FUNDING AND FINANCING INFRASTRUCTURE: THE JOINT-USE OF PUBLIC AND PRIVATE FINANCE [1]

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Abstract. Attracting private financing is high on the agenda of policy makers concerned with closing the infrastructure gap in developing countries. To date, however, private finance represents a minor share of overall infrastructure financing and the poorest countries struggle to attract any private investors. This paper develops a model that rationalizes these facts. We characterize the structure of financial and regulatory infrastructure contracts and derive conditions under which public and private finance coexist. This requires a combination of regulated prices and public subsidies sufficiently attractive for outside financiers pointing at a fundamental trade-off between financial viability and social inclusion. While improvements in the efficiency of bankruptcy procedures facilitate access to private finance, institutional changes lowering the cost of public funds make public finance more attractive.

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1. INTRODUCTION

There is a large infrastructure gap in developing countries. Approximately one billion people lack access to electricity, 2.4 billion people, about one third of the world population, are using unimproved sanitation facilities, and 660 million lack access to an improved water source. Most of this infrastructure deficit is found in the poorest countries in Africa and Asia.

At current rates of investment, this gap is not going to be closed anytime soon. Available figures show ongoing investments that are way below estimated needs. For example, Fay et al. (2019) report annual spending estimates of between 1.9 and 3.5 percent of GDP for Africa , between 3.7 and 4.7 percent of GDP for South Asia, and between 2 and 3.2 percent of GDP for Latin America. This is way below average estimated needs, which stand at 9.2 for Africa, 7.5 for South Asia, and 4.5 for Latin America (Rozenberg and Fay, 2019).[1] Excluding China, the Asian Development Bank (2017) estimates a 5 percent of GDP investment gap in Developing Asia. Using detailed harmonized household survey data covering 1.6 million households in 14 Latin American countries from 1992 to 2012, Fay and Straub (2019) show that under current trajectories, it would take countries of the region between three and more than nine decades to provide a water connection to the poorest 10 percent of households.

This situation stems in part from many of the poorest countries being unable to mobilize on their own the resources needed to bring key services to the population. They often rely on substantial concessional lending from international institutions and bilateral donors. Given the magnitude of the investments required, these players, however, are also unable to bridge the spending gap. In Developing Asia, for example, Multilateral Development Banks (MDBs) have contributed a mere 2.5 percent to overall infrastructure investment, and 10 percent when excluding China and India (ADB, 2017).

This has led MDBs and major international donors to put their hope on attracting resources from the private sector. The idea is that public finance may under certain conditions crowd-in private finance towards infrastructure projects through so-called blended

[1]While these numbers are far from perfect, they are the result of recent research efforts to use public budget, national account, and micro-level data to improve the quality of available investments and needs figures, moving away from crude cross-country estimates.

finance arrangements, also sometimes referred to as a 'billions to trillions' strategy. A refined version of this argument is found in the World Bank's 'Maximizing Finance for Development' guidelines, which establish a step-by-step approach to prioritize and attract non-government guaranteed, commercial financing, by assessing its viability and addressing needed regulatory and institutional reforms (World Bank, 2017). To date, however, of an approximate global amount of $1 trillion invested annually in developing countries, only 9 to 13 percent corresponds to the private sector (Fay et al., 2019).

This paper aims at assessing the viability of this strategy by providing a theoretical framework to analyze the issue of the feasible level of private finance in a simple model of infrastructure funding and financing, relevant to the different infrastructure sectors (transport, water, energy). It explains the stylized facts above regarding the dearth of private finance for infrastructure despite the outstanding needs, by making clear the key trade-offs faced by policy makers when involving private finance in infrastructure projects. Specifically, it sheds light on how the feasibility and desirability of outside finance is affected by the nature of the projects and by the characteristics of the economic and institutional environment.

The key intuitions for our results can be illustrated with examples from a few developing and emerging countries' experiences. The World Bank poster child for the private sector based strategy is Turkey. There, a $5.6 billion set of loans over more than a decade aimed at energy sector reform, transmission lines upgrading, reduction in distribution losses, as well as technical and policy assistance, led to ten times as much in private investments (World Bank, 2017). This happened as Turkey's average income level increased rapidly, making it an upper-middle-income country with a per capita GDP of approximately $10,000.

The story went differently for other developing countries. Liberia, one of the poorest countries on earth, with more than half of its population under the poverty line and a per capita GDP of $580, has a huge infrastructure deficit, with in particular less than 20 percent of the population having access to electricity, and a crumbling road network. Following the Millennium Challenge Corporation (MCC) Compact preparation in 2014, the country's authorities solicited support for a road project. The MCC rejected the project because the baseline internal rate of return was too low to justify the investment, let alone attract private investors.[2]

The main culprit in this case is the combination of an impossibility to charge meaningful user-fees without entirely suppressing demand given the low levels of individual income, and the inability of the government to extend subsidies to substitute for these fee-based resources. This story has been repeated across many of the poorest sub-Saharan countries, where most infrastructure projects are financed by the public sector and international donors.

Lebanon is another example of a country where poor infrastructure holds back economic development.[3] The energy sector combines tremendous inefficiencies, with high generation costs because of a reliance on expensive diesel fuel, up to 40 percent transmission and distribution losses, and prices well below cost recovery levels. As a result, it imposes rolling blackouts of several hours a day throughout the country, at a high welfare cost to consumers and firms alike. In addition, the government routinely covers the state electricity company losses, which generated about half of the total fiscal deficit of the country between 2008 and 2017. Similar inefficiencies plague the telecom services and the transport network.

As the country is unable to find the resources to invest in these sectors, it called on international support through the 2018 Paris CEDRE conference, which ended with an $11 billion pledge from the international community to fund Lebanon's Capital Investment Program (CIP). At least 35 percent of these resources were supposed to come from the private sector. However, more than two years later, no such investments have materialized. This should come as no surprise, in a country where the previous experience with public-private partnerships has been mixed at best.

In particular, several telecom projects involving private foreign firms in the early 1990s ended up in high-profile failures. In this case, despite the telecom PPPs being initially very successful, there were clear governance failures, including corruption, government

[2]See Guyde Moore, former Liberia minister of public works: https://www.cgdev.org/blog/rethinking-infrastructure-gap-poorest-countries, accessed August 28, 2020.

[3]The analysis of the Lebanon case is based on Straub (2019). As of 2019, Lebanon is an upper-middle income country with a per capita GDP of US$7,800

opportunism and internal division, the absence of credible creditors' rights,[4] and the lack of an independent regulator. As a result, despite the profitability and the potential availability of fee-based funding, since 2001 the Lebanese government has been unable to fix the sector's governance –the telecom regulatory board was not renewed since 2008– and attract new private investors, leaving customers to face one of the most expensive and inefficient services of the region. Most projects in the energy and transport sectors are likewise on standby.

These examples show that the reliance on private investors is hardly a simple and straightforward solution to attract resources towards infrastructure projects. In fact, as stressed above, this may be hardest in countries and sectors that need those investments the most, because of a lack of funding potential, governance failures, or a lack of basic creditors' rights related guarantees. In Section 2, we illustrate the insights above further with data from a sample of infrastructure public-private partnerships (PPPs) projects from the World Bank Private Participation in Infrastructure (PPI) database. The data reveals that the actual share of private debt in these PPPs is only 39%, and is quite dispersed within countries across projects, but also across countries at similar levels of development. While richer countries seem to attract slightly higher levels of private finance, some upper middle-income countries have been quite unsuccessful at doing so, a finding reminiscent of the difference between Turkey and Lebanon discussed above. Finally, the share of private debt correlates with a number of key institutional characteristics. It increases in the quality of bankruptcy procedures and in the quality of institutions, and decreases as the cost of public fund becomes smaller.

Based on the country examples and the stylized facts discussed so far, the next section presents the building blocks of the model.

Overview of the Model. Mobilizing resources to deliver infrastructure investments involves two related questions. First, how infrastructure is funded, i.e., who eventually pays for the full cost of delivering the service. The range of possibilities goes from full cost recovery through users' fees to full tax funding. Second, how infrastructure is financed, i.e., who puts the money upfront to build and start operating it. Here the range goes from

[4] An international arbitration process granted the expropriated companies $286 million in compensation, but this was never paid by the Lebanese government.

fully public to fully private finance, and involves a large variety of instruments (budgetary expenditures, bond markets, bank loans, equity issuance, etc.).

Funding is in essence a cash flow issue: Where does the money that will pay for the construction and operation of the service-providing piece of infrastructure ultimately come from? Typically, in the case of long-lived infrastructure investments such as roads or energy networks, such cash flows accrue gradually through time and are often significantly back-loaded, in the sense that very little is available in the initial construction phase. Financing, on the other hand, refers to the mechanisms and arrangements by which the equivalent of this potential accumulated cash flow is made available upfront to build the infrastructure and ensure it provides services to users.

In the context of a costly state verification contracting model involving three main agents -the government, the firm that delivers the service, and the financier- we characterize the structure of financial contracts, deriving the conditions under which public and private finance coexist. A central aspect of the model is that access to outside finance and the regulatory decision on pricing and the amount of public subsidy, hence the extent of cost recovery, are jointly determined.

First and foremost, attractive funding is needed to make outside financing possible: private finance requires a combination of price for the service and subsidy to the service provider that is sufficiently appealing to outside financiers. Private financiers have a comparative advantage in auditing the service-providing firm under non-verifiability of costs. However, benefiting from this auditing capacity implies leaving a socially costly rent to the firm. To keep this rent in check, the optimal financial contract requires both increasing the audit probability, at a cost, and raising the price above the level under verifiability, at the risk of excluding the poorest consumers. The optimal balance between the cost of audit and the informational rent of the firm is achieved through a reduction in the scale of the firm's activities. It requires reducing demand, and thus a price increase. As a result, the model displays a correlation between consumer prices and the feasible amount of private finance.

When demand is more elastic, for example in contexts where consumers are very poor or credit constrained and react strongly to price increases,[5] it is harder to move the price above the expected cost to raise revenues from user fees. Ceteris paribus, we therefore expect projects in those contexts to rely on greater shares of public subsidies. This tension between financial viability and inclusion, in the very places where the share of poor consumers to whom one would wish to extend the service is higher, is the first fundamental challenge of private infrastructure finance in developing countries.

Our model then shows how different parameters of the economic and institutional environment affect this trade-off. The viability condition for private finance implies that the cost to society of the service-providing firm's rent increases with the cost of audit and bankruptcy. When resolving insolvency becomes cheaper and faster, i.e., the cost of audit decreases, a combination of higher private financing and lower price for the service becomes feasible, easing the trade-off highlighted above. Ensuring stronger creditors' rights thus appears to be a crucial step in the quest to facilitate access to private finance.

On the other hand, there are a number of characteristics for which improvements, sometimes counterintuitively, decrease the optimal level of private finance as well as the price of the service. These include the cost of public funds as well as financial competitiveness. As budgetary pressures and the cost of taxation decrease, it becomes relatively more attractive for public authorities to rely on transfers rather than on user fees, and both service prices and the share of private finance are reduced. Similarly, more efficient financial markets mean an enhanced sensitivity of finance to the regulated price. This results in both lower levels of private finance and lower prices.

Other key parameters have more mixed effects. Better institutional environments in the sense of reduced corruption, greater bureaucratic efficiency, or regulatory quality allow for higher levels of private finance, but this comes at the cost of higher prices for the service. Indeed, these improvements also mean that governments optimally (from an economic efficiency point of view) raise prices to obtain more financing but also to restrict demand and leave lower rents to the regulated firms.

Literature and Contribution. The role of private partners in infrastructure ventures has dominated the infrastructure policy debate at least since the revival of public-private

[5]See for example Lee, Miguel, and Wolfram (2019) on demand for rural electrification in Kenya.

partnerships (PPPs) in the 1980s. The recurrent narrative in policy circles on the need to generate a growing stream of PPPs to boost infrastructure investment levels relies both on the need to mobilize additional resources, and on the belief that the involvement of private counterparts improves project efficiency. However, the private part of PPPs still represents at most 5 percent of the yearly investments in infrastructure by developing countries governments (Fay et al., 2017). Similarly, the jury is still out on the efficiency argument.[6]

The recent policy literature has mostly been concerned with the creation of an 'infrastructure asset class' as a way to attract more private financing to infrastructure projects (Blanc-Brude at al., 2017), with identifying the types of project that could be privately financed (Cordella, 2018), with the need to generate significant pipelines of such projects to lure investors and with issues of pooling risk in portfolio of projects (Ketterer and Powell, 2018), or has approached the question of the private sector involvement mostly from the management perspective (Arezki et al., 2017).

Closer to us, Estache et al. (2015) analyze one specific aspect, namely how the optimal combination of public finance, private debt, and private equity depends on the institutional environment and its limitations (capture, limited commitment, limited resources, etc.). Also relevant is Eichengreen's (1994) analysis of the development of railways in the 19^{th} century. Consistent with our analysis, he shows that due to heavy information asymmetries, securing needed private investments was in most cases only possible through the granting of different forms of public subsidies, but also that these had consequences in terms of information rents left to railway entrepreneurs.

Insights on the feasibility of private finance have to be looked for in the theoretical literature. Even though the topic is of considerable importance, very few papers have analyzed the interaction between financial constraints and the regulatory, or more generally the competitive, environment in which the firm evolves.

A first branch of the literature assumes that debt contracts strategically impact the relationship between the firm and its regulator or its competitors. In a seminal paper that

[6]See Fabre and Straub (2019) for a recent review of the empirical literature on the efficiency of PPPs, and Engel, Fischer, and Galetovic (2014, Chapter 5) for a general discussion.

triggered a more general literature on how industrial organization and corporate finance interact,[7] Brander and Lewis (1986) show that debt contracts, because they make firms focus on the upside risk of their payoff, also render them more aggressive on the product market, the so called *limited liability effect*.[8] This idea was further adapted to regulatory contexts by Spiegel and Spulber (1994) and Dasgupta and Nanda (1993), who show how a regulated firm may choose to increase its debt to grasp more of the surplus when it bargains ex post on regulated prices with its regulator. This conclusion is consistent with empirical evidence to the extent that firms in regulated infrastructure sectors are often highly leveraged although the direction of causality remains unclear.[9] Our model also predicts that higher regulated prices may come with more debt leverage. However, the mechanisms we highlight are fundamentally different from the *liability effect* stressed by this body of work. Indeed, we reverse the causality. Outside investors are ready to invest in infrastructure sectors because the regulatory environment is stable and guarantees high returns; an argument consistent with practitioners' views.[10]

Reversing the timing assumption, a premise of the New Regulatory Economics -and one that we adopt here- is that public authorities set up the regulatory environment *before* firms approach financiers.[11] These timing and information structure choices then imply that there is no room for a limited liability effect in our analysis. As in Iossa and Martimort (2012, 2015), the main usefulness of financiers is that they bring expertise in evaluating the firm's performance. However, the optimal debt contracts signed with financiers change the preferences of the firm and thus modify its behavior with respect to the regulator. Roughly speaking, the cost structure of the firm must now account not only for the cost of bankruptcy but also for the agency cost of debt.

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[7] See Harris and Raviv (1992) for an introduction.

[8] Brander and Lewis (1988) investigate the robustness of these results when bankruptcy costs are no longer fixed costs but proportional to the firm's output.

[9] See Corria da Silva et al. (2006) for early evidence of a shift towards debt in the financing structure of utilities. The literature has stressed how the limited liability effect might also have consequences on incentives to make specific investments (Spiegel, 1996), how it helps predict the link between the power of regulatory incentives and the capital structure of firms in emerging markets (Moore et al., 2014), or how it might be exacerbated in some institutional context, noticeably when regulators are independent agencies (Bortolotti et al. 2011, Cambini and Spiegel, 2016).

[10] See Bitsch, Buchner and Kaserer (2010) and Inderst (2010) for instance.

[11] Baron and Myerson (1982), Laffont and Tirole (1993), Armstrong and Sappington (2006).

Lewis and Sappington (1995) analyze how the financial structure of a regulated firm itself can be regulated under various informational scenarios.[12] Our analysis contrasts with theirs because public authorities are now interested by the economic regulation of the service and not only by its financial aspects. This allows us to uncover a new set of interactions between regulation and financial contracting. On the one hand, public subsidies act as a (public) equity stake that facilitates access to the financial market. On the other hand, the regulated prices for the services must account for the change in the firm's cost structure that is induced by its relationship with financiers. Regulated prices and profits respond to the presence of outside financiers.

By endogenizing the financing structure, our framework thus also helps understand how different institutional reforms affect the optimal financing mix, much in the spirit of Laffont's (2005, Chapter 2) applied theory of the design of regulatory institutions at different stages of development, and shape access to the infrastructure service.

ORGANIZATION. The paper is structured as follows. Section 2 develops the model and provides the main results on the link between regulatory prices and financial contracts. Section 3 develop extensions of our main framework, with an eye on various comparative statics that illustrate empirical regularities. There, we assess how our results change when there is limited commitment on the part of public authorities and more general governance issues such as corruption, and when we move away from perfectly competitive financial markets or introduce players with more complex objectives such as sovereign wealth funds. In each Section, we related the findings to empirical stylized facts and case studies. Finally, Section 4 discusses policy implications and concludes. Proofs of the main results, additional model extensions, the description of the data, and additional stylized facts are relegated to appendices.

2. THE MODEL

2.1. *Main Ingredients*

A Public Agency wants to develop a public service. It requires building an essential facility worth an outlay investment I. Whether it relies on the private sector or on a publicly-managed firm to provide the service, the Public Agency still regulates its price.

[12]Spiegel (1994) also studies how the capital structure of the firm depends on the regulatory regime.

To finance such infrastructure, the Public Agency may also rely on private financiers. In this case, in a first phase of contracting, the cashless service provider approaches outside financiers to obtain the required funds. In a second stage, the firm operates the service under the regulatory constraints. At this level of abstraction, the model is consistent with the kind of private-public partnerships that are now fairly common throughout many sectors (transport, energy, water, environmental services, etc.) in both developing and developed countries. It may also conveniently represent the case of highly leveraged state-owned enterprises (SOEs) in regulated sectors (say a power plant or a water utility).

Consumers. There is a continuum of potential consumers with mass one. For simplicity, each of those consumers expresses demand for only one unit of the service. We denote by p the price of the service, which is regulated by the Public Agency. Consumers differ according to their valuation $\tilde{v}$ for this unit of service. We assume that $\tilde{v}$ is drawn on $\mathbb{R}_+$ according to an atomless cumulative distribution H. We denote by $h = H'$ the corresponding density function which is positive on $\mathbb{R}_+$.

A consumer with valuation $\tilde{v}$ buys this unit of service when $p < \tilde{v}$. The probability $1 - H(p)$ can thus be viewed as the aggregate demand for the service. Alternatively, it is a measure of access to the service; a higher price means less coverage. This perspective is essential to understand how our model can explain a trade-off between social coverage and financing. From this, it follows that consumers' (aggregate) surplus can be defined as:

$$\mathcal{S}(p) = \int_p^{\infty} (\tilde{v} - p)h(\tilde{v})d\tilde{v} = \int_p^{\infty} (1 - H(\tilde{v}))d\tilde{v}. \tag{2.1}$$

Accordingly, we might also define the elasticity of demand as $\frac{1}{\varepsilon(p)} = \frac{1-H(p)}{ph(p)}$. For future reference, we will impose the familiar condition on the monotonicity of the hazard rate:[13]

Assumption 1

$$\frac{1 - H(p)}{h(p)} \quad \textit{non-increasing.}$$

The Firm. Beyond the fixed cost I of building a key facility, the firm has a marginal cost $\tilde{c}$ of providing one unit of service. This random variable is drawn on $\mathbb{R}_+$ according

[13] See Bagnoli and Bergstrom (2005).

to the atomless cumulative distribution F. We denote the corresponding positive density function by $f = F'$. Let $c_e = \mathbb{E}_{\tilde{c}}(\tilde{c})$ be the average cost. We assume that the price for the service is fixed before costs are realized. A justification is that costs are hard to describe *ex ante* at the time of fixing the regulated price.

Mimicking what we already did on the demand side, we also impose (but somewhat refine) another standard monotonicity condition of the hazard rate that applies to the cost distribution:

Assumption 2

$$\frac{F(c)}{f(c)} \quad \textit{increasing with} \lim_{c \to \mathbb{R}_+} \frac{F(c)}{f(c)} = +\infty.$$

For future reference, we also require that $1 - F(c)$ should be integrable at $+\infty$, i.e., $\int_c^{\infty}(1 - F(\tilde{c}))d\tilde{c}$ converges towards zero as c goes to infinity.[14]

The Public Agency may offer a lump-sum subsidy T to the firm for providing the service. Subsidies are familiar tools for governments to ensure that firms break even in regulated sectors that involve large fixed costs.[15]

Therefore, the firm breaks even in expectations over possible realizations of its cost when its profit $\mathcal{U}(p, T)$, including public subsidies, remains non-negative:

$$\mathcal{U}(p, T) = (p - c_e)(1 - H(p)) + T - I \geq 0. \tag{2.2}$$

Observe that the regulatory instruments (p, T) are fixed before the realization of costs. Echoing real world practices, the firm may indeed get bankrupt when large costs realize if the public subsidies or the price of the service are not large enough to cover losses.

[14]Of course, this assumption always holds when $\tilde{c}$ has finite support. This possibility is allowed by our model although we keep a slightly more general formulation to capture settings where shocks on costs may be very large due, for instance, to extreme adverse conditions on some input markets or significant shortages of key resources that might arise when input supply gets disrupted as in the case of a sanitary crisis for instance.

[15]T could also be interpreted as any kind of commitment to invest public resources in the project. Possible rationales for such a commitment are that the Public Agency may help screening among many potential projects the ones with the highest social value, or that it may allocate expertise or human capital into the day-to-day management of the project

The Public Agency. Following a by now standard approach in New Regulatory Economics, the Public Agency maximizes a social welfare function which reflects not only the consumers surplus and the firm's profit but also the distortionary cost of taking the subsidy/public investment T from the public budget. Denoting by $\lambda > 0$ the cost of public funds, this social welfare function writes as:

$$\mathcal{W}(p,T) = \mathcal{S}(p) + \mathcal{U}(p,T) - (1+\lambda)T.$$

Expressing T in terms of the firm's profit from (2.2) and using (2.1) yields a more convenient expression of social welfare as:

$$\omega(p) - (1+\lambda)I - \lambda\mathcal{U} \tag{2.3}$$

where

$$\omega(p) = \int_p^{\infty}(1 - H(\tilde{v}))d\tilde{v} + (1+\lambda)(p - c_e)(1 - H(p)).$$

The Public Agency's problem is to find $(p, \mathcal{U})$ so as to maximize this expression of social welfare subject to the break-even condition (2.2), a standard Ramsey-Boîteux problem.

This expression of welfare shows that the cost of public funds plays a critical role on several fronts. First, increasing revenues by raising the price above the expected marginal cost allows to save on public subsidies and relieves the budgetary burden. Second, the social cost of investment $(1+\lambda)I$ differs from the private cost borne by the firm precisely because the cost of public funds λ is positive. It does not mean that private money is cheaper than public money. One dollar which is not directly invested by the Public Agency into the project ends up being nevertheless paid to the firm under the form of public subsidies. The cost of public funds in itself offers no justification for relying on private finance– a point already forcefully made by Engel, Fischer, and Galetovic (2013). Third, as a consequence of the cost of public funds, transferring wealth to the firm is socially costly and the Public Agency wants to minimize the firm's profit $\mathcal{U}$, while satisfying the break-even condition (2.2).

2.2. *Self-Finance and/or Traditional Procurement*

Consider a first scenario where the firm has initially enough cash to make the upfront investment I by itself without relying on outside financiers. Alternatively, this setting is formally equivalent to a scenario where the Public Agency invests I by itself, owns the asset and rents its use to the firm; i.e., a model of traditional procurement and management contract.

At the optimum, the break-even constraint (2.2) is necessarily binding ($\mathcal{U} = 0$). The optimal regulated price p^{sf} is thus chosen so as to maximize

$$\omega(p) - (1 + \lambda)I.$$

The Positive Wedge Between Price and Expected Cost. Observe that ω is quasi-concave thanks to Assumption 1. The optimal price p^{sf} is thus immediately obtained from the following first-order condition:

$$\frac{p^{sf} - c_e}{p^{sf}} = \frac{\lambda}{1 + \lambda} \frac{1}{\varepsilon(p^{sf})}. \tag{2.4}$$

The optimal price p^{sf} follows a Ramsey-Boîteux pricing formula, which is by now familiar from the New Regulatory Economics.[16] The Lerner index is proportional to the inverse elasticity of demand, conveniently weighted by an increasing function of the cost of public funds. The logic for such positive price-cost margin is simple. Indeed, the Public Agency faces a trade-off between reducing budgetary pressures and mitigating allocative distortions. On the one hand, keeping the regulated price above marginal cost leaves more revenues to the firm and it saves on public subsidies that would ensure that the firm breaks even. On the other hand, increasing price above (expected) marginal cost creates allocative distortions.

Comparative Statics. Simple comparative statics are immediately obtained from (2.4). As aggregate demand becomes more elastic, the optimal regulated price comes closer to the expected cost for the service. It is indeed harder to raise revenues from sales under those circumstances and public subsidies become a more attractive way of covering the upfront investment. When budgetary pressures are more stringent (i.e., for greater values of the cost of public funds λ), saving on public funds becomes more attractive. The wedge between price and (expected) cost thus increases. Finally, when the expected cost itself increases, part of it is passed on to the regulated price.

[16] See Laffont and Tirole (1993, Chapter 1).

Optimality of Investment. The investment I is socially valuable when:

$$\omega(p^{sf}) \geq (1+\lambda)I. \tag{2.5}$$

The Complementarity Between Revenues and Public Subsidies. The binding break-even constraint (2.2) gives us the expression of the optimal public subsidy T^{sf} as

$$T^{sf} = I - (p^{sf} - c_e)(1 - H(p^{sf})) = I - \frac{\lambda}{1+\lambda}\frac{(1-H(p^{sf}))^2}{h(p^{sf})} < I \tag{2.6}$$

Because of the cost of public funds, the investment should not be entirely financed through public subsidies and part of it is recouped with revenues from the service, which is socially cheaper. This gain is represented by the term $\frac{\lambda}{1+\lambda}\frac{(1-H(p^{sf}))^2}{h(p^{sf})}$ on the right-hand side of Equation (2.6). There thus exists a complementarity between the two modes of financing either through revenues from the service or through public subsidies.

2.3. *The Benefits of Outside Finance*

There are two main reasons why relying on outside financiers could *a priori* be attractive. First, the operator may not have retained enough earnings from previous operations to bear the initial cost of investment by itself. Although public subsidies could be used to relieve such constraint, when large investments are at stake, sharp increases in the marginal cost of public funds[17] could render such solution difficult to implement in practice, especially at times of increasing budgetary discipline.[18]

Second, and more importantly, outside financiers may bring their unique expertise to improve the terms of financing.[19] Given the underlying uncertainty on costs and thus on profits, outside financiers might use their comparative advantage in monitoring the

[17]One *ad hoc* modeling of such non-linearity would consist in assuming that the cost of public funds is increasing and convex, say $\Lambda(T)$ (with $\Lambda' \geq 0$ and $\Lambda'' \geq 0$).

[18]Schmitz (2013) analyzes how budgetary pressures may affect the traditional theory of PPP, which following Hart (2003), Bennet and Iossa (2006), Iossa and Martimort (2015), and Martimort and Pouyet (2008), supposes that task bundling under PPP saves on various agency and transaction costs. Under such circumstances, while public subsidies might be preferred for investments of small size, relying on outside financiers might be a least costly solution to build the infrastructure if the upfront outlay is of significant magnitude.

[19]This point was already suggested by Iossa and Martimort (2012, 2015) in the context of $PPPs$ plagued with moral hazard problems.

project and offer better estimates of its prospects. In the sequel, we will thus assume that private financiers bring their auditing expertise to evaluate costs.

To understand why such knowledge on costs can be useful, observe that the solution (p^{sf}, T^{sf}) obtained in Section 2.2 is only feasible if the firm can run losses. Indeed, when the realized cost $\tilde{c}$ is above its expectation c_e, the subsidy T^{sf} is no longer sufficient to ensure a positive profit, and the subsidy and/or the price of the service must be raised to ensure that the firm does not go bankrupt. One potential solution would be to offer a cost-contingent subsidy $T^{sf}(\tilde{c})$ of the form:

$$T^{sf}(\tilde{c}) = I - (p^{sf} - \tilde{c})(1 - H(p^{sf})).$$

This subsidy ensures that the firm breaks even under all circumstances and of course it leaves the budgetary burden unchanged since $\mathbb{E}_{\tilde{c}}(T^{sf}(\tilde{c})) = T^{sf}$. Unfortunately, a contingent subsidy might be hard to enforce in practice. First, it might be committed in advance, before costs realize and although observable *ex post*, costs may be hard to describe *ex ante*. Regulatory contracts are certainly incomplete under those circumstances and only a fixed subsidy might be feasible. Second, even when costs are verifiable, the Public Agency may find it difficult to commit *ex ante* to such contingent subsidy. Incentives to cut it *ex post*, especially when costs are high, may open the door to costly renegotiation, haggling, influence activities and other political costs, which may be of significant magnitude.[20]

In the sequel, we instead assume that costs *are non-observable and thus non-verifiable.* In those circumstances, the cost-contingent subsidy $T^{sf}(\tilde{c})$ is no longer immune to manipulations. The firm may indeed inflate its costs to grab more subsidies; maybe up to the point where the Public Agency may prefer to give up the project. An audit becomes useful to check the adequacy of the firm's reported costs and discipline such manipulations.

Debt contracts. Following the literature on costly state verification (Townsend, 1979; Gale and Hellwig, 1985; Border and Sobel, 1987), outside financiers, who bring funds I, can also audit the firm's claim on its non-verifiable cost (and thus on its profits) to secure some reimbursement of their loan. Auditing the firm is itself costly. We assume that

[20] See Guasch, Laffont, and Straub (2008) for an empirical assessment of those costs and Engel et al. (2006) for a theoretical model that addresses those issues.

outside financiers enjoy a comparative advantage and that the Public Agency faces an infinite cost of doing so.[21]

We follow Gale and Hellwig (1985, p. 648) and the corporate finance literature (Leland, 1994) and interpret the probability of an audit as the probability of bankruptcy and the corresponding auditing cost as a bankruptcy cost. This view is consistent with an interpretation of the firm's strategy of under-reporting profits (or exaggerating costs in our framework) as a move of going into strategic bankruptcy to avoid repaying debt (Bolton and Scharfstein, 1990). More generally, bankruptcy costs should be viewed as costs of re-organizing the ventures, terminating contracts with clients and input providers, severing employment relationships, selling dedicated assets at less than their market value, etc.

If audit takes place on a subset $\mathcal{A} \subseteq \mathbb{R}_+$ of claims $\tilde{c}$ regarding realized cost, the expected total cost of audit can be written as

$$\mu(1 - H(p))Proba\{\tilde{c} \in \mathcal{A}\}$$

where μ is the marginal cost of audit.

We normalize the cost of audit and evaluate this cost per unit of demand. This assumption avoids any scale effect that would arise if audit becomes comparatively more attractive as the firm's scale of activity expands.[22]

The costly state verification literature has demonstrated that an optimal financial contract in such an environment is a debt contract. The set of claims that trigger an audit is thus of the form $\mathcal{A} = \{\tilde{c} \geq \hat{c}\}$ whose probability measure is $1 - F(\hat{c})$. Financiers capture all existing assets and exert control when their audit reveals that the firm has exaggerated

[21] Our results could of course be extended to a context where the Public Agency can also audit cost, but at a cost disadvantage, because it lacks the required expertise or because close public control may be subject to capture. There exists a literature on the interaction between multiple auditors and how it might structure their respective financial claims. See Winton (1995) and Khalil, Martimort and Parigi (2007) among others.

[22] In the corporate finance and costly state verification literature, auditing/verification costs are most often introduced as fixed costs because the scale of activity is normalized. The assumption that auditing costs increase with the scale of activities is consistent with our interpretation of those costs as bankruptcy costs. To illustrate, the (presumably lower) resale value of dedicated assets like machines depend on their size, and thus on the firm's production scale.

its costs and cannot repay a debt whose face value is D. An audit therefore takes place when the firm's claimed cost $\tilde{c}$ lies above the threshold $\hat{c}$ which is defined as:

$$D = (p - \hat{c})(1 - H(p)) + T. \tag{2.7}$$

Outside financiers are competitive so that the following break-even condition must hold:

$$I = DF(\hat{c}) + \int_{\hat{c}}^{\infty} ((p - c)(1 - H(p)) + T)dF(c) - \mu(1 - H(p))(1 - F(\hat{c})). \tag{2.8}$$

The right-hand side is the benefit obtained by lenders net of the cost of audit. When the firm claims that its cost exceeds the threshold $\hat{c}$, lenders audit the firm, verify that the claim is correct at a truthful revelation equilibrium (thanks to the Revelation Principle that applies to that context[23]) but seize the corresponding profit $(p - c)(1 - H(p)) + T$. When the firm instead claims that its cost is below the threshold $\hat{c}$, audit does not take place. Lenders then receive a fixed repayment, the face value of debt D.[24]

The regulatory environment affects the face value of debt D and the audit threshold $\hat{c}$ through the choice of the regulatory instruments p and T, which are fixed once and for all by the Public Agency and define a stable playing field for investors and firms.[25]

[23]See Border and Sobel (1987) for a formal proof.

[24]Observe that the profit $(p-c)(1-H(p))+T$ eventually becomes negative for realizations of c, which lie beyond $\hat{c}$ (especially when this random variable has unbounded support). When the firm truthfully reports such large costs and cannot repay its debt, it gets bankrupt. Since the firm is protected by limited liability and we assume full commitment to the financial contract, financiers must keep the corresponding losses for themselves. To do so, they compensate the firm with some funds (to maintain current activities, fulfill contractual obligation vis-à-vis suppliers, pay workers' wages and the like), which becomes an extra cost of audit. Hence, the optimal debt contract is such that, once the firm is bankrupt, financiers can still pocket positive profits when c remains close to but below $\hat{c}$, while they incur losses when c is above $\hat{c}$. They are thus exposed to some risk when they commit to audit the firm. Since financiers are risk neutral and are not cash constrained, that risk is costless. In the case of risk aversion, financiers would benefit from taking insurance. The implicit coalition they would form with insurers would participate when the same break-even condition as (2.8) holds. Finally, in the case of cash constraints for the financiers, the public authority should take residual losses. It is akin to an increase in the public subsidy and our analysis carries over to that scenario.

[25]This assumption is consistent with the view, often held by practitioners, that infrastructure offers long-term stable and predictable returns with low correlation to other assets. See Bitsch, Buchner and Kaserer (2010) and Inderst (2010) for instance.

In the context of our model, the Public Agency acts as a Stackelberg leader in designing regulatory contracts for the firm before investors show up. The fact that regulatory instruments do not depend on what can be learned by outside financiers in the process of auditing the firm is consistent with the view that such *ex post* information may be costly to incorporate *ex ante* into prices and subsidies since it is highly manipulable by investors so as to ensure greater revenues for the infrastructure they contribute to finance.

2.4. *The Optimal Regulatory Charter*

The optimal regulatory charter consists of a regulated price p, a public subsidy T, together with a financial contract, which is optimally designed by competitive lenders in response to this regulatory environment. Using the expression of $\mathcal{U}(p, \hat{c})$, social welfare becomes:

$$\mathcal{W}(p, \hat{c}) = \omega(p) - (1+\lambda)I - \lambda\mathcal{U}(p, \hat{c}) - (1+\lambda)(1-H(p))\mu(1-F(\hat{c}))$$

where we again make explicit the dependence on $(p, \hat{c})$. This expression highlights how the social cost of audit is counted in the Public Agency's objectives. Since financiers are competitive, the private cost of audit ends up being borne by the firm itself and it can be recouped through extra public subsidies, whose social cost is $(1+\lambda)(1-H(p))\mu(1-F(\hat{c}))$.

The properties of the optimal regulatory charter are now summarized.

PROPOSITION 1 *Suppose that financiers are competitive. The optimal regulatory charter* $(p^*, \hat{c}^*)$ *entails the following properties.*

1. *Price is above its value under traditional procurement,* $p^* > p^{sf}$*, with*

$$\frac{p^* - c^*}{p^*} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon(p^*)} \tag{2.9}$$

where

$$c^* = c_e + \mu(1 - F(\hat{c}^*)) + \frac{\lambda}{1+\lambda}\int_0^{\hat{c}^*} F(c)dc > c_e. \tag{2.10}$$

2. *The firm's profit is always positive:*

$$\mathcal{U}(p^*, \hat{c}^*) = (1 - H(p^*))\int_0^{\hat{c}^*} F(c)dc > 0. \tag{2.11}$$

3. Bankruptcy arises for $c \geq \hat{c}^*$ *where*

$$\mu = \frac{\lambda}{1+\lambda}\frac{F(\hat{c}^*)}{f(\hat{c}^*)}. \tag{2.12}$$

Higher prices and more bankruptcy. Leaving a positive information rent to the firm is now socially costly. Reducing it requires acting both on the regulated price and on the probability of bankruptcy.

As far as price distortions are concerned, observe that increasing the price of the service depresses demand. It thus has a direct impact in reducing the overall cost of audit but also the cost of the firm's information rent since both costs are counted per unit of demand. The pricing formula (2.9) has the same structure as that obtained with traditional procurement except for a change in the value of the costs. Everything indeed happens as if the firm's average cost c_e was now replaced by a *virtual cost* c^*, which is strictly greater. This virtual cost incorporates not only the cost of audit (the second term on the right-hand side of (2.10)) but also the extra cost of leaving information rent to the firm (the third term on the right-hand side of (2.10)).[26]

The presence of the cost of audit in formula (2.10) comes from the fact that the cost of bankruptcy borne by financiers is actually passed onto the Public Agency since it must be covered through subsidies to ensure that financiers break even and that the firm operates.[27]

[26]This last term is properly weighted to account for the fact that transferring one dollar of extra subsidy to the firm costs $1+\lambda$ from the general budget but yields 1 unit of social welfare. Hence the rate of transformation of public money into the marginal social cost of the firm's rent is $\frac{1+\lambda-1}{1+\lambda} \equiv \frac{\lambda}{1+\lambda}$.

[27]At this stage, it is interesting to stress the role played by our assumption that auditing costs are proportional to the scale of activities. Suppose, to the contrary, that auditing costs are fixed, independent of the scale of activities. The social cost of audit would now be $(1+\lambda)\mu(1-F(\hat{c}))$. This alternative specification would imply a lower virtual cost in Proposition 1. Namely, (2.10) would be modified as

$$c^* = c_e + \frac{\lambda}{1+\lambda}\int_0^{\hat{c}^*} F(c)dc > c_e$$

while keeping the expression of the firm's information rent in (2.11) unchanged. As a consequence, the complementarity between higher prices and more audit would remain while price distortions would be lower thanks to the absence of any direct impact of audit on virtual costs. A last, more specific result, is that the probability of bankruptcy would increase with the scale of activities while in our main framework the two are independent as shown in (2.12).

Finally, interpreting (2.9) as a pass-through formula also shows that with private finance the regulated price increases with the cost of public funds (higher values of λ increase the regulated price p^*; as budgetary pressures increase, it becomes more attractive to use the firm's revenue to facilitate its access to outside finance and less attractive to rely on direct subsidies) and with the cost of bankruptcy (higher values of μ), because their impact on the firm's virtual cost is similar. While the first implication is already found with self-finance, the second one is specific to the scenario with outside finance.

Applications. The proposition above can be linked to the examples put forward in the introduction.

Extensive margin: When is private finance feasible? In Appendix A, we derive a 'feasibility condition' describing all public policies (p, T) that render outside finance feasible (see (A5) and (A6)). These policies are those ensuring a non-negative payoff to the private financier. The first result is that, everything else equal, fewer projects will be undertaken with outside finance than if self-finance was possible. Indeed, the fact that the cost of audit and the information rent left to the firm are passed on to prices make the condition for the social optimality of the investment more difficult to satisfy.

Beyond that, if the monopolistic financier described there cannot break even given prevailing subsidies and regulated price, then private finance is simply not feasible. This is the case in Liberia, where very low levels of income made it impossible to charge road toll fees high enough to sustain the above condition while ensuring a sufficient level of demand. A similar situation arose regarding the extension of the electric grid in Kenya, where demand dropped to zero when the actual connection price set by the company ($398) was charged, and less than a quarter of households took it up when offered a 57 percent subsidy (Lee, Miguel, and Wolfram, 2019).[28] In Appendix A we derive comparative statics showing that a lower demand schedule leads to a smaller share of private finance.[29]

In addition, these projects have very large fixed costs. Transmission grid extension is estimated around $10,000 per kilometer, while a km of road may cost between $30,000 and $100,000 (Collier, Kirchberger, and Söderbom, 2015). Both the Liberian and the Kenyan

[28]The real cost was actually estimated to be above $1,400.

[29]This is for example the case for a demand schedule H' such that $\frac{1-H'}{h'} FOSD \frac{1-H}{h}$.

governments are likely to face budgetary conditions that impede them from extending sizable subsidies.

Intensive margin: When feasible, how does private finance affect the nature of projects? The model also brings lessons for the case where the feasibility conditions are satisfied. We then expect the extending of private financing to vary with a number of key variables identified in our framework. Comparative statics based on Proposition 1 are developed in Appendix A regarding a number of parameters. Let us focus here on the cost of bankruptcy and the cost of public funds. Other parameters are addressed in the extensions Section 3 below.

The model shows that the level of debt decreases as the cost of bankruptcy increases, i.e., when it becomes harder to recoup financial claims, and as the cost of public funds decreases. Both outcomes are intuitive. Lenders will audit less, and debt will therefore decrease, as the cost of doing so increases. In addition, auditing is no free lunch, as its cost is passed on to the public budget through subsidies or to fee-paying customers. On the other hand, a lower cost of public funds reduces the virtual cost of the firms and lowers the pressure to decrease its information rent. As a result, in both cases the Public Agency chooses a combination of price and subsidy that induces less debt.

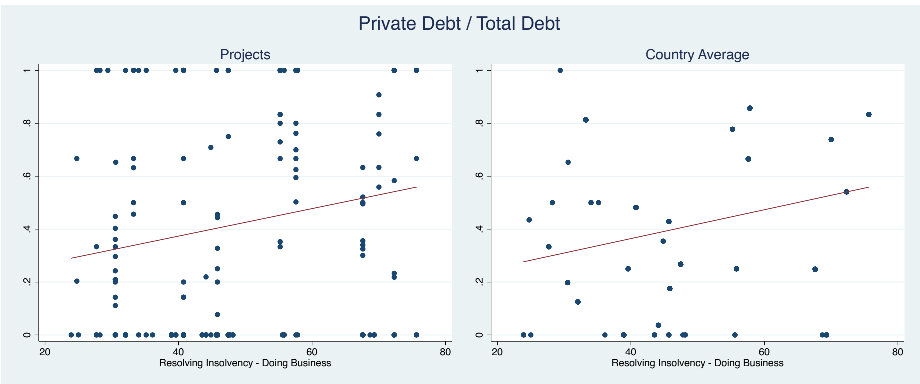

FIGURE 1.— Projects' Share of Private Debt as a Function of the Cost of Bankruptcy

These correlations show up clearly in the financial composition of our sample of PPI projects.[30] Figures 1 and 2 show cross-projects and cross-country correlations between, respectively, the cost of bankruptcy and the cost of public funds, and the share of private

[30]See the precise definition of all variables in Appendix C.

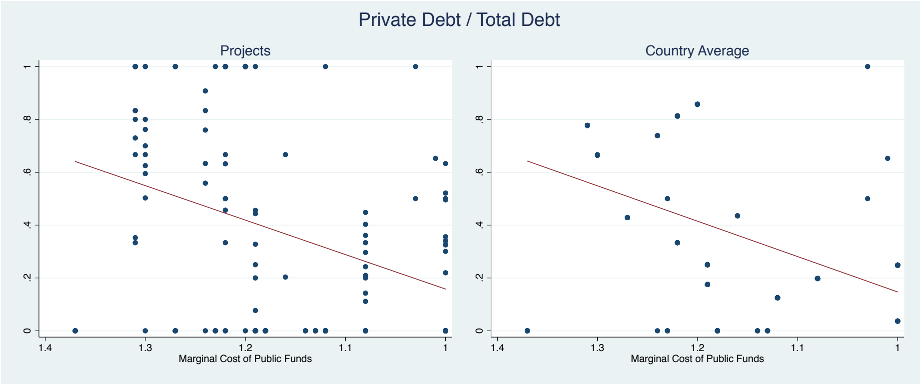

FIGURE 2.— Projects' Share of Private Debt as a Function of the Cost of Public Funds

to total debt.[31] The level of private finance increases in the quality of bankruptcy procedures, and decreases as the cost of public fund becomes smaller. Again, the Figures make clear the large degree of heterogeneity both within and across developing countries.

Table I displays the regression equivalent of these plots. We regress the project-level share of private debt on a number of characteristics, controlling for subsector and region fixed effects. Column 1 first shows that the prevalence of private finance increases with the level of development, with per capita GDP being positive and strongly significant. However, the effect eventually levels out: in column 2 including a squared term supports a non-linear, concave relationship. In column 3, the coefficient for resolving insolvency is positive and significant, indicating that projects in countries with more efficient insolvency frameworks, i.e., those in which collecting assets in case of bankruptcy is easier, attract higher shares of private debt as predicted by our model. Finally, in column 5, projects in countries with a higher marginal cost of public funds also boast higher shares of private debt.[32]

In addition, the PPI data allows us to perform a direct comparison of the two middle income countries already discussed in the introduction, Turkey and Lebanon. While there are too few projects with financial information to draw meaningful inferences, we can observe the broader pattern of private participation from the PPI database.

[31] The panels on the left use the project-level data. To adjust for the fact that countries with a lot of projects such as Brazil, India, or Mexico, are potentially biasing the pattern, the right panels use country averages pooled over the three years.

[32] The result for corruption is discussed below in Section 3.1.

TABLE I

Share of Private Debt: Conditional Correlations

	(1)	(2)	(3)	(4)	(5)
pcGDP	0.0493***	0.149***			
	(0.0113)	(0.0333)			
pcGDP squared		-0.00802***			
		(0.00251)			
Resolving Insolvency			0.00684**		
			(0.00264)		
Control of corruption				0.161**	
				(0.0722)	
Cost of Public Funds					1.538***
					(0.293)
Subsector F.E.	Yes	Yes	Yes	Yes	Yes
Region F.E.	Yes	Yes	Yes	Yes	Yes
Observations	326	326	278	326	141
R-squared	0.180	0.206	0.131	0.144	0.403

Standard errors in parentheses: *** p<0.01, ** p<0.05, * p<0.1

Turkey has both a lower cost of bankruptcy, as proxied by the Doing Business resolving insolvency index, and a higher cost of public funds. We thus expect that it is an environment more conducive to private finance in infrastructure. Indeed, between 1990 and 2019, Turkey had 246 PPP projects recorded in the PPI database, for a total committed investment of $146 billion. In contrast, Lebanon had only 9 projects for a total of $383 million.[33]

3. ALTERNATIVE ASSUMPTIONS

We now assess the robustness of our findings to various alternative assumptions and derive related implications. We do so following an approach pioneered by Laffont (2005) that consists in looking at specific comparative statics that prevail for developing countries in the context of a model which might have a broader appeal.

We focus on two main issues, which are directly empirically relevant. First, in Section 3.1 we address issues related to governance, namely limited commitment (Section 3.1.1) and congruence between the Public Authorities and the firm, taking for example the form of corruption and capture (Section 3.1.2). Next, Sections 3.2.1 and 3.2.2 explore specific assumptions regarding the competitiveness of financial markets and the role of sovereign

[33] See https://ppi.worldbank.org/en/visualization, accessed September 3, 2020. Of course, other parameters like governance are also likely to matter for this sharp difference in attractiveness. These are discussed below in the extensions Section 3.

wealth funds.[34] In both cases, we discuss how these extensions help understanding the practical situations observed in developing countries

3.1. *Governance*

3.1.1. *Limited Commitment*

An important assumption of our main analysis is that the Public Agency has the ability to commit to the regulatory instruments (p, T). In practice, the Public Agency's commitment ability is certainly more limited, which in turn makes it harder to attract outside financiers. These financiers will then request a premium for the risk of changes in the rules of the game.

To view how this premium translates into distortions of prices and levels of debt, we now analyze how the regulatory contract is modified under limited commitment. Our starting assumption is that, once a debt contract with outside financiers has been signed, the Public Agency may benefit from decreasing the price p below what financiers expected, for example because of short-term electoral incentives. Let us denote by (p^e, T^e) the regulatory contract that is expected at equilibrium. The Public Agency enjoys an extra gain $B(p^e - p)$ from decreasing the price *ex post*. We assume that B is increasing, concave and satisfies $B(0) = 0$. The parameter $B'(0)$ captures the magnitude of short-term incentives to decrease the price for electoral reasons. Of course, reducing the price in response to those incentives while at the same time keeping the face-value of debt unchanged would increase the probability of bankruptcy. It means that the Public Agency must also increase the public subsidy T in order to guarantee that outside financiers still break even.[35]

The following proposition summarizes the impact of limited commitment on the optimal regulatory charter.

[34] Other extensions, such as the role of consumption subsidies and the social cost of bankruptcy, are relegated to Appendix A.

[35] An increase in the public subsidy is needed as long as the debt contract itself cannot be renegotiated. Our implicit assumption here is that such a renegotiation is impossible. In practice, reorganizing the firm's activity in times of financial distress, selling some of the firm's assets on short notice, and more generally restructuring claims among various claimholders is certainly a costly process as has been argued in the corporate finance literature (Harris and Raviv, 1991).

Proposition 2 *Suppose that the Public Agency has limited commitment. Necessary conditions for a regulatory charter $(p^{lc}, \hat{c}^{lc})$ to be an equilibrium are as follows.*[36]

1. *The regulated price decreases with the magnitude of ex post incentives $B'(0)$:*

$$\frac{p^{lc} - c^{lc}}{p^{lc}} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon(p^{lc})} - \frac{B'(0)}{(1+\lambda)p^{lc}h(p^{lc})} \tag{3.1}$$

where

$$c^{lc} = c_e + \mu(1 - F(\hat{c}^{lc})) + \frac{\lambda}{1+\lambda}\int_0^{\hat{c}^{lc}} F(c)dc. \tag{3.2}$$

2. *Bankruptcy is less likely than under full commitment:*

$$\hat{c}^{lc} > c^*. \tag{3.3}$$

There exists in fact a multiplicity of rational expectations equilibria to the game under limited commitment. Those equilibria differ in terms of the price and the level of debt of the firm, but all of them are characterized by a lower price-cost margin, less debt and a lower likelihood of bankruptcy.[37] The Public Agency's incentives to reduce the price for the service are a direct consequence of the fact that the marginal electoral gain from doing so is positive. If that effect is strong enough ($B'(0)$ large), the price-cost margin can even be negative and subsidies have to be increased to ensure that the firm breaks even. Less intuitive is the fact that bankruptcy is less likely and that debt decreases. Indeed, the set of possible deviations from any putative equilibrium allocation is constrained by the requirement of keeping financiers on board. This means that any cut on price has also to come with an increase in the direct subsidy to the firm so as to pay for its debt commitment. This puts an extra burden on the public budget and such constraint is of course released by having lower levels of debt and less bankruptcy.

3.1.2. *Congruence Between the Public Agency and the Industry*

Our baseline model can be extended to allow for the possibility that the Public Agency and the firm have more congruent objectives than in our baseline scenario. For instance,

[36] We assume that the Public Agency's objective remains quasi-concave in that limited commitment scenario so that necessary conditions for optimality are also sufficient. The proof is in the Appendix.

[37] See the Appendix for details.

the Public Agency may want to cajole the firm by giving its profit a weight in its own objective function that goes beyond what social welfare maximization would recommend. To keep a high level of generality and broaden the interpretation, we will be somewhat agnostic on the source of such alignment. Congruence of interests may be due to direct corruption of public officials as in Dixit (2010). It may also follow from the electoral pressure that political principals exert on the Public Agency if they want to favor a constituency which has a stake in the firm, a point made by Baron (1988). In general, congruence can be thought of as encapsulating several aspects of weak governance.

Following Baron and Myerson (1982), we thus now suppose that the Public Agency's objective is

$$\mathcal{S}(p) + (1+\alpha)\mathcal{U}(p,T) - (1+\lambda)T,$$

where $\alpha \in [0,\lambda]$ denotes the degree of congruence between the Public Agency and the firm. Expressing again T in terms of the firm's profit $\mathcal{U}(p,T)$ from (2.2) and using (2.1), we obtain the new expression of the Public Agency's objective function as

$$\mathcal{W}(p,T) = \omega(p) - (1+\lambda)I - (\lambda-\alpha)\mathcal{U}. \tag{3.4}$$

This expression highlights that the firm's information rent remains costly as long as the congruence parameter is not too large. Yet, the incentives to distort prices and increase the likelihood of bankruptcy for rent extraction reasons diminish in this context. Indeed, the firm's virtual cost is lower. As a result, the price decreases.

Proposition 3 *Suppose that financiers are competitive but that the Public Agency's and the firm's objectives are congruent. The optimal regulatory charter $(p^\alpha, \hat{c}^\alpha)$ entails the following properties.*

1. *Price is above its value under traditional procurement, $p^\alpha > p^{sf}$ but lower as α increases:*

$$\frac{p^\alpha - c^\alpha}{p^\alpha} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon(p^\alpha)} \tag{3.5}$$

where

$$c^\alpha = c_e + \mu(1-F(\hat{c}^\alpha)) + \frac{\lambda-\alpha}{1+\lambda}\int_0^{\hat{c}^\alpha} F(c)dc > c_e. \tag{3.6}$$

2. Bankruptcy becomes less likely as α increases:

$$\mu = \frac{\lambda - \alpha}{1 + \lambda - \alpha} \frac{F(\hat{c}^{\alpha})}{f(\hat{c}^{\alpha})}. \tag{3.7}$$

Propositions 2 and 3 have similar implications. Indeed, both when the Public Agency has a limited commitment ability, and when firms and public authorities are more congruent, regulated prices are lower, firms have less debt and bankruptcy is less likely.

Applications. These results help illustrate the impact of governance on the feasibility of private finance in infrastructure projects. Figure 3 displays a negative correlation between an aggregate measure of corruption, here the World Bank Worldwide Governance Indicators (WGI) Control of Corruption indicator, and the share of private debt in infrastructure projects.

Figure 3.— Projects' Share of Private Debt as a Function of Corruption

In column 4 of Table I above, we regress the project-level share of private debt on the corruption indicator, controlling for subsector and region fixed effects. The coefficient for Control of Corruption is indeed positive and significant, indicating that it is easier to attract higher shares of private debt in less corrupt environments, as predicted by our model.

One reason that resonates with our model is that worse governance translates into higher costs. Collier, Kirchberger, and Söderbom (2015) show that the cost of building one kilometer of road is about 30% higher in countries that are in conflict, and 15% higher in countries above the median corruption level in the sample in the World Governance Indicators. These differences remain when controlling for the governments' public investment capacity and for the quality of the business environment. Comparative statics in Appendix A show that higher cost schedules ceteris paribus lead to a lower share of private finance.[38]

In specific contexts, where corruption is serious and worsening, it is likely that this marginal impact on private finance is quite strong. Consider again the case of Turkey vs Lebanon. The two countries are on very different trajectories since the 1990s. Turkey has mostly displayed an improving governance rating over the last 20 years, and it is currently around the 50th percentile of the world distribution. Lebanon, in contrast, has experienced the opposite evolution. Its corruption rating, already at quite low levels before 2005, dropped markedly after the assassination of prime minister Rafic Hariri, and has lingered around the 15th percentile ever since. In the last few years, Lebanon has been characterized by endemic corruption and extreme uncertainty, with extended periods without formal government, implying a total inability to commit to long term contracts with private firms. As a result, as argued above, contrary to Turkey, the country has been unable to attract significant private investments.

3.2. *Financial Markets*

3.2.1. *Financiers with Market Power*

Assuming that financiers are competitive might be better suited to the context of well-developed financial markets. In most developing countries, various forms of transaction costs and entry barriers might instead lead to some financial institutions having significant market power. We now investigate how our previous findings are modified in that case. To see the consequences of market power on financial markets in its most extreme form, we consider for simplicity the polar case where funds are now provided by a monopolistic lender.[39]

We already know that a monopolistic lender always chooses the highest possible prob-

[38]This is for example the case when the cost distribution F' shifts towards the right such that F' dominates F in the sense of first-order stochastic dominance.

[39]Our motivation for focusing on the monopolistic scenario comes from the fact that there exists a plethora of models of imperfect competition in financial markets with different combinations of ingredients (see for example Winton (1995), Parlour and Rajan (2001) and Khalil et al. (2007)) and there is no specific reason to pick one rather than the other in our context.

ability of bankruptcy since it wants to extract as much information rent as possible from the firm.[40] Such lender offers a debt contract which defines a threshold for the audit zone at $\hat{c} = \hat{c}^m$. Importantly, this choice is independent of the regulatory instruments (p, T) used by the Public Agency. While with competitive financiers the Public Agency was able to indirectly control the optimal probability of bankruptcy, this control is now lost in the case of a monopolistic lender.

Anticipating the added restriction that the probability of bankruptcy is now responsive to the monopolistic lender's sole objectives, the Public Agency must still choose his regulatory instruments (p, T) with a view on the participation of this lender.[41] Since public subsidies are socially costly, this constraint is again binding at the optimum and even a monopolistic lender ends up making zero profit.[42] Of course, social welfare is always lower in that scenario.

Mutatis mutandis, we can derive the main features of the optimal regulatory charter as follows.

Proposition 4 *Suppose that financiers are monopolists. The optimal regulatory charter* $(p^m, \hat{c}^m)$ *entails the following properties.*

1. *Price is above its value with competitive financiers,* $p^m > p^*$, *with:*

$$\frac{p^m - c^m}{p^m} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon(p^m)} \tag{3.8}$$

   where

$$c^m = c_e + \mu(1 - F(\hat{c}^m)) + \frac{\lambda}{1+\lambda}\int_0^{\hat{c}^m} F(c)dc > c^*. \tag{3.9}$$

2. *The firm's profit is lower than with competitive financiers:*

$$\mathcal{U}(p^m, \hat{c}^m) = (1 - H(p^m))\int_0^{\hat{c}^m} F(c)dc < \mathcal{U}(p^*, \hat{c}^*). \tag{3.10}$$

[40]See Lemma 1 in Appendix A.

[41]In other words, the feasibility requirement (A5) still applies (See Appendix).

[42]From this, it follows that the firm's rent is still given by (A8) with the additional constraint that $\hat{c} = \hat{c}^m$.

3. Bankruptcy is more likely than with competitive financiers:

$$\hat{c}^m < \hat{c}^*. \tag{3.11}$$

Pushing a bit further the interpretation of our results, we conjecture that, when financial markets are not well developed, and market power is a concern, even in a less crude way than modeled here, the firm has more debt and bankruptcy becomes more likely. The *virtual cost* of the firm increases, which further raises prices and depresses demand. The firm's information rent also diminishes as a result of both a demand reduction and a higher likelihood of bankruptcy.

3.2.2. *Sovereign Wealth Funds*

Sovereign wealth funds ($SWFs$) are pools of capital publicly owned by governments. Nowadays, those funds represent a significant share of all investments made in developed countries.[43] They are concerned not only with profit maximization but also with stabilizing domestic revenues.[44] To model these additional concerns, we suppose that $SWFs$ are risk-averse, assuming for simplicity a $CARA$ utility function with a degree of constant absolute risk aversion denoted by r. A SWF is endowed with some initial wealth w and invests in an infrastructure project if doing so yields a non-negative net benefit. When $SWFs$ adopt a competitive behavior, this net benefit is driven to zero, a condition which writes as

$$(1 - H(p))(p - \varphi(\hat{c})) + T - I = \rho(p, \hat{c}) \tag{3.12}$$

where

$$\rho(p,\hat{c}) = (1-H(p))(\hat{c}-\varphi(\hat{c}))+\frac{1}{r}\ln\left(F(\hat{c})+\int_{\hat{c}}^{+\infty} exp(-r((1-H(p))(\hat{c}-c-\mu)))dF(c)\right). \tag{3.13}$$

To induce competitive risk-averse $SWFs$ to take the risk of bankruptcy associated to the project, their net return must now also cover a (non-negative) risk-premium $\rho(p, \hat{c})$.

[43] For a discussion of those issues, see Bortolotti et al. (2015).

[44] This concern is particularly relevant when the sponsoring countries enjoy significant rent from natural resources endowments, oil being a primary example.

To get further insights on how this risk-premium varies with p and $\hat{c}$, it is useful to consider the case where $SWFs$ almost behave as pure profit maximizers, i.e., the case where r is small.

PROPOSITION 5 *Suppose that the investment is financed by competitive $SWFs$, which are also concerned with stabilizing their revenues. Up to terms of order more than 2 in their common degree of risk aversion r, the optimal regulatory charter $(p^s, \hat{c}^s)$ entails the following properties.*

1. *Price is above its value with profit-maximizing competitive financiers, $p^m > p^*$, with:*

$$\frac{p^s - c^s}{p^s} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon(p^s)} \tag{3.14}$$

where

$$c^s = c_e + \mu(1 - F(\hat{c}^s)) + \frac{\lambda}{1+\lambda}\int_0^{\hat{c}^s} F(c)dc + r(1 - H(p^s))\Delta(\hat{c}^s) \tag{3.15}$$

2. *Bankruptcy is less likely than with profit-maximizing competitive financiers:*

$$\hat{c}^s > \hat{c}^* \tag{3.16}$$

where

$$\mu = \frac{\lambda}{1+\lambda}\frac{F(\hat{c}^s)}{f(\hat{c}^s)} + \frac{r}{2}(1 - H(p^s))\Delta'(\hat{c}^s). \tag{3.17}$$

Sovereign funds are harder to attract than pure profit-maximizing financiers because they have to be compensated for a risk-premium associated with the possibility of bankruptcy, an event that would put their diversification strategy at risk. The Public Agency thus needs to play on both p and T to reduce this risk-premium. To understand this, it is useful to rewrite the risk-premium $\rho(p, \hat{c})$ in the limit of r small enough. In the Appendix, we show that this risk-premium admits the following second-order Taylor approximation:

$$\rho(p, \hat{c}) = \frac{r}{2}(1 - H(p))^2\Delta(\hat{c}) \tag{3.18}$$

where

$$\Delta(\hat{c}) = \int_{\hat{c}}^{+\infty} (\hat{c} - c - \mu)^2 dF(c) - \left(\int_{\hat{c}}^{+\infty} (\hat{c} - c - \mu) dF(c) \right)^2 > 0. \tag{3.19}$$

First, remember that the cost of bankruptcy is proportional to demand in our setting. Henceforth, the risk-premium diminishes when demand is reduced as can be seen in (3.18); this pushes the Public Agency to choose higher regulated prices. Second, raising the public subsidy T also reduces the probability of bankruptcy and thus pleases foreign investors.

We draw from Propositions 4 and 5 above some immediate implications. In the case of the assumption that private financiers evolve in the more competitive environments characteristic of more developed financial markets, the optimal contract involves lower debt, as financiers extract less of the firms' rent, but also lower price and a higher level of welfare. Regarding financiers with objectives that may differ from strict profit-maximization, such as SWFs, their involvement also leads to lower levels of debt, but higher prices.

Applications. The implications above can again be related to stylized facts regarding the involvement of different categories of financial players in infrastructure projects in developing countries.

Monopolistic lenders have the potential to induce higher private debt level, but this comes at the cost of increasing the price for the service. Doing so, in developing countries environments risks hitting the feasibility constraint already discussed in Section 2.4. Involving sophisticated private institutional investors such as sovereign wealth funds or pension funds raises similar issues.

Despite the popular idea that a larger part of the close to \$100 trillion of assets under management of institutional investors such as pension funds, sovereign wealth funds, and insurance companies, could somehow be channeled towards infrastructure projects (Arezki et al., 2017), no such thing has happened. Inderst and Stewart (2014), relying on several available databases on institutional investors, estimate that the scope for such inflow should not be overestimated and that under optimistic assumptions at most an additional \$60 billion could flow every year to infrastructure investments in emerging and developing countries. While not negligible, this represents at most 5 percent of the total flow of current investments in developing countries, and even less of the needed investments.

4. CONCLUDING REMARKS

We have developed a model of infrastructure funding and financing, in which access to outside finance, the extent of cost recovery resulting from the regulatory decision on pricing, and the amount of public subsidy, are jointly determined. This model highlights some key trade-offs faced by policy makers when seeking to involve private finance in infrastructure projects. A number of lessons stand out.

First, mobilizing private finance for infrastructure projects in developing countries requires setting a combination of price for the service and subsidy to the service provider that are sufficiently attractive. When bankruptcy costs are very high, as is indeed the case in most developing countries, the feasibility constraints may simply be impossible to satisfy, precluding access to private finance. This rationalizes the fact that relatively little private money flows into infrastructure. Clearly, providing more efficient mechanisms to resolve bankruptcy cases tops the list of desirable policy reforms in that respect.

In addition, feasibility also relies on sufficient levels of demand being available. Even if this feasibility constraint is satisfied, the implications of private finance in term of higher prices and lower access are likely to restrict the social acceptability of private arrangements and fragilize the social contract between the state and the population, including non-poor segments.[45]

Second, this feasibility trade-off is affected by the nature of the projects and by characteristics of the economic and institutional environment. Beyond the efficiency of bankruptcy procedures, aspects such as the cost of public funds, regulatory capacity, and corruption all appear to matter.

Interestingly, while improvements along some of these dimensions ease access to private finance, for others it actually makes public finance more efficient. The latter is particularly true for reductions in the cost of public funds, which tilt the balance towards more public

[45] See Martimort and Straub (2009), and Bonnet et al. (2012) for such an argument in the context of infrastructure privatization.

subsidies. As a result, and consistently with existing stylized facts, the evolution of the desirability of private finance for infrastructure as countries transit the development path appears to be less obvious than posited by the standard policy narratives.

Countries with infrastructure deficit should therefore not consider the expansion of private finance as the sole way to increase investment. In fact, policy reforms meant to improve the business environment are likely to have competing effects. Some will make infrastructure ventures more attractive for private financiers, while others will improve public sectors' ability to raise taxes and spend efficiently. Our analysis suggests that both paths should be pursued.

REFERENCES

[1] Armstrong, M. and D. Sappington (2006). Regulation, Competition and Liberalization. *Journal of Economic Literature,* **44:** 325-366.

[2] Asian Development Bank. (2017). Meeting Asia's Infrastructure Needs. Manila: ADB.

[3] Auriol, E. and M. Warlters (2012). The Marginal Cost of Public Funds and Tax Reform in Africa, *Journal of Development Economics*, **97:** 5872.

[4] Arezki, R., Bolton, P., Peters, S., Samama, F., and J. Stiglitz (2017). From Global Savings Glut to Financing Infrastructure. *Economic Policy*, **32:** 221-261.

[5] Bagnoli, C. and T. Bergstrom (2005). Log-Concave Probability and its Applications. *Economic Theory*, **26:** 445-469.

[6] Baron, D. (1988). Regulation and Legislative Choice. *The RAND Journal of Economics,* **19:** 467-477.

[7] Baron, D. and R. Myerson (1982). Regulating a Monopolist with Unknown Costs. *Econometrica*, **50:** 911-930.

[8] Bennett, J. and E. Iossa (2006). Building and Managing Facilities for Public Services. *Journal of Public Economics*, **90:** 2143-2160.

[9] Bitsch, F., Buchner, A. and C. Kaserer (2010). Risk, Return and Cash Flow Characteristics of Infrastructure Fund Investments. *EIB papers,* **15: 106-136**.

[10] Blanc-Brude, F., Whittaker, T. and S. Wilde (2017). Searching for a Listed Infrastructure Asset Class: Mean-Variance Spanning Tests of 22 Listed Infrastructure

Proxies. *Financial Markets and Portfolio Management.*

[11] Bolton, P. and D. Scharfstein (1990). A Theory of Predation Based on Agency Problems in Financial Contracting. *The American Economic Review,* **80:** 93-106.

[12] Bonnet, C., Dubois, P., Martimort, D. and S. Straub (2012). Empirical Evidence on Satisfaction with Privatization in Latin America. *The World Bank Economic Review*, **26:** 133.

[13] Border, K. and J. Sobel (1987). Samurai Accountant: A Theory of Auditing and Plunder. *The Review of Economic Studies*, **54:** 525-540.

[14] Bortolotti, B., Cambini, C., Rondi, L., and Y. Spiegel (2011). Capital Structure and Regulation: Do Ownership and Regulatory Independence Matter? *Journal of Economics and Management Strategy,* **20:** 517-564.

[15] Bortolotti, B., Fotak, V., and W. Megginson. (2015). The Rise of Sovereign Wealth Funds: Definition, Organization, and Governance. In *Public Private Partnerships for Infrastructure and Business Development* 295-318. Palgrave Macmillan, New York.

[16] Brander, J. and T. Lewis (1986). Oligopoly and Financial Structure: The Limited Liability Effect. *The American Economic Review,* **76:** 956-970.

[17] Brander, J. and T. Lewis (1988). Bankruptcy Costs and the Theory of Oligopoly. *The Canadian Journal of Economics,* **21:** 221-243

[18] Cambini, C. and Y. Spiegel (2016). Investment and Capital Structure of Partially Private Regulated Firms. *Journal of Economics and Management Strategy*, **25:** 487-515.

[19] Collier, P., Kirchberger, M. and M. Söderbom (2016). The Cost of Road Infrastructure in Low- and Middle-Income Countries. *The World Bank Economic Review.* **30(3):** 522-548.

[20] Cordella, T. (2018). Optimizing Finance for Development. Policy Research working paper WPS 8320. Washington, D.C.: World Bank Group.

[21] Dasgupta, S. and V. Nanda (1993). Bargaining and Brinkmanship: Capital Structure Choice by Regulated Firms. *International Journal of Industrial Organization.* **11:**475-497

[22] Dixit, A. (2010). Democracy, Autocracy and Bureaucracy. *Journal of Globalization*

and Development, **1**.

[23] Eichengreen (1994). Financing Infrastructure in Developing Countries: Lessons from the Railway Age. *Economics Department WP 94-230*.

[24] Engel, E., Fischer, R., and A. Galetovic (2001). Least-Present-Value-of-Revenue Auctions and Highway Franchising. *Journal of Political Economy,* **109:** 993-1020.

[25] Engel, E., Fisher, R., and A. Galetovic (2006). Renegotiation Without Hold-Up: Anticipating Spending in Infrastructure Concessions, *Cowles Foundation Discussion Paper* 1567.

[26] Engel, E., Fischer, R., and A. Galetovic (2013). The Basic Public Finance of Public-Private-Partnerships. *Journal of the European Economic Association,* **11:** 83-111.

[27] Engel, E., Fischer, R., and A. Galetovic (2014). *The Economics of Public-Private Partnerships: A Basic Guide*, Cambridge University Press.

[28] Ensor, W. (2016). Global Tax Structures and the Marginal Cost of Funds. Mimeo, Clemson University.

[29] Estache, A., Serebrisky, T., and L. Wren-Lewis (2015). Financing Infrastructure in Developing Countries. *Oxford Review of Economic Policy,* **31:** 279-304.

[30] Fabre, A. and S. Straub (2019). The Economic Impact of public private partnerships (PPPs) in Infrastructure, Health and Education: A Review. TSE working paper 986.

[31] Fay, M., Andres, L.A., Fox, C., Narloch, U., Straub, S., and M. Slawson (2017). *Rethinking Infrastructure in Latin America and the Caribbean. Spending Better to Achieve More.* Directions in Development. Washington, DC: World Bank.

[32] Fay, M., Han. S., Lee, H., Mastruzzi, M. and Cho, M. (2019). *Hitting the Trillion Mark a Look at How Much Countries are Spending on Infrastructure.* Policy Research Working Paper WPS No. 8730. Washington, D.C: World Bank Group.

[33] Fay, M. and MS. Straub (2019). IRising incomes and inequality of access to infrastructure among Latin American Households. *Journal of Infrastructure Policy and Development.* **3:1** 76-99.

[34] Gale, D. and M. Hellwig (1985). Incentive-Compatible Debt Contracts: The One-Period Problem. *The Review of Economic Studies.* **52:** 647-663.

[35] Guasch, J. L., Laffont, J.J., and S. Straub (2008). Renegotiation of Concession Contracts in Latin America: Evidence from the Water and Transport Sectors. *International Journal of Industrial Organization,* **26:** 421-442.

[36] Hart, O. (2003). Incomplete Contracts and Public Ownership: Remarks and an Application to Public-Private Partnerships. *Economic Journal*, **119:** 69-76.

[37] Harris, M. and A. Raviv (1991). The Theory of Capital Structure. *The Journal of Finance,* 46: 297-355.

[38] Harris, M. and A. Raviv (1992). Financial Contracting Theory. In *Advances in Economic Theory, 6th World Congress,* vol 2, J.J. Laffont ed. Econometric Society Monographs.

[39] Inderst, G. (2010). Infrastructure as an Asset Class. *EIB papers,* **15:** 70-105.

[40] Inderst, G. and F. Stewart (2014). Institutional Investment in Infrastructure in Emerging Markets and Developing Economies. PPIAF publication.

[41] Iossa, E. and D. Martimort (2012). Risk Allocation and the Costs and Benefits of Public-Private Partnerships. *The RAND Journal of Economics*, **43:** 442-474.

[42] Iossa, E. and D. Martimort (2015). The Simple Micro-Economics of Public-Private Partnerships, *Journal of Public Economic Theory,* Special Issue on Public-Private Partnerships, **17:** 4-48.

[43] Iwaro, J. and A. Mwasha (2010). Towards Energy Sustainability in the World: The Implications of Energy Subsidy for Developing Countries, *International Journal of Energy and Environment,* **1:** 705-714.

[44] Ketterer, J., and A. Powell (2018). Financing Infrastructure: On the Quest for an Asset Class. IDB discussion paper 622.

[45] Khalil, F., Martimort, D., and B. Parigi (2007). Monitoring a Common Agent: Implications for Financial Contracting. *Journal of Economic Theory,* **1:** 35-67.

[46] Laffont, J.J. and J. Tirole (1993). *A Theory of Incentives in Regulation and Procurement.* MIT Press.

[47] Laffont, J.J. (2005). *Regulation and Development.* Cambridge University Press.

[48] Leland, H. (1994). Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *The Journal of Finance,* **49:** 1213-1252.

[49] Lee, K., Miguel, T., and C. Wolfram (2019). Experimental Evidence on the

Economics of Rural Electrification. *The Journal of Political Economy, forthcoming.*

[50] Lewis, T. and D. Sappington (2000). Contracting with Wealth-Constrained Agents. *International Economic Review,* **41:** 743-767.

[51] Martimort, D. and J. Pouyet (2008). Build It Not: Normative and Positive Theories of Public-Private Partnerships. *International Journal of Industrial Organization*, Special Issue on PPPs , **26:** 393-411.

[52] Martimort, D., Pouyet, J. and W. Sand-Zantman (2014). How to Regulate a Firm under Financial Constraint? Mimeo.

[53] Martimort, D. and S. Straub (2009). Infrastructure Privatization and Changes in Corruption Patterns: The Roots of Public Discontent. *Journal of Development Economics,* **90:** 69-84

[54] Moore, A., Straub, S., and J.-J. Dethier (2014). Regulation, Renegotiation and Capital Structure: Theory and Evidence from Latin American Transport Concessions. *Journal of Regulatory Economics*, **45:** 209232.

[55] Parlour, C. and U. Rajan (2001). Competition in Loan Contracts. *American Economic Review,* **91:** 1311-1328.

[56] Rozenberg, J., and M. Fay, eds. (2019). Beyond the Gap: How Countries Can Afford the Infrastructure They Need while Protecting the Planet. Sustainable Infrastructure Series. Washington, DC: World Bank.

[57] Schmitz, P. (2013). Public Procurement in Times of Crisis: The Bundling Decision Reconsidered. *Economics Letters,* **121:** 533-536.

[58] Spiegel, Y. (1994). The Capital Structure and Investment of Regulated Firms under Alternative Regulatory Regimes. *Journal of Regulatory Economics,* **6:** 297-319.

[59] Spiegel, Y. (1996). The Role of Debt in Procurement Contracts. *Journal of Economics and Management Strategy,* **5:** 379-407.

[60] Spiegel, Y. and D. Spulber (1994). The Capital Structure of a Regulated Firm. *The RAND Journal of Economics,* **25:** 424-440.

[61] Townsend, R. (1979). Optimal Contracts and Competitive Markets with Costly State Verification. *Journal of Economic Theory,* **21:** 265-293.

[62] Winton, A. (1995). Costly State Verification and Multiple Investors: The Role of Seniority. *The Review of Financial Studies,* **8:** 91-123.

[63] World Bank (2017). A Cascade-Decision Making Approach. Infrastructure Finance: Guiding Principles for the World Bank Group.

[64] World Energy Outlook (2006). Paris: OECD/IEA.

APPENDIX A: MODEL EXTENSIONS

Remark on the Irrelevance of Assets Ownership

The expression of social welfare (2.3) nicely illustrates the irrelevance of assets ownership in our setting. Indeed, the contractual environment under scrutiny may actually capture a scenario where, instead of the firm looking for outside finance, the government is keeping ownership of the assets while, at the same time, renting its usage to the least-costly service provider through tenders as in Engel, Fischer and Galetovic (2001). This scenario prevails under most traditional forms of procurement. Indeed, (2.3) indicates that everything happens as if the Public Agency is paying I out of public budget. The Public Agency then chooses to delegate operating tasks to the firm but the latter is no longer concerned with the financial side of the project. Because of the cost of public funds, raising that amount with public finance has a social cost worth $(1+\lambda)I$. However, public finance also allows to cut on the direct subsidy paid to the firm, which is now reduced to $T' = T - I$, It thus save an equal amount $(1+\lambda)I$. In contrast, standard PPP's model would require to bundle operations and finance altogether. Public subsidies are then substituted to direct public ownership of the investment. As far as the overall social cost of the investment is concerned, there is a complete Ricardian equivalence between public and private funds and the two institutional arrangements entail *in fine* the same social cost.

General Case: The Feasibility of Outside Finance

Manipulating (2.7) and (2.8) and denoting by $I_{pr} = I - T$ the share of the investment that ends up being financed by the private sector, we may define the threshold value $\hat{c}$ (or alternatively the face value of debt through (2.7)) as the solution to the following equation

$$\text{(A1)} \qquad I_{pr} = (1 - H(p)) \left(p - \hat{c} - \int_{\hat{c}}^{\infty} (1 - F(c))dc - \mu(1 - F(\hat{c})) \right).$$

The right-hand side is the revenue that investors may recoup. Because lenders are competitive, this revenue just covers the requested level of private investment. This revenue can be decomposed into two different components. On the one hand, for each unit of demand, financiers have to pay a cost of audit over the verification zone $[\hat{c}, +\infty)$. On the other hand, everything happens as if, for each unit of demand, a competitive lender could only recoup a price-cost margin $p - \hat{\hat{c}}$ where the expected (marginal) cost perceived by the lender can be defined as

$$\hat{\hat{c}} = \hat{c} + \int_{\hat{c}}^{\infty} (1 - F(c))dc = \hat{c}F(\hat{c}) + \int_{\hat{c}}^{\infty} cdF(c).$$

This perceived cost actually amounts to the average cost conditional on information, which has been collected through the audit. For all cost realizations below $\hat{c}$, there is no audit and lenders only know that the firm's cost is below $\hat{c}$, an event that occurs with probability $F(\hat{c})$. For all cost realizations above $\hat{c}$, audit takes place and lenders perfectly learn the firm's cost, which makes a contribution $\int_{\hat{c}}^{\infty} cdF(c)$ to the perceived cost $\hat{\hat{c}}$.

It is useful to rewrite (A1) as:

$$\text{(A2)} \qquad \varphi(\hat{c}) = p - \frac{I - T}{1 - H(p)},$$

where

$$\varphi(\hat{c}) = \hat{c} + \int_{\hat{c}}^{\infty} (1 - F(\tilde{c}))d\tilde{c} + \mu(1 - F(\hat{c})).$$

Observe that $\varphi(\hat{c})$ is the sum of the expected (marginal) cost perceived by the lender (namely $\hat{c} + \int_{\hat{c}}^{\infty}(1 - F(\tilde{c}))d\tilde{c}$) and the cost of bankruptcy $\mu(1 - F(\hat{c}))$ per unit of demand.

Next Lemma highlights some important properties satisfied by the function φ.

Lemma 1 *φ is quasi-convex and achieves a minimum at $\hat{c}^m$ defined by:*

$$\text{(A3)} \qquad \frac{F(\hat{c}^m)}{f(\hat{c}^m)} = \mu.$$

To better understand the meaning of φ, suppose that there is a single potential lender who thus has monopoly power in fixing the terms of a debt contract. The net payoff to such monopolistic lender could be written as:

$$\text{(A4)} \qquad (1 - H(p))(p - \varphi(\hat{c})) + T - I.$$

In other words, a minimum of φ also maximizes the profit of this monopolistic lender. This financier chooses an audit zone that optimally trades off the cost of audit against the benefit of a greater repayment when audit is more likely. It is a familiar intuition from the work of Townsend (1978) and Gale and Hellwig (1985). Increasing the audit zone when reducing $\hat{c}^m$ by a small amount dc increases the cost of audit by $(1-H(p))\times\mu f(\hat{c}^m)dc$. At the same time, the repayment can be increased by $(1-H(p))\times dc$ for all cost realizations $c\leq\hat{c}^m$; a gain worth $(1-H(p))F(\hat{c}^m)dc$. A monopolistic lender chooses a threshold $\hat{c}^m$ for the audit zone so as to equate those marginal gains and costs.

From Lemma 1, it follows that (A2) has a unique solution $\hat{c}\geq\hat{c}^m$ only if the following feasibility condition is satisfied:

$$p-\frac{I-T}{1-H(p)}\geq\varphi(\hat{c}^m).$$

By means of (2.7), this solution defines the corresponding face value of debt D. As the face value of debt D increases, the audit zone also increases ($\hat{c}$ decreases).

The feasibility condition amounts to requiring a positive payoff to the monopolistic financier and can also be written as:

$$\text{(A5)}\qquad (p-\varphi(\hat{c}^m))(1-H(p))+T-I\geq 0.$$

This feasibility condition describes all public policies (p,T) that render outside finance feasible. Were the Public Agency able to choose a pair (p,T) that instead violates (A5), by picking either a subsidy level or imposing a regulated price that are too small, outside finance would no longer be feasible. We would then be back to the case where the expertise of financiers to assess costs cannot be used. Given that the firm is protected by limited liability, the subsidy T or the regulated price p would have to be raised sufficiently to induce participation from the firm even for the worst realization of its costs.

By means of (2.7), the threshold $\hat{c}^m$ thus corresponds to the greatest audit zone and the maximal amount of debt D^m that makes outside finance feasible, namely:

$$D^m=(p-\hat{c}^m)(1-H(p))+T.$$

A pair (p,T) that would strictly satisfy (A5) corresponds to a solution to (A2) (on the increasing branch of φ) with a higher cut-off $\hat{c}>\hat{c}^m$ (a smaller audit zone) and a lower face value of debt $D<D^m$.[46]

[46] There is also another solution to (A2) that lies on the decreasing part of φ (i.e., such that $\hat{c}_-\leq\hat{c}^m$). Because the audit zone is greater with $\hat{c}_-\leq\hat{c}^m$ than with the solution $\hat{c}$ on the increasing part of φ, such solution does not minimize the cost of audit and cannot be selected in any welfare maximizing policy.

Next Lemma allows us to write the feasibility condition (A5) in a simpler form.

Lemma 2 *For a feasible debt contract, the threshold $\hat{c}$ beyond which audit takes place necessarily satisfies:*

$$\hat{c} \geq \hat{c}^m. \tag{A6}$$

In other words, the maximal audit is achieved with financiers who are monopolists. More competitive financiers will exert exert less audit and the optimal level of debt will be lower.

Feasibility condition in terms of the firm's profit. Because lenders are competitive and make zero profit with a debt contract inducing a cut-off $\hat{c}$, the firm's net profit also stands for the overall profit of the implicit coalition it forms with its lenders including the cost of audit. It leads us to write the firm's expected profit as:

$$(p - c_e)(1 - H(p)) + T - I - \mu(1 - H(p))(1 - F(\hat{c})). \tag{A7}$$

Slightly abusing notations and now making the dependence on $(p, \hat{c})$ explicit allows us to give an alternative and very compact expression of the firm's expected profit.

Lemma 3 *The firm's expected profit under a debt contract with audit for $c \geq \hat{c}$ satisfies:*

$$\mathcal{U}(p, \hat{c}) = (1 - H(p)) \int_0^{\hat{c}} F(c)dc. \tag{A8}$$

This expression is remarkable. While under self-finance or traditional procurement, the firm is making zero profit, it now always enjoys some positive rent from having signed a debt contract with outside financiers while being privately informed on its costs. The firm's rent is proportional to the demand for the service, namely $1-H(p)$. That it depends positively on its scale of activities is a rather intuitive result since the proportionality follows from the fact that audit costs are also counted per unit of demand. In addition, the rent is also proportional to a second term $\int_0^{\hat{c}} F(c)dc$, which comes from the fact that, with a debt contract, the firm repays financiers less than what it gets whenever its cost realization falls below the threshold $\hat{c}$. For all cost realizations $c \leq \hat{c}$ (that thus induce no audit) and for each unit of demand, the firm enjoys some gain $\hat{c} - c$ from such low repayment. Integrating those benefits over all infra-marginal costs then gives a payoff $\int_0^{\hat{c}}(\hat{c}-c)dF(c) = \int_0^{\hat{c}} F(c)dc$ where the equality follows from a simple integration by parts.

The firm's information rent is of course socially costly. Both the pricing of the service and the debt contract will be modified accordingly to account for this extra cost. Equation (A8) already shows how to do so. First, the Public Agency is able to reduce this rent by increasing the regulated price and decreasing demand: a direct effect under its own control. Second, increasing the audit zone and choosing a lower threshold also helps: an indirect effect that is left to financiers.

On more technical grounds, we may notice that Condition (A8) also establishes a one-to-one relationship between the firm's payoff $\mathcal{U}(p,\hat{c})$ and the face value of debt, or alternatively the threshold $\hat{c}$ above which outside financiers audit the project. In other words, controlling the firm's profit through prices and subsidies allows the Public Agency to indirectly command the firm's level of debt. Although the Public Agency has no direct control of the debt contract in practice, it may make the venture more attractive for outside financiers by simply increasing the public subsidy or raising the regulated prices so as to boost revenues from the service. This simple observation will later allow us to write the social welfare maximization in terms of p and $\hat{c}$; understanding that the Public Agency indirectly commands access to outside finance. For the time being, it is important to stress that less audit (i.e., a greater value of $\hat{c}$) and less debt also means that the firm gets more information rent.

Finally, a few simple comparative statics regarding the share of the investment covered by private finance $I - T^*$ can be derived. We expect a a smaller share of private finance under the following conditions:

- An increase in the cost of providing the service such that the new cost distribution $F^{'}$ shifts towards the right, i.e., one such that $F^{'}$ dominates F in the sense of first-order stochastic dominance.
- A lower demand schedule $H^{'}$, i.e., one such that $\frac{1-H^{'}}{h^{'}} \geq \frac{1-H}{h}$.

Optimal level of debt. The Public Agency designs a regulatory environment so as to induce outside financiers to ask for a debt level, which remains below what a monopolistic lender would chose. It thus induces less audit: $\hat{c}^* > \hat{c}^m$. The intuition for this result is straightforward. A monopolistic lender cares about extracting the firm's rent and to do so is more likely to audit ($\hat{c} = \hat{c}^m$) and ask for the highest possible level of debt D^m. Instead, the Public Agency is less concerned by rent extraction and thus induces less audit and less debt. As the cost of public funds increases, the Public Agency's and the financiers' objectives become more aligned; both want to extract the firm's information rent at the same rate. The hierarchical structure of the game with the Public Agency moving first, before financiers, amounts to having financiers being delegated the decision to audit because of their comparative advantage in doing so. When concerns for rent extraction are similar along the hierarchy, the Public Agency chooses a regulatory environment that induces the same level of audit than what a monopolistic lender would do.

Optimality of investment. The investment I is now socially valuable when:

$$\text{(A9)} \qquad \omega(p^*) \geq (1+\lambda)I + (1 - H(p^*)) \left((1+\lambda)\mu(1 - F(\hat{c}^*)) + \lambda \int_0^{\hat{c}^*} F(c)dc \right).$$

This condition is clearly harder to satisfy than its counterpart (2.5) in the scenario with self-finance. Indeed, both the social cost of audit and the social cost of the firm's rent must now be covered by the net social value of the project $\omega(p^*) - (1+\lambda)I$. As a result, fewer projects will be undertaken with outside finance than if self-finance was possible. To understand this, observe that, under self-finance, only the expected cost matters to evaluate the feasibility of the project and *in fine* the standard Ramsey-Boîteux price distortion is optimal. When self-finance is no longer possible, the firm could take advantage of the fact that financiers remain uninformed about costs to minimize repayments for its loan by reporting very large costs. In case reported costs seem too high, financiers have to rely on an audit to unveil their true value, ease incentive compatibility and thus reduce the burden of the information rent. However, the cost of audit and the remaining part of information rent left to the firms are both passed on to prices.[47]

The mix between public subsidies and outside finance. Using (A7) and (2.11) now yields an expression of the optimal public subsidy as:

$$(1 - H(p^*)) \int_0^{\hat{c}^*} F(c)dc = T^* - I + (1 - H(p^*)) \left(p^* - c_e - \mu(1 - F(\hat{c}^*))\right).$$

[47] Of course, if we had instead assumed that the Public Agency, uninformed on cost, could audit the firm on its own but at a higher cost than private financiers, the corresponding scenario of traditional procurement would implement fewer projects than outside financing.

Simplifying further by using (2.10), we obtain the following expression of the optimal public subsidy as

$$T^* = I - (1 - H(p^*))(p^* - c^*) + \frac{1}{1+\lambda}(1 - H(p^*)) \int_0^{\hat{c}^*} F(c)dc,$$

and finally, using (2.9), a more compact definition

$$T^* = I - \frac{\lambda}{1+\lambda}\frac{(1 - H(p^*))^2}{h(p^*)} + \frac{\mathcal{U}(p^*, \hat{c}^*)}{1+\lambda}. \tag{A10}$$

This condition is similar to what was found under self-finance, namely Equation (2.6), although some differences appear. The first one comes from the fact now the real cost of the project is increased. Beyond the investment outlay I, this cost must also account for the firm's information rent $\mathcal{U}(p^*, \hat{c}^*) = (1 - H(p^*)) \int_0^{\hat{c}^*} F(c)dc$, which cannot be captured and is socially costly. This increase in the cost of the project gives a first reason to raise public subsidies.

The second difference is that, when the regulated price p^* is greater than p^{sf} and demand decreases accordingly, the quantity $(1 - H(p^*))(p^* - c^*)$ is lower than the similar quantity $(1-H(p^{sf}))(p^{sf} - c_e)$, the revenues from the service in the self-finance scenario.[48] With outside finance, the quantity $(1 - H(p^*))(p^* - c^*)$ could similarly be viewed as some kind of revenues from the service, taking into account that, from a social welfare point of view, costs and price-cost margins should now include the cost of bankruptcy and the cost of the firm's information rent. The above ranking simply means that less revenues can be used to cover the investment outlay with outside finance. It gives a second reason to raise public subsidy.

Alternative Assumption: Social Cost of Bankruptcy

Firms in infrastructure sectors are often big players involved in projects of prime importance for the economy. When their projects get bankrupt, it generates a social cost that goes beyond the simple private cost of bankruptcy borne by financiers. Unemployment, disruption of major public services, domino effects on smaller enterprises are likely to impose a cost on society that goes beyond the sole cost of bankruptcy incurred by private parties. Those costs are likely to be of significant magnitude in developing countries.

[48] It can be readily verified that, thanks to Assumption 1, $\frac{d}{d\theta}\left(\frac{(1-H(p))^2}{h(p)}\right) = (1 - H(p))\left(-2 - \frac{h'(p)(1-H(p))}{h^2(p)}\right) = -1 - \frac{d}{d\theta}\left(\frac{1-H(p)}{h(p)}\right) < 0$ and thus $\frac{(1-H(p^{sf}))^2}{h(p^{sf})} > \frac{(1-H(p^*))^2}{h(p^*)}$.

To illustrate the consequences of that wedge between the private and the social costs of bankruptcy in our framework, let us suppose that in case of bankruptcy, the Public Agency must also account for a loss, which, expressed in terms of compensatory payments and still per unit of demand, is worth

$$(1+\lambda)\kappa(1-H(p))(1-F(\hat{c}))$$

where $\kappa > 0$.

It is straightforward to check that Proposition 1 carries over *mutatis mutandis* with the only modification coming from replacing μ by $\mu + \kappa$. We obtain immediately that firms have less debt and bankruptcy is less likely when the social cost of bankruptcy increases. In addition, public subsidies are more important.

Alternative Assumption: Consumption Subsidy

In many circumstances, the Public Agency does not provide lump-sum subsidies to the firm but instead directly subsidizes consumption. As noticed by the World Energy Outlook (2006), consumption subsidies which have been to a large extent eliminated in the $OECD$, remain significant in some non-$OECD$ countries, going up to 220 billion per year, according to 2005 data. For instance, energy is commonly subsidized through price controls, often through state owned companies.[49] Given this, it is particularly relevant to understand how the lessons of our model carry over when consumption subsidies are allowed.

Actually, consumption subsidies boost demand and indirectly the firm's revenues much like what a direct lump-sum subsidy T would do. To see the impact of a per unit of consumption subsidy s on demand and profits, observe that the aggregate demand becomes $1 - H(\tilde{p})$, where $\tilde{p} = p - s$ is now the perceived price paid by consumers, while p remains the price charged by the firm. With these notations, we may rewrite the firm's profit as:

$$(\tilde{p} - c^e)(1 - H(\tilde{p})) + s(1 - H(\tilde{p})) - I - \mu(1 - H(\tilde{p}))(1 - F(\hat{c})).$$

This expression shows that everything happens as if the price charged by the firm was now $\tilde{p}$ itself while the firm also receives an overall subsidy $s(1 - H(\tilde{p}))$, which plays the same

[49] See Iwaro and Mwasha (2010) for the role and importance of energy consumption subsidies in some African countries.

role as the lump-sum subsidy T used previously. Of course, with such an implementation, there is no direct transfer given to the firm, which could a priori save on the cost of public funds. On the other hand, this consumption subsidy remains taken from taxpayers elsewhere in the economy and it still has a social cost worth $(1+\lambda)s(1-H(\tilde{p}))$, another instance of Ricardian equivalence. It follows that welfare is now maximized when the perceived price and the consumption subsidy satisfy

$$\tilde{p}^{cs} = p^*, \quad p^{cs} = p^* + s^{cs} \text{ and } s^{cs} = \frac{T^*}{1-H(p^*)}.$$

Anticipating that consumption is subsidized, the firm charges a higher price that undoes the impact of such subsidy on demand. An immediate corollary is thus that with outside finance, and in the absence of direct transfers to the firm, both consumption subsidies and prices are of greater magnitude.

Suppose now that there is a hidden extra social cost of using lump-sum transfers to directly subsidize the firm; i.e., raising T now costs $(1+\lambda')T$ with $\lambda' > \lambda$. Explanations for such an extra cost might come from the need to prevent corrupt political decision-makers and bureaucrats from embezzling public funds and to increase transparency of the process, an issue particularly significant in most developing countries as already discussed in Section 3.1.2.[50] Clearly, the above equivalence fails and using a consumption subsidy now dominates from a welfare point of view. This remark suggests that consumption subsidies might still be a good vehicle to carry public investment in developing countries with weak governance.

APPENDIX B: PROOFS

Proof of Lemma 1: Differentiating φ w.r.t. c yields:

$$\frac{\varphi'(c)}{f(c)} = -\mu + \frac{F(c)}{f(c)},$$

From Assumption 1, the right-hand side above is increasing and thus has a unique zero for any μ. Hence, φ is quasi-convex with a minimum at $\hat{c}^m$ defined in (A3). *Q.E.D.*

[50]On this issue, see Martimort and Straub (2009) and Engel et al. (2013).

Proof of Lemma 2: We first rewrite the feasibility condition (A5) in terms of $\mathcal{U}(p,\hat{c})$, once the private investment $I-T$ has been eliminated using (A7) to get:

$$\mathcal{U}(p,\hat{c}) \geq (1-H(p))(\varphi(\hat{c}^m) - c_e - \mu(1-F(\hat{c})).$$

Using (A8) to express the left-hand side and simplifying, this condition becomes:

$$\text{(A11)} \quad \int_0^{\hat{c}} F(c)dc - \mu F(\hat{c}) \geq \int_0^{\hat{c}^m} F(c)dc - \mu F(\hat{c}^m).$$

Observe that for $c \geq \hat{c}^m$ we have:

$$\frac{d}{dc}\left(\int_0^c F(\tilde{c})d\tilde{c} - \mu F(c)\right) = F(c) - \mu f(c) \geq 0.$$

Thus, the feasibility condition (A11) holds when (A6) is satisfied. *Q.E.D.*

Proof of Lemma 3: From the fact that lenders are competitive, the firm's profit can be expressed as in (A7). Using (A1), we can now rewrite:

$$\frac{\mathcal{U}(p,\hat{c})}{1-H(p)} = \hat{c} - c_e - \int_{\hat{c}}^{\infty}(1-F(c))dc.$$

Integrating by parts the right-hand side, we finally obtain (A8). *Q.E.D.*

Proof of Proposition 1: The Public Agency's maximization problem can now be written as:

$$(\mathcal{P}): \quad \max_{(p,\hat{c})} \mathcal{W}(p,\hat{c}) \text{ subject to (A6) and (A8).}$$

First, observe that (A8) is necessarily binding at the optimum of $(\mathcal{P})$. The objective function can thus be written as:

$$\text{(A12)} \quad \mathcal{W}(p,\hat{c}) = \omega(p) - (1+\lambda)I - (1+\lambda)\mu(1-H(p))(1-F(\hat{c})) - \lambda(1-H(p))\int_{\hat{0}}^{\hat{c}} F(c)dc.$$

We compute

$$\frac{\partial \mathcal{W}}{\partial \hat{c}}(p,\hat{c}) = (1-H(p))((1+\lambda)\mu f(\hat{c}) - \lambda F(\hat{c})).$$

By an argument similar to that made in the Proof of Lemma 1, we can easily show that $\mathcal{W}(p,\hat{c})$ is quasi-concave in $\hat{c}$ and maximum for $\hat{c}^*$ defined in (2.12).

From this, and using the monotonicity condition in Assumption 2, it follows that $\hat{c}^* > \hat{c}^m$. Thus, Condition (A6) is slack. We immediately obtain (2.11). Finally, the first-order condition w.r.t. p yields (2.9). *Q.E.D.*

Proof of Proposition 2: Let us still denote by $\hat{c}$ the threshold value beyond which bankruptcy arises if the debt level is fixed at D and the Public Agency deviates to a new regulatory contract (p, T):

$$\text{(A13)} \quad \hat{c} = p - \frac{D - T}{1 - H(p)}.$$

To ensure that financiers break even after a deviation to (p, T), it must also be that

$$\text{(A14)} \quad \varphi(\hat{c}) \leq p - \frac{I - T}{1 - H(p)}.$$

Taken together (A13) and (A14) give us

$$\text{(A15)} \quad \varphi\left(p - \frac{D - T}{1 - H(p)}\right) \leq p - \frac{I - T}{1 - H(p)}.$$

Of course, (A13) and (A14) also hold at the equilibrium value (p^e, T^e) with (A14) being an equality. We may accordingly define $\hat{c}^e$ as the equilibrium value of the bankruptcy threshold:

$$\text{(A16)} \quad \hat{c}^e = p^e - \frac{D - T^e}{1 - H(p^e)},$$

$$\text{(A17)} \quad \varphi(\hat{c}^e) = p^e - \frac{I - T^e}{1 - H(p^e)}.$$

For any possible deviation (p, T), keeping D constant, we may also express the firm's payoff in terms of the threshold on the audit zone $\hat{c}$ and the putative deviation (p, T) as

$$\mathcal{U}(p, \hat{c}) = (p - c_e)(1 - H(p)) + T - I - \mu(1 - H(p))\left(1 - F\left(p - \frac{D - T}{1 - H(p)}\right)\right).$$

Proceeding as in Lemma 3, we also obtain

$$\text{(A18)} \quad \mathcal{U}(p, \hat{c}) \geq (1 - H(p)) \int_0^{\hat{c}} F(c)dc.$$

Finally, we observe that eliminating D from (A13), (A14), (A16) and (A17) gives us an equivalent expression for Condition (A15) as:

$$\text{(A19)} \quad (\varphi(\hat{c}^e) - \hat{c}^e)(1 - H(p^e)) \geq (\varphi(\hat{c}) - \hat{c})(1 - H(p)).$$

Taking into account its *ex post* incentives to decrease the price, the Public Agency's maximization problem can now be written as:

$$(\mathcal{P}): \quad \max_{(p,\hat{c})} \mathcal{W}(p,\hat{c}) + B(p^e - p) \text{ subject to (A18) and (A19).}$$

Of course, at equilibrium, the solution to $(\mathcal{P})$ must be $(p^e, \hat{c}^e)$ because conjectures are correct. Notice also that (A18) must be binding. Inserting the value of $\mathcal{U}(p,\hat{c})$ so obtained into the maximand, we may form the Lagrangean for this problem. We denote by ζ the non-negative Lagrange multiplier for (A19). Assuming that this Lagrangean is quasi-concave in $(p,\hat{c})$, the first-order condition for optimality w.r.t. p is then given by:

$$-(1+\lambda)(p-c_e)h(p)+\lambda(1-H(p))+\lambda h(p)\int_0^{\hat{c}} F(c)dc-(1+\lambda)\mu h(p)(1-F(\hat{c}))+B'(p^e-p)$$
$$+h(p)\zeta\left(\varphi(\hat{c})-\hat{c}-(\varphi(\hat{c}^e)-\hat{c}^e)\right)=0.$$

Taking into account that conjectures are correct at equilibrium so that $\hat{c}^{lc} = \hat{c}^e$ and $p^{lc} = p^e$ and simplifying yields (3.1).

The first-order condition for optimality w.r.t. $\hat{c}$ writes as

$$\text{(A20)} \quad (1+\lambda)\mu f(\hat{c}^{lc}) - \lambda F(\hat{c}^{lc}) = \zeta(\varphi'(\hat{c}^{lc}) - 1).$$

Taking into account that $\varphi'(\hat{c}^{lc})-1 = F(\hat{c}^{lc})-1-\mu f(\hat{c}^{lc}) < 0$ and $\zeta \geq 0$, inequality (3.3) holds since $\zeta \geq 0$. Finally, observe that any $\zeta \geq 0$ is consistent with an equilibrium with rational expectations.

As ζ varies, all values $\hat{c}^{lc} \geq \hat{c}^m$ becomes consistent with a rational expectations equilibrium. The equilibrium price is given by (3.1) while the debt level D^{lc} and the corresponding public subsidy T^{lc} satisfy:

$$\hat{c}^{lc} = p^{lc} - \frac{D^{lc} - T^{lc}}{1 - H(p^{lc})},$$

and

$$\varphi(\hat{c}^{lc}) = p^{lc} - \frac{I - T^{lc}}{1 - H(p^{lc})}.$$

Q.E.D.

Proof of Proposition 4: Remember from (A5) that the monopolistic lender's profit $\mathcal{V}^m(p,T)$ satisfies

(A21) $$\mathcal{V}^m(p,T) = (p - \varphi(\hat{c}^m))(1 - H(p)) + T - I = (p - c_e - \mu)(1 - H(p)) - \mathcal{U}^m(p).$$

with the break-even condition for the financier being now

(A22) $$\mathcal{V}^m(p,T) \geq 0$$

and where the firm's expected profit under monopolistic lending, $\mathcal{U}^m(p)$, is now defined as

(A23) $$\mathcal{U}^m(p) = (1 - H(p)) \int_0^{\hat{c}^m} F(c)dc.$$

Under monopolistic lending, social welfare can be expressed in terms of p and $\hat{c}^m$ as

$$\int_p^{\infty} (1 - H(\tilde{v}))d\tilde{v} - (1 + \lambda)T + \mathcal{V}^m(p,T) + \mathcal{U}^m(p).$$

Observe that the monopolistic lender's profit can be fully captured by the Public Agency by setting T large enough so that (A22) is binding. Expressing T from (A22) being binding, social welfare becomes

$$\omega(p) - (1 + \lambda)I - (1 + \lambda)\mu(1 - H(p))(1 - F(\hat{c}^m)) - \lambda(1 - H(p)) \int_0^{\hat{c}^m} F(c)dc.$$

The formula (3.8) to (3.10) are then directly obtained by optimization with respect to p. Turning now to some comparative statics, observe that c^* achieves the minimum of the quasi-convex function:

$$(1 + \lambda)\mu(1 - F(c)) + \lambda \int_0^{c} F(c)dc.$$

Hence, the inequality in (3.9) follows. From which it also follows that $p^m > p^*$. Using (3.11), (3.10) is readily obtained. *Q.E.D.*

Proof of Proposition 5: The SWF invests whenever the corresponding expected utility yields a greater expected payoff than its reservation value. With a $CARA$ specification, this condition becomes:

(A24) $$-\int_0^{\hat{c}} exp(-r((1 - H(p))(p - \hat{c}) + T - I + w))dF(c)$$

$$-\int_{\hat{c}}^{+\infty} exp(-r((1-H(p))(p-c-\mu)+T-I+w))dF(c) \geq -exp(-rw).$$

The certainty equivalent of the SWF's payoff, namely $\mathcal{V}^s(p,T)$, can now be defined as

$$\text{(A25)}\quad \mathcal{V}^s(p,\hat{c},T) = -\frac{1}{r}\ln\Big(\int_0^{\hat{c}} exp(-r((1-H(p))(p-\hat{c})+T-I+w))dF(c)$$

$$+\int_{\hat{c}}^{+\infty} exp(-r((1-H(p))(p-c-\mu)+T-I+w))dF(c)\Big).$$

The zero-net benefit condition for a SWF then writes as

$$\mathcal{V}^s(p,\hat{c},T) = w$$

which can be further expressed as (3.12).

The firm's expected profit $\mathcal{U}^s(p,\hat{c})$ being again defined as (A8), social welfare with SWF finance can be expressed as

$$\int_p^{\infty}(1-H(\tilde{v}))d\tilde{v} - (1+\lambda)T + \mathcal{U}(p,\hat{c}).$$

Observe that the certainty equivalent of SWF's payoff does not enter into social welfare because the SWF is owned abroad. Expressing thus T from (3.12) yields a new expression of social welfare in terms of $(p,\hat{c})$ only as

$$\text{(A26)}\quad \omega(p)-(1+\lambda)I-(1+\lambda)\mu(1-H(p))(1-F(\hat{c}))-\lambda(1-H(p))\int_0^{\hat{c}} F(c)dc-(1+\lambda)\rho(p,\hat{c}).$$

Maximizing (A26) with respect to $(p,\hat{c})$ yields the following first-order conditions (assuming strict quasi-concavity of the objective function)

$$\text{(A27)}\quad \frac{p^s-c^s}{p^s} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon(p^s)}$$

where

$$\text{(A28)}\quad c^s = \hat{c}^s + \frac{\frac{\partial\rho}{\partial p}(p^s,\hat{c}^s)}{h(p^s)}$$

and

$$\text{(A29)}\quad \mu = \frac{\lambda}{1+\lambda}\frac{F(\hat{c}^s)}{f(\hat{c}^s)} + \frac{\frac{\partial\rho}{\partial\hat{c}}(p^s,\hat{c}^s)}{1-H(p^s)}.$$

A second-order Taylor expansion then gives a simpler expression of $\rho(p,\hat{c})$ (up to terms of order more than 2 in r) as (3.18). Observe in particular that

$$\Delta(\hat{c}) = \int_{\hat{c}}^{+\infty}\left(\hat{c}-c-\mu-\left(\int_{\hat{c}}^{+\infty}(\hat{c}-c-\mu)dF(c)\right)\right)^2 dF(c) > 0.$$

Inserting the corresponding derivatives of $\rho(p,\hat{c})$ into (A28) and (A29) respectively yields the approximations (3.15) and (3.17).

Turning now to the comparative statics. Observe that

$$\Delta'(\hat{c}) = -\mu^2(\hat{c})f(\hat{c}) - 2\left(\int_{\hat{c}}^{+\infty}(\hat{c}-c-\mu)dF(c)\right)(\mu f(\hat{c}) - F(\hat{c}))$$

and thus

$$\Delta'(\hat{c}^*) = -\mu^2(\hat{c})f(\hat{c}^*) + \frac{2F(\hat{c}^*)}{1+\lambda}\left(\int_{\hat{c}^*}^{+\infty}(\hat{c}^*-c-\mu)dF(c)\right) < 0.$$

From this and the definition (A28), it immediately follows that

$$\hat{c}^s > \hat{c}^*$$

as stated in (3.15). *Q.E.D.*

APPENDIX C: DATA SOURCES AND ADDITIONAL STYLIZED FACTS

Data Description. The paper uses a sample of infrastructure projects from the World Bank Private Participation in Infrastructure (PPI) database to further illustrate the results of the model. Starting in 2015, the PPI team collected, whenever possible, information on the financing structure of the projects. The PPI database includes any infrastructure project in low- and middle-income countries for which information is found in publicly available sources, which have at least 20% private participation in the project contract, and in which private parties assume operating risk, by either operating the project alone or jointly with a public counterpart, or owning an equity share in the project.

The PPI database contains detailed financial information for 361 projects, including the share of public and private debt and equity, across four infrastructure sectors (energy, transport, water and sanitation, and ICT backbone) and three years (2015-2017). The projects span 55 countries across 6 regions of the world. Tables II and III below provide the list of countries and the sector distribution of projects.

For each of these projects, the data reports the total financing amount, disaggregated between public and private financing. These two categories are further disaggregated in the following way:

$$\text{(A30)} \quad PublicFinancing = PublicDebt + PublicEquity + DirectGovernmentSupport$$

with Public Debt = Multilateral Debt + Bilateral Debt + Institutional Debt + Public National Debt, and

$$\text{(A31)} \quad PrivateFinancing = CommercialDebt + PrivateEquity.$$

Based on this, we compute the following financing indicators:

$$\text{(A32)} \quad ShareofPrivateFinancing = PrivateFinancing/TotalFinancing,$$

where Total Financing = Public Financing + Private Financing, and

$$\text{(A33)} \quad ShareofPrivateDebt = PrivateDebt/TotalDebt,$$

where Total Debt = Public Debt + Commercial Debt.

Following the model, the availability of private project financing across this sample is likely to be dependent on features of the legal environment, and specifically on the perceived cost of bankruptcy. We consider the following proxy: The Doing Business Resolving Insolvency indicator, as a measure of the cost of audit / bankruptcy facing financial institutions that lend to the projects.

Second, to capture the feasibility of public subsidies, we consider the cost of public funds, due to distortionary taxation. We use the marginal cost of public funds (MCF) estimates computed by Ensor (2016) for 106 countries, following Auriol and Warlters' (2012) model.

Finally, we consider a measure of institutional quality from the World Bank Worldwide Governance Indicators (WGI): the Control of Corruption index.[51]

The details of these data sources are the following:

Cost of audit / bankruptcy: Doing Business Resolving Insolvency indicator. It is a country-level proxy for "the time, cost and outcome of insolvency proceedings involving

[51] Similar results obtain using the WGI Regulatory Quality index instead.

domestic entities as well as the strength of the legal framework applicable to judicial liquidation and reorganization proceedings." We use the Resolving Insolvency DTF (distance to frontier) variable, which measures an economy's distance to frontier indicated on a scale from 0 to 100, where 0 represents the lowest performance and 100 the frontier.[52]

Cost of Public Funds: We use the marginal cost of public funds (MCF) estimates computed by Ensor (2016) for 106 countries, following the methodology of Auriol and Warlters (2012). The variable measures the change in social welfare associated with raising an additional unit of tax revenue using a particular tax instrument. The paper reports estimates corresponding to the average marginal cost of public funds for five key tax instruments: domestic sales taxes, import and export taxes, and corporate and personal income taxes.

Institutional Quality: We use World Bank Worldwide Governance Indicators (WGI) country level governance indicators for 2015 and 2016. We focus on a specific dimensions of governance: Control of Corruption. The indicator ranges from approximately -2.5 to 2.5, with higher values corresponding to better outcomes.[53]

Additional Stylized Facts. Note first that in this three-year sample, the average share of private financing in PPP investment is only 57%, the rest thus coming from government direct support, public banks loans, and International Financial Institutions' (IFIs) loans or grants. Zooming in on the share of private debt, the corresponding number is 39%.

Figure A1 illustrates the correlations between the project-level ratio of private to total debt and the level of development of countries where projects are located, measured with per capita GDP.[54] Overall, private finance prevalence is quite dispersed within countries across projects, but also across countries at similar levels of development. Even though richer countries seem to attract slightly higher levels of private finance, some upper middle-income countries have been quite unsuccessful at doing so, a finding reminiscent of the difference between Turkey and Lebanon discussed above.[55]

[52] See http://www.doingbusiness.org/Methodology/resolving-insolvency for more details (accessed 03/21/18).

[53] See http://info.worldbank.org/governance/wgi/ for more details (accessed 03/21/18).

[54] The sample covers 327 projects, excluding 34 projects for which there is no debt financing.

[55] The conclusions are unchanged when using instead the ratio of private to total financing.

FIGURE A1.— Projects' Share of Private Debt as a Function of per capita GDP

TABLE II

PROJECTS BY COUNTRY

Country	Number of projects
Afghanistan	1
Argentina	3
Bangladesh	4
Brazil	39
Burkina Faso	1
Cambodia	2
China	11
Colombia	6
Costa Rica	1
Dominican Republic	1
Ecuador	1
Egypt, Arab Rep.	24
El Salvador	3
Gabon	1
Georgia	2
Ghana	2
Honduras	8
India	67
Indonesia	16
Iran, Islamic Rep.	2
Iraq	1
Jamaica	4
Jordan	16
Lao PDR	2
Lebanon	1
Madagascar	1
Malaysia	2
Mali	1
Mexico	18
Mongolia	2
Montenegro	1
Morocco	2
Mozambique	2
Myanmar	1
Namibia	2
Nepal	2
Nigeria	1
Pakistan	10
Panama	4
Peru	10
Philippines	13
Romania	1
Russian Federation	7
Rwanda	1
Senegal	7
Serbia	2
South Africa	10
Sri Lanka	1
St. Lucia	1
Thailand	15
Turkey	12
Uganda	7
Ukraine	2
Vietnam	2
Zambia	2
Total	361

TABLE III
Infrastructure Projects with Financial Information

	Energy	ICT	Transport	Water & sanitation	All
2015	93	0	14	9	116
2016	77	1	14	1	93
2017	117	1	29	5	152
Total	287	2	57	15	361