On Sunspots, Habits and Monetary Facts*

Stéphane Auray
Université de Lille 3
GREMARS (EA 2459) and CIRPÉE
Phone: (+33) 3 20 41 61 82, Fax: (+33) 3 20 41 61 71
email: stephane.auray@univ-lille3.fr

Patrick Fève
University of Toulouse
GREMAQ (UMR CNRS 5604) and IDEI
Banque de France (Research Division)
Phone: (+33) 5 61 12 85 75, Fax: (+33) 5 61 22 55 63
email: patrick.feve@univ-tlse1.fr

Submitted July 2005. Revised July 2006.

*We would like to thank the editor, William Barnett, an associate editor and two referees for insightful comments that lead to a substantial revision of the paper. We would also like to thank Paul Beaudry, Andreas Beyer, Fabrice Collard, Arianna Degan, Julien Matheron and Franck Portier for helpful comments. This work has benefited from discussions during presentations at various conferences. The traditional disclaimer applies. The views expressed herein are those of the authors and not necessarily those of the Banque de France.

Running title:

On Sunspots, Habits and Monetary Facts

Corresponding Author:

Patrick Fève
University of Toulouse
Manufacture des tabacs
Aile Jean-Jacques Laffont, Bft. F
21 allée de Brienne
31000 Toulouse
France

Phone: (+33) 5 61 12 85 75
Fax: (+33) 5 61 22 55 63
email: patrick.feve@univ-tlse1.fr

Abstract

This paper proposes a sunspot-based mechanism that quantitatively accounts for the main monetary facts. In particular, we propose a cash-in-advance-model with habit persistence and local durability in consumption decisions. In this context when habit persistence is strong enough there is real indeterminacy. We show that when sunspots positively correlate with money injections, the model generates a persistent response of inflation, a hump shaped response of output, and the *price puzzle*. We then take the model to the U.S data and we show that it performs well in reproducing the monetary transmission mechanism and the *price puzzle* in the short-run.

Keywords: Habit Persistence, Cash–in–Advance, Real Indeterminacy, Sunspots, *Liquidity Effect*, *Price Puzzle*

JEL Classification: E40, E50

1 Introduction

This paper proposes a sunspot and habits based mechanism that quantitatively accounts for monetary facts. A cash–in–advance model (CIA) with habit persistence and local durability in consumption is considered. In this economy, the equilibrium is indeterminate provided habit formation is large enough. We then study the empirical relevance of the real indeterminacy phenomenon and investigate the quantitative performance of the model in reproducing monetary facts.

The empirical literature that has studied the short–run effects of a monetary shock (in particular the Structural Vector Autoregressions (SVAR) literature) reports the following monetary facts. After an expansionary monetary policy, *(i)* there is a persistent and hump–shaped increase in real GDP, *(ii)* prices decrease in the very short–run but then persistently increase and *(iii)* the nominal interest rate declines in the short–run. Points *(i)* and *(iii)* together define the *liquidity effect*. The short–run response of inflation is described as the *price puzzle* in the literature. Indeed, when prices are almost non responsive in the very short–run, the points *(i)* and *(ii)* together define the *monetary transmission mechanism* whereas the *price puzzle* is defined as a decrease of the price level after an expansionary monetary policy shock. These results seem to be robust against different identification schemes (see e.g. Sims (1992), Leeper, Sims and Zha (1996), Christiano, Eichenbaum and Evans (1999) and (2005)). Consequently, any structural model that could plausibly be used for monetary policy analysis should be able to account for these facts.

A large strand of the literature has developed Dynamic Stochastic General Equilibrium (DSGE) models to account for these facts (see Rotemberg and Woodford (1997), Ireland (2001), Smets and Wouters (2003), Boivin and Giannoni (2005) and Christiano, Eichenbaum and Evans (2005)). These models include real frictions (habit formation, adjustments costs, etc ...) as well as nominal rigidities (price stickiness, wages stickiness, price indexation, etc ...). Due to their empirical success, these models are gaining credibility in policy making institutions. However, it is worth noting that to empirically perform well, these large scale models have to pay a high fee in terms of sophistication and lose a lot in terms of understanding the economic mechanism. It is well known that the intertemporal elasticity of substitution needs to be weakened to match the result of theoretical models with data. For this reason, large scale models consider habit persistence as a key ingredient in intertemporal complementarities in consumption decisions. Thus, it seems natural to investigate deeper the role of this mechanism.

We assume that intertemporal substitution motives are weakened by including habit formation over preferences. We also depart from the monetary literature (see Rotemberg and Woodford (1997), Boivin and Giannoni (2005) and Christiano, Eichenbaum and Evans (2005)) and stick to the financial one in assuming the existence of a local durability effect (see Heaton (1995), Hindy, Huang and Zhu (1997) and Giannikos and Shi (2004)). Durability captures the notion that consumption is substitutable over time whereas habit persistence implies intertemporal complementarities in consumption decisions. We consider a specification of the utility function that implies both long run habit persistence and short run durability effect.

The paper provides conditions on the elasticity of intertemporal substitution in consumption decisions for real indeterminacy to occur when the central bank follows an exogenous money growth rule. Real indeterminacy results from the interplay of habit formation and the cash–in–

advance constraint when the parameter indexing habit persistence exceeds a particular threshold. When the equilibrium is indeterminate, we take into account sunspots. We investigate types of sunspots that are consistent with monetary facts. We follow a large part of the literature on real indeterminacy that introduces a correlation structure between sunspots and fundamentals shocks to replicate observed business cycle facts.[1] We consider a sunspot function correlated to a money injection and we impose restrictions in order to match the above–mentioned monetary facts.

When the equilibrium is indeterminate and sunspots display a positive correlation structure with money injections, the model accounts for the humped–shaped response of output, the *price puzzle* and to a lower extent the *liquidity effect*. The ability to match these facts is not a trivial consequence of the degrees of freedom that are provided by the property of real indeterminacy and the form of the sunspots function. First, the sunspots function is consistent with the rational expectations equilibrium. Second, we restrict attention to a time invariant linear sunspot function. By assuming that the martingale difference sequence on aggregate variables is a linear and stable function of the money injection, the approach is kept parsimonious.

We quantitatively evaluate and test the ability of our model to match the data. The habit formation and monetary sunspot parameters are estimated using the empirical strategy proposed by Rotemberg and Woodford (1997), Boivin and Giannoni (2005) and Christiano, Eichenbaum and Evans (2005). This approach consists in minimizing the distance between the impulse response functions generated by a SVAR model (*i.e.* "the monetary facts") and the ones computed from our monetary model. It is worth noting that the number of moments to be matched

[1] Footnote 1 about here.

greatly exceeds the number of estimated parameters. Consequently, the monetary model under indeterminacy can be tested on the basis of over–identifying restrictions.

We show that our model matches well the persistent and hump–shaped response of output, it is able to reproduce the puzzling behavior of prices as well as the *monetary transmission mechanism*. In our model, only three parameters matter to do the job: the habit persistence parameter must be large enough, a local durability effect must be significant and the monetary sunspot must be positively correlated with money injections.[2] Despite this empirical success, the model has a hard time at quantitatively reproducing the behavior of the short–run nominal interest rate. However, the model is able to reproduce its impact response.

The paper is organized as follows. Section 1 presents the monetary facts. Section 2 presents the model and characterizes its local dynamic properties. More precisely, this section underlies conditions for real indeterminacy and discusses the role of sunspots in generating the monetary facts. Section 3 presents a quantitative evaluation of our economy. A last section concludes.

2 Monetary Facts

This section presents the strategy used to identify the monetary policy shock in a SVAR model, the data and the monetary facts.

[2] Footnote 2 about here.

2.1 The Monetary SVAR

We start our analysis by characterizing the actual economy's response to a monetary policy shock. As is now standard, this is done by estimating a monetary SVAR in line with Christiano, Eichenbaum and Evans (1996) and (1999) so as to identify monetary policy shocks.[3] We first assume that monetary authorities set their instrument, $\hat{\imath}_t$ (here the Federal Funds rate), according to the policy rule

$$\hat{\imath}_t = f\left(\Omega_t\right) + \sigma_i \epsilon_t,$$

where Ω_t is the information set available at the time monetary authorities take their decisions, σ_i is a positive scalar, and ϵ_t is a white noise monetary shock orthogonal to the elements generating Ω_t. Formally, let Z_t denote the data vector of dimension m

$$Z_t = \left(Z'_{1,t}, \hat{\imath}_t, Z'_{2,t}\right)'.$$

The vector $Z_{1,t}$ is a $n_1 \times 1$ vector composed of variables whose current and past realizations are included in Ω_t and that are assumed to be predetermined with respect to ϵ_t. $Z_{2,t}$ is a $n_2 \times 1$ vector containing variables that are allowed to respond contemporaneously to ϵ_t but whose value is unknown to monetary policy authorities at t. Thus only lagged values of $Z_{2,t}$ appear in Ω_t. Accordingly, $m = n_1 + n_2 + 1$.

In order to implement this identification strategy, we first estimate an unconstrained Vector Autoregression (VAR) of the form

$$Z_t = B_1 Z_{t-1} + \cdots + B_\ell Z_{t-\ell} + u_t, \quad \mathrm{E}\{u_t u_t'\} = \Sigma,$$

where ℓ is the maximal lag, which we determine by minimizing the Hannan-Quinn information

[3] Footnote 3 about here.

criterion. In our empirical analysis, we found that $\ell = 4$. Then, to recover the structural shock to monetary policy ϵ_t, we assume that the canonical innovations u_t are linear combinations of the structural shocks η_t, i.e.

$$u_t = S\eta_t,$$

for some non singular matrix S. As usual, we impose an orthogonality assumption on the structural shocks, which combined with a scale normalization implies $\mathrm{E}\{\eta_t\eta_t'\} = I_m$, where I_m is the identity matrix and m is the number of variables in Z_t.

With the above recursiveness assumptions, a monetary policy shock can be recovered as follows. Let S be the Cholesky factor of Σ, so that $SS' = \Sigma$. Then, σ_i is the (n_1+1, n_1+1) element of S, and ϵ_t is the shock appearing in the (n_1+1) equation of the system

$$A_0 Z_t = A_1 Z_{t-1} + \cdots + A_\ell Z_{t-\ell} + \eta_t,$$

where $A_0 = S^{-1}$ and $A_i = S^{-1}B_i$, $i = 1, \ldots, \ell$.

2.2 Data and Findings

We apply the SVAR methodology to US quarterly data[4] (see Figure 5 in the appendix) over the period running from the first quarter of 1960 to the last quarter of 2002. The vector Z_t includes the log of real GDP in deviation from a linear trend, the log of the implicit GDP deflator, the federal funds rate and the log of M1.[5]

As in Christiano, Eichenbaum and Evans (1999), the federal funds rate is taken to be the main instrument of monetary policy. We refer to a monetary shock as a shock on the nominal interest

[4]Footnote 4 about here.
[5]Footnote 5 about here.

rate. Recursiveness assumes, among other things, that while the policymaker observes current production and prices when he sets the federal funds rate, private agents do not observe the current monetary shock. Another implication is that GDP and prices do not react to a monetary policy shock on impact.

— FIGURE 1 ABOUT HERE —

Figure 1 reports the estimated IRF for all the variables in the SVAR model after a contractionary shock to monetary policy — that is a positive shock to the federal fund rate –. The solid line reports the point estimates of the various dynamic response functions. The dashed lines correspond to the 95% confidence interval obtained through Monte-Carlo simulations. The main consequences of a contractionary monetary policy shock are similar to those obtained by previous studies. Following a contractionary shock to monetary policy, figure 1 indicates *(i)* a persistent decline in real GDP. Moreover, outputs exhibits a hump shaped response with a trough effect after about 2.5 years. Furthermore, *(ii)* the aggregate price level exhibits a positive response in the very short–run but then persistently increase. Finally, *(iii)* the federal fund rate rises and the money growth rate decreases. The points *(i)* and *(iii)* together define the so–called *liquidity effect*. The response of inflation is described as the *price puzzle* in the literature (see Sims (1992)). Indeed, when prices are almost non responsive in the very short–run but decrease, the points *(i)* and *(ii)* together define the *monetary transmission mechanism* whereas the *price puzzle* is defined as an increase of the price level after a contractionary monetary policy shock. This is explained by bad times that bring a high price level as money demand falls. In this paper, we focus on the hump–shaped response of output, the prize puzzle and the *liquidity effect*. Our SVAR results confirm a host of previous studies and show that the hump and persistent responses of output, the nominal interest rate, and the delayed and persistent response of inflation are key stylized monetary facts that any monetary DSGE model should be able to reproduce.

We compute the variance decomposition in order to briefly document the contribution of monetary policy shocks to the variability of the different economic aggregates under consideration. Table 1 shows that monetary policy shocks account for a small portion of the variance of output and inflation when the forecast horizons are short, and a substantially bigger portion at longer horizons, comprised between 13% and 41%. However, what turns out to be important when focusing on monetary effects is to obtain precisely estimated responses of aggregate variables, this is the case especially when it comes to the typical hump shape and persistent patterns previously emphasized.

— TABLE 1 ABOUT HERE —

3 The Monetary Model

In this section, we present a flexible price model with a cash–in–advance constraint and habit formation. We also describe the dynamic properties of the economy and discuss its qualitative implications.

3.1 The economy

This section describes a cash–in–advance model with habit persistence and local substitution in consumption. Some assumptions on the functional forms of preferences and technology make it possible to determine analytically the approximate solution of the model.

3.1.1 Households

The setup is standard. The economy is comprised of a unit mass continuum of identical infinitely lived agents. A representative household enters period t with nominal balances M_t brought from the previous period and nominal bonds B_t. The household supplies labor at the real wage W_t/P_t. During the period, the household also receives a lump–sum transfer from the monetary authorities in the form of cash equal to N_t, profit from the firm Π_t and real interest rate payments from bond holdings ($(R_{t-1}-1)B_t/P_t$). These revenues are used to buy a consumption good (C_t), money balances (M_{t+1}) and nominal bonds (B_{t+1}) for the next period. Therefore, the budget constraint takes the form,

$$B_{t+1} + M_{t+1} + P_t C_t = W_t h_t + R_{t-1} B_t + M_t + N_t + \Pi_t. \tag{1}$$

Money is held for transaction motives. The household must carry cash to purchase goods and faces the following cash–in–advance constraint:

$$P_t C_t \leqslant M_t + R_{t-1} B_t - B_{t+1}. \tag{2}$$

We restrict our attention to equilibria with a strictly positive nominal interest rate, so that the cash constraint is binding. Following Abel (1990), Carroll, Overland and Weil (2000) and Fuhrer (2000) consumers' utility at time t is affected by habits expressed as a ratio. Each household has preferences over consumption and leisure represented by the following intertemporal utility function:

$$E_{\Phi_t} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{(1-\eta)} \left(\frac{C_\tau}{Z_\tau^{\varphi}} \right)^{(1-\eta)} - h_\tau, \tag{3}$$

where $\beta \in (0,1)$ is the discount factor and h_t denotes the number of hours supplied by the household. Following Hansen (1985) and Rogerson (1988), we assume that utility is linear in leisure. E_{Φ_t} denotes the expectation operator conditional on the information set Φ_t available in period t. Since, later on, we will seek to compare the model with the monetary SVAR model of section 2.1, it is important to make sure that both models embed the same timing restrictions. To achieve this, we make assumptions about the timing of various decision variables. For instance, consumption is decided prior to observing the monetary shock whereas bond holdings are decided after observing this shock. Z_τ refers to the level of habit and the parameter φ rules the sensitivity of individual consumption to this level of habit. Notice that when $\varphi = 0$, we retrieve the standard model. The stock of habits in the utility function is defined as

$$Z_\tau^{\varphi} = \bar{C}_{\tau-1}\bar{C}_{\tau-2}^{\varsigma}, \tag{4}$$

where $\bar{C}_\tau$ is aggregate consumption in period τ. The second parameter ς represents the potential existence of durability in consumption behavior. Our modeling choice allows us to represent different consumption behaviors in a parsimonious way. Indeed, depending on the values of φ and ς, the model may generate pure habit ($\varphi > 0$ and $\varsigma \geq 0$) or habit persistence with local durability ($\varphi > 0$ and $\varsigma < 0$). As emphasized by Heaton (1993), one problem with the habit persistence specification (as in Constantinides (1990)) is that it ignores a local substitution effect in consumption decisions (see also Hindy and Huang (1992) and Hindy, Huang and Kreps (1993)). Our modeling choice is simpler than the one used by these authors. However, it allows us to account for both the long run persistence of consumption and for the durability effect in the short run.

We consider external habit specified in ratio form. Aggregate consumption $\bar{C}_\tau$ is unaffected by any one agent's decision, exhibiting the "catching up with the Joneses" form of habit formation (see ABEL (1990)).[6] At this stage, no further restriction will be placed on either φ or ς.

The household determines her optimal consumption/saving choice, labor supply and money and bond holdings plans by maximizing utility (3) subject to the budget (1) and cash–in–advance (2) constraints. The timing is of importance in this framework. Households decide on consumption and money holdings before observing the shock, whereas bonds holdings are decided after. Therefore, consumption behavior together with labor supply yields.

$$E_{t-1}\frac{1}{P_t W_t} = \beta E_{t-1}\frac{1}{P_{t+1}}\frac{1}{C_{t+1}^{\eta}} Z_{t+1}^{\varphi(\eta-1)}, \tag{5}$$

whereas nominal return of bond holdings is given by:

$$R_t = \frac{1}{\beta} E_t \left(\frac{C_{t+1}}{C_t}\right)^{\eta} \frac{P_{t+1}}{P_t} \left(\frac{Z_t}{Z_{t+1}}\right)^{\varphi(\eta-1)}. \tag{6}$$

Equations (2) and (6) determine money demand where the real balances are a decreasing function of the nominal interest rate for a given real wage.

3.1.2 Firms

The representative firm produces the homogenous consumption good by means of labor according to the following constant returns–to–scale technology

$$y_t = h_t.$$

Profit maximization implies that, in equilibrium, the real wage will be constant and equal to 1.

[6] Footnote 6 about here.

3.1.3 The Government Budget Constraint and the monetary policy

The government issues nominal bonds B_t to finance open market operations.[7] The government budget constraint is

$$M_{t+1} + B_{t+1} = M_t + N_t + R_{t-1}B_t,$$

with M_0 and B_0 given. As in Christiano, Eichenbaum and Evans (2005), money is exogenously supplied according to the following money growth rule

$$M_{t+1} = \gamma_t M_t,$$

where the gross rate of money growth γ_t follows a stationary stochastic process:

$$\log(\gamma_t) = \rho \log(\gamma_{t-1}) + (1 - \rho) \log(\bar{\gamma}) + \varepsilon_t.$$

ε_t is a white noise with a variance σ^2 and $|\rho| < 1$.

3.1.4 Equilibrium Conditions

An equilibrium is a sequence of prices and allocations, such that given prices, allocations maximize profits (when taking technology into account) and utility (subject to the budget constraint), and all markets clear. In a symmetric equilibrium, all households have the same consumption and $C_t = \bar{C}_t$ every period. Goods market clearing conditions require $C_t = Y_t$ for all t. The equilibrium conditions are approximated by log–linearization about the deterministic steady

[7] Footnote 7 about here.

state:

$$
\begin{aligned}
\widehat{y}_t &= \left(\frac{1}{\varphi} - \eta + 1\right)\left(E_{t-1}\widehat{y}_{t+1} - \varphi\varsigma\widehat{y}_{t-1} + \frac{\rho}{\eta - 1}\widehat{\gamma}_{t-1}\right), && (7)\\
\widehat{\pi}_t &= \widehat{\gamma}_{t-1} + \widehat{y}_{t-1} - \widehat{y}_t, && (8)\\
\widehat{R}_t &= (1-\eta)E_t\widehat{y}_{t+1} + (1-\eta)(1+\varphi)\widehat{y}_t + (1-\eta)(\varphi\varsigma - \varphi)\widehat{y}_{t-1} + (\eta-1)\varphi\varsigma\widehat{y}_{t-2} + \widehat{\gamma}_t, && (9)\\
\widehat{\gamma}_t &= \rho\widehat{\gamma}_{t-1} + \varepsilon_t, && (10)
\end{aligned}
$$

where $\widehat{y}_t$, $\widehat{\pi}_t$, $\widehat{R}_t$ and $\widehat{\gamma}_t$ correspond to the percentage deviation from steady state of output, the inflation rate, the gross nominal interest rate and the money growth rate. Equations (7)–(10) describe the equilibrium conditions of the monetary model with a CIA constraint and habit formation. Note that the dynamics of output is only affected by the exogenous money supply. It can thus be solved independently from inflation and the nominal interest rate. At the same time, the dynamics of inflation and the interest rate depend on those of output.

3.2 Dynamic Properties

This section establishes the dynamic properties of our model economy. We characterize conditions on the level of habits, and discuss the economic mechanisms, that yield real indeterminacy.

3.2.1 Real Indeterminacy

The local dynamic properties of output are strongly related to the perfect foresight version of the model. Importantly, the dynamic properties of the monetary model can be summarized by the behavior of output. Holding the rate of growth of the money supply constant, equation (7) reduces to the following linear second order finite difference equation:

$$\widehat{y}_t = \left(\varphi + \frac{1}{1-\eta}\right)\widehat{y}_{t-1} + \varphi\varsigma\widehat{y}_{t-2}.$$

The model satisfies a saddle path property if and only if one root of the characteristic equation

$$P(\lambda) = \lambda^2 - \left(\varphi + \frac{1}{1-\eta}\right)\lambda - \varphi\varsigma,$$

has modulus greater than one, *i.e.* the number of eigenvalues whose modulus exceeds one must be equal to the number of non–predetermined variables. In the model, the next period consumption levels are free. Conversely, if the modulus of the eigenvalue is less than one, the equilibrium is locally indeterminate, *i.e.* there exists a continuum of equilibrium paths that converge to the steady state. The following Proposition establishes conditions for the existence of real indeterminacy.

Proposition 1 *If φ, ς and η satisfy*

$$\begin{aligned} \varphi(1+\varsigma) &< \frac{\eta}{\eta-1}, \\ \varphi(1-\varsigma) &> \frac{2-\eta}{\eta-1}, \\ |\varphi\varsigma| &< 1, \end{aligned}$$

then the equilibrium is locally indeterminate.

Proof : The roots of the characteristic polynomial have modulus lower than 1 when $P(\lambda)$ satisfies: $P(1) > 0$, $P(-1) > 0$ and $|P(0)| < 1$. The result follows immediately. $\square$

Proposition 1 shows that there exist values of the habit formation parameters (φ, ς) and η that yield real indeterminacy. Notice that the model can exhibit negative or positive real roots as well as complex roots. These conditions show that habit persistence must be strong enough to generate real indeterminacy. They also state that, in order to guarantee the stationarity of the solution, the habit effect can not be too high. Further, one may notice that Proposition 1 nests previous conditions for real indeterminacy in CIA economies. For instance, when φ and ς are equal to 0, then $\eta > 2$ yields indeterminacy (see *e.g.* Woodford (1994), Farmer (1999) or Carlstrom and Fuerst (2003)). In addition, when $\eta = 2$ and $\varsigma = 0$, the conditions can be rewritten as $\varphi \in (0, 2)$ which is very similar to the results of Auray, Collard and Fève (2005).

3.2.2 Discussion and Robustness

This section attempts to shed light on the underlying forces that are at work in generating indeterminacy. Further, it proposes some extensions of the model to show that the indeterminacy results are robust.

First note that whatever happens in this economy, labor demand takes the simple form $W_t/P_t = 1$. Therefore, the only way for an individual to increase her income is to supply more labor. The intuition for real indeterminacy is the following.[8] Let us assume that individuals behavior is characterized by a high intertemporal elasticity of substitution (η, φ small for example) and that they all expect an increase in future inflation. This leads every individual to increase current consumption. However, as intertemporal substitution is high, individual consumption drops in the next period. Since all individuals are identical and have the same expectations, aggregate consumption drops as well in the next period. Therefore, the inflation tax will decrease, which cannot support the original inflation expectations. Any change in expectations can only be due

[8] Footnote 8 about here.

to monetary policy, and is therefore related to fundamental shocks.

Let us now consider the case where intertemporal substitution is low ($\eta > 1$ and $\varphi \gg 0$ for example) and all individuals again have the same expectations on future inflation. As in the previous case, individuals consume more today. But, contrary to the preceding case, the irreversibility in consumption decisions associated with habit persistence leads the agents to increase their future individual consumption too. Since, they are all identical and have the same expectations, aggregate future consumption eventually increases. It follows that the aggregate inflation tax increases, therefore supporting the initial individual expectations. These expectations can now depart from fundamentals — even though they may be arbitrarily correlated with fundamentals.

The above discussion shows how the interplay between habit persistence and cash–in–advance, given a specific environment on the labor and asset markets, can give rise to real indeterminacy and persistence. One may then question the robustness of our results to modifications in the labor and asset markets arrangements.

First, rather than using a linear utility in leisure, we assume that preferences are represented by the following expression:

$$U(C_t, h_t) = \left(\frac{C_t}{Z_t^{\varphi}}\right)^{(1-\eta)} - \frac{h_t^{1+\chi}}{1+\chi}.$$

With this specification of the utility function, the conditions of Proposition 1 can be rewritten as:

$$\begin{aligned}\varphi(1+\varsigma) &< \frac{\eta+\chi}{\eta-1}, \\ \varphi(1-\varsigma) &> \frac{2+\chi-\eta}{\eta-1}, \\ |\varphi\varsigma| &< 1.\end{aligned}$$

These conditions show that the results on real indeterminacy are maintained when the elasticity of labor supply is finite. The main difference is that the conditions on the parameters φ and ς are more stringent. To see this, consider the case where $\varsigma = 0$ and $\eta = 2$. The condition for indeterminacy reduces to $\chi < \varphi < 2 + \chi$. When the elasticity of labor supply decreases – *i.e.* χ increases –, the threshold value for φ that yields indeterminacy increases. In other words, the intertemporal complementarities in consumption decisions must be higher in order to obtain indeterminacy, since labor supply is less responsive. Indeed, if the labor supply does not respond sufficiently, the expectations based willingness to consume more cannot be supported and this may weaken the mechanism that creates indeterminacy.

We next consider a second departure from the original model for which the production function displays decreasing returns to scale to labor. As aforementioned, the response of labor income is crucial in generating real indeterminacy. One of the implications of our previous technology is that the real wage is constant in equilibrium. Therefore, labor income can increase following an increase in the labor supply. One may question the robustness of our previous results to a non–constant endogenous real wage. To address this issue, we investigate the case of a more general production function given by

$$y_t = h_t^{\alpha},$$

where $\alpha \in (0, 1]$. The conditions for indeterminacy become

$$\begin{aligned} \varphi(1+\varsigma) &< \frac{\alpha(\eta-1)+1}{\alpha(\eta-1)}, \\ \varphi(1-\varsigma) &> \frac{1+\alpha-\eta}{\alpha(\eta-1)}, \\ |\varphi\varsigma| &< 1. \end{aligned}$$

Results on real indeterminacy are left qualitatively unaffected by the non–constancy of the endogenous real wage, unless α is equal to zero. When α is close to 1, the real wage is not very responsive to increases in hours worked, and we retrieve the results of Proposition 1. Consider now a rather extreme experiment, where α is set close to 0. In this case, the real wage drops by a huge amount after an increase in hours worked, such that the labor income does not respond. This implies that φ has to be large to support inflation expectations. But real indeterminacy continues to occur.

This discussion illustrates the robustness of our results to relaxing the assumption of a constant real wage. We next check the robustness of our results to the introduction of a new good (an asset), as a mean to escape the inflation tax. In our simple framework, the household can only use leisure to avoid paying the tax. We consider, instead, an economy where the household can use physical capital to avoid it. We use a monetary optimal growth model *à la* Cooley and Hansen (1989) augmented with habit formation. Each household has preferences over consumption and leisure represented by the intertemporal utility function (3). We allow for capital accumulation and assume a constant depreciation rate ($\delta \in (0, 1)$), so that the intertemporal budget constraint of the household can be rewritten as

$$\frac{m_{t+1}}{P_t} + c_t + k_{t+1} \leq (Q_t + 1 - \delta)k_t + W_t h_t + \frac{m_t + N_t}{P_t}, \tag{11}$$

where Q_t is the real rental rate of capital. As in the previous model, money is held because the household faces a cash–in–advance constraint (2). The problem of the representative household is to choose her consumption–savings, labor and real balances plans to maximize (3) subject to (2) and (11). Monetary arrangements are assumed to be the same as in our benchmark framework. The representative firm produces an homogeneous good that can be either invested or consumed using the constant returns to scale technology, represented by the Cobb–Douglas production function:

$$y_t = Ak_t^{1-\alpha}h_t^{\alpha},$$

where $A > 0$ is a scale parameter. The firm determines its production plans maximizing its profit. We keep the preceding assumption concerning the determination of the labor demand, implying that the real wage does not remain constant when hours vary. Finally market clearing imposes $y_t = c_t + i_t$. The labor supply takes the form,

$$h_t = \alpha\lambda_t y_t,$$

where λ_t is the lagrange multiplier associated with the budget constraint (11). Together with the production function, it implies that the output/capital ratio is a function of λ_t only:

$$\frac{y_t}{k_t} = A^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}\lambda_t^{\alpha/(1-\alpha)}.$$

The Euler equation associated with capital decisions ($Q_t = (1-\alpha)y_t/k_t$) can be written as

$$\lambda_t = \beta E_t\lambda_{t+1}\left((1-\alpha)\frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right).$$

Plugging the labor market clearing condition into the Euler equation, we obtain

$$\lambda_t = \beta E_t\lambda_{t+1}\left(1 - \delta + (1-\alpha)A^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}\lambda_{t+1}^{\alpha/(1-\alpha)}\right),$$

which can be solved for λ_t independently from the rest of the dynamic system. It follows that the model with capital accumulation generates the same conditions for real indeterminacy as the simple model. Hence all our previous results still apply. In other words, letting the agent escape the inflation tax using another asset does not eliminate the possibility of real indeterminacy of the equilibrium.

3.2.3 Qualitative results

When the equilibrium is indeterminate, the dynamics of the economy is described by the following set of equations:

$$\begin{aligned}
\widehat{y}_t &= \frac{\rho}{1-\eta}\widehat{\gamma}_{t-2} + \left(\frac{1}{1-\eta} + \varphi\right)\widehat{y}_{t-1} + \varphi\varsigma\widehat{y}_{t-2} + \varepsilon_t^y, \\
\widehat{\pi}_t &= \widehat{\gamma}_{t-1} + \widehat{y}_{t-1} - \widehat{y}_t, \\
\widehat{R}_t &= \left[(1-\eta)(1+2\varphi)+1\right]\widehat{y}_t + \left[2(1-\eta)\varphi\varsigma - \varphi(1-\eta)\right]\widehat{y}_{t-1} \\
&\quad -(1-\eta)\varphi\varsigma\widehat{y}_{t-2} + b_2(1-\eta)\varepsilon_t + \widehat{\gamma}_t + \rho\widehat{\gamma}_{t-1}, \\
\widehat{\gamma}_t &= \rho\widehat{\gamma}_{t-1} + \varepsilon_t,
\end{aligned}$$

where ε_t^y is a martingale difference sequence that satisfies $E_{t-2}\varepsilon_t^y = 0$. Let us consider the following sunspot function:

$$\varepsilon_t^y = b_1\varepsilon_t + b_2\varepsilon_{t-1}. \tag{12}$$

This linear time invariant function is consistent with rational expectations equilibrium since $E_{t-2}(b_1\varepsilon_t + b_2\varepsilon_{t-1}) = 0$.[9]

Due to our timing restrictions (*i.e.* consumption must be decided before the observation of the

[9]Footnote 9 about here.

money shock) the output equation displays two types of sunspots ($b_1\varepsilon_t$ and $b_2\varepsilon_{t-1}$). For compatibility purposes with the timing of the SVAR model, we set $b_1 = 0$. In this case, output is allowed to react to the monetary shock only one period later. In addition, the interplay of this timing restriction with the CIA constraint (2) leads inflation to respond with one lag. Conversely, the nominal interest rate (bond holdings are decided after observing the shock) and the money growth are free to instantaneously react to the shock.

We now investigate the ability of this model to match the stylized facts that emerged from the SVAR. For simplicity of exposition, we consider an *i.i.d* process for the money growth rate. This leads to the following proposition:

Proposition 2 *When money growth is i.i.d., the model matches monetary facts through sunspots if*

(i) φ, ς *and* η *satisfy Proposition 1,*

(ii) $\varphi > \frac{1}{\eta-1}$,

(iii) $b_2 > 1$,

(iv) $\eta > \frac{1+b_2}{b_2}$.

The proof is straightforward and is implicitly given in the following discussion. The first condition (*i*) is related to real indeterminacy. In this case, the model may generate some persistence adjustment paths for output. We retrieve these persistent effects in the dynamics of inflation and the nominal interest rate given the assumption of the CIA constraint. The second requirement (*ii*) implies that the habit persistence parameter must be large enough to generate a hump pattern in output. It is important to notice that condition (*ii*) is consistent with Proposition 1. Consequently, getting persistent and hump shaped responses is more than empirically plausible in our indeterminate economy. Condition (*iii*) is necessary to get the *monetary transmission mechanism* and the *price puzzle*. These three conditions highlight that this model is able to reproduce the persistent and hump–shaped responses of the variables that characterized the monetary SVAR.[10] These results come in part from the monetary sunspot that creates a large supply side effect. This positive supply effect implies a decrease of prices in equilibrium. Indeed, following a monetary injection, demand shifts upward, which if supply were non responsive, would solely trigger an increase in prices. This corresponds to the situation when money is neutral. If sunspots are positively and sufficiently correlated with the money injection, labor supply shifts upward to sustain the increase in consumption. This corresponds to a positive supply shock that shifts supply, which offsets the upward pressure on price. The last condition of Proposition 2 shows that the model can qualitatively reproduce the *liquidity effect* provided η is large enough or b_2 is not too large. The intuition for this latter restriction is as follows. When b_2 is too large, the increase in labor supply is so important that it allows households to sustain an increase both in bonds and consumption. Thus, the nominal interest rate rises.

Propositions 1 and 2 show that a high value for the habit persistence parameter φ is needed to match the monetary facts. Consequently, the marginal utility of consumption is very responsive to an unexpected shock leading to a too high reactivity of the nominal interest rate. We end up this section by arguing again that these results are not trivial. As previously explained, the sunspots function is fully consistent with the rational expectations equilibrium. Furthermore,

[10] Footnote 10 about here.

we restrict attention to a time invariant linear sunspot function. Therefore, our approach is kept parsimonious. More importantly, the model is formally taken to the data. Notice that, at this stage, there is no guarantee that our economy performs well quantitatively. This point is examined in the next section.

4 Quantitative analysis

In this section, we present our empirical strategy, the estimation results and then discuss the empirical performance of the model.

4.1 Econometric methodology

As in most of the literature that follows the original work by Rotemberg and Woodford (1997), we estimate the model parameters ψ by minimizing a measure of the distance between the empirical responses of key aggregate variables obtained from the monetary SVAR (see section 2) and their model counterparts.[11]

More precisely, we focus our attention on the responses of the vector of actual variables Z_t. We let θ_k be the vector of responses to a monetary shock at horizon $k \geq 0$, as implied by the above SVAR estimated on actual data, *i.e.*

$$\theta_j = \frac{\partial Z_{t+j}}{\partial \epsilon_t}, \;\; j \geq 0,$$

where ϵ_t is the monetary policy shock previously identified.

Given a selected horizon k, we seek to match $\theta = \mathrm{vec}([\theta_0, \theta_1, \ldots, \theta_k])'$ where we exclude from

[11] Footnote 11 about here.

θ_0 the responses corresponding to the elements in Z_t that belong to Ω_t. As previously mentioned the monetary DSGE model embeds the same exclusion restrictions as the SVAR model. Then let $h(\cdot)$ denote the mapping from the structural parameters $\psi = (\eta, \varphi, \varsigma, b_2, \rho, \sigma)'$ to the monetary model counterpart of θ. Our estimate of ψ is solution to the following problem

$$\hat{\psi}_T = \arg\min_{\psi\in\Psi}(h(\psi_2) - \hat{\theta}_T)V_T(h(\psi_2) - \hat{\theta}_T)',$$

where $\hat{\theta}_T$ is an estimate of θ, T is the sample size, Ψ is the set of admissible values of ψ, and V_T is a weighting matrix which we assume to be the inverse of the diagonal matrix containing the variances of each element of θ. These variances are obtained from the SVAR parameters.

For further references, let us define the objective function at convergence

$$\mathcal{J} = (h(\widehat{\psi}_T) - \hat{\theta}_T)V_T(h(\widehat{\psi}_T) - \hat{\theta}_T)'.$$

Under the null hypothesis, as shown in Hansen (1982), $\mathcal{J} \sim \chi^2(\dim(\theta) - \dim(\psi))$. Given our choice of weighting matrix, we can further decompose $\mathcal{J}$ into components pertaining to each element of Z_t, according to

$$\mathcal{J} = \sum_{i=1}^{\dim(Z)} \mathcal{J}_i.$$

The latter decomposition provides a simple diagnostic tool allowing us to locate those dimensions on which the model succeeds or fails to replicate the impulse response functions implied by the SVAR.

4.2 Results

The model parameters are partitioned into two subsets. A first subset contains the parameters that are calibrated prior to estimation, η and σ. A second subset contains the parameters that are estimated: the habit formation parameters, φ and ς; the sunspot parameter, b_2, and the money growth parameter that defines the persistency of the process, ρ.

Due to numerical failures incurred when all parameters are jointly estimated, we set the value for the parameters η and σ.[12] The standard–error of the shock to monetary policy is fixed to 0.0007, such that the money growth process is consistent to what is observed in the data. For the parameter η, we resort on a fine grid-search and select the value, $\eta = 3$ which provides the smallest loss function.[13] We, therefore use this value in the estimation. The loss function at convergence is reported in Figure 2. Notice that whatever the selected value of η, the model is not rejected by the data. In addition, the flatness of the objective function when η is greater than 3 indicates that some identification problem may occur when we freely estimate η.

— FIGURE 2 ABOUT HERE —

The estimation of $\{\varphi, \varsigma, b_2, \rho\}$ is performed for different choices of Z_t: *(i)* output and the money growth rate ($Z_t = \{y_t, \gamma_t\}$), *(ii)* inflation and the money growth rate ($Z_t = \{\pi_t, \gamma_t\}$) and *(iii)* output, inflation and the money growth rate ($Z_t = \{y_t, \pi_t, \gamma_t\}$). The first choice allows us to evaluate the ability of the model to mimic the persistency of the real effect of monetary policy shocks on output. The second choice aims at examining whether the model is able to replicate the *price puzzle* as well as the persistent and delayed response of the inflation rate. Finally, the last choice allows us to study the dynamics of output and inflation following a monetary policy shock.

As previously mentioned, the model has a hard time in reproducing the dynamics of the nominal

[12] Footnote 12 about here.
[13] Footnote 13 about here.

interest rate. Indeed, the excessive response of the nominal interest rate leads to the rejection of the model. Furthermore, in this case the estimates of $\{\varphi, \varsigma, b_2, \rho\}$ take implausibly high values with very large standard errors. For these reasons, we discarded the nominal interest rate from our estimation. We will discuss this empirical issue later.

We present the estimation results in Table 2 for different Z_t and different restrictions on habit parameters. In each case, we set the impulse response functions horizons k to 41.[14] The first three columns report the results when the vector $\{\varphi, \varsigma, b_2, \rho\}$ is freely estimated. The last three columns provide the outcome of the estimation when ς is set to zero. In addition, this table includes the value of the objective function $\mathcal{J}$ at convergence and its associated decomposition. It also reports the eigenvalues of the polynomial that describes the dynamics properties of the model.

Let us first concentrate on the unconstrained estimation of $\{\varphi, \varsigma, b_2, \rho\}$. The model is in no case rejected by the data. Indeed, the P value of the $\mathcal{J}$ statistics is very large whatever the choice of Z_t. Consequently, the model does a very good job reproducing both the *monetary transmission mechanism* and the *price puzzle*. When looking at the decomposition of the $\mathcal{J}$ statistics, it appears that the model performs well in terms of the persistent and hump–shaped responses of output and inflation after a monetary policy shock. In contrast, though not rejected by the data, the model matches poorly the dynamics of money growth. This is a direct consequence of our simple money growth rule.

The estimates of the habit persistence parameter φ lies between 2.39 and 2.42 and are always significant in the different cases. As already mentioned, this parameter captures the sensitivity

[14] Footnote 14 about here.

of individual consumption to the stock of habits. The higher is φ the more the agent will take into account habits in his consumption decisions over time. We previously showed that a large value of this parameter is necessary to match the monetary stylized facts. This high value clearly helps us to obtain a quantitatively persistent response to the monetary shock.

— TABLE 2 ABOUT HERE —

The estimate of the parameter ς lies between -0.37 and -0.39 and is also significant. The negative estimated value of this parameter suggests a local durability effect in consumption behavior. It expresses a substitution (or saturation) effect that is associated with local substitution of consumption over time. The value obtained is similar to the ones obtained in many different empirical studies focusing on the moments of US assets returns (see Heaton (1993) and (1995), Hindy, Huang and Zhu (1997), Allais (2004) and Giannikos and Shi (2004)). For instance, Heaton (1993) and (1995) finds that both durability and habits help to improve the explanatory power of his model. The values we obtain express both the existence of a strong habit persistence effect and of a significant durability effect. High complementarity and substitutability of consumption coupled with a high sensitivity of consumption to habits allow the model to reproduce persistent and hump shaped responses to the monetary shock.

The estimate of the sunspot parameter b_2 exceeds 2 and it is always significant. Note that the estimates of b_2 lies in a very tiny range whatever impulse response functions are used to estimate the parameters. As shown in Proposition 2, when b_2 is greater than zero, the model replicates the *monetary transmission mechanism* and when b_2 is greater than one, the model reproduces the *price puzzle*. Our results suggest that, as long as people believe that sunspots may exist, sunspots may affect the economy. Although our estimates are not easily comparable, our findings are in line with previous studies that point out the quantitative relevance of the correlation structure between sunspots and fundamentals shocks to replicate observed business cycle facts (see Benhabib and Farmer (1996) and (2000), Farmer and Guo (1995), Perli (1998), and Schmitt-Grohe (2000)).

Our estimate of the money growth parameter defining the persistence of the process is between 0.70 and 0.74. This value is high but not so far from previous estimates (see for instance Christiano, Eichenbaum and Evans (1997) and (2005)). Notice also that imposing lower values for this parameter in the estimation procedure does not affect our results.

Table 2 reports the modulus of eigenvalues associated to the characteristic polynomial $P(\lambda)$ summarizing the dynamics of the economy. These eigenvalues are complex conjugates but the complex part remains very small compared to the real one. The large values obtained show that the persistency generated in this model may be, in accordance with the data, very high. Notice also that the ability of the model to reproduce the hump–shaped response is strongly related to these high eigenvalues.

For illustrative purposes, we now present the impulse response functions of output, inflation and money growth rate from our monetary model (solid line with bullet) and report the impulse response functions of the SVAR model (solid line) in figure 3. The figure also includes a 95% confidence interval. The persistent and hump shaped response of output is particularly well reproduced. Furthermore, as discussed at length in Woodford (2003), the delayed response of inflation is a key stylized fact that any monetary model should accurately mimic. The figure shows that on this dimension our model does a very good job, as it precisely accounts for the delayed and persistent response of inflation and for the *price puzzle* in the short–run.

— FIGURE 3 ABOUT HERE —

Let us now consider the constrained estimation that is $\varsigma = 0$. The results are reported in the last three columns of Table 2. The $\mathcal{J}$ statistics as well as the $\mathcal{J}$ statistics associated with output, inflation and the money growth rate lead unambiguously to a rejection of the model. This experiment is important since it shows that both complementarity and substitutability of consumption over time are necessary to match US actual data. The estimates of φ are still significant, meaning that the consumption is sensitive to the habit persistence effect. However, the model will never be able to perform well in terms of persistence. Furthermore, the estimates of the money growth parameter indicate that this parameter may be negative. We therefore join Heaton (1995) (though focusing on different data), in arguing that habit persistence and durability help improve the ability of the model to match US data. The relevance of this assumption is also highlighted by the plot of output, inflation, the nominal interest rate and the money growth rate after a monetary policy shock when $\varsigma = 0$ (see figure 6 in the appendix). In this case, the model dramatically fails to reproduce the hump shaped and persistent responses of output and the inflation rate.

— FIGURE 4 ABOUT HERE —

We next address the issue of how well our model can explain the *liquidity effect*. As discussed above, our model is able to qualitatively match the dynamics of the nominal interest rate but is rejected by the data when this variable is taken into account in the estimation. The failure occurs because its response is too strong compared to the data. This is a direct consequence of the high stock of habits that implies a high volatility of the marginal utility of consumption and thus of the nominal interest rate. In order to better compare the two responses, we report in figure 4, the nominal interest rate behavior extracted from the model on the left vertical axis and the Federal Funds Rate dynamics extracted from the SVAR on the right vertical axis. The responses from the model are obtained using the parameters estimates of Table 2 and $Z_t = \{y_t, \pi_t, \gamma_t\}$. Such comparison indicates that, although the model fails quantitatively on this dimension, it provides a similar shape to the data. Furthermore, it is of importance to notice that the model reproduces well the response of the nominal interest rate at the impact of the shock. The impact response implied by the model is 0.24 whereas the one implied by the SVAR is 0.19. In addition, the value of the nominal interest rate at the impact of the shock in the model is within the confidence interval of the Federal Funds Rate response provided by the SVAR.

5 Concluding remarks

This paper considered a cash–in–advance economy with long run complementary and short run substitutability in consumption decisions and aimed to match the monetary facts that emerge from a SVAR model. The hump shaped and persistent responses of output and inflation are considered as key empirical features that the theoretical model should be able to reproduce. In addition, we focused on the puzzling behavior of the inflation rate after a monetary shock. We first studied the dynamic properties of our economy and determined under which conditions on habits, real indeterminacy may occur. We did not try to avoid indeterminacy but instead we took advantage of it to investigate whether our monetary economy with sunspot fluctuations can account for the monetary stylized facts. Using a minimum distance estimation method, we compared the model to the data. Our findings suggest that the model replicates the *monetary transmission mechanism* and the *price puzzle* identified in the data. However, the model overestimates the response of the nominal interest rate.

Footnotes

[1] For example, Benhabib and Farmer (1996), Perli (1998) and Schmitt-Grohe (2000) calibrate the correlation between the sunspots and the technology shock to reproduce the co–movements of aggregate US data. See Farmer and Guo (1995) for an applied econometric study.

[2] Notice that with only three key parameters, the dynamics of the economy is rich enough for the model to be considered quantitatively relevant.

[3] See also Christiano, Eichenbaum and Evans (1997) and (2005), and Rotemberg and Woodford (1997) and (1999) for other examples of this identifying strategy.

[4] The data are extracted from the Bureau of Labor Statistics website, except for the Fed Funds rate and M1 which are obtained from the FREDII database.

[5] We also experimented with quadratically detrended or first-differenced output, without quantitatively altering our findings.

[6] External habit has been preferred to internal habit only for tractability purposes. However, the mechanism at work plays in the same direction for these two specifications of habit. Indeed, whatever the form of habit is, the model generates real indeterminacy because of the interplay between the CIA constraint and the habit formation assumption. In addition, the dynamic properties of the model are very similar.

[7] These nominal bonds could be used to finance government consumption. Nevertheless, this issue is beyond the scope of the paper.

[8] The mechanisms at the core of the indeterminacy phenomenon are similar to those presented in Auray, Collard and Fève (2005) in a cash–in–advance economy with pure internal habit persistence.

[9] This function may introduce an additional variable (consistent with rational expectations equilibrium) that accounts for pure extrinsic sunspots that are unrelated to fundamentals but this variable is meaningless in our quantitative analysis.

[10] One may remark that money is neutral when $b_2 = 0$.

[11] See also Altig, Christiano, Eichenbaum and Linde (2005), Boivin and Giannoni (2005), Christiano, Eichenbaum and Evans (2005) and Giannoni and Woodford (2004).

[12] Most likely, those parameters are not identified.

[13] For each value of η, the remaining model parameters are estimated in order to minimize the loss function.

[14] Notice that the results are left qualitatively unaffected by a modification of the horizon, provided that the impulse response functions contain the hump pattern of output and inflation.

References

Abel, Andrew B. (1990) Asset Prices under Habit Formation and Catching up with the Joneses. *American Economic Review, Papers and Proceedings*, 80, 38-42.

Allais, Olivier (2004) Local Substitution and Habit Persistence: matching the Moments of the Equity Premium and the Risk-free Rate. *Review of Economic Dynamics*, 7, 265-296.

Altig, David, Lawrence Christiano, Martin Eichenbaum, and Jesper Linde (2005) *Firms-Specific Capital, Nominal Rigidities and the Business Cycle*, mimeo.

Auray, Stéphane, Fabrice Collard, and Patrick Fève (2005) Habit Persistence, Money Growth Rule and Real Indeterminacy. *Review of Economic Dynamics*, 8, 48-67.

Benhabib, Jess and Robert E. A. Farmer (1996) Indeterminacy and Sector Specific Externalities. *Journal of Monetary Economics*, 37, 397-419.

Benhabib, Jess and Robert E. A. Farmer (2000) The Monetary Transmission Mechanism. *Review of Economic Dynamics*, 3, 523-550.

Boivin, Jean and Marc Giannoni (in press) Has Monetary Policy Become More Effective?. *Review of Economics and Statistics*.

Carlstrom, Charles T. and Timothy S. Fuerst (2003) Money Growth Rules and Price Level Determinacy. *Review of Economic Dynamics*, 6, 263-275.

Carroll, Christopher D. and Jody Overland and David N. Weil (2000) Saving and Growth with Habit Formation. *American Economic Review*, 90, 341-355.

Christiano, Lawrence J., Martin Eichenbaum and Charles Evans (1996) The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds. *The Review of Economics and Statistics*, 78, 16-34.

Christiano, Lawrence J., Martin Eichenbaum and Charles Evans (1997) Sticky Price and Limited Participation Models of Money: A Comparison. *European Economic Review*, 41, 1201-1249.

Christiano, Lawrence J., Martin Eichenbaum and Charles Evans (1999) Monetary Policy Shocks: What Have we Learned and to What End?. In Michael Woodford and John Taylor (eds.), *Handbook of Macroeconomics*, pp. 65-148. Amsterdam: North-Holland.

Christiano, Lawrence J., Martin Eichenbaum and Charles Evans (2005) Nominal Rigidities and the Dynamics Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113, 1-45.

Constantinides, Georges M. (1990) Habit Formation: A Resolution of the Equity Premium Puzzle. *Journal of Political Economy*, 98, 519-543.

Cooley, Thomas F. and Gary D. Hansen (1989) The Inflation Tax in a Real Business Cycle Model. *American Economic Review*, 79, 733-48.

Farmer, Roger E. A. (1999) *The Macroeconomics of Self-Fulfilling Prophecies*. Cambridge, (MA): MIT Press.

Farmer, Roger E. A. and Jang-Ting Guo (1995) The Econometrics of Indeterminacy: an Applied Study. *Carnegie-Rochester Conference Series on Public Policy*, 43, 225-271.

Fuhrer, Jeffrey C. (2000) Habit Formation in Consumption and Its Implications for Monetary-

Policy Models. *American Economic Review*, 90, 367-390.

Giannikos, Christos and Zhihong Shi (2004) *Does Durability Help Asset Pricing with Habit Formation to conform with U.S. Data?*, mimeo.

Giannoni, Marc and Michael Woodford (2004) Optimal inflation targeting rules. In Ben Bernanke and Michael Woodford (eds.), *The Inflation Targeting Debate*, pp. 93-162. Chicago: University of Chicago Press.

Hansen, Lars Peter (1982) Large Sample Properties of Generalized Method of Moments Estimators. *Econometrica*, 50, 1029-54.

Hansen, Gary D. (1985) Indivisible Labor and the Business Cycles. *Journal of Monetary Economics*, 16, 309-327.

Heaton, John C. (1993) The Interaction between Time–nonseparable Preferences and Time Aggregation. *Econometrica*, 61, 353-385.

Heaton, John C. (1995) An Empirical Investigation of Asset Pricing with Temporally Dependent Preferences Specification. *Econometrica*, 63, 681-717.

Hindy, Ayman and Chi-fu Huang (1992) Intertemporal Preferences fo Uncertain Consumption: A Continuous Time Approach. *Econometrica*, 60, 781-801.

Hindy, Ayman, Chi-fu Huang and David Kreps (1993) On Intertemporal Preferences with a Continuous Time Dimension I: The Case of Certainty. *Journal of Mathematical Economics*, 21, 401-440.

Hindy, Ayman, Chi-fu Huang and Steven H. Zhu (1997) Optimal Consumption and Portfolio Rules with Durability and Habit Formation. *Journal of Economic Dynamics and Control*, 21, 525-550.

Ireland, Peter N. (2001) Sticky-price models of the business cycle: Specification and stability. *Journal of Monetary Economics*, 47, 3-18.

Leeper, Eric M., Christopher A. Sims and Tao Zha (1996) What Does Monetary Policy Do?. *Brookings Papers on Economic Activity*, 2, 1-63.

Perli, Roberto (1998) Indeterminacy, Home Production, and the Business Cycle: A Calibrated Analysis. *Journal of Monetary Economics*, 41, 105-125.

Rogerson, Richard (1988) Indivisible Labor, Lotteries and Equilibrium. *Journal of Monetary Economics*, 21, 3-16.

Rotemberg, Julio and Michael Woodford (1997) An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy. In NBER Macroeconomics Annual, 297-346.

Rotemberg, Julio and Michael Woodford (1999) The Cyclical Behavior of Prices and Costs. In Michael Woodford and John Taylor (eds.), *Handbook of Macroeconomics*, pp. 1051-1135. Amsterdam: North-Holland.

Schmitt-Grohe, Stephanie (2000) Endogenous Business Cycles and the Dynamics of Output, Hours, and Consumption. *American Economic Review*, 90, 1136-1159.

Sims, Christopher A. (1992) Interpreting the Macroeconomic Time Series Facts : The Effects of Monetary Policy. *European Economic Review*, 36, 975-1000.

Smets, Frank and Rafael Wouters (2003) An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1, 1123-1175.

Woodford, Michael (1994) Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy. *Economic Theory*, 4, 345-80.

Woodford, Michael (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.

Appendix

— FIGURE 5 ABOUT HERE —

— FIGURE 6 ABOUT HERE —

Figure 1: impulse response functions from the monetary SVAR

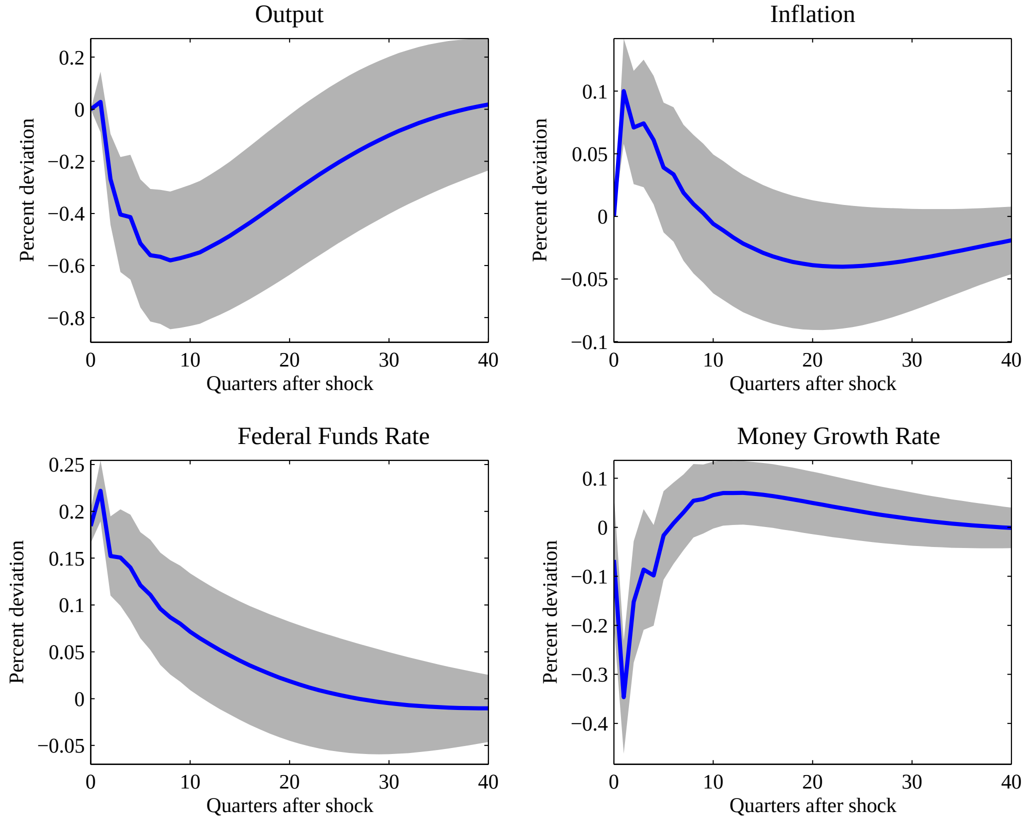


Table 1: variance decomposition

$Quarters$	Output	Inflation	Federal funds rate	Money growth rate
0	0.00	0.00	0.91	0.01
			[0.79;0.98]	[0.00;0.05]
4	0.08	0.10	0.58	0.16
	[0.02;0.21]	[0.03;0.19]	[0.39;0.70]	[0.07;0.28]
8	0.22	0.08	0.49	0.16
	[0.07;0.42]	[0.03;0.19]	[0.28;0.63]	[0.08;0.29]
20	0.33	0.08	0.43	0.18
	[0.11;0.50]	[0.04;0.24]	[0.21;0.59]	[0.10;0.30]
40	0.30	0.13	0.41	0.18
	[0.11;0.48]	[0.05;0.28]	[0.21;0.59]	[0.10;0.30]

Note: Confidence intervals in brackets. These confidence intervals are obtained by simulation.

Figure 2: $\mathcal{J}$ Statistics for different values of η and with $Z_t = \{y_t, \pi_t, \gamma_t\}$

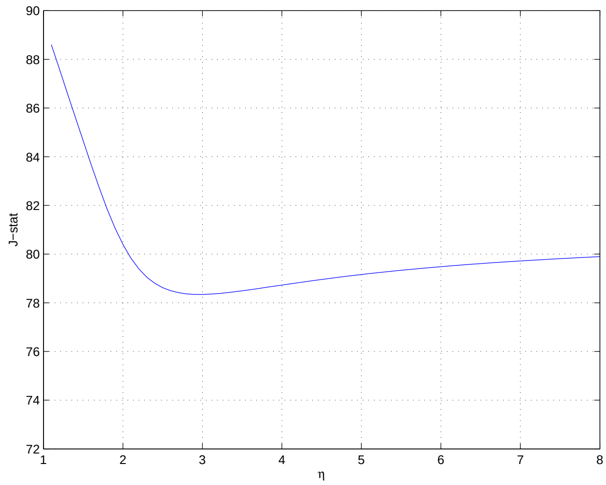
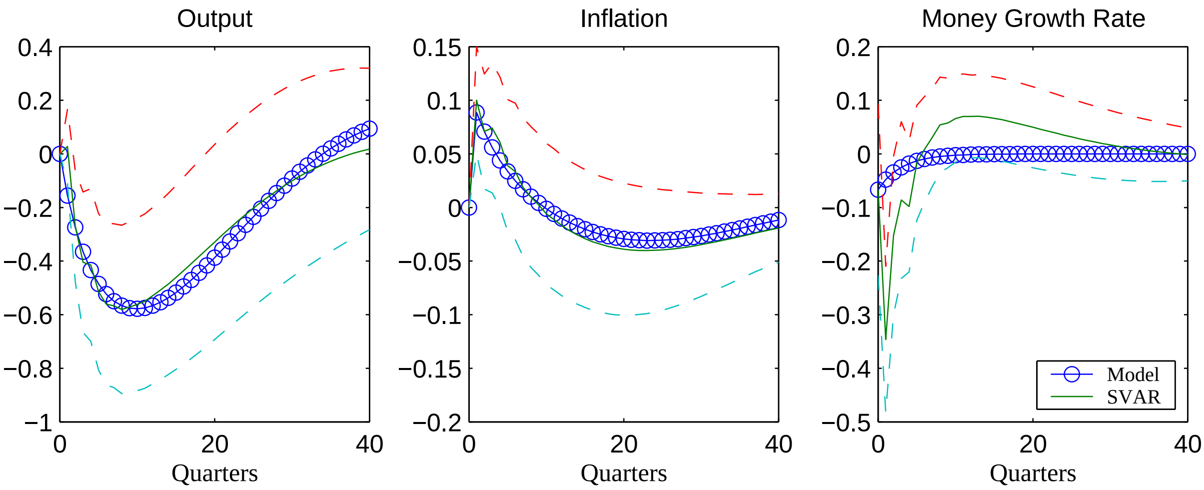


Table 2: Quantitative evaluation

	Estimated Parameters					
	Selected Z_t			Selected Z_t		
	$\{y_t,\gamma_t\}$	$\{\pi_t,\gamma_t\}$	$\{y_t,\pi_t,\gamma_t\}$	$\{y_t,\gamma_t\}$	$\{\pi_t,\gamma_t\}$	$\{y_t,\pi_t,\gamma_t\}$
φ	2.3987	2.4372	2.4198	1.4809	1.4775	1.4887
	(0.2711)	(0.1592)	(0.1459)	(0.0242)	(0.3209)	(0.0238)
ς	-0.3779	-0.3877	-0.3832	–	–	–
	(0.0412)	(0.0376)	(0.0320)			
b_2	2.3261	2.4917	2.3311	5.3250	2.5915	3.4204
	(0.7982)	(1.6797)	(0.6384)	(2.7039)	(2.3964)	(1.4686)
ρ	0.7050	0.7435	0.7212	0.3381	-0.2198	-0.3173
	(0.7390)	(0.4110)	(0.4116)	(2.5042)	(1.4201)	(1.2518)
$\mathcal{J}$-stat	72.4207	65.8488	78.3422	138.6674	132.4428	212.1225
	[99.9]	[99.9]	[99.8]	[9.4]	[17.2]	[0.0]
$\mathcal{J}_y$	7.5545	–	9.2312	69.3718	–	73.8247
$\mathcal{J}_\pi$	–	0.7847	4.2211	–	57.5593	62.6048
$\mathcal{J}_\gamma$	64.8662	65.0641	64.8899	69.2956	74.8834	75.6929
Eigenvalues	0.9521	0.9721	0.9630	–	–	–
	0.9521	0.9721	0.9630	0.9809	0.9775	0.9887

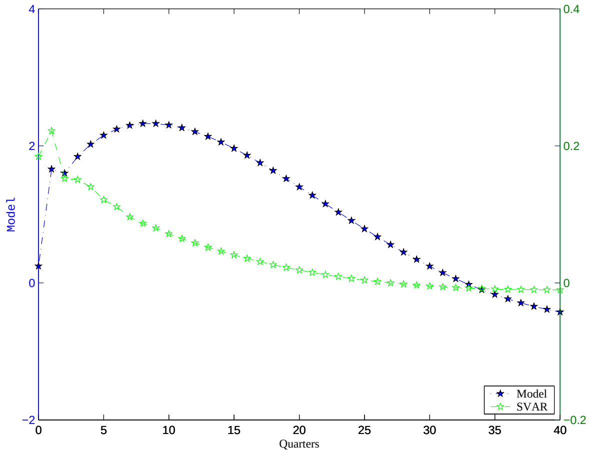
Note: s.e. in parentheses and P values in brackets. The first three columns correspond to the case where the estimation is conducted with $\eta = 3$ and the last three columns to the case where the estimation is conducted with $\eta = 3$ and imposing $\varsigma = 0$.

Figure 3: impulse response functions from the monetary and the SVAR models



Note: These figures are drawn for $\eta = 3$, $\sigma = 0.0007$, and $Z_t = \{y_t, \pi_t, \gamma_t\}$.

Figure 4: IRF of the nominal interest rate from the monetary and the SVAR models



Note: This figure is drawn for $\eta = 3$, $\sigma = 0.0007$, and $Z_t = \{y_t, \pi_t, \gamma_t\}$.

Figure 5: Data used for estimation

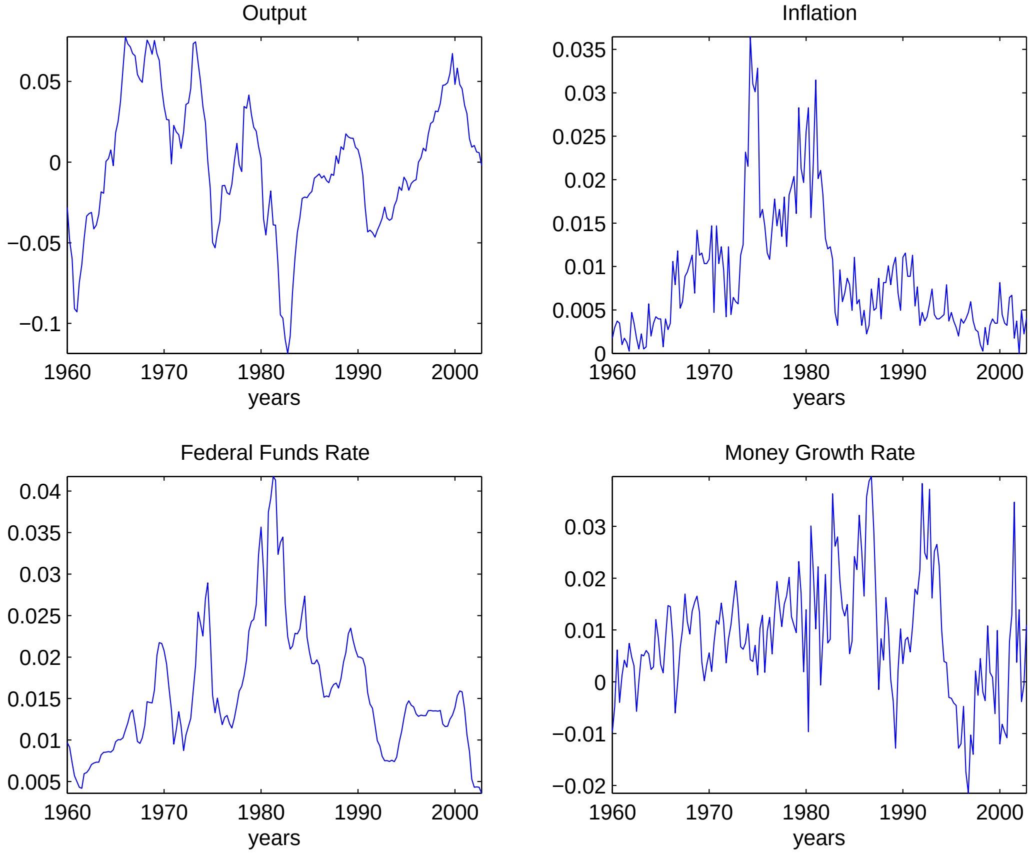
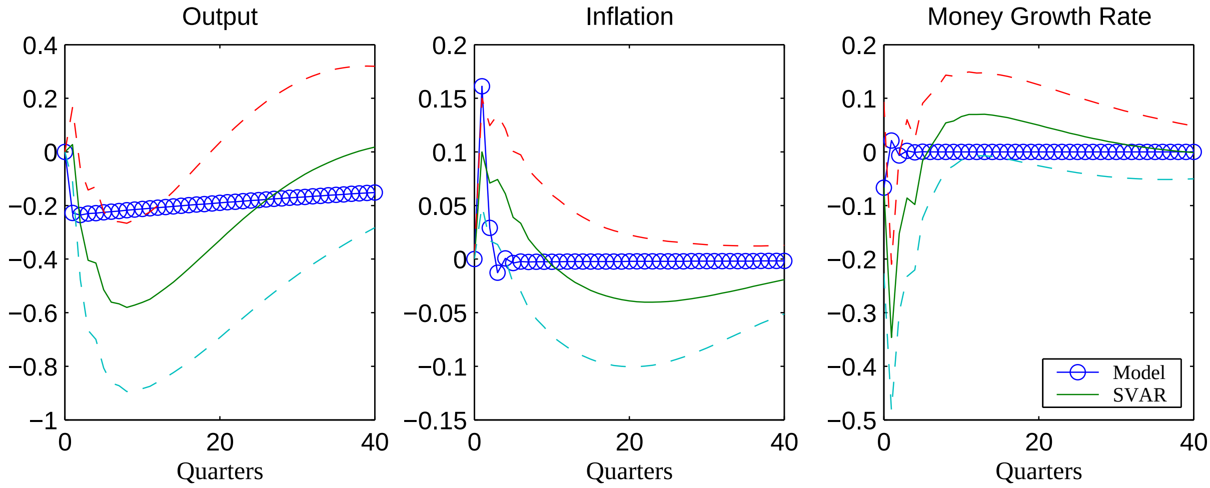


Figure 6: impulse response functions from the Monetary and the SVAR models



Note: Theses figures are drawn for $\eta = 3$, $\sigma = 0.0007$, $\varsigma = 0$, and $Z_t = \{y_t, \pi_t, \gamma_t\}$.