp. > 1) ; clearly then, when $n^o > n^e, g_Y^o > g_Y^e$, and $g_R^o > g_R^e$ (resp. $g_R^o < g_R^e$) for $\varepsilon < 1$ (resp. $\varepsilon > 1$), without any economic policy, then σ will be positive. Indeed, a subvention on the wage paid to workers in the $R\&D$ sector will conduce $R\&D$ firms to hire more of them, and thus to perform more research. That is why n^e and g_Y^e

will increase, and get closer to their optimal values. This first case (under-optimal equilibrium growth) corresponds to the unique case of Schou: we may also find the same results in a Schou/Scholz-Ziemes/Grimaud framework if we implement the optimal path.

In the opposite case ($n^o < n^e, g_Y^o < g_Y^e$ and $g_R^o < g_R^e$ (resp. $g_R^o > g_R^e$) if $\varepsilon < 1$ (resp. $\varepsilon > 1$), of course, σ will be negative. The reverse occurs: a tax on the wage paid to $R\&D$ workers will induce the firm to hire less workers, thus n^e and g_Y^e will decrease and approach their optimal values. We could probably have the same in Aghion-Howitt (1998) chapter 5, if we interpret their model as a vertical innovations model.

6 Conclusion

In this paper, we have considered a simple endogenous growth model with creative destruction. First, we studied the optimal steady-state growth path ; more specifically, we gave the conditions under which growth is positive along this path. Our aim was also to analyze the steady-state equilibrium. In particular, we characterized the economic policies necessary to implement the optimum.

We showed that, at the steady-state, both optimal and equilibrium growth can be either positive or negative, depending on the value of the psychological discount rate of the economy, relative to the values of the $R\&D$ technology parameters. We also proved that the equilibrium growth path is not optimal. In fact, we distinguished between two cases. In the first one, equilibrium growth is under-optimal, and the equilibrium resource extraction growth rate is under (over) optimal if the elasticity of marginal utility is lower (higher) than one; this case corresponds to the results established by Schou (1996). But we found a second case in which we obtained the opposite result, i.e., equilibrium growth is over-optimal and so is the extraction growth rate if the elasticity of marginal utility is smaller than one.

Next, we proposed an economic policy which allows the implementation of the optimum. We showed that in the first case mentioned above, it corresponds to a subsidy for the wage paid to $R\&D$ workers; moreover, an increase in this subsidy will make the equilibrium growth rate of resource extraction higher or lower, depending, once again, on whether the elasticity of marginal utility is higher or lower than one. In the second case (over-optimal equilibrium growth), economic policy consists in a tax on the $R\&D$ wage (here, the effect on the resource extraction growth rate depends also on the elasticity of marginal utility).

Finally, we showed that increasing the subsidy (or tax) has the same effects on the steady-state equilibrium variables as an increase of the technical progress parameters in Stiglitz (1974).

Further research could, for instance, study the transitional dynamics of the model. Several other extensions are also possible. For instance, we could extend our analysis to frameworks that take capital into account, or generalize our model by considering a continuum of intermediate goods, instead of only one (see for instance Aghion-Howitt, chapter 5, for an analysis of the optimum in this case).

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