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"Opting out and topping up reconsidered: informal care under uncertain altruism"

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Opting out and topping up reconsidered: informal care under uncertain altruism*

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Abstract

We study the design of public long-term care (LTC) insurance when the altruism of informal caregivers is uncertain. We consider non-linear policies where the LTC transfer depends on the level of informal care, which is assumed to be observable, while children's altruism is not. Our policy encompasses two policies traditionally considered in the literature: topping up policies consisting of a transfer independent of informal care, and opting out policies entailing a positive transfer only if children fail to provide care. We show that both total and informal care should increase with the children's level of altruism. This obtains under full and asymmetric information. Public LTC transfers, on the other hand, may be non-monotonic. Under asymmetric information, public LTC transfers are lower than their full information level for the parents whose children are the least altruistic, while it is distorted upward for the highest level of altruism. This is explained by the need to provide incentives to highly-altruistic children. In contrast to both topping up and opting out policies, the implementing contract is always such that social care *increases* with informal care.

JEL classification: H2, H5.

Keywords: Long-term care, uncertain altruism, private insurance, public insurance, topping up, opting out.

1 Introduction

Old-age dependency and the associated need for long-term care (LTC) represent a lifetime risk that has recently received a lot of media attention. LTC needs arise when the elderly depend on help to carry out their daily activities. LTC is different from—albeit often complementary to—health care, and in particular, terminal care or hospice care. Dependent individuals do not only need medical care but also help with their normal daily routines. Providing this type of assistance is labor intensive and often quite costly, especially in severe cases of dependency that call for institutional care.

Dependency represents a significant financial risk of which, in most countries, only a small part is currently covered by social or private insurance.[1] As a consequence, individuals often have to rely on their own private savings or on care provided by their family. Informal care is estimated to play a dominant role in the provision of LTC (see Norton, 2016). However, not all elderly individuals can count on family assistance, and informal care is subject to many random shocks. There are pure demographic factors such as widowhood, absence or the loss of children; divorce and migration can also be put in this category. In addition, conflicts within the family or financial problems incurred by children may prevent them from helping their parents.

This paper studies the role and design of LTC policy when informal care is uncertain. This uncertainty is represented by a single parameter, referred to as the child's degree of altruism, which is randomly distributed over some interval.[2] This parameter is not

[1]See, for instance, Lipszyc *et al.* (2012). In most European countries (with the exception of Germany), social health insurance does not pay for LTC. The specific programs that exist cover only a small fraction of actual costs. For instance, in France, APA (Personalized Autonomy Allocation) is a means' tested program that contributes at most around 500 euros per month, while even a low-quality public facility (the so-called EPHAD) costs more than 2,000 euros. High-quality institutions cost up to 5,000 euros. Similarly, in the US, where costs can be twice as large, Medicare does not cover LTC. Medicaid provides some means-tested coverage.

The failure of private LTC insurance markets has been extensively studied, for instance, by Brown and Finkelstein (2009, 2011).

[2]We assume that altruism is exogenous and cannot be affected by the parents' behavior, either through fertility choices (see for instance Pestieau and Ponthière, 2016) or through socialization efforts

publicly observable and—while parents may eventually observe it—this comes at a time when their cognitive abilities no longer allow them to make significant financial decisions. We concentrate on a single generation of parents over their life cycle. When young, they work, consume, and save for their retirement. When old, they face the risk of becoming dependent. In the case of dependence, the level of informal care depends on the children's degree of altruism.

The original feature of our analysis is that we consider nonlinear policies linking the public LTC transfer to the level of informal care, assuming that the latter is observable while children's altruism is not. In reality, family care—for which by definition there is no formal market—is typically not *perfectly* observable but can be ascertained to some extent. This is the case, in particular, when public services are provided at the local level and care arrangements are coordinated by social workers in direct contact with the families.

The issue of uncertain altruism and the role of social insurance in that context has previously been studied by Cremer *et al.* (2014, 2017) and Canta *et al.* (2017). The first two of these papers concentrate on the case where altruism is a binary variable. Children are either altruistic at some known degree or not altruistic at all. The third paper—like this paper—considers a continuous distribution. These papers all consider a restricted set of policies and are agnostic about the information structure, and focus on two types of LTC policies. The first one, referred to as "topping up" (TU), provides a transfer to dependent elderly, which is non-exclusive and can be supplemented by informal care. The second one is an "opting out" scheme (OO); it provides care which is exclusive and cannot be topped up. In other words, if the family provides informal assistance, this amounts to opting out from any public LTC benefit.[3] One of the main messages that

(see Ponthière, 2014). We also rule out descending altruism and the possibility that altruistic parents may induce optimal behavior from their selfish children through appropriate transfers (Becker's (1974) rotten kid theorem).

[3] One can think of some examples of informal care that may still be compatible with OO policies, such as visits or phone calls. Here we consider actual assistance rather than attention or socialization.

emerges from these studies is that none of these schemes appears to dominate in all circumstances. In particular, the distribution of the degree of altruism is an important factor.

Both policies are *special cases* of the nonlinear schemes we consider in this paper. Under TU, the policy is independent of informal care and can, in fact, be implemented even when informal care is not observed. Under OO, we have a decreasing scheme but with rather extreme features: LTC provided by social insurance is positive when informal care is zero and then jumps to zero as soon as some informal care is provided.[4] By revisiting this problem with our information structure, we thus also have a fresh look at the TU vs. OO issue. Since none of these schemes appears to systematically dominate the other, one may expect our problem to yield an "intermediate" solution, which would imply that public care is not flat but decreases less drastically than in the OO case. Interestingly, we can show that such an intermediate solution is never optimal. The optimal policy drastically differs from both of these schemes because social LTC is always strictly *increasing* with informal care.

The specification of our formal model remains agnostic about the exact nature of the transfers—cash or in-kind. Care, whether formal or informal, is expressed in monetary equivalent and the price of formal care is normalized to one. However, the interpretation we have in mind is that informal care is mostly in-kind and represents the time that children or other relatives devote to care. While some children may assist their parents financially, the monetary transfer typically goes in the opposite direction; and gifts or bequests are often viewed as implicit payment for informal care; see Norton (2016) or Canta and Cremer (2019) for a more detailed discussion. Public LTC, on the other hand, can be of either type. In reality, it is often a mixture of monetary allowance and formal care services paid or subsidized by the government.

The distinction between TU and OO has been widely studied in the literature

[4] In the remainder of the paper we shall refer to LTC provided by social insurance simply as social LTC or public LTC insurance.

about in-kind vs cash transfers.[5] For instance, it has been shown to be relevant in the context of education and health, both from a normative and a positive perspective.[6] In this paper we remain agnostic about the exact nature of the public transfer—cash or in-kind—and instead focus on its interaction with transfers within the family and specifically informal care provided by children.[7] Furthermore, this literature typically focuses on redistributive concerns, while we concentrate on insurance issues. Individuals are identical *ex ante*, but they face two types of risk: that of becoming dependent and, when dependent, the risk of having children with a low degree of altruism.

A major concern raised by LTC policies is the crowding out of informal care.[8] This may make public LTC insurance ineffective for some persons and more expensive overall. Within the context of informal care, crowding out may occur both at the intensive and the extensive margins. Crowding out at the intensive margin refers to the reduction of informal care, possibly on a one-by-one basis, for parents who receive aid from their children alongside social LTC. Crowding out at the extensive margin, on the other hand, occurs when some children are dissuaded from providing any informal care. The specific policies considered in the earlier literature have different effects on informal care. TU involves crowding out both at the intensive and the extensive margins, whereas OO crowds out informal care solely at the extensive margin.

Since we assume that informal care is observable and can thus to some extent be controlled, one might at first be tempted to think that crowding out is not an issue in our setting. However, even under full information (about altruism) informal care

[5] For a review of the literature, see Currie and Gahvari (2008).

[6] On the normative side, for instance, Blomquist and Christiansen (1998) show that both regimes can be optimal (to supplement an optimal income tax) depending on whether the demand for the publicly provided good increases or decreases with labor. From a positive perspective, TU regimes may emerge from majority voting rules, as shown by Epple and Romano (1996).

[7] As shown by Cremer and Gahavari (1997), the distinction between in-kind and cash is irrelevant when individual consumption levels of the good are observable and non-linear instruments are available. In-kind transfers make a difference only when individual consumption is not observable so that the transfer imposes a minimum consumption level.

[8] See, for instance, Cremer *et al.* (2012) and Grabowski *et al.* (2012).

is limited by the children's participation constraints. When the degree of altruism is not observable, children have to be given the appropriate incentives to provide informal care. The design of the social insurance scheme will then involve a tradeoff between efficient provision of care and informational rents.

Throughout the paper, we concentrate on intra-generational issues; the cost of the LTC program is borne by the generation that also benefits from it. In other words, we consider a single generation of parents. The role of children is limited to their decision on the provision of informal care to their parents. The welfare of the adult children does not figure in social welfare, which accounts only for the expected life-cycle utility of parents. However, since children are subject to a participation constraint, their utility does also matter. In other words, we maximize parents' utilities for a given (minimum) utility level for children, which amounts to characterizing a Pareto-efficient allocation. The participation constraints are relevant because family exchanges are voluntary. Children do have the option not to provide any care and forego a transfer from their parents.[9] We first characterize the full information allocation, then we turn to the case where neither the government nor the parents observe the children's level of altruism.

We show that, not surprisingly, both the consumption of dependent elderly and the level of informal care should increase with the children's level of altruism. This is the case under both full and asymmetric information. Social LTC transfers, however, may be non-monotonic. On the one hand, children with a higher level of altruism may provide more informal care, which reduces the need for social insurance. On the other hand, the more altruistic the children, the higher the optimal level of consumption of the parents, since a high parental consumption relaxes the altruistic children's participation constraint.

Under asymmetric information, the consumption of the dependent elderly is distorted down for all levels of children's altruism, except for the highest one. This is

[9]In other words, parents cannot simply impose a level of care. The only leverage they have to obtain a positive level of care is to pay for it via the transfer.

the rather standard result of no distortion at the top, while the total care provided to all other parents is distorted downward to reduce informational rents. The level of informal care is also distorted down, except for the lowest level of altruism. Since informal care can be considered as a transfer from children to parents, this result is also intuitive and in line with traditional findings in contract theory. Finally, social LTC under asymmetric information is lower than its full information level for the lowest level of altruism. The opposite is true for the higher level of altruism, where social LTC is distorted upward. This may be surprising at first as it goes against the role of social care as insurance against children with low levels of altruism. Intuitively, under asymmetric information, social LTC has to be distorted up at the top of the distribution in order to elicit the appropriate level of informal care from highly-altruistic children. This is more complicated under asymmetric information where incentive constraints have to be accounted for (rather than just the participation constraints under full information). Consequently, the most altruistic children have to be "bribed" into providing informal care by an increase in social care.

The paper is organized as follows. In Section 2, we present the model and derive the laissez faire allocation, and we characterize the optimal allocation under full information in Section 3. In Section 4, we turn to the case where the government does not observe the children's degree of altruism, while Section 5 provides some numerical illustrations of the results. In Section 6, we examine their implications for the TU vs. OO debate. Finally, we summarize and conclude in Section 7.

2 The model

Consider a single generation of parents facing the risk of becoming dependent when they are elderly and retired. If they become dependent, elderly parents may or may not receive informal care from their grown-up children. This in turn depends on how altruistic the children are. All parents are identical *ex ante* and each of them has one child to rely upon.

The sequence of events is as follows. Period 0 is when the government designs and announces its tax/transfer policy; this is the first stage of our game. In period 1, young working parents each have one child, pay taxes, and save an exogenously fixed amount, s. No decision is made in this period. Next, in period 2, parents have grown old, are retired, and may be dependent. For parents who remain healthy in old age, the game is over; they simply consume their savings. Dependent parents may receive informal care from their children, who have by now become working adults. The latter decide if and how much informal care, a, they want to provide their parents with. Dependent parents also receive from the government an LTC transfer g conditional on a.[10]

All parents face two sources of uncertainty. The first concerns their state of health in old age; they may either remain healthy or become dependent. The probability of dependence, π, is exogenously given and known. The second source of uncertainty relates to the degree of altruism of their adult children, which is represented by $\beta \in [0, \overline{\beta}]$.[11] The higher β is, the more altruistic a child is. Children with $\beta = 0$ have no altruistic feelings towards their parents. The random variable β is distributed according to the distribution function $F(\beta)$, with density $f(\beta)$. We assume that the hazard rate is non-decreasing in β.[12]

The public policy consists of a menu of contracts $(a(\beta), g(\beta))$ offered to children, where $a \geq 0$ denotes the informal care that children provide to their dependent elderly parents and g is the public LTC transfer. In order to finance the LTC transfers, a tax τ is levied on young parents.

[10]The transfer is not conditional on parental wealth, which is natural in this setup where parents are homogeneous *ex ante*. We discuss this assumption in the Conclusion.

[11]We rule out $\beta < 0$, which represents a case where children are happier if their parents are worse off.

[12]The assumption that the hazard rate is non-decreasing is common in the literature and is satisfied by many classical probability distributions, such as the normal, the uniform, and the exponential distribution. In our context, the hazard rate $f(\widehat{\beta})/(1 - F(\widehat{\beta}))$ is the conditional probability that a child's altruism does not exceed $\widehat{\beta}$ given that that child's altruism is not smaller than $\widehat{\beta}$. Our assumption implies that this probability increases with altruism. For a discussion of the monotone hazard rate property, see for instance Laffont and Tirole, (1989), pp. 66-67.

Parents provide a fixed labor supply when young, and they have no disutility associated with working. Their labor income is exogenously given and equal to w. Preferences are quasilinear in consumption when young; risk aversion is introduced through the concavity of second-period, state-dependent utilities. Denote the utility function for consumption when old and healthy by U and when old and dependent by H. The parent's life-time expected utility is

$$EU = w - \tau - s + (1 - \pi) U(s) + \pi E[H(m)],$$

where E is the expectation operator. Parents pay the tax τ in the first period. In case of dependence, they consume $m = s + a + g$. Assume that $U' > 0$, $U'' < 0$, $U > 0$, $U'(0) = \infty$, and that the same properties hold for H. We also assume that $H'(s) > 1$. This ensures that, if parents do not receive any informal care or LTC transfers, their marginal utility of consumption when dependent is greater than their first-period marginal utility of consumption. In other words, parents would benefit from LTC insurance, for which a premium is paid in the first period. We express all types of care in monetary equivalent, assuming that the price of formal care purchased on the market is normalized to one. We also assume that all types of care are perfect substitutes so that $m = s + a + g$ can be interpreted as total care, which is the sum of formal care (bought from savings), informal care, and social LTC.

Assume that adult children also have quasilinear preferences and that their altruism towards their parents comes into play only if the parents become dependent. The children's utility function is represented by

$$u = \begin{cases} y - a + \beta H(m) & \text{if the parent is dependent,} \\ y & \text{if the parent is nondependent,} \end{cases} \tag{1}$$

where y denotes the working children's fixed income. No transfers are made to the healthy elderly parents regardless of the size of their savings, s. Note that children do not make any decisions under uncertainty (they choose levels of informal care and not lotteries). When they decide on the level of informal care, they know the health status of their parents, as well as their own β. Consequently, the risk neutrality implied by the linearity of preferences in income has no impact on their equilibrium choices.

2.1 Laissez faire

We first consider the equilibrium, referred to as *laissez faire*, where the government does not intervene, so that $g = 0$. The altruistic children allocate an amount a of their income y to assist their dependent parents, given the parents' savings s. Its optimal level, a^{lf}, is found through the maximization of equation (1). The first-order condition with respect to a is (assuming an interior solution),

$$-1 + \beta H'(s + a) = 0. \tag{2}$$

The concavity of H ensures that the second-order condition is satisfied and that $H'(s + a)$ is decreasing in a. Set $a = 0$ in the above equation and define $\beta_0(s)$ such that

$$\beta_0(s) \equiv \frac{1}{H'(s)}. \tag{3}$$

This function represents the minimum level of β, for a given s, at which a positive level of care is provided. We refer to a child with a β equal to the threshold level as the "marginal child".

Condition (2) implies that whenever $\beta \geq \beta_0 > 0$, a^{lf} satisfies[13]

$$s + a^{lf} = (H')^{-1}\left(\frac{1}{\beta}\right) \equiv m(\beta).$$

Conversely, when $\beta < \beta_0$, we have $a^{lf} = 0$. The consumption of dependent parents is then equal to

$$m^{lf}(\beta) = \begin{cases} s & \text{if } \beta < \beta_0, \\ m(\beta) & \text{if } \beta \geq \beta_0. \end{cases} \tag{4}$$

[13] A corner solution at $a = y$, cannot be ruled out. To avoid a tedious and minimally insightful multiplication of cases, we assume throughout the paper that the constraint $a \leq y$ is not binding in equilibrium.

Differentiating (4) yields

$$\frac{dm}{d\beta} = \begin{cases} 0 & \text{if } \beta < \beta_0, \\ \frac{-1}{\beta^2 H''(m)} > 0 & \text{if } \beta \geq \beta_0, \end{cases}$$

where the second line is positive because H is concave.[14] As expected, a dependent parent's total consumption increases with the degree of altruism of their child. In other words, parents are exposed to the risk of having a child with a low degree of altruism. Figure 1 represents equation (4). It illustrates the consumption level of dependent parents as a function of the child's degree of altruism.

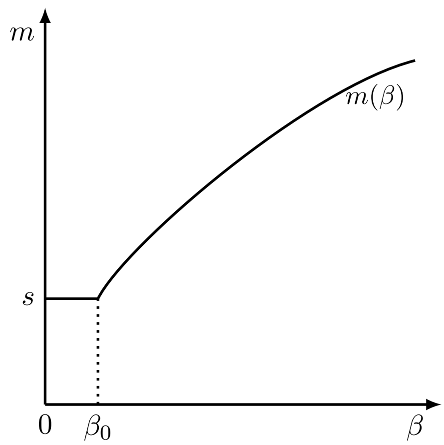


Figure 1: *Laissez-faire allocation: consumption of dependent parents as a function of the degree of altruism of the children.*

3 Full information

Under full information, the government observes the type β of each child and sets a menu $(a(\beta), g(\beta))$ maximizing the *ex ante* utility of parents. The problem of the government

[14]The function m is not differentiable at $\beta = \beta_0$. To avoid cumbersome notation, we use $dm/d\beta$ for the right derivative at this point.

is

$$\begin{aligned}
\max_{a(\beta),g(\beta)} \quad & EU = w - s - \tau + (1-\pi)u(s) + \pi \int_0^{\overline{\beta}} H(s + a(\beta) + g(\beta)) f(\beta) d\beta \\
\text{s.t.} \quad & \tau \geq \pi \int_{\beta} g(\beta) f(\beta) d\beta \qquad (5) \\
& y - a(\beta) + \beta H\left(s + a(\beta) + g(\beta)\right) \geq y + \beta H(s) \quad \forall \beta \\
& a(\beta) \geq 0, \quad g(\beta) \geq 0 \quad \forall \beta,
\end{aligned}$$

where the first constraint is the government's resource constraint and the second one is the children's participation constraint, which stipulates that their utility must be at least equal to a minimal level. We set this equal to their utility when they do not participate in the provision of care, but setting it at any other arbitrary level would not change the results. Under full information the participation constraint is binding for all types of children, so that we effectively calculate a Pareto-efficient allocation. In other words, we maximize the expected utility of the parents for a given utility level for all potential types of children. Specifying a different utility level for children would yield another allocation on the utility possibility curve, with otherwise similar properties.[15] As an alternative, one could have set the children's minimum utility at their laissez-faire level. Though intuitively appealing this would be not be consistent from a contract theory perspective. Specifically, once a government program is in place, the laissez faire may no longer be an option.

The budget constraint must be binding, and equation (5) holds with equality. Substituting it into the objective function, the problem of the government can be rewritten

[15] With general preferences the inter-generational sharing of the burden would affect the solution and our solution may not be Pareto efficient for the lack of instruments. However, with quasi-linear preferences Pareto efficiency and intergenerational redistribution are separable. To be more precise, the utility possibility curve is a straight line (as usual in a quasi-linear world) and the allocations differ only in the consumption levels of the numeraire by parents and children.

as

$$\begin{aligned}
\max_{a(\beta),g(\beta)} \quad & EU = w - s - \pi \int_0^{\overline{\beta}} g(\beta) f(\beta) d\beta + (1-\pi) u(s) \\
& + \pi \int_0^{\overline{\beta}} H(s + a(\beta) + g(\beta)) f(\beta) d\beta \\
\text{s.t.} \quad & - a(\beta) + \beta H\left(s + a(\beta) + g(\beta)\right) - \beta H(s) \geq 0 \quad \forall \beta \\
& a(\beta) \geq 0, \quad g(\beta) \geq 0 \quad \forall \beta.
\end{aligned} \tag{6}$$

The first order conditions with respect to $a(\beta)$ and $g(\beta)$ are, respectively

$$\frac{\partial EU}{\partial a} = \pi H'(s + a(\beta) + g(\beta)) f(\beta) + \lambda_\beta [\beta H'(s + a(\beta) + g(\beta)) - 1] + \mu_\beta = 0, \tag{7}$$

and

$$\frac{\partial EU}{\partial g} = -\pi f(\beta) + \pi H'(s + a(\beta) + g(\beta)) f(\beta) + \lambda_\beta \beta H'(s + a(\beta) + g(\beta)) + \nu_\beta = 0, \tag{8}$$

where λ_β, μ_β, and ν_β are the Lagrange multipliers associated with the participation constraint and the non-negativity constraints for the β-type child, respectively. Equation (7) implies that $\lambda_\beta > 0$ for all β: under full information the participation constraint is binding for all children. Intuitively, if this were not the case, it would be possible to increase EU by increasing $a(\beta)$ until the constraint becomes binding.

We shall now use these conditions to study the properties of the solutions. In particular, we examine the possibility of both interior and corner solutions as well as the shape of $a(\beta)$ and $g(\beta)$.

3.1 Interior solutions

When the solution is interior ($\mu_\beta = \nu_\beta = 0$), combining (7) and (8) yields $\lambda_\beta = \pi f(\beta)(1 - H')/\beta H'$. Substituting in (7) we obtain

$$(1+\beta) H'(s + a(\beta) + g(\beta)) = (1+\beta) H'(m(\beta)) = 1. \tag{9}$$

This equation determines total care $m^*(\beta)$ and because $H'(m) < 1$, it shows that parents receive *more* than full insurance (which would imply $H' = 1$), as long as $\beta > 0$. This is because m contributes to children's utility. This matters from the parent's and the government's perspective because it increases the level of informal care that can be provided by the children required to meet participation constraints.

Totally differentiating (9) yields

$$\frac{\partial(g^*(\beta) + a^*(\beta))}{\partial\beta} = -\frac{H'}{(1+\beta)H''} > 0. \tag{10}$$

Consequently, under symmetric information, the *total transfer* to dependent parents $a+g$ increases as the level of children's altruism increases. This is in line with the interpretation of (9): the children's benefits from m increase with their degree of altruism.

When the solution is interior, the optimal policy under symmetric information is such that (9) holds and the participation constraints of the children hold with equality. Using (9), for each β the public transfer g can be expressed as a function of a of the form

$$g^*(\beta) = m^*(\beta) - s - a^*(\beta), \tag{11}$$

where $m^*(\beta) = H'^{-1}(1/(1+\beta))$ is the optimal consumption of dependent parents. Substituting for g in the participation constraints we obtain

$$a^*(\beta) = \beta H(m^*(\beta)) - \beta H(s). \tag{12}$$

Differentiating this expression with respect to β yields, using (10)

$$\frac{\partial a^*(\beta)}{\partial\beta} = H(m^*(\beta)) - H(s) + \beta H'(m^*(\beta))\frac{\partial m^*(\beta)}{\partial\beta} = H(m^*(\beta)) - H(s) - \frac{\beta}{(1+\beta)^3 H''(m^*(\beta))},$$

which is positive as long as $m(\beta) \geq s$. Not surprisingly, informal care increases with the children's degree of altruism. For a given level of utility (the reservation level), they are willing to provide more informal care the more altruistic they are.

Observe that equations (9) and (12) fully determine the solution for $m^*(\beta)$ and $a^*(\beta)$. The profile of public LTC $g^*(\beta)$ is then simply determined as a "residual" via equation (11). Differentiating this expression we obtain

$$\frac{\partial g^*(\beta)}{\partial \beta} = -H(m(\beta)) + H(s) + [1 - \beta H'(m(\beta))]m'(\beta) = -H(m(\beta)) + H(s) - \frac{1}{(1+\beta)^3 H''(m(\beta))},$$

which does not appear to have an unambiguous sign. On the one hand, an increase in β entails an increase of the optimal level of consumption when dependent, $m(\beta)$, since altruistic children partially internalize it. This tends to increase g. On the other hand, as β increases, $a(\beta)$ increases, which reduces the government's transfer. The sign of the derivative depends on whether $a^*(\beta)$ grows faster than $m^*(\beta)$. This argument of course simply proves that $g^*(\beta)$ is not *necessarily* monotonic. However, the illustrations presented in Section 5 show that a non-monotonic profile is indeed possible. The results so far are summarized in the following proposition:

Proposition 1 *Assume that private savings are such that $H'(s) > 1$. Under full information when the optimal policy $(a^*(\beta), g^*(\beta))$ is given by an interior solution, it has the following properties:*

(i) when $\beta > 0$ we have $H'(m) < 1$, so that parents are more than fully insured against dependence;

(ii) $m^*(\beta)$ and $a^*(\beta)$ are always increasing;

(iii) $g^*(\beta)$ may or may not be monotonically increasing.

Intuitively, the first two properties arise because the benefits of m increase with β via the altruistic term in children's utility. This, in turn, makes a "cheaper," in that a larger level can be extracted while satisfying the participation constraint. Finally g^* is determined as a "residual," and it is not necessarily monotonic.

3.2 Corner solutions

So far, we have assumed an interior solution. However, a corner solution may also arise for some levels of β; the study of which is rather tedious, and mainly consists of technicalities, but there are a few properties worth reporting. This is done in the following proposition, which is established in the Appendix.

Proposition 2 *Assume that private savings are such that $H'(s) > 1$. Under full information, the optimal policy $(a^*(\beta), g^*(\beta))$ has the following properties:*

(i) there exists no level of altruism $\beta \geq 0$ such that $(a^*(\beta), g^*(\beta)) = (0, 0)$, which would imply that both types of care are equal to zero;

(ii) $a^*(0) = 0$ and $g^*(0) > 0$ is implicitly defined by $H'(s + g^*(0)) = 1$;

(iii) for all $\beta \in (0, \beta_0]$, with β_0 defined by (3), $(a^*(\beta), g^*(\beta))$ is interior, and given by (9) and (6);

(iv) for $\beta \in (\beta_0, \overline{\beta}]$, corner solutions such that $a(\beta) > 0$ and $g(\beta) = 0$ are possible. Otherwise, $(a^*(\beta), g^*(\beta))$ is given by (9) and (6).

Intuitively, the first two properties are due to the assumption that $H'(s) > 1$, so that savings are not "sufficient" to cover LTC. Consequently it is never optimal to set both public and informal care at zero and, when $a = 0$, we necessarily have $g > 0$. This leads to point *(iii)* which simply states that this is the case when the laissez faire implies $a = 0$. Consequently, a corner solution with $g = 0$ is possible only when children are sufficiently altruistic to provide care in the laissez faire; which confirms point *(iv)*.

3.3 Implementation

This solution can be achieved by directly "imposing" the optimal level of a and g. However, as a reference case for interpreting the asymmetric information solution, it is interesting to study how the solution can be decentralized (i.e., implemented), by letting children choose the level of informal care they provide while accounting for the impact that this will have on the level of g received by their parents. To achieve this, it is necessary to offer a different contract, $g(a, \beta)$ to each type; in other words, $g(a)$ is type specific.[16] Implementing the symmetric information policy requires (assuming that the solution of the government's problem is interior for all $\beta > 0$)

$$g'_a(a, \beta) = 1/\beta > 0 \tag{13}$$

and

$$g(a^*(\beta), \beta) = g^*(\beta). \tag{14}$$

To derive condition (13), note that the first-order condition of a child facing a schedule $g(a)$ is

$$-1 + \beta H'(m)[1 + g'_a(a, \beta)] = 0.$$

This expression is satisfied by the full information solution, as defined by (9), when $g'_a(a, \beta) = 1/\beta > 0$. In words, social LTC must *increase* with the level of informal care. While there is no asymmetric information here, the implementation nevertheless requires providing incentives to children for choosing a larger level of care than in the LF. Indeed, comparing (2) and (9) shows that $m^*(\beta) > m^{lf}(\beta)$. Since $a^{lf}(\beta) = m^{lf}(\beta) - s$, it then follows from (12) that $a^*(\beta) > a^{lf}(\beta)$.

Combining (13) and (14), it then follows that, under symmetric information, the first-best allocation can be decentralized by assigning to each type β a function

$$g(a, \beta) = \frac{1}{\beta} a + [g^*(\beta) - \frac{1}{\beta} a^*(\beta)].$$

The implementing function for any given β is thus an affine function consisting of two parts. The first part depends on the level of informal care actually chosen by the children. The second part is a fixed transfer that depends on the optimal levels of a

[16] It is nevertheless "decentralized" in the sense that it is the children who choose a and thus indirectly g. This is in contrast to the mechanism design approach under which a and g are both directly imposed.

and g but not on the actual level of informal care.[17]

We now turn to the case of asymmetric information where the contract can no longer be type specific and where we must have a unique $g(a)$. As usual, we first study the optimal incentive compatible allocation and then characterize the implementing policy.

4 Asymmetric information

Under asymmetric information, the utility of a child with altruism parameter β reporting β' is

$$u(\beta, \beta') = y + \beta H(s + a(\beta') + g(\beta')) - a(\beta').$$

Incentive compatibility implies that this function is maximized for $\beta' = \beta$; that is, when the children report their true type. The FOC for truthful reporting then requires

$$\left.\frac{\partial u(\beta, \beta')}{\partial \beta'}\right|_{\beta'=\beta} = 0. \tag{15}$$

Differentiating $u(\beta) = y+\beta H(s+a(\beta)+g(\beta))-a(\beta)$ with respect to β and using (15) implies that an incentive compatible policy has to satisfy the local incentive compatibility constraint

$$\frac{\partial u(\beta)}{\partial \beta} = H(s + a + g) > 0. \tag{16}$$

In words, under asymmetric information children should receive a rent that is increasing with their level of altruism.[18]

[17]Its slope decreases in β as well as its vertical intercept because

$$\frac{\partial[g^*(\beta) - \frac{1}{\beta}a^*(\beta)]}{\partial \beta} = m'(\beta)[1 - (1 + \beta)H'(m(\beta))] - \frac{1 + \beta}{\beta}[H(m(\beta)) - H(s)] \leq 0,$$

where the sign follows from (9) and from $m(\beta) \geq s$.

[18]The second order condition of the individual revelation problem holds if $m(\beta)$ is non-decreasing. We assume for the moment that this condition is satisfied, and show that this is indeed the case for the asymmetric information solution.

The problem of the government is now

$$
\begin{aligned}
\max_{a(\beta),g(\beta)} \quad & EU = w - s - \tau + (1-\pi)U(s) + \pi \int_{\underline{\beta}}^{\overline{\beta}} H(s + a(\beta) + g(\beta)) f(\beta) d\beta \\
s.t. \quad & \tau \geq \pi \int_{\beta} g(\beta) f(\beta) d\beta \\
& u(\beta) = y + \beta H(s + a(\beta) + g(\beta)) - a(\beta) \quad \forall \beta \qquad (17) \\
& u(\beta) \geq y + \beta H(s) \quad \forall \beta \\
& \dot{u} = H(s + a(\beta) + g(\beta)).
\end{aligned}
$$

In addition to the participation constraint of the children, the government now has to fulfill the local incentive compatibility constraint. As long as $a+g>0$, since u increases faster than $y+\beta H(s)$, the participation constraint is binding only for the lowest altruism type. When $a+g=0$, on the other hand, we have bunching over the relevant interval. We neglect this issue in the analytical part, but this possibility is illustrated in the second example provided in Section 5.

The problem of the government reduces to

$$
\begin{aligned}
\max_{a(\beta),g(\beta)} \quad & \pi \int_{\beta} [H(s + a(\beta) + g(\beta)) - g(\beta)] f(\beta) d\beta \\
s.t. \quad & u = y + \beta H(s + a(\beta) + g(\beta)) - a(\beta) \quad \forall \beta \\
& \dot{u} = H(s + a(\beta) + g(\beta)).
\end{aligned}
$$

We show in the Appendix that the FOC characterizing $m(\beta)$ is now given by

$$
\left[(1+\beta) - \frac{1-F(\beta)}{f(\beta)}\right] H'(s+a+g) = 1. \qquad (18)
$$

This expression is the counterpart to (9) under full information. Since the second term in brackets is strictly positive, except for $\overline{\beta}$ where it vanishes, $m(\beta)$ and thus the sum $a(\beta) + g(\beta)$ is now distorted down in order to relax incentive compatibility constraints. Furthermore, the wedge between the full- and asymmetric-information solutions decreases with β. This is a classical result in contract theory. Formally it obtains because, from the local incentive constraint (16), rents increase at a lower rate with β when $m(\beta)$ (and thus H) is smaller. To put it more intuitively, the benefit of $m(\beta)$ to children increases with their degree of altruism. Roughly speaking, the downward distortion in m and the decreasing wedge makes the consumption bundle of a low-β child (the mimicked) less attractive to a high-β child (the potential mimicker). Denoting by AI asymmetric information, $m^{AI}(\overline{\beta}) = m^*(\overline{\beta})$, and $m^{AI}(\beta) < m^*(\beta)$ for all $\beta < \overline{\beta}$.

We now study the profile of informal care under asymmetric information. Assuming an interior solution for all $\beta \geq 0$, using the children incentive compatibility and participation constraint of the lowest type, we obtain

$$u(\beta) = y + \int_0^\beta H(m(z))dz. \tag{19}$$

Substituting $u(\beta)$ with its value in (17) yields

$$a(\beta) = \beta H(m(\beta)) - \int_0^\beta H(m(z))dz. \tag{20}$$

We want to compare this expression to (12). We know from (18) that $m^{AI}(\beta) < m^*(\beta)$, so that $\beta H(m^{AI}(\beta)) < \beta H(m^*(\beta))$ for all $\beta < \overline{\beta}$. Using the participation constraints along with (17) and (19) we also have

$$\int_0^\beta H(m(z))dz \geq \beta H(s),$$

with the condition holding with strict inequality for all $\beta > 0$. Combining this expression with (20) yields

$$a(\beta) \leq \beta H(m(\beta)) - \beta H(s),$$

with the condition holding with strict inequality for all $\beta > 0$. This condition implies that, for all $\beta > 0$, $a^{AI}(\beta) < a^*(\beta)$.

Social LTC, g, is once again determined as a residual. Since $m^{AI}(\overline{\beta}) = m^*(\overline{\beta})$, while $a^{AI}(\beta) < a^*(\beta)$ for all $\beta > 0$, one “mechanically” has $g^{AI}(\overline{\beta}) > g^*(\overline{\beta})$. Intuitively, the amount of informal care that can be extracted from the most altruistic children is limited by the incentive constraint (their utility must be larger than under full information). But since the level of total care is the same as under FI, g has to be larger to make up the difference. Conversely, for the lowest altruism type, $m^{AI}(0) < m^*(0)$ and $a^{AI}(0) = a^*(0) = 0$, so that $g^{AI}(0) < g^*(0)$.

To sum up, informal care is distorted downwards, even for the most altruistic child. This may come as a surprise at first because it appears to violate the traditional "no distortion at the top" property. The latter property applies to m but not to its components a and g. To understand this result, recall that to establish it we have used participation and incentive constraints. There are two effects at play. First, recall that informal care is more expensive since m is distorted downwards. Second, and more crucially, the most altruistic child receives a rent which in any event limits the amount of care that can be extracted. The two effects go in the same direction, and while the first one is not relevant for $\overline{\beta}$, the second effect is also at play for the most altruistic child.

Equation (18) also implies that the consumption of dependent parents, $m^{AI}(\beta)$, is strictly increasing. Then, differentiating (20) shows that a^{AI} is also strictly increasing.

Thus far, we have concentrated on the case where the solution is interior, however, a corner solution is also possible. Unlike under full information, we may now have $g^{AI}(\beta) + a^{AI}(\beta) = 0$, even though insurance is desirable (recall that under full information at least one of the transfers was always strictly positive). Intuitively, this is due to the downward distortion of a and, for low levels of β, of g. When this distortion is sufficiently large the non-negativity constraints may become binding. This is formally established in the Appendix and stated in points *(v)* and *(vi)* of the following proposition. The other points of the proposition summarize the results that we have obtained when the solution is interior.

Proposition 3 *Assume that private savings are such that $H'(s) > 1$. When β is not* *publicly observable, the optimal policy* $(a^{AI}(\beta), g^{AI}(\beta))$ *has the following properties:*

(i) for all $\beta < \overline{\beta}$*, we have* $m^{AI}(\beta) < m^*(\beta)$ *(downward distortion of total care), while* $m^{AI}(\overline{\beta}) = m^*(\overline{\beta})$ *(no distortion at the top);*

(ii) for all $\beta > 0$*, we have* $a^{AI}(\beta) < a^*(\beta)$*, while* $a^{AI}(0) = a^*(0) = 0$*;*

(iii) social LTC satisfies $g^{AI}(0) < g^*(0)$ *and* $g^{AI}(\overline{\beta}) > g^*(\overline{\beta})$*;*

(iv) parents' consumption when old and dependent, $m^{AI}(\beta)$*, is non-decreasing in* β*, and so is informal care,* $a^{AI}(\beta)$*;*

(v) if $(1 - 1/f(0))H'(s) < 1$*, there exist a threshold* $\widetilde{\beta} \in (0, \overline{\beta})$ *such that* $g^{AI}(\beta) + a^{AI}(\beta) = 0$ *for all* $\beta \leq \widetilde{\beta}$*, and* $g^{AI}(\beta) + a^{AI}(\beta)$ *is strictly positive and increasing for* $\beta > \widetilde{\beta}$*;*

(vi) if $(1 - 1/f(0))H'(s) > 1$*, then* $g^{AI}(\beta) + a^{AI}(\beta)$ *is positive and increasing for all* β*.*

4.1 Implementation

The implementation of the above allocation (assuming that the solution of the government's problem is interior for all β) implies a function $g(a)$, such that the solution to the children's problem is equal to $a^{AI}(\beta)$. Unlike in the full information case, g cannot be conditioned on β; the same function $g^{AI}(a)$ applies to all types. Combining (18) with the children's first order condition we obtain

$$\beta(1 + g^{AI\prime}(a)) = (1 + \beta(a)) - \frac{1 - F(\beta(a))}{f(\beta(a))} \iff g^{AI\prime}(a) = \frac{1}{\beta(a)}\left\{1 - \frac{1 - F(\beta(a))}{f(\beta(a))}\right\}, \tag{21}$$

where $\beta(a) = (a^{AI})^{-1}(a)$ is the level of β for which the optimal level of informal care is $a(\beta)$. According to (18), $1 - 1/f(0) \geq 0$ is a necessary condition for the solution to be interior for all β. If this is the case, the non-decreasing hazard rate property implies that $1 - (1 - F(\beta))/f(\beta) > 0$ for all β, which in turn implies that $g^{AI\prime}(a) > 0$ for all levels of a. In words, the consumption of the parents in the laissez faire would be too low with respect to the asymmetric information optimal allocation. Then, the LTC policy provides extra incentives to children to provide informal care. It is also easy to verify that $g'(a^{AI}(\overline{\beta})) = 1/\overline{\beta}$.

Under asymmetric information, the slope $g'(a)$ as defined in (21) is always smaller than the slope of the schedule $g(a)$ under full information, $g'_a(a,\beta) = 1/\beta$, except for the most highly altruistic type. This is due to the fact that a is distorted down under asymmetric information. As a consequence, any given a has to be chosen by a child with a larger β (more altruistic) than under full information. Consequently, the marginal incentive $g'(a)$ faced by the child must be smaller under asymmetric information.

5 Illustration

In this section, we illustrate the results summarized in Propositions 1–3 using explicit functional forms for H and F.

First, we assume $H(m) = \ln(m) + C$, and that $F(\beta) = 2\beta$, so that β is uniformly distributed between 0 and 1/2. We also assume that $s = 0.5$, so that $H'(s) > 1$, and that $C = -\ln(0.4)$, so that $H > 0$ for all β. These assumptions ensure that the solution of the full and asymmetric information problems are both interior. Figure 2(a) presents the full and asymmetric information levels of dependent parents' consumption (net of savings), informal care, and LTC transfer as functions of children's altruism. Parental consumption and informal care are both increasing in β, as it was shown analytically. Furthermore, both m and a are distorted down under asymmetric information. The optimal LTC transfer displays a different pattern. Under full information, it is first increasing and then decreasing in β. Under asymmetric information, this is no longer the case, as g^{AI} increases in β. In this case, the need to provide incentives to informal care givers prevails over the insurance motive of the LTC policy.

In the illustration above, there are no corner solutions, so that m is positive for all β under both full and asymmetric information. We now consider the case where the utility function and the level of savings are the same as above, but where $F(\beta) = \beta$. In words, we consider a uniform distribution of β between zero and one, thus expanding the support of the distribution. In this case, under asymmetric information, it may be useful to set m to zero when children have a relatively low level of altruism, in order to contain the rents for highly-altruistic children. Corner solutions may then arise for low levels of β. The optimal policy is illustrated in Figure 2(b). In this case $\widetilde{\beta} = 0.25$. Below this threshold $a^{AI} + g^{AI} = 0$. Except for these corner solutions, the pattern is similar to that depicted in Figure 2(a).[19]

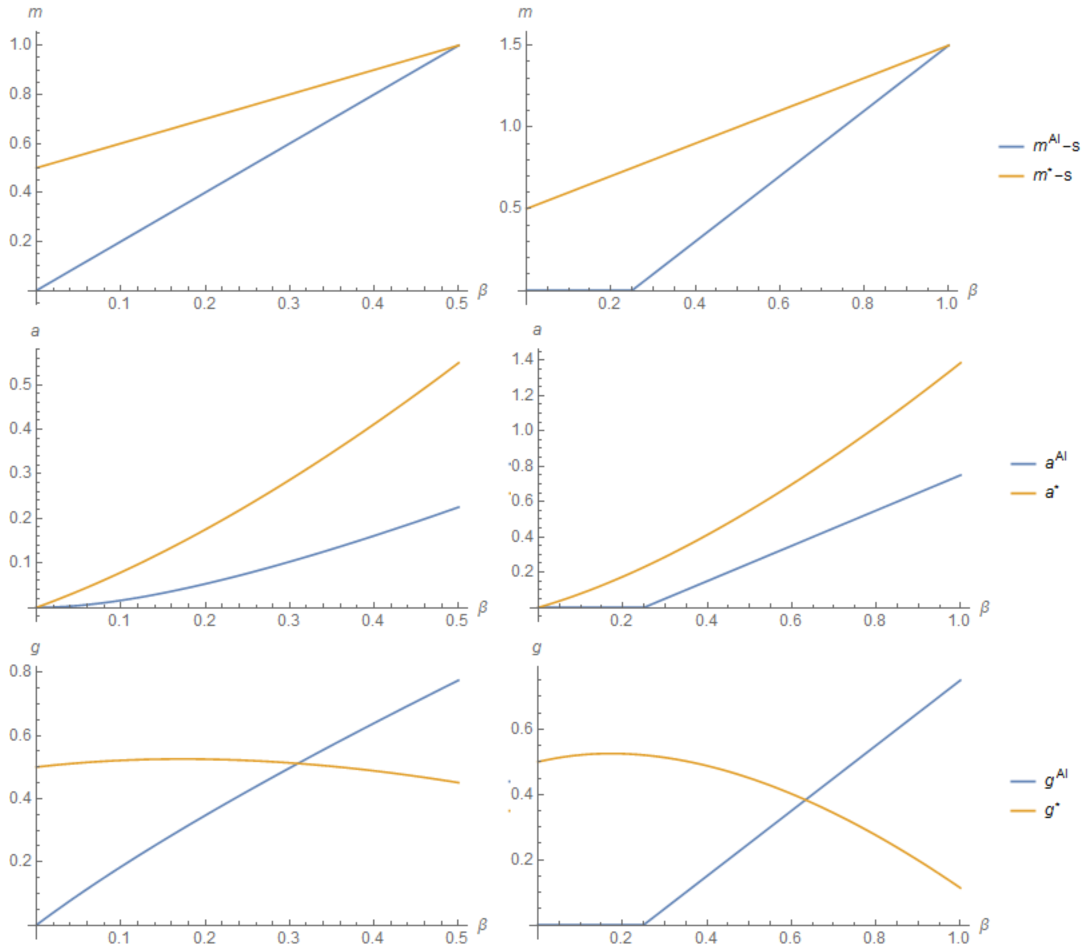
6 Opting out and topping up reconsidered

In the previous section, we have considered non-linear policies where the social LTC transfer depends on the amount of informal care received, assuming that the latter is observable but that children's altruism is not. The traditional *TU* and *OO* policies are special cases of our own.

The optimal policy involves a LTC transfer that is always increasing in the level of informal care provided. This obviously corresponds to neither of these special cases. First, the optimal LTC transfer is never flat with respect to informal care, as it would be the case in a *TU* regime. Under full information, it may eventually drop to zero for sufficiently large levels of a (and β). However, the schedule does not resemble an *OO* policy either, which would have a discontinuity at zero or at the very least decrease drastically when children provide care.

Under asymmetric information, on the other hand, g will be larger than under full information for large levels of β and our illustration suggests that it may even be increasing all along. This is because altruistic children have to be provided with incentives not to mimic children with lower levels of altruism. This draws us even further away from *TU* or *OO*.

[19] We have considered a different functional form for the utility function of the dependent elderly ($H(m) = m^{\varepsilon}/\varepsilon$, with $\varepsilon = 0.5$, keeping $s = 0.5$ and $F(\beta) = \beta$). The optimal policy is qualitatively similar to those in Figure 2.



(a) $F(\beta) = 2\beta$

(b) $F(\beta) = \beta$

Figure 2: *Optimal consumption of dependent parents (net of private savings), informal care, and LTC insurance as a function of the children' s altruism. Case with* $H(m) = \ln(m) - \ln(0.4)$ *and* $s = 0.5$*. Superscript AI denotes the asymmetric information case, * denotes the full information case.*

Since the existing literature does not establish that either TU or OO systematically dominates, one could expect our problem to yield intermediate solutions. Interestingly, this is not the case, as the optimal social LTC transfer does not decrease with the level of informal care provided.

7 Conclusion

This paper has studied the design of public LTC insurance when the children's altruism—and thus informal care—is uncertain. We have shown that both total care and informal care received by parents should increase with the children's level of altruism. This is the case both under full and asymmetric information. Social LTC, on the other hand, may be non-monotonic. Our examples suggest that while it may first be increasing in β, it may decrease for larger levels of child altruism. This is in line with the idea that social LTC provides insurance against the risk of having children with a low degree of altruism. However, under asymmetric information, the government limits the provision of insurance in order to provide incentives to caregivers.

From a practical perspective, the main lesson is that we do not need a social system that can be topped up by informal care but, rather, we should provide social LTC to supplement or top up private care. In other words, there should be no penalty for informal care through a reduction of social care. Quite the opposite; social care has to increase sufficiently with informal care to provide the appropriate incentives to caregivers. In other words, our results point towards a policy that rewards informal care with extra public care, rather than reducing public care for individuals receiving assistance from their family. This is not the case for most policies traditionally used around the world. However, many countries have now started to establish policies aimed at promoting informal care, including cash transfers for individuals being taken care of by family members (Germany), LTC leaves, possibly part time, allowing children (now, typically in their 50's) to combine care with a professional activity (the Netherlands), respite care offering short-term relief for families taking care of dependent individuals (Sweden), and training and support services for caregivers.[20] Another example is provided by Canadian system of non-refundable tax credits for caregivers.

While informal care is in reality difficult to observe, so that the implementation of such a policy is not trivial, some degree of observability, albeit imperfect, is not unrealistic. When informal care is "rewarded" by extra social care, families have no incentive to hide it. As long as it is difficult to report a level of informal care higher than the actual one (for instance because the government observes the caregivers' labor supply), our policy could then be implemented even under imperfect observability. Furthermore, informal care can be subsidized through labor market policies, such as LTC leaves.

It is important to highlight, however, that all parents are *ex ante* identical in our model, so that the optimal LTC policy characterized in this paper does not have any redistributive role. The reason why we abstract from redistributive issues is that we want to focus on the risk related to uncertain altruism and its impact on the optimal LTC policy. With heterogeneous wealth levels, the optimal policy would depend on the observability of parents' wealth and on the correlation between wealth and children's altruism. If wealth was observable, our optimal policy rule would be preserved for each level of wealth. With unobservable wealth levels, the correlation between the wealth and the altruism distribution would matter for the optimal policy (Rochet, 1991). For instance, if wealth and altruism were positively correlated, redistribution motives would push towards lower aggregate levels of care (public and family aid) for parents with more altruistic children. These questions, while interesting, are left for further research.

Our analysis relies on a number of assumptions. First, we assume that savings are exogenously determined, so that parents do not respond to the public LTC policy. This greatly simplifies the analysis and allows us to focus on the children's responses. It would be interesting to see how the optimal policy would change should this hypothesis

[20] For a survey of these policies in OECD countries, see Gori *et al.* (2016).

be relaxed. We leave this for future research. Second, we assume that the utility functions are quasilinear in young-age consumption. Relaxing this assumption would make the analysis more complex by introducing income effects. However, the main tradeoffs faced by the government when setting the optimal LTC policy would remain unchanged. Finally, the children's altruism parameter is exogenously determined in our model. There is no role for the parent's socialization effort aimed at fostering the altruism of their children (Ponthière, 2014) or for parents' transfers shaping their children's behavior (Becker, 1974), and introducing mechanisms such as these is outside the scope of this paper. However, our optimal policy does not seem to completely discourage or crowd out socialization efforts. To the contrary, our optimal policy implies that the consumption of dependent parents increases with the children's level of altruism, which makes children's altruism valuable to parents (at least to some extent), even in the presence of social LTC.

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Appendix

Proof of Proposition 2: First, note that there exists no β such that the solution implies neither informal nor social care: $a^*(\beta) = g^*(\beta) = 0$. Suppose this was the case. Then, the participation constraint of type β would be binding. But this constraint would continue to be satisfied when setting $g(\beta) = \varepsilon$, with $\varepsilon > 0$ arbitrarily small, a policy which provides parents with a higher welfare as long as $H'(s) > 1$, so that parents enjoy insurance. This contradicts the fact that (a^*, g^*) is a solution to the government problem.

Second, for $\beta = 0$, the solution implies $a^*(0) = 0$ and $g^*(0) > 0$. Irrespective of the value of g, the participation constraint is satisfied only for $a^* = 0$, which is then the optimal value of informal care for the non-altruistic child. The first-order condition with respect to g for $\beta = 0$ is then given by

$$\frac{\partial EU}{\partial g} = -\pi f(0) + \pi H'(s+g) f(0) + \nu_0 = 0,$$

with $g^* = 0 \Rightarrow \nu_0 \geq 0$. Suppose that $g^* = 0$ is a solution to this equation. Then, under the assumption that $H'(s) > 1$, $-\pi f(0) + \pi H'(s) f(0) + \nu_0 > 0$, which is a contradiction with $g^* = 0$ being a solution. The transfer g is always interior, so that $H'(s+g) = 1$. When the child is not altruistic, no informal care should (or could) be provided and parents are insured through g.

Third, when $\beta > 0$, we never have $a^*(\beta) = 0$. To see this, assume that for some $\beta > 0$ the solution to the government problem is (a^*, g^*) with $a^* = 0$. But then, the participation constraint is satisfied with strict inequality, and it would continue to be satisfied for $a = \varepsilon$, with $\varepsilon > 0$ arbitrarily small, a solution which implies a higher welfare for the parents. This contradicts the fact that (a^*, g^*) is a solution to the government problem.

Fourth, for any $\beta \in (0, \beta_0)$, where β_0 is the marginal child in the laissez faire defined by (3), $a^* > 0$ and $g^* = 0$ cannot be a solution. Suppose that for some β the solution (a^*, g^*) is such that $a^* > 0$ and $g^* = 0$. This implies that there exists an $a^* > 0$ such that

$$a^* = \beta H\left(s + a^*\right) - \beta H(s). \tag{A1}$$

However, such a level a^* exists if and only if $\beta > \beta_0$, the threshold above which children provide informal care in the laissez faire. To see this, notice that the LHS of (A1) is a linear function of a with slope 1 and intercept equal to zero. The RHS is increasing and concave in a, and it is equal to zero if $a = 0$. Then, (A1) is always satisfied at $a = 0$, and there exists another positive solution if and only if the slope of the RHS evaluated at $a = 0$ is greater than one, that is if and only if $\beta H'(s) > 1$. But this in turn is the case only when $\beta > \beta_0$, with β_0 defined by (3)

Finally, when $\beta > \beta_0$, we may have either an interior solution or a corner solution where all care is provided by children, so that $a > 0$ and $g = 0$. A solution with $a^* > 0$ and $g^* = 0$ is characterized by

$$\frac{\partial EU}{\partial a} = \pi H'(s+a^*)f(\beta) + \lambda_\beta[\beta H'(s+a^*) - 1] = 0 \iff \lambda_\beta = \frac{\pi H'(s+a^*)f(\beta)}{1 - \beta H'(s+a^*)}, \tag{A2}$$

$$\frac{\partial EU}{\partial g} = -\pi f(\beta) + \pi H'(s+a^*)f(\beta) + \lambda_\beta \beta H'(s+a^*) < 0, \tag{A3}$$

and (A1). Condition (A2) implies that $1 - \beta H'(s+a^*) > 0$. Comparing this expression with (2) shows that, when g is equal to zero, the level of informal care implied by a binding participation constraint is larger than its laissez-faire level. Substituting (A2) into (A3) yields

$$-1 + (1+\beta)H'(s+a^*) < 0,$$

so that an increase in g starting from 0 would indeed decrease welfare.

Proof of Expression 18: The corresponding Hamiltonian is

$$\mathcal{H} = [H(s+a+g) - g]f(\beta) + \eta_\beta(u - y - \beta H(s+a+g) + a) + \mu_\beta H(s+a+g)$$

where u is the state variable, a and g are the control variables, η_β is the Lagrange multiplier associated with the first constraint, and μ_β is the costate variable.

The first order conditions are

$$\frac{\partial \mathcal{H}}{\partial a} = \pi H'(s+a+g)f(\beta) + \eta_\beta(1 - \beta H'(s+a+g)) + \mu_\beta H'(s+a+g) = 0, \tag{A4}$$

$$\frac{\partial \mathcal{H}}{\partial g} = -\pi f(\beta) + \pi H'(s+a+g) f(\beta) - \eta_\beta \beta H'(s+a+g) + \mu_\beta H'(s+a+g) = 0, \quad \text{(A5)}$$

$$\frac{\partial \mathcal{H}}{\partial u} = \eta_\beta = -\dot{\mu}_\beta,$$

and

$$\frac{\partial \mathcal{H}}{\partial \mu} = H(s+a+g) = \dot{u}_c.$$

Substituting (A4) into (A5) yields $-\pi f(\beta) = \eta_\beta$. Since $\dot{\mu}_\beta = -\eta_\beta = \pi f(\beta)$ and $\mu_{\overline{\beta}} = 0$ from the transversality condition, the costate can be rewritten as

$$\mu_\beta = -\int_\beta^{\overline{\beta}} \pi f(\beta) d\beta = -\pi(1 - F(\beta)).$$

Using the expression above and proceeding by substitution, conditions (A4) and (A5) yield

$$-1 + (1+\beta) H'(s+a+g) + \frac{\mu_\beta}{\pi f(\beta)} H'(s+a+g) = 0 \quad \text{(A6)}$$

which directly implies (18).

Proof of Proposition 3, (v) and (vi): Under the non-decreasing hazard rate property, $1 + \beta - (1 - F(\beta))/f(\beta)$ increases in β and is positive if $\beta = \overline{\beta}$. Then, two cases are possible. If $(1 - 1/f(0))H'(s) \geq 1$, then the solution of (18) is always interior for $\beta = 0$ and consequently $g(\beta) + a(\beta) > 0$ for all levels of β and increasing in β. If $(1 - 1/f(0))H'(s) < 1$, then there is a corner solution for $\beta = 0$ and there exists a threshold $\widetilde{\beta} \in (0, \overline{\beta})$ such that $g(\beta) + a(\beta) = 0$ for all $\beta \leq \widetilde{\beta}$, and $g(\beta) + a(\beta) > 0$ and increasing for $\beta > \widetilde{\beta}$.

If $(1 - 1/f(0))H'(s) < 1$, then

$$u(\beta) = \begin{cases} y + \beta H(s) & \text{if } \beta \leq \widetilde{\beta}, \\ y + \widetilde{\beta} H(s) + \int_{\widetilde{\beta}}^{\beta} H(m(z)) dz & \text{if } \beta > \widetilde{\beta}, \end{cases}$$

and

$$a(\beta) = \begin{cases} 0 & \text{if } \beta \leq \widetilde{\beta}, \\ \beta H(m(\beta)) - \widetilde{\beta} H(s) - \int_{\widetilde{\beta}}^{\beta} H(m(z)) dz. & \text{if } \beta > \widetilde{\beta}, \end{cases}$$