Allocating essential inputs*

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Abstract

Regulators must often allocate essential inputs, such as spectrum rights, transmission capacity or airport landing slots, which can transform the structure of the downstream market. These decisions involve a trade-off, as provisions aimed at fostering competition and lowering prices for consumers, also tend to limit the proceeds from the sale of the inputs. We first characterize the optimal allocation, from the standpoints of consumer and total welfare. We then note that standard auctions yield substantially different outcomes. Finally, we show how various regulatory instruments can be used to implement the desired allocation.

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1 Introduction

In many industries regulators rely on competition rather than direct price control to discipline consumer prices. To do so, regulators must determine how best to allocate essential inputs, such as spectrum bandwidth, transmission capacity or airport landing slots, which affect firms' costs or the quality of their offerings, and can transform market structure. Regulators face a tradeoff. On the one hand, they may seek to maximize

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consumer or social welfare. On the other hand, they may face political pressures to maximize revenue from the sale or lease of such resources. This tension can be particularly acute when the competing firms start with different levels of inputs and market shares, as the more established firms are then likely to be willing to outbid the weaker rivals in order to strengthen their market position. As a result, regulators adopt auctions that include specific provisions such as caps, set-asides and bidding credits, to balance these concerns.

We study this problem in a simplified and stylized model in which a single incumbent initially enjoys a cost advantage over a potential entrant. A regulator can allocate a divisible amount of newly released essential input, which can either widen or narrow the cost differential. In most spectrum auctions, the market structure includes three or four asymmetric incumbents, as well as potential challengers; for airline slots between city pairs or electric transmission rights connecting two nodes, there can be as few as one or two incumbents. Our two-firm model captures a key aspect of regulators' tradeoff: awarding the newly released input to the incumbent tends to generate higher revenue, whereas awarding it to the challenger promotes competition in the market.

We start by characterizing the optimal allocation of this input among the two firms, assuming that the regulator has full information about their costs. When the regulator's only objective is maximizing consumer surplus, the optimal policy is to allocate the resource so as to equalize the costs of the two firms, as consumer prices are lowest in that case. However, equalizing costs tends to minimize profits. As a result, the willingness and ability of the firms to pay for the resource are also the lowest. Hence, when the regulator also puts weight on profits or auction revenues, the optimal allocation limits the cost advantage of the incumbent, but no longer tries to equalize the firms' costs; it instead leaves an advantage to one firm – either one, if the incumbent's advantage can be overcome, and the incumbent otherwise. The winner then pays an amount equal to its operating profit.

We then characterize the optimal allocation for the case in which the challenger's cost is private information. We find that the objective of the regulator and of the challenger are so conflicting that it is typically impossible to induce the challenger to reveal its costs in a useful way. Instead, the optimal allocation is similar to the previous one, except that the regulator must base its allocation decision on the expected value of its objective.

Next, we contrast this optimal allocation with the outcome that would arise in the types of auctions commonly used to allocate radio spectrum licenses. In particular, we compare the outcomes of sequential auctions, sealed-bid Vickrey-Clarke-Groves ("VCG") auctions, and simultaneous multi-round ascending ("SMRA") or clock auctions. In our framework, all these auctions exacerbate the incumbency advantage, as the incumbent always ends up winning all the newly released input. Moreover, revenues are the same in the VCG and clock auctions, and lower in a sequential auction.

Finally, we consider various measures that the regulator can adopt to promote competition, such as caps and set-asides, and discuss how they can be used to implement the optimal allocation.

The remainder of this section discusses the related literature. Section 2 presents the model; Section 3 characterizes the optimal allocation in the case of perfect information, and Section 4 extends the analysis to the case where the challenger's cost is private information. Section 5 examines the outcome of standard auction formats used in these settings. Section 6 discusses the use of regulatory instruments to implement the optimal allocation. Section 7 concludes.

Related Literature

Our insights are reminiscent of the literature on second-sourcing. Although the focus there was mostly on competition "for the market" rather than "in the market", it was recognized that the awarding of a contract or a procurement decision could affect the purchaser's ability to switch to alternative suppliers later on, or the suppliers' ability to compete effectively for subsequent contracts.[1] A few papers however consider the impact of the chosen market structure (e.g., monopoly or duopoly) on prices and welfare.[2] We build on this literature and study how the allocation of a (divisible) essential input can further affect the market structure and the outcome of competition.

Our paper relates to the large literature on optimal auction design, starting with

[1] For instance, Anton and Yao (1987) show that second-sourcing can be used to reduce suppliers' informational rents. Rob (1986), Laffont and Tirole (1988) and Riordan and Sappington (1989) consider the trade-off between such *ex post* savings and suppliers' *ex ante* R&D incentives.

[2] See, e.g., Dana and Spier (1994), McGuire and Riordan (1995) and Auriol and Laffont (1992).

Myerson (1981), the classic paper for single-object auctions.[3] Post-auction interaction generates externalities not only between the firms and their customers in the downstream market,[4] but also among the bidders in the auction: each bidder's payoff depends not only on what it wins, but also on what its rivals win. A series of papers, most notably by Jehiel and Moldovanu, have explored single-object auctions with such externalities.[5]

More closely related are the works Jehiel and Moldovanu (2003), Cramton et al. (2011) and Klemperer (2004), as well as a couple of papers by Janssen and Karamychev – see Salant (2014) for a more extensive discussion. Jehiel and Moldovanu (2003) consider several examples of auctions of fixed-size spectrum licenses, and discuss the likely market outcomes.[6] Klemperer (2004) warns regulators against the temptation of taking measures to increase auction revenues at the cost of discouraging entry, and suggests instead the Anglo-Dutch hybrid auction as a way to balance the trade-off between revenues and post-auction concentration. Cramton et al. (2011) note that provisions favoring entrants need not always sacrifice auction revenues; they provide one example, with one incumbent and several symmetric potential entrants, in which setting a license aside for the entrants does not affect auction revenues. Cramton et al. (2011) also argue that, absent provisions to handicap large bidders, entrants and small participants are unlikely to win new spectrum; hence, regulators should concentrate their efforts on achieving an efficient allocation rather than revenue maximization. Finally, Janssen and Karamychev (2009, 2010) study the impact of auctions on *ex post* prices.[7] Their analysis, however, focuses on firms' risk preferences rather than on the impact on market structure.[8]

There is also some empirical work that sheds light on the benefits of competition.

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[3]See also Maskin et al. (1989) and Armstrong (2000). Also, Milgrom (2004) provides sufficient conditions for the simultaneous ascending auction to result in a Pareto-optimal equilibrium.

[4]Borenstein (1988) shows that the resulting discrepancy between private and social benefits can lead to inefficient outcomes.

[5]See Jehiel and Moldovanu (2001), Jehiel and Moldovanu (2000), and Jehiel et al. (1996). Also, Varma (2003) and Goeree (2003) consider auctions in which bids convey signals that affect rivals' behavior after the auction.

[6]See also Hoppe et al. (2006), who show that limiting the number of licenses to be auctioned may foster entry, by exacerbating free-riding among incumbents' preemption strategies.

[7]See also Janssen and Karamychev (2007), who show that auctions do not always select the most efficient firm.

[8]Other papers include Moldovanu and Sela (2001) in which a seller is conducting an all pay auction so as to maximize the sum of the bidder payments (or efforts). Eső et al. (2010) examines efficient capacity allocations when there is Cournot competition in the downstream market and Brocas (2013) examines optimal auction design of a single, indivisible object when there are externalities.

Landier and Thesmar (2012) evaluate the macroeconomic impact of the entry of the fourth telecom operator in France, Free. They find that entry benefited the population in several ways. First, it had an immediate effect on consumer prices, which increased the purchasing power of the population. Second, the price shock induced by the enhancement of competition created between 16,000 and 30,000 jobs in France. The authors argue that far from distressing the financial position of incumbents, the increased competition encouraged investments in the sector. Hazlett and Muñoz (2009) conducted a large-scale cross-country analysis of spectrum awards and found a significant positive relation between market concentration and consumer prices. This suggests that the social benefits from encouraging entry can more than offset the loss of auction revenue from spectrum withholding or concentration.

2 Model

For the sake of exposition, we consider the case of mobile communication services, where spectrum constitutes an essential input. It should be clear that the analysis can be readily transposed to the access to key inputs in other industries.

Two firms, an incumbent I and a new entrant E, compete à la Bertrand for a consumer demand $D(p)$. The operators have constant returns to scale, but their costs depend on how much bandwidth they have: the more spectrum a mobile operator has, the more data it can carry at a given cell-site; it can thus maintain a given network capacity with fewer cells, and thus at lower costs. The incumbent starts with more spectrum, and thus enjoys a lower cost:

$$\bar{c}_I = c(B_I) < \bar{c}_E = c(B_E),$$

where B_i denotes the bandwidth initially available to firm i, $\bar{c}_i$ denotes its initial unit cost, and $c(\cdot)$ is a strictly decreasing function that is common to both firms. The entrant thus obtains no profit, whereas the incumbent obtains a profit which, assuming that the entrant exerts effective competitive pressure (i.e., $\bar{c}_I < \bar{c}_E$), is equal to:

$$\Pi(B_I, B_E) \equiv [c(B_E) - c(B_I)]\, D(c(B_E)), \tag{1}$$

which increases with the bandwidth advantage of the incumbent:

$$\frac{\partial \Pi}{\partial B_I}(B_I, B_E) = -D(c(B_E))\, c'(B_I) > 0, \tag{2}$$

$$\frac{\partial \Pi}{\partial B_E}(B_I, B_E) = \{D(c(B_E)) + [c(B_E) - c(B_I)]\, D'(c(B_E))\}\, c'(B_E) < 0. \tag{3}$$

New spectrum becomes available, in amount Δ. Each firm i can thus obtain an additional bandwidth $b_i \geq 0$, with $b_I + b_E \leq \Delta$. With this additional bandwidth, the cost of firm i can lie anywhere in the range $[\underline{c}_i, \bar{c}_i]$, where

$$\underline{c}_i = c(B_i + \Delta)$$

denotes the lowest cost that firm i can achieve with all the additional spectrum. We assume that cost differences are never so drastic that competition is ineffective; that is, the incumbent cannot charge its monopoly price, even if it obtains all the additional spectrum:

$$\bar{c}_E < p^m(\underline{c}_I),$$

where $p^m(c) = \min_p \{p \mid p \in \arg\max (\tilde{p} - c)\, D(\tilde{p})\}$. This assumption ensures that the competitive price is always equal to the higher of the two costs:

- As long as $B_I + b_I > B_E + b_E$, I maintains a cost advantage (that is, $c_I = c(B_I + b_I) < c_E = c(B_E + b_E)$) and thus wins the downstream market. The profits are then $\pi_E = 0$ and $\pi_I = \Pi(B_I + b_I, B_E + b_E)$, and consumer surplus is equal to $S(c(B_E + b_E))$, where

$$S(p) \equiv \int_p^{+\infty} D(x)\, dx.$$

- If instead $B_I + b_I < B_E + b_E$, E obtains a lower cost; the profits of the two firms are then $\pi_I = 0$ and $\pi_E = \Pi(B_E + b_E, B_I + b_I)$, and consumer surplus is equal to $S(c(B_I + b_I))$.

3 Complete Information

As a benchmark, this section characterizes the optimal spectrum allocation when costs are public information. We first consider the case where the regulator aims at maximizing consumer surplus, before considering the case where it aims at maximizing social welfare, possibly accounting for a social cost of public funds.

3.1 Consumer Surplus

We first note that a regulator maximizing consumer surplus should seek to minimize the cost asymmetry among the two firms:

Proposition 1 *To maximize consumer surplus, it is optimal to allocate all the additional spectrum among the two firms so as minimize their cost difference. The associated consumer price is*

$$p^S = \max\{\underline{c}_E, \hat{c}\}$$

where $\underline{c}_E = c(B_E + \Delta)$ *and*

$$\hat{c} \equiv c\left(\frac{B_I + B_E + \Delta}{2}\right).$$

Proof. See Appendix A. ■

The intuition is straightforward. Maximizing consumer surplus amounts to minimizing the competitive price, which is equal to the lower of the two costs, $c_I = c(B_I + b_I)$ and $c_E = c(B_E + b_E)$. Hence, it is always optimal to distribute all the additional spectrum, and if there is enough spectrum to offset the initial cost difference, it is optimal to allocate this spectrum so as to equate the two costs (leading to $p = \hat{c}$). If instead it is impossible to do so, then it is optimal to minimize the cost asymmetry by allocating all the additional spectrum to the entrant (leading to $p = \underline{c}_E$). Interestingly, a former FCC Chief Technology Office indeed argued that equalizing spectrum holdings is essential for effective competition among carriers.[9]

[9] See Peha (2017).

3.2 Social Welfare

In practive, industry regulators need to pay attention to firms' profitability. First, firms would not operate at a loss absent socially costly subsidies. This concern does not affect the findings of Proposition 1, however: the described allocation remains optimal even when taking into account firms' budget constraints, as the entrant always obtains zero profit and the incumbent obtains a non-negative profit. Second, as public funds are costly, firms' financial contributions (e.g., in the form of – lump-sum – spectrum licensing fees) reduce the public budget deficit and/or lower distortionary taxes. To account for this concern, we now suppose that the regulator aims at maximizing social welfare, defined as the sum of the industry profit and of consumer surplus, taking into account a cost $\lambda \geq 0$ of public funds and firms' viability constraints; that is:

- Any transfer t from firms to consumers generates an additional benefit λt, representing the social gain from reducing budget deficit or distortionary taxes; social welfare is thus given by

$$W = (\pi_I - t_I) + (\pi_E - t_E) + (S + t_I + t_E) + \lambda (t_I + t_E),$$

  where, as before, π_i denotes firm i's profit and $S = S(p)$ denotes consumer surplus.

- The regulator must accommodate the firms' profitability constraints: for $i = I, E$,

$$\pi_i - t_i \geq 0.$$

It follows that it is optimal to choose $t_i = \pi_i$ for $i = I, E$; social welfare can thus be expressed as

$$W = (1 + \lambda)(\pi_I + \pi_E) + S.$$

Obviously, it is again optimal to allocate all the additional bandwidth Δ:

Lemma 1 *It is socially optimal to allocate all the additional spectrum.*

Proof. It suffices to note that giving any residual bandwidth to the firm with the lower cost (or to either firm, if both have the same cost) would further reduce its cost and increase industry profit, without any adverse effect on consumers. ■

Therefore, without loss of generality, we can restrict attention to spectrum allocations of the form $b_I = \Delta - b_E$, for some $b_E \in [0, \Delta]$. Furthermore:

- If $b_E < (B_I + B_E + \Delta)/2$, this spectrum allocation yields a competitive equilibrium of the form

$$p = c_E > \hat{c} > c_I = \gamma(p), \tag{4}$$

  where

$$\gamma(p) \equiv c\left(B_I + B_E + \Delta - c^{-1}(p)\right)$$

  denotes the lower cost among the two firms, when all the bandwidth is allocated so as to set the higher cost to p. The resulting social welfare that can be expressed as:

$$W(p) = (1 + \lambda)(p - \gamma(p)) D(p) + S(p). \tag{5}$$

- If instead $b_E > (B_I + B_E + \Delta)/2$ (which requires $\Delta > B_I - B_E$), then

$$p = c_I > \hat{c} > c_E = \gamma(p), \tag{6}$$

  which, keeping p constant, generates the same social welfare as the equilibrium described by (4) (the roles of the two firms are simply swapped).

Hence, looking for the optimal spectrum allocation amounts to maximizing $W(p; \lambda)$ in the range $p \in [\max\{\underline{c}_E, \hat{c}\}, \bar{c}_E]$, with the caveat that any price $p \leq \bar{c}_I$ can be achieved in two equivalent ways, namely, by conferring the same cost advantage to either firm. Maximizing $W(p)$ with respect to the equilibrium price p yields the first-order condition

$$p = \gamma(p) + \left[\frac{\lambda}{1 + \lambda} - \gamma'(p)\right] \mu(p), \tag{7}$$

where

$$\mu(p) = -\frac{D(p)}{D'(p)}$$

represents the market power attached to the demand function – see Weyl and Fabinger (2013).

To ensure that the first-order condition (7) yields a unique solution, we will assume the following regularity conditions:

Assumption A.

1. The market power function is decreasing in the relevant range: $\mu'(p) \leq 0$ for any $p \in [\max\{\underline{c}_E, \hat{c}\}, \bar{c}_E]$.

2. The unit cost function $c(\cdot)$ is convex: $c''(B) \geq 0$ for any $B \geq 0$.

Assumption A.1 amounts to assuming that the demand function is log-concave; it also implies that the monopoly pass-through rate is lower than one. Assumption A.2 asserts that, while using more spectrum enables the firms to reduce their costs (i.e., $c'(\cdot) < 0$), this is less and less so as more bandwidth becomes available; it ensures that, while $\gamma(p)$ decreases as p increases, it does so at a decreasing rate:

Lemma 2 *Under Assumption A.2, for any $p \in [\max\{\hat{c}, \underline{c}_E\}, \bar{c}_E]$:*

$$\gamma'(p) < 0 \leq \gamma''(p).$$

Proof. See Appendix B. ■

The following Proposition characterizes the socially optimal allocation:

Proposition 2 *Under Assumption A, the spectrum allocation that maximizes social welfare yields an equilibrium price, p^W, which is uniquely defined and lies strictly above $p^S = \hat{c}$.*

Specifically, using

$$\phi(p) \equiv \gamma(p) + \left[\frac{\lambda}{1+\lambda} - \gamma'(p)\right]\mu(p) - p, \tag{8}$$

we have $\phi'(\cdot) < 0$ and:

- *If $\phi(\bar{c}_E) \geq 0$, then it is optimal to allocate all the additional bandwidth to the incumbent: $p^W = \bar{c}_E$.*

- *If $\phi(\underline{c}_E) \leq 0$ (which can only arise when $\underline{c}_E > \hat{c}$), then it is optimal to allocate all the additional bandwidth to the entrant: $p^W = \underline{c}_E$.*

- *Otherwise, p^W is the unique solution to $\phi(p) = 0$ lying (strictly) between $\max\{\hat{c}, \underline{c}_E\}$ and $\bar{c}_E$. Furthermore:*

– *When* $p^W > \max\{\bar{c}_I, \underline{c}_E\}$*, the optimal spectrum allocation is unique and maintains a cost advantage to the incumbent.*

– *When instead* $p^W < \max\{\bar{c}_I, \underline{c}_E\}$*, there are two optimal spectrum allocations, conferring the same cost advantage to either firm.*

Proof. See Appendix C. ■

Maximizing social welfare thus never leads to equating the costs of the two firms: it is instead optimal to maintain a cost advantage (for the incumbent when $p^W > \max\{\bar{c}_I, \underline{c}_E\}$, for either firm otherwise),[10] even when $\lambda = 0$ (that is, when profits and consumer surplus are given the same weight). This insight is quite robust: in any setting in which firms compete in prices, cost equalization is a local minimum of total welfare. To see this, note that total welfare can be expressed as

$$\Pi + S = U(D(p)) - cD(p),$$

where $c = \hat{c} - \varepsilon$ and $p = \hat{c} + \varepsilon$, for some $\varepsilon > 0$; it follows that, starting from cost equalization ($\varepsilon = 0$), where $U'(.) = p = c = \hat{c}$, introducing a slight asymmetry ($d\varepsilon > 0$) increases total welfare:

$$d(\Pi + S) = [U'(\cdot) - \hat{c}]\, D'(\hat{c})\, d\varepsilon + D(\hat{c})\, d\varepsilon = D(\hat{c})\, d\varepsilon > 0.$$

As intuition suggests, an increase in the bandwidth initially available to either firm, or in the additional bandwidth made available, leads to a reduction in the socially optimal price, whereas an increase in the weight λ placed on profit tends to increase in that price. More precisely:

• If $\phi(c(B_E)) \geq 0$, then $p^W = c(B_E)$; it thus only depends on B_E, and strictly decreases as B_E increases.

• If $\phi(c(B_E + \Delta)) \leq 0$, then $p^W = c(B_E + \Delta)$; it thus does not depend on B_I, but strictly decreases as B_E or Δ increases.

• Otherwise, it is optimal to divide the additional bandwidth, and we have:

[10] When $\underline{c}_E > \bar{c}_I$, there is not enough bandwidth to offset I's initial cost advantage anyway; when instead $\underline{c}_E < \bar{c}_I$, conferring the same cost advantage to either firm is optimal when $\phi(\bar{c}_I) \leq 0$.

Corollary 1 *If $\phi(c(B_E)) < 0 < \phi(c(B_E + \Delta))$, the socially optimal price strictly increases with λ, but strictly decreases as the total bandwidth, $B_I + B_E + \Delta$, increases.*

Furthermore, when $p^W > \max\{c(B_I), c(B_E + \Delta)\}$, the unique optimal spectrum allocation maintains a cost advantage to the incumbent and is such that:

- *Any increase in λ leads to a re-allocation of the additional bandwidth Δ in favor of the incumbent,*
- *Any increase in the additional bandwidth Δ is shared between the two firms.*
- *Any increase in the bandwidth initially available to one firm, B_E or B_I, leads to a re-allocation of the additional bandwidth Δ in favor of the other firm, which is however limited so as to ensure that both firms end-up with a larger total bandwidth.*

Proof. See Appendix D. ■

4 Incomplete information

This section studies how the above allocation must be adjusted when firms have private information about their costs. As we will see, eliciting this private information is made difficult here, as the usual techniques rely on monotonicity conditions that do not hold here. In particular, the impact of the initial cost handicap on firms' willingness to pay for additional spectrum depends critically on which firm ends-up facing a lower cost. To see this, we focus on the simple case where:

- Only one firm has private information; to fix ideas, we assume that the cost of the incumbent is common knowledge, whereas only the entrant knows its own cost.
- The regulator aims at maximizing consumer surplus (or equivalently, minimizing the equilibrium price), subject to the firms' budget constraints; the previous analysis suggests that this case is easier to address than when total welfare is the objective.

It will be convenient to denote by

$$\theta = B_I - B_E \, (\geq 0)$$

the initial "bandwidth handicap" of the entrant. This parameter θ should not be interpreted literally as the difference in spectrum holdings (which is likely to be public information); rather, we use it as a proxy for the initial cost asymmetry between the two firms. In practice, an incumbent benefits from scale economies arising from its existing spectrum holdings and from its denser network of cell sites; it may also benefit from a better bargaining position when dealing with equipment suppliers, and possibly from superior know-how and expertise (due, e.g., from learning-by-doing). Firm i' cost can thus be expressed as $C(A_i)$, where A_i denotes firm i's total asset and is of the form $A_i = K_i + B_i$, where K_i reflects firm i's accumulated capital other than spectrum, and $K_I > K_E$. In this setting, the relevant cost handicap of the entrant is given by $\theta = K_I - K_E + B_I - B_E$, and is likely to be subject of private information even if the spectrum holdings B_I and B_E are public knowledge. For the sake of exposition, and in line with our previous analysis, we will simply denote here by B_I and B_E the two firms' initial "total assets".

As before, it is optimal to allocate all the additional bandwidth; thus, if E obtains $b_E = b$, I obtains $\Delta - b$. The costs of the two firms are they respectively given by

$$c_I = c(B_I + \Delta - b) \text{ and } c_E = c(B_I - \theta + b),$$

and they coincide when

$$b = \hat{b}(\theta) \equiv \frac{\theta + \Delta}{2}.$$

From the previous analysis, the first-best allocation aims at equalizing costs:

$$b^{FB}(\theta) \equiv \begin{cases} \hat{b}(\theta) & \text{if } \theta \leq \Delta, \\ \Delta & \text{otherwise.} \end{cases}$$

Suppose now that the handicap θ is: (i) drawn from a cumulative distribution function $F(\theta)$, with continuous density $f(\theta)$ on a support $\Theta = [\underline{\theta}, \bar{\theta}]$, where $\bar{\theta} > \underline{\theta} \geq 0$; and: (ii) only observed by E, who thus has private information. The regulator then wishes to maximize expected consumer surplus, $E_\theta[S(p(\theta))]$, where the market price is given by

$$p(\theta) = \max\{c_I = c(B_I + \Delta - b), c_E = c(B_I - \theta + b)\}.$$

Without loss of generality, the regulator can offer a direct mechanism, which determines the allocation of the additional spectrum, $b(\theta)$, and a monetary transfer from the entrant, $t(\theta)$, as a function of the cost handicap reported by the entrant. This direct mechanism must be individually rational:

$$\forall\theta \in \Theta, \pi_E(b(\theta), \theta) - t(\theta) \geq 0, \tag{(IR)}$$

and incentive compatible:

$$\forall\theta, \tilde{\theta} \in \Theta, \pi_E(b(\theta), \theta) - t(\theta) \geq \pi_E\left(b\left(\tilde{\theta}\right), \theta\right) - t\left(\tilde{\theta}\right), \tag{(IC)}$$

where $\pi_E(b, \theta)$ denotes the gross profit of the entrant and is given by

$$\pi_E(b, \theta) \equiv \begin{cases} \pi(b, \theta) & \text{if } b > \hat{b}(\theta) = \frac{\Delta+\theta}{2}, \\ 0 & \text{otherwise,} \end{cases}$$

where[11]

$$\begin{aligned} \pi(b, \theta) &\equiv \Pi(B_I - \theta + b, B_I + \Delta - b) \\ &= [c(B_I + \Delta - b) - c(B_I - \theta + b)]\, D(c(B_I + \Delta - b)). \end{aligned}$$

This profit function satisfies:

$$\begin{aligned} \frac{\partial\pi}{\partial\theta}(b, \theta) &= c'(B_I - \theta + b)\, D(c(B_I + \Delta - b)) < 0, \\ \frac{\partial^2\pi}{\partial b\partial\theta}(b, \theta) &= c''(B_I - \theta + b)\, D(c(B_I + \Delta - b)) - c'(B_I - \theta + b)\, D'(c(B_I + \Delta - b))\, c'(B_I + \Delta - b) \\ &> 0, \end{aligned}$$

[11] As before, we assume that cost differences are never so drastic that competition is ineffective.

and

$$
\begin{aligned}
\frac{\partial \pi}{\partial b}(b,\theta) =& -\left[c'\left(B_I+\Delta-b\right)+c'\left(B_I-\theta+b\right)\right] D\left(c\left(B_I+\Delta-b\right)\right) \\
& -\left[c\left(B_I+\Delta-b\right)-c\left(B_I-\theta+b\right)\right] D'\left(c\left(B_I+\Delta-b\right)\right) c'\left(B_I+\Delta-b\right) \\
>& -c'\left(B_I+\Delta-b\right) D\left(c\left(B_I+\Delta-b\right)\right) \\
& -c'\left(B_I+\Delta-b\right)\left[c\left(B_I+\Delta-b\right)-c\left(B_I-\theta+b\right)\right] D'\left(c\left(B_I+\Delta-b\right)\right) \\
>& 0,
\end{aligned}
$$

where the last inequality stems from the fact that competition remains effective (and thus E's profit increases with the consumer price).

The second-best coincides with the first-best whenever the latter is implementable, that is, whenever there exists transfers $\{t(\theta)\}_{\theta\in\Theta}$ satisfying (IR) and (IC). This is so in the trivial case in which the entrant suffers from such a large initial handicap that it can never win the competition, that is, when $\underline{\theta} > \Delta$. In that case, the first-best consists in allocating all the additional spectrum to the entrant, regardless of the size of its handicap, and this can trivially be implemented with null transfers $\{t(\theta)=0\}_{\theta\in\Theta}$. From now on, we focus on the case where

$$\underline{\theta} < \Delta.$$

In this case, the first-best is not implementable. Indeed, for any θ we have

$$b^{FB}\left(\underline{\theta}\right) = \hat{b}\left(\underline{\theta}\right) = \frac{\underline{\theta}+\Delta}{2} < b^{FB}(\theta) = \min\left\{\hat{b}(\theta), \Delta\right\} = \min\left\{\frac{\theta+\Delta}{2}, \Delta\right\}.$$

Hence, the regulator would have to give a subsidy to an entrant of type $\underline{\theta}$ in order to convince that type to accept $b^{FB}\left(\underline{\theta}\right) = \hat{b}\left(\underline{\theta}\right)$ (and obtain zero profit in the market) rather than $b^{FB}(\theta) > \hat{b}\left(\underline{\theta}\right)$ (and making a profit in the market). Thus, the transfer $t\left(\underline{\theta}\right)$ should satisfy

$$-t\left(\underline{\theta}\right) \geq -t(\theta) + \left[c\left(B_I+\Delta-b^{FB}(\theta)\right) - c\left(B_I-\theta+b^{FB}(\theta)\right)\right] D\left(c\left(B_I+\Delta-b^{FB}(\theta)\right)\right) > -t(\theta).$$

But then, an entrant with a higher handicap, $\theta > \underline{\theta}$ (who cannot earn any profit in the market anyway, as $b^{FB}(\underline{\theta}) < b^{FB}(\theta) \leq \hat{b}(\theta)$) would pick the offer designed for type $\underline{\theta}$, in order to pocket the subsidy (that is, to obtain $-t(\underline{\theta})$ rather than $-t(\theta)$).

The next lemma shows that the divergence of interest between the regulator and the entrant substantially limits the extent to which the regulator can make use of the private information of the entrant:

Lemma 3 *If the socially optimal allocation $b^{SB}(\theta)$ coincides with the first-best for some handicap $\hat{\theta}$, then it features full bunching: $b^{SB}(\theta) = b^{FB}\left(\hat{\theta}\right)$ for every $\theta \in \Theta$.*

Proof. See Appendix E. ■

The intuition is as follows. From a first-best perspective, the regulator wishes to allocate more bandwidth to the entrant when it suffers from a larger handicap. However, incentive compatibility implies that, if the handicap of the entrant is exactly offset for some particular type $\hat{\theta}$ (i.e., $b\left(\hat{\theta}\right) = b^{FB}\left(\hat{\theta}\right)$), then $b(\theta)$ must lie below $b\left(\hat{\theta}\right)$ for $\theta < \hat{\theta}$, and above $b\left(\hat{\theta}\right)$ for $\theta > \hat{\theta}$. The best such schedule is therefore $b(\theta) = b^{FB}\left(\hat{\theta}\right)$.

The following proposition shows that when demand is inelastic,[12] the objectives of the regulator and of the entrant are so conflicting that the same bandwidth must indeed be allocated to the entrant, regardless of its handicap:

Proposition 3 *If $\underline{\theta} < \Delta$, the second-best optimal allocation differs from the first-best allocation. Furthermore, if demand is inelastic, then the second-best optimal allocation consists of offering the same bandwidth b^{SB} to the entrant, regardless of its type, where*

$$b^{SB} \equiv \min\left\{\hat{b}^S, \Delta\right\} \left(> b^{FB}(\underline{\theta})\right),$$

with $\hat{b}^S$ denoting the unique bandwidth that lies between $b^{FB}(\underline{\theta})$ and $b^{FB}\left(\bar{\theta}\right)$ that satisfies:

$$\int_{\underline{\theta}}^{2\hat{b}-\Delta} c'\left(B_I + \Delta - \hat{b}\right) f(\theta)\, d\theta = \int_{2\hat{b}-\Delta}^{\bar{\theta}} c'\left(B_I - \theta + \hat{b}\right) f(\theta)\, d\theta.$$

Proof. See Appendix F. ■

The lessons from this analysis are two-fold. On the one hand, the qualitative insights of the previous section for the case of complete information appear robust: it is socially

[12] That is, $D(p)$ is constant in the relevant price range.

desirable to allocate some of the bandwidth to the entrant (even if the incumbent ends up serving the market), and to do so in a way that limits the cost asymmetry between the two firms. On the other hand, accounting for firms' private information leads to the adoption of somewhat coarser mechanisms, because of the conflict of interests that arises between the regulator and the firms. As a result, the second-best mechanism exhibits full "bunching:" the allocation is the same, regardless of the magnitude of the handicap. Hence, the previous insights carry over, but rely on the overall distribution of the cost handicap, rather than on its actual realization.

We show in the Online Appendix that this insight extends to the case of an elastic demand when the handicap is sufficiently diverse (namely, $\underline{\theta} < \Delta < \bar{\theta}$) and attention is, moreover, restricted to bandwidth allocations that vary continuously, or monotonically, with the handicap of the entrant (see Online Appendix A). However, incentive compatibility does allow for discontinuous as well as non-monotonous allocations (see Online Appendix B). We also provide an example where the handicap can take two, not too diverse values (namely, $\underline{\theta} < \bar{\theta} < \Delta$), in which case the optimal allocation varies with the value of the handicap (see Online Appendix C).

5 Standard Auctions

Regulators most commonly use three types of auction for allocating multiple blocks of spectrum (which may be heterogenous): sequential auctions, combinatorial clock auctions (CCAs), simultaneous multi-round ascending auctions (SMRAs). To compare these auction formats, we adopt a discretized version of our framework and assume that the additional spectrum Δ is divided into k equal blocks of size $\delta \equiv \Delta/k$; for the sake of exposition, we moreover focus on the case of complete information among the two bidders.

As regulators often allocate different tranches of spectrum at different times, we first consider sequential auctions (Section 5.1). To approximate CCAs, we then turn to simultaneous Vickrey-Clarke-Groves (VCG) auctions (Section 5.2).[13] Finally, to approximate

[13]A CCA starts with multiple clock rounds, each involving single package bids, and ends with a supplementary round with multipe package bids. VCG allocation rules then apply, except when the VCG outcome would not be in the core.

SMRAs, we discuss the case of simultaneous clock auctions (Section 5.3).[14]

We show that the outcomes of these auction formats depart drastically from the optimal allocations characterized above: while all the available spectrum is allocated, it is likely to go to the incumbent, thus reinforcing its initial advantage.

5.1 Sequential Auctions

We start with the case of sequential auctions. Specifically, we assume that k successive auctions are organized, one for each of block, and that the outcome of each auction is publicly announced before the next auction takes place. As bidders have perfect information about each other, all classic auction formats (first-price or second-price sealed-bid auctions, as well as ascending or descending auctions) yield the same outcome. For the sake of exposition, we will refer to these auctions as "classic auctions". It is well-known that these auctions can generate multiple equilibria, as the losing bidder may bid more than its value without incurring a loss; to address this issue, we focus on coalition-proof Nash equilibria – see Bernheim et al. (1987); in our two-bidder setting, this amounts to focusing on Pareto-undominated Nash equilibria.

The following proposition shows that the incumbent wins all the auctions, and may even do so at zero price if the initial handicap of the entrant exceeds the size of the individual blocks:

Proposition 4 *Suppose that k blocks are sold sequentially using any classic auction format. At any coalition-proof Nash equilibrium, the incumbent wins all k auctions; furthermore, if $B_I - B_E > \Delta/k$, then the incumbent acquires each block for free.*

Proof. See Appendix G. ■

The intuition is simple, and reminiscent of the insight of Vickers (1986) for patent races.[15] When a monopolistic incumbent bids against a potential entrant for a better technology, the incumbent's gain from preserving its monopoly position (and enjoying the better technology) exceeds the profit that the entrant would obtain in a duopoly

[14] In SMRAs and clock auctions, prices increase across multiple rounds of bidding. The main difference is that prices are set by bidders in SMRAs, and by the auctioneer in clock auctions. With homogenous blocks and symmetric information, the two auctions result in essentially the same outcome.

[15] Also see Gilbert and Newbery (1982) and Riordan and Salant (1994).

situation, even if it benefits from a better technology than the incumbent. Likewise, here the incumbent, who by assumption is the initial leader, gains more from maintaining its incumbency advantage than an entrant gains from overtaking the incumbent.

Proposition 4 moreover shows that, with sequential auctions, the incumbent can obtain all the additional spectrum for free, if this spectrum is divided into sufficiently many blocks. However, when the size of the blocks exceeds the initial handicap of the entrant, the incumbent pays for the first block a price equal to:

$$p^S(B_I, B_E) = \begin{cases} \sum_{h=0}^{m-1} \phi_h^k(B_E, B_I) - \sum_{h=1}^{m} \phi_h^k(B_I, B_E) & \text{if } k = 2m, \\ \sum_{h=0}^{m} \phi_h^k(B_E, B_I) - \sum_{h=1}^{m} \phi_h^k(B_I, B_E) & \text{if } k = 2m+1. \end{cases}$$

where

$$\phi_h^k(B_1, B_2) \equiv \Pi(B_1 + (k-h)\delta, B_2 + h\delta).$$

In the particular case where $B_I = B_E = B$, both firms bid the full benefit generated by the additional spectrum and the equilibrium price is thus equal to

$$p^S(B, B) = \Pi(B + \Delta, B).$$

5.2 VCG Auctions

This section considers a single, simultaneous VCG auction for all k blocks, in which each bidder submits a sealed bid demand schedule specifying how much it would offer for every number of blocks it may wish to purchase. That is:

- Each firm $i = I, E$ submits a bid of the form[16]

$$\beta_i = \{\beta_i(n_I, n_E)\}_{(n_I, n_E) \in \mathcal{A}},$$

where $n_i \in \mathcal{K} \equiv \{0, 1, 2, ..., k\}$ denotes the number of blocks assigned to firm $i \in$

[16]In theory, bids should be made for each allocation (n_I, n_E). In practice, firm i is often asked to submit bids for the various combinations of slots assigned to it (that is, $\beta_i = \{\beta_i(n_i)\}_{n_i \in \mathcal{K}}$). However, in our simple two-bidder setting, in which all k blocks are always allocated, the distinction is moot.

$\{I, E\}$, and

$$\mathcal{A} \equiv \{(n_I, n_E) \in \mathcal{K} \times \mathcal{K} \mid n_I + n_E \leq k\}.$$

- The resulting spectrum allocation, $n^V(\beta_I, \beta_E) = \left(n_I^V(\beta_I, \beta_E), n_E^V(\beta_I, \beta_E)\right)$ maximizes the sum of the offers over feasible allocations, i.e.,

$$n^V(\beta_I, \beta_E) = \arg\max_{n \in \mathcal{A}} \{\beta_I(n) + \beta_E(n)\}.$$

- Finally, the price paid by each bidder i is the value that the other bidder would offer for bidder i's blocks, and is thus equal to (where the subscript "$-i$" refers to firm i's rival)

$$p_i^V(\beta_I, \beta_E) = \max_{n \in \mathcal{A}} \{\beta_{-i}(n)\} - \beta_{-i}\left(n^V(\beta_I, \beta_E)\right).$$

It is well-known that it is a dominant strategy for each firm to bid its full value for each package. The following proposition shows that, in this equilibrium, the incumbent again wins all the blocks. However, it always pays a positive price whenever the additional spectrum is large enough to offset the handicap of the entrant:

Proposition 5 *In a simultaneous VCG auction for the k blocks, the incumbent wins all the blocks and pays a price equal to the entrant's profits from winning all the blocks:*

$$p^V(B_I, B_E) = \begin{cases} \Pi(B_E + \Delta, B_I) & \text{if } \Delta > B_I - B_E, \\ 0 & \text{otherwise.} \end{cases}$$

Proof. See Appendix H. ■

The underlying logic is the same as for sequential auctions, and leads again to allocate all the additional spectrum to the incumbent. Furthermore, if the additional bandwidth is not large enough to fully offset the initial handicap of the entrant, then revenues are zero in both auctions. Otherwise, Proposition 5 shows that the incumbent must now pay a positive price, independent of the block size. The following proposition moreover shows that this price is typically higher than in sequential auctions:

Proposition 6 *Revenues are always at least as high in a VCG auction than in a sequential auction, and strictly higher in the case where* $\Delta > B_I - B_E$*; furthermore, VCG revenues are independent of the block size, whereas a sequential auction brings no revenue if the size of the blocks is sufficiently small.*

Proof. See Appendix I. ■

5.3 Clock Auctions

Now consider a two-bidder clock auction. The auctioneer posts a price per block, and bidders announce how many blocks they want at that price. The posted price is initially set to zero and increases by increments as long as there is excess demand; as price increases, bidders can maintain or reduce the number of blocks they demand, but not augment it. When the market clears, each bidder obtains its desired number of blocks at the clearing price.[17]

In this auction, it is a dominant strategy for the entrant to bid for all k blocks as long as the clock price p satisfies

$$p < p^E \equiv \frac{\Pi\left(B_E + \Delta, B_I\right)}{k},$$

and to drop out once the posted price tops p^E. By contrast, the dominant strategy of the incumbent is to bid for all k blocks as long as the clock price p satisfies

$$p < p^I \equiv \frac{\Pi\left(B_I + \Delta, B_E\right)}{k}.$$

As $p^I > p^E$ whenever $B_I > B_E$, the incumbent wins the clock auction at price p^E, which is the same outcome as with a VCG auction. Thus, we have:

Proposition 7 *Auction outcomes are the same with a simultaneous VCG auction and a clock auction.*

[17] In case demand abruptly drops below the clearing level, various tie-breaking rules apply, which often involve a random element.

6 Policy Implications

The above analysis shows that the outcome of standard auctions typically differs drastically from the optimal allocation. While we focus for simplicity on a setting in which a single incumbent faces a single challenger, these insights can shed some light on regulators' actual policy choices. In a number of recent spectrum auctions, regulators have faced quite explicit choices between running a competitive auction and avoiding consolidation in the market. This is now the case in Switzerland, in which three competitors are vying for the 30 MHz available in the 700 MHz band (a relevant band for 5G networks).[18] The smallest challenger, SALT, stated that "with less than 10 MHz [in that band] no competitive and nationwide 5G network can be operated", thus suggesting that, in order to maintain competition, the regulator should allocate one third of the spectrum to each incumbent.[19]

This section discusses regulatory instruments and policy to address this issue.

6.1 Regulatory instruments

Regulators can adopt different measures to promote competition – the most common being spectrum caps, set-asides and bidding preferences or discounts. A spectrum cap is a limit on the total amount of spectrum a firm can have. A set-aside reserves some spectrum for target groups such as "challengers" or entrants. A bidding preference provides a discount off the final auction price to such challengers. Each of these provisions can take different forms. For example, a cap can be on the amount available in the auction or on overall spectrum holdings, including what bidders have prior to the auction. A set-aside can also take the form of floors, i.e., minimum spectrum packages.[20]

[18] A similar situation arose in the US Incentive Auction, where the regulator (the FCC) had to determine how much spectrum to reserve for Sprint and T-Mobile, the two firms with limited holdings; the FCC reserved a spectrum adequate for one, but not both, of the two firms: the reserve was limited to three blocks, and each laggard needed to win two blocks in order to become economically efficient. In a 2013 auction, the Austrian regulator had to decide on caps in the allocation of 28 blocks among three incumbents. The adopted cap of 14 blocks resulted in very high revenues and nearly forced market consolidation.

[19] See https://www.handelszeitung.ch/unternehmen/telekommunikation-vertragswidrig-salt-pruft-klage-gegen-upc. At the time of writing, the regulator's proposal ensures that each of the two smaller operators could obtain 5MHz, and that at least one could get 10MHz.

[20] For instance, floors have been used in the UK – see https://www.ofcom.org.uk/about-ofcom/latest/media/media-releases/2011/ofcom-prepares-for-4g-mobile-auction; for a discussion of this case, see Myers (2013).

Caps and set-asides impose similar restrictions on the set of feasible allocations. To see this, let $B_E \geq 0$ and $B_I > B_E$ denote, as before, the amount of spectrum initially owned by the entrant and the incumbent, and by Δ the additional amount made available. Introducing an overall cap K limits the additional bandwidth b_i that firm i can obtain, that is:

$$B_i + b_i \leq K.$$

Obviously, a cap K has no effect if it exceeds $B_I + \Delta \,(> B_E + \Delta)$. Introducing instead a "binding" cap $K < B_I + \Delta$ *de facto* reserves a bandwidth

$$S(K) \equiv \Delta - (K - B_I)$$

for the entrant. However, compared with the imposition of such a cap K, introducing a set-aside $S(K)$ further restricts the set of feasible allocations, as it also prevents the entrant from having less than $S(K)$ of additional bandwidth; that is, both instruments can be used to put the same *upper* bound on I's share of the additional spectrum, b_I, but in addition a set-aside puts a *lower* bound on E's share of this spectrum, b_E. As a result, as discussed below, caps and set-asides have different impacts on the outcome of an auction.

Some auctions, such as the early US auctions, included spectrum caps on overall spectrum holdings, of the form $B_i + b_i \leq K$, as discussed above.[21] Other auctions have used caps on the amount of spectrum each firm can acquire in the auction, of the form $b_i \leq k$. However, these two variants can have very different effects on product market competition. Indeed, whereas an overall cap can be effective in limiting the acquisition of additional spectrum by the incumbent, an auction cap puts more stringent limitations on the entrant, and may actually keep it from overtaking the incumbent.

Set-asides are often accompanied by reserve prices, which tend to discourage entry – in our Bertrand setting, any positive reserve price would deter entry. Entry was indeed discouraged in the 2013 4G auctions in Austria and limited in the UK, which relied on various forms of set-asides accompanied by reserve prices. In the same vein, while our analysis suggests that all spectrum should be allocated, this does not always occur in practice, as reserve prices sometimes result in unsold blocks. For instance, this has been

[21]See https://www.fcc.gov/node/189694 for a discussion of US spectrum caps.

the case in 4G auctions in Spain and Portugal, in which some of the most valuable (900 MHz) spectrum remained unsold.[22]

When the initial handicap of the entrant is very large, it is optimal to allocate all the additional spectrum to the entrant. Overall caps, set-asides and bidding credits can all be used to accomplish this.[23] In what follows, we concentrate on the more interesting case where the regulator finds it optimal to share the additional spectrum between the two firms.

Consider first the case where the regulator focuses on consumer surplus. From Proposition 1, it is optimal to equalize the costs of the two firms; that is, the optimal allocation $\left(b_I^S, b_I^S\right)$ is such that:

$$c_I\left(B_I + b_I^S\right) = c_E\left(B_E + b_E^S\right) = \hat{c} = c\left(\frac{B_I + B_E + \Delta}{2}\right).$$

This could be achieved with an overall cap set to $K^S = \left(B_I + B_E + \Delta\right)/2$. By contrast, a set-aside $S^S = S\left(K^S\right)$ would not work, as it would put the entrant ahead of the incumbent (as $B_E + S^S \left(= K^S\right) > B_I$), and thus result in the entrant winning all the additional spectrum. A bidding credit would not be effective either, as it would result in either firm winning all the additional spectrum (I if the bidding credit is too low, and E otherwise).

When instead the regulator also cares about revenues, Proposition 2 applies, and two cases arise.

• If the handicap of the entrant and/or the weight placed on revenues is not too large, then there are two optimal allocations, which: (i) confer a competitive advantage to one firm and lead to the same consumer price, p^W – more specifically, one firm (the "loser") ends-up with an overall holding equal to $B_l^W \equiv c^{-1}\left(p^W\right)$, whereas the other firm (the "winner") accumulates a total amount of spectrum equal to $B_w^W \equiv c^{-1}\left(\gamma\left(p^W\right)\right) > B_l^W$;

[22]See http://www.minetad.gob.es/telecomunicaciones/es-ES/ResultadosSubasta/Informe_Web_29072011_fin_de_subasta.pdf and http://www.anacom.pt/streaming/Final_Report_Auction.pdf?contentId=1115304&field=ATTACHED_FILE for results of the 2011 Spanish and Portuguese multi-band auctions. Also, in France, the regulator took a number of years to reduce the reserve price before awarding a fourth 3G license. See https://www.arcep.fr/?id=8562.

[23]Specifically, an overall cap $K = B_I$, a set-aside $S = \Delta$, or a bidding credit reflecting I's profit with the additional spectrum, would all result in E winning all the additional spectrum. When the regulator cares about revenues as well, however, this would need to be complemented with a tax (e.g., an unconditional tax on I's equilibrium profit).

and: (ii) appropriate the winner's equilibrium profit, $\pi^W \equiv \left(p^W - c\left(K^W\right)\right) D\left(p^W\right)$. This optimal allocation could be achieved by setting aside an amount $S^W = B_I - B_E$ for the entrant, and introducing an overall cap set to $K^W = B_w^W$. The set-aside is designed to offset the initial handicap of the entrant; as a result, both firms are willing to bid up to π^W to reach the overall cap.[24] Interestingly, neither instrument alone suffices to achieve a desired outcome. Relying only on a set-aside would again result in one firm (either one) winning all the additional spectrum (the same would be obtained with a bidding credit). Relying only on a cap could achieve the desired spectrum allocation (by setting the cap to K^W), but it would leave a positive rent to the incumbent;[25] a second instrument would be required to deal with this issue.[26]

• If the handicap of the entrant and/or the weight placed on revenues is large enough, the unique optimal allocation is such that $c_E^W = c\left(B_l^W\right) > \max\{\bar{c}_I, \underline{c}_E\}$, and thus satisfies: $B_E^W = B_l^W < B_I$ – that is, it is no longer optimal to offset the initial handicap of the entrant. In this case, setting aside an amount $S^W = B_l^W - B_E$ for the entrant, or alternatively introducing a cap set to $K^W = B_w^W$, would both achieve the desired spectrum allocation. However, either instrument would again need to be complemented with another instrument designed to limit the rent left to the incumbent.[27]

6.2 Regulatory Experience

Most countries have adopted auctions and other spectrum assignment procedures, including caps and set-asides, designed to promote *ex post* competition in the market for mobile communications services. However, these provisions have tended to have limited long-run impact, once the first set of spectrum allocations was completed.

The three initial waves of spectrum allocations resulted in 4 - 5 mobile operators in

[24]Consider, for instance, a simple clock auction with the set-aside S^W and the overall cap K^W (with the rule that any unbid spectrum is allocated to the losing bidder). As E secures an amount $B_I - B_E$ for free, both firms are then willing to bid up to $\pi^W / \left(K^W - B_I\right)$ to obtain the additional amount $K^W - B_I$. As a result, either firm wins and accumulates an overall holding of $K^W = B_w^W$, and the other firm obtains an overall holding of B_l^W, leading to the market price p^W.

[25]For instance, in a simple clock auction I would be willing to bid up to $\pi^W / \left(K^W - B_I\right)$, whereas E is only willing to bid up to $\pi^W / \left(K^W - B_E\right)$. The conclusion follows from $B_E < B_I$.

[26]This, for instance, could be an unconditional tax on I's profit, as described in footnote 23. Alternatively, it could take the form of a reserve price – a non-linear or discriminatory reserve price may however be needed.

[27]Combining a set-aside with a cap no longer suffices here to achieve the desired allocation *and* to appropriate the profit π^W; that is, the additional instrument should again be a tax on the incumbent's equilibrium profit, and/or some form of a reserve price.

each European country and at least 4 operators in almost all of the US and Canada,[28] and often as many as 5 or 6.[29] Since the 3G auctions, however, consolidation has been the rule in much of Europe, including in Austria, Germany, Italy, the Netherlands, Switzerland and the UK, despite various measures employed to promote competition.[30] The Netherlands set aside two prime, low frequency 4G blocks for entrants,[31] which did attract two new bidders; however, only one entrant won any blocks, and after the auction it signed a network-sharing agreement with the one incumbent that failed to win any low frequency blocks. Austria failed to attract any bidders for the two blocks provisionally set aside for entrants. The UK's provisions for a floor mentioned in footnote 20 also failed to induce meaningful changes in the competitive structure.

The US, too, has seen continuing consolidation.[32] Since the FCC abandoned overall spectrum caps in 2003, the two largest MNOs have acquired most of the spectrum that has been auctioned. The HHI has increased from 2151 in 2003 to 3027 at the end of 2013. In a very recent auction for AWS-3 spectrum, AT&T and Verizon spent 6 and 10 times as much as the third largest MNO in the auction (T-Mobile), and no other MNO spent even 1% of what AT&T spent. In the most recent 600 MHz auction, the reserve price of $1.25 per MHzPOP[33] has deterred the weakest incumbent, Sprint, from even participating.[34]

[28]For the first generation of analog cellular in the Americas and in Europe, regulators tended to award one license to the incumbent local exchange carrier and one to an entrant. Additional operators entered during the second and third waves of spectrum allocations, starting in the late 1980's and running through the early 2000's (first for 2G, and then for 3G spectrum).

[29]The US and Canada, unlike European countries, awarded regional and not national licenses.

[30]The UK had 5 mobile network operators ("MNOs"), but even after set-asides in the recent 4G auction, only 4 remain, and there is talk of further consolidation. Germany had 6 winners after their 3G auction, but two winners abandoned their licenses, and subsequent to a recent merger, there are now only 3 MNOs left. The Netherlands had 5 MNOs, but mergers resulted in 3 players. The Austrian 3G auction had 6 winners. By the time the Austrians auctioned off 4G spectrum, a little over 10 years after the 3G auctions, there were only three MNOs remaining in the market. Finally, the 2013 re-auction of legacy spectrum in Austria nearly left Austria with two viable MNOs. See Salant (2014)) for a discussion.

[31]By contrast, Germany turned down requests of entrants for set-asides.

[32]Among the large regional and national carriers that at one time existed: (i) Cingular, BellSouth, Ameritech, and Leapwireless have all been absorbed by AT&T; (ii) BellAtlantic, NYNEX, USWest, Airtouch , GTE, Cincinnati Bell, and Alltel have been absorbed by Verizon; (iii) Western Wireless, Voicestream, Omnipoint and Powertel formed T-Mobile, which subsequently acquired MetroPCS; (iv) Sprint merged with Nextel; and (v) US Cellular is still independent, but has sold off most of its larger markets.

[33]The term "per MHzPOP" is used to compare prices of different sized blocks in different countries or regions. Literally, one MHzPOP is a license of one MHz covering an area with a population of one.

[34]In addition, the spectrum reserved for challengers sold eventually for essentially the same price than the non-reserved spectrum - less than 1% overall difference, and, in nearly 20% of the PEAs the

Finally, Canada at one time had 4 national operators, which was reduced to 3 via merger. Despite having conducted a series of auctions with provisions including caps and regulations on wholesale prices, in the hope of attracting more competitors, no fourth national operator has emerged.

7 Conclusion

This paper characterizes the optimal allocation of a scarce resource (e.g., spectrum rights) between an incumbent and a challenger, for a regulator seeking to maximize the social surplus. The main insight is that the regulator wants to limit the dominance of the incumbent, and ensure that the challenger exerts an effective competitive pressure. More specifically, when the regulator focuses on consumer surplus, and does not care about auction revenues, it tries to equalize firms' competitiveness. When instead the regulator seeks to maximize social welfare, taking into account auction revenues, it finds it optimal to maintain some asymmetry among the competitors.

Further, we find that there is a tension between the regulator's objective and the challenger's incentives to report its handicap. More specifically, a regulator may want to provide a weaker challenger more spectrum than a stronger challenger. However, doing so gives the stronger challenger an incentive to try to act as if it is weak. As a result of this tension, the optimal auction is likely to exhibit "bunching", in that the challenger ends up with the same allocation, regardless of its initial handicap.

The finding that the regulator wishes to limit dominance contrasts sharply with the outcome of standard types of auctions, such as sequential, clock and VCG auctions, which all result in increasing dominance: in the Bertrand competition setting that we consider, the incumbent always obtains all the additional spectrum.[35] Furthermore, while the spectrum allocations are the same, revenues are lower in a sequential auction.

Finally, we examine some policy implications. When the regulator's objective only includes consumer surplus, a cap on firms' overall spectrum holdings can suffice to achieve the desired allocation. By contrast, neither a cap on the amount of spectrum that any firm can win in the auction, nor a set-aside reserved for the challenger are

reserved spectrum was more expensive than the non-reserved spectrum.

[35] This may not always hold when firms compete à la Cournot competition, in which case the packaging of blocks can also affect the outcome.

helpful – auction-specific caps could actually be counter-productive, as they may limit the challenger's ability to reduce its handicap. When the regulator also cares about auction revenues, an overall cap needs to be complemented with a set-aside or with another instrument designed to limit the incumbent's rent.

While this paper has focused on spectrum auctions, similar issues arise in many other sectors. We discuss a few below.

Sports broadcasting rights.

Sports broadcasting rights, and, in particular, soccer rights are often regulated, especially in Europe. The reason for competition authorities to intervene is the concern that a concentration of broadcasting rights could create or reinforce the dominance of the rights holder.[36] Indeed, the fraction of rights owned by a provider affects the perceived value of its offering relative to that of its rivals. Suppose, for example, that the value a consumer derives from provider i's offering is of the form:

$$u + v(r_i),$$

where r_i denotes the fraction of rights own by firm i. Formally, the increase in firm i's perceived quality stemming from an increase in r_i then has the same effect as the cost reduction resulting from an increase in the bandwidth b_i in our model.

Train scheduling.

Another application is the allocation of train slots. The frequency of service offered on a given route will affect the average wait time, and thus the cost imposed on customers. Many riders, e.g., because they purchased their tickets in advance or because they benefit from loyalty programs, will therefore favor the carrier offering the most frequent service. Suppose, for example, that the net value offered by firm i is of the form:

$$u - c(s_i),$$

[36] See, e.g., `http://ec.europa.eu/competition/elojade/isef/case_details.cfm?proc_code=1_38173`.

where s_i denotes the number of train slots allocated to firm i. The reduction in customers' costs stemming from an increase in s_i then has the same effect as the reduction in firm i's own operating cost in our model.

Electric transmission.

Electricity transmission rights raises similar issues in regions in which energy supply is limited and relatively costly.[37] The particular details of electricity markets differ quite a bit from spectrum,[38] but the basic message that the allocation of transmission rights can affect post-auction competition applies.

Similar issues arise with the allocation of many other scarce resources, such as landing slots (and other airport facilities, such as gates, kerosene tanks, and so forth), gas pipeline capacity, concessions to operate in given areas (e.g., highway service stations), or when zoning regulations put constraints on commercial activities or on the number (and/or the size) of supermarkets.[39]

[37]See Joskow and Tirole (2000) and Loxley and Salant (2004).

[38]See for example `http://pjm.com/markets-and-operations/ftr.aspx`, and also Salant (2005).

[39]In France, for instance, zoning regulations have prevented the entry of new retail chains.

Appendix

A Proof of Proposition 1

It is obviously optimal to allocate all the additional spectrum: Allocating any residual bandwidth equally among the two firms reduces for sure the resulting competitive price and thus benefits consumers.

Without loss of generality, we can thus restrict attention to spectrum allocations of the form $b_E \in [0, \Delta]$, $b_I = \Delta - b_I$, yielding a competitive price equal to

$$\begin{aligned} p &= \max\left\{c\left(B_E + b_E\right), c\left(B_I + b_I\right)\right\} \\ &= \max\left\{c\left(B_E + b_E\right), c\left(B_I + \Delta - b_E\right)\right\}. \end{aligned}$$

Maximizing consumer surplus amounts to minimizing this competitive price; therefore:

- If $\Delta \geq B_I - B_E$, there is enough spectrum to offset the initial cost asymmetry; the optimal spectrum allocation thus equates the costs of the two firms:

  $$b_E = \hat{b} \equiv \frac{B_I - B_E + \Delta}{2} \text{ and } b_I = \Delta - \hat{b},$$

  leading to an equilibrium price equal to (where the superscript S refers to consumer surplus):

  $$p^S = c_I = c_E = \hat{c}.$$

- If instead $\Delta < B_I - B_E$, there is not enough spectrum to offset the initial cost disadvantage of the entrant; to minimize this disadvantage, it it then optimal to give all the additional spectrum to the entrant:

  $$b_E = \Delta \text{ and } b_I = 0,$$

  leading to

  $$p^S = c_E = \underline{c}_E = c\left(B_E + \Delta\right) > c_I = \bar{c}_I = c\left(B_I\right).$$

B Proof of Lemma 2

For $p \in [\max\{\hat{c}, \underline{c}_E\}, \bar{c}_E]$, let $b_E \in \left[0, \hat{b}\right]$ and $b_I = \Delta - b_E$ denote the spectrum allocation such that $c_E = c(B_E + b_E) = p$ and $c_I = c(B_I + b_I) = \gamma(p)$. We have:

$$\gamma'(p) = \gamma'(c_E) = -\frac{c'(B_I + B_E + \Delta - c^{-1}(c_E))}{c'(c^{-1}(c_E))} = -\frac{c'(B_I + b_I)}{c'(B_E + b_E)} < 0$$

and

$$\gamma''(p) = \gamma''(c_E) = \frac{c''(B_I + b_I) + \frac{c'(B_I+b_I)}{c'(B_E+b_E)} c''(B_E + b_E)}{\left[c'(B_E + b_E)\right]^2} \geq 0,$$

where the inequality follows from Assumption $A.2$.

C Proof of Proposition 2

The derivative of the welfare function $W(p)$ can be expressed as

$$W'(p) = -(1 + \lambda) D'(p) \phi(p), \tag{9}$$

where the function $\phi(p)$, defined by (8), is positive for $p = \hat{c}$, as then $\gamma(p) = p = \hat{c}$ and thus

$$\phi(\hat{c}) = \left[\frac{\lambda}{1+\lambda} - \gamma'(\hat{c})\right] \mu(\hat{c}) \geq -\gamma'(\hat{c}) \mu(\hat{c}) > 0,$$

where the first inequality stems from $\lambda \geq 0$ and the second one from $\mu > 0$ and $\gamma' < 0$ (from Lemma 2). The function $\phi(p)$ is moreover strictly decreasing in the relevant range $p \in [\max\{\underline{c}_E, \hat{c}\}, \bar{c}_E]$:

$$\phi'(p) = \gamma'(p) - \gamma''(p) \mu(p) + \left[\frac{\lambda}{1+\lambda} - \gamma'(p)\right] \mu'(p) - 1 < 0, \tag{10}$$

where the inequality stems from $\gamma' < 0$, $\gamma'' \geq 0$ (from Lemma 2) and $\mu' \leq 0$ (from Assumption A). Therefore:

- If $\phi(\bar{c}_E) \geq 0$, then welfare strictly increases with p as long as $c_E < \bar{c}_E$, and thus

$$p^W = \bar{c}_E \, (> \hat{c}).$$

That is, it is optimal to allocate all the additional bandwidth to the incumbent.

- If $\phi(\underline{c}_E) \leq 0$ (which is possible only if $\underline{c}_E > \hat{c}$, as $\phi(\hat{c}) > 0$), then welfare strictly decreases with p in the feasible range $p \in [\underline{c}_E, \bar{c}_E]$, and thus

$$p^W = \underline{c}_E \, (> \hat{c}) \, .$$

That is, it is optimal to allocate all the additional bandwidth to the entrant.

- Otherwise, the first-order condition $\phi(p) = 0$ has a unique solution, p^W, lying (strictly) between $\max\{\hat{c}, \underline{c}_E\}$ and $\bar{c}_E$ (it thus satisfies $p^W > \hat{c}$); from (9) and (10), social welfare moreover increases with p in the range $p < p^W$, whereas it decreases as p further increases in the range $p > p^W$. It follows that social welfare is maximal for $p = p^W$.

D Proof of Corollary 1

Building on the previous analysis, and using

$$\phi(p; \lambda) \equiv \gamma(p) + \left[\frac{\lambda}{1+\lambda} - \frac{\partial \gamma}{\partial p}(p)\right] \mu(p) - p,$$

the optimal price can be expressed as $p^W = p_\phi(\lambda)$, where $p_\phi(\lambda)$ is the unique solution to:

$$\phi(p; \lambda) = 0.$$

Differentiating this condition with respect to p and λ yields:

$$\frac{\partial p^W}{\partial \lambda}(\lambda) = \frac{\frac{\partial \phi}{\partial \lambda}\left(p^W, \lambda\right)}{-\frac{\partial \phi}{\partial p}\left(p^W, \lambda\right)},$$

where the denominator is positive and the numerator is equal to

$$\frac{\partial \phi}{\partial \lambda}\left(p^W, \lambda\right) = \mu\left(p^W\right) \frac{d}{d\lambda}\left(\frac{\lambda}{1+\lambda}\right) > 0.$$

It follows that p^W strictly increases with λ. When the optimal spectrum allocation maintains a cost advantage to the incumbent, this implies a re-allocation of Δ which further favors the incumbent.

Turning to the impact of bandwidth, and using

$$\gamma(p;B) \equiv c\left(B - c^{-1}(p)\right)$$

and

$$\phi(p;B) \equiv \gamma(p;B) + \left[\frac{\lambda}{1+\lambda} - \frac{\partial \gamma}{\partial p}(p;B)\right]\mu(p) - p,$$

the optimal price can be expressed as $p^W = p_\phi(B)$, where $p_\phi(B)$ is the unique solution to

$$\phi(p;B) = 0. \tag{11}$$

The optimal price thus only depends on total available bandwidth, $B = B_I + B_E + \Delta$. In addition,

$$\frac{\partial p^W}{\partial B} = -\frac{\frac{\partial \phi}{\partial B}}{\frac{\partial \phi}{\partial p}}\left(p^W, B\right),$$

where

$$\begin{aligned}
\frac{\partial \phi}{\partial B}\left(p^W, B\right) &= \frac{\partial \gamma}{\partial B}\left(p^W; B\right) - \frac{\partial^2 \gamma}{\partial B \partial p}\left(p^W; B\right)\mu\left(p^W\right) \\
&= c'\left(B - c^{-1}\left(p^W\right)\right) + \frac{c''\left(B - c^{-1}\left(p^W\right)\right)}{c'\left(c^{-1}\left(p^W\right)\right)}\mu\left(p^W\right) \\
&= c'(S_I) + \frac{c''(S_I)}{c'(S_E)}\mu\left(c\left(S_E\right)\right),
\end{aligned}$$

where $S_I = B_I + b_I$ and $S_E = B_E + b_E$, respectively, denote the overall amount of spectrum eventually assigned to the incumbent and to the entrant, assuming that the incumbent is favored when the optimal price p^W can be achieved in two symmetric ways, and

$$\begin{aligned}
\frac{\partial \phi}{\partial p}\left(p^W, B\right) &= \left[\frac{\lambda}{1+\lambda} - \frac{\partial \gamma}{\partial p}\left(p^W; B\right)\right] \mu'\left(p^W\right) - 1 \\
&\quad + \frac{\partial \gamma}{\partial p}\left(p^W; B\right) - \frac{\partial^2 \gamma}{\partial p^2}\left(p^W; B\right) \mu\left(p^W\right) \\
&= \left[\frac{\lambda}{1+\lambda} - \frac{\partial \gamma}{\partial p}\left(p^W; B\right)\right] \mu'\left(p^W\right) - 1 - \frac{c'\left(B - c^{-1}\left(p^W\right)\right)}{c'\left(c^{-1}\left(p^W\right)\right)} \\
&\quad - \left\{\frac{c''\left(B - c^{-1}\left(p^W\right)\right)}{\left[c'\left(c^{-1}\left(p^W\right)\right)\right]^2} + \frac{c'\left(B - c^{-1}\left(p^W\right)\right) c''\left(c^{-1}\left(p^W\right)\right)}{\left[c'\left(c^{-1}\left(p^W\right)\right)\right]^3}\right\} \mu\left(p^W\right) \\
&= \left[c\left(S_E\right) - c\left(S_I\right)\right] \frac{\mu'\left(c\left(S_E\right)\right)}{\mu\left(c\left(S_E\right)\right)} - 1 - \frac{c'\left(S_I\right)}{c'\left(S_E\right)} - \left\{\frac{c''\left(S_I\right)}{\left[c'\left(S_E\right)\right]^2} + \frac{c'\left(S_I\right) c''\left(S_E\right)}{\left[c'\left(S_E\right)\right]^3}\right\} \mu\left(c\left(S_E\right)\right),
\end{aligned}$$

where the last equality uses condition (11) and the fact that, by construction, $p^W = c_E = c\left(S_E\right)$ and $\gamma\left(c_E\right) = c_I = c\left(S_I\right)$. It follows that the derivative of p^W with respect to total bandwidth can be expressed as:

$$\begin{aligned}
\frac{\partial p^W}{\partial B} &= -\frac{\frac{\partial \phi}{\partial B}}{\frac{\partial \phi}{\partial p}}\left(p^W, B\right) \\
&= -\frac{c'\left(S_I\right) + \frac{c''\left(S_I\right)}{c'\left(S_E\right)} \mu\left(c\left(S_E\right)\right)}{-\frac{c'\left(S_I\right)}{c'\left(S_E\right)} - \left\{\frac{c''\left(S_I\right)}{\left[c'\left(S_E\right)\right]^2} + \frac{c'\left(S_I\right) c''\left(S_E\right)}{\left[c'\left(S_E\right)\right]^3}\right\} \mu\left(c\left(S_E\right)\right) + \left[c\left(S_E\right) - c\left(S_I\right)\right] \frac{\mu'\left(c\left(S_E\right)\right)}{\mu\left(c\left(S_E\right)\right)} - 1} \\
&= \frac{c'\left(S_E\right)\left[c'\left(S_I\right) + \frac{c''\left(S_I\right)}{c'\left(S_E\right)} \mu\left(c\left(S_E\right)\right)\right]}{c'\left(S_I\right) + \frac{c''\left(S_I\right)}{c'\left(S_E\right)} \mu\left(c\left(S_E\right)\right) + \frac{c'\left(S_I\right) c''\left(S_E\right)}{\left[c'\left(S_E\right)\right]^2} \mu\left(c\left(S_E\right)\right) + c'\left(S_E\right)\left\{1 - \left[c\left(S_E\right) - c\left(S_I\right)\right] \frac{\mu'\left(c\left(S_E\right)\right)}{\mu\left(c\left(S_E\right)\right)}\right\}} \\
&= \frac{c'\left(S_E\right)}{1 + A}
\end{aligned}$$

with

$$A = \frac{\left[c'\left(S_E\right)\right]^2\left\{1 - \left[c\left(S_E\right) - c\left(S_I\right)\right] \frac{\mu'\left(c\left(S_E\right)\right)}{\mu\left(c\left(S_E\right)\right)}\right\} + \frac{c'\left(S_I\right)}{c'\left(S_E\right)} c''\left(S_E\right) \mu\left(c\left(S_E\right)\right)}{c'\left(S_E\right) c'\left(S_I\right) + c''\left(S_I\right) \mu\left(c\left(S_E\right)\right)} > 0,$$

where the inequality follows from the fact that, in the numerator and the denominator, the first terms are positive and the second terms are non-negatives. Using

$$p^W = c\left(S_E\right),$$

We have:

$$0 < \frac{\partial S_E}{\partial B} = \frac{1}{1+A} < 1.$$

Therefore:

- An increase in Δ leads to an increase in both b_E (as $\partial S_E/\partial B > 0$) and b_I (as $\partial S_E/\partial B < 1$).
- An increase in B_I leads to an increase in b_E (as $\partial S_E/\partial B > 0$) and a reduction in b_I (as $\partial S_E/\partial B < 1$).
- An increase in B_E leads to an increase in b_I (as $\partial S_E/\partial B < 1$) and thus to a reduction in b_E (as $b_E + b_I = \Delta$).

E Proof of Lemma 3

From the revelation principle, we can restrict attention to direct incentive-compatible mechanisms (DICMs for short) of the form $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$. We first show that if a DICM satisfies $b\left(\tilde{\theta}\right) \leq \hat{b}\left(\tilde{\theta}\right)$ for some $\tilde{\theta} \in \Theta$, then the profile $\{b(\theta)\}_{\theta\in\Theta}$ remains (weakly) below $\hat{b}\left(\tilde{\theta}\right)$ for any θ higher than $\tilde{\theta}$:

Lemma 4 *For any given $\tilde{\theta} \in \Theta$, if a DICM satisfies $b\left(\tilde{\theta}\right) \leq \hat{b}\left(\tilde{\theta}\right)$ then, for any $\theta > \tilde{\theta}$, the DICM satisfies $b(\theta) \leq \hat{b}\left(\tilde{\theta}\right)$.*

Proof. Consider a DICM satisfying $b\left(\tilde{\theta}\right) \leq \hat{b}\left(\tilde{\theta}\right)$ for some $\tilde{\theta} \in \Theta$ (implying that the entrant is expected to lose the competition in the market when of type $\tilde{\theta}$), and suppose that there exists $\theta > \tilde{\theta}$ such that $b(\theta) > \hat{b}\left(\tilde{\theta}\right)$ (implying that the entrant of type $\tilde{\theta}$ would win the competition if it were to pick instead the option designed for this particular type θ). Two cases can be distinguished:

Case a: $b(\theta) \leq \hat{b}(\theta)$. In this case, incentive compatibility requires that:

- an entrant of type $\tilde{\theta}$ should not prefer reporting a type θ, that is:

$$-t\left(\tilde{\theta}\right) \geq \left[c(B_I + \Delta - b(\theta)) - c\left(B_I - \tilde{\theta} + b(\theta)\right)\right] D(c(B_I + \Delta - b(\theta))) - t(\theta);$$

- conversely, an entrant of type θ should not want to report $\tilde{\theta}$, that is:

$$-t(\theta) \geq -t\left(\tilde{\theta}\right).$$

Combining these two conditions yields:

$$\left[c(B_I + \Delta - b(\theta)) - c\left(B_I - \tilde{\theta} + b(\theta)\right)\right] D(c(B_I + \Delta - b(\theta))) \leq t(\theta) - t\left(\tilde{\theta}\right) \leq 0,$$

and thus

$$c(B_I + \Delta - b(\theta)) - c\left(B_I - \tilde{\theta} + b(\theta)\right) \leq 0,$$

contradicting the assumption $b(\theta) > \hat{b}\left(\tilde{\theta}\right)$.

Case b: $b(\theta) > \hat{b}(\theta)$. In this case, the above incentive-compatibility conditions become:

$$-t\left(\tilde{\theta}\right) \geq \left[c(B_I + \Delta - b(\theta)) - c\left(B_I - \tilde{\theta} + b(\theta)\right)\right] D(c(B_I + \Delta - b(\theta))) - t(\theta),$$

and

$$[c(B_I + \Delta - b(\theta)) - c(B_I - \theta + b(\theta))] D(c(B_I + \Delta - b(\theta))) - t(\theta) \geq -t\left(\tilde{\theta}\right).$$

Adding-up these two conditions yields:

$$\left[c\left(B_I - \tilde{\theta} + b(\theta)\right) - c(B_I - \theta + b(\theta))\right] D(c(B_I + \Delta - b(\theta))) \geq 0,$$

and thus

$$c\left(B_I - \tilde{\theta} + b(\theta)\right) \geq c(B_I - \theta + b(\theta)),$$

contradicting the condition $\theta > \tilde{\theta}$. ■

Building on this, we now proceed to prove Lemma 3. Consider first the case $\hat{\theta} = \underline{\theta}$. Lemma 4 implies that the profile $\{b(\theta)\}_{\theta\in\Theta}$ then lies everywhere below $\hat{b}(\underline{\theta})$; but then, replacing the DICM $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ with the "full bunching" DICM $\left\{\tilde{b}(\theta) = \hat{b}(\underline{\theta}), \tilde{t}(\theta) = 0\right\}_{\theta\in\Theta}$ increases expected consumer surplus:

- The alternative allocation is obviously incentive compatible and individually rational, as it allocates the same option to every type of entrant (full bunching), and this option gives a non-negative net profit to any type of entrant.

- The alternative allocation (weakly) increases consumer surplus: it coincides with the first-best for $\theta = \underline{\theta}$ and, for $\theta > \underline{\theta}$, it is closer to the first-best, as it satisfies

$$\hat{b}(\theta) > \hat{b}(\underline{\theta}) \geq \tilde{b}(\theta) \geq b(\theta).$$

Consider now the case where $\hat{\theta} > \underline{\theta}$, and suppose that there exists $\theta < \hat{\theta}$ (for which we thus have $b(\theta) > \hat{b}(\theta)$) such that $b(\theta) < \hat{b}\left(\hat{\theta}\right) \left(= b\left(\hat{\theta}\right)\right)$. Incentive compatibility then requires that an entrant with handicap θ does not want to report a handicap of $\hat{\theta}$, and conversely, which in this case respectively amounts to:

$$\pi(b(\theta), \theta) - t(\theta) \geq \pi\left(\hat{b}\left(\hat{\theta}\right), \theta\right) - t\left(\hat{\theta}\right),$$
$$-t\left(\hat{\theta}\right) \geq -t(\theta).$$

Combining these two conditions yields:

$$\pi(b(\theta), \theta) - \pi\left(\hat{b}\left(\hat{\theta}\right), \theta\right) \geq t(\theta) - t\left(\hat{\theta}\right) \geq 0,$$

contradicting the assumption $b(\theta) < \hat{b}\left(\hat{\theta}\right)$ (as $\partial\pi/\partial b > 0$).

Therefore, in the range $\theta < \hat{\theta}$, $b(\theta)$ must lie above $\hat{b}\left(\hat{\theta}\right)$. But then, incentive compatibility requires:

$$\pi(b(\theta), \theta) - t(\theta) \geq \pi\left(\hat{b}\left(\hat{\theta}\right), \theta\right) - t\left(\hat{\theta}\right),$$
$$\pi\left(\hat{b}\left(\hat{\theta}\right), \hat{\theta}\right) - t\left(\hat{\theta}\right) \geq \pi\left(b(\theta), \hat{\theta}\right) - t(\theta),$$

and thus:

$$\begin{aligned}
0 &\leq t(\theta) - t\left(\hat{\theta}\right) \\
&\leq \left[\pi(b(\theta), \theta) - \pi\left(\hat{b}\left(\hat{\theta}\right), \theta\right)\right] - \left[\pi\left(b(\theta), \hat{\theta}\right) - \pi\left(\hat{b}\left(\hat{\theta}\right), \hat{\theta}\right)\right] \\
&= \int_{\hat{b}(\hat{\theta})}^{b(\theta)} \left[\frac{\partial \pi}{\partial b}(b, \theta) - \frac{\partial \pi}{\partial b}\left(b, \hat{\theta}\right)\right] db \\
&= \int_{\hat{b}(\hat{\theta})}^{b(\theta)} \int_{\hat{\theta}}^{\theta} \frac{\partial^2 \pi}{\partial b \partial \theta}(b, s)\, ds db.
\end{aligned}$$

As $\partial^2 \pi / \partial b \partial \theta > 0$ and $\theta < \hat{\theta}$, this in turn implies $b(\theta) \leq \hat{b}\left(\hat{\theta}\right)$.

Therefore, we must have $b(\theta) = \hat{b}\left(\hat{\theta}\right)$ in the range $\theta \leq \hat{\theta}$, and $b(\theta) \leq \hat{b}\left(\hat{\theta}\right)$ in the range $\theta \geq \hat{\theta}$. The best such profile corresponds to the bunching mechanism where $b(\theta) = \hat{b}\left(\hat{\theta}\right)$ for every $\theta \in \Theta$.

F Proof of Proposition 3

That the first-best is not implementable has been established in the text. We now show that full bunching obtains when demand is inelastic. Without loss of generality, we will normalize to 1 the size of that demand in the relevant price range, namely for $p \in \left[c(B_I + \Delta), c\left(B_I - \bar{\theta}\right)\right]$. When the entrant acquires enough spectrum to win the market, its gross profit is thus equal to:

$$\pi(b, \theta) = c(B_I + \Delta - b) - c(B_I - \theta + b).$$

Building on Lemma 4, the following lemma shows that "full bunching" is optimal:

Lemma 5 *Without loss of generality, we can restrict attention to direct mechanisms* $\{(b(\theta), t(\theta))\}_{\theta \in \Theta}$ *such that, for every* $\theta \in \Theta$, $t(\theta) = 0$ *and* $b(\theta) = \hat{b}$, *for some* $\hat{b} \in [0, \Delta]$.

Proof. Consider a DICM $\{b(\theta), t(\theta)\}_{\theta \in \Theta}$ and let

$$\hat{\theta} \equiv \begin{cases} \bar{\theta} & \text{if } b(\theta) > \hat{b}(\theta) \text{ for all } \theta \in \Theta, \\ \inf\left\{\theta \mid b(\theta) \leq \hat{b}(\theta)\right\} & \text{otherwise,} \end{cases}$$

denote the threshold beyond which the profile $\{b(\theta)\}_{\theta\in\Theta}$ remains below the first-best profile $\left\{\hat{b}(\theta)\right\}_{\theta\in\Theta}$. From Lemma 4, $b(\theta)$ remains below $\hat{b}\left(\hat{\theta}\right)$ in the range $\theta > \hat{\theta}$.[40] Therefore, any type $\theta > \hat{\theta}$ would obtain a net profit of $-t\left(\tilde{\theta}\right)$ by picking the option designed for type $\tilde{\theta} > \hat{\theta}$ (as $\hat{b}(\theta) > \hat{b}\left(\hat{\theta}\right) \geq b\left(\tilde{\theta}\right)$). Incentive compatibility then implies that the profile $t(\theta)$ is constant in the range $\theta > \hat{\theta}$; that is, there exists $\hat{t}$ such that $t(\theta) = \hat{t}$ for $\theta > \hat{\theta}$, and any type $\theta > \hat{\theta}$ obtains a net payoff equal to $-\hat{t}$ by picking any option $\left(b\left(\tilde{\theta}\right), t\left(\tilde{\theta}\right)\right)$ designed for any type $\tilde{\theta} > \hat{\theta}$.

Consider first the case $\hat{\theta} = \underline{\theta}$ (that is, the profile $\{b(\theta)\}_{\theta\in\Theta}$ lies everywhere below $\hat{b}(\underline{\theta})$, except possibly for $\theta = \underline{\theta}$), and let

$$\underline{b} \equiv \sup\{b(\theta) \mid \theta > \underline{\theta}\}$$

denote the upper bound of the profile $\{b(\theta)\}_{\theta>\underline{\theta}}$.[41] Note that, by construction, $b(\theta) \leq \underline{b} \leq \hat{b}(\underline{\theta})$ for any $\theta > \underline{\theta}$. Hence, replacing the DICM $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ with $\left\{\tilde{b}(\theta) = \hat{b}(\underline{\theta}), \tilde{t}(\theta) = 0\right\}_{\theta\in\Theta}$ can only increase expected consumer surplus:

- The alternative mechanism is trivially incentive compatible, as it does not depend on the handicap, and it is individually rational, as it gives zero profit to the entrant, regardless of its handicap.

- The alternative allocation coincides with the first-best for $\theta = \underline{\theta}$ and, for $\theta > \underline{\theta}$, it is (weakly) closer to the first-best, as it satisfies

$$\hat{b}(\theta) > \hat{b}(\underline{\theta}) \geq \tilde{b}(\theta) \geq b(\theta).$$

Thus, if $\hat{\theta} = \underline{\theta}$ then we can restrict attention to a DICM of the form described in the statement of the Lemma.

We now focus on the case where $\hat{\theta} > \underline{\theta}$, and first consider the range $\theta < \hat{\theta}$, where

[40]By definition, either $b\left(\hat{\theta}\right) \leq \hat{b}\left(\hat{\theta}\right)$, in which case Lemma 4 directly implies that $b(\theta) \leq \hat{b}(\theta)$ for all $\theta > \hat{\theta}$, or for any $\theta > \hat{\theta}$ there exists $\theta' \in \left(\hat{\theta}, \theta\right)$ such that $b(\theta') \leq \hat{b}(\theta')$, in which case Lemma 4 implies that $b(\theta) \leq \hat{b}(\theta)$.

[41]This supremum exists and is finite, as $b(.)$ is bounded above by $\hat{b}(\underline{\theta})$ in this range.

$b(\cdot) > \hat{b}(\cdot)$; in this range, the entrant obtains a net profit equal to:

$$r(\theta) \equiv \pi(b(\theta), \theta) - t(\theta),$$

where the gross profit is here given by

$$\pi(b, \theta) \equiv c(B_I + \Delta - b) - c(B_I - \theta + b).$$

A revealed preference argument implies that the net profit $r(\theta)$ decreases with θ (as $\partial\pi/\partial\theta < 0$), and that $\lim_{\theta \to \hat{\theta}^-} r(\theta) = -\hat{t}$. Therefore, individual rationality boils down to $\hat{t} \leq 0$, and without loss of generality we can set $\hat{t} = 0$.

Furthermore, by choosing the option designed for a type $\tilde{\theta}$ "close enough" to its own type θ (so that $b\left(\tilde{\theta}\right)$ not only exceeds $\hat{b}\left(\tilde{\theta}\right)$, but also exceeds $\hat{b}(\theta)$), an entrant of type θ would obtain:

$$\varphi\left(\theta, \tilde{\theta}\right) \equiv \pi\left(b\left(\tilde{\theta}\right), \theta\right) - t\left(\tilde{\theta}\right).$$

The usual reasoning can then be used to show that incentive compatibility requires the profiles $\{b(\theta)\}_{\theta < \hat{\theta}}$ and $\{t(\theta)\}_{\theta < \hat{\theta}}$ to be (weakly) increasing (as the profit function satisfies Mirrlees' single-crossing property: $\partial^2\pi/\partial\theta\partial b > 0$) and to satisfy:

$$t(\theta) = \pi(b(\theta), \theta) + \int_{\theta}^{\hat{\theta}} \frac{\partial\pi}{\partial\theta}(b(s), s)\, ds.$$

Next, we show that without loss of generality we can restrict attention to DICMs such that $b(\underline{\theta}) \leq \hat{b}\left(\hat{\theta}\right)$. To see this, it suffices to note that if $b(\underline{\theta}) > \hat{b}\left(\hat{\theta}\right)$ (which implies that the profile $\{b(\theta)\}_{\theta \in \Theta}$ lies above $\hat{b}\left(\hat{\theta}\right)$ for $\theta < \hat{\theta}$, and below $\hat{b}\left(\hat{\theta}\right)$ for $\theta > \hat{\theta}$), then replacing the DICM $\{(b(\theta), t(\theta))\}_{\theta \in \Theta}$ with $\left\{\tilde{b}(\theta) = \hat{b}\left(\hat{\theta}\right), \tilde{t}(\theta) = 0\right\}_{\theta \in \Theta}$ increases expected consumer surplus:

- The alternative mechanism is trivially incentive compatible (full bunching); it is moreover individually rational, as the single option gives every type of entrant a non-negative net profit (it is actually positive for $\theta < \hat{\theta}$, and null for $\theta \geq \hat{\theta}$).
- The alternative allocation is closer to the first-best (and strictly so for $\theta < \hat{\theta}$):
  - for $\theta < \hat{\theta}$, the alternative allocation is such that $\hat{b}(\theta) < \hat{b}\left(\hat{\theta}\right)\left(= \tilde{b}(\theta)\right) <$

$b(\underline{\theta}) \leq b(\theta)$;

– for $\theta > \hat{\theta}$, the alternative allocation is such that $b(\theta) \leq \hat{b}\left(\hat{\theta}\right) \left(= \tilde{b}(\theta)\right) < \hat{b}(\theta)$.

Finally, consider a DICM $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ such that $b(\underline{\theta}) \leq \hat{b}\left(\hat{\theta}\right)$, and let

$$\left(b^-, t^-\right) \equiv \lim_{\theta\to\hat{\theta}^-} (b(\theta), t(\theta))$$

denote the left-sided limit of the profile $\{b(\theta), t(\theta)\}_{\theta\in\Theta}$ at $\hat{\theta}$ (this limit exists, as incentive-compatibility implies that $b(\theta)$ and $t(\theta)$ are both non-decreasing in this range). Note that, by construction, $b^- \geq \hat{b}\left(\hat{\theta}\right)$ (as $b(\theta) > \hat{b}(\theta)$ for $\theta < \hat{\theta}$). We can distinguish two cases, according to whether or not b^- lies above $\hat{b}\left(\hat{\theta}\right)$.

Case a: $b^- = \hat{b}\left(\hat{\theta}\right)$. The same revealed preference argument as in the proof of Lemma 3 implies that the profile $\{b(\theta)\}_{\theta\in\Theta}$ must then coincide with $\hat{b}\left(\hat{\theta}\right)$ for $\theta < \hat{\theta}$; furthermore, from Lemma 4 $b(\theta)$ lies below $\hat{b}\left(\hat{\theta}\right)$ for $\theta > \hat{\theta}$. It follows that replacing the DICM $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ with the alternative DICM $\left\{\tilde{b}(\theta) = \hat{b}\left(\hat{\theta}\right), \tilde{t}(\theta) = 0\right\}_{\theta\in\Theta}$ can only increase expected consumer surplus, as the alternative mechanism is trivially incentive compatible and individually rational, and is closer to the first-best.[42]

Case b: $b^- > \hat{b}\left(\hat{\theta}\right)$. As by construction $\hat{b}(\underline{\theta}) < b(\underline{\theta}) \leq \hat{b}\left(\hat{\theta}\right)$, there exists a (unique) type $\tilde{\theta} \in (\underline{\theta}, \hat{\theta}]$ such that

$$\hat{b}\left(\tilde{\theta}\right) = b(\underline{\theta}),$$

and the first-best profile $\left\{\hat{b}(\theta)\right\}_{\theta\in\Theta}$ lies strictly below $b(\underline{\theta})$ for $\theta < \tilde{\theta}$, whereas it lies strictly above $b(\underline{\theta})$ for $\theta > \tilde{\theta}$. Consider replacing the DICM $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ with the alternative mechanism $\left\{\tilde{b}(\theta) = b(\underline{\theta}), \tilde{t}(\theta) = 0\right\}_{\theta\in\Theta}$. This alternative mechanism is again trivially incentive compatible and individually rational, and outside the range $\theta \in \left[\tilde{\theta}, \hat{\theta}\right]$, it is closer to the first-best and thus increases consumer surplus:

- for $\underline{\theta} < \theta < \tilde{\theta}$, the alternative allocation is such that $\hat{b}(\theta) < b(\underline{\theta}) = \tilde{b}(\theta) \leq b(\theta)$;
- for $\theta > \hat{\theta}$, the alternative allocation is such that $b(\theta) \leq \hat{b}\left(\hat{\theta}\right) = \tilde{b}(\theta) < \hat{b}(\theta)$.

[42] If the distribution of the handicap θ is atomless at $\hat{\theta}$, a quicker argument consists in noting that the DICM is then equivalent to a DICM where $b(\theta) = b^- = \hat{b}\left(\hat{\theta}\right)$; Lemma 3 then applies.

Therefore, if $\tilde{\theta} = \hat{\theta}$, the alternative mechanism exhibits full bunching and outperforms the original DICM.[43] We now show that, if instead $\tilde{\theta} < \hat{\theta}$, then the alternative mechanism also increases consumer surplus in the range $\theta \in \left[\tilde{\theta}, \hat{\theta}\right]$. To see this, note first that, for any θ such that $\tilde{\theta} < \theta < \hat{\theta}$, implying $(b(\theta) >) \, \hat{b}(\theta) > b(\underline{\theta}) \left(> \hat{b}(\underline{\theta})\right)$, incentive-compatibility requires:

$$\pi(b(\theta), \theta) - t(\theta) \geq -t(\underline{\theta}),$$
$$\pi(b(\underline{\theta}), \underline{\theta}) - t(\underline{\theta}) \geq \pi(b(\theta), \underline{\theta}) - t(\theta).$$

Combining these two conditions implies:

$$\pi(b(\underline{\theta}), \underline{\theta}) \geq \pi(b(\theta), \underline{\theta}) - \pi(b(\theta), \theta),$$

which can be expressed as

$$\phi(b(\theta), \theta) \leq 0, \tag{12}$$

where

$$\phi(b, \theta) \equiv \pi(b, \underline{\theta}) - \pi(b, \theta) - \pi(b(\underline{\theta}), \underline{\theta})$$

satisfies, for $\theta > \underline{\theta}$ (using the fact that $c'(\cdot)$ is negative but increasing):

$$\frac{\partial \phi}{\partial b}(b, \theta) = \frac{\partial \pi}{\partial b}(b, \underline{\theta}) - \frac{\partial \pi}{\partial b}(b, \theta) = c'(B_I - \theta + b) - c'(B_I - \underline{\theta} + b) < 0, \tag{13}$$

and:

$$\frac{\partial \phi}{\partial \theta}(b, \theta) = -\frac{\partial \pi}{\partial \theta}(b, \theta) = -c'(B_I - \theta + b) > 0.$$

From (13), the necessary condition (12) amounts to $b(\theta) \geq \beta(\theta)$, where, for any $\theta > \underline{\theta}$, the function $\beta(\theta)$ is the implicit solution to $\phi(\beta, \theta) = 0$. Note that, by construction:

- $\beta\left(\tilde{\theta}\right) = \hat{b}\left(\tilde{\theta}\right)$; to see this, it suffices to note that

$$\phi\left(\hat{b}\left(\tilde{\theta}\right), \tilde{\theta}\right) = \pi\left(\hat{b}\left(\tilde{\theta}\right), \underline{\theta}\right) - \pi\left(\hat{b}\left(\tilde{\theta}\right), \tilde{\theta}\right) - \pi(b(\underline{\theta}), \underline{\theta}) = \pi\left(\hat{b}\left(\tilde{\theta}\right), \underline{\theta}\right) - \pi(b(\underline{\theta}), \underline{\theta}) = 0,$$

[43] While the above inequalities only establish that $\tilde{b}(\theta) \leq b(\theta)$ for $\theta < \hat{\theta}$, in the particular case where $\tilde{\theta} = \hat{\theta}$ (which can arise only when $b(\underline{\theta}) = b\left(\hat{\theta}\right) (< b^-)$), we have $\tilde{b}(\theta) < b(\theta)$ for θ close enough to $\hat{\theta}$ (as $\lim_{\theta \to \hat{\theta}^-} b(\theta) = b^- > \hat{b}\left(\hat{\theta}\right)$). Hence, the alternative mechanism does strictly better than the original DICM.

where the second equality stems from $\pi\left(\hat{b}\left(\tilde{\theta}\right),\tilde{\theta}\right)=0$) and the last one from $\hat{b}\left(\tilde{\theta}\right)=b\left(\underline{\theta}\right)$.

- For $\theta\in\left[\tilde{\theta},\hat{\theta}\right]$:

$$\beta'\left(\theta\right)=-\frac{\frac{\partial\phi}{\partial\theta}\left(b,\theta\right)}{\frac{\partial\phi}{\partial b}\left(b,\theta\right)}=\frac{c'\left(B_I-\theta+\beta\left(\theta\right)\right)}{c'\left(B_I-\theta+\beta\left(\theta\right)\right)-c'\left(B_I-\underline{\theta}+\beta\left(\theta\right)\right)}=\frac{1}{1-\frac{c'\left(B_I-\underline{\theta}+\beta\left(\theta\right)\right)}{c'\left(B_I-\theta+\beta\left(\theta\right)\right)}}>1.$$

Therefore, for $\theta\in(\tilde{\theta},\hat{\theta})$ (using $\hat{b}\left(\theta\right)=\left(\Delta+\theta\right)/2$):

$$b\left(\theta\right)-b\left(\underline{\theta}\right)=b\left(\theta\right)-\hat{b}\left(\tilde{\theta}\right)>\theta-\tilde{\theta}=2\left[\hat{b}\left(\theta\right)-\hat{b}\left(\tilde{\theta}\right)\right],$$

which in turn implies:

$$\hat{b}\left(\tilde{\theta}\right)-\hat{b}\left(\theta\right)>\hat{b}\left(\theta\right)-b\left(\theta\right).$$

It follows that in the range $\theta\in(\tilde{\theta},\hat{\theta})$, replacing $b\left(\theta\right)$ with $\hat{b}\left(\tilde{\theta}\right)=b\left(\underline{\theta}\right)$ increases consumer surplus, as it reduces the price from

$$c_I|_{b=b(\theta)}=c\left(B_I+\Delta-b\left(\theta\right)\right)=c\left(B_I+\frac{\Delta-\theta}{2}+\hat{b}\left(\theta\right)-b\left(\theta\right)\right)$$

to

$$c_E|_{b=\hat{b}(\tilde{\theta})}=c\left(B_I-\theta+\hat{b}\left(\tilde{\theta}\right)\right)=c\left(B_I+\frac{\Delta-\theta}{2}+\hat{b}\left(\tilde{\theta}\right)-\hat{b}\left(\theta\right)\right).$$

■

We can therefore restrict attention to "bunching" mechanisms which allocate the same bandwidth $\hat{b}$ to the entrant, regardless of its type. The resulting price is equal to

$$p\left(\hat{b},\theta\right)=\begin{cases}c\left(B_I+\Delta-\hat{b}\right) & \text{if }\theta\leq\hat{\theta},\\ c\left(B_I-\theta+\hat{b}\right) & \text{otherwise,}\end{cases}$$

where

$$\hat{\theta}=\hat{b}^{-1}\left(\hat{b}\right)=2\hat{b}-\Delta.$$

Therefore, the optimal bandwidth lies between $\hat{b}\left(\underline{\theta}\right)$ (which is lower than Δ by assumption) and $\min\left\{\hat{b}\left(\bar{\theta}\right),\Delta\right\}$ and aims at minimizing the expected market price:

$$\min_{\hat{b}\leq\Delta}\hat{p}\left(\hat{b}\right)\equiv\int_{\underline{\theta}}^{\bar{\theta}}p\left(\theta,\hat{b}\right)f\left(\theta\right)d\theta=\int_{\underline{\theta}}^{2\hat{b}-\Delta}c\left(B_I+\Delta-\hat{b}\right)f\left(\theta\right)d\theta+\int_{2\hat{b}-\Delta}^{\bar{\theta}}c\left(B_I-\theta+\hat{b}\right)f\left(\theta\right)d\theta.$$

We have:

$$\begin{aligned}\hat{p}'\left(\hat{b}\right)&=-\int_{\underline{\theta}}^{2\hat{b}-\Delta}c'\left(B_I+\Delta-\hat{b}\right)f\left(\theta\right)d\theta+\int_{2\hat{b}-\Delta}^{\bar{\theta}}c'\left(B_I-\theta+\hat{b}\right)f\left(\theta\right)d\theta,\\ \hat{p}''\left(\hat{b}\right)&=\int_{\underline{\theta}}^{2\hat{b}-\Delta}c''\left(B_I+\Delta-\hat{b}\right)f\left(\theta\right)d\theta+\int_{2\hat{b}-\Delta}^{\bar{\theta}}c''\left(B_I-\theta+\hat{b}\right)f\left(\theta\right)d\theta\\ &\quad-4c'\left(B_I+\Delta-\hat{b}\right)f\left(2\hat{b}-\Delta\right)\\ &\quad>0,\end{aligned}$$

and:

$$\begin{aligned}\hat{S}'\left(\hat{b}\left(\underline{\theta}\right)\right)&=\int_{\underline{\theta}}^{\bar{\theta}}c'\left(B_I-\theta+\hat{b}\right)f\left(\theta\right)d\theta<0,\\ \hat{S}'\left(\hat{b}\left(\bar{\theta}\right)\right)&=-\int_{\underline{\theta}}^{\bar{\theta}}c'\left(B_I+\Delta-\hat{b}\right)f\left(\theta\right)d\theta>0.\end{aligned}$$

Therefore, the socially optimal threshold is equal to:

$$\hat{\theta}^{SB}=\min\left\{\hat{\theta}^S,\hat{b}^{-1}\left(\Delta\right)\right\},$$

where $\hat{\theta}^S$ is the unique threshold lying between $\hat{b}\left(\underline{\theta}\right)$ and $\hat{b}\left(\bar{\theta}\right)$ satisfying:

$$\frac{d\hat{S}}{d\hat{\theta}}\left(\hat{\theta}^S\right)=0.$$

G Proof of Proposition 4

As is well-known, in each (classic) auction, the higher-valuation bidder wins and pays a price equal to the lower-valuation bidder, where all valuations take into account the expected equilibrium outcome of subsequent auctions.

The proof proceeds by induction. We will label "auction h", for $h=1,...,k$, the auction taking place when h blocks remain to be allocated (hence, auction "k" is the first auction, and auction "1" is the auction for the last block). Let $p_0(B_L, B_l) \equiv 0$ and $\Pi_0(B_L, B_l) \equiv \Pi(B_L, B_l)$, where $\Pi(\cdot,\cdot)$ is given by (1), and for every $h = 1, ..., k$, let L_h and l_h respectively denote the leader and the laggard (i.e., the firm with the larger and with the smaller bandwidth) at the beginning of auction h – if both firms have the same bandwidth at the beginning of auction h, then select either firm as leader with probability 1/2.

We will use the following induction hypothesis H_h:

1. If $B_{L_h} > B_{l_h}$, then L_h wins auction h and obtains an expected net profit equal to $\Pi_h(B_{L_h}, B_{l_h}) = \Pi(B_{L_h} + h\delta, B_{l_h}) - p_h(B_{L_h}, B_{l_h})$, where

$$p_h(B_{L_h}, B_{l_h}) = \begin{cases} \Pi_{h-1}(B_{l_h} + \delta, B_{L_h}) & \text{if } B_{L_h} - B_{l_h} < \delta, \\ 0 & \text{otherwise.} \end{cases}$$

   whereas l_h obtains zero expected net profit.

2. If $B_{L_h} = B_{l_h}$, then either firm wins auction h and pays a price

$$p_h(B_{L_h}, B_{L_h}) = \Pi(B_{L_h} + h\delta, B_{L_h}).$$

   Both firms obtain zero expected net profit.

We first check that H_1 holds:

- If $B_{L_1} \geq B_{l_1} + \delta$, then the laggard cannot obtain any profit in the product market, regardless of whether it wins the auction; hence, the leader obtains the last block for free.

- If instead $B_{L_1} < B_{l_1} + \delta$, then winning the auction gives the laggard a profit (gross of the price paid in the last auction) equal to $\Pi_0(B_{l_1} + \delta, B_{L_1}) = \Pi(B_{l_1} + \delta, B_{L_1})$, and gives the leader a (gross) profit equal to $\Pi_0(B_{L_1} + \delta, B_{l_1}) = \Pi(B_{L_1} + \delta, B_{l_1})$. Therefore:

  - If $B_{L_1} > B_{l_1}$, then the leader has a greater willingness to pay, as

$$\Pi(B_{L_1} + \delta, B_{l_1}) > \Pi(B_{l_1} + \delta, B_{l_1}) > \Pi(B_{l_1} + \delta, B_{L_1}),$$

where the first and second inequalities respectively stem from (2) and (3). Hence, the leader obtains the last block for a price equal to $p_1(B_{L_1}, B_{l_1}) = \Pi(B_{l_1} + \delta, B_{L_1})$.

– If instead $B_{L_1} = B_{l_1}$, then both firms obtains the same (gross) profit from winning the auction, and thus bid the same amount, equal to this profit. Hence, $p_1(B_{L_1}, B_{L_1}) = \Pi(B_{L_1} + \delta, B_{L_1})$, either firm wins at that price, and both firms obtain zero net profit.

Suppose now that H_t holds for $t = 1, ..., h$, and consider auction $h+1$. If the leading firm L_{h+1} wins, then it will be again the leader in the next round, and will enjoy a bandwidth advantage of at least δ; therefore, according to the induction hypothesis, its profit from winning (gross of the price paid in auction $h+1$) is given by (taking into account that $p_h\left(B_{L_{h+1}} + \delta, B_{l_{h+1}}\right) = 0$, as $\left(B_{L_{h+1}} + \delta\right) - B_{l_{h+1}} \geq \delta$):

$$\hat{\Pi}_L = \Pi\left(B_{L_{h+1}} + (h+1)\,\delta, B_{l_{h+1}}\right).$$

If instead the laggard firm l_{h+1} wins auction $h+1$, it then becomes the leader in the next round if $B_{L_{h+1}} - B_{\ell_{h+1}} < \delta$, and otherwise remains the laggard (or becomes equally efficient as its rival, in which case it also obtains zero profit in the product market); therefore, according to the induction hypothesis, it obtains a profit (gross of the price paid in auction $h+1$) equal to:

$$\hat{\Pi}_l = \begin{cases} \Pi\left(B_{l_{h+1}} + (h+1)\,\delta, B_{L_{h+1}}\right) - p_h\left(B_{l_{h+1}} + \delta, B_{L_{h+1}}\right) & \text{if } B_{L_{h+1}} - B_{l_{h+1}} < \delta, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore:

- If $B_{L_{h+1}} = B_{l_{h+1}}$,

$$\Pi\left(B_{l_{h+1}} + (h+1)\,\delta, B_{L_{h+1}}\right) = \Pi\left(B_{L_{h+1}} + (h+1)\,\delta, B_{l_{h+1}}\right) = \Pi\left(B_{L_{h+1}} + (h+1)\,\delta, B_{L_{h+1}}\right)$$

and

$$p_h\left(B_{l_{h+1}} + \delta, B_{L_{h+1}}\right) = p_h\left(B_{L_{h+1}} + \delta, B_{L_{h+1}}\right) = 0,$$

and thus $\hat{\Pi}_L = \hat{\Pi}_l$. Hence, both firms bid

$$p_{h+1} = \Pi\left(B_{L_{h+1}} + (h+1)\,\delta, B_{L_{h+1}}\right),$$

either firm wins, and both firms obtain zero net profit.

- If instead $B_{L_{h+1}} > B_{l_{h+1}}$, then $\hat{\Pi}_L > \hat{\Pi}_l$, as $p_h(\cdot) \geq 0$ and

  $$\Pi\left(B_{L_{h+1}} + (h+1)\,\delta, B_{l_{h+1}}\right) > \Pi\left(B_{l_{h+1}} + (h+1)\,\delta, B_{l_{h+1}}\right) > \Pi\left(B_{l_{h+1}} + (h+1)\,\delta, B_{L_{h+1}}\right),$$

  where the first and second inequalities respectively stem again from (2) and (3), and so the leading firm L_{h+1} wins auction $h+1$. Furthermore:

  - When $B_{L_{h+1}} - B_{l_{h+1}} > \delta$, the lagging firm l_{h+1} would remain behind and thus obtain zero profit even if it were to win; hence, it bids zero, that is, $p_{h+1} = 0$.
  - When instead $B_{L_{h+1}} - B_{l_{h+1}} < \delta$, the lagging firm l_{h+1} is willing to bid up to

    $$\Pi\left(B_{l_{h+1}} + (h+1)\,\delta, B_{L_{h+1}}\right) - p_h\left(B_{l_{h+1}} + \delta, B_{L_{h+1}}\right) = \Pi_h\left(B_{l_{h+1}} + \delta, B_{L_{h+1}}\right).$$

    The equilibrium price is thus equal to $p_{h+1} = \Pi_h\left(B_{l_{h+1}} + \delta, B_{L_{h+1}}\right)$, as in the induction hypothesis. It follows that the equilibrium payoffs are also as in the induction hypothesis.

Therefore, H_{h+1} holds when H_t holds for $t = 1, ..., h$. It follows that the incumbent firm I wins all successive rounds. Furthermore, if $B_I - B_E \geq \delta$, then it obtains all the bandwidth at zero price. If instead $B_I - B_E < \delta$, then using the induction hypothesis we have:

$$p_k(B_I, B_E) = \begin{cases} \sum_{h=0}^{m-1} \phi_h^k(B_E, B_I) - \sum_{h=1}^{m} \phi_h^k(B_I, B_E) & \text{if } k = 2m, \\ \sum_{h=0}^{m} \phi_h^k(B_E, B_I) - \sum_{h=1}^{m} \phi_h^k(B_I, B_E) & \text{if } k = 2m+1. \end{cases}$$

where

$$\phi_h^k(B_1, B_2) \equiv \Pi\left(B_1 + (k-h)\,\delta, B_2 + h\delta\right).$$

Note that when $B_I = B_E = B$,

$$\phi_m^{2m}(B,B) = \Pi(B + m\delta, B + m\delta) = 0$$

and thus the equilibrium price is equal to

$$p_k(B,B) = \phi_0^k(B,B) = \Pi(B + \Delta, B).$$

H Proof of Proposition 5

We now show that, for each firm $i = I, E$, it is a dominant strategy to bid $\beta_i^*(n) = \pi_i(n)$, where (using the subscript "$-i$" to refer to firm i's rival):

$$\pi_i(n) \equiv \begin{cases} \Pi(B_i + n_i\delta, B_{-i} + n_{-i}\delta) & \text{if } B_i + n_i\delta > B_{-i} + n_{-i}\delta, \\ 0 & \text{otherwise.} \end{cases}$$

with $\Pi(\cdot,\cdot)$ given by (1).

To see this, consider an alternative strategy $\hat{\beta}_i$, and suppose that, for some bidding strategy of the other firm, β_{-i}, the bidding strategies β_i^* and $\hat{\beta}_i$ lead to different outcomes. As the payments only depend on the bids through the spectrum allocation, this implies that β_i^* and $\hat{\beta}_i$ lead to different spectrum allocations, which we will respectively denote by n^* and $\hat{n}$. Likewise, let Π_i^* and $\hat{\Pi}_i$ denote the net payoffs of firm i associated with the bidding strategies β_i^* and $\hat{\beta}_i$. We have:

$$\begin{aligned} \Pi_i^* - \hat{\Pi}_i &= \left\{\pi_i(n^*) - p_i^V(\beta_i^*, \beta_{-i})\right\} - \left\{\pi_i(\hat{n}) - p_i^V\left(\hat{\beta}_i, \beta_{-i}\right)\right\} \\ &= \left\{\pi_i(n^*) - \left[\max_{n\in\mathcal{A}}\{\beta_{-i}(n)\} - \beta_{-i}(n^*)\right]\right\} - \left\{\pi_i(\hat{n}) - \left[\max_{n\in\mathcal{A}}\{\beta_{-i}(n)\} - \beta_{-i}(\hat{n})\right]\right\} \\ &= \pi_i(n^*) - \pi_i(\hat{n}) - [\beta_{-i}(\hat{n}) - \beta_{-i}(n^*)] \\ &= \beta_i^*(n^*) - \beta_i^*(\hat{n}) - [\beta_{-i}(\hat{n}) - \beta_{-i}(n^*)]. \end{aligned}$$

But, by construction, as the bidding strategy β_i^* leads to n^*, it must be the case that

$$\beta_i^*(n^*) + \beta_{-i}(n^*) \geq \beta_i^*(\hat{n}) + \beta_{-i}(\hat{n}).$$

It follows that $\Pi_i^* \geq \hat{\Pi}_i$, establishing that bidding $\beta_i^*(n) = \pi_i(n)$ is a dominant strategy for firm i.

Given these bidding strategies, the outcome maximizes

$$\max_{n \in \mathcal{A}} \Pi\left(B_I + n_I \delta, B_E + n_E \delta\right),$$

which is achieved for $n_I = k$ and $n_E = 0$. That is, the incumbent firm I obtains all k blocks, and pays a price equal to:

$$\max_{n \in \mathcal{A}} \{\beta_E(n)\} - \beta_E(k, 0) = \begin{cases} \Pi\left(B_E + \Delta, B_I\right) & \text{if } \Delta > B_I - B_E, \\ 0 & \text{otherwise.} \end{cases}$$

I Proof of Proposition 6

When the handicap of the entrant is too large to be offset by the additional spectrum (i.e., when $B_I - B_E \geq \Delta$), the equilibrium prices are zero in both types of auctions. We now focus on the more interesting case where $B_I - B_E < \Delta$. In the case of a multi-unit VCG auction, the price is then always positive and equal to

$$p^V = \Pi\left(B_E + \Delta, B_I\right) > 0.$$

By contrast, in the case of a sequential auction, the price remains zero when the lagging firm cannot catch-up with a single block of size $\delta = \Delta/k$. Hence, for any given $\Delta > 0$, the price remains zero when the spectrum is divided in sufficiently many blocks, namely, when

$$k \geq \bar{k} = \frac{\Delta}{B_I - B_E}.$$

Finally, when instead the lagging firm could catch up with a single block (i.e., $B_I - B_E < \delta = \Delta/k$), the price is of the form (using the induction hypothesis):

$$p_k\left(B_I, B_E\right) = \Pi_{k-1}\left(B_E + \delta, B_I\right) = \Pi\left(B_E + \Delta, B_I\right) - p_{k-1}\left(B_E + \delta, B_I\right),$$

where $p_{k-1}\left(B_E + \delta, B_I\right) > 0$. Hence, the revenue is again lower with sequential auctions.

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Online Appendix
(Not for Publication)

This online Appendix first extends the bunching result of Proposition 3 (Section A). It then provides a partial characterization of socially optimal incentive-compatible mechanisms (Section B), before studying a counter-example in which bunching is not optimal (Section C).

A Optimal bunching

To extend the result of Proposition 3, we now suppose that the handicap is sufficiently diverse, namely: $(\underline{\theta} <)\, \Delta < \bar{\theta}$.[44] Under this assumption, the first-best allocation gives some additional bandwidth to the incumbent when the handicap is low (θ close enough to $\underline{\theta}$), but gives instead the entire additional bandwidth to the entrant when its handicap is large (θ close enough to $\bar{\theta}$).

We now show that, under this assumption, the optimal mechanism exhibits again "full bunching" when attention is restricted to continuous or monotonic allocations:

Proposition 8 *If $\underline{\theta} < \Delta < \bar{\theta}$, then within the set of direct incentive-compatible mechanisms (DICMs) $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ such that the profile $\{b(\theta)\}_{\theta\in\Theta}$ varies either continuously or monotonically with θ, the optimal mechanism exhibits full bunching; that is, it is optimal to offer the same bandwidth to the entrant, regardless of its type.*

Proof. Consider a direct mechanism $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ satisfying feasibility (i.e., $b(\theta) \in [0, \Delta]$), individual rationality and incentive compatibility. We first note that $\bar{\theta} > \Delta$ implies $b(\theta)\,(\leq \Delta) < \hat{b}(\theta)$ whenever θ is sufficiently close to $\bar{\theta}$. Let

$$\hat{\theta} \equiv \inf\left\{\theta \mid b(\theta) \leq \hat{b}(\theta)\right\}$$

denote the threshold beyond which the schedule $b(\cdot)$ remains below the first-best schedule $\hat{b}(\cdot)$. If $\hat{\theta} = \underline{\theta}$ then, as in Lemma 5, replacing the DICM $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ with

[44] Recall that when $\underline{\theta} > \Delta$, the second-best allocation trivially coincides with the first-best allocation, and consists of always allocating the entire additional bandwidth, Δ, to the entrant.

$\left\{\tilde{b}(\theta)=\hat{b}(\underline{\theta}),\tilde{t}(\theta)=0\right\}_{\theta\in\Theta}$ can only increase expected consumer surplus, as the alternative mechanism is trivially incentive-compatible (full bunching) as well as individually rational (it gives zero profit to the entrant, regardless of its handicap), and it is (weakly) closer to the first-best (it actually coincides with the first-best for $\theta=\underline{\theta}$). In what follows, we therefore focus on the case where $\hat{\theta}>\underline{\theta}$.

By construction, $b(\theta)>\hat{b}(\theta)$ for $\theta<\hat{\theta}$. Also, from Lemma 4, $b(\theta)\leq\hat{b}\left(\hat{\theta}\right)$ for $\theta>\hat{\theta}$. Hence, when the schedule $b(\cdot)$ is continuous, we must have $b\left(\hat{\theta}\right)=\hat{b}\left(\hat{\theta}\right)$, and Lemma 3 establishes that full bunching is optimal.

Consider now the case where the schedule $b(\cdot)$ is monotonic. If $b\left(\hat{\theta}\right)=\hat{b}\left(\hat{\theta}\right)$, then Lemma 3 establishes again that full bunching is optimal. If instead $b\left(\hat{\theta}\right)\neq\hat{b}\left(\hat{\theta}\right)$, then monotonicity implies that the profile $\{b(\theta)\}_{\theta\in\Theta}$ must be (weakly) decreasing:

- If $b\left(\hat{\theta}\right)<\hat{b}\left(\hat{\theta}\right)$ then, for θ smaller than but close enough to $\hat{\theta}$ (and thus, $\hat{b}(\theta)$ close enough to $\hat{b}\left(\hat{\theta}\right)$), we have:

$$b(\theta)>\hat{b}(\theta)>b\left(\hat{\theta}\right).$$

- If instead $b\left(\hat{\theta}\right)>\hat{b}\left(\hat{\theta}\right)$, then, for θ larger than $\hat{\theta}$ we have:

$$b\left(\hat{\theta}\right)>\hat{b}\left(\hat{\theta}\right)>b(\theta).$$

It follows that the profile $\{b(\theta)\}_{\theta\in\Theta}$ (i) lies strictly above $\hat{b}\left(\hat{\theta}\right)$ for $\theta<\hat{\theta}$, and below $\hat{b}\left(\hat{\theta}\right)$ for $\theta>\hat{\theta}$, and (ii) it does strictly so in at least one of the ranges (for $\theta>\hat{\theta}$ when $b\left(\hat{\theta}\right)<\hat{b}\left(\hat{\theta}\right)$, and for $\theta<\hat{\theta}$ when $b\left(\hat{\theta}\right)>\hat{b}\left(\hat{\theta}\right)$). But then, replacing the DICM $\{(b(\theta),t(\theta))\}_{\theta\in\Theta}$ lies with the bunching mechanism $\left\{\tilde{b}(\theta)=\hat{b}\left(\hat{\theta}\right),\tilde{t}(\theta)=0\right\}_{\theta\in\Theta}$ strictly increases expected consumer surplus, as $\tilde{b}(\theta)$ is weakly closer to $\hat{b}(\theta)$ for every $\theta\in\Theta$, and strictly so in one of the ranges. ■

B Incentive-compatible mechanisms

The following proposition provides a complete characterization of direct incentive-compatible mechanisms; it shows in particular that:

- The entrant overtakes the incumbent when the handicap is sufficiently small, and the incumbent wins the market otherwise;

- Incentive compatibility allows for discontinuous and non-monotonic bandwidth allocations.

The proposition also provides a partial characterization of the optimal mechanism.

Proposition 9 *(i) Any direct incentive-compatible mechanism (DICM)* $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ *is such that, for some* $\hat{\theta} \in \Theta$ *and some* $\hat{t} \in \mathbb{R}$*:*

- *For* $\theta < \hat{\theta}$, $b(\theta) > \hat{b}(\theta)$, $b(\theta)$ *(weakly) increases with* θ*, and*

$$t(\theta) = \hat{t} + \pi(b(\theta), \theta) + \int_{\theta}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}(b(s), s)\, ds. \tag{14}$$

- *For* $\theta > \hat{\theta}$, $b(\theta) \leq \hat{b}\left(\hat{\theta}\right)$ *and* $t(\theta) = \hat{t}$.

(ii) Without loss of generality, we can further restrict attention to DICMs $\{(b(\theta), t(\theta))\}_{\theta\in\Theta}$ *such that:*

- *For* $\theta > \hat{\theta}$, $b(\theta) = b(\underline{\theta}) \left(\leq \hat{b}\left(\hat{\theta}\right)\right)$ *and* $t(\theta) = 0$*; and*
- *For* $\theta = \hat{\theta}$, $b\left(\hat{\theta}\right)$ *is the closest to* $\hat{b}\left(\hat{\theta}\right)$ *between* $\lim_{\theta\to\hat{\theta}^-} b(\theta)$ *and* $b(\underline{\theta})$*.*

Conversely, any direct mechanism satisfying the above conditions is individually rational, and it is incentive-compatible if and only:

$$\pi(b(\underline{\theta}), \underline{\theta}) = -\int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}(b(\theta), \theta)\, d\theta. \tag{15}$$

Proof. To establish part (i) of the proposition, consider a DICM $\{b(\theta), t(\theta)\}_{\theta\in\Theta}$ and let

$$\hat{\theta} \equiv \begin{cases} \bar{\theta} & \text{if } b(\theta) > \hat{b}(\theta) \text{ for any } \theta \in \Theta, \\ \inf\left\{\theta \mid b(\theta) \leq \hat{b}(\theta)\right\} & \text{otherwise,} \end{cases}$$

denote the threshold beyond which the profile $\{b(\theta)\}_{\theta\in\Theta}$ remains below the first-best profile $\left\{\hat{b}(\theta)\right\}_{\theta\in\Theta}$.

From Lemma 4, we know that $b(\theta)$ remains below $\hat{b}\left(\hat{\theta}\right)$ in the range $\theta > \hat{\theta}$. Therefore, any type $\theta > \hat{\theta}$ would obtain a net profit of $-t\left(\tilde{\theta}\right)$ by picking the option designed for another type $\tilde{\theta} > \hat{\theta}$ (as $\hat{b}(\theta) > \hat{b}\left(\hat{\theta}\right) \geq b\left(\tilde{\theta}\right)$). Incentive compatibility then implies that the profile $t(\theta)$ is constant in the range $\theta > \hat{\theta}$; that is, there exists $\hat{t}$ such that $t(\theta) = \hat{t}$ for $\theta > \hat{\theta}$, and any type $\theta > \hat{\theta}$ obtains a net payoff equal to $-\hat{t}$ by picking any option $\left(b\left(\tilde{\theta}\right), t\left(\tilde{\theta}\right)\right)$ designed for any type $\tilde{\theta} > \hat{\theta}$.

We now turn to the range $\theta < \hat{\theta}$, where $b(\theta) > \hat{b}(\theta)$. By choosing the option designed for a type $\tilde{\theta}$ "close enough" to its own type θ (so that $b\left(\tilde{\theta}\right)$ not only exceeds $\hat{b}\left(\tilde{\theta}\right)$, but also exceeds $\hat{b}(\theta)$), an entrant of type θ would obtain:

$$\varphi\left(\theta, \tilde{\theta}\right) \equiv \pi\left(b\left(\tilde{\theta}\right), \theta\right) - t\left(\tilde{\theta}\right),$$

where:

$$\pi(b, \theta) = \left[c\left(B_I + \Delta - b\right) - c\left(B_I - \theta + b\right)\right] D\left(c\left(B_I + \Delta - b\right)\right).$$

The usual reasoning can then be used to show that incentive compatibility requires the profiles $\{b(\theta)\}_{\theta<\hat{\theta}}$ and $\{t(\theta)\}_{\theta<\hat{\theta}}$ to be (weakly) increasing (as the profit function satisfies Mirrlees' single-crossing property: $\partial^2\pi/\partial\theta\partial b > 0$) and such that, by opting for the option $(b(\theta), t(\theta))$, a type $\theta < \hat{\theta}$ obtains a net profit equal to

$$r(\theta) \equiv \pi(b(\theta), \theta) - t(\theta) = r\left(\hat{\theta}\right) - \int_{\theta}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}(b(s), s)\, ds, \tag{16}$$

which is decreasing in the range $\left[\underline{\theta}, \hat{\theta}\right]$, and such that $\lim_{\theta \to \hat{\theta}^-} r(\theta) = r\left(\hat{\theta}\right)$.

To complete the proof of part (i), we now show that $r\left(\hat{\theta}\right) = -\hat{t}$. Indeed, if $r\left(\hat{\theta}\right) < -\hat{t}$, then an entrant of type $\hat{\theta}$ would strictly prefer any option designed for $\theta > \hat{\theta}$ to the option $\left(b\left(\hat{\theta}\right), t\left(\hat{\theta}\right)\right)$. If instead $r\left(\hat{\theta}\right) > -\hat{t}$, two cases can be distinguished:

- If $b\left(\hat{\theta}\right) > \hat{b}\left(\hat{\theta}\right)$, then a type θ slightly above $\hat{\theta}$ would strictly prefer the option $\left(b\left(\hat{\theta}\right), t\left(\hat{\theta}\right)\right)$ to the option $\left(b(\theta) \leq \hat{b}\left(\hat{\theta}\right), t(\theta) = \hat{t}\right)$.

- If $b\left(\hat{\theta}\right) \leq \hat{b}\left(\hat{\theta}\right)$, then $r\left(\hat{\theta}\right) = -t\left(\hat{\theta}\right)$, and incentive compatibility implies $t\left(\hat{\theta}\right) = \hat{t}$.

Replacing $r\left(\hat{\theta}\right)$ by its value $-\hat{t}$ in (16) yields (14).

To establish part (ii) of the proposition, we first show that without loss of generality, we can restrict attention to profiles $\{b(\theta)\}_{\theta\in\Theta}$ that remain constant in the range $\theta > \hat{\theta}$. To see this, it suffices to note that replacing the profile $\{b(\theta)\}_{\theta>\hat{\theta}}$ with the constant profile $\left\{\tilde{b}(\theta)) = \underline{b}\right\}_{\theta>\hat{\theta}}$, where

$$\underline{b} \equiv \sup\left\{b(\theta) \mid \theta > \hat{\theta}\right\},$$

weakly improves expected consumer surplus:

- This does not affect incentive compatibility in the range $\theta > \hat{\theta}$, as any such type obtains $-\hat{t}$ anyway.
- This does not affect incentive compatibility in the range $\theta \leq \hat{\theta}$ either, as by construction we have, for any $\theta \leq \hat{\theta}$:

$$\pi(b(\theta), \theta) - t(\theta) \geq \sup_{\tilde{\theta}>\theta}\left\{\pi\left(b\left(\tilde{\theta}\right), \theta\right) - t\left(\tilde{\theta}\right)\right\} = \pi(\underline{b}, \theta) - \hat{t}.$$

- Finally, this can only reduce the consumer price in the range $\theta > \hat{\theta}$; indeed, for any $\theta > \hat{\theta}$, the consumer price is initially given by

$$c_E = c(B_I - \theta + b(\theta)),$$

  and thus can only decrease when $b(\theta)$ is replaced with $\underline{b} \geq b(\theta)$.

When $b\left(\hat{\theta}\right) \leq \hat{b}\left(\hat{\theta}\right)$, the same reasoning applies to the entire range $\theta \geq \hat{\theta}$ (that is, including the type $\hat{\theta}$), and thus $b\left(\hat{\theta}\right) = \underline{b}$. When instead $b\left(\hat{\theta}\right) > \hat{b}\left(\hat{\theta}\right)$, incentive compatibility implies that $b\left(\hat{\theta}\right)$ cannot lie below $\lim_{\theta\to\hat{\theta}^-} b(\theta)$. Furthermore, as $b(\theta) > \hat{b}(\theta)$ for $\theta < \hat{\theta}$, it must be case that $\lim_{\theta\to\hat{\theta}^-} b(\theta) \geq \hat{b}\left(\hat{\theta}\right)$; therefore, setting $b\left(\hat{\theta}\right) = \lim_{\theta\to\hat{\theta}^-} b(\theta)$ is better than any higher value for $b\left(\hat{\theta}\right)$.

So far we have shown that any type $\theta \in \Theta$ obtains a net profit that is continuous in θ and weakly decreases as θ increases: it coincides with $r(\theta)$ given by (16) for $\theta \leq \hat{\theta}$, and with $r\left(\hat{\theta}\right) = -\hat{t}$ for $\theta \geq \hat{\theta}$. Therefore, individual rationality boils down to $\hat{t} \leq 0$, and without loss of generality we can set $\hat{t} = 0$.

To conclude the proof of part (ii), it remains to show that without loss of generality, we can further restrict attention to DICMs such that $\underline{b} = b(\underline{\theta}) \leq \hat{b}\left(\hat{\theta}\right)$ and (15) holds. We do through a sequence of claims.

We first note that incentive compatibility requires $\underline{b} \leq b(\underline{\theta})$:

Claim 1 *$\underline{b} \leq b(\underline{\theta})$.*

Proof. *By construction, $\underline{b} \leq \hat{b}\left(\hat{\theta}\right)$ (as $b(\theta)$ lies below $\hat{b}\left(\hat{\theta}\right)$ in the range $\theta > \hat{\theta}$). Suppose now that $\underline{b} > b(\underline{\theta}) \left(> \hat{b}(\underline{\theta})\right)$. As type $\underline{\theta}$ should prefer $(b(\underline{\theta}), t(\underline{\theta}))$ to $\left(\underline{b}, \hat{t} = 0\right)$, we have:*

$$\pi(b(\underline{\theta}), \underline{\theta}) - t(\underline{\theta}) \geq \pi(\underline{b}, \underline{\theta}).$$

Conversely, any type $\theta > \hat{\theta}$ should prefer $(\underline{b}, 0)$ to $(b(\underline{\theta}), t(\underline{\theta}))$, and thus (using $\underline{b} \leq \hat{b}\left(\hat{\theta}\right) < \hat{b}(\theta)$):

$$0 \geq -t(\underline{\theta}).$$

Combining these two incentive compatibility conditions yields, $\pi(b(\underline{\theta}), \underline{\theta}) \geq \pi(\underline{b}, \underline{\theta})$, implying $\underline{b} \leq b(\underline{\theta})$, a contradiction. Hence, we must have $\underline{b} \leq b(\underline{\theta})$. ■

Next, we show that we can restrict attention to DICMs such that $b(\underline{\theta}) \leq \hat{b}\left(\hat{\theta}\right)$:

Claim 2 *$b(\underline{\theta}) \leq \hat{b}\left(\hat{\theta}\right)$.*

Proof. *To see this, note that incentive compatibility requires $b(\theta)$ to be non-decreasing in the range $\theta < \hat{\theta}$. Hence, if $b(\underline{\theta}) > \hat{b}\left(\hat{\theta}\right)$, then the alternative mechanism where $(b(\theta), t(\theta))$ is replaced with $\left(\tilde{b}(\theta), \tilde{t}(\theta)\right) = \left(\hat{b}\left(\hat{\theta}\right), \hat{t} = 0\right)$ in the range $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$, is trivially incentive-compatible and individually rational, and would dominate the original DICM, as it gets closer to the first-best in the range $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$.* ■

The next step is to show that incentive compatibility imposes some bounds on $b(\underline{\theta})$ and $\underline{b}$:

Claim 3 *Incentive compatibility requires the profile $\{b(\theta)\}_{\theta \in \Theta}$ to satisfy:*

$$\pi(\underline{b}, \underline{\theta}) \leq -\int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}(b(\theta), \theta)\, d\theta \leq \pi(b(\underline{\theta}), \underline{\theta}). \tag{17}$$

Proof. *To ensure that the type $\underline{\theta}$ does not prefer the option designed for a type $\theta > \hat{\theta}$, we must have:*

$$\pi\left(\underline{b}, \underline{\theta}\right) \leq r\left(\underline{\theta}\right) = -\int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}\left(b\left(\theta\right), \theta\right) d\theta,$$

which establishes the first condition in (17).

Conversely, to ensure that a type $\theta > \hat{\theta}$ does not prefer the option designed for $\underline{\theta}$, we must have (using $b\left(\underline{\theta}\right) \leq \hat{b}\left(\hat{\theta}\right)$):

$$0 \geq -t\left(\underline{\theta}\right) = -\pi\left(b\left(\underline{\theta}\right), \underline{\theta}\right) - \int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}\left(b\left(s\right), s\right) ds,$$

and thus:

$$\pi\left(b\left(\underline{\theta}\right), \underline{\theta}\right) \geq -\int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}\left(b\left(s\right), s\right) ds,$$

which establishes the second condition in (17). ■

Conversely, we now show that a mechanism $\{(b(\theta), t(\theta))\}_{\theta \in \Theta}$ is incentive-compatible and individually rational if it satisfies the above properties:

Claim 4 *A mechanism $\{(b(\theta), t(\theta))\}_{\theta \in \Theta}$ is incentive-compatible and individually rational whenever, for some $\hat{\theta} \in \Theta$ and some $\underline{b} \leq \hat{b}\left(\hat{\theta}\right)$:*

(a) *for $\theta < \hat{\theta}$, $b(\theta) > \hat{b}(\theta)$, $b(\theta)$ increases with θ, and $t(\theta)$ satisfies* (14)*; and*

(b) *for $\theta \geq \hat{\theta}$, $b(\theta) = \underline{b}$ and $t(\theta) = 0$; and*

(c) *the profile $\{b(\theta)\}_{\theta \in \Theta}$ satisfies $b\left(\underline{\theta}\right) \leq \hat{b}\left(\hat{\theta}\right)$ and* (17).

Proof. *We first note that, from the above analysis, such a DICM is individually rational:*

- *For $\theta \geq \hat{\theta}$, opting for the option $(\underline{b}, 0)$ guarantees a net profit of zero.*
- *For $\theta < \hat{\theta}$, opting for the option $(b(\theta), t(\theta))$ gives a net profit equal to*

$$r\left(\theta\right) = \pi\left(b\left(\theta\right), \theta\right) - t\left(\theta\right) = -\int_{\theta}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}\left(b\left(s\right), s\right) ds,$$

which is decreasing in the range $\left[\underline{\theta}, \hat{\theta}\right]$ and satisfies $r\left(\hat{\theta}\right) = 0$.

We now turn to incentive compatibility. Consider first a type $\theta < \hat{\theta}$. Condition (14) *ensures that such a type (weakly) prefers the option designed for it to any option designed for another $\tilde{\theta} < \hat{\theta}$. Therefore, incentive compatibility holds if, in addition, it does not prefer the option $(\underline{b}, 0)$ designed for $\theta > \hat{\theta}$, that is, if:*

$$r(\theta) \geq \pi(\underline{b}, \theta).$$

We have:

$$\begin{aligned} \frac{d}{d\theta}\left[r(\theta) - \pi(\underline{b}, \theta)\right] &= \frac{\partial \pi}{\partial \theta}(b(\theta), \theta) - \frac{\partial \pi}{\partial \theta}(\underline{b}, \theta) \\ &= \int_{\underline{b}}^{b(\theta)} \frac{\partial^2 \pi}{\partial b \partial \theta}(b(s), s)\, ds \\ &\geq 0, \end{aligned}$$

where the inequality stems from $\underline{b} \leq b(\underline{\theta}) \leq b(\theta)$ and $\partial^2\pi/\partial b\partial\theta > 0$. Therefore, incentive compatibility holds for any type $\theta < \hat{\theta}$ if it holds for $\underline{\theta}$, that is, if

$$\pi(\underline{b}, \theta) \leq r(\underline{\theta}) = -\int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}(b(s), s)\, ds,$$

which amounts to the first inequality in (17)*.*

Next, consider a type $\theta > \hat{\theta}$:

- *By selecting an option designed for $\tilde{\theta} < \hat{\theta}$ such that $b\left(\tilde{\theta}\right) > \hat{b}(\theta)$ (which is feasible if $\lim_{\theta \to \hat{\theta}^-} b(\theta) > \hat{b}(\theta)$), it obtains*

  $$\varphi\left(\theta, \tilde{\theta}\right) = \pi\left(b\left(\tilde{\theta}\right), \theta\right) - t\left(\tilde{\theta}\right) = r\left(\tilde{\theta}\right) + \pi\left(b\left(\tilde{\theta}\right), \theta\right) - \pi\left(b\left(\tilde{\theta}\right), \tilde{\theta}\right),$$

  where:

  $$\begin{aligned} &\frac{\partial \varphi}{\partial \tilde{\theta}}\left(\theta, \tilde{\theta}\right) = \left[\frac{\partial \pi}{\partial b}\left(b\left(\tilde{\theta}\right), \theta\right) - \frac{\partial \pi}{\partial b}\left(b\left(\tilde{\theta}\right), \tilde{\theta}\right)\right] \frac{db}{d\theta}\left(\tilde{\theta}\right) = \int_{\tilde{\theta}}^{\theta} \frac{\partial^2 \pi}{\partial \theta \partial b}\left(b\left(\tilde{\theta}\right), s\right) ds \frac{db}{d\theta}\left(\tilde{\theta}\right) \geq 0, \\ &\frac{\partial \varphi}{\partial \theta}\left(\theta, \tilde{\theta}\right) = \frac{\partial \pi}{\partial \theta}\left(b\left(\tilde{\theta}\right), \theta\right) < 0. \end{aligned}$$

  Therefore, incentive compatibility holds, as $\varphi\left(\theta, \tilde{\theta}\right) \leq \varphi\left(\theta, \hat{\theta}\right) < \varphi\left(\hat{\theta}, \hat{\theta}\right) = 0$.

- *By selecting instead an option designed for* $\tilde{\theta} < \hat{\theta}$ *such that* $b\left(\tilde{\theta}\right) \leq \hat{b}(\theta)$, *an entrant of type* θ *obtains a net profit of* $-t\left(\tilde{\theta}\right)$*; as* $t\left(\tilde{\theta}\right)$ *increases with* $\tilde{\theta}$, *this net profit is maximal for* $\tilde{\theta} = \underline{\theta}$. *It follows that incentive compatibility holds if:*

$$0 \geq -t\left(\underline{\theta}\right) = -\pi\left(b\left(\underline{\theta}\right), \underline{\theta}\right) - \int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}\left(b\left(s\right), s\right) ds,$$

which amounts to the second inequality in (17).

■

Note that (17) implies $\underline{b} \leq b\left(\underline{\theta}\right)$. Thus, so far we have shown that attention could be restricted to DICMs such as described in the Proposition, except that we allow for $\underline{b} \leq b\left(\underline{\theta}\right)$ and the bounds on transfers are given by (17).

We now show that we can further restrict the relevant class of DICMs:

Claim 5 *Without loss of generality, we can restrict attention to DICMs such as described by Claim 4 that moreover satisfy*

$$\pi\left(b\left(\underline{\theta}\right), \underline{\theta}\right) = -\int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}\left(b\left(s\right), s\right) ds.$$

Proof. *Suppose that the DICM* $\left\{\left(b\left(\theta\right), t\left(\theta\right)\right)\right\}_{\theta \in \Theta}$ *is instead such that*

$$\pi\left(b\left(\underline{\theta}\right), \underline{\theta}\right) > r\left(\underline{\theta}\right) = -\int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}\left(b\left(s\right), s\right) ds,$$

and consider now the following alternative mechanism, where for every $\theta < \hat{\theta}$, $b\left(\theta\right)$ *is replaced with*[45]

$$b\left(\theta, \alpha\right) \equiv \alpha \hat{b}\left(\theta\right) + \left(1 - \alpha\right) b\left(\theta\right).$$

In this alternative mechanism, a type $\underline{\theta}$ *obtains a net profit equal to*

$$r\left(\underline{\theta}, \alpha\right) \equiv -\int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}\left(b\left(\theta, \alpha\right), \theta\right) d\theta,$$

[45] In the alternative mechanism, the bandwidth allocated to a type $\hat{\theta}$ can be taken equal to $b\left(\hat{\theta}, \alpha\right)$ when $b\left(\hat{\theta}\right) = \lim_{\theta \to \hat{\theta}^-} b\left(\theta\right)$, and to $\underline{b}$ otherwise.

where:

$$\frac{\partial r}{\partial \alpha}(\underline{\theta},\alpha)=\left[b(\underline{\theta})-\hat{b}(\underline{\theta})\right]\int_{\underline{\theta}}^{\hat{\theta}}\frac{\partial^2 \pi}{\partial b \partial \theta}(b(\theta,\alpha),\theta)\,d\theta>0.$$

It follows that starting from $\alpha=0$, an increase in α:

- *Reduces the consumer price when the entrant is of type $\theta<\hat{\theta}$, from $c_I|_{b=b(\theta)}=c(B_I+\Delta-b(\theta))$ to $c_I|_{b=b(\theta,\alpha)}=c(B_I+\Delta-b(\theta,\alpha))$.*
- *Relaxes the second constraint in (17), which becomes:*

  $$\pi(\underline{b},\underline{\theta})\leq r(\underline{\theta},\alpha),$$

  where $r(\underline{\theta},\alpha)>r(\underline{\theta},0)=r(\underline{\theta})$.
- *Keeps satisfying the first constraint in (17), for α small enough.*

Therefore, without loss of generality, we can set restrict attention to DICMs satisfying

$$\pi(b(\underline{\theta}),\underline{\theta})=-\int_{\underline{\theta}}^{\hat{\theta}}\frac{\partial \pi}{\partial \theta}(b(s),s)\,ds.$$

■

Note that the equality stated in Claim 5 implies $t(\underline{\theta})=0\left(=\hat{t}\right)$. This, in turn, yields:

Claim 6 *Without loss of generality, we can restrict attention to DICMs such as described by Claim 4 that moreover satisfy*

$$\pi(\underline{b},\underline{\theta})=-\int_{\underline{\theta}}^{\hat{\theta}}\frac{\partial \pi}{\partial \theta}(b(s),s)\,ds.$$

Proof. *Suppose that the DICM $\{(b(\theta),t(\theta))\}_{\theta\in\Theta}$ satisfies is such that*

$$\pi(b(\underline{\theta}),\underline{\theta})=-\int_{\underline{\theta}}^{\hat{\theta}}\frac{\partial \pi}{\partial \theta}(b(s),s)\,ds>\pi(\underline{b},\underline{\theta}),$$

implying $\underline{b}<b(\underline{\theta})\left(\leq\hat{b}\left(\hat{\theta}\right)\right)$, and consider the alternative direct mechanism where, for $\theta>\hat{\theta}$, $b(\theta)=\underline{b}$ is replaced with $\tilde{b}(\theta)=b(\underline{\theta})$ – transfers being unchanged: $\tilde{t}(\theta)=t(\theta)=t(\underline{\theta})=\hat{t}=0$. The alternative mechanism is remains individually rational and *incentive-compatible and it increases consumers' expected surplus (as the price decreases from* $c_E|_{b=\underline{b}} = c(B_I - \theta + \underline{b})$ *to* $c_E|_{b=b(\underline{\theta})} = c(B_I - \theta + b(\underline{\theta})))$*. Therefore, without loss of generality, we can set* $\underline{b} = b(\underline{\theta})$*, implying*

$$\pi(b(\underline{\theta}), \underline{\theta}) = -\int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial \pi}{\partial \theta}(b(s), s)\, ds = \pi(\underline{b}, \underline{\theta}).$$

■

This concludes the proof of part (ii). ■

C An example where bunching is not optimal

We provide here an example where bunching is no longer optimal.

C.1 Setup

C.1.1 Demand and supply conditions

We will adopt the following specifications:

- *Linear demand:* Letting p denote the market price (i.e., the lowest of the firms' prices), consumer demand is given by:

$$D(p) = 1 - p.$$

  It follows that consumer surplus is equal to:

$$S(p) = \frac{(1-p)^2}{2},$$

  and the industry monopoly profit, based on a constant marginal cost γ, is equal to:

$$p^m(\gamma) = \frac{1+\gamma}{2}.$$

- *Linear unit cost:* If a firm benefits from a bandwidth $\tilde{B}$, its unit cost is given by:

$$c\left(\tilde{B}\right) = C - \tilde{B}.$$

C.1.2 Two types of entrant

We denote the bandwidth initially available to the incumbent by

$$B_I = B$$

and that initially available to the entrant by

$$B_E = B - \tilde{\theta}.$$

Thus, as before, the parameter $\tilde{\theta}$ reflects the handicap of the entrant. We assume that this handicap can take two values:

- With probability $\rho \in (0, 1)$, the handicap is given by $\tilde{\theta} = 0$; that is, the entrant is initially as efficient as the incumbent.
- With probability $\rho' = 1 - \rho$, the handicap is given by $\tilde{\theta} = B$; that is, the entrant has initially no bandwidth.

C.1.3 Calibration

For the sake of exposition, we further assume that:

- The additional bandwidth is large enough to enable both types of entrant to win the market competition; that is:

  $$\Delta = B + 2\varepsilon,$$

  where $\varepsilon > 0$. With this notation, the relevant values of the critical bandwidth threshold,

  $$\hat{b}(\theta) = \frac{\Delta + \theta}{2},$$

  are equal to:

  $$\hat{b} = \hat{b}(0) = \frac{\Delta}{2} = \frac{B}{2} + \varepsilon,$$
  $$\hat{b}' = \hat{b}(B) = \frac{B + \Delta}{2} = B + \varepsilon.$$

- The cost function is normalized such that

$$C = B + \Delta = 2\left(B + \varepsilon\right).$$

This ensures that all unit costs remain non-negative (the incumbent benefits from a zero unit cost if it obtains all the additional bandwidth, and both unit costs are positive otherwise), and also simplifies some of the exposition.[46]

C.1.4 Prices

When an entrant of type $\tilde{\theta}$ obtains an additional bandwidth $\tilde{b} \geq \hat{b}\left(\tilde{\theta}\right)$, the market price $p_{\tilde{\theta}}\left(\tilde{b}\right)$ is equal to the cost of the incumbent:

$$c\left(B_I + \Delta - \tilde{b}\right) = C - \left(B + \Delta - \tilde{b}\right).$$

Using $C = B + \Delta$, this simplifies to

$$p_{\tilde{\theta}}\left(\tilde{b}\right) = \tilde{b}.$$

When instead the incumbent wins the competition (i.e., when $\tilde{b} < \hat{b}\left(\theta\right)$), the market price is determined by the cost of the entrant, and is thus equal to:

$$p_{\tilde{\theta}}\left(\tilde{b}\right) \equiv \begin{cases} B + 2\varepsilon - \tilde{b} & \text{if } \tilde{\theta} = \theta = 0, \\ 2\left(B + \varepsilon\right) - \tilde{b} & \text{if } \tilde{\theta} = \theta' = B. \end{cases}$$

C.1.5 Profit

When an entrant of type $\tilde{\theta}$ obtains an additional bandwidth $\tilde{b} \in \left[\hat{b}\left(\tilde{\theta}\right), 1\right]$, it wins the product-market competition and obtains a profit equal to:

$$\begin{aligned} \pi\left(\tilde{b}, \tilde{\theta}\right) &\equiv \left[c\left(B + \Delta - \tilde{b}\right) - c\left(B - \tilde{\theta} + \tilde{b}\right)\right] D\left(c\left(B + \Delta - \tilde{b}\right)\right) \\ &= 2\left[\tilde{b} - \hat{b}\left(\tilde{\theta}\right)\right]\left(1 - \tilde{b}\right). \end{aligned}$$

[46] See for instance below the derivation of the market price $p_{\tilde{\theta}}\left(\tilde{b}\right)$ for $\tilde{b} \geq \hat{b}\left(\tilde{\theta}\right)$.

We have:

$$\frac{\partial \pi}{\partial \tilde{\theta}} \left(\tilde{b}, \tilde{\theta} \right) = - \left(1 - \tilde{b} \right) \leq 0,$$

with a strict inequality for $\tilde{b} < 1$, and:

$$\frac{\partial^2 \pi}{\partial \tilde{\theta} \partial \tilde{b}} \pi \left(\tilde{b}, \tilde{\theta} \right) = 1 > 0.$$

Finally,

$$\partial_{\tilde{b}} \pi \left(\tilde{b}, \tilde{\theta} \right) = 2 \left(1 - \tilde{b} \right) - 2 \left(\tilde{b} - \hat{b} \left(\tilde{\theta} \right) \right) = 4 \left(\frac{1 + \hat{b} \left(\tilde{\theta} \right)}{2} - \tilde{b} \right) = 4 \left[\hat{p}^m \left(\hat{b} \left(\tilde{\theta} \right) \right) - \tilde{b} \right],$$

where

$$\hat{p}^m \left(\gamma \right) = \frac{1 + \gamma}{2}$$

denotes the monopoly price based on a unit cost γ. Hence, $\partial_{\tilde{b}} \pi \left(\tilde{b}, \tilde{\theta} \right)$ is positive as long as:

$$p_{\tilde{\theta}} \left(\tilde{b} \right) = \tilde{b} < \hat{p}^m \left(\hat{b} \left(\tilde{\theta} \right) \right) = \frac{1 + \hat{b} \left(\tilde{\theta} \right)}{2} = \frac{1 + \frac{\Delta + \tilde{\theta}}{2}}{2} = \frac{1}{2} \left(1 + B + \varepsilon + \frac{\tilde{\theta}}{2} \right).$$

In particular, when the entrant has no handicap ($\theta = 0$), in order to ensure that the price ($p_\theta \left(b \right) = b$) remains below the monopoly level $\left(\hat{p}^m \left(\hat{b} \right) \right)$ in the relevant range $\left(b \in \left[\hat{b}, \hat{b}' \right] \right)$, we need:

$$\begin{aligned} \hat{b}' = B + \varepsilon < \hat{p}^m \left(\hat{b} \right) = \frac{1 + \frac{B}{2} + \varepsilon}{2} \\ \Longleftrightarrow B < \frac{2}{3} \left(1 - \varepsilon \right). \end{aligned} \tag{18}$$

C.2 Bunching mechanisms

When considering bunching mechanisms, which allocate the same additional bandwidth b to both types of entrant, without loss of generality we can restrict attention to $b \in \left[\hat{b}, \hat{b}' \right]$, as any lower value $\left(b < \hat{b} \right)$ is dominated by $b = \hat{b}$, and any higher value $\left(b > \hat{b}' \right)$ is dominated by $b = \hat{b}'$. For any value b in that range:

- when the entrant has no handicap, the market price is equal to the cost of the incumbent; and

- otherwise, the market price is equal to the cost of the entrant.

Hence, expected consumer surplus is equal to:

$$\begin{aligned} S_B(b) &= \rho S(c(B+\Delta-b)) + \rho' S(c(B-\theta'+b)) \\ &= \rho S(b) + \rho' S(2B+2\varepsilon-b). \end{aligned}$$

This expected surplus is convex in b:

$$\begin{aligned} S_B'(b) &= -\rho D(b) + \rho' D(2B+2\varepsilon-b), \\ S_B''(b) &= \rho+\rho' > 0. \end{aligned}$$

It follows that the best bunching mechanism consists of allocating either $\hat{b}$ or $\hat{b}'$ to the entrant; both options are moreover equivalent when:

$$\begin{aligned} & S_B\left(\hat{b}'\right) = S_B\left(\hat{b}\right) \\ &\Longleftrightarrow \rho S\left(C-s\left(B+\Delta-\hat{b}'\right)\right) + \rho' S\left(C-s\left(B-\theta'+\hat{b}'\right)\right) = \rho S\left(C-s\left(B+\Delta-\hat{b}\right)\right) + \rho' S\left(C - \right. \\ &\Longleftrightarrow \frac{\rho}{\rho'} = \frac{S\left(C-\left(B-\theta'+\hat{b}'\right)\right) - S\left(C-\left(B-\theta'+\hat{b}\right)\right)}{S\left(C-\left(B+\Delta-\hat{b}\right)\right) - S\left(C-\left(B+\Delta-\hat{b}'\right)\right)} \\ &\Longleftrightarrow \frac{\rho}{\rho'} = \frac{1-\varepsilon-\frac{5B}{4}}{1-\varepsilon-\frac{3B}{4}}. \end{aligned} \tag{19}$$

C.3 Discriminating mechanisms

We will consider a candidate discriminating mechanism which gives the non-handicapped entrant a bandwidth $b \in \left[\hat{b}, \hat{b}'\right]$ (in exchange for a transfer t), and the handicapped entrant a higher bandwidth $b' > \hat{b}'$ (in exchange for a transfer t').[47] To be incentive-

[47] By inspecting the incentive constraints for all possible cases (where $b \gtrless \hat{b}$ and $b' \gtrless \hat{b}'$), it can be checked that the best discriminating mechanism has indeed these features.

compatible, the mechanism must satisfy:

$$\pi\left(b,\theta\right)-t\geq\pi\left(b',\theta\right)-t',$$
$$\pi\left(b',\theta'\right)-t'\geq-t.$$

Combining these conditions imposes:

$$\pi\left(b,\theta\right)\geq\pi\left(b',\theta\right)-\pi\left(b',\theta'\right). \tag{20}$$

The right-hand side of this inequality decreases as b' increases:

$$\begin{aligned}\frac{d}{db'}\left(\pi\left(b',\theta\right)-\pi\left(b',\theta'\right)\right)&=\frac{\partial\pi}{\partial\tilde{b}}\left(b',\theta\right)-\frac{\partial\pi}{\partial\tilde{b}}\left(b',\theta'\right)\\&=-\int_{\theta}^{\theta'}\frac{\partial^2\pi}{\partial\tilde{\theta}\partial\tilde{b}}\left(b',x\right)dx\\&<0.\end{aligned}$$

Hence, for any given bandwidth, $b\in\left[\hat{b},\hat{b}'\right]$, that an entrant with no handicap would receive, the best value for the bandwidth, b', that a handicapped entrant should receive is the lowest one that is compatible with (20); that is, b' should be chosen such that:

$$\begin{aligned}&\pi\left(b,\theta\right)=\pi\left(b',\theta\right)-\pi\left(b',\theta'\right)\\&\quad\Longleftrightarrow 2\left(b-\hat{b}\right)\left(1-b\right)=2\left(b'-\hat{b}\right)\left(1-b'\right)-2\left(b'-\hat{b}'\right)\left(1-b'\right)\\&\quad\Longleftrightarrow b'=\beta\left(b\right)\equiv\left[1-\frac{b-\hat{b}}{\hat{b}'-\hat{b}}\left(1-b\right)\right]=2-b-\frac{2}{B}\left(b-\varepsilon\right)\left(1-b\right)=\frac{B\left(2-b\right)-2\left(b-\varepsilon\right)\left(1-b\right)}{B}.\end{aligned}$$

This optimal value is such that:

$$\beta\left(\hat{b}'\right)=\frac{\hat{b}'\left(\hat{b}'-\hat{b}\right)}{\hat{b}'-\hat{b}}=\hat{b}',$$

and, for $b \in \left[\hat{b}, \hat{b}'\right]$:

$$\beta'(b) = \frac{d}{db}\left(\frac{B(2-b) - 2(b-\varepsilon)(1-b)}{B}\right)$$
$$= -\frac{2 + B + 2\varepsilon - 4b}{B}$$
$$= -\frac{1 + \hat{b} - 2b}{\hat{b}' - \hat{b}} = -2\frac{\hat{p}^m\left(\hat{b}\right) - b}{\hat{b}' - \hat{b}},$$

which is negative as long as

$$b < \hat{p}^m\left(\hat{b}\right) = \frac{1 + \hat{b}}{2} = \frac{1}{2}\left(1 + \frac{B}{2} + \varepsilon\right).$$

In particular:

$$\beta'\left(\hat{b}'\right) = -2\frac{\hat{p}^m\left(\hat{b}\right) - \hat{b}'}{\hat{b}' - \hat{b}} = \left[-2\frac{\frac{1}{2} + \frac{\Delta}{4} - \frac{\Delta + B}{2}}{\frac{B}{2}}\right]_{\Delta = B + 2\varepsilon} = -\frac{2(1-\varepsilon) - 3B}{B},$$

which is thus negative as long as B satisfies (18). Hence, as long as B satisfies this condition, $\beta(b)$ is indeed higher than $\hat{b}'$ for b lower than but close to $\hat{b}'$.

Expected consumer surplus is then equal to:

$$S_D(b) = \rho S(c(B + \Delta - b)) + \rho' S(c(B + \Delta - \beta(b)))$$
$$= \rho\frac{(1-b)^2}{2} + \rho'\frac{(1-\beta(b))^2}{2}.$$

Therefore:

$$S_D'(b) = -\rho(1-b) - \rho'(1-\beta(b))\beta'(b),$$

Bunching will for instance not be optimal if:

- the probabilities of the two types are such that expected consumer surplus is the same in the situation where both types receive $\hat{b}$ and in the situation where they both receive $\hat{b}'$; and,
- starting from the latter situation, where both types receive $\hat{b}'$, a small reduction in the bandwidth b allocated to the entrant in case of no handicap, together with an

increase in the bandwidth b' allocated to the entrant in case of a large handicap, up to $b' = \beta(b)$, increases expected consumer surplus.

Hence, to exhibit an example where bunching is not optimal, it suffices to find parameters B and ε such that $S_D'\left(\hat{b}'\right) < 0$ for the probabilities ρ and ρ' that satisfy (19). As:

$$\begin{aligned} S_D'\left(\hat{b}'\right) &= -\rho\left(1-\hat{b}'\right) - \rho'\left(1-\hat{b}'\right)\beta'\left(\hat{b}'\right) \\ &= -\rho\left(1-\hat{b}'\right)\left[1+\frac{\rho'}{\rho}\beta'\left(\hat{b}'\right)\right], \end{aligned}$$

this amounts to finding parameters B and ε such that the terms within square brackets is positive, that is:

$$-\beta'\left(\hat{b}'\right) = \frac{2-3B-2\varepsilon}{B} < \frac{\rho}{\rho'} = \frac{1-\varepsilon-\frac{5B}{4}}{1-\varepsilon-\frac{3B}{4}}.$$

This requires:

$$\begin{aligned} &\frac{2-3B-2\varepsilon}{B} < \frac{1-\frac{5B}{4}-\varepsilon}{1-\frac{3B}{4}-\varepsilon} \\ &\Longleftrightarrow (2-3B-2\varepsilon)\left(1-\frac{3B}{4}-\varepsilon\right) < B\left(1-\frac{5B}{4}-\varepsilon\right) \\ &\Longleftrightarrow 0 < B\left(1-\frac{5B}{4}-\varepsilon\right) - (2-3B-2\varepsilon)\left(1-\frac{3B}{4}-\varepsilon\right) = -2\left(1-\varepsilon-\frac{7B}{4}\right)(1-\varepsilon-B), \end{aligned}$$

which amounts to:

$$\frac{4}{7}(1-\varepsilon) < B < 1-\varepsilon.$$

Combining these conditions with (18), it suffices to choose B and ε such that:

$$\frac{4}{7}(1-\varepsilon) < B < \frac{2}{3}(1-\varepsilon).$$

C.4 Numerical example

C.4.1 Parameter values

For $\varepsilon = 0$, the above conditions boil down to:

$$\frac{4}{7} = \frac{12}{21} < B < \frac{2}{3} = \frac{14}{21}.$$

We will thus consider the case

$$B = \frac{13}{21},$$

and choose ε "small enough" to satisfy (18), namely, such that:

$$B < \frac{2}{3}(1-\varepsilon) \Longleftrightarrow \varepsilon < 1 - \frac{3B}{2} = \frac{1}{14} \simeq 0.07.$$

We will thus take $\varepsilon = 0.05\,(= 1/20)$. We then have:

$$\begin{aligned}
\hat{b} &= \frac{13}{42} + \frac{1}{20} = \frac{151}{420} \simeq 0.36, \\
\hat{b}' &= \frac{13}{21} + \frac{1}{20} = \frac{281}{420} \simeq 0.67, \\
\Delta &= \frac{13}{21} + \frac{1}{10} = \frac{151}{210} \simeq 0.72, \\
\beta(b) &= \left[\frac{B(2-b) - 2(b-\varepsilon)(1-b)}{B}\right]_{B=\frac{13}{21},\varepsilon=\frac{1}{20}} = \frac{281}{130} - \frac{571}{130}b + \frac{42}{13}b^2, \\
p^m\left(\hat{b}\right) &= \left[\frac{1+\hat{b}}{2}\right]_{\hat{b}=13/42+1/20} = \frac{571}{840} \simeq 0.68, \\
p^m\left(\hat{b}'\right) &= \left[\frac{1+\hat{b}'}{2}\right]_{\hat{b}'=13/21+1/20} = \frac{701}{840} \simeq 0.83, \\
\rho &= \frac{37}{139} \simeq 0.27, \\
\rho' &= \frac{102}{139} \simeq 0.73,
\end{aligned}$$

and:

$$-\beta'\left(\hat{b}'\right) = \frac{9}{130} \simeq 0.06 < \frac{\rho}{\rho'} = \frac{37}{102} \simeq 0.36.$$

The function $\beta(b)$ is depicted by the following figure (for $b \in \left[\hat{b}, \hat{b}'\right] \simeq [0.36, 0.67]$):

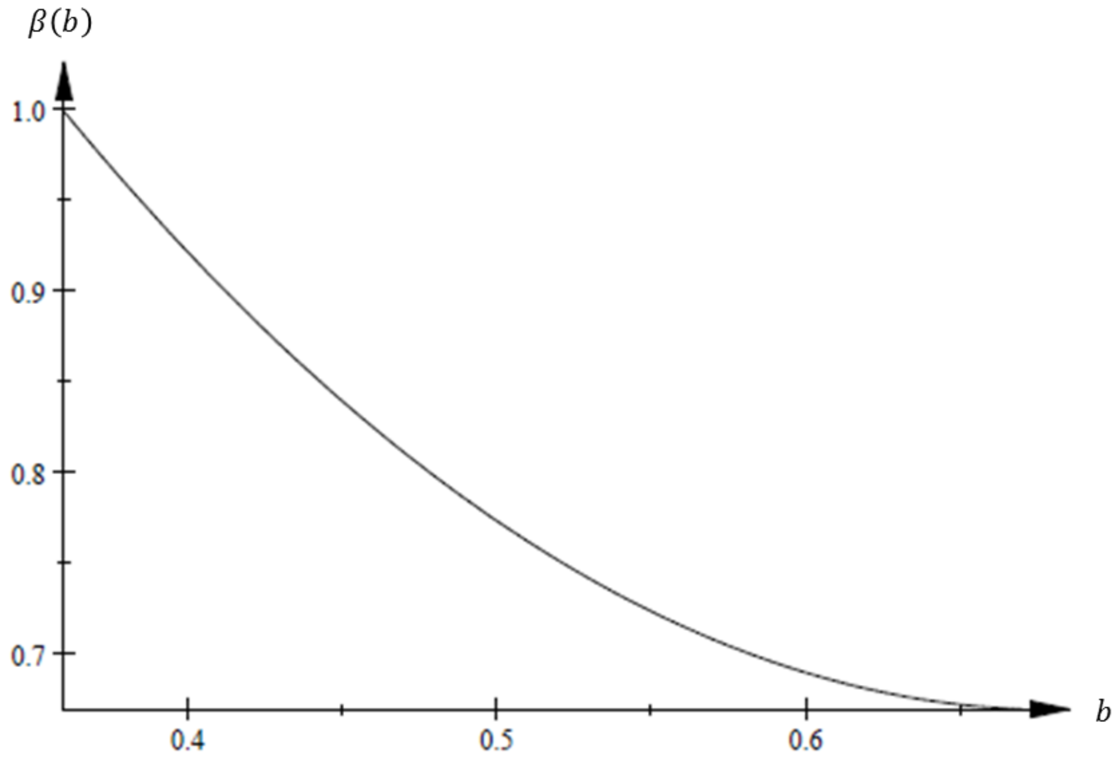

Figure 3: Bandwidth for handicapped entrant

In particular, we have:

$$\begin{aligned}\beta(b) \leq \Delta &\Longleftrightarrow \frac{281}{130} - \frac{571}{130}b + \frac{42}{13}b^2 \leq \frac{151}{210} \\ &\Longleftrightarrow b \geq \bar{b} = \frac{571}{840} - \frac{\sqrt{3}\sqrt{3667}}{840} \simeq 0.55.\end{aligned}$$

It can be checked that the above conditions are satisfied in this example; in particular:

- Demand is positive (i.e., $b < 1$) in the relevant ranges $b \leq \hat{b}'$ (as $\hat{b}' \simeq 0.67 < 1$) and $b' \leq \Delta$ (as $\Delta \simeq 0.72 < 1$). It follows that $\partial_\theta \pi(b, \theta) < 0$ in the relevant ranges.

- We also have $\partial_b \pi(b,\theta) > 0$ (i.e., $b < \hat{p}^m\left(\hat{b}(\theta)\right)$) in these ranges:

$$\begin{aligned} b \leq \hat{b}' &= \frac{281}{420} \simeq 0.67 < p^m\left(\hat{b}\right) = \frac{571}{840} \simeq 0.68, \\ b' \leq \Delta &= \frac{151}{210} \simeq 0.72 \leq p^m\left(\hat{b}'\right) = \frac{701}{840} \simeq 0.83. \end{aligned}$$

C.4.2 Prices

In case of bunching, for $b = b' \in \left[\hat{b}, \hat{b}'\right] \simeq [0.36, 0.67]$, the price is equal to b if the entrant faces no handicap, and it is otherwise equal to:

$$\begin{aligned} p_{\theta'}(b') &= [C - (B - \theta' + b)]_{\theta'=B,\Delta=B+2\varepsilon,C=2(B+\varepsilon)} \\ &= \frac{281}{210} - b. \end{aligned}$$

In case of discrimination, for $b \in \left[\bar{b}, \hat{b}'\right] \simeq [0.55, 0.67]$ and $b' = \beta(b) = \frac{281}{130} - \frac{571}{130}b + \frac{42}{13}b^2$, the price is the same as in the previous scenario (i.e., it is equal to b) if the entrant faces no handicap, and it is otherwise equal to:

$$p_{\theta'}(b') = \beta(b) = \frac{281}{130} - \frac{571}{130}b + \frac{42}{13}b^2.$$

The following figure depicts the price in case of handicap, in the two scenarios: bunching (thin line, for $b \in \left[\hat{b}, \hat{b}'\right] \simeq [0.36, 0.67]$) and discrimination (bold curve, for $b \in \left[\bar{b}, \hat{b}'\right] \simeq$

[0.55, 0.67]):

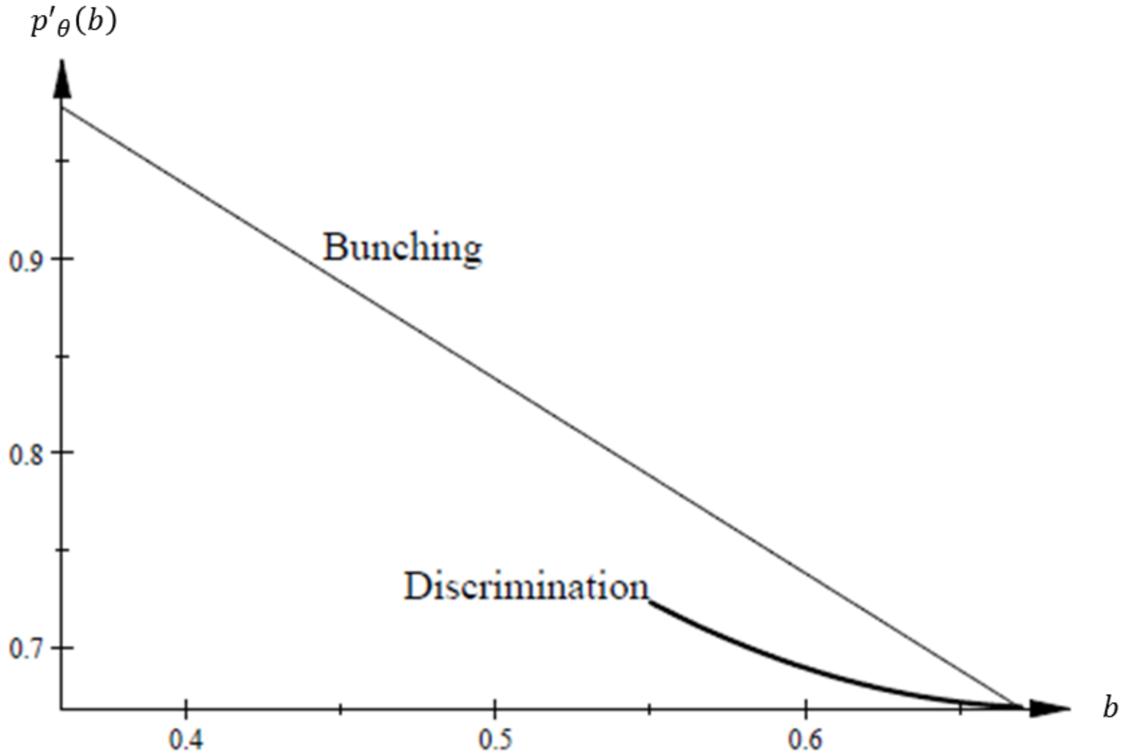

Figure 4: Market price when the entrant is handicapped

The figure confirms that the price $p_{\theta'}(b)$ is lower in the discriminating scenario than in the bunching scenario.

C.4.3 Consumer surplus

$$p_{\theta'}(b') = \begin{cases} \frac{281}{210} - b & \text{if} \quad b' = b, \\ \frac{281}{130} - \frac{571}{130}b + \frac{42}{13}b^2 & \text{if} \quad b' = \beta(b). \end{cases}$$

Building on the above analysis, and using

$$\rho = \frac{37}{139} \simeq 0.27 \text{ and } \rho' = \frac{102}{139} \simeq 0.73,$$

consumers' expected surplus is given by:

- If $b' = b$ ("Bunching"), then for $b = b' \in \left[\hat{b}, \hat{b}'\right] \simeq [0.36, 0.67]$:

$$\begin{aligned} S_B(b) &= \rho \frac{(1-b)^2}{2} + \rho' \frac{(1 - p_{\theta'}(b))^2}{2} \\ &= \frac{37}{139} \frac{(1-b)^2}{2} + \frac{102}{139} \frac{\left(1 - \left(\frac{281}{210} - b\right)\right)^2}{2} \\ &= \frac{1}{2} b^2 - \frac{18}{35} b + \frac{2573}{14\,700}. \end{aligned}$$

From the above analysis, this expected consumer is maximal for $b = \hat{b}$ and $b = \hat{b}'$, where it is equal to:

$$S_B\left(\hat{b}\right) = S_B\left(\hat{b}'\right) = \frac{19321}{352800} \simeq 0.05.$$

- If $b' = \beta(b)$ ("Discriminating"), then for $b \in \left[\bar{b}, \hat{b}'\right] \simeq [0.55, 0.67]$ and $b' \in \left[\hat{b}', \Delta\right] \simeq [0.67, 0.72]$):

$$\begin{aligned} S_D(b) &= \rho \frac{(1-b)^2}{2} + \rho' \frac{(1 - \beta(b))^2}{2} \\ &= \frac{37}{139} \frac{(1-b)^2}{2} + \frac{102}{139} \frac{\left(1 - \left(\frac{281}{130} - \frac{571}{130} b + \frac{42}{13} b^2\right)\right)^2}{2} \\ &= \frac{8996400 b^2 - 6468840 b + 1475501}{2349100} (1-b)^2. \end{aligned}$$

The following figure depicts expected consumer surplus in the bunching scenario (thin curve, for $b \in \left[\hat{b}, \hat{b}'\right] \simeq [0.36, 0.67]$) and the discriminating scenario (bold curve, for $b \in \left[\bar{b}, \hat{b}'\right] \simeq [0.55, 0.67]$); it shows that discriminating is indeed optimal:

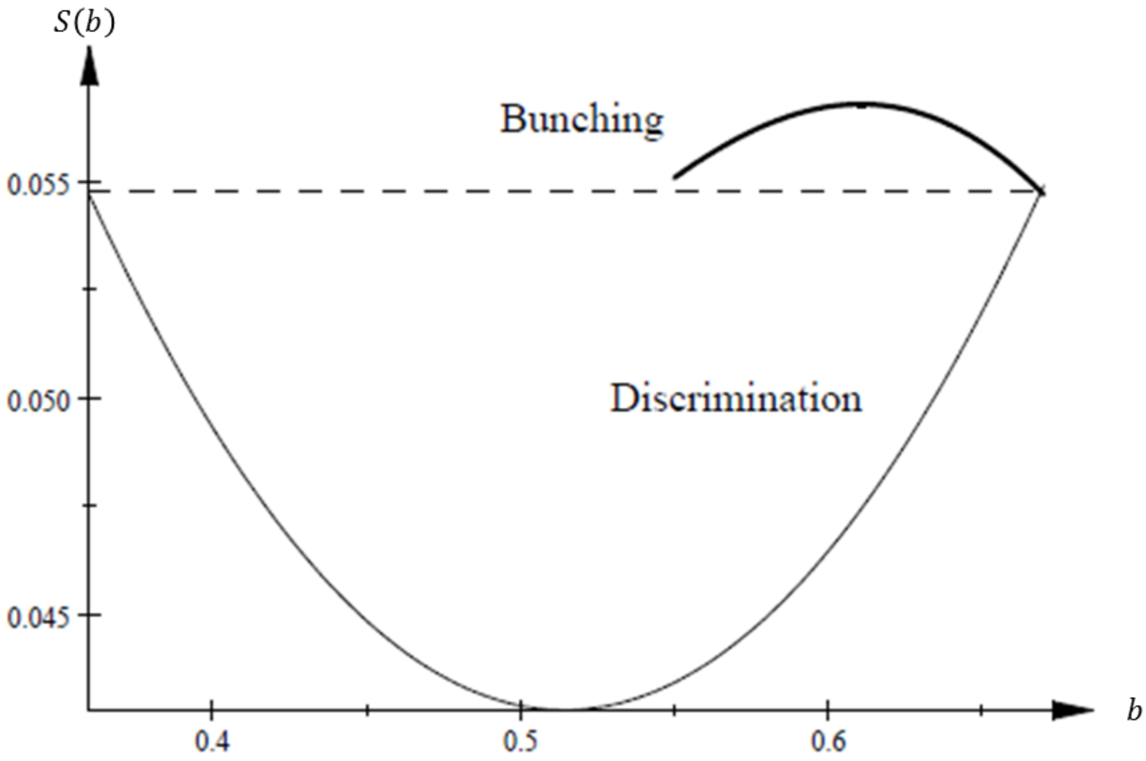

Figure 5: Consumer surplus

To determine the socially optimal mechanism, it suffices to maximize $S_D(b)$, which yields:

$$S'_S(b) = \frac{359856}{23491}b^3 - \frac{3669246}{117455}b^2 + \frac{1800737}{90350}b - \frac{4709921}{1174550} = 0,$$

leading to

$$b^* = \frac{291}{560} + \frac{\sqrt{51}\sqrt{1189\,211}}{85\,680} \simeq 0.610\,54.$$