Refunding Emissions Taxes: The Case For A Three-Part Policy

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Abstract

This paper examines theoretically whether by combining both output based refunding and abatement expenditures based refunding it is possible to limit the negative consequences that a pollution tax imply for a polluting industry. We actually show that this is indeed the case by using such a three-part policy where emissions are subject to a fee and where output and abatement expenditures are subsidized. In particular, when the industry is homogenous, it is possible to replicate the standard emission tax outcome by inducing a polluting firm to choose the production and emission levels obtained under any emission tax, without departing from budget balance. By construction, any polluter earns strictly more than under the standard tax alone without rebate, making this proposal highly acceptable to the industry.

When firms are heterogenous, the refunding policy needed to replicate the standard emission tax outcome is personalized in the sense that at least the output subsidy should be type dependent. Another result is that this three-part policy is strictly prefered only from the industry's point of view to a standard environmental tax. We also explore the implications of uniform three-part refunding policies for a heterogenous industry.

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1 Introduction

Taxing sources of emissions is often difficult to implement in practice despite its theoretical appeal, since Pigou, to solve pollution problems in a cost-effective way. Traditional arguments against emission taxes encompass that they may harm industry's competitiveness, they may hit low income households hard and last but not least reducing domestic emissions may have zero effect on global pollution as increased foreign emissions may follow. As a consequence, it is not surprising that emissions taxes tends to be usually low and this typically undermines the efficiency of environmental policies (OECD, 2015).

It has long been recognized that earmarking the product of taxes to polluters is a way to diminish the economic burden to them and hence to potentially lessen political opposition to emissions taxes (see e.g. Fischer 2001, Gersbach and Requate 2004, Cato 2010). For instance, Sweden is refunding the taxes collected on NOx emissions in proportion of output (Bonilla et al. 2015). In Norway, these emissions payments for Nox are refunded in proportion to (observable) abatement expenditures. In a recent paper, Hagem et al. (2015) compare both schemes and show that refunding necessarily implies a distortion on abatement (and production) decisions at the individual level compared to the situation where only emissions are taxed (at the same rate). More precisely, both output based and abatement expenditures based refunding induce a cost-ineffective provision of abatement as polluters put relatively too much effort into reducing emissions via abatement compared with reducing output. It follows that both schemes are welfare inferior to the optimal standard tax on emissions, because they lead to inferior output reductions.

This paper examines theoretically whether by combining both output based refunding and abatement expenditures based refunding it is possible to reduce these distortions. We actually show that this is indeed the case by using such a three-part policy where emissions are subject to a fee and where output and abatement expenditures are subsidized. In particular, when the industry is homogenous, it is possible to replicate the standard emission tax outcome using such a policy: with the appropriate definition of the fee and of the output and abatement subsidies, the three-part refunding policy induces the polluting firm to choose the production and emission levels obtained under any emission tax, without departing from budget balance. By construction, the proposed regulation scheme entails that the polluter earns strictly more than under the standard tax alone without rebate, thereby making this proposal highly acceptable.

When the industry is heterogenous, the refunding policy needed to replicate the standard emission tax outcome is now personalized in the sense that at least the output subsidy should be type dependent. Another result is that this three-part policy is strictly prefered only from the industry's point of view to a standard environmental tax.

We then explore the implications of uniform three-part refunding policies in this context and find, by using an example, that it is possible for the refunding policy to replicate the standard tax outcome in the aggregate at the sector's level with respect to production and emissions without departing from the budget constraint. Abatement expenditures differ in general from those spent under a standard emission tax without refunding.

The paper is organized as follows. Section 2 lays out assumptions and notations as well as the two benchmarks of a standard emission tax and of output based versus abatement expenditures based refunding policies. In section 3, we consider the three-part refunding policy in a context with homogenous firms. Section 4 explores the case of heterogenous firms. Section 5 concludes.

2 The model

2.1 Assumptions and notations

Consider a competitive industry whose size is fixed to n firms indexed by $i = 1...n$. Product is homogenous and sold at (constant) price p. Production is also polluting and any firm has the possibility to control and reduce it by employing its optimal mix of output reduction and specific input use for abatement. We decompose the total cost of firm i as the sum of first a production cost $c_i(q_i)$ strictly increasing in production q_i and strictly convex and second an abatement cost function $a_i(q_i, e_i)$, strictly increasing in q_i, strictly decreasing in emissions e_i up to some level $e_i^{\circ}(q_i)$ and then increasing. Under *laissez-faire*, i.e. in the absence of pollution regulation, firm i would pollute up to the "selfish" level $e_i^{\circ}(q_i)$ that minimizes abatement costs.

We also assume that $a_i(.,.)$ is strictly quasi-convex, which together with the convexity of c_i, ensures that the profit function $\pi_i(q_i, e_i)$ is strictly quasi-concave. Denoting a_{ix} and a_{ixy} the corresponding first-order partial derivative and the second-order partial derivative respectively for $x, y \in \{e, q\}$, quasi-convexity means that $a_{iee} > 0$, $a_{iqq} > 0$ and $a_{iee}a_{iqq} - a_{iqe}^2 > 0$.

We will also make use of the two following assumptions that would prove useful in obtaining some comparative statics results.

Assumption 1 *The abatement cost function $a_i(.,.)$ is such that $a_{ieq} < 0$.*

Assumption 1 says that the marginal benefit of pollution for the firm, namely $-a_{ie} > 0$, is increasing in q_i. The more the firm produces the higher the cost savings from polluting are.

Assumption 2 *The abatement cost function $a_i(.,.)$ is such that the ratio $\frac{a_{ie}}{a_{iq}}$ is increasing in q_i and in e_i.*

It can be checked that assumption 2 holds whenever $|a_{ieq}|$ is sufficiently small.[1]

2.2 Standard emission tax as a benchmark

Let us first define the benchmark situation of a standard emission tax t without refunding (or when refunding corresponds to a lump sum transfer). When facing the emission tax t, firm i maximizes its (strictly quasi-concave) profit function with respect to production and emissions:

$$\max_{q_i, e_i} \pi_i = pq_i - c_i(q_i) - a_i(q_i, e_i) - te_i$$

and this leads to the following necessary and sufficient optimality conditions (assuming interior solutions):

$$\begin{aligned} p &= c_i'(q_i^*) + a_{iq}(q_i^*, e_i^*) \\ -a_{ie}(q_i^*, e_i^*) &= t. \end{aligned} \tag{1}$$

Firm i's optimal reaction to the emission tax t is governed by marginal cost pricing and equality between marginal abatement cost and marginal pollution price. For further reference, aggregate production is denoted $Q^*(t) = \sum_i q_i^*(t)$ and aggregate pollution is $E^*(t) = \sum_i e_i^*(t)$. Furthermore, aggregate abatement expenditures are denoted $A^*(t) = \sum_i a_i(q_i^*(t), e_i^*(t))$.

In terms of net profit, the regulated firm earns:

$$\pi_i^* = pq_i^* - c_i(q_i^*) - a_i(q_i^*, e_i^*) - te_i^*$$

which is assumed positive for any i and for the values of t considered.

In Appendix A, we show that comparative statics results with respect to price, emission tax and increase in abatement costs scale can be established as follows. First, quasi-concavity of profit implies that production is increasing in output price p whereas

[1]Indeed, dropping the index i for clarity, we note that $\frac{\partial}{\partial q}(\frac{a_e}{a_q}) = \frac{a_q a_{qe} - a_e a_{qq}}{a_q^2}$ and $\frac{\partial}{\partial e}(\frac{a_e}{a_q}) = \frac{a_q a_{ee} - a_e a_{qe}}{a_q^2}$. Hence, it is sufficient that $|a_{qe}|$ is sufficiently close to 0 for assumption 2 to hold.

emissions are decreasing in the tax level t. Second, assumption 1 ensures that $\pi_{iqe} > 0$ or equivalently that the marginal profitability of q_i increases in emissions e_i. This in turn implies that production is decreasing in the tax level t while emissions are increasing in output price. Third, under assumption 2, an increase in the scale of abatement cost $a_i(q_i, e_i)$ leads to decreasing production and increasing emissions.

2.3 Output based versus abatement expenditures based refunding policies

Now consider the possibility for the government to refund emission payments collected back to the firms. We here revisit results obtained by Hagem et al. in a competitive setting with our notations (see also Gersbach and Requate (2004) for an analysis in markets with imperfect competition).

For a given emission tax t, it is interesting to compare a policy that entails refunding through a subsidy τ on output and an alternative policy refunding occurs through a subsidy s on abatement cost to a policy with no refunding of the fee. Whenever there is a subsidy, and whether it is output or abatement based, its level is given by the budget constraint that writes:

$$tE = \tau Q$$

in the output based refunding case and

$$tE = sA$$

in the abatement expenditures refunding case. The total emissions tax collected are returned back to polluters either proportionnally to output or to abatement cost.

Given the above comparative statics results presented above, it is easy to recover the following results due to Hagem et al. (2015).

Proposition 1 (Hagem et al.) *Under assumptions 1 and 2, (i) for a given emission tax t and compared to a policy without refunding, an output based refunding policy (τ, t)*

implies too much pollution and an abatement expenditures based refunding policy (s, t) *implies too few pollution, and (ii) for a given pollution target, the emission tax needed under output based refunding policy is larger than the tax without refunding whereas the tax needed under abatement expenditures based refunding policy is lower.*

Proof: Part (i) directly follows from Appendix A which states that, for a fixed price of pollution, subsidizing the abatement cost decreases pollution while subsidizing production will increase emissions. Similarly, part (ii) holds because Appendix A states that, in order to keep pollution constant, it is necessary to increase the pollution fee if one is going to subsidize production, while it is necessary to decrease the pollution fee if abatement expenditures are to be subsidized. ■

A two-part refunding policy generally involves a distortion on the pollution level for any firm and thus in aggregate, compared to what prevails under an emission tax without refunding. Intuitively, an output subsidy increases the marginal benefit of production which in turn increases the marginal incentives to pollute. Conversely, an abatement subsidy decreases the marginal benefit of pollution which in turn induces a downward distortion on pollution.

The opposite nature of the distortions brought by the two ways to refund emission taxes leads to study whether by combining both subsidies it is possible to reduce these distortions. We will first examine this possibility for a homogenous industry in the next section while the study of heterogenous industries is postponed to section 4.

3 Three-part refunding policies for a homogenous industry

When the industry is composed of identical firms, the equilibrium outcome under a standard emission tax t is described by the system (1) that reduces to:

$$\begin{aligned} p &= c'(q^*) + a_q^* \\ -a_e^* &= t \end{aligned} \tag{2}$$

where we denote, for the sake of exposition, $a^* \equiv a(q^*, e^*)$, $a_q^* \equiv a_q(q^*, e^*)$ and $a_e^* \equiv a_e(q^*, e^*)$.

The question we ask is whether it is possible to replicate this equilibrium outcome for a given tax t by using instead a three-part policy based on a per unit fee f on emissions, an output subsidy τ per unit and an abatement subsidy at rate s, without departing from budget balance. The natural advantage of the three-part policy would then be to leave a higher profit to firms and thus to make the regulation more acceptable to them. It turns out that the answer to this question is positive as we now show. We proceed by first defining the budget constraint of the government and then the optimal reaction of a representative polluter to a three-part refunding policy, before establishing our main result in this section.

We depart from Hagem et al. (2015) by assuming that there is also a revenue requirement R asked by the government from the regulation policy.[2] To avoid an arbitrary choice of R, we assume that

$$R = (1 - \delta)tE^*$$

with $\delta \in (0, 1)$. Hence, the government requires the refunding policy to collect at least a portion $1 - \delta$ of the emissions taxes $tE^*(t)$ that would be collected in the absence of refunding. When $\delta = 1$ there is complete refunding of emissions taxes towards polluters

[2] Gersbach and Requate (2004) and Cato (2010) also consider the possibility of partial refunding.

while for $\delta = 0$, the revenue requirement will actually impose the absence of refunding ($\tau = s = 0$) as it will be clear below. When $0 < \delta < 1$, there is only partial refunding of taxes to polluters and parameter δ represents the rate of refunding. This representation of the budget constraint allows for a richer modelling of possible refunding policies that can be only partial with a portion of taxes being spent purposedly elsewhere in the economy.

To sum up, the budget constraint of the regulation schedule writes as:

$$fE = \tau Q + sA + R \tag{3}$$

where industry's emissions are E, total production is Q and total abatement expenditures A. Hence, all emissions taxes collected net of the revenue requirement R are given back to polluters under the form of output and abatement subsidies and this implicitly defines a relationship between emission tax, production and abatement subsidies.

Facing a three-part policy (τ, s, f), the optimal decisions for a representative firm are

$$(\hat{q}(\tau, s, f), \hat{e}(\tau, s, f)) \in \arg\max_{q,e} \pi(q, e) \equiv (p + \tau)q - c(q) - (1 - s)a(q, e) - fe \tag{4}$$

and the corresponding profit is

$$\hat{\pi}(\tau, s, f) = \pi(\hat{q}(\tau, s, f), \hat{e}(\tau, s, f)$$

Let us denote

$$\hat{a} \equiv a(\hat{q}, \hat{e}) \text{ with } \hat{a}_q = \left.\frac{\partial a}{\partial q}\right|_{q=\hat{q},\ e=\hat{e}} \text{ and } \hat{a}_e = \left.\frac{\partial a}{\partial e}\right|_{q=\hat{q},\ e=\hat{e}}$$

The FOCs are (for interior solutions):

$$\begin{aligned} p + \tau &= c'(\hat{q}) + (1 - s)\hat{a}_q \\ -(1 - s)\hat{a}_e &= f \end{aligned} \tag{5}$$

and second order conditions are (assuming $s < 1$):

$$
\begin{aligned}
\left.\frac{\partial^2 \pi}{\partial q^2}\right|_{q=\hat{q},\ e=\hat{e}} &= -c''(\hat{q}) - (1-s)\hat{a}_{qq} < 0 \\
\left.\frac{\partial^2 \pi}{\partial e^2}\right|_{q=\hat{q},\ e=\hat{e}} &= -(1-s)\hat{a}_{ee} < 0 \\
\left.\frac{\partial^2 \pi}{\partial q^2}\frac{\partial^2 \pi}{\partial e^2} - \left(\frac{\partial^2 \pi}{\partial q \partial e}\right)^2\right|_{q=\hat{q},\ e=\hat{e}} &= (1-s)\left[\hat{a}_{ee}c''(\hat{q}) + (1-s)(\hat{a}_{qq}\hat{a}_{ee} - \hat{a}_{qe}^2)\right] > 0
\end{aligned}
$$

Comparing (2) and (5), we can establish the following result. For this, denote $\varepsilon_{a,e}^* = a_e^* e^*/a^* < 0$ as the elasticity of a w.r.t. e taken at the standard emission tax optimum q^*, e^* while $\varepsilon_{a,q}^* = a_q^* q^*/a^* > 0$ denotes the elasticity of a w.r.t. q taken at the optimum q^*, e^*.

Proposition 2 *For a homogeneous industry of fixed size, there exists a unique three-part policy that entails any firm to choose $\hat{e} = e^*$ and $\hat{q} = q^*$ while sustaining the budget constraint (provided firm's problem concavity is preserved). It is characterized by*

$$
\begin{aligned}
f &= (1-s)t \\
\tau &= -sa_q^* \\
s &= -\frac{\delta \varepsilon_{a,e}^*}{\sigma^*}
\end{aligned}
$$

where

$$\sigma^* = 1 - \varepsilon_{a,e}^* - \varepsilon_{a,q}^*.$$

Moreover, any firm earns $\hat{\pi}(\tau, s, f) = \pi^* + \delta t e^* > \pi^*$ so that the three-part policy is strictly prefered by all firms to a standard emission tax.

Proof: See Appendix B. ■

The intuition is clear: with the three instruments contained in the three-part policy, one can mimic the standard emission tax outcome in terms of production and pollution without sacrifying the budget balance. Moreover, due to the (at least partial) refunding of taxes, the three-part policy (τ, s, f) is stricly prefered to the standard emission tax by all firms. When the government requires $\delta = 0$, the three-part policy boils down to the no-refunding policy with $s = \tau = 0$ and $f = t$.

To interpret the policy exhibited, note that τ and s necessarily have opposite signs (as $a_q > 0$). Note also that the sign of s depends on the sign of $\sigma^* = 1 - \varepsilon^*_{a,e} - \varepsilon^*_{a,q}$. Hence, we get that abatement should be taxed ($s < 0$) iff $\sigma^* < 0$ or equivalently $\varepsilon^*_{a,q} > 1 - \varepsilon^*_{a,e} (> 0)$, and conversely abatement should be subsidized iff $0 \leq \varepsilon^*_{a,q} < 1 - \varepsilon^*_{a,e}$. To exclude the possibility of having $s > 1$ in order to preserve concavity, we exclude the situation where $\varepsilon^*_{a,q} \in (0, 1 - (1-\delta)\varepsilon^*_{a,e})$. Hence, to sum up, we have $s \in (0, 1)$ when $\varepsilon^*_{a,q} \in (1 - (1-\delta)\varepsilon^*_{a,e}, 1 - \varepsilon^*_{a,e})$ and $s < 0$ when $\varepsilon^*_{a,q} > 1 - \varepsilon^*_{a,e}$.

We deduce from this the following Corollary.

Corollary 1 *For $\delta \in (0, 1)$, the three-part policy is characterized as follows:*

(i) *if $\varepsilon^*_{a,q} \in (1 - (1-\delta)\varepsilon^*_{a,e}, 1 - \varepsilon^*_{a,e})$, then there is under-taxation of emissions ($f < t$), a subsidy on abatement ($s > 0$) and a tax on production ($\tau < 0$)*

(ii) *if $\varepsilon^*_{a,q} > 1 - \varepsilon^*_{a,e}$, then there is over-taxation of emissions ($f > t$), a tax on abatement ($s < 0$) and a subsidy on production ($\tau > 0$).*

To illustrate, consider the following specification of the model: Assume that all firms share the following cost and abatement functions, $c(q) = cq^2/2$ and $a(q, e) = k(q - e/\gamma)^2/2$. This formulation entails the possibility of inverted U shaped emissions curves as positive parameter γ decreases and consequently when production grows. Hence, emissions are low when γ is small or large while they are higher for intermediary level of γ. Straightforward computations lead to:

$$\begin{aligned}
q^* &= \frac{p - t\gamma}{c} > 0 \text{ if } t < p/\gamma \\
e^* &= \frac{p\gamma}{c} - t\gamma^2 \left(\frac{k+c}{kc}\right) > 0 \text{ if } t < \frac{k}{k+c}\frac{p}{\gamma} \\
q^* - e^*/\gamma &= \frac{t\gamma}{k} > 0 \\
\sigma^* &= -1 \\
\varepsilon^*_{a,e} &= \frac{a^*_e e}{a^*} = -\frac{2}{\gamma}\frac{e}{(q - e/\gamma)} = -\frac{2k}{t\gamma}\left(\frac{p - t\gamma}{c} - \frac{t\gamma}{k}\right) \underset{t \to 0^+}{\to} -\infty
\end{aligned}$$

and pollution is first increasing and then decreasing in γ (or equivalently in q). The maximum of pollution is in $1/\gamma = 2t(k + c)/(pk)$. Abatement effort $q^* - e^*/\gamma$ is increasing in γ as long as $e^* > 0$. We thus assume that the condition $t < \frac{k}{k+c}\frac{p}{\gamma}$ holds so that pollution remains strictly positive.

Applying straightforwardly Proposition 2, we obtain the following result.

Proposition 3 *Assume that all firms share the following cost and abatement functions, $c(q) = cq^2/2$ and $a(q, e) = k(q - e/\gamma)^2/2$. Then the unique three-part policy is such that:*

$$\begin{aligned}
s &= \frac{2\delta}{c}\left(k + c - k\frac{p}{t\gamma}\right) < 0 \\
\tau &= -2\delta\gamma\frac{k+c}{c}\left(t - \frac{k}{k+c}\frac{p}{\gamma}\right) > 0 \\
f &= t(1 - 2\delta\frac{k+c}{c}) + 2\delta\frac{p}{\gamma} > t
\end{aligned}$$

where one has to over-tax emissions, to tax abatement and to refund taxes through a production subsidy.

4 Extension to heterogenous industries

4.1 Personalized three-part refunding policies

In this section, we examine the case of heterogenous industries with respect to cost conditions. The three-part policy able to replicate the standard emission outcome while sustaining the budget constraint is a priori *personalized* and should thus be indexed by i : f_i, τ_i and s_i. Faced with this three-part policy, firm i chooses $\hat{q}_i(\tau_i, s_i, f_i)$ and $\hat{e}_i(\tau_i, s_i, f_i)$ such that:

$$\begin{aligned} p + \tau_i &= c_i'(\hat{q}_i) + (1 - s_i)\hat{a}_{iq} \\ -(1 - s_i)\hat{a}_{ie} &= f_i \end{aligned} \tag{6}$$

As above, comparing (1) with (6) allows to derive the following result.

Proposition 4 *For a heterogenous industry of fixed size, there exists at least a three-part policy that entails any firm i to choose $\hat{e}_i = e_i^*$ and $\hat{q}_i = q_i^*$ while sustaining the budget constraint (provided firm's problem concavity is preserved). They are characterized by:*

$$\begin{aligned} f_i &= (1 - s_i)t \\ \tau_i &= -s_i a_{iq}^* \\ \sum_i \sigma_i^* a_i^* s_i &= \delta t E^* \end{aligned}$$

where

$$\sigma_i^* = 1 - \varepsilon_{a_i,e}^* - \varepsilon_{a_i,q}^*.$$

Moreover, aggregate net profit is larger than the one without rebate ($\hat{\Pi} - \Pi^* = \delta t E^* > 0$) so that the three-part policy is strictly prefered from the industry's point of view to a standard environmental tax.

Proof: See Appendix C. ■

One particular policy of interest is where one takes $s_i = s \equiv tE^*/\sum_i \sigma_i^* a_i^*$ and $f_i = f = (1 - s)t$ for any i and where $\tau_i = -s a_{iq}^*$. For this policy, the sign of $\sum_i \sigma_i^* a_i^*$, weighted sum of abatement expenditures, is the key element. Assume that $\sum_i \sigma_i^* a_i^* > 0$. In that case, the policy prescribes a uniform subsidy $s > 0$ on abatement, a uniform tax on emissions that under-taxes emissions ($f < t$) and a personalized tax

$\tau_i < 0$ on production. Conversely, if $\sum_i \sigma_i^* a_i^* < 0$, then the policy prescribes a tax on abatement ($s < 0$), a tax on emissions that over-taxes emissions ($f > t$) and a personalized subsidy on production $\tau_i > 0$.

Two remarks are to be made. First, at the individual level, refunding is profitable if only if $s\sigma_i^* > 0$, that is when s and σ_i^* have the same sign. While it is always profitable for the industry as a whole, it may not be profitable for all members of the industry. Second, incomplete information will impede the implementation of such personalized output subsidies. The search for incentive compatible refunding policies is beyond the scope of this paper. Nevertheless, it is interesting to investigate what can be done with a uniform policy (τ, s, f). This is the purpose of the next section.

4.2 Uniform three-part refunding policies

We now consider a uniform policy of the type (τ, s, f) implemented in a sector composed of n heterogenous polluting firms as described in section 4. Non-personalized policies will not make it possible to have firms choosing q_i^* and e_i^* in general. Nevertheless, it is still possible to find a three-part refunding policy that satisfies the budget constraint and that leads to the same aggregate pollution and production levels that prevail under any emission tax t without refunding. For this, a solution (τ, s, f) to the following system for given t must exist:

$$\hat{E}(\tau, s, f) - E^*(t) = 0 \tag{7}$$

$$\hat{Q}(\tau, s, f) - Q^*(t) = 0 \tag{8}$$

$$f\hat{E}(\tau, s, f) - \tau\hat{Q}(\tau, s, f) - s\hat{A}(\tau, s, f) - (1-\delta)tE^*(t) = 0 \tag{9}$$

Existence of a solution can be deduced from using the Implicit Function Theorem and so by checking that the Jacobian matrix of the system is non singular. Studying the properties of this system of equations with this level of generality is not very informative. Hence, we instead prove that such a solution exists for a particular example relying on the specification used in the previous section, which is done below.

More importantly, a uniform three-part refunding policy solution of the system above is unable in general to always ensure that the industry benefits from the refund, because the uniform policy makes it impossible to have firms choosing individually q_i^* and e_i^*. As a consequence, the aggregate costs of production and abatement under the three-part refundng policy and under the standard tax policy generally differ. To see this, recall that the aggregate profit of the industry under the three-part refunding policy can be written as follows:

$$\hat{\Pi} = (p+\tau)\hat{Q} - \hat{C} - (1-s)\hat{A} - f\hat{E}$$

where $\hat{C} = \sum_i c_i(\hat{q}_i)$, $\hat{A} = \sum_i a_i(\hat{q}_i, \hat{e}_i)$ with $\hat{q}_i(\tau, s, f)$ and $\hat{e}_i(\tau, s, f)$ being the optimal decisions for polluter i facing the policy (τ, s, f). Recall also that the aggregate profit under the standard emission tax t is:

$$\Pi^* = pQ^* - C^* - A^* - tE^*$$

where $C^* = \sum_i c_i(q_i^*)$. Computing the difference and using (7), (8), (9) and simplifying, we obtain finally:

$$\hat{\Pi} - \Pi^* = \delta t E^* + C^* + A^* - \hat{C} - \hat{A}$$

It follows that $\hat{\Pi} - \Pi^* > 0$ only if the uniform refunding policy does not increase too much the aggregate cost of production (gross of emissions payments) compared to the benchmark case of a standard emission tax t. Although the refunding policy ensures stability of aggregate production and emission by definition, it will reallocate these quantities among the different polluters compared to the benchmark case. If this reallocation of production and emissions ends up by increasing too much the sum of production and abatement costs for the industry, the refunding policy may not be beneficial to the firms as a whole.

To illustrate these results, we now specify the model and assume that there are n firms indexed by $i = 1...n$. The cost of production is $c_i(q_i) = cq_i^2/2$ and abatement cost is $a_i(q_i, e_i) = k(q_i - e_i/\gamma_i)^2/2$. Firms are heterogeneous according to parameter γ_i. The number of firms of type γ_i is n_i and $\sum_i n_i = n$. Let us denote the mean $\bar{\gamma}$ and the variance σ_γ^2 of parameter γ.

Using (6), we immediately obtain that:

$$\begin{aligned} \hat{q}_i &= \frac{p + \tau - f\gamma_i}{c} \\ \hat{e}_i &= \gamma_i(\hat{q}_i - \frac{f\gamma_i}{k(1-s)}) \end{aligned}$$

This in turn allows to compute aggregate values for production, emissions and abatement expenditures as follows:

$$\begin{aligned} \hat{Q} &= \frac{n\,(p + \tau - f\bar{\gamma})}{c} \\ \hat{E} &= n\bar{\gamma}\frac{p+\tau}{c} - nf(\frac{1}{c} + \frac{1}{k(1-s)})(\sigma_\gamma^2 + \bar{\gamma}^2) \\ \hat{A} &= \frac{nf^2(\sigma_\gamma^2 + \bar{\gamma}^2)}{2k(1-s)^2}. \end{aligned}$$

We look for τ, s and f such that the system (7),(8),(9) hold for the above values of $\hat{Q}$, $\hat{E}$ and $\hat{A}$. We show in Appendix D that there are two solutions (τ_1, s_1, f_1) and (τ_2, s_2, f_2). Following the remark above, we also impose the additional constraint that a refunding policy should benefit the industry as a whole to our search and it appears that this holds only for solution indexed by 1 as explained below. We depict the tax t and the two solutions in Figures 1 and 2 as a function of the initial tax t for a specific set of parameters.[3] Figure 1 indicates that the solution (τ_1, s_1, f_1) entails a tax on abatement and tax refunding through output subsidy along with emissions being overtaxed compared to the initial tax t. On the contrary, Figure 2 shows that the second solution entails a tax on output and taxe refunding through abatement subsidy along with emissions being undertaxed. By construction, both solutions lead to same aggregate production Q^* and emissions E^* depicted on Figure 3, but they

[3]We use $p = 15$, $c = 1$, $k = 1/2$, $\bar{\gamma} = 0.7$, $\sigma_\gamma = 0.3$, $\delta = 1$ and $n = 1$. Tax t is allowed to vary between 1 and 6. A tax $t > 1$ ensures a corresponding abatement subsidy $s < 1$. At $t = 6$, aggregate pollution cancels out.

differ in terms of aggregate abatement expenditures and cost of production: While solution 1 entails lower abatement expenditures but higher production cost ($\hat{C}_1 \geq C^*$, $\hat{A}_1 \leq A^*$), solution 2 entails higher abatement expenditures but lower production costs ($\hat{C}_2 \leq C^*$, $\hat{A}_2 \geq A^*$) at the aggregate level. This is because the solution 1 implies a reallocation of production (and pollution) towards firms with low values of γ while it is the reverse for the solution 2. As shown by Figure 4, it turns out that only the three-part refunding policy (τ_1, s_1, f_1) is beneficial for the industry compared to the standard tax benchmark.

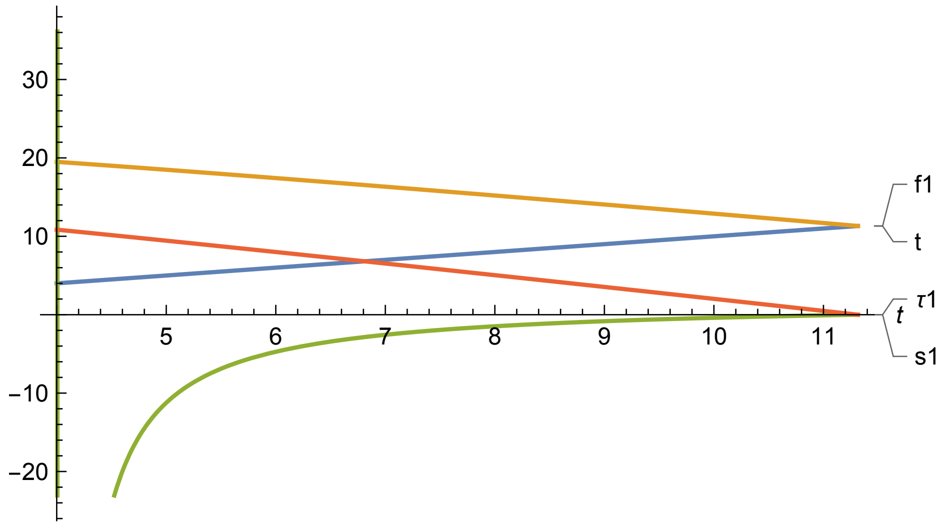

Figure 1: Tax on abatement ($s < 0$), emissions are overtaxed ($f_1 > t$) and refunding through output subsidy ($\tau_1 > 0$).

5 Conclusion

In this paper, we have shown that a three-part refunding policy can help to alleviate the drawbacks of either pure output based refunding or pure abatement expenditure based refunding. In particular, when the regulated industry is homogenous, it is possible to replicate the standard emission tax outcome using such a policy: with the appropriate definition of the fee and of the output and abatement subsidies, the three-part refunding

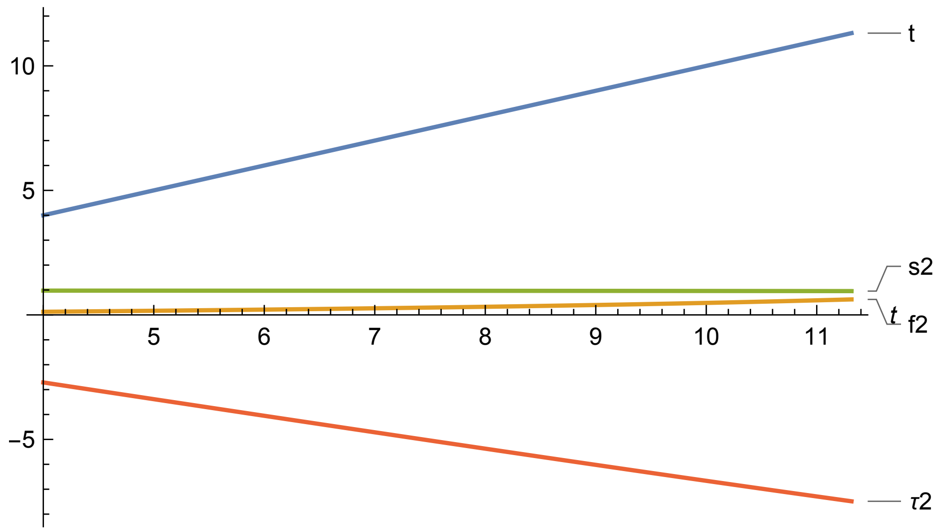

Figure 2: Tax on output ($\tau_2 < 0$), emissions are undertaxed ($f_2 < t$) and refunding through abatement subsidy ($s > 0$).

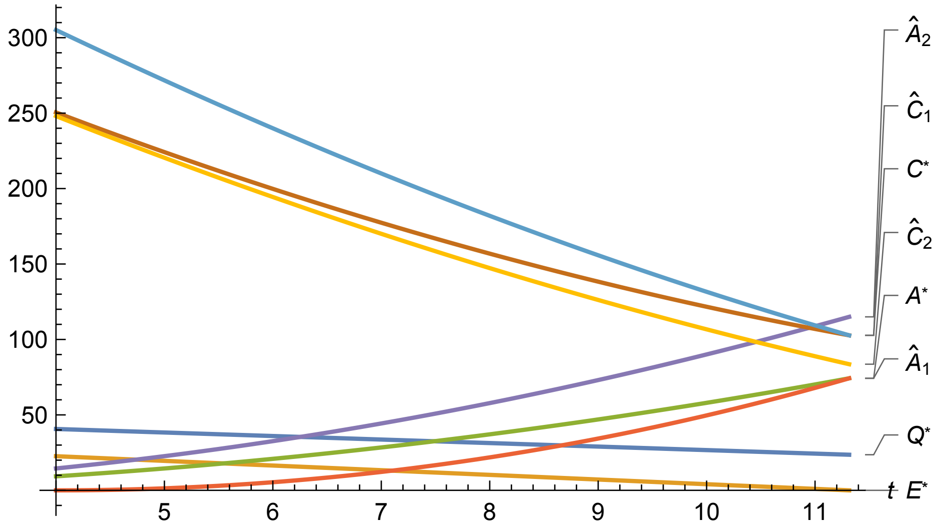

Figure 3: Production (Q^*), emissions (E^*) and abatement expenditures (A^*, $\hat{A}_1$ and $\hat{A}_2$).

policy induces the polluting firm to choose the production and emission levels obtained under any emission tax, without departing from budget balance. By construction, the proposed regulation scheme entails that the polluter earns strictly more than under the standard tax alone without rebate, thereby making this proposal highly acceptable.

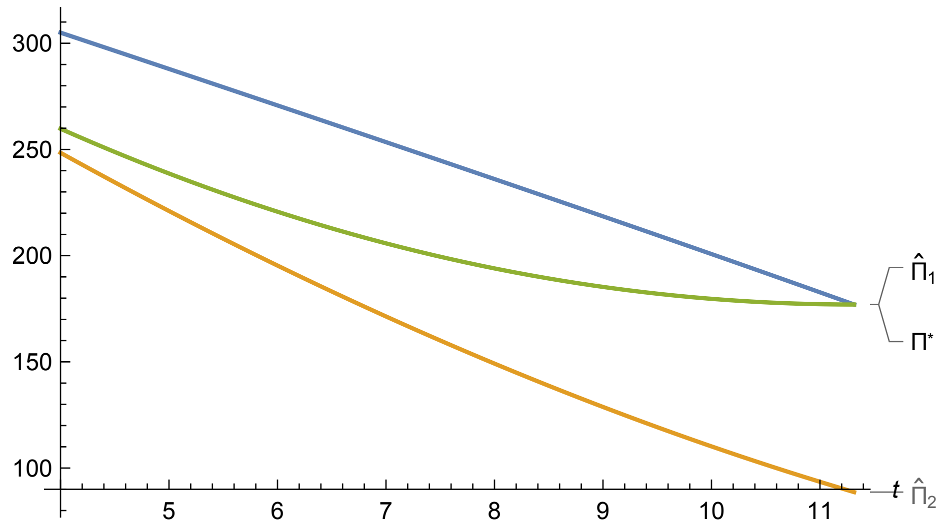

Figure 4: Industry's profit (Π^*, $\hat{\Pi}_1$ and $\hat{\Pi}_2$).

When the industry is heterogenous, the refunding policy needed to replicate the standard emission tax outcome is now personalized in the sense that at least the output subsidy should be type dependent. Another result is that this three-part policy is strictly prefered only from the industry's point of view to a standard environmental tax. By specifying the model, we also show that a uniform three-part refunding policies can also replicate standard tax outcome in the aggregate at the sector's level with respect to production and emissions without departing from the budget constraint.

Overall, in this paper, we have shown that there are ways to design refunded pollution taxes so that negative impacts on the industry's profit are limited while still inducing firms to limit emissions. Such policies however require information on emissions, outputs and abatement expenditures. It would thus be interesting to investigate the design of such refunding policies when these informations are private and/or can be observed at a cost by the regulator (see e.g. Bontems and Bourgeon 2005 and Macho-Stadler and Perez-Castrillo for analysis of pollution taxes under costly observability of pollution). We leave this to future research.

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Appendix

A Comparative statics

Dropping the firm's index for the sake of clarity and introducing a positive scale parameter θ for abatement cost function, the system (1) rewrites as:

$$\begin{aligned} p &= c'(q^*) + \theta a_q(q^*, e^*) \\ -\theta a_e(q^*, e^*) &= t. \end{aligned}$$

Differentiating totally this system and dropping arguments, we obtain:

$$\begin{aligned} dq &= \frac{1}{\Delta}\left[a_{qe}dt + a_{ee}dp - (a_q a_{ee} - a_e a_{qe})d\theta\right] \\ de &= \frac{1}{\Delta}\left[(a_e c'' - \theta(a_e a_{qq} - a_q a_{qe}))d\theta - \left(c'' + \theta a_{qq}\right)dt - \theta a_{qe}dp\right] \end{aligned}$$

where $\Delta = a_{ee}c'' + \theta(a_{ee}a_{qq} - a_{qe}^2) > 0$ under quasi-convexity of $a(.,)$ and convexity of $c(.)$. We obtain that:

$$\frac{\partial q}{\partial p} = \frac{1}{\Delta}a_{ee} > 0 \text{ and } \frac{\partial e}{\partial t} = -\frac{c'' + \theta a_{qq}}{\Delta} < 0.$$

Also, under assumption 1, we obtain that:

$$\frac{\partial q}{\partial t} = \frac{1}{\Delta}a_{qe} < 0 \text{ and } \frac{\partial e}{\partial p} = -\frac{a_{qe}}{\Delta} > 0.$$

Finally, assumption 2 allows to state that:

$$\frac{\partial q}{\partial \theta} = -\frac{(a_q a_{ee} - a_e a_{qe})}{\Delta} < 0 \text{ and } \frac{\partial e}{\partial \theta} = -\frac{(a_e a_{qq} - a_q a_{qe}))}{\Delta} > 0.$$

B Proof of Proposition 2

To get $\hat{q}(\tau, s, f) = q^*$ and $\hat{e}(\tau, s, f) = e^*$, in view of the FOCs (2) and (5), it is sufficient to take

$$\begin{aligned} f &= t + sa_e^* = (1-s)t \\ \tau &= -sa_q^*. \end{aligned}$$

Also, the subsidy s is given by the budget constraint (3) that now writes:

$$nfe^* = n\tau q^* + nsa^* + R \tag{10}$$

or equivalently with $R = (1-\delta)tne^*$

$$\begin{aligned}
(t + sa_e^*)\, e^* &= -sa_q^* q^* + sa^* + (1-\delta)te^* \\
s &= \frac{\delta te^*}{a^* - a_e^* e^* - a_q^* q^*} = \frac{\delta te^*}{\sigma^* a^*} = -\frac{\delta \varepsilon_{a,e}^*}{\sigma^*}
\end{aligned}$$

where $\sigma^* = 1 - \varepsilon_{a,e}^* - \varepsilon_{a,q}^*$ with $\varepsilon_{a,e}^* = a_e^* e^*/a^* < 0$ and $\varepsilon_{a,q}^* = a_q^* q^*/a^* > 0$ and recalling that $t = -a_e^*$ from (1).

Finally, by construction, the difference in terms of net profit is

$$\begin{aligned}
\hat{\pi}(\tau, s, f) - \pi^* &= (p+\tau)q^* - c(q^*) - (1-s)a^* - fe^* - pq^* + c(q^*) + a^* + te^* \\
&= \tau q^* + sa^* + (t-f)e^* \\
&= fe^* - (1-\delta)te^* + (t-f)e^* = \delta te^*
\end{aligned}$$

by using the budget constraint.

C Proof of Proposition 4

To get $\hat{q}_i = q^*$ and $\hat{e}_i = e^*$, in view of the FOCs (1) and (6), it is sufficient to take

$$\begin{aligned}
f_i &= (1-s_i)t \\
\tau_i &= -s_i a_{iq}^*.
\end{aligned} \tag{11}$$

Moreover, the budget constraint writes:

$$\sum_i f_i e_i^* = \sum_i \tau_i q_i^* + \sum_i s_i a_i^* + (1-\delta)tE^* \tag{12}$$

Using (11) and replacing in (12), we get

$$\sum_i (1-s_i)te_i^* = -\sum_i s_i a_{iq}^* q_i^* + \sum_i s_i a_i^* + (1-\delta)tE^*$$

which implies

$$\sum_i \sigma_i^* a_i^* s_i = \delta t E^* \tag{13}$$

with $\sigma_i^* = 1 - \varepsilon^*_{a_i,e} - \varepsilon^*_{a_i,q}$.

Moreover, the difference between net profits is:

$$\begin{aligned}\hat{\pi}_i - \pi_i^* &= (p+\tau_i)\, q_i^* - c_i(q_i^*) - (1-s_i)a_i(q_i^*, e_i^*) - f_i e_i^* - (pq_i^* - c_i(q_i^*) - a_i(q_i^*, e_i^*) - te_i^*) \\ &= \tau_i q_i^* + s_i a_i^* + (t - f_i)e_i^*\end{aligned}$$

Replacing with the values obtained for the instruments τ_i and f_i and recalling that $t = -a_{ie}^*$, we obtain:

$$\hat{\pi}_i - \pi_i^* = s_i \left(a_i^* - a_{iq}^* q_i^* - a_{ie}^* e_i^*\right) = s_i a_i^* \sigma_i^*$$

Summing over i and using (13), we thus get $\hat{\Pi} - \Pi^* = \sum_i \sigma_i^* a_i^* s_i = \delta t E^* > 0$.

D Example

From the expression for individual production,

$$\hat{q}_i = \frac{p + \tau - f\gamma_i}{c}$$

we get by summing over i :

$$\hat{Q} = \frac{n\,(p + \tau - f\bar{\gamma})}{c}$$

Also from

$$\hat{e}_i = \gamma_i(\hat{q}_i - \frac{f\gamma_i}{k(1-s)})$$

we obtain similarly

$$\begin{aligned}\hat{E} &= \sum_i n_i \gamma_i (\hat{q}_i - \frac{f\gamma_i}{k(1-s)}) \\ &= \sum_i n_i \gamma_i (\frac{p+\tau - f\gamma_i}{c} - \frac{f\gamma_i}{k(1-s)}) \\ \hat{E} &= n\bar{\gamma}\frac{p+\tau}{c} - nf(\frac{1}{c} + \frac{1}{k(1-s)})(\sigma_\gamma^2 + \bar{\gamma}^2)\end{aligned}$$

Finally, for abatement expenditures we obtain:

$$\hat{A} = \sum_i n_i \frac{k(\hat{q}_i - \hat{e}_i/\gamma_i)^2}{2} = \frac{f^2}{2k(1-s)^2} \sum_i n_i \gamma_i^2 = \frac{nf^2(\sigma_\gamma^2 + \bar{\gamma}^2)}{2k(1-s)^2} \tag{14}$$

The system to be solved in (τ, s, f) rewrites as follows:

$$\hat{Q} = Q^* \tag{15}$$

$$\hat{E} = E^* \tag{16}$$

$$f\hat{E} = \tau\hat{Q} + s\hat{A} + (1-\delta)tE^* \tag{17}$$

Equation (15) can be expressed as

$$\frac{n\,(p + \tau - f\bar{\gamma})}{c} = \frac{n\,(p - t\bar{\gamma})}{c}$$

from which we deduce that

$$\tau = (f - t)\bar{\gamma} \tag{18}$$

Also, equation (16) writes as

$$n\bar{\gamma}\frac{p+\tau}{c} - nf(\frac{1}{c} + \frac{1}{k(1-s)})(\sigma_\gamma^2 + \bar{\gamma}^2) = n\bar{\gamma}\frac{p}{c} - nt(\frac{1}{c} + \frac{1}{k})(\sigma_\gamma^2 + \bar{\gamma}^2)$$

and replacing τ using (18) allows to obtain

$$\bar{\gamma}^2\frac{(f-t)}{c} - f(\frac{1}{c} + \frac{1}{k(1-s)})(\sigma_\gamma^2 + \bar{\gamma}^2) = -t(\frac{1}{c} + \frac{1}{k})(\sigma_\gamma^2 + \bar{\gamma}^2)$$

which simplifies into

$$\frac{f}{1-s} = t + \lambda(t - f) \tag{19}$$

where $\lambda \equiv \frac{k}{c}\frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \bar{\gamma}^2} \geq 0.$ We can deduce f as a function of s :

$$f = t\frac{(1+\lambda)(1-s)}{1+\lambda(1-s)}$$

When there is homogeneity, $\sigma_\gamma^2 = 0$ and thus $\lambda = 0$ and we recover $f = (1-s)t$ as in Proposition 2.

Last, using (15) and (16), the budget constraint (17) rewrites simply as

$$\begin{aligned}
\left[f - t(1-\delta)\right] E^* &= \tau Q^* + s\hat{A} \\
\left[f - t(1-\delta)\right] E^* &= (f-t)\bar{\gamma} Q^* + s\hat{A}
\end{aligned}$$

or using (14) and the definition of E^* and Q^*:

$$\left[f - t(1-\delta)\right] E^* = (f-t)\bar{\gamma} Q^* + s\frac{nf^2(\sigma_\gamma^2 + \bar{\gamma}^2)}{2k(1-s)^2}$$

This allows to compute $s/(1-s)$ as a function of f :

$$\frac{s}{1-s} = \frac{2k}{\sigma_\gamma^2 + \bar{\gamma}^2} \frac{f(E^* - \bar{\gamma}Q^*) + t(\bar{\gamma}Q^* - (1-\delta)E^*)}{t + \lambda(t-f)}$$

Hence, it follows that:

$$\frac{1}{1-s} = 1 + \frac{2k}{\sigma_\gamma^2 + \bar{\gamma}^2} \frac{f(E^* - \bar{\gamma}Q^*) + t(\bar{\gamma}Q^* - (1-\delta)E^*)}{t + \lambda(t-f)}$$

and replacing in (19):

$$f\left(1 + \frac{2k}{\sigma_\gamma^2 + \bar{\gamma}^2} \frac{f(E^* - \bar{\gamma}Q^*) + t(\bar{\gamma}Q^* - (1-\delta)E^*)}{t + \lambda(t-f)}\right) = t + \lambda(t-f)$$

which amounts to solve a polynomial equation of degree 2 in f. We denote the two solutions f_1 and f_2. And the corresponding values of τ and s by respectively τ_1, τ_2, s_1 and s_2.