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Abstract

We study the impact of horizontal mergers on the incentives of merging firms to invest in incremental innovation. We provide a decomposition of this impact that clarifies the various forces at work and the differences between demand-enhancing and cost-reducing innovation. Moreover, we derive sufficient conditions for a merger to either reduce or raise the merging firms' incentives to innovate, and show that the comparison of the price diversion ratio and the innovation diversion ratio can help screen mergers. We also uncover a useful connection between the level of production synergies induced by a merger and its impact on innovation.

Keywords: Horizontal Mergers, Innovation, Competition.

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1 Introduction

Competition authorities have been paying great attention to the effects of horizontal mergers on innovation over the past two decades. Gilbert and Greene (2015) find that the US Department of Justice and the Federal Trade Commission identified innovation concerns in about one-third of their merger challenges between 2004 and 2014. The European Commission has also taken action in many merger cases over the past decade on grounds of adverse effects on innovation.[1]

Policy debates and the academic literature on the impact of horizontal mergers on innovation have highlighted several potentially conflicting effects.[2] They have also shown that the effects of mergers on demand-enhancing innovation are more complex than their effects on cost-reducing innovation.[3] However, the existing papers analyzing the impact of mergers on demand-enhancing innovation have focused on specific demand functions.[4] In this paper, we consider a setting with a general demand function, allowing for both cost-reducing and demand-enhancing innovation, and provide sufficient conditions under which the net impact of a merger is to decrease or increase incentives to innovate.

Our contribution is as follows. First, our unified approach for studying the effects of mergers on both cost-reducing and demand-enhancing innovation clarifies the differences between the impact of horizontal mergers on these two types of innovation. Second, we provide a decomposition of the forces at play through first-order conditions. By applying this decomposition to various families of demand functions, we gain insights into why the existing literature on the impact of mergers on innovation presents conflicting results. Third, we show that the mere comparison of two diversion ratios can help screen mergers in industries where innovation plays a key role. In particular, this comparison allows us to identify scenarios where the impact of the merger on prices (for given innovation levels) can inform about the impact of the merger on innovation. Fourth, we use our approach to

[1] See, for instance, Novartis/GSK (case no. COMP/M.7276), GE/Alstom (case no. COMP/M.7278), Pfizer/Hospira (case no. COMP/M.7559), Dow/DuPont (case no. COMP/M. 7932), Bayer/Monsanto (case no. COMP/M.8084), and Bayer/BASF (case no. COMP/M.8851). The European Commission identified innovation concerns in all of these mergers and cleared them subject to the implementation of remedies addressing these concerns.

[2] See, e.g., Baker (2007), Katz and Shelanski (2007), Shapiro (2012), Federico (2017), Federico et al. (2017, 2018), Motta and Tarantino (2021), Jullien and Lefouili (2018), Denicolò and Polo (2019), Régibeau and Rockett (2019), Federico et al. (2020), and Gilbert (2020).

[3] See Motta and Tarantino (2021) and Jullien and Lefouili (2018). Relatedly, Greenstein and Ramey (1998) and Chen and Schwartz (2013) have shown that the seminal result on the effect of competition on process innovation by Arrow (1962) does not always extend to the case of product innovation.

[4] See the discussion of Motta and Tarantino (2021) and Federico et al. (2018) in the related literature section.

investigate the effect of a merger on consumer surplus.

In our baseline model, we study the impact of a merger between two symmetric duopolists on their incentives to innovate in an environment with no production synergies, no R&D synergies, and no R&D spillovers. Firms set their prices and innovation levels simultaneously. Innovation is incremental: it affects the cost and/or demand of an *existing* product. Thus, our model encompasses both cost-reducing and demand-enhancing innovation. We show that the overall impact of the merger on innovation is the sum of two effects: the *market power effect* and the *externality effect*.

The *market power effect* subsumes two effects driven by the impact of the merger on the merging firms' output. First, a reduction in output reduces the merging firms' incentives to innovate when innovation increases their margins.[5] This *margin expansion effect* is negative. Second, a change in output may lead to a change in the return to investment per unit of output. This effect can be either positive or negative.

The *externality effect* subsumes two effects related to price and innovation externalities between firms. First, the merged entity internalizes the impact of each merging firm's innovation on the other merging firm's demand. We call this the *innovation diversion effect*. We focus on the case where the innovation externality is negative, as this is the scenario that competition authorities are most concerned about. Second, the merger affects the merging firms' margins and, therefore, their incentives to innovate when innovation increases their sales. This *demand expansion effect* is positive. We find that, whenever the externality effect is not zero, it is negative if and only if the *price diversion ratio*–commonly used by competition authorities to assess the impact of mergers on prices–is less than the *innovation diversion ratio*–its counterpart for innovation analysis (Farrell and Shapiro, 2010; Salinger, 2019).

When innovation is *(purely) cost-reducing* (i.e., it reduces marginal production costs but does not affect demands), the externality effect vanishes. Moreover, the market power effect is negative and, therefore, a merger reduces innovation.

When innovation is *(purely) demand-enhancing* (i.e., it affects demands but not production costs), the externality effect generally differs from zero.[6] Using our decomposition, we provide sufficient conditions for such mergers to reduce or raise innovation incentives and apply our approach to several commonly used models.

[5]Of course, whether innovation increases the margin or the sales volume is endogenous and results from the firms' price optimization.

[6]Examples of a demand-enhancing innovation include the discovery of additional therapeutic applications for an existing drug, the addition of new features to videogames, increasing the capacity of hard disks, etc.

We also investigate the effect of a merger on consumer surplus, accounting for the change in both prices and innovation induced by the merger. We show that a merger always harms consumers if it leads to less innovation. The impact of the merger on consumer surplus becomes ambiguous if it boosts innovation but, importantly, our simulations indicate that it is unlikely that the merger benefits consumers, absent synergies and spillovers.

We then consider the impact of a merger on innovation when it induces production synergies (still assuming away R&D synergies and spillovers). To highlight the consequences of production synergies, we define *P-neutral mergers* as mergers that do not affect prices when the level of innovation of the merging firms is *fixed* at the pre-merger equilibrium level. The fact that a merger is P-neutral does *not* mean that it does not affect equilibrium prices, but that any merger-induced changes in equilibrium prices are driven by the effect of the merger on innovation incentives. Studying the impact of P-neutral mergers on innovation allows us to determine the conditions for applying a stand-alone innovation theory of harm, i.e., a theory stipulating that even a merger that does not affect prices (at a given level of innovation) can have a negative impact on innovation (Denicolò and Polo, 2019). The market power effect vanishes for P-neutral mergers, which implies that their overall impact on innovation is fully driven by the comparison of the price diversion ratio and the innovation diversion ratio. Specifically, a P-neutral merger has a negative effect on innovation if and only if the innovation diversion ratio is greater than the price diversion ratio.

Next, we incorporate R&D synergies and R&D spillovers in our model and show that our decomposition can be adapted in a very natural way to account for them. Moreover, we find that the comparison between the innovation diversion ratio and the price diversion ratio remains relevant in environments with R&D synergies or spillovers as long as the diversion ratios are adjusted accordingly.

We also extend our analysis to an oligopolistic setting with merging and non-merging firms and show that in this context as well, determining the impact of a P-neutral merger on the merging firms' innovation level boils down to comparing the innovation diversion ratio and the price diversion ratio.

Finally, we briefly discuss two other extensions to our baseline model, which are detailed in the Online Appendix. First, we allow for observable investment in innovation, which creates a strategic effect of innovation on prices. Second, we consider asymmetric demand and cost functions.

Related literature. While there is a vast and long-standing literature on the effect of competition on innovation,[7] the literature addressing the specific question of how mergers affect firms' incentives to innovate is more recent.

Motta and Tarantino (2021) focus on the impact of horizontal mergers on process innovation and show that they reduce merging firms' incentives to engage in cost-reducing investment in the absence of spillovers and efficiency gains.[8] They also establish that this result extends to quality-improving investments for two specific demand functions under which a quality-improving investment is isomorphic to a cost-reducing investment. Our unified analysis of cost-reducing and quality-improving investments offers new insights with respect to Motta and Tarantino (2021) due the general nature of the demand function in our model.

Federico et al. (2018) study the effect of a horizontal merger on firms' incentives to engage in incremental product innovation. Using simulations, they find that absent spillovers and efficiency gains, a merger is detrimental to innovation and consumer surplus for the three demand functions they consider. Our approach differs in that we use a novel decomposition of the impact of a merger on innovation to provide sufficient conditions for the merger to reduce or raise the merging firms' incentives to innovate in a model with a general demand function. In particular, we show that the comparison of the innovation diversion ratio and the price diversion ratio is a key determinant of the net impact of a merger on the merging firms' incentives to invest in demand-enhancing innovation. In this respect, our work is related to the paper by Gaudin (2024), who shows that these two ratios are also useful for the characterization of quality distortions under imperfect competition.

Federico et al. (2017) also analyze the effect of a merger on product innovation but focus on the case where firms invest in R&D to develop new products. The authors find that the merger has a negative impact on innovation and consumer surplus. Considering a similar setting, Denicolò and Polo (2018) show that a merger between two firms can lead to an increase in their innovation incentives and consumer surplus if the merged entity does not find it optimal to spread its R&D expenditure evenly across the research units of the two merged firms.[9] Furthermore, Denicolò and Polo (2021) show that a merger may increase the merging firms' incentives to innovate because it allows them to share R&D knowledge and technologies.

[7] See Gilbert (2006) for a recent survey and Schmutzler (2013) for a unified approach to this issue.

[8] See also Matsushima et al. (2013) for an analysis of the effects of a merger when heterogeneous oligopolists compete both in process innovation and in the product market.

[9] See also Jullien and Lefouili (2020) for an extension of Federico et al. (2017) to the case of differentiated products.

Considering a setting where firms can undertake more than one research project, Letina (2016) and Gilbert (2019) show that a horizontal merger can reduce the variety of projects that are developed, and Moraga-González et al. (2022) find that a merger can either increase or decrease consumer welfare depending on whether the most profitable projects are also the most appropriable ones.[10] In the context of markets with buyer power, Loertscher and Marx (2019a, 2019b) show that a merger raises rivals' investment incentives and can raise the merging parties' investment incentives. Considering an environment with overlapping ownership, López and Vives (2019) show that increasing partial ownership interest in rivals decreases (resp., increases) R&D if spillovers are sufficiently small (resp., large).[11] Finally, Mermelstein et al. (2020) consider a dynamic model in which firms can reduce costs either by investing in building capital or by merging, and show that merger policy can strongly affect firms' investment behavior and vice versa.

A related but distinct strand of the literature examines the impact of merger policy on firms' pre-merger incentives to innovate in settings where an incumbent may acquire an entrant.[12] Finally, there is a growing empirical literature showing that the effects of mergers on innovation are mixed.[13]

The paper proceeds as follows. We lay out our baseline model in Section 2.1. In Section 2.2, we present our decomposition of the impact of a merger on innovation. We apply our framework to demand-enhancing innovation in Section 2.3, and discuss the impact of the merger on consumer welfare in Section 2.4. We consider production synergies and study the effects of P-neutral mergers in Section 3. We allow for the possibility of R&D synergies in Section 4.1 and incorporate R&D spillovers into our setting in Section 4.2. In Section 4.3, we consider a merger between two firms in an oligopoly setting. In Section 4.4, we discuss the case when investment is observed before firms set prices, and when firms have asymmetric demand and cost functions. Section 5 concludes.

[10] In the same vein, Letina et al. (2024) examine how the possibility of acquiring entrants affects the R&D incentives of both incumbents and entrants in a model where firms are allowed to choose which innovation projects to invest in and how much to invest in those projects.

[11] See also Vives (2020).

[12] See e.g., Jaunaux et al. (2017), Fumagalli et al. (2020), Hollenbeck (2020), Kamepalli et al. (2020), Cabral (2021), Motta and Peitz (2021), Gilbert and Katz (2021, 2022), Denicolò and Polo (2023), Letina et al. (2024), and Dijk et al. (2024a, 2024b). Lefouili and Madio (2024) provide an overview of this strand of literature.

[13] See, e.g., Grabowski and Kyle (2008), Ornaghi (2009), Guadalupe et al. (2012), Szücs (2014), Haucap et al. (2019), Bennato et al. (2021), and Igami and Uetake (2020).

2 Baseline Model

2.1 Setup

Consider two single-product firms, 1 and 2, producing differentiated goods. The firms compete in prices and can invest in innovation. In our baseline model, we suppose that the firms set their prices $p_i \geq 0$ and innovation levels $\gamma_i \in [0, \bar{\gamma}]$, $i = 1, 2$, simultaneously.[14]

Firm i's innovation level affects the demand for both products and/or the production cost of firm i. Let $D_i(p_i, p_j, \gamma_i, \gamma_j)$ denote the demand addressed to firm $i = 1, 2$ when it sets a price p_i and an innovation level γ_i and the rival firm $j \neq i$ sets a price p_j and an innovation level γ_j. We assume that the demand functions are symmetric – i.e., $D_i(p, p', \gamma, \gamma') = D_j(p, p', \gamma, \gamma')$ for any (p, p', γ, γ') – and continuously differentiable. A firm's demand is decreasing in its own price and increasing in its rival's price. Moreover, we assume that, whenever innovation affects demands, an increase in a firm's innovation level leads to an increase in its own demand and a decrease in its rival's demand. Our analysis also applies to the case where innovation by one firm has a positive effect on the rival's demand (see, e.g., Lin and Saggi, 2002), but we focus on the case where the impact is negative, as this is the scenario that is the most likely to raise anticompetitive concerns. Finally, we make the standard assumption that $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ (i.e., own effects dominate cross effects) at symmetric prices $p_i = p_j$ and innovation levels $\gamma_i = \gamma_j$. We also make a similar (reasonable) assumption regarding the effect of a uniform increase in innovation levels on demands: $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j \geq 0$ at symmetric prices $p_i = p_j$ and innovation levels $\gamma_i = \gamma_j$.[15] We can summarize these assumptions as follows:

Assumption 1: For any $i, j = 1, 2$, $j \neq i$, $(p_i, p_j) \in \mathbb{R}^2_+$, $(\gamma_i, \gamma_j) \in [0, \bar{\gamma}]^2$: (i) $\partial D_i/\partial p_i < 0 < \partial D_i/\partial p_j$; (ii) $\partial D_i/\partial \gamma_i \geq 0 \geq \partial D_j/\partial \gamma_i$; (iii) for any symmetric prices and innovation levels, $\partial D_i/\partial p_i + \partial D_i/\partial p_j < 0$ and $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j \geq 0$.

Let $C(\gamma, Q)$ be the production cost for quantity Q, which we assume to be twice continuously differentiable. The production cost is gross of the cost of innovation. At

[14] This is equivalent to saying that each firm does not observe its rival's innovation level (and price) before setting its own price (and innovation level). Oligopoly models with a simultaneous choice of price and innovation levels have been studied by Dasgupta and Stiglitz (1980), Levin and Reiss (1988), Ziss (1994), Leahy and Neary (1997), Cabral (2000), Vives (2008), and López and Vives (2019), among others. In the Online Appendix, we also consider the case where innovation levels are observed before prices are set.

[15] Notice that the assumption that $\partial D_i/\partial \gamma_i + \partial D_i/\partial \gamma_j \geq 0$ at symmetric prices and innovation levels is equivalent to the assumption that an increase in one firm's innovation level (starting from a symmetric situation) has a (weakly) positive effect on aggregate demand, i.e., $\partial D_i/\partial \gamma_i + \partial D_j/\partial \gamma_i \geq 0$ at symmetric prices and innovation levels.

this stage, we make no assumptions about the effect of innovation on the production cost. For instance, a cost-reducing innovation will reduce the production cost, but a quality-improving innovation may lead to a higher production cost.

Finally, we denote by $\Phi(\gamma)$ the investment cost that a firm must incur to achieve an innovation level $\gamma \in \mathbb{R}_+$, and assume that $\Phi(\gamma)$ is increasing, continuously differentiable, and such that $\Phi(0) = 0$, $\Phi'(0) = 0$ and $\lim_{\gamma \to \bar{\gamma}} \Phi'(\gamma) = +\infty$.

The profit function of firm i, gross of investment cost, can then be written as

$$\begin{aligned} \Pi_i (p_i, p_j, \gamma_i, \gamma_j) &= p_i Q_i - C(\gamma_i, Q_i) \\ \text{where } Q_i &= D_i (p_i, p_j, \gamma_i, \gamma_j). \end{aligned}$$

Consider first the benchmark scenario in which firms act independently. In a symmetric equilibrium, the first-order condition for the pricing decision is:

$$\left(p - \frac{\partial C(\gamma, Q)}{\partial Q}\right) \frac{\partial D_i (p, p, \gamma, \gamma)}{\partial p_i} + D_i (p, p, \gamma, \gamma) = 0. \tag{1}$$

$$\text{where } Q = D_i (p, p, \gamma, \gamma) \tag{2}$$

For the sake of exposition, we assume that this condition has a unique solution denoted by $\tilde{p}^* (\gamma)$.[16]

Likewise, the first-order condition for the innovation decision in a symmetric equilibrium is:

$$\begin{aligned} \left(p - \frac{\partial C(\gamma, Q)}{\partial Q}\right) \frac{\partial D_i (p, p, \gamma, \gamma)}{\partial \gamma_i} - \frac{\partial C(\gamma, Q)}{\partial \gamma} &= \Phi'(\gamma) \\ \text{where } Q &= D_i (p, p, \gamma, \gamma). \end{aligned} \tag{3}$$

We now make the following assumption regarding the price-innovation game.

Assumption 2: The duopoly price-innovation game has a unique symmetric equilibrium $(p^*, p^*, \gamma^*, \gamma^*)$ satisfying first-order conditions (1) and (3).[17]

To simplify the exposition, we adopt the following convention.

[16] All of our results hold without this uniqueness assumption, as only the equilibrium price p^* (defined below) matters. However, the function $\tilde{p}^* (\gamma)$ helps with the interpretation of our findings.

[17] Note that this assumption requires some degree of product differentiation, as existence of a symmetric (pure-strategy) equilibrium may fail otherwise. By contrast, uniqueness of the symmetric equilibrium is only assumed for conciseness. Absent this assumption, our analysis would apply to the comparison between the post-merger equilibrium and any symmetric equilibrium of the duopoly game.

Convention: We denote with the superscript $*$ any function evaluated at symmetric innovation levels γ and the independent firms' equilibrium prices $\tilde{p}^*(\gamma)$ and with the superscript M any function evaluated at symmetric innovation levels γ and the merged entity's profit-maximizing prices $\tilde{p}^M(\gamma)$. In particular,

$$D_i^*(\gamma) \equiv D_i(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma) \text{ and } D_i^M(\gamma) \equiv D_i\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right),$$

and for any $x \in \{p_i, \gamma_i, p_j, \gamma_j\}$,

$$\frac{\partial D_i^*(\gamma)}{\partial x} \equiv \frac{\partial D_i(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma)}{\partial x}, \quad \frac{\partial D_i^M(\gamma)}{\partial x} \equiv \frac{\partial D_i\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)}{\partial x}.$$

The profit-maximizing price of the merged entity, $\tilde{p}^M(\gamma)$, is formally defined below.

Consider now a merger between the two firms, and suppose that the merged entity continues to sell both products.[18] For now, we assume that the merger does not generate any production or R&D synergies.

The (monopoly) profit of the merged entity for levels of innovation γ_1 and γ_2 is given by

$$\begin{aligned} \Pi^M(\gamma_1, \gamma_2) &\equiv \max_{p_1, p_2} p_1 Q_1 - C(\gamma_1, Q_1) + p_2 Q_2 - C(\gamma_2, Q_2) - \Phi(\gamma_1) - \Phi(\gamma_2) \\ \text{where } Q_1 &= D_1(p_1, p_2, \gamma_1, \gamma_2) \text{ and } Q_2 = D_2(p_2, p_1, \gamma_2, \gamma_1). \end{aligned}$$

We assume that the maximization problem of the merged entity with respect to innovation levels is well behaved in the following sense:

Assumption 3: The profit function $\Pi^M(\gamma_1, \gamma_2)$ is continuously differentiable and strictly quasi-concave in (γ_1, γ_2).

This assumption, combined with the symmetric nature of the demand system, implies that the merged entity's optimal innovation strategy is symmetric. Therefore, we can restrict our attention to a uniform level of innovation for both units of the merged entity, i.e., $\gamma_1 = \gamma_2 = \gamma$. For any given innovation level γ that applies to both products, the merged entity's optimal symmetric price $\tilde{p}^M(\gamma)$ is then assumed to be defined by the

[18] See Johnson and Rhodes (2021) for an analysis of the effects of a merger in a setting where firms can reposition their product lines by adding or removing products of different qualities after the merger.

following first-order condition:

$$\left(\tilde{p}^M(\gamma) - \frac{\partial C(\gamma, Q)}{\partial Q}\right)\left[\frac{\partial D_i^M(\gamma)}{\partial p_i} + \frac{\partial D_j^M(\gamma)}{\partial p_i}\right] + D_i^M(\gamma) = 0. \tag{4}$$

Consistent with the standard effect of a merger on prices in the absence of efficiency gains, we make the very mild assumption that the merged entity's optimal price is higher than the independent firms' equilibrium prices for a *given* (symmetric) innovation level. Formally:

Assumption 4: $\tilde{p}^M(\gamma) > \tilde{p}^*(\gamma)$ for all γ.

2.2 Decomposition of the effect of the merger on innovation

The general idea behind the following analysis is to use first-order conditions (1) and (3) to eliminate marginal costs and focus on equilibrium prices, innovation levels and demands. To formally express the terms capturing the incentives to expand margins, we define the *per unit return to innovation* as

$$r_i(p_i, p_j, \gamma_i, \gamma_j) \equiv \frac{\partial}{\partial \gamma_i}\left(p_i - \frac{C(\gamma_i, D_i(p_i, p_j, \gamma_i, \gamma_j))}{D_i(p_i, p_j, \gamma_i, \gamma_j)}\right)\Bigg|_{D_i, p_j, \gamma_j}.$$

The per unit return to innovation $r_i(p_i, p_j, \gamma_i, \gamma_j)$ measures the marginal gain that firm i can achieve when it increases its level of innovation and raises its price so as to keep the volume of sales D_i constant, holding the level of innovation and the price of firm j fixed at γ_j and p_j, respectively. We can rewrite this as:

$$r_i(p_i, p_j, \gamma_i, \gamma_j) = -\frac{\frac{\partial D_i(p_i, p_j, \gamma_i, \gamma_j)}{\partial \gamma_i}}{\frac{\partial D_i(p_i, p_j, \gamma_i, \gamma_j)}{\partial p_i}} - \frac{\frac{\partial C(\gamma_i, D_i(p_i, p_j, \gamma_i, \gamma_j))}{\partial \gamma_i}}{D_i(p_i, p_j, \gamma_i, \gamma_j)}.$$

Moreover, we define $r_i^*(\gamma)$ and $r_i^M(\gamma)$ using the same convention as above.

The *marginal gain from innovation of an independent firm* can then be written as the product of the volume of output and the per unit return to innovation, $D_i^*(\gamma) r_i^*(\gamma)$. The symmetric equilibrium thus satisfies

$$D_i^*(\gamma^*) r_i^*(\gamma^*) = \Phi'(\gamma^*). \tag{5}$$

Turning to the merged entity's innovation choice, the optimal level of innovation given symmetric prices is the solution to the following first-order condition:

$$\left(\tilde{p}^M(\gamma) - \frac{\partial C\left(\gamma, D_i^M(\gamma)\right)}{\partial Q}\right)\left[\frac{\partial D_i^M(\gamma)}{\partial \gamma_i} + \frac{\partial D_j^M(\gamma)}{\partial \gamma_i}\right] - \frac{\partial C\left(\gamma, D_i^M(\gamma)\right)}{\partial \gamma} = \Phi'(\gamma). \quad (6)$$

An optimal symmetric price-innovation pair $\left(p^M, \gamma^M\right)$ for the merged entity satisfies conditions (4) and (6).

Similar to what we have done with independent firms, we define the *per unit return to innovation for the merged entity* as:

$$\rho_i\left(p_i, p_j, \gamma_i, \gamma_j\right) \equiv -\frac{\frac{\partial D_i\left(p_i,p_j,\gamma_i,\gamma_j\right)}{\partial \gamma_i} + \frac{\partial D_j\left(p_i,p_j,\gamma_i,\gamma_j\right)}{\partial \gamma_i}}{\frac{\partial D_i\left(p_i,p_j,\gamma_i,\gamma_j\right)}{\partial p_i} + \frac{\partial D_j\left(p_i,p_j,\gamma_i,\gamma_j\right)}{\partial p_i}} - \frac{\frac{\partial C\left(\gamma_i, D_i\left(p_i,p_j,\gamma_i,\gamma_j\right)\right)}{\partial \gamma_i}}{D_i\left(p_i, p_j, \gamma_i, \gamma_j\right)}.$$

The *merged entity's marginal gain from innovation* is then equal to $D_i^M(\gamma)\,\rho_i^M(\gamma)$, and the first-order condition for the post-merger level of innovation can be written as:

$$D_i^M\left(\gamma^M\right)\rho_i^M\left(\gamma^M\right) = \Phi'\left(\gamma^M\right). \quad (7)$$

From (4), we can see that the left-hand side in the expression above corresponds to the slope of the merged entity's profit (gross of investment cost) with respect to γ_i (at $\gamma_i = \gamma^M$) when all prices are optimally set, holding constant the innovation level of the other unit (at $\gamma_j = \gamma^M$). Based on these definitions, the following proposition shows that the impact of the merger on innovation depends on the relative magnitude of the marginal gain from innovation of the independent firms and the merged entity, evaluated at the independent firms' innovation level.[19]

Proposition 1 *The impact of the merger on innovation, given by $\gamma^M - \gamma^*$, has the same sign as $D_i^M(\gamma^*)\,\rho_i^M(\gamma^*) - D_i^*(\gamma^*)\,r_i^*(\gamma^*)$.*

Proof. See Appendix. ■

Proposition 1 shows that the merger increases (resp., decreases) innovation if the merged entity's marginal gain from innovation is larger (resp., smaller) than the independent firms' marginal gain from innovation. The comparison involves direct changes in incentives due

[19]It is crucial to evaluate the relative marginal gains from innovation at the independent firms' innovation level γ^*. In particular, we cannot use the same approach to evaluate the marginal gain difference at γ^M because in the duopoly case, p^* and γ^* are determined simultaneously at the symmetric equilibrium outcome. Therefore, we cannot simplify the incentive to innovate under a duopoly into a single equation, evaluated at γ^M, as we do when using γ^* as a point of comparison.

to price and innovation externalities but also changes related to the difference between the merged entity's and independent firms' prices.

We now show that the impact of the merger on innovation is a combination of two effects: the *market power effect* and the *externality effect*.

To obtain this decomposition, we isolate the terms in the merged entity's marginal gain from innovation, $D_i^M(\gamma)\,\rho_i^M(\gamma)$, which captures the impact of innovation in product i on the demand for that product. Eliminating the terms related to the impact of innovation on the demand for the other product, product j, we define

$$\psi^M(\gamma) \equiv -\frac{\frac{\partial D_i^M(\gamma)}{\partial \gamma_i} D_i^M(\gamma)}{\frac{\partial D_i^M(\gamma)}{\partial p_i} + \frac{\partial D_j^M(\gamma)}{\partial p_i}}.$$

Using the first-order condition (4), we have

$$\psi^M(\gamma) = \left(\tilde{p}^M(\gamma) - \frac{\partial C\left(\gamma, D_i^M(\gamma)\right)}{\partial Q}\right)\frac{\partial D_i^M(\gamma)}{\partial \gamma_i},$$

which shows that this term can be interpreted as a measure of the post-merger marginal revenue from innovation resulting from the sales of product i at constant prices. In other words, $\psi^M(\gamma)$ captures the marginal gain from innovation due to the direct effect of innovation γ_i on the demand for product i. In particular, $\psi^M(\gamma) = 0$ if innovation does not affect demand.

The following proposition provides a decomposition of the impact of the merger on innovation.

Proposition 2 *The change in innovation incentives induced by the merger can be decomposed as follows:*

$$D_i^M(\gamma^*)\,\rho_i^M(\gamma^*) - D_i^*(\gamma^*)\,r_i^*(\gamma^*) = H_P + H_E,$$

where

$$H_P \equiv D_i^M(\gamma^*)\,r_i^M(\gamma^*) - D_i^*(\gamma^*)\,r_i^*(\gamma^*),$$

and

$$H_E \equiv \psi^M(\gamma^*) \times \left(\frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial p_i}} + \frac{\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}\right)$$

if a firm's innovation affects its demand (i.e., $\partial D_i/\partial \gamma_i \neq 0$), whereas $H_E = 0$ if a firm's innovation only affects its production cost (i.e., $\partial D_i/\partial \gamma_i = \partial D_j/\partial \gamma_i = 0$).

Proof. See Appendix. ■

We interpret below the two terms H_P and H_E.

Market power effect. The term H_P captures the change in the incentives to innovate for a given product that is associated with the change in output induced by increased market power. We refer to it as the *market power effect.*

To interpret this term, it is useful to first consider the case where innovation affects only the cost of production, that is, where

$$\frac{\partial D_i\left(p_i,p_j,\gamma_i,\gamma_j\right)}{\partial \gamma_i}=\frac{\partial D_j\left(p_i,p_j,\gamma_i,\gamma_j\right)}{\partial \gamma_i}=0.$$

In this case, we have $H_E = 0$, so H_P captures the overall effect of the merger on innovation incentives. Moreover, we have

$$H_P=-\frac{\partial C\left(\gamma^*,D_i^M\left(\gamma^*\right)\right)}{\partial \gamma}+\frac{\partial C\left(\gamma^*,D_i^*\left(\gamma^*\right)\right)}{\partial \gamma}=\int_{D_i^M(\gamma^*)}^{D_i^*(\gamma^*)}\frac{\partial^2 C\left(\gamma^*,Q\right)}{\partial\gamma\partial Q}dQ.$$

Under our assumptions, since the merger leads to higher prices for a given level of innovation, the merger reduces the output, so $D_i^M(\gamma^*) < D_i^*(\gamma^*)$. Hence, H_P is negative whenever innovation reduces the marginal cost of production, which leads to the following result.

Corollary 1 *If innovation only affects production costs (cost-reducing innovation), the merger reduces incentives to innovate.*

Proof. Follows immediately from the above discussion. ■

Note that the term $-\frac{\partial C(\gamma,Q)}{\partial\gamma}$ captures the gain from increasing the margin $m_i = p_i - \frac{C(\gamma_i,Q_i)}{Q_i}$ for a given level of output. Since the post-merger output is lower than the pre-merger output, innovation efforts to increase the margin become less profitable post-merger and, consequently, innovation is lower.[20] Following Motta and Tarantino (2021), we refer to the fact that lower post-merger output reduces the incentives of merging firms to innovate as the *margin expansion effect.*

In the case where innovation affects demands, the same interpretation holds. Holding $\left(p_j,\gamma_j\right)$ constant, firm i can maintain the level of output D_i by changing its price p_i along with the level of innovation γ_i. The corresponding change in margin m_i for a change $d\gamma_i$ at a constant level of output is precisely $r_i\left(p,p,\gamma,\gamma\right)d\gamma_i$. Thus, the marginal gain in terms of

[20] Lower innovation, in turn, reinforces the output contraction effect of the merger by raising post-merger costs relative to pre-merger levels.

revenue derived from innovating on product i is equal to $r_i^*(\gamma^*) D_i^*(\gamma^*)$ for the independent firm and to $r_i^M(\gamma^*) D_i^M(\gamma^*)$ for the merged entity, where the former is evaluated at the Nash equilibrium prices $\tilde{p}^*(\gamma^*)$ and the latter at the coordinated prices $\tilde{p}^M(\gamma^*)$.

We can further decompose the market power effect into two terms:

$$H_P = H_Q + H_R \text{ where } \begin{cases} H_Q \equiv \left[D_i^M(\gamma^*) - D_i^*(\gamma^*)\right] r_i^*(\gamma^*) \\ H_R \equiv D_i^M(\gamma^*) \left[r_i^M(\gamma^*) - r_i^*(\gamma^*)\right] \end{cases}.$$

Since $D_i^M(\gamma^*) < D_i^*(\gamma^*)$ under Assumption 4, the term H_Q is negative; it captures the margin expansion effect. The second term H_R captures the impact of the quantity reduction on the per unit return to innovation discussed above; it can be either negative or positive. In contrast to the case where innovation only reduces (marginal) production costs, the market power effect H_P can now be either positive or negative. However, note that

Lemma 1 *$H_P < 0$ if $h(p, \gamma^*) \equiv D_i(p, p, \gamma^*, \gamma^*) r_i(p, p, \gamma^*, \gamma^*)$ is decreasing in p.*

Proof. Follows from $H_P = h\left(\tilde{p}^M(\gamma^*), \gamma^*\right) - h\left(\tilde{p}^*(\gamma^*), \gamma^*\right)$ and Assumption 4. ■

Hence, under our assumption that the merger raises prices (for given innovation levels) and thus reduces output, the market power effect is negative whenever the merger does not increase the return to innovation too much.

Externality effect. The term H_E captures the effect associated with the internalization of externalities between the two products. Hence, we refer to it as the *externality effect*. This effect is present only if a firm's innovation affects its demand, and in this case, it can be further decomposed as

$$H_E = H_D + H_I,$$

where

$$H_D \equiv \psi^M(\gamma^*) \times \left(\frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial p_i}} \right) > 0 \text{ and } H_I \equiv \psi^M(\gamma^*) \times \left(\frac{\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}} \right) < 0.$$

The terms H_D and H_I capture the effect of the merger on innovation with respect to the price and the innovation externality between the two products, respectively. Both terms are proportional to $\psi^M(\gamma^*)$, which is equal to zero if innovation does not affect demand.[21]

[21] Recall that $\psi^M(\gamma^*)$ is the marginal gain from innovation due to the direct effect of innovation γ_i on the demand for product i.

The term H_D reflects the interaction between pricing decisions and innovation. It is proportional to the price diversion ratio and can be interpreted as follows. Everything else been equal, the price diversion ratio is a determinant of the price-increasing effect of the merger (Farrell and Shapiro, 2010). Consider the marginal revenue $m_i dD_i$ derived by increasing demand through innovation for a constant margin m_i. Since the merger increases price-cost margins at a constant level of innovation, it raises this marginal revenue, which increases the incentives to innovate. We call this effect the *demand expansion effect.*[22] A higher price diversion ratio implies a higher increase in margins and thus a stronger demand expansion effect.

The term H_I captures the internalization by the merged entity of the *diversion* of sales that innovation in one product may induce for the other product. Following Farrell and Shapiro (2010), we refer to this externality as *innovation diversion*. Accordingly, we call H_I the *innovation diversion effect*. This term is negative because the underlying innovation externality is negative.

A key insight of the paper is that the externality effect H_E has the same sign as the difference between the price diversion ratio and the innovation diversion ratio whenever a firm's innovation affects its demand.

Proposition 3 *If a firm's innovation affects its demand, then the sign of the externality effect is governed by the comparison of the price and innovation diversion ratios:*

$$H_E \lesseqgtr 0 \text{ if } \underbrace{-\frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial p_i}}}_{\text{price diversion ratio}} \lesseqgtr \underbrace{-\frac{\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}}_{\text{innovation diversion ratio}}. \tag{8}$$

Thus, the sign of H_E captures whether the *price externality* that firms exert on each other is stronger or weaker than the *innovation externality* that they exert on each other. If the price externality is stronger, the merger induces a relatively large increase in margins, leading to a demand expansion effect large enough to outweigh the effect on firms' incentives of sales cannibalization resulting from innovation. By contrast, if the price diversion ratio is small relative to the innovation diversion ratio, the merged entity gains little from increasing demand through innovation but has a strong incentive to reduce cannibalization. In this case, the merger tends to reduce innovation.

[22] This effect has been highlighted by Bourreau and Jullien (2018) in the context of the geographic development of a new technology.

2.3 Application: Demand-enhancing innovation

In this section, we focus on (pure) demand-enhancing innovation by assuming away any effect of innovation on production costs:

Assumption 5: $\frac{\partial C(\gamma,Q)}{\partial \gamma} \equiv 0$ for all γ and Q.

It is well known that in some setups, demand-enhancing innovation is equivalent to cost-reducing innovation. This means that one can redefine the strategic variables so that innovation only affects production costs in the new reduced form of profits. For instance, Motta and Tarantino (2021) mention two cases where this holds. The first is the case of *hedonic prices*, where demand is given by $D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = Q\left(p_i - \gamma_i, p_j - \gamma_j\right)$ and $\hat{p}_i = p_i - \gamma_i$ is the hedonic price of firm i. The second is the case of *quality-adjusted prices*, where demand is given by $D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{1}{\gamma_i} Q\left(\frac{p_i}{\gamma_i}, \frac{p_j}{\gamma_j}\right)$ and $\hat{p}_i = p_i/\gamma_i$ is the quality adjusted price of firm i. In both cases, we can define a cost function $\hat{C}(\gamma_i, Q)$ such that the profit function of firm i can be written as:

$$\Pi_i = \hat{p}_i Q\left(\hat{p}_i, \hat{p}_j\right) - \hat{C}(\gamma_i, Q\left(\hat{p}_i, \hat{p}_j\right)).$$

Specifically, $\hat{C}(\gamma_i, Q) = C\left(Q\right) - \gamma_i Q$ in the model with hedonic prices and $\hat{C}(\gamma_i, Q) = C\left(Q\right)/\gamma_i$ in the model with quality-adjusted prices.

This formulation shows that for these two models, the effect of a merger on the incentives to innovate is the same as for a cost-reducing innovation. Thus, the merger reduces innovation. However, not all models of demand-enhancing innovation can be transformed into models of cost-reducing innovation.

The following corollary provides sufficient conditions for the overall effect of the merger on innovation to be negative or positive.

Corollary 2 *In the case of a demand-enhancing innovation, the merger reduces (resp., raises) incentives to innovate if the following two conditions hold:*

(i) $h\left(p, \gamma^*\right) = D_i\left(p, p, \gamma^*, \gamma^*\right) r_i\left(p, p, \gamma^*, \gamma^*\right)$ *is decreasing (resp., increasing) in* p*;*

(ii) The price diversion ratio is smaller (resp., larger) than the innovation diversion ratio.

Proof. Follows immediately from the above discussion. ■

Part (i) of the corollary determines whether the market power effect H_P is negative or positive (from Lemma 1), while part (ii) determines the sign of the externality effect

H_E (from (8)). In what follows, we examine the sign of $H_P + H_E$ for several demand functions commonly used in the literature. The next table lists the demand functions that we consider and the corresponding sign of H_E, which captures the internalization of price and innovation externalities after the merger.

Model	$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right)$	H_E
Constant expenditures	$\frac{\eta(p_i,\gamma_i)}{p_i\eta(p_i,\gamma_i)+p_j\eta\left(p_j,\gamma_j\right)+K}$	negative
Price-innovation index	$Q\left(\eta\left(p_i,\gamma_i\right),\eta\left(p_j,\gamma_j\right)\right)$	zero
Quality-augmented linear demand	$\frac{\gamma_i\left[2\gamma_i(1-p_i)-\rho\gamma_j(1-p_j)\right]}{4-\rho^2}$	positive
Augmented Singh and Vives demand	$\frac{a(\gamma_i)-a\left(\gamma_j\right)\rho(\gamma_1,\gamma_2)-p_i+\rho(\gamma_1,\gamma_2)p_j}{1-\rho(\gamma_1,\gamma_2)^2}$	positive

Table 1: Externality effect for various demand models.

First, consider the class of **models with constant expenditures**, where:[23]

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = \frac{\eta\left(p_i, \gamma_i\right)}{p_i\eta\left(p_i, \gamma_i\right) + p_j\eta\left(p_j, \gamma_j\right) + K}.$$

In these models, the demand functions depend on a price-innovation index $\eta(p,\gamma)$, decreasing in p and increasing in γ, and K represents expenditures on other goods. We find that the innovation diversion ratio is greater than the price diversion ratio for these models, which implies $H_E < 0$. Therefore, it follows from Corollary 2 that the merger reduces innovation if $h\left(p, \gamma^*\right)$ is decreasing in p. This leads to the following statement.

Corollary 3 *In models with constant expenditures, a merger reduces incentives to innovate if the elasticities* $\frac{\gamma}{\eta}\frac{\partial\eta}{\partial\gamma}$ *and* $\frac{p}{\eta}\frac{\partial\eta}{\partial p}$ *are nonincreasing in* p*. This is true, for example, for the CES demand function with* $\eta\left(p,\gamma\right) = \gamma^{\alpha}p^{\beta}$, $\alpha > 0$ *and* $\beta < 0$.

Proof. See Appendix. ■

Second, consider models that subsume the effect of prices and innovation into a **price-innovation index** $\eta(p,\gamma)$, decreasing in p but increasing in γ, where the demand of firm i is given by

$$D_i\left(p_i, p_j, \gamma_i, \gamma_j\right) = Q\left(\eta(p_i,\gamma_i), \eta(p_j,\gamma_j)\right).$$

In this case, the price diversion ratio is equal to the innovation diversion ratio, hence $H_E = 0$. Therefore, the merger reduces innovation if the market power effect is negative.

[23] For a discussion of these demand functions, see Vives (1999).

For example, the *multinomial logit* (MNL) model belongs to this class of models. In this case, $\eta(p,\gamma) = \exp u(\gamma, y-p)$ where u increasing in both its arguments,[24] and the demand of firm i is given by

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = \frac{\exp u(\gamma_i, y-p_i)}{\exp u(\gamma_i, y-p_i) + \exp u\left(\gamma_j, y-p_j\right) + \exp u(0,y)}.$$

For this demand function, $h(p,\gamma^*)$ can be either increasing or decreasing in p. Denoting by u_1 and u_2 the derivatives of u with respect to its first and second arguments, respectively, and u_{12} and u_{22} the cross-derivative of u and the second derivative of u with respect to its second argument, respectively, we find the following sufficient conditions for the merger to reduce (resp., raise) the incentives to innovate.

Corollary 4 *In the MNL model, a merger reduces incentives to innovate if*

$$\frac{-u_{12}(\gamma, y-p)}{u_1(\gamma, y-p)} + \frac{u_{22}(\gamma, y-p)}{u_2(\gamma, y-p)} \leq 0$$

for all γ *and* $p \in \left[p^*(\gamma), \tilde{p}^M(\gamma)\right]$.
The merger raises the incentives to innovate if

$$\frac{-u_{12}(\gamma, y-p)}{u_1(\gamma, y-p)} + \frac{u_{22}(\gamma, y-p)}{u_2(\gamma, y-p)} \geq u_2(\gamma, y-p)$$

for all γ *and* $p \in \left[p^*(\gamma), \tilde{p}^M(\gamma)\right]$.

Proof. See Appendix. ■

The above corollary shows that a merger may either reduce or raise incentives to innovate. For example, it reduces incentives in the case of a Cobb-Douglas utility function $u(\gamma, y-p) = \gamma^\alpha (y-p)^\beta$, with $\alpha, \beta \in (0,1)$, and raises them in the case of a quasi-linear utility function $u(\gamma, y-p) = v(\gamma) + f(\gamma)(y-p)$ satisfying $-f'(\gamma)/f(\gamma) > v'(\gamma) + yf'(\gamma) > 0$.

For the last two models in Table 1, the price diversion ratio is greater than the innovation diversion ratio, and $h(p,\gamma)$ is decreasing in p (over the relevant range). Thus, we have $H_E > 0$ on the one hand and $H_P < 0$ on the other hand. Consequently, the effect of a merger on innovation is *a priori* ambiguous for these two models. Below we discuss this effect further considering in turn each model.

[24] See, e.g., Dubé (2019). In this model, y represents income, and $u(\gamma_i, y-p_i)$ is the mean utility from consuming one unit of product of quality γ_i paid at price p_i.

The **quality-augmented linear demand**

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = \frac{\gamma_i \left[2\gamma_i (1 - p_i) - \rho\gamma_j (1 - p_j)\right]}{4 - \rho^2}$$

was introduced by Sutton (1997, 1998) and used *inter alia* by Symeonidis (2000, 2003) and Federico et al. (2018). Assuming a constant marginal cost, i.e., $C(Q) = cQ$, this model has the interesting property that the competitive equilibrium price and the monopoly price do not depend on the innovation level γ. Specifically,[25]

$$\widetilde{p}^* (\gamma) = c + (1 - c) \frac{2 - \rho}{4 - \rho} < 1 \text{ and } \widetilde{p}^M (\gamma) = \frac{1 + c}{2}.$$

Therefore, innovation is monetized only through an increase in demand. We obtain the following result.

Corollary 5 *In the model with quality-augmented linear demand, a merger reduces incentives to innovate.*

Proof. See Appendix. ■

Let us now consider the model with an **augmented Singh and Vives demand**, where:

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = \frac{a(\gamma_i) - a\left(\gamma_j\right) \rho(\gamma_1, \gamma_2) - p_i + \rho(\gamma_1, \gamma_2) p_j}{1 - \rho(\gamma_1, \gamma_2)^2}.$$

In this model, used for instance by Lin and Saggi (2002), innovation has both vertical and horizontal dimensions. First, innovation increases product quality, which we capture by assuming that $a(\gamma) = \alpha + \tau\gamma$, with $\alpha > 0$ and $\tau \geq 0$. Second, innovation increases the horizontal differentiation between the firms' products. Specifically, suppose that the degree of substitutability between the products is given by $\rho(\gamma_1, \gamma_2) = 1 - \delta(\gamma_1 + \gamma_2)$, where $\delta > 0$. Assuming a constant marginal cost $c < \alpha$, we obtain the following result.

Corollary 6 *In the model with augmented Singh and Vives demand, a merger raises incentives to innovate if*

$$\alpha - c > 2\frac{\tau}{\delta}. \tag{9}$$

If this condition does not hold, then, denoting $\Phi(\gamma) \equiv \eta\Phi_0(\gamma)$, *there exists* $\eta_0 > 0$ *such that a merger reduces the incentives to innovate if* $\eta > \eta_0$.

[25] See Symeonidis (2003) for the derivation of equilibrium prices.

Proof. See Appendix. ■

When the value of the market $(\alpha - c)$ is higher or the investment cost is lower, firms tend to invest more in innovation. As a result, the externality effect increases, in particular due to a lower innovation diversion ratio. At the same time, the market power effect decreases in absolute terms, in particular because the margin expansion effect is reduced due to increased differentiation. Therefore, in these cases, the merger is more likely to raise the incentives to innovate.

The application of our approach to standard models thus shows that, absent synergies and spillovers, a merger reduces the incentives to innovate in many, but not all, cases.

2.4 Consumer welfare

We have shown that a merger can either reduce or increase innovation. However, most competition authorities are ultimately interested in the impact of a merger on the welfare of consumers, which depends on both innovation and prices. In this section, we investigate the impact of a merger on consumer surplus in the absence of spillovers and synergies.

Denote by $CS(p, \gamma)$ consumer surplus for a symmetric price p and a symmetric innovation level γ and assume that $\partial CS/\partial p < 0$ and $\partial CS/\partial \gamma \geq 0$.[26] The effect of the merger on consumer surplus is given by the sign of $\Delta CS \equiv CS(p^M, \gamma^M) - CS(p^*, \gamma^*)$.

We generally expect that, for a *given* market structure, an exogenous increase in innovation benefits consumers. Under the post-merger market structure, this formally amounts to making the following assumption:

Assumption 6: $\frac{\partial CS(\tilde{p}^M(\gamma),\gamma)}{\partial \gamma} + \frac{\partial CS(\tilde{p}^M(\gamma),\gamma)}{\partial p} \frac{\partial \tilde{p}^M(\gamma)}{\partial \gamma} > 0$ for all $\gamma \in [0, \bar{\gamma}]$.

The effect of innovation on consumer surplus under the post-merger market structure must be combined with the effect of the price increase on consumer surplus induced by the merger for a given innovation level. Specifically, the overall effect of a merger on consumer surplus can be decomposed as follows:

$$\Delta CS = \left[CS(\tilde{p}^M(\gamma^M), \gamma^M) - CS(\tilde{p}^M(\gamma^*), \gamma^*)\right] + \left[CS(\tilde{p}^M(\gamma^*), \gamma^*) - CS(p^*, \gamma^*)\right]. \quad (10)$$

Under Assumption 6, the first term in this decomposition has the same sign as the difference $\gamma^M - \gamma^*$. In other words, this term is positive (resp., negative) if the merger

[26] The special case $\partial CS/\partial \gamma = 0$ corresponds to the scenario of a (pure) cost-reducing innovation.

leads to an increase (resp., decrease) in innovation. The second term has the same sign as $p^* - \tilde{p}^M(\gamma^*)$ and is therefore negative.[27]

A first immediate conclusion is that a merger that leads to less innovation and does not reduce prices for a *given* level of innovation necessarily harms consumers. Notice that this happens even though such a merger could induce lower equilibrium prices (driven by less innovation).

By contrast, the overall effect of a merger on consumer surplus is *a priori* ambiguous if the merger increases innovation, since the first term of the decomposition is positive in this case, while the second term is (weakly) negative. The impact of a merger on consumer surplus can be negative even if it increases innovation. It is only when the effect on innovation is strong enough that the impact of the merger on consumer surplus becomes positive.

To derive further insights into whether a merger can lead to higher consumer surplus in the absence of (production and R&D) synergies and spillovers, we conducted simulations for two specific demand models in the context of demand-enhancing innovation: the MNL model and the augmented Singh and Vives model. For these two models, we have indeed shown that a merger can lead to more innovation (see Corollaries 4 and 6, respectively). We want to see whether the positive effect of innovation on consumer surplus can outweigh the negative effect of higher prices.

To estimate the impact of the merger on innovation, we need to compute the equilibrium prices and innovation levels before and after the merger. For both models, we cannot obtain analytical solutions, so we resort to simulations. We assume a quadratic cost of innovation, $\Phi(\gamma) = (k/2)\gamma^2$.

In the Singh and Vives model,[28] we set $\alpha = 1$ and $c = 0$ and simulate the impact of the merger on consumer surplus for different values of δ and τ, which capture the horizontal and vertical dimensions of the innovation, respectively, and k, the innovation cost parameter. We take ranges of parameters such that $\rho(\gamma^*) > 0$, $H_I < 0$, and the second order conditions hold. We find that prices always increase with the merger, while the level of innovation can either decrease or increase. But even in the latter case, consumer surplus decreases with the merger.

For the MNL model, we consider the mean utility $u(\gamma, y - p) = v_0(1 + \gamma) + e^{-\tau\gamma}(y - p)$. We set $c = 0$ and then choose values for v_0, τ, y and k such that the second order conditions

[27] Note that a similar decomposition and qualitatively similar insights can be obtained if we consider total welfare instead of consumer surplus as long as we assume that, for a given market structure, an exogenous increase in innovation raises total welfare. This is the case when the equilibrium level of innovation is suboptimal due to insufficient appropriability.

[28] We describe the simulations in more detail in the Online Appendix.

hold and $v_0 > \tau y$ to ensure that $u_1 > 0$ for all values of p and γ. We find that prices always increase with the merger, while the level of innovation can either decrease or increase. However, even if the merger boosts innovation, we find that consumer surplus decreases.

In sum, our simulations show a negative effect of the merger on consumer surplus, suggesting that in the absence of (production or R&D) synergies and spillovers, a merger is unlikely to benefit consumers even if it leads to higher levels of innovation. A possible explanation for this result is that a positive effect of a merger on innovation is likely to be driven by a strong demand expansion effect, which requires a large price diversion ratio and, therefore, implies a large adverse effect of the merger on prices.

3 Production synergies and P-neutral mergers

So far we have focused on mergers that do not entail any synergies. In this section, we highlight the interplay between production synergies and innovation incentives (we consider R&D synergies in Section 4.1).

Our goal is twofold. First, we want to contrast the case where the externality effect vanishes (which happens when demand is not affected by innovation) with the case where the market power effect vanishes, which can only occur if there are production synergies. Second, we want to provide some insights on the level of production efficiencies that would eliminate the negative impact of the merger on consumers, accounting for the effect on innovation.

As discussed above, the change in margin induced by a merger affects the incentives to innovate. The question then is whether a "simple" merger analysis that examines the effect on prices for a given level of innovation (fixed at γ^*) can shed some light on the effect of the merger on innovation. To address this question, we now assume that the merger may generate production synergies, i.e., the merger reduces the marginal cost of production by $\sigma \geq 0$. The post-merger cost for a production Q then becomes

$$C(\gamma, Q) - \sigma Q.$$

We further assume that the production synergies σ are independent of the level of innovation γ and of the output Q. Under this assumption, the decomposition of the effect of the merger on the incentives to innovate described in Section 2.2 still applies. The only change is that the post-merger coordinated price is now $\tilde{p}^M(\gamma, \sigma)$, evaluated at the new marginal cost $\frac{\partial C(\gamma,Q)}{\partial Q} - \sigma$. This price decreases with the level of synergies σ and is therefore lower than in the baseline case without synergies.

While there is no direct relationship between production synergies and the externality effect H_E, production synergies reduce the market power effect H_P whenever the function $h(p, \gamma)$ defined in Lemma 1 decreases in p. This suggests that our conclusions regarding the effect of the merger on innovation incentives may change if there are sufficient production synergies.

Of particular interest is the case of *compensating synergies*, defined as production synergies at a level that maintains the price at its pre-merger level for a fixed innovation level. Specifically, we say that a merger is *P-neutral* if the merger does not affect prices when the innovation level of both firms is *fixed* at the level chosen by independent firms, that is,

Definition 1 *A merger is* P-neutral *if* $\tilde{p}^M(\gamma^*, \sigma) = p^*$.

The fact that a merger is P-neutral does *not* mean that it does not affect equilibrium prices. Instead, it means that any merger-induced changes in equilibrium prices are driven by the effect of the merger on innovation incentives and the effect of innovation on prices.[29]

Our focus on the special case of P-neutral mergers is also motivated by their policy relevance. Studying the impact of such mergers helps to determine the conditions under which a merger that does not affect prices (at a given level of innovation) has a negative impact on innovation (Denicolò and Polo, 2019).

Consider a P-neutral merger. It is straightforward to see that the market power effect (H_P) vanishes as it stems from changes in pricing behavior.[30] Therefore, the effect of a P-neutral merger on innovation is governed solely by the externality effect (H_E).[31] This yields the following result.

Proposition 4 *If a firm's innovation affects its demand, then a P-neutral merger reduces (resp., raises) incentives to innovate whenever the price diversion ratio is lower (resp., higher) than the innovation diversion ratio, where both ratios are evaluated at* $(p^*, p^*, \gamma^*, \gamma^*)$. *If a firm's innovation only affects its production cost, then a P-neutral merger has no effect on the incentives to innovate.*

[29] A P-neutral merger would be CS-neutral in the terminology of Nocke and Whinston (2010) if the demand functions were not affected by innovation.

[30] We have $H_P = D_i(\tilde{p}^M(\gamma^*, \sigma), \tilde{p}^M(\gamma^*, \sigma), \gamma^*, \gamma^*) \times r_i(\tilde{p}^M(\gamma^*, \sigma), \tilde{p}^M(\gamma^*, \sigma), \gamma^*, \gamma^*) - D_i(p^*, p^*, \gamma^*, \gamma^*) \times r_i(p^*, p^*, \gamma^*, \gamma^*) = 0$ since $\tilde{p}^M(\gamma^*, \sigma) = p^*$ for a P-neutral merger.

[31] Note that a P-neutral merger leads to a higher margin due to production synergies, even though it does not affect prices, which increases incentives to expand demand as discussed above.

Proof. When a firm's innovation affects its demand, the result follows from (8) and $H_P = 0$. When a firm's innovation only affects its production cost, the result follows from $H_P = H_E = 0$. ∎

When a firm's innovation affects its demand, the sign of the externality effect H_E captures whether the *price externality* that firms exert on each other is stronger or weaker than the *innovation externality* that they exert on each other. If the price externality is stronger, a P-neutral merger requires large production synergies σ, which induces a relatively large increase in margins. This leads to a demand expansion effect that is large enough to outweigh the effect of sales cannibalization resulting from innovation on firms' incentives. As a result, the merged entity invests more in innovation. By contrast, if the price diversion ratio is small relative to the innovation diversion ratio, a small amount of production synergies σ is sufficient to maintain the price at the pre-merger level. Hence, the merged entity gains little from increasing its demand but has a strong incentive to reduce cannibalization. In this case, the merger reduces innovation.

When a firm's innovation only affects its production cost, there is no externality effect and the incentive to innovate is solely determined by the impact of the merger on output, which is zero in the case of a P-neutral merger.

Let us now consider the impact of a P-neutral merger on consumer surplus. For such a merger, the second term in the decomposition of ΔCS given by equation (10) is equal to zero and, therefore, the impact of the merger on consumer surplus is entirely determined by its impact on innovation. Therefore, we get the following result.

Corollary 7 *The impact of a P-neutral merger on consumer surplus has the same sign as its impact on incentives to innovate. In particular, if a firm's innovation affects its demand, a P-neutral merger reduces (resp. raises) consumer surplus if the price diversion ratio is lower (resp. higher) than the innovation diversion ratio, both ratios being evaluated at* $(p^*, p^*, \gamma^*, \gamma^*)$.

Proof. See Appendix. ∎

Proposition 4 and Corollary 7 suggest that whenever the price diversion ratio weakly exceeds the innovation diversion ratio (as is the case for the quality-augmented model, the MNL model, and the augmented Singh and Vives model), a merger evaluation concluding that the merger generates enough production synergies to remove price concerns (holding products and technology constant) should also conclude that there are no concerns regarding the merger hindering innovation and harming consumers. However, if the price diversion ratio is smaller than the innovation diversion ratio, the absence of price concerns is not sufficient to remove concerns about the impact of the merger on innovation and consumer welfare.

4 Extensions

In this section, we consider potential R&D-related benefits of a merger that may alter the conclusions of the baseline model regarding the effect of a merger on innovation and consumer welfare. First, we consider R&D synergies that could result, for instance, from the redeployment of assets or the voluntary exchange of knowledge between the R&D units of the merging firms. Second, we discuss the implications of involuntary knowledge spillovers between firms. Third, we consider the case where there are more than two firms in the industry. Finally, we briefly discuss two other extensions that we analyze in detail in the Online Appendix. For the sake of conciseness, we focus on the case of a (pure) demand-enhancing innovation in this section.

4.1 R&D synergies

Suppose that the merger leads to a reduction in the cost of R&D investment.[32] More specifically, assume that the post-merger cost of R&D is given by $\Phi(\gamma)/(1+\mu)$, where $\mu \geq 0$ is a measure of the magnitude of the efficiency gains in R&D. To simplify the exposition, we abstract from any efficiency gains in production. We also assume that firms have a constant marginal cost, c.

The only first-order condition that is affected by efficiency gains in R&D is related to the merged entity's innovation level, i.e., equation (6), which becomes

$$(1+\mu)(p-c)\left[\frac{\partial D_i}{\partial \gamma_i}(p,p,\gamma,\gamma)+\frac{\partial D_j}{\partial \gamma_i}(p,p,\gamma,\gamma)\right]=\Phi'(\gamma).$$

The equilibrium price of the independent firms and the optimal price of the merged entity for a given (symmetric) innovation level are still given by $\tilde{p}^*(\gamma)$ and $\tilde{p}^M(\gamma)$, respectively. Therefore, the result in Proposition 1 can be extended to the case of efficiency gains in R&D of size μ as follows: the impact of the merger on innovation has the same sign as

$$(1+\mu)\,D_i^M(\gamma^*)\,\rho_i^M(\gamma^*)-D_i^*(\gamma^*)\,r_i^*(\gamma^*)=H_P+H_{E\mu},$$

[32] Davidson and Ferrett (2007) emphasize the importance of R&D synergies in shaping the profitability of a merger. In contrast, we focus on how they affect innovation efforts.

where

$$H_P \equiv D_i^M(\gamma^*)\, r_i^M(\gamma^*) - D_i^*(\gamma^*)\, r_i^*(\gamma^*)$$

and

$$H_{E\mu} \equiv \psi^M(\gamma^*) \times \left(\frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial p_i}} + \mu + (1+\mu) \frac{\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}} \right). \tag{11}$$

As in the baseline model, we can thus decompose the impact of the merger on innovation into a market power effect, captured by the term H_P, and an externality effect, captured by the (adjusted) term $H_{E\mu}$.

Consider now a merger where $H_P < 0$. In this case, the overall impact of the merger on innovation is negative if $H_{E\mu} < 0$. It is straightforward to show that $H_{E\mu}$ has the same sign as the difference between a synergy-adjusted price diversion ratio and the innovation diversion ratio. Specifically, we have the following result:

Proposition 5 *Assume that the merged entity's R&D cost function is given by $\Phi(\gamma)/(1+\mu)$, where μ measures efficiency gains in R&D. The sign of $H_{E\mu}$ is the same as the sign of:*

$$\underbrace{\frac{\frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial p_i}} + \mu}{1+\mu}}_{\text{synergy-adjusted price diversion ratio}} - \underbrace{\frac{\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{-\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}}_{\text{innovation diversion ratio}}.$$

Proof. Follows immediately from equation (11). ■

This shows that the comparison of the price diversion ratio and the innovation diversion ratio remains a key determinant of the impact of a merger on innovation in the presence of efficiency gains in R&D as long as the price diversion ratio is adjusted to account for these efficiency gains.

Notice that the synergy-adjusted price diversion ratio is larger than the price diversion ratio and increases with the level μ of efficiency gains. Since the market power effect is not affected by these efficiency gains, we conclude that R&D synergies increase the likelihood of a positive effect of the merger on innovation.

4.2 Technological spillovers

It is well known that a firm's R&D may benefit other firms, including its rivals, through technological spillovers (d'Aspremont and Jacquemin, 1988; Bloom et al., 2013; López and

Vives, 2019). In this section, we show how our baseline model can be adapted to account for such spillovers.

Let us assume that there exists a degree of (involuntary) spillovers $\lambda \in [0,1]$ such that the demand for firm i is given by $D_i\left(p_i, p_j, \gamma_i + \lambda\gamma_j, \gamma_j + \lambda\gamma_i\right)$. In other words, a share λ of the innovation efforts of firm i spills over to firm j (and vice versa). To abstract from efficiencies stemming from the merging firms' ability to increase spillovers between them after the merger, we assume that the degree of spillovers is not affected by the merger.[33]

Let $\hat{\gamma}_i \equiv \gamma_i + \lambda\gamma_j$ for $i = 1, 2$ and $\hat{\gamma} \equiv (1+\lambda)\gamma$, and denote by $\hat{\gamma}^*$ the (symmetric) independent firms' equilibrium level of innovation. It is straightforward to show that Proposition 1 extends to the scenario with R&D spillovers. Specifically, the impact of the merger on innovation has the same sign as $D_i^M(\hat{\gamma}^*)\,\rho_\lambda^M(\hat{\gamma}^*) - D_i^*(\hat{\gamma}^*)\,r_\lambda^*(\hat{\gamma}^*)$, where $\rho_\lambda^M(.)$ and $r_\lambda^*(.)$ are obtained from $\rho_i^M(.)$ and $r_i^*(.)$ by replacing $\frac{\partial D_i}{\partial \gamma_i}$ and $\frac{\partial D_j}{\partial \gamma_i}$ with $\frac{\partial D_i}{\partial \hat{\gamma}_i} + \lambda\frac{\partial D_i}{\partial \hat{\gamma}_j}$ and $\frac{\partial D_j}{\partial \hat{\gamma}_i} + \lambda\frac{\partial D_j}{\partial \hat{\gamma}_j}$, respectively, and replacing the arguments $\left(\tilde{p}^*(\gamma), \tilde{p}^*(\gamma), \gamma, \gamma\right)$ and $\left(\tilde{p}^M(\gamma), \tilde{p}^M(\gamma), \gamma, \gamma\right)$ with $\left(\tilde{p}^*(\hat{\gamma}), \tilde{p}^*(\hat{\gamma}), \hat{\gamma}, \hat{\gamma}\right)$ and $\left(\tilde{p}^M(\hat{\gamma}), \tilde{p}^M(\hat{\gamma}), \hat{\gamma}, \hat{\gamma}\right)$, respectively.

We can again decompose the overall impact of the merger on incentives to innovate into several effects:

$$D_i^M(\hat{\gamma}^*)\,\rho_\lambda^M(\hat{\gamma}^*) - D_i^*(\hat{\gamma}^*)\,r_\lambda^*(\hat{\gamma}^*) = H_{P\lambda} + H_{E\lambda} + H_{S\lambda},$$

where $H_{P\lambda}$ and $H_{E\lambda}$ are obtained from H_P and H_E by making the replacements specified above and

$$H_{S\lambda} \equiv \psi^M(\hat{\gamma}^*)\,\lambda\left[\frac{\frac{\partial D_j^M(\hat{\gamma}^*)}{\partial \hat{\gamma}_i}}{\frac{\partial D_i^M(\hat{\gamma}^*)}{\partial \hat{\gamma}_i}} \times \frac{\frac{\partial D_j^M(\hat{\gamma}^*)}{\partial p_i}}{-\frac{\partial D_i^M(\hat{\gamma}^*)}{\partial p_i}} + 1\right] > 0.$$

The term $H_{S\lambda}$ captures a *spillover effect* and is positive.[34]

Furthermore, we find that the sum of the externality effect and the spillover effect, $H_{E\lambda} + H_{S\lambda}$, has the same sign as the difference between the price diversion ratio and a spillover-adjusted innovation diversion ratio. Specifically, we have the following result.

Proposition 6 *Assume that there are R&D spillovers and denote the spillover rate as* λ*.*

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[33] See Denicolò and Polo (2021) for an analysis of the case where the merger increases spillovers between the merging parties.

[34] To see why this term is positive, note that $\frac{\partial D_j}{\partial \hat{\gamma}_i} \times \frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} + \frac{\partial D_i}{\partial \hat{\gamma}_i} > \min\left[\frac{\partial D_i}{\partial \hat{\gamma}_i}, \frac{\partial D_j}{\partial \hat{\gamma}_i} + \frac{\partial D_i}{\partial \hat{\gamma}_i}\right] > 0$, because $\partial D_i/\partial p_i + \partial D_i/\partial p_j$ is negative and $\partial D_i/\partial \hat{\gamma}_i + \partial D_i/\partial \hat{\gamma}_j$ is positive.

The sign of $H_{E\lambda} + H_{S\lambda}$ is the same as the sign of:

$$\underbrace{\frac{\frac{\partial D_j^M(\hat{\gamma}^*)}{\partial p_i}}{-\frac{\partial D_i^M(\hat{\gamma}^*)}{\partial p_i}}}_{\textit{price diversion ratio}} - \underbrace{\frac{-\frac{\frac{\partial D_j^M(\hat{\gamma}^*)}{\partial \hat{\gamma}_i}}{\frac{\partial D_i^M(\hat{\gamma}^*)}{\partial \hat{\gamma}_i}} - \lambda}{1 + \lambda \frac{\frac{\partial D_j^M(\hat{\gamma}^*)}{\partial \hat{\gamma}_i}}{\frac{\partial D_i^M(\hat{\gamma}^*)}{\partial \hat{\gamma}_i}}}}_{\textit{spillover-adjusted innovation diversion ratio}}.$$

Proof. Follows from the expressions of $H_{E\lambda}$ and $H_{S\lambda}$. ■

Note that the denominator of the spillover-adjusted innovation diversion ratio is always positive. This follows from the assumption that $\lambda \in [0, 1]$ and $\partial D_i/\partial \hat{\gamma}_i + \partial D_i/\partial \hat{\gamma}_j > 0$. Thus, the sign of the spillover-adjusted innovation diversion ratio is given by the sign of the difference between the innovation diversion ratio and the spillover rate:

$$\left(-\frac{\frac{\partial D_j^M(\hat{\gamma}^*)}{\partial \hat{\gamma}_i}}{\frac{\partial D_i^M(\hat{\gamma}^*)}{\partial \hat{\gamma}_i}} \right) - \lambda.$$

This sign can be related to the magnitude of the net innovation pressure (NIP) defined by Salinger (2019). Considering an environment with no price competition, Salinger (2019) shows that a merger reduces innovation if and only if

$$NIP \equiv \frac{\left(\frac{\partial D_i}{\partial \gamma_i} + \frac{\partial D_j}{\partial \gamma_i} \right)(1 + \lambda)}{\frac{\partial D_i}{\partial \gamma_i} + \lambda \frac{\partial D_j}{\partial \gamma_i}} > 1.$$

It is straightforward to see that this condition holds if and only if the spillover-adjusted innovation diversion ratio is positive, i.e., $\lambda < -\frac{\partial D_j/\partial \gamma_i}{\partial D_i/\partial \gamma_i}$.

Finally, note that the spillover-adjusted innovation diversion ratio is less than the innovation diversion ratio and decreases with the spillover rate λ. Hence, it is more likely to be less than the price diversion ratio, confirming the intuition that the existence of spillovers can turn an otherwise innovation-decreasing merger into an innovation-increasing one.

4.3 Oligopoly

In this section, we extend our analysis to a merger between two firms in an oligopoly. We assume that there are N firms, indexed by $i \in \{1, 2, ...N\}$. Each firm chooses a price p_i and a level of innovation γ_i, and we again assume that these choices are simultaneous.

Firms 1 and 2 are the merging firms, and firms 3 to N are the outsiders. Building on our baseline model, we assume that the two merging firms have symmetric demands and the same production and innovation cost functions. We also suppose that Assumptions 1 and 4 hold for any given strategy $(p_3, \gamma_3, ..., p_N, \gamma_N)$ of the outsiders. Finally, we assume that firms 1 and 2 have the same constant marginal cost c and we allow for merger-induced production synergies $\sigma \geq 0$.

For conciseness, we assume that the outsiders have the same marginal production cost c_o, investment cost Φ_o and symmetric demand for given symmetric strategy (p, γ) of firms 1 and 2. Also to simplify notations, we denote by $\Sigma_o = (p_o, \gamma_o)$ a symmetric strategy of outsiders.

Following Deneckere and Davidson (1985), we aggregate outsiders' reaction into a joint reaction to the strategy of firm 1 and 2. For this purpose, for any given symmetric strategy (p, γ) of firms 1 and 2, consider the game where players are only firms 3 to N and firms choose simultaneously prices and innovation levels. Assume that for each (p, γ), this reduced game has a unique symmetric equilibrium, where each outsider chooses the same price p_o and innovation γ_o. This assumption is the counterpart of Assumption 2 for the outsiders and allows to reduce the dimension of the strategy space.[35]

Denote the mapping from merging firms' strategy (p, γ) to equilibrium of the game reduced to outsiders by $R_o(p, \gamma) : \mathbb{R}_+^2 \to \mathbb{R}_+^2$. We refer to $R_o(p, \gamma)$ as the aggregate reply for outsiders.

In the benchmark scenario in which firms 1 and 2 act independently, the symmetric first-order conditions for prices and innovation levels of firms 1 and 2, given the outsiders strategy Σ_o, can now be written as

$$(p - c) \frac{\partial D_i}{\partial p_i} (p, p, \gamma, \gamma, \Sigma_o) + D_i (p, p, \gamma, \gamma, \Sigma_o) = 0 \tag{12}$$

for the price and

$$(p - c) \frac{\partial D_i}{\partial \gamma_i} (p, p, \gamma, \gamma, \Sigma_o) = \Phi' (\gamma) \tag{13}$$

for the innovation level. We extend Assumption 2 to this oligopoly setting as follows:

Assumption 2': The oligopoly price-innovation game has a unique equilibrium, in which firms 1 and 2 play symmetric strategies (p^*, γ^*) satisfying first-order conditions (12) and (13) and outsiders play strategy $\Sigma_o^* = R_o(p^*, \gamma^*)$.

[35] In an asymmetric setting, the strategy of outsiders would be of dimension $2N - 4$. However, the analysis that follows would still apply.

The results of Section 2 can be interpreted as characterizing the behavior of firms 1 and 2 holding constant the strategy of outsiders. In particular, equation (12) defines the equilibrium price of firms 1 and 2 as a function $\tilde{p}^* (\gamma, \Sigma_o)$ of their innovation level γ and strategy Σ_o of outsiders. As in the baseline model, we can define the independent firm's marginal gain from innovation conditional on the behavior Σ_o of outsiders and conclude that the equilibrium innovation level of firms 1 and 2 satisfies equation (5) evaluated at the equilibrium strategy Σ_o^*. Considering the situation where firms 1 and 2 merge, we can similarly define the merged entity's marginal gain from innovation conditional on the behavior of outsiders. If the post-merger equilibrium strategy of outsiders is denoted $\Sigma_o^M = R_o \left(p^M, \gamma^M\right)$, the symmetric equilibrium level of innovation of each of the merging firms after the merger satisfies equation (7) evaluated at marginal cost $c - \sigma$ and strategy Σ_o^M.

For the analysis of the post-merger equilibrium, we define the *post-merger accessory game* for any given γ as the game where the innovation level of the merged firms is fixed at $\gamma_1 = \gamma_2 = \gamma$ and the merged entity chooses only the prices $p_i, \ i = 1, 2$, while the outsiders choose both their price and innovation level. For the oligopolistic setting that we consider in this section, we cannot rely on the global optimality of the choices of the merged entity to compare the post-merger and pre-merger situations because the outsiders' strategy changes. We therefore replace Assumption 3 with

Assumption 3': (i) The post-merger accessory game has an equilibrium $\left(\hat{p}^M (\gamma), R_o \left(\hat{p}^M (\gamma), \gamma\right)\right)$, which is unique and continuous in γ. (ii) The post-merger game equilibrium $\left(p^M, \gamma^M, \Sigma_o^M\right)$ is symmetric in products 1 and 2 and uniquely characterized by equilibrium conditions: $p^M = \hat{p}^M \left(\gamma^M\right)$, $\Sigma_o^M = R_o \left(p^M, \gamma^M\right)$ and $D_i^M \left(\gamma^M, \Sigma_o^M\right) \rho_i^M \left(\gamma^M, \Sigma_o^M\right) = \Phi' \left(\gamma^M\right)$.

The difficulty in extending our analysis to a setting with more than two firms is that we do not know *a priori* how the behavior of the non-merging firms is affected by the merger, in part because this behavior is two-dimensional. However, we may gain some insight by noticing that our definition and interpretation of a P-neutral merger extends to the case of an oligopoly. We say that a merger is P-neutral if the efficiency gains in production σ are such that holding the merged entity's innovation level fixed at the pre-merger equilibrium level γ^*, and outsiders' strategy at the pre-merger equilibrium level Σ_o^*, the merger would not affect the merged entity's price. In other words, a merger is P-neutral if for (γ^*, Σ_o^*), the merger entity's profit-maximizing price is

$$\tilde{p}^M (\gamma^*, \sigma, \Sigma_o^*) = p^*.$$

Note that if the merging firms' innovation levels were fixed at γ^*, a P-neutral merger would not affect the price nor the innovation level of the outsiders. Indeed, outsiders would still choose $\Sigma_o^* = R_o(p^*, \gamma^*)$ in this case so that the equilibrium of the post-merger accessory game, at innovation level γ^*, coincides with the pre-merger equilibrium. In the oligopoly environment, as in the baseline model, a P-neutral merger affects equilibrium prices (and quantities) if and only if it affects the equilibrium innovation level of the merged entity. The following result extends Proposition 4 to an oligopoly setting.

Proposition 7 *A P-neutral merger reduces (resp., raises) the merging firms' innovation level if the innovation diversion ratio is greater (resp., lower) than the price diversion ratio, where both ratios are evaluated at* $(p^*, p^*, \gamma^*, \gamma^*, \Sigma_o^*)$.

Proof. See Appendix. ■

Therefore, the mere comparison of the diversion ratios allows us to sign the effect of a P-neutral merger on the merging firms' innovation level, even in the presence of outsiders. A caveat is that the diversion ratios are evaluated at the equilibrium of the post-merger accessory game, holding innovation and the price of outsiders fixed at their pre-merger levels.

4.4 Further extensions

We consider two additional extensions in the Online Appendix, which we briefly discuss below.

In the first extension, we assume that a firm's investment in R&D is observed by its rival before prices are set. In this scenario, each firm considers not only the direct effect of its investment on its own profit but also the strategic effect that operates through its rival's pricing reaction. Due to this additional effect, a new term appears in the decomposition of the overall effect of the merger on the merging firms' incentives to innovate. We show that the sign of this term is the opposite of the sign of the strategic effect of innovation on the rival's price. Under the natural assumption that this strategic effect is negative,[36] the new term is positive. The intuition behind this is that when firms are independent, the benefit from innovation is lower if R&D is observable because greater innovation leads to more aggressive pricing by the rival. The internalization of this effect is a new channel through which a merger can stimulate innovation.

[36] In the Online Appendix, we provide general conditions on the demand functions under which this holds.

In the second extension, we aim to determine if the approach from our baseline model can be applied to a setting with asymmetric demand and/or cost functions. Specifically, we examine whether comparing the merged entity's marginal gain from innovation for each product with the corresponding gain for the independent firm producing this product is sufficient to determine the effect of the merger on incentives to innovate. We show that the answer is positive under the assumption that the merged entity's innovation levels are strategic complements (as precisely defined in the Online Appendix).

5 Conclusion

In this paper, we provide a novel decomposition of the impact of a merger on merging firms' incentives to invest in incremental innovation, and use it to establish sufficient conditions under which this impact is negative or positive.

Focusing first on the scenario in which there are no (production or R&D) synergies or spillovers, our analysis suggests that the impact on their incentives to invest in demand-enhancing innovation is likely to be negative if the innovation diversion ratio is greater than the price diversion ratio, but can be either positive or negative if the innovation diversion ratio is less than the price diversion ratio. Applying our approach to several specific demand functions, we show that both cases can arise. While these findings indicate that there may be a trade-off between the impact of a merger on innovation and its effect on prices, our simulations strongly suggest that in the absence of synergies and spillovers, a merger is likely to reduce consumer surplus.

We also show how our approach can be extended to account for spillovers and both production and R&D synergies. Interestingly, production synergies matter not only for the effect of the merger on prices, but also for its effect on innovation.

The key role played by the comparison between the price diversion and innovation diversion ratios has interesting policy implications. If the innovation diversion ratio is larger, a merger is likely to negatively impact (demand-enhancing) innovation, thus exacerbating the standard adverse effect of a merger on prices. This situation calls for even larger merger-specific efficiencies (relative to the case of fixed innovation) for the merger to be consumer-friendly. Conversely, if the price diversion ratio is larger, a level of (production) efficiency gains that removes concerns about an adverse effect of the merger on prices (holding innovation fixed) also addresses concerns about adverse effects of the merger on innovation, as shown in our analysis of P-neutral mergers. Therefore, a practical implication for competition authorities is that an analysis of the unilateral effects of the merger on prices showing an adverse effect (for a given innovation level) should be sufficient to block a merger if the innovation diversion ratio is larger than the price diversion ratio. Similarly, an analysis of the unilateral effects of the merger on prices showing no adverse effects on prices (for a given innovation level) should be sufficient to clear a merger if the price diversion ratio is larger than the innovation diversion ratio.

While many competition authorities have developed expertise in measuring and using price diversion ratios for merger control, the innovation diversion ratio is a recent tool for which expertise is still to be developed. Recent empirical research can provide guidance on the practical measurement of the innovation diversion ratio. In particular, Conlon and Mortimer (2021) have developed a methodology for a class of discrete choice models to empirically evaluate diversion ratios for prices, as well as for non-price variables such as quality. More work along these lines is needed.

Appendix

Proof of Proposition 1

From (5), we have $D_i^*(\gamma^*) r_i^*(\gamma^*) = \Phi'(\gamma^*)$. Moreover, using (6) and (7), it follows that $\frac{d\Pi^M}{d\gamma}(\gamma,\gamma) = 2\left[D_i^M(\gamma)\rho_i^M(\gamma) - \Phi'(\gamma)\right]$. Assumption 3 implies that $\gamma^M > \gamma^*$ if and only if $\frac{d\Pi^M}{d\gamma}(\gamma^*,\gamma^*) > 0$, which yields the result.

Proof of Proposition 2

From Proposition 1, the impact of the merger on innovation is given by the sign of

$$D_i^M(\gamma^*)\rho_i^M(\gamma^*) - D_i^*(\gamma^*) r_i^*(\gamma^*) = \underbrace{D_i^M(\gamma^*) r_i^M(\gamma^*) - D_i^*(\gamma^*) r_i^*(\gamma^*)}_{=H_P} + D_i^M(\gamma^*)\left[\rho_i^M(\gamma^*) - r_i^M(\gamma^*)\right].$$

Moreover, we have

$$\rho_i^M(\gamma^*) - r_i^M(\gamma^*) = -\frac{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i} + \frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial p_i} + \frac{\partial D_j^M(\gamma^*)}{\partial p_i}} + \frac{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial p_i}}.$$

If $\frac{\partial D_i}{\partial \gamma_i} = \frac{\partial D_j}{\partial \gamma_i} = 0$ then $\rho_i^M(\gamma^*) - r_i^M(\gamma^*) = 0$. If $\frac{\partial D_i}{\partial \gamma_i} \neq 0$ then

$$\rho_i^M(\gamma^*) - r_i^M(\gamma^*) == -\underbrace{\frac{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial p_i} + \frac{\partial D_j^M(\gamma^*)}{\partial p_i}}}_{=\frac{\psi^M(\gamma^*)}{D_i^M(\gamma^*)}} \times \left[\frac{\frac{\partial D_j^M(\gamma^*)}{\partial \gamma_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial \gamma_i}} - \frac{\frac{\partial D_j^M(\gamma^*)}{\partial p_i}}{\frac{\partial D_i^M(\gamma^*)}{\partial p_i}}\right],$$

which completes the proof.

Proof of Corollary 3

For the class of models with constant expenditures, we find that the price diversion ratio is lower than the innovation diversion ratio at symmetric prices and innovation levels:

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = \frac{p\eta\frac{\partial \eta}{\partial p} + \eta^2}{(p\eta + K)\frac{\partial \eta}{\partial p} - \eta^2} < \frac{p\eta}{p\eta + K} = \frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}},$$

where the inequality follows from the fact that $\partial\eta/\partial p < 0$. Therefore, we have $H_E < 0$. We can then apply Corollary 2: if $h(p,\gamma^*)$ is decreasing, the merger reduces innovation.

Denoting $\eta_1 = \partial\eta/\partial p$ and $\eta_2 = \partial\eta/\partial\gamma$, we find that

$$\begin{aligned} h(p,\gamma) &= \frac{1}{2p\eta+K}\frac{(p\eta+K)\,\eta\eta_2}{\eta^2-(p\eta+K)\,\eta_1} \\ &= \frac{1}{2p\eta+K}\frac{(p\eta+K)\,p\eta}{p\eta-(p\eta+K)\left(-\frac{p\eta_1}{\eta}\right)}\left(\frac{\eta_2}{\eta}\right). \end{aligned}$$

Let us define $x \equiv p\eta$ and $\beta \equiv -p\eta_1/\eta$, where $\beta > 1$ as $\eta + p\partial\eta/\partial p < 0$. We have

$$\frac{\partial}{\partial x}\frac{1}{2x+K}\frac{(x+K)\,x}{(x+K)\,\beta+x} = \frac{K\left[(x+K)^2\beta - x^2\right]}{(2x+K)^2\,(x+\beta(x+K))^2} > 0.$$

The variations of $h(p,\gamma)$ with respect to p are then given by:

$$\frac{dh(p,\gamma)}{dp} = \underbrace{\frac{\partial h}{\partial x}}_{(+)}\underbrace{\frac{\partial x}{\partial p}}_{(-)} + \underbrace{\frac{\partial h}{\partial \beta}}_{(-)}\underbrace{\frac{\partial \beta}{\partial p}}_{(+)\text{ or }(-)} + \underbrace{\frac{\partial h}{\partial(\eta_2/\eta)}}_{(+)}\underbrace{\frac{\partial(\eta_2/\eta)}{\partial p}}_{(+)\text{ or }(-)}.$$

Therefore, $h(p,\gamma)$ is decreasing in p if $\frac{p\eta_1}{\eta}$ and $\frac{\eta_2}{\eta}$ are non-increasing in p.

For example, consider the CES demand with $\eta(p,\gamma) = \gamma^\alpha p^\beta$, $\alpha > 0$ and $\beta < 0$. We have $\frac{p}{\eta}\frac{\partial\eta}{\partial p} = \beta$ and $\frac{\gamma}{\eta}\frac{\partial\eta}{\partial\gamma} = \alpha$, which yields the result.

Proof of Corollary 4

In models based on a price-innovation index $\eta(p_i,\gamma_i)$, with $\partial\eta/\partial p_i > 0$ and $\partial\eta/\partial\gamma_i < 0$, the demand of firm i is given by

$$D_i\left(p_i,p_j,\gamma_i,\gamma_j\right) = Q\left(\eta(p_i,\gamma_i),\eta(p_j,\gamma_j)\right).$$

Let Q_1 and Q_2 denote the derivatives of Q with respect to its first and second arguments, respectively, and assume that $Q_1 < 0$ and $Q_2 > 0$. The innovation diversion ratio and the price diversion ratio are both equal to $-Q_2/Q_1$ at symmetric prices and innovation levels, so we have $H_E = 0$.

Now, consider the MNL model, where demand is given by:

$$D_i(p_i,p_j,\gamma_i,\gamma_j) = \frac{\exp u\,(\gamma_i, y-p_i)}{\exp u\,(\gamma_i,y-p_i)+\exp u\left(\gamma_j,y-p_j\right)+\exp u\,(0,y)}.$$

We have

$$r_i(p,p,\gamma,\gamma) = \frac{u_1(\gamma, y-p)}{u_2(\gamma, y-p)},$$

and therefore,

$$h(p,\gamma) = \frac{u_1(\gamma, y-p)\exp u(\gamma, y-p)}{2u_2(\gamma, y-p)\exp u(\gamma, y-p) + u_2(\gamma, y-p)\exp u(0,y)}.$$

The derivative of $h(p,\gamma^*)$ with respect to p has the same sign as

$$[2(u_1u_{22} - u_{12}u_2)(\gamma^*, y-p)]\exp u(\gamma^*, y-p) + \left[(u_1u_{22} - u_{12}u_2 - u_1u_2^2)(\gamma^*, y-p)\right]\exp u(0,y).$$

If $u_1u_{22} - u_{12}u_2 \leq 0$, or equivalently, $-u_{12}/u_1 + u_{22}/u_2 < 0$, then the first term into brackets is nonpositive and the second term into brackets is negative, so $h(p,\gamma^*)$ decreases with p. In this case, the merger reduces the incentives to innovate. For instance, the condition $-u_{12}/u_1 + u_{22}/u_2 < 0$ holds for the Cobb-Douglas utility function $u(\gamma, y-p) = \gamma^\alpha(y-p)^\beta$, with $\alpha, \beta \in (0,1)$, since $u_{12} = \alpha\beta\gamma^{\alpha-1}(y-p)^{\beta-1} > 0$ and $u_{22} = \beta(\beta-1)\gamma^\alpha(y-p)^{\beta-2} < 0$.

If $-u_{12}u_2 - u_1u_2^2 + u_1u_{22} \geq 0$, or equivalently, $-u_{12}/u_1 + u_{22}/u_2 \geq u_2$, then the first term into brackets is positive and the second term into brackets is nonnegative, so $h(p,\gamma^*)$ increases with p. In this case, the merger raises the incentives to innovate.

In the special case where utility is given by $u(\gamma, y-p) = v(\gamma) + f(\gamma)(y-p)$, the sufficient condition under which the merger reduces incentives to innovate is equivalent to

$$\frac{f'(\gamma^*)}{v'(\gamma^*) + f'(\gamma^*)(y-p)} > 0,$$

and, therefore, it holds whenever $f(\gamma)$ is increasing.

The sufficient condition under which the merger raises incentives to innovate is equivalent to

$$\frac{-f(\gamma^*)[v'(\gamma^*) + f'(\gamma^*)(y-p)] - f'(\gamma^*)}{v'(\gamma^*) + f'(\gamma^*)(y-p)} > 0,$$

for all $p \in \left[p^*, \tilde{p}^M(\gamma^*)\right]$, which holds if and only if

$$-f(\gamma^*)[v'(\gamma^*) + f'(\gamma^*)(y-p)] - f'(\gamma^*) > 0$$

for all $p \in \left[p^*, \tilde{p}^M(\gamma^*)\right]$. For this inequality to hold, it is necessary that $f'(\gamma^*) < 0$. Using this, a sufficient condition for the inequality to hold is that $-f(\gamma^*)[v'(\gamma^*) + f'(\gamma^*)y] -$

$f'(\gamma^*) > 0$, which we can write as

$$\frac{-f'(\gamma^*)}{f(\gamma^*)} > v'(\gamma^*) + yf'(\gamma^*).$$

Proof of Corollary 5

In the model with quality-augmented linear demand, the price diversion ratio and the innovation diversion ratio, at symmetric prices and innovation levels, are such that:

$$\frac{\frac{\partial D_j}{\partial p_i}}{-\frac{\partial D_i}{\partial p_i}} = \frac{\rho}{2} > \frac{\rho}{4-\rho} = \frac{-\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}}.$$

So, we have $H_E > 0$. Besides, we find that

$$h(p,\gamma) = \frac{(4-\rho)\gamma(1-p)^2}{2(2+\rho)}$$

is decreasing in p, so $H_P < 0$. Since $H_E > 0$, while $H_P < 0$, the sign of $H_E + H_P$ is ambiguous. So, we apply Proposition 1 and directly compare $D_i^M(\gamma^*)\rho_i^M(\gamma^*)$ and $D_i^*(\gamma^*)r_i^*(\gamma^*)$. We find that

$$D_i^*(\gamma)r_i^*(\gamma) = \frac{2}{4-\rho}\frac{\gamma(1-c)^2}{2+\rho} > D_i^M(\gamma)\rho_i^M(\gamma) = \frac{1}{2}\frac{\gamma(1-c)^2}{2+\rho}.$$

Therefore, the merger reduces innovation.

Proof of Corollary 6

The demand for firm i is given by

$$D_i(p_i, p_j, \gamma_i, \gamma_j) = \frac{(\alpha + \tau\gamma_i) - (\alpha + \tau\gamma_j)\rho(\gamma_1, \gamma_2) - p_i + \rho(\gamma_1, \gamma_2)p_j}{1 - \rho(\gamma_1, \gamma_2)^2}. \tag{14}$$

The condition $\partial D_j^M(\gamma^*)/\partial \gamma_i < 0$ holds if the parameters of the model are such that

$$\frac{(a(\gamma^*) - c)(1 - \rho(\gamma^*))}{2\rho(\gamma^*)(1 + \rho(\gamma^*))} < \frac{\tau}{\delta}. \tag{15}$$

Under this specification, the innovation diversion ratio is lower than the price diversion ratio at symmetric prices and innovation levels, as:

$$-\frac{\frac{\partial D_j^M}{\partial \gamma_i}}{\frac{\partial D_i^M}{\partial \gamma_i}} = \rho - \frac{(a-\tilde{p}^M)\delta(1-\rho^2)}{(a-\tilde{p}^M)\delta(1-\rho)+\tau(1+\rho)} < -\frac{\frac{\partial D_j^M}{\partial p_i}}{\frac{\partial D_i^M}{\partial p_i}} = \rho,$$

so $H_E > 0$. Besides, $h(p,\gamma^*)$ is decreasing in p, since

$$\frac{\partial h(p,\gamma^*)}{\partial p} = -\frac{2(a-p)\delta(1-\rho)+\tau(1+\rho)}{(1+\rho)^2} < 0.$$

Therefore, $H_P < 0$.

We find that (we drop the argument of $\rho(\gamma^*)$ to simplify the exposition)

$$H_P + H_E = \frac{a(\gamma^*)-c}{2(1+\rho)^2(2-\rho)^2}\left[(a(\gamma^*)-c)\delta\left[2-\rho(2-\rho)\right]-\rho(1+\rho)(2-\rho)\tau\right],$$

which is positive if and only if

$$(a(\gamma^*)-c)\frac{2-\rho(\gamma^*)(2-\rho(\gamma^*))}{\rho(\gamma^*)(1+\rho(\gamma^*))(2-\rho(\gamma^*))} > \frac{\tau}{\delta}. \tag{16}$$

The merger increases innovation if this condition holds; otherwise, it reduces innovation.

The function $(a(\gamma)-c)\frac{2-\rho(\gamma)(2-\rho(\gamma))}{\rho(\gamma)(1+\rho(\gamma))(2-\rho(\gamma))}$ is increasing in γ because (i) $a(\gamma)$ is increasing, and (ii) $\rho(\gamma)$ is decreasing, while $\frac{2-\rho(2-\rho)}{\rho(1+\rho)(2-\rho)}$ is decreasing in ρ. To see why the latter holds, note that:

$$\frac{d}{d\rho}\frac{2-\rho(2-\rho)}{\rho(1+\rho)(2-\rho)} = \frac{-4\rho+10\rho^2-4\rho^3+\rho^4-4}{\rho^2(\rho+1)^2(2-\rho)^2},$$

where the numerator is negative, as can be verified with a simple plot.

Hence, condition (16) holds regardless of the value of γ^* if it holds for the value $\gamma = 0$, which gives condition (9) in the corollary.

If condition (9) does not hold, the merger reduces the incentive to innovate if γ^* is small enough, which is the case if η is above some threshold.[37]

[37] We have focused on the case where the demand is negatively affected by the rival's innovation ($\partial D_j/\partial\gamma_i < 0$), which is the case if condition (15) holds. First, suppose that condition (9) holds. Since the LHS of (15) goes to zero when γ^* goes to 0, for any τ and δ, we can find an R&D cost high enough for condition (15) to hold. If condition (9) does not hold, condition (15) and condition (16) can hold

Proof of Corollary 7

Consider a P-neutral merger. From Definition 1, we have $\tilde{p}^M(\gamma^*, \sigma) = p^*$. Therefore, we can rewrite ΔCS as

$$\begin{aligned}\Delta CS &= \left[CS(\tilde{p}^M(\gamma^M, \sigma), \gamma^M) - CS(\tilde{p}^M(\gamma^*, \sigma), \gamma^*)\right] + \left[CS(\tilde{p}^M(\gamma^*, \sigma), \gamma^*) - CS(p^*, \gamma^*)\right] \\ &= CS(\tilde{p}^M(\gamma^M, \sigma), \gamma^M) - CS(\tilde{p}^M(\gamma^*, \sigma), \gamma^*).\end{aligned}$$

Under Assumption 6, $CS(\tilde{p}^M(\gamma, \sigma), \gamma)$ is increasing in γ, which implies that $\Delta CS > 0$ if and only if $\gamma^M > \gamma^*$, that is, if and only if the price diversion ratio is greater than the innovation ratio (from Proposition 4).

Proof of Proposition 7

The proposition follows immediately from the following lemma.

Lemma 2 *The impact of a P-neutral merger on the merging firms' innovation level has the same sign as* $D_i^M(\gamma^*, \Sigma_o^*)\, \rho_i^M(\gamma^*, \Sigma_o^*) - D_i^*(\gamma^*, \Sigma_o^*)\, r_i^*(\gamma^*, \Sigma_o^*)$.

Proof. By Assumptions 2' and 3', γ^M is the unique solution to

$$D_i^M\left(\gamma, R_o\left(\hat{p}^M(\gamma), \gamma\right)\right) \rho_i^M\left(\gamma, R_o\left(\hat{p}^M(\gamma), \gamma\right)\right) = \Phi'(\gamma).$$

Consider a P-neutral merger. Then $\hat{p}^M(\gamma^*) = p^*$ and $R_o\left(\hat{p}^M(\gamma^*), \gamma^*\right) = \Sigma_o^*$. The function $D_i^M\left(\gamma, R_o\left(\hat{p}^M(\gamma), \gamma\right)\right) \rho_i^M\left(\gamma, R_o\left(\hat{p}^M(\gamma), \gamma\right)\right) - \Phi'(\gamma)$ is continuous by Assumption 2', positive at $\gamma = 0$ and negative at $\gamma = \bar{\gamma}$ by Assumption 2'. This, combined with the uniqueness of γ^M, implies that the function $D_i^M\left(\gamma, R_o\left(\hat{p}^M(\gamma), \gamma\right)\right) \rho_i^M\left(\gamma, R_o\left(\hat{p}^M(\gamma), \gamma\right)\right) - \Phi'(\gamma)$ crosses the horizontal axis before γ^* if and only if it is negative at γ^*, that is (using $D_i^*(\gamma^*, \Sigma_o^*)\, r_i^*(\gamma^*, \Sigma_0^*) = \Phi'(\gamma^*)$) if and only if $D_i^M(\gamma^*, \Sigma_o^*)\, \rho_i^M(\gamma^*, , \Sigma_o^*) - D_i^*(\gamma^*, \Sigma_o^*)\, r_i^*(\gamma^*, \Sigma_o^*) < 0$. The latter holds if and only if the innovation diversion ratio is lower than the price diversion ratio (with both ratios evaluated at $(p^*, p^*, \gamma^*, \gamma^*, \Sigma_o^*)$). ■

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simultaneously because we have

$$\frac{2 - \rho(\gamma^*)(2 - \rho(\gamma^*))}{\rho(\gamma^*)(1 + \rho(\gamma^*))(2 - \rho(\gamma^*))} > \frac{1 - \rho(\gamma^*)}{2\rho(\gamma^*)(1 + \rho(\gamma^*))}.$$

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Online Appendix

Impact of the merger on consumer surplus

In this section, we study the impact of a merger on consumer surplus for two specific demand models: (i) the augmented Singh and Vives model, and (ii) the MNL model. Since we cannot obtain analytical results, we perform simulations assuming throughout that $C(\gamma, Q) = cQ$ for all γ and Q.

Augmented Singh and Vives model

Consumer surplus. In the Augmented Singh and Vives model, consumer surplus is given by the net surplus of the representative consumer:

$$CS = a_1 q_1 + a_2 q_2 - (q_1^2 + q_2^2)/2 - \rho q_1 q_2 - p_1 q_1 - p_2 q_2.$$

We use our parameterization where $a_i(\gamma_i) = a(\gamma_i) = \alpha + \tau\gamma_i$ and $\rho(\gamma_1, \gamma_2) = 1 - \delta(\gamma_1 + \gamma_2)$. After replacing the quantity q_i for the demand given by (14), we obtain the following expression of consumer surplus for a symmetric outcome where $p_1 = p_2 = p$ and $\gamma_1 = \gamma_2 = \gamma$, (to simplify the exposition, we write $\rho(\gamma) = 1 - 2\delta\gamma$ for the symmetric outcomes with a little abuse of notation):

$$CS(p, \gamma) = \frac{(a(\gamma) - p)^2}{1 + \rho(\gamma)}.$$

When firms are independent, at the symmetric equilibrium in prices for a given innovation level γ, consumer surplus is given by

$$CS^*(\gamma) = \frac{(a(\gamma) - c)^2}{(1 + \rho(\gamma))(2 - \rho(\gamma))^2}.$$

After the merger, at the symmetric equilibrium prices for a given γ, consumer surplus becomes

$$CS^M(\gamma) = \frac{(a(\gamma) - c)^2}{4(1 + \rho(\gamma))}.$$

Simulations. We assume a quadratic cost of innovation: $\Phi(\gamma) = (k/2)\gamma^2$. The first-order conditions for an innovation level γ cannot be solved analytically (e.g., it is a polynomial of degree 5 in γ when firms are independent). Therefore, we resort to simulations. At the equilibrium, we check that the second order conditions for profit maximization are satisfied, that $\rho(\gamma^*) = 1 - 2\delta\gamma^* > 0$ and that $H_I < 0$.

Simulations show that (i) prices increase with the merger; (ii) innovation increases with the merger if k is sufficiently low; and (iii) consumer surplus decreases.

For example, suppose that $\alpha = 1$, $c = 0$, and $\delta = 1$. If $\tau = 0.4$, then Condition (9) in Corollary 5 holds, so that $\gamma^M > \gamma^*$ for all k, while the merger leads to higher prices ($p^M > p^*$). Nevertheless, we find that $CS^M < CS^*$ for all $k \in [2, 6]$. If $\tau = 0.8$, Condition (9) in Corollary 5 does not hold, and we find that $\gamma^M > \gamma^*$ if $k < 3.15$. However, for all $k \in [2.5, 6]$, we have $CS^M < CS^*$. We obtain similar results for other parameter constellations. For example, we find that $CS^M < CS^*$ if $k = 4$ and $\{\tau, \delta\} \in [0.2, 1] \times [0.2, 1]$. This is also true if $\delta = 1$ and $\{\tau, k\} \in [0.2, 1] \times [2.5, 10]$.

Multinomial-logit model

Consumer surplus. In the multinomial logit model, consumer surplus is given by

$$\begin{aligned}\mathbb{E}[CS] &= \frac{1}{\alpha}\mathbb{E}\left[\max_j \left(u\left(\gamma_j, y - p_j\right) + \epsilon_j\right)\right] \\ &= \frac{1}{\alpha}\log\left(\sum_j \exp u\left(\gamma_j, y - p_j\right)\right) + C,\end{aligned}$$

where α represents the marginal utility of income and C is an integration constant that we can ignore for our comparison.

In our setting, we have (we drop the expectations to simplify the exposition):

$$CS^* = \frac{\log\left(\exp u(0, y) + 2\exp u(\gamma^*, y - p^*)\right)}{1 + \gamma^*} \quad \text{and} \quad CS^M = \frac{\log\left(\exp u(0, y) + 2\exp u(\gamma^M, y - p^M)\right)}{1 + \gamma^M}.$$

Simulations. As above, we adopt a quadratic cost of innovation, $\Phi(\gamma) = (k/2)\gamma^2$, and consider the following specification:

$$u(\gamma, y - p) = v_0(1 + \gamma) + e^{-\tau\gamma}(y - p).$$

We assume that $v_0 > \tau y$, which ensures that $u_1(\gamma, y - p) > 0$ for all values of γ and p. Moreover, we have $u_1 u_{22} - u_{12} u_2 > 0$. We compute the equilibrium before and after the merger. To do so, first, we numerically compute the equilibrium prices for a given level of innovation, $\tilde{p}^*(\gamma)$ and $\tilde{p}^M(\gamma)$. Replacing for these prices in the first-order conditions for the levels of innovation, we then compute the equilibrium levels of innovation γ^* and γ^M. We check that the second-order conditions hold in equilibrium.

Simulations show that prices increase with the merger, while innovation can either increase or decrease. For example, consider the following parameter values: $v_0 = 1$, $y = 2$, $\tau = 0.5$, and $c = 0$. We find that $p^* < p^M$ for $k \in [0.5, 2]$, and that $\gamma^* > \gamma^M$ for $k \in [0.5, 0.75]$ and $\gamma^* < \gamma^M$ for $k \in (0.75, 2]$. Thus, the merger leads to higher prices, but it can increase innovation. However, in this numerical example, consumer surplus decreases with the merger ($CS^* > CS^M$ for $k \in [0.5, 2]$). We obtain similar results for other parameter constellations.

Further extensions

In this section, we extend our analysis to settings with observable investment in R&D and asymmetric demand and cost functions. For conciseness, we assume throughout that $C(\gamma, Q) = cQ$ for all γ and Q.

Observable investments

In the baseline model, we assume that firms make their price and innovation decisions simultaneously or equivalently, that a firm cannot observe its rival's investment before setting its price. We now assume that a firm's investment in R&D is observed by its rival before prices are set. At given investment levels, the profit-maximizing price for an independent firm i, $\tilde{p}_i^*(\gamma_i, \gamma_j)$, is the solution to the following first-order condition:

$$(p_i - c)\frac{\partial D_i}{\partial p_i}(p_i, p_j, \gamma_i, \gamma_j) + D_i(p_i, p_j, \gamma_i, \gamma_j) = 0.$$

With observable investments, the first-order condition with respect to γ_i becomes

$$(p_i - c)\left(\frac{\partial D_i}{\partial \gamma_i} + \frac{\partial \tilde{p}_j^*}{\partial \gamma_i}\frac{\partial D_i}{\partial p_j}\right) = \Phi'(\gamma_i), \tag{17}$$

where $\partial D_i/\partial p_j$ is evaluated at $\left(\tilde{p}_i^*(\gamma_i, \gamma_j), \tilde{p}_j^*(\gamma_i, \gamma_j), \gamma_i, \gamma_j\right)$ and $\partial \tilde{p}_j^*/\partial \gamma_i$ is evaluated at (γ_i, γ_j). Therefore, firm i takes into account not only the direct effect of its investment on its profit but also the strategic effect that operates through firm j's pricing reaction. The first-order conditions associated with the merged entity's maximization program remain the same as before. Therefore, the decomposition in our baseline setting remains valid as long as we replace the partial derivative $\partial D_i/\partial \gamma_i$ with $\partial D_i/\partial \gamma_i + \partial \tilde{p}_j^*/\partial \gamma_i \times \partial D_i/\partial p_j$ in the independent firm's marginal gain from innovation. The change in incentives to innovate due to the merger has the same sign as:

$$H_P + H_E + H_O,$$

where

$$H_O = -D_i(\gamma^*) r_i^*(\gamma^*) \frac{\partial D_i}{\partial p_j} \frac{\partial \tilde{p}_j^*}{\partial \gamma_i}.$$

The sign of the additional term H_O is the opposite of the sign of the strategic effect on the rival's price, $\partial \tilde{p}_j^* / \partial \gamma_i$. It seems natural to assume that when firm i invests more in innovation, firm j reacts by setting a lower price. We can show that $\partial \tilde{p}_j^* / \partial \gamma_i \leq 0$ under the following conditions:[38]

$$\frac{\partial^2 D_i}{\partial p_i^2} \leq 0, \frac{\partial^2 D_i}{\partial p_i^2} + \frac{\partial^2 D_i}{\partial p_i \partial p_j} \geq 0, \text{ and } \frac{\partial^2 D_i}{\partial p_i \partial \gamma_i} + \frac{\partial^2 D_i}{\partial p_i \partial \gamma_j} \geq 0.$$

In this case, the last term of the decomposition, H_O, is positive. When investment is observable, a merger allows firms to internalize the strategic effect of their investments on profits, which tends to stimulate innovation.

Asymmetric demand and cost functions

We now extend our analysis to a setting in which the demand functions D_i and the innovation cost functions Φ_i are potentially asymmetric. We maintain the assumptions of the baseline model on the cost of innovation Φ_i of firm $i = 1, 2$, and Assumption 1 on demand. We allow firms to have different marginal costs, c_i for $i = 1, 2$.

Consider first the scenario in which the two firms are independent. Assume that the pricing game derived from the price-innovation game by fixing the innovation levels of firms 1 and 2 to γ_1 and γ_2, respectively, has a unique equilibrium. The corresponding equilibrium price pair $(\tilde{p}_1^*(\gamma_1, \gamma_2), \tilde{p}_2^*(\gamma_1, \gamma_2))$ is the solution to the following system of first-order conditions:

$$\begin{cases} (p_1 - c_1) \frac{\partial D_1}{\partial p_1} + D_1 = 0 \\ (p_2 - c_2) \frac{\partial D_2}{\partial p_2} + D_2 = 0. \end{cases} \tag{18}$$

Likewise, the system of first-order conditions for the equilibrium pair of innovation levels of firms 1 and 2 in the price-innovation game is:

$$\begin{cases} (p_1 - c_1) \frac{\partial D_1}{\partial \gamma_1} = \Phi_1'(\gamma_1) \\ (p_2 - c_2) \frac{\partial D_2}{\partial \gamma_2} = \Phi_2'(\gamma_2). \end{cases} \tag{19}$$

[38] The proof is available upon request.

Consider now the post-merger situation. For any given innovation levels γ_1 and γ_2, the merged entity's optimal price pair $\left(\tilde{p}_1^M(\gamma_1,\gamma_2), \tilde{p}_2^M(\gamma_1,\gamma_2)\right)$, assumed to be positive, is defined by the following system of first-order conditions:

$$\begin{cases} (p_1 - c_1)\frac{\partial D_1}{\partial p_1} + (p_2 - c_2)\frac{\partial D_2}{\partial p_1} + D_1 = 0 \\ (p_1 - c_1)\frac{\partial D_1}{\partial p_2} + (p_2 - c_2)\frac{\partial D_2}{\partial p_1} + D_2 = 0. \end{cases}$$

Combining these two equations leads to

$$\begin{cases} p_1 - c_1 = \frac{D_2\frac{\partial D_2}{\partial p_1} - D_1\frac{\partial D_2}{\partial p_2}}{\frac{\partial D_1}{\partial p_1}\frac{\partial D_2}{\partial p_2} - \frac{\partial D_2}{\partial p_1}\frac{\partial D_1}{\partial p_2}} \\ p_2 - c_2 = \frac{D_1\frac{\partial D_1}{\partial p_2} - D_2\frac{\partial D_1}{\partial p_1}}{\frac{\partial D_1}{\partial p_1}\frac{\partial D_2}{\partial p_2} - \frac{\partial D_2}{\partial p_1}\frac{\partial D_1}{\partial p_2}}. \end{cases}$$

We can now state the counterparts to Assumptions 2-3 for the current setting.

Assumption 2": The duopoly price-innovation game has an equilibrium $(p_1^*, p_2^*, \gamma_1^*, \gamma_2^*)$ satisfying first-order conditions (18).

The main role of Assumption 2" is to ensure the existence of an equilibrium. Moreover, it rules out mixed strategy equilibria as well as corner equilibria in the price game. Notice that our assumption that $\Phi_i'(0) = 0$, $i = 1, 2$, implies that the optimal levels of innovation are positive.

Assumption 3": The profit function $\Pi^M(\gamma_1,\gamma_2)$ is continuously differentiable and strictly quasi-concave in (γ_1,γ_2), where $\Pi^M(\gamma_1,\gamma_2)$ is the merged entity's profit for levels of investments γ_1 and γ_2:

$$\begin{aligned} \Pi^M(\gamma_1,\gamma_2) \equiv \; & \max_{p_1,p_2}\{(p_1 - c_1) D_1(p_1,p_2,\gamma_1,\gamma_2) + \\ & (p_2 - c_2) D_2(p_2,p_1,\gamma_2,\gamma_1) - \Phi(\gamma_1) - \Phi(\gamma_2)\}. \end{aligned}$$

Assumption 3" ensures that the merged entity's optimization problem with respect to innovation is well behaved and has a unique solution. Notice, however, that it does not guarantee that the merger entity's optimal innovation levels are positive for both products. In what follows we allow for the scenario in which one of the innovation levels is zero.[39]

[39] Under our assumptions, at least one of the merged entity's optimal innovation levels is positive.

We define the *independent firm's marginal gain from innovation* of firm $i = 1, 2$ as:

$$h_i^*(\gamma_1, \gamma_2) \equiv -D_i \frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i}},$$

where all functions are evaluated at $(\tilde{p}_1^*(\gamma_1, \gamma_2), \tilde{p}_2^*(\gamma_1, \gamma_2), \gamma_1, \gamma_2)$. We also define the *merged entity's marginal gain from innovation* in product $i = 1, 2$ as:

$$l_i^M(\gamma_1, \gamma_2) \equiv \frac{\left(D_j \frac{\partial D_j}{\partial p_i} - D_i \frac{\partial D_j}{\partial p_j}\right) \frac{\partial D_i}{\partial \gamma_i} + \left(D_i \frac{\partial D_i}{\partial p_j} - D_j \frac{\partial D_i}{\partial p_i}\right) \frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i} \frac{\partial D_j}{\partial p_j} - \frac{\partial D_j}{\partial p_i} \frac{\partial D_i}{\partial p_j}},$$

where all functions are evaluated at $\left(\tilde{p}_1^M(\gamma_1, \gamma_2), \tilde{p}_2^M(\gamma_1, \gamma_2), \gamma_1, \gamma_2\right)$.

The merged entity's innovation levels are such that for $i = 1, 2$:

$$l_i^M\left(\gamma_1^M, \gamma_2^M\right) \leq \Phi_i'\left(\gamma_i^M\right), \text{ with equality if } l_i^M\left(\gamma_1^M, \gamma_2^M\right) > 0.$$

In order to apply the methodology developed in our baseline model to this setup, we need to make an additional assumption.

Assumption 4": The merged entity's innovation efforts are *strategic complements,* that is, $l_1^M(\gamma_1, \gamma_2)$ is increasing in γ_2 and $l_2^M(\gamma_1, \gamma_2)$ is increasing in γ_1.

This assumption ensures that the optimal level of innovation on product i, denoted $R_i^M(\gamma_j)$ is weakly increasing in the innovation level γ_j.[40] To understand the role of Assumption 4", we can decompose the effect of the merger on the merging firms' incentives to innovate into a direct effect and an indirect one. The direct effect for product i is the effect of the merger on the incentive to innovate on product i holding constant γ_j fixed at the level γ_j^*. The indirect effect is the impact of the change in γ_j on the incentive to innovate on product i and the feedback loop that ensues. Assumption 4" ensures that the direct effect and the indirect effect do not conflict, so that the sign of the overall effect is the same as the sign of the direct effect.[41]

The next proposition shows that under Assumption 4", the comparison of an independent firm's marginal gain from innovation and the merged entity's marginal gain from

[40] Notice that it is possible that $R_i^M(\gamma_j) = 0$ or $R_i^M(\gamma_j) = \bar{\gamma}_i$, where $\bar{\gamma}_i$ is the upper bound on firm i's innovation level.

[41] Note that this can be the case even if the merged entity's innovation efforts are strategic substitutes as long as the indirect effect is dominated by the direct effect.

innovation (as defined above) still determines the impact of the merger on the merging firms' incentives to innovate.

Proposition 8 *A merger reduces innovation in both products if* $l_i^M(\gamma_1^*, \gamma_2^*) < h_i^*(\gamma_1^*, \gamma_2^*)$ *for* $i = 1, 2$*, and a merger boosts innovation in both products if* $l_i^M(\gamma_1^*, \gamma_2^*) > h_i^*(\gamma_1^*, \gamma_2^*)$ *for* $i = 1, 2$.

Proof. First, note that (γ_1^*, γ_2^*) is positive and solution of the following system of equations

$$\begin{cases} h_1^*(\gamma_1, \gamma_2) = \Phi_1'(\gamma_1) \\ h_2^*(\gamma_1, \gamma_2) = \Phi_2'(\gamma_2) \end{cases}, \tag{20}$$

and, whenever interior, $\left(\gamma_1^M, \gamma_2^M\right)$ is the unique solution of:

$$\begin{cases} l_1^M(\gamma_1, \gamma_2) = \Phi_1'(\gamma_1) \\ l_2^M(\gamma_1, \gamma_2) = \Phi_2'(\gamma_2) \end{cases}. \tag{21}$$

Notice that

$$\frac{\partial \Pi^M(\gamma_1, \gamma_2)}{\partial \gamma_i} = l_i^M(\gamma_1, \gamma_2) - \Phi_i'(\gamma_i).$$

Denote $R_1^M(\gamma_2) = \arg\max_\gamma \Pi^M(\gamma, \gamma_2)$. Whenever interior, it is the unique solution of $l_1^M(\gamma_1, \gamma_2) = \Phi_1'(\gamma_1)$ in γ_1. Denote similarly $R_2^M(\gamma_1) = \arg\max_\gamma \Pi^M(\gamma_1, \gamma)$. Again, whenever interior, it is the unique solution of $l_2^M(\gamma_1, \gamma_2) = \Phi_2'(\gamma_2)$ in γ_2.

Thus, we have

$$\gamma_1^* = R_1^*(\gamma_2^*) \quad ; \quad \gamma_2^* = R_2^*(\gamma_1^*)$$

and

$$\gamma_1^M = R_1^M\left(\gamma_2^M\right) \quad ; \quad \gamma_2^M = R_2^M\left(\gamma_1^M\right).$$

Differentiating $h_1^*(R_1^*(\gamma_2), \gamma_2) = \Phi_1'(R_1^*(\gamma_2))$ with respect to γ_2 yields

$$\frac{dR_1^*}{d\gamma_2} = \frac{-\frac{\partial h_1^*}{\partial \gamma_2}(R_1^*(\gamma_2), \gamma_2)}{\frac{\partial h_1^*}{\partial \gamma_2}(R_1^*(\gamma_2), \gamma_2) - \Phi_1''(R_1^*(\gamma_2))},$$

which has the same sign as $\frac{\partial h_1^*}{\partial \gamma_2}(R_1^*(\gamma_2), \gamma_2)$, since the denominator is negative by assumption. Likewise, $\frac{dR_2^*}{d\gamma_1}$ has the same sign as $\frac{\partial h_2^*}{\partial \gamma_1}(\gamma_1, R_2^*(\gamma_1))$. Moreover, whenever R_1^M is positive, $\frac{dR_1^M}{d\gamma_2}$ has the same sign as $\frac{\partial h_2^*}{\partial \gamma_1}(\gamma_1, R_2^*(\gamma_1))$ and whenever R_2^M is positive, $\frac{dR_2^M}{d\gamma_1}$ has the same sign as $\frac{\partial l_2^M}{\partial \gamma_1}\left(\gamma_1, R_2^M(\gamma_1)\right)$.

Assume now that $l_1^M(\gamma_1,\gamma_2)$ is increasing in γ_2 and $l_2^M(\gamma_1,\gamma_2)$ is increasing in γ_1. This implies that $R_1^M(.)$ and $R_2^M(.)$ are non-decreasing. Consider first the scenario in which $l_i^M(\gamma_1^*,\gamma_2^*) < h_i^*(\gamma_1^*,\gamma_2^*)$ for $i=1,2$. In this case, $\gamma_1^* > R_1^M(\gamma_2^*) \geq 0$ and $\gamma_2^* > R_2^M(\gamma_1^*) \geq 0$. To see why the latter inequalities hold, notice that

$$\frac{\partial \Pi^M(\gamma_1^*,\gamma_2^*)}{\partial \gamma_i} = l_i^M(\gamma_1^*,\gamma_2^*) - \Phi_i'(\gamma_1^*) < h_i^*(\gamma_1^*,\gamma_2^*) - \Phi_i'(\gamma_1^*) = 0,$$

which implies by strict quasi-concavity in γ_i that $\gamma_i^* > R_i^M(\gamma_j^*)$.

We have then

$$\begin{aligned}\gamma_2 &\leq R_2^M(\gamma_1^*) \Rightarrow \gamma_2 < \gamma_2^* \Longrightarrow R_1^M(\gamma_2) \leq R_1^M(\gamma_2^*)\\ \gamma_1 &\leq R_1^M(\gamma_2^*) \Rightarrow \gamma_1 < \gamma_1^* \Longrightarrow R_2^M(\gamma_1) \leq R_2^M(\gamma_1^*).\end{aligned}$$

Consider the mapping

$$\begin{aligned}\left[0, R_1^M(\gamma_2^*)\right] \times \left[0, R_2^M(\gamma_1^*)\right] &\longmapsto \left[0, R_1^M(\gamma_2^*)\right] \times \left[0, R_2^M(\gamma_1^*)\right]\\ (\gamma_1,\gamma_2) &\longmapsto \left(R_1^M(\gamma_2), R_2^M(\gamma_1)\right).\end{aligned}$$

This mapping is continuous on a compact support, hence it has a fixed point. Then, Assumption 3" ensures that it represents the merged entity's optimal innovation levels. This implies that $\gamma_1^M \leq R_1^M(\gamma_2^*) < R_1^*(\gamma_2^*) = \gamma_1^*$ and $\gamma_2^M \leq R_2^M(\gamma_1^*) < R_2^*(\gamma_1^*) = \gamma_1^*$.

Consider now the scenario in which $l_i^M(\gamma_1^*,\gamma_2^*) > h_i^*(\gamma_1^*,\gamma_2^*)$ for $i=1,2$. In this case, $\gamma_1^* < R_1^M(\gamma_2^*) \leq \bar{\gamma}_1$ and $\gamma_2^* < R_2^M(\gamma_1^*) \leq \bar{\gamma}_2$. We have then

$$\begin{aligned}\gamma_2 &\geq R_2^M(\gamma_1^*) \Rightarrow R_1^M(\gamma_2) \geq R_1^M(\gamma_2^*)\\ \gamma_1 &\leq R_1^M(\gamma_2^*) \Rightarrow R_2^M(\gamma_1) \geq R_2^M(\gamma_1^*).\end{aligned}$$

Consider the mapping

$$\begin{aligned}\left[R_1^M(\gamma_2^*), \bar{\gamma}_1\right] \times \left[R_2^M(\gamma_1^*), \bar{\gamma}_2\right] &\longmapsto \left[R_1^M(\gamma_2^*), \bar{\gamma}_1\right] \times \left[R_2^M(\gamma_1^*), \bar{\gamma}_2\right]\\ (\gamma_1,\gamma_2) &\longmapsto \left(R_1^M(\gamma_2), R_2^M(\gamma_1)\right).\end{aligned}$$

This is continuous on a compact support, hence it has a fixed point. Then, Assumption 3" ensures that it represents the merged entity's optimal innovation levels and Assumption 1 with $\Phi'(\bar{\gamma}_i) = +\infty$ implies that it less than $\bar{\gamma}_i$. This implies that $\bar{\gamma}_1 > \gamma_1^M \geq R_1^M(\gamma_2^*) > \gamma_1^*$ and $\bar{\gamma}_2 > \gamma_2^M \geq R_2^M(\gamma_1^*) > \gamma_1^*$. ■

As an illustration, a sufficient condition under which the merged entity's innovation efforts are strategic complements in the augmented Singh and Vives model with asymmetric cost is:[42]

$$2\alpha - c_1 - c_2 > \frac{\tau}{\delta}\left(\frac{3\rho - 1}{1-\rho}\right).$$

It is easy to see that this condition is more likely to hold the smaller ρ and the larger α.

Following the same logic as that for symmetric mergers, we can define a P-neutral merger as a merger with synergies σ_1 and σ_2 such that at constant innovation levels γ_1^* and γ_2^*, the merger does not affect prices. In other words, a merger is P-neutral if

$$\left(\tilde{p}_1^M\left(\gamma_1^*, \gamma_2^*\right), \tilde{p}_2^M\left(\gamma_1^*, \gamma_2^*\right)\right) = \left(p_1^*, p_2^*\right).$$

We define price and innovation diversion ratios for each product $i = 1, 2$ as in the baseline model. Our main result on the impact of P-neutral mergers on innovation extends to the asymmetric setting considered here as follows:

Corollary 8 *Assume that the merged entity's innovation efforts are strategic complements. A P-neutral merger reduces (raises) innovation in both products if the price diversion ratio is lower (higher) than the innovation diversion ratio for both products, where both ratios are evaluated at* $(p^*, p^*, \gamma^*, \gamma^*)$.

Proof. We have

$$\begin{aligned} l_i^M\left(\gamma_1^*, \gamma_2^*\right) - h_i^*\left(\gamma_1^*, \gamma_2^*\right) &= \frac{\left(D_j \frac{\partial D_j}{\partial p_i} - D_i \frac{\partial D_j}{\partial p_j}\right)\frac{\partial D_i}{\partial \gamma_i} + \left(D_i \frac{\partial D_i}{\partial p_j} - D_j \frac{\partial D_i}{\partial p_i}\right)\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i}\frac{\partial D_j}{\partial p_j} - \frac{\partial D_j}{\partial p_i}\frac{\partial D_i}{\partial p_j}} + D_i \frac{\frac{\partial D_i}{\partial \gamma_i}}{\frac{\partial D_i}{\partial p_i}} \\ &= \frac{\left(D_i \frac{\partial D_i}{\partial p_j} - D_j \frac{\partial D_i}{\partial p_i}\right)\frac{\partial D_i}{\partial \gamma_i}\left(\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} - \frac{\frac{\partial D_j}{\partial p_i}}{\frac{\partial D_i}{\partial p_i}}\right)}{\frac{\partial D_i}{\partial p_i}\frac{\partial D_j}{\partial p_j} - \frac{\partial D_j}{\partial p_i}\frac{\partial D_i}{\partial p_j}}. \end{aligned}$$

This is negative for both products if

$$\frac{\frac{\partial D_j}{\partial \gamma_i}}{\frac{\partial D_i}{\partial \gamma_i}} - \frac{\frac{\partial D_j}{\partial p_i}}{\frac{\partial D_i}{\partial p_i}} < 0 \text{ for } i, j = 1, 2,\ i \neq j,$$

and is positive for both products if the reverse holds. ■

Thus, the comparison of the innovation diversion ratios with the corresponding price

[42] The proof is available upon request.

diversion ratios still determines the impact of a P-neutral merger on innovation (in both products) as long as the outcome of the comparison is the same for both products.