June 21, 2003

Beyond the Good and the Evil: Anarchy, Commitment, and Peace.

by

Jean-Paul Azam,

ARQADE, IDEI, University of Toulouse,

Institut Universitaire de France, and CSAE, Oxford.

Abstract: A game-theoretic model of anarchy, allowing for conflict is presented, which brings out the possibility of peace and prosperity in an equilibrium without government. It sheds some light on the issue of peace among nations without a supra-national government. Peace can be supported either by the promise of income transfers, which requires some commitment credibility, or by the gift of productive resource. This highlights the role of territorial concessions in securing peace among warlords or nations. It also brings out the link between the allocation of property rights and peace among unreliable players. An extension to the n-player case is presented as well.

Key Words: Conflict; Anarchy; Peace; Gift Exchange; Property Rights.

JEL Classification Numbers: D74, C78, H19, P48.

Corresponding Address: Prof Jean-Paul Azam

ARQADE, University of Toulouse 1,

21 Allée de Brienne, 31000 Toulouse, France.

Tel.: (33) 561 12 85 35. Fax: (33) 561 12 85 38.

Email: azam@univ-tlse1.fr

1. Introduction

The belief in Heaven and Hell is widespread in prosperous societies (Barro and McCleary, 2002). In *The Genealogy of Morals*, Nietzsche (1999) emphasized how clever had the Christian religion been in moralizing "the mechanism of debt in moral duty and in bad conscience, in conscience as guilt" (discussed in Derrida, 1995, p.114). Derrida summarizes this aphorically as "… the irreducible experience of belief, between credit and faith, the *believing* suspended between the credit of the creditor and the credence of the believer" (Derrida, 1995, p.115). Broken promises thus entail intimate suffering, due to the feeling of guilt, and creditors often rely on this to found their belief in repayment. Similar effects resulted from other religions. Moral codes and religions, and even other forms of ideology, have thus emerged for enhancing the credibility of promises. Sophisticated societies have thus found some mechanisms for enforcing contracts, while reducing the costs of punishing default by transposing most of its repression in the spiritual or ideological sphere. Kant's "categorical imperative", another form of moral restraint on disloyal behavior, thus played a large part in founding the enlightenment, the philosophy of freedom and responsibility. Its potential importance for economics has been highlighted by Laffont (1975). Kant explicitly defined enlightenment as the way for human beings to escape from the tutelage of others. Adam Smith similarly emphasized the role of self-esteem as a preferable substitute for the pressure of others in his *Theory of Moral Sentiments*: "… the prudent man is always both supported and rewarded by the entire approbation of the impartial spectator, and of the representative of the impartial spectator, the man within the breast" (cited by Charlier, 1996, p.278).

Ekelund *et al*. (2002) discuss how the Catholic religion differentiated finely the "price of sin", for creating some opportunity for rent extraction in return for redemption. They explain the "entry" of the Protestant Reformation as the emergence of a rival firm in the redemption market, which drastically reduced the ability of the Catholic church to extract rents from its members. This discussion highlights the fact that the enforcement mechanisms that make broken promises more or less costly for the defaulter, and hence make commitments more or less credible, depend at least as much on ideological factors, as on institutional ones like judicial courts and police, or the clergy. This fact plays undoubtedly an important part in shaping the working of modern society. Most of the transactions take place without litigation nor conflict, and most agents are committed to keep their promises, even without the threat of the judiciary. Moral duty is thus an efficient substitute to the shadow of the judge. It is well known that even gangsters obey generally a "code of honor", which forces them to perform certain duty, even if they reap no immediate benefit from it. Many promises are thus kept in the real world, without the pressure of a formal institution.

The aim of the present paper is to analyze the difference that it makes, in a state of anarchy, when the players are able to make credible promises in such a way. The economic theory of anarchy has been recently developed by Hirshleifer (1995), using a model where the players can use their resources either for production or for appropriation, i.e. for capturing by force the output of others. He emphasizes the part played by the technology of appropriation, i.e. of fighting, in shaping the social equilibrium resulting from such a setting. In his mind, anarchy (as opposed to amorphy), necessarily involves conflict. He defines it as follows: "… anarchy is a social arrangement in which contenders struggle to conquer and defend durable resources, without effective regulation by either higher authorities or social pressures" (Hirshleifer, 1995, p.27). This involves a typical waste of resources, as some production factors are diverted from productive use, and are used up instead for appropriation or defense. By contrast, the present model shows that an anarchical equilibrium can emerge where no fighting takes place, for some values of the parameters. No resources are wasted in appropriation and defense, and output is maximized, when such an equilibrium prevails. This is done by extending Hirshleifer's model by adding a third technology, beside the production technology and the appropriation one, which are present in his model. A commitment technology is added here, which allows the players to make some promises, with a given probability of keeping their word. This probability is not modeled as a characteristics of the equilibrium of the game, but as a personal characteristics of each player. They promise to make a transfer, in return for refraining from engaging resources for fighting, i.e. to pay the price of peace. The peaceful anarchy equilibrium is thus modeled here as a gift-exchange equilibrium, somehow related to the classic work by Akerlof (1982). As discussed above, the subjective cost of breaking a promise depends to a large extent on the system of beliefs present in any given society, be they religious or ideological. There does not necessarily need to exist a third party for enforcing contracts, at least in an institutional sense. The effect of such a system of beliefs is captured in the model presented below by some exogenous probabilities of the agents keeping their promises. This is not more of a "black box" than assuming an exogenous level of labor productivity, for example. Both encapsulate the result of complex processes of learning and understanding of a specific type of technology. The foundations of honesty have been discussed, among economists, by Akerlof (1983) and Sen (1976). The latter emphasizes commitment, defined in a broader sense than here. More recently, Basu (2000) discusses this issue in an evolutionary setting, using the concept of group selection. His argument can be adapted to the present context as follows: honesty may emerge as a dominant characteristic, because groups with a high enough degree of honesty can avoid violence, and thus prosper, while those afflicted with low credibility will be decimated by internecine violence. The present paper does not attempt to model this group selection effect, and takes as given the degree of reliability of the promises made. Moreover, one may regard the conflict between two agents analyzed here as a metaphor used for shedding some light on issues involving more complex players, like ethnic groups or countries. Then, the players' ability to commit credibly does not rest only on purely moral characteristics. It involves in this case organizational or political systems. What is at stake here is the modeling of a peaceful equilibrium relying on some internal characteristics of the players, rather than on the presence of an external enforcer.

Grossman (2002) follows a different route for pulling an economy out of a state of violent anarchy, where a fraction of the population is engaging in predation on the rest of the population. He shows that a proprietary state whose sole function is to enforce the collective choice to allocate resources to deterring predation may be Pareto superior to violent anarchy, even if this state is maximizing the consumption of the ruling elite. In the words of Hirshleifer (1995), this corresponds to the Hobbesian vertical social contract. By contrast, the present model may be viewed as developing the horizontal Lockean social contract, in the same terminology. Interestingly, Grossman refers to the Bible, citing the people of Israel requesting that the prophet Samuel "make us a king … [who] may judge us, and fight our battles" (Grossman, 2002, p.31). He interprets the passage "… may judge us " as the "request for the enforcement of a collective choice to allocate more resources to guarding against predators" (p.32). Another interpretation is possible for the need to establish a judge, namely that the people of Israel had realized that their capacity to commit credibly was low at that point in time, and that some external enforcement mechanism was needed to avoid violence. This is the interpretation favored here, although the issue of the potential role for a Leviathan, or any other form of third party enforcer, falls outside the scope of the present paper. The analysis presented below is not claiming that the creation of an external enforcer would be inefficient for correcting, or preventing, a situation of violent anarchy. It is claiming that under some conditions, a peaceful anarchy situation can prevail, without any external intervention.

This issue of anarchical peace is of more than purely academic interest. In the field of international relations, for example, it bears on the problem of peace among nations, in the absence of a supra-national, or world-wide, government. Whether or not the United Nations Organization should see its powers extended so as to make it a world-wide government is often discussed. While many governments from smaller countries are in favor of such a solution, other countries, like the USA and the UK, are more reluctant to follow this path. The issue of peace among symmetrically positioned players, in the absence of a third party above them, is obviously relevant for this debate. In his work on "Perpetual Peace", Immanuel Kant tackled exactly this question (Kant, 1983). He emphasizes that such a peace would follow if the potential opponents were signing an agreement in good faith. His argument seems a bit outdated at first, because of its emphasis on good faith and moral restraint. However, when this is interpreted in terms of ability to commit, it gains a renewed interest, in game-theoretic terms. According to Kant, the ability of the governments involved to commit would be based on two elements. First, the governments involved should have a republican organization, by which he means having a political regime characterized by the separation of powers between the executive and the legislature. This anticipates to some extent the argument of North and Weingast (1989), which bases the government ability to commit on checks and balances, i.e. the existence of multiple veto points. Second, he relies on commercial interests to ensure that peace brings about a larger opportunity set than war, so that a large enough constituency exists in favor of peace in his peaceful republic. He thus contributed to the issue of trade and conflict. A recent introduction to this field is presented in Barbieri and Schneider (1999).

The model is presented in the next section, which also discusses its outcome in case of fighting. It turns out that most of the properties of the anarchical equilibrium discussed by Hirshleifer (1995) are also present in this case. However, this is not the only type of equilibrium that can prevail in the present model. Hence, section 3 discusses the conditions under which a peaceful equilibrium prevails instead, using a model of bargaining with a restricted set of possible transfers. These restrictions are derived from the assumed commitment technology. Roughly speaking, the condition for peace to prevail says that this happens if the player who is "too rich" is "credible enough" to make a transfer workable. In this instance, "too rich" is given a precise meaning, by reference to a benchmark case that makes transfers unnecessary for peace to prevail. The existence of this benchmark opens the way for an interesting solution to the credibility problem. Instead of relying on the promise of income transfers, which is quite demanding in terms of credibility, the players can rely on an initial gift of productive resources. This can typically be interpreted as territorial concessions, in the field of inter-ethnic or international relations. Section 4 develops this argument, and discusses how it provides some theoretical foundations for the endogenous allocation of property rights at the beginning of the game.

In both cases, be it the promise of an income transfer, or the ex ante gift of productive resources, the rationale for giving has nothing to do with any form of altruism. Redistribution is here founded on self interest, as peace is more profitable than war, and is determined without neglecting the possibility of cheating, by failing to deliver the promised transfer. The resulting peace is unarmed, and is established among symmetrically positioned players. It can thus be labeled "eleutheristic peace", by analogy with Kolm's concept of eleutheristic justice, although it does not entail the equality of income or wealth (Kolm, 1996).

The players are equal in the deeper sense that the outcome is determined by bargaining, and only reflects each player's personal characteristics. They are equal in freedom, or formal rights, and each one of them can reject the peaceful outcome, and choose war instead. Section 5 extends the result of section 3 to the case with more than two players, under several simplifying assumptions. Section 6 concludes.

2. The Model, and its Violent Anarchy Outcome

This section presents the model, and sketches the analysis of its equilibrium in case of fighting. The comparative statics exercises performed reproduce the most important features of Hirshleifer's model (Hishleifer, 1995).

The Three Technologies

There are two agents in this model, indexed by $i \in \{1,2\}$. They have an initial endowment of productive resources N_i, given exogenously, which can be used either for producing output, with a constant productivity α_i, or for fighting. The quantity devoted to the latter is denoted F_i, and *i*'s output is thus $Y_i = \alpha_i (N_i - F_i)$. The quantity of resources devoted to fighting is assumed perfectly observable by the two parties. More precisely, the two players are assumed to know all the parameters of the model, and the only uncertainty involved is due to the random outcome of a fight and the imperfect credibility of the players' commitment to deliver the promised transfers.

The conflict technology is modeled here in the standard fashion, by assuming that the probability of agent 1 winning a fight against agent 2, if any agent spares any resource for fighting, is given by the following strictly increasing and concave function:

$$p = p\left(\frac{F_1}{F_2}\right). \tag{1}$$

Agent 2 wins with the complementary probability. This is very similar to the fighting technology assumed in Azam (1995, 2002), Grossman (1991, 2002), Grossman and Kim (1995), Hirshleifer (1991, 1995), and Skaperdas (1992, 1996, 2002), among others. A discussion of this type of assumption, with a comparison with rent-seeking models, is provided by Neary (1997). Esteban and Ray (1999) generalize this setting to more than two players, and bring out very clearly the link between conflict and distribution. They measure conflict by the total amount of resources that are dissipated in the struggle for preferred outcomes.

The new feature introduced in the present model, relative to the classic literature on conflict, is the commitment technology. Each agent is thus assumed to be able to make the following promise: "I will give you a transfer $g_i \geq 0$ if you do not arm, i.e. keep your $F_i = 0$"; however, it is common knowledge that each agent has an imperfect commitment technology, and will only deliver the promised transfer at best with probability λ_i, if the other player does not arm. He might even give less, or with a lower probability, if he is prevented from doing so by his budget constraint, in the worst case. With the complementary probability, the debtor will simply default, and deliver nothing; but then, it is too late for arming. Economists are more used to assuming the two extreme values of this type of probability than to take a continuum of values into account, as is done here. Traditionally, in one-shot games like the present one, game theory teaches us to assume either no commitment at all, in a simultaneous-move Nash equilibrium, or to assume perfect commitment, in the Stackelberg equilibrium framework. Azam (2001) contrasts the outcomes resulting from these two extreme assumptions, in an asymmetric game where a government is facing a potential insurgency. By contrast, in the present setting, the two players are treated symmetrically, by assuming that no government does exist at the beginning of the game, which thus starts in a state of anarchy, as in Hirshleifer (1995). Azam (2002) analyzes also a symmetric model, where involuntary transfers take place in equilibrium, because of looting and taxation, but no gift is allowed, and no commitment problem arises.

Now, the game takes place as follows. In the first stage, the two agents meet and reciprocally offer the contract described above to each other. If any one of the players rejects the offer, the two sides get armed. Their remaining resources are used for producing output. Notice that these assumptions do not allow for either side getting armed in a hidden way, without the other side noticing. If any one of them gets armed, the other one can get armed as well before any fighting occurs. In the second stage, either the exchange of gifts takes place, with the known probabilities, if the contracts have been accepted and implemented by both parties, or the fight takes place if the two players have devoted a positive amount of resources to arming, and the winner takes the whole output.

The Outcome in Case of Fighting

In the latter case, the outcome is very similar to the state of anarchy analyzed by Hirshleifer (1995). Define total output as:

$$Y = \alpha_1 (N_1 - F_1) + \alpha_2 (N_2 - F_2) . \tag{2}$$

Then, the conflict equilibrium can be characterized as in proposition 1.

Proposition 1: If fighting breaks out, the two players get the following payoffs:

$$B_1 = p^* Y^* \tag{3}$$

and

$$B_2 = (1 - p^*) Y^* , \tag{4}$$

where *p** is only a function of α_2 / α_1, implicitly determined by (1) and:

$$\frac{p^*}{1 - p^*} \frac{F_1}{F_2} = \frac{\alpha_2}{\alpha_1} . \tag{5}$$

Lastly, equilibrium output is then given by:

$$Y^* = \frac{\alpha_1 N_1 + \alpha_2 N_2}{1 + \omega^*} , \tag{6}$$

where, defining *p′** as the derivative of *p* evaluated at *p**, the following definition is used:

$$\omega^* = \frac{p'^*}{p^{*2}} \frac{\alpha_2}{\alpha_1} > 0 \ . \tag{7}$$

Proof: The best-response functions for F_1 and F_2 are determined by maximizing the following two payoff functions, taking due account of (2):

$$B_1 = \max_{F_1} p\left(\frac{F_1}{F_2}\right) Y \text{ and } B_2 = \max_{F_2} \left(1 - p\left(\frac{F_1}{F_2}\right)\right) Y , \tag{8}$$

They must thus solve simultaneously the following two first-order conditions, in the Nash equilibrium:

$$p'\left(\frac{F_1}{F_2}\right)\frac{Y}{F_2} = p\left(\frac{F_1}{F_2}\right)\alpha_1 = \left(1 - p\left(\frac{F_1}{F_2}\right)\right)\frac{\alpha_2 F_2}{F_1}. \tag{9}$$

The concavity of the function $p(.)$ is sufficient for ensuring that the second-order condition holds. Equation (5) is easily derived from (9). The latter allows to derive implicitly p^* as a function of α_2/α_1 only, using definition (1). The left-hand part of (9) is then used along with (2) to derive (6). **QED**

Several comments come to mind about proposition 1. First, (6) shows that output is less than its maximum potential value, as some resources are diverted from production, and wasted in fighting. The coefficient $1/(1+\omega^*)$ acts like a discount factor, applied to potential output, and ω^* captures the rate of resource diversion due to fighting. It can be checked that $\omega^* Y^* = \alpha_1 F_1 + \alpha_2 F_2$. Therefore, using (6), one can write:

$$\frac{\alpha_1 F_1 + \alpha_2 F_2}{\alpha_1 N_1 + \alpha_2 N_2} = \frac{\omega^*}{1+\omega^*}. \tag{10}$$

Hence, the share of potential output which is wasted because of the diversion of productive resources for fighting is an increasing function of ω^*. Notice that this parameter is positively related to Hirshleifer's "decisiveness parameter", which determines the marginal impact on the probability of winning of an increase in the relative levels of forces engaged (Hirshleifer, 1995). Therefore, as in Hirshleifer's model, more resources are wasted for fighting, the more "decisive" they are. Second, (5) embeds a result which is akin to Hirshleifer's "paradox of power" (Hirshleifer, 1991). Its left-hand side is an increasing function of F_1/F_2, so that (5) shows that the relative engagement of forces by the two sides is decreasing in their relative productivity. As in Hirshleifer's analysis, this reflects the impact of the opportunity cost of labor: the more productive side will divert less resources into fighting than the less productive side. Lastly, (3) and (4) may be combined with the latter result to show that the share of output obtained by each side is a decreasing function of its relative productivity. This does not entail necessarily that its payoff has the same property, without further specification. The impact on Y^* might in some cases offset the impact on the shares.

Therefore, this model leads to several familiar predictions, in the case where the conflict breaks out. However, such an event is not necessarily the equilibrium outcome of the game, as specified above. The next section derives the conditions for a peaceful anarchical equilibrium to prevail instead, where the players quietly devote all their resources to production. Then, when neither player devotes any resources to fighting, the probability of winning or loosing is zero for both players, and the status quo prevails.

3. Peaceful Anarchy, as a Gift Exchange Equilibrium

The existence of the fighting technology in the model creates an externality, whereby the sparing of resources for fighting by one player entails an expected loss of income for the other player. The problem of peace in this setting is thus simply that of finding an arrangement for internalizing this externality. The aim of the present section is to determine the conditions for a solution based on promised transfers to be effective. It shows how low credibility may interact with the budget constraints to make such a peaceful equilibrium impossible. The next section will enlarge the set of possible actions, by allowing for an unconditional redistribution of property rights up front.

The Credibility Constraint and the Violent Equilibrium

In the absence of fighting, in a peaceful equilibrium where the two sides refrain from arming, each player *i* promises a transfer g_i and expects to receive a transfer $\hat{g}_i$ with some probability, depending on λ_i and $\hat{\lambda}_i$, the probabilities for each player to keep his promise. More precisely, the latter probabilities are defined as the probabilities that each player delivers either the promised amount, or the maximum that is allowed by his budget constraint. Hence, ex post, the maximum amount that player *i* will deliver is $\min\{g_i, \alpha_i N_i + \hat{g}_i\}$. With the complementary probability, he simply delivers nothing. Let Eg_i and $E\hat{g}_i$ denote the mathematical expectations of these transfers.

Player *i*'s expected utility is thus in this case:

$$U_i = \alpha_i N_i - E g_i + E \hat{g}_i . \tag{11}$$

Because of their budget constraints, no player can promise a transfer whose expected value exceeds the expected value of his income, augmented with the expected value of the transfer received. Hence, Eg_i and $E\hat{g}_i$ are to some extent determined jointly, in equilibrium, as they must satisfy consistently both players' budget constraints. Table 1 represents these budget-constraint consistent expected values of the transfers. They are computed using the following formula:

$$E\,g_i = \lambda_i \min\{g_i, \alpha_i N_i + E\,\hat{g}_i\}. \qquad (12)$$

Table 1: Determination of $\{E\,g_1, E\,g_2\}$

	$g_1 \le \alpha_1 N_1$	$g_1 > \alpha_1 N_1$
$g_2 \le \alpha_2 N_2$	$E\,g_1 = \lambda_1 g_1$ $E\,g_2 = \lambda_2 g_2$	$E\,g_1 = \lambda_1(\alpha_1 N_1 + \lambda_2 g_2)$ $E\,g_2 = \lambda_2 g_2$
$g_2 > \alpha_2 N_2$	$E\,g_1 = \lambda_1 g_1$ $E\,g_2 = \lambda_2(\alpha_2 N_2 + \lambda_1 g_1)$	$E\,g_1 = \dfrac{\lambda_1(\alpha_1 N_1 + \lambda_2 \alpha_2 N_2)}{1-\lambda_1\lambda_2}$ $E\,g_2 = \dfrac{\lambda_2(\lambda_1 \alpha_1 N_1 + \alpha_2 N_2)}{1-\lambda_1\lambda_2}$

These results allow to establish the following lemma.

Lemma 1: (i) Any value of $0 \le E\,g_1 - E\,g_2 \le \lambda_1 \alpha_1 N_1$, or of $0 \le E\,g_2 - E\,g_1 \le \lambda_2 \alpha_2 N_2$, can be attained by a one-sided gift exchange;

(ii) any value of the net expected transfer strictly below these upper bounds can be attained by a two-sided gift exchange.

(iii) no net expected transfer can be attained outside these ranges.

Table 2: Determination of $E\,g_1 - E\,g_2$.

	$g_1 \le \alpha_1 N_1$	$g_1 > \alpha_1 N_1$
$g_2 \le \alpha_2 N_2$	$\lambda_1 g_1 - \lambda_2 g_2$	$\lambda_1 \alpha_1 N_1 - (1-\lambda_1)\lambda_2 g_2$
$g_1 > \alpha_1 N_1$	$(1-\lambda_2)\lambda_1 g_1 - \lambda_2 \alpha_2 N_2$	$\dfrac{(1-\lambda_2)\lambda_1 \alpha_1 N_1 - (1-\lambda_1)\lambda_2 \alpha_2 N_2}{1-\lambda_1\lambda_2}$

Proof: Simple calculations allow to derive table 2 from table 1. It is easily seen from this table that the net expected transfer $E g_1 - E g_2$ is bounded from above by $\lambda_1 \alpha_1 N_1$ and from below by $-\lambda_2 \alpha_2 N_2$. Moreover, it is an increasing function of g_1, and a decreasing function of g_2, both not strictly so. This allows to derive easily the three points of lemma 1. **QED**

Now, the choice of peace rather than war depends on the ability of the two parties to find a pair of promised gifts $\{g_1, g_2\}$ that gives them a higher expected payoff than the prospect of fighting. If they fail to strike an agreement, the players will settle their problem by fighting. Their expected payoffs in this case, analyzed in the previous section, determine their outside options.

Assume that the two players bargain over the sharing of the surplus output due to peace. The latter is easily shown to be equal to $\alpha_1 F_1 + \alpha_2 F_2$, i.e. the output produced by the resources saved from being engaged in fighting. This can be called the total peace dividend. Notice the difference with the bargaining framework discussed by Anbarci *et al.* (2002). The latter use the "guns and butter" paradigm, where the players get armed first, and then bargain to avoid the fight. Here, by contrast, the players bargain in order to avoid wasting resources for arming. As the peace dividend is positive, there exists some possible transfers ensuring that peace is the preferred outcome by both sides, whenever they are feasible. Proposition 2 below spells out the restrictions that the imperfect credibility of the promises made by the two players imposes on the set of feasible outcomes.

Proposition 2: A violent anarchy equilibrium prevails if either:

$$\lambda_1 < 1 - \frac{(\omega^* + p^*)(\alpha_1 N_1 + \alpha_2 N_2)}{(1 + \omega^*)\alpha_1 N_1}, \tag{13}$$

or:

$$\lambda_2 < 1 - \frac{(1 + \omega^* - p^*)(\alpha_1 N_1 + \alpha_2 N_2)}{(1 + \omega^*)\alpha_2 N_2}. \tag{14}$$

Peaceful anarchy prevails otherwise.

Proof: We know from lemma 1 and (11) that peaceful anarchy cannot deliver to each player higher payoffs than:

$$\overline{U}_1 = \alpha_1 N_1 + \lambda_2 \alpha_2 N_2, \tag{15}$$

and:

$$\overline{U}_2 = \lambda_1 \alpha_1 N_1 + \alpha_2 N_2, \tag{16}$$

Then, one player or the other prefers violence to peace if either $B_1 > \overline{U}_1$ or $B_2 > \overline{U}_2$. Substituting from (15) and (16), and re-arranging the terms yields (13) and (14). **QED**

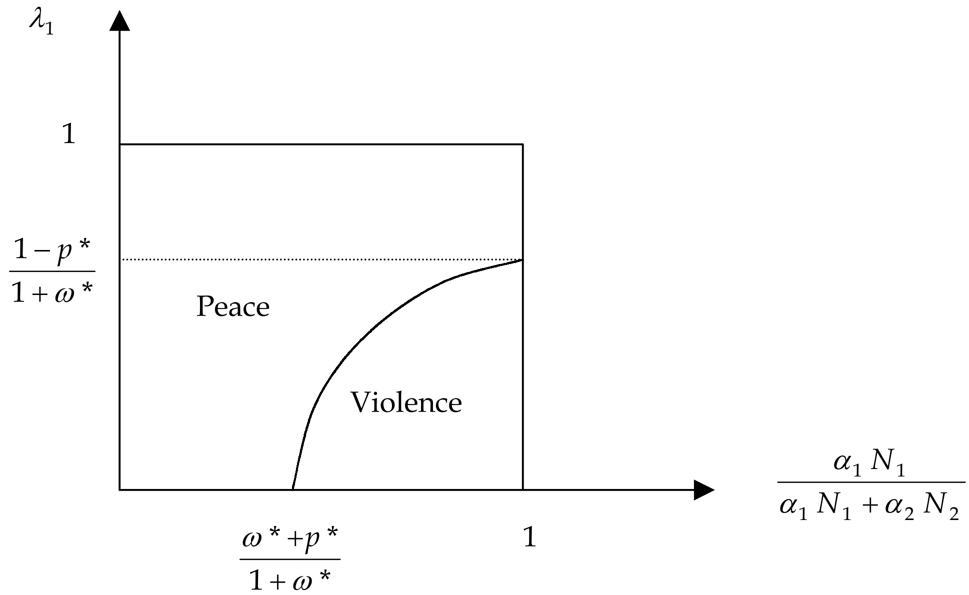

Figure 1: The Partition of the Parameter Space

Conditions (13) and (14) show very neatly how the degree of credibility of the two players interacts with the initial distribution of income to determine whether peace or violence prevails in equilibrium. Notice that violence erupts even if only one of the players rejects the prospect of peace, because of the assumption ruling out secret arming. Figure 1 depicts condition (13), and shows how it partitions the space into an area where peace may prevail, unless player 1 chooses to arm, above the curve, and another one, where violence erupts, at player 2's initiative, below the curve. This diagram brings out the role of the parameters of the fighting technology in determining the choice between war and peace. The more decisive are the forces engaged in fighting, the smaller is the sub-set of parameter values in figure 1 for which violence prevails. This results from the fact that the fraction of output lost because of the diversion of resources for fighting is larger, the more decisive they are, as seen in the previous section. Hence, violence is more expensive, the larger is the decisiveness parameter. This makes peace more attractive. A very similar diagram could of course be drawn for representing (14), and identifying in the same way the sub-set of parameter values for which player 1 would trigger the eruption of violence.

The Peaceful Anarchy Equilibria

Two different types of peaceful anarchy equilibria may arise in this model, because of the restriction on transfers imposed by the lack of full credibility. The latter sets an upper limit on the maximum payoff accessible for each player, and described by (15) and (16). This creates “kinks” in the bargaining set, such that we may either find a corner equilibrium or an interior one. It can be checked that the corner solution would not arise if the chosen solution concept was the one presented by Kalai and Smorodinsky (1975). Let us begin by presenting the interior equilibrium.

The most standard assumption regarding the outcome of a bargaining process is the generalized Nash bargaining solution, introduced by Rubinstein (1982). It is used here for solving the reciprocal gift problem of the two players. Let us now characterize this equilibrium, before inquiring about its existence.

If it exists, the peaceful equilibrium must solve the following maximization problem, where $\eta > 0$ denotes player 1’s bargaining power:

$$\max_{g_1, g_2} \left(\alpha_1 N_1 - E g_1 + E g_2 - B_1\right)^{\eta} \left(\alpha_2 N_2 + E g_1 - E g_2 - B_2\right)^{1-\eta}. \tag{17}$$

Additionally, the bargaining solution must fit in the budget constraints described above at tables 1 and 2, which no credible promise can exceed. Using (15) and (16), together with lemma 1, they may be written as:

$$\alpha_1 N_1 - \left(E g_1 - E g_2\right) \leq \overline{U}_1 \text{ and } \alpha_2 N_2 + \left(E g_1 - E g_2\right) \leq \overline{U}_2. \tag{18}$$

Forget the latter for a moment, and let us characterize an interior solution. The first-order condition for the generalized Nash bargaining solution, which solves (17), may be written as:

$$E g_1 + \eta\left[\alpha_2 N_2 - B_2\right] = E g_2 + (1-\eta)\left[\alpha_1 N_1 - B_1\right]. \tag{19}$$

Examination of (19) shows immediately that some indeterminacy is involved in the reciprocal gift exchange, as what is really determined is the difference $E g_1 - E g_2$, which may be positive or negative. Table 2 may be used to find the different possible values of this net expected transfer, as a function of the promised transfers. In other words, what the generalized Nash bargaining solution determines here is the surplus of expected transfer that the net giver, whoever that is, must promise over and above the mere compensation of the expected value of the gift received from the other player. Cases can be constructed where the two values of g_1 and g_2 can be determined separately, using table 1. In general, however, some indeterminacy remains. Then, taking due account of the budget constraints (18), as well as the definition of the payoffs (11), allows to characterize the interior solution to this bargaining problem as in proposition 3.

Proposition 3: A peaceful anarchy interior equilibrium exists where the two players share the total peace dividend among themselves as follows:

$$U_1 = B_1 + \eta\left(\alpha_1 F_1 + \alpha_2 F_2\right), \tag{20}$$

and:

$$U_2 = B_2 + (1-\eta)\left(\alpha_1 F_1 + \alpha_2 F_2\right), \tag{21}$$

if and only if either one of the following conditions holds:

(i) $\alpha_1 N_1 > \left(p^* + \eta\,\omega^*\right)Y^*$ and $\lambda_1 \geq 1 - \left(p^* + \eta\,\omega^*\right)\dfrac{Y^*}{\alpha_1 N_1}$, or (22)

(ii) $\alpha_1 N_1 < \left(p^* + \eta\,\omega^*\right)Y^*$ and $\lambda_2 \geq 1 - \left(1 - p^* + (1-\eta)\,\omega^*\right)\dfrac{Y^*}{\alpha_2 N_2}$, or (23)

(iii) $\alpha_1 N_1 = \left(p^* + \eta\,\omega^*\right)Y^*$. (24)

In case (iii), no expected net transfer is required, and no credibility condition is bearing on the outcome.

Proof: In case (i), the net expected gift must come from player 1, as can be checked by re-arranging the terms in (19), after substituting for the different variables from above. Then, the net expected gift from player 1 must fulfill both (19), and the budget constraint derived from table 2. This is only possible if the credibility constraint on the right-hand side of (i) holds. Otherwise, credibility is deficient, and no feasible transfer can implement the interior solution. In case (ii), the net expected gift must come from player 2, and a symmetric argument applies. Then, it is the credibility constraint on the right-hand side of (ii) that must hold for ensuring existence of a peaceful interior equilibrium. Case (iii) is the limiting case between the two previous ones, and does not demand much comment. **QED**

Between the interior solution just described and the violent anarchy equilibrium analyzed at proposition 2, there exist possible peaceful anarchy corner equilibria, described in proposition 4 below. This type of equilibria is constrained by the imperfect credibility of the promises made by the players, which makes some transfers impossible. This occurs when the right-hand sides of either condition (22) or (23) does not hold. This type of equilibrium would not exist if the Kalai-Smorodinsky solution to the bargaining problem was used instead of the generalized Nash bargaining one.

Proposition 4: The corner peaceful-anarchy equilibrium yields the following allocation of payoffs to the two players. We have either:

(i) $U_1 = (1-\lambda_1)\alpha_1 N_1 > B_1 + \eta(\alpha_1 F_1 + \alpha_2 F_2)$, and (25)

$U_2 = \alpha_2 N_2 + \lambda_1 \alpha_1 N_1$, with $B_2 + (1-\eta)(\alpha_1 F_1 + \alpha_2 F_2) > U_2 \geq B_2$, (26)

if:

$$1-(\omega^* + p^*)\frac{Y^*}{\alpha_1 N_1} \leq \lambda_1 < 1-(p^* + \eta\,\omega^*)\frac{Y^*}{\alpha_1 N_1}; \tag{27}$$

or:

(ii) $U_1 = \alpha_1 N_1 + \lambda_2 \alpha_2 N_2$, with $B_1 + \eta(\alpha_1 F_1 + \alpha_2 F_2) > U_1 \geq B_1$, (28)

$U_2 = (1-\lambda_2)\alpha_2 N_2 > B_2 + (1-\eta)(\alpha_1 F_1 + \alpha_2 F_2)$, and (29)

if:

$$1-(1+\omega^* - p^*)\frac{Y^*}{\alpha_2 N_2} \leq \lambda_2 < 1-(1-p^* + (1-\eta)\,\omega^*)\frac{Y^*}{\alpha_2 N_2}; \tag{30}$$

Proof: The credibility conditions for (i) and (ii), numbered (27) and (30) respectively, are derived immediately from (13) and (22), or (14) and (23), respectively. The comparison of the allocation of payoffs with those of proposition 3, given by (20) and (21) are easily derived from the fact that the relevant budget constraint is binding in each case. **QED**

The Role of Transfers

Propositions 3 and 4 put a lot of emphasis on the redistribution of income in a peaceful equilibrium, and bring out the credibility constraint that bears on the set of feasible transfers. Notice that the income condition $\alpha_1 N_1 > (p^* + \eta\,\omega^*)Y^*$ in proposition 3 (i) can be interpreted more symmetrically than it looks, as it holds if and only if $\alpha_2 N_2 < (1 - p^* + (1-\eta)\,\omega^*)Y^*$. Hence, this case can be equivalently interpreted as requiring 1 to be "too rich", relative to the benchmark provided by the right-hand side of the former income constraint, or 2 to be "too poor", relative to the latter income constraint. Case (ii) can thus also be interpreted as requiring player 2 to be too rich, while case (iii) provides the benchmark for the income distribution to be "just right" for avoiding peace to be dependent on the credibility of one or the other player. This benchmark case can be understood more easily by re-writing the allocation of income that prevails in this case, using (3), (4) and (6), as:

$$\alpha_1 N_1 = B_1 + \eta(\alpha_1 F_1 + \alpha_2 F_2), \tag{31}$$

and:

$$\alpha_2 N_2 = B_2 + (1-\eta)(\alpha_1 F_1 + \alpha_2 F_2). \tag{32}$$

This benchmark income distribution is such that each player produces exactly an income flow equal to the sum of his expected gain in case of fighting plus his share of the total peace dividend, as determined by his bargaining power. Promised transfers are required to ensure peace when the distribution of income in case of peace differs from this benchmark, in the interior equilibrium. Then, the limited credibility of the players may combine with the budget constraints to make the interior solution impossible in the cases identified at proposition 3. The corner solution described at proposition 4 prevails in this case, provided the condition for avoiding violence, easily derived from proposition 2 above, holds. Hence, proposition 2 can be re-phrased as saying that peace prevails in equilibrium when the "too rich" agent is "credible enough" to make a compensating transfer workable.

The corner solution requires one additional comment. In case (i), this equilibrium involves a one-sided transfer by player 1, while case (ii) involves a one-sided transfer by player 2, as can be inferred from lemma 1. These cases thus introduce in this model an asymmetry between the players, which could be misleadingly interpreted as the end of anarchy. The transfer from the "too rich" player to the other one could thus be interpreted as a form of taxation. The "too poor" player thus could be viewed as an emerging government, extracting a tribute from the "too rich" player, as a compensation for not engaging in fighting. This is again akin to Hirshleifer's paradox of power, cited above (Hirshleifer, 1991). However, with probability $1-\lambda_1$, in case (i), or $1-\lambda_2$, in case (ii), the "too rich" player does not deliver the promised transfer, while the other one does not impose any punishment for that. The analogy with a government is thus quite limited. Moreover, we have seen above that this one-sided transfer equilibrium is just an interim case between the violent anarchy equilibria, on one side, and the interior generalized Nash bargaining peaceful equilibrium, on the other side. Conditions (27) and (30) show that they only prevail within some interval of parameter values. Above this range, the equilibrium transfers are generically indeterminate, as it is only their expected difference that is determined. We have then a continuum of gift-exchange equilibria. Therefore, there exists generically a large number of peaceful equilibria where the two players are giving positive transfers to each other, so that the interpretation of the "too poor" player as an emerging government, and of the transfers as a tax, becomes somewhat contrived in these cases. It is only tenable as a limit case, when the smaller of the two transfers tends to zero. Moreover, the existence of the corner-solution equilibrium depends on the bargaining equilibrium concept used. It can be easily checked that it does not occur if the Kalai-Smorodinsky solution is used as the solution concept, except again as a limiting case.

As a corollary to proposition 2, a state of violent anarchy, as described in Hirshleifer's paper cited above (Hirshleifer, 1995), comes out here as a special case, valid only for a sub-set of potential parameter values. It requires credibility to be "too low", on the part of the "too rich" player, for a peaceful regime to prevail in equilibrium. As peace entails a positive surplus of output over the state of fighting, the two players would rather have the former than the latter. Therefore, war is here predicted to occur because of a failure of the commitment technology available to the "too rich" player. The latter is quite willing to promise the transfer that could prevent war from breaking out, but is unable to do it in a credible enough fashion. Fighting is thus the direct effect of a deficient commitment technology, and not the result of any irrational behavior.

Case (iii) of proposition 3 is particularly interesting, as it suggests that it is possible to relax the credibility constraint, by an appropriate redistribution of productive resources among the players, made unconditionally at the beginning of the game. In the real world, this type of redistribution of productive resources is typically observed as territorial concession, or other forms of redistribution of property rights over some assets. This is discussed in the next section.

4. Redistribution of Property Rights for Relaxing the Credibility Constraint

In this section, the players have access to an enlarged set of possible actions. In addition to offering the contracts described in the previous sections, they can now make an unconditional gift of some productive resources at the beginning of the game. In the case of international relations, or of warlord competition in a collapsed-state society, this can be interpreted as territorial concessions. This is a very common occurrence in contemporary history. For example, in Colombia, the government has surrendered the control of a large chunk of the national territory to the FARC, the insurgent guerrilla movement. In Africa, we also have several examples (Reno, 1998). Charles Taylor in Liberia and Jonas Savimbi in Angola have been left *de facto* in control of large parts of the national territory for long periods of time, without any serious challenge by the government.

At a more theoretical level, the type of initial reallocation of productive resources that are analyzed in this section may be interpreted as the definition of property rights by the two players before the start of the game. Whatever the initial endowment, the two players agree to an equilibrium distribution of productive resources that ensures a peaceful anarchy equilibrium. This provides some eleutheristic foundations for property rights on productive resources, insofar as they are decided by symmetrically positioned players, and are not based either on exogenously given initial endowments, or on a coercive redistribution by any kind of government, or other form of third party.

The Impact of the Re-allocation of Property Rights

Let us focus on case (i) of propositions 3 or 4, where player 1 is the "too rich" one, who must make a positive net transfer in order to avoid fighting. The other case can be analyzed in a symmetric fashion, without yielding any additional insight. Assume now that the credibility constraint (13) does not hold. In the previous section, this would entail that violence would break out, triggered by player 2, with a large negative impact on incomes. In the present section, player 1 has one more possible action, namely to make at the beginning of the game the unconditional gift of some productive resources to player 2. Following the same steps as in the previous section, I first characterize the resulting peaceful equilibrium, before analyzing its existence. Define δ as the quantity of productive resource that player 1 is so giving. Then, denote:

$$\widetilde{N}_1 = N_1 - \delta \qquad (33)$$

as the quantity of productive resources left to player 1 at the second stage, when the game analyzed in the previous section begins.

For the sake of generality, assume that the gift technology might be imperfect, so that some costs are attached to the transfer of property rights from one player to the other one. A simple way to capture this idea is to make the following "melting ice" assumption: if one player parts with δ for making a gift to the other one, the latter can only enjoy the benefit of $\mu\delta, 0 \leq \mu \leq 1$. Therefore, after receiving the gift, his increased stock of productive resources is only:

$$\widetilde{N}_2 = N_2 + \mu\delta\,. \qquad (34)$$

This parameterization is quite convenient for encompassing many different situations, depending on the assumed value of μ. The standard case of a simple transfer of property rights can be described by assuming $\mu = 1$. However, the model can also capture other situations, where some of the productive characteristics of the asset are lost in the transfer. For example, in many societies the contending groups could master specific techniques, and own specific assets, which could not be usefully transferred to the other group. For example, the northerners in most Sahelian countries in Africa own large herds of cattle, while the southerners practice some sedentary agriculture in tse-tse fly ridden areas. This parasite is lethal for cattle, so that the northerners could certainly not come easily and look after cocoa plantations in the south, were the southerners to give them up, unless they could leave their herd behind. If the model was used to analyze the potential conflict between two clans of nomadic agriculturists, then the main productive resource would be family labor. This would be the case for example of those who practice the "slash-and-burn" type of agriculture, with very long fallow periods, in open land areas. Then the rules of the clan may preclude the gift of a son to the potentially opposing group. However, in most pre-industrial societies, the gift of women as brides for the sons of the other groups is a common occurrence. This model can also encompass a very different type of gift, when $\mu = 0$. In pre-historical times, as well as in many primitive societies in the contemporaneous world, people used to destroy productive resources, as a sacrifice. Among the philosophers, Girard (1979) discussed the links between the practice of sacrifice and the control of violence in different societies. Most modern religions refer to a specific past when human sacrifice was abolished. It seems that it is still practiced occasionally in some societies (see Touré and Konaté, 1990). Similarly, in ancient Iran, the Zoroastrian religion was explicitly targeted at eradicating the practice of sacrificing bulls, which could be more productively used as traction animals, and at enhancing simultaneously the virtues of the believers. The present model sheds some light on why the sacrifice of some productive resources might be practiced as an imperfect substitute for the transfer of property rights on them, when the latter is not feasible. The only difference with the previous cases, where $\mu > 0$, is that while the "too rich" player can reduce his own stock of productive resources by this method, there is no simultaneous increase in the other player's stock.

For determining the equilibrium condition for this two-stage game, it is also useful to distinguish $\widetilde{B}_1$ and $\widetilde{B}_2$, which define the breakpoint in the second-stage game, after the redistribution of property rights has been performed, from B_1 and B_2, the payoffs from fighting if player 1 chooses not to give up any resource ex ante to player 2, and goes for a direct fight instead.

If peace is to be secured by the type of gift captured by (33) and (34), the following modified credibility constraint, derived from (13), must hold:

$$\lambda_1 \geq 1 - \frac{\alpha_1 \widetilde{N}_1 + \alpha_2 \widetilde{N}_2}{\xi \alpha_1 \widetilde{N}_1}, \tag{35}$$

where:

$$\xi = \frac{1 + \omega *}{p * + \omega *}. \tag{36}$$

Re-arranging the terms, using (33) and (34), (35) may be written as:

$$\lambda_1 \geq \frac{[\alpha_2 \mu + (\xi - 1)\alpha_1]\widetilde{N}_1 - \alpha_2 (N_2 + \mu N_1)}{\xi \alpha_1 \widetilde{N}_1}. \tag{37}$$

As $\xi > 1$, condition (37) can be seen as putting an upper bound on the acceptable value of $\widetilde{N}_1$ consistent with peace. In case of peace, player 1 will chose the minimum value of the gift δ that ensures that (37) holds, as it affects his income prospect negatively.

Figure 2 helps to understand the determination of the reallocation of property rights on resources that secures peace when player 1's credibility is low. The modified amount of resources owned by player 1 after transferring the required property rights to player 2 is measured on the horizontal axis. It is obviously bounded from above by *N*. His level of credibility is measured on the vertical axis. The upward sloping concave curve represents condition (37). Peace prevails for all the $\{\lambda_1, \widetilde{N}_1\}$ pairs located on or above this curve. Notice that the definition of player 1 being "too rich", the case to which the analysis of this section is restricted, implies that:

$$\frac{\alpha_2\,(\mu N_1 + N_2)}{\alpha_2\,\mu + (\xi - 1)\,\alpha_1} < N_1\,. \tag{38}$$

Violence erupts for the whole set of points located below it. Point A is the assumed initial situation, where player 1 is not credible enough to offer the required transfer for securing peace, with a too small value λ_{1L} of his probability of keeping his word, given his exogenous endowment N_1. Then, the minimum gift required for securing peace can be read off the diagram as the horizontal distance of A to the credibility constraint.

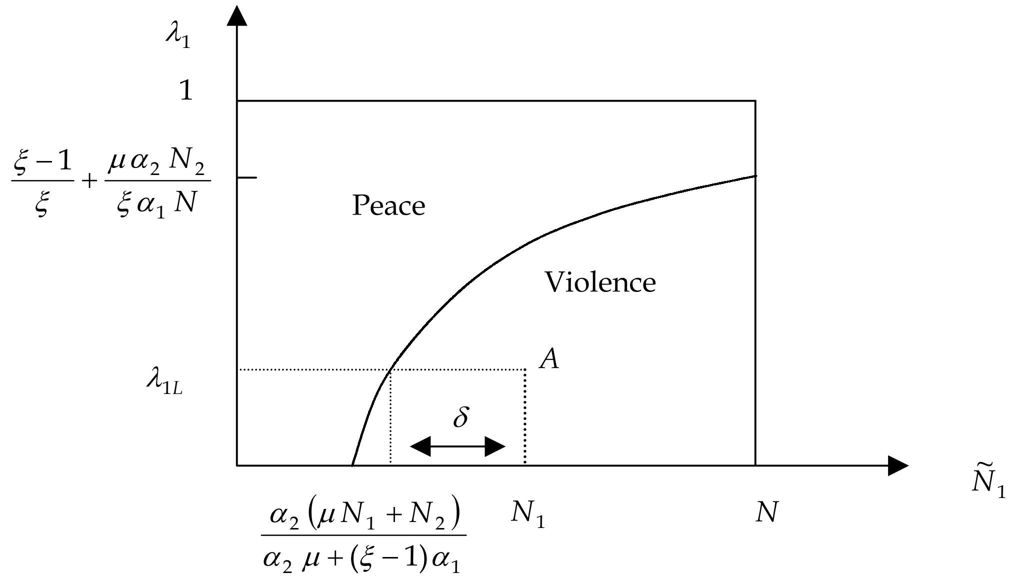

Figure 2: Determination of the Resource Redistribution

Putting (37) to equality, it can be used for computing the value of this minimum gift of resource consistent with peace as:

$$\delta^* = N_1 - \tilde{N}_1 = \frac{[\xi\,(1 - \lambda_1) - 1]\alpha_1\,N_1 - \alpha_2\,N_2}{[\xi\,(1 - \lambda_1) - 1]\alpha_1 + \alpha_2\,\mu}\,. \tag{39}$$

This minimal reallocation of property rights results in a peaceful anarchy equilibrium, of the corner-solution type, which allows the two players to get the following payoffs:

$$U_1 = (1 - \lambda_1)\alpha_1\,\tilde{N}_1 \geq \tilde{B}_1\,, \tag{40}$$

and:

$$U_2 = \alpha_2\,\tilde{N}_2 + \lambda_1\,\alpha_1\,\tilde{N}_1 = \tilde{B}_2\,. \tag{41}$$

Hence, if such an outcome is chosen by player 1, it yields a second-stage peaceful equilibrium which is supported by a one-sided expected gift from player 1 to player 2. As discussed above, this is only the limiting case of a gift-exchange equilibrium where the gift from player 2 tends to zero.

Peaceful Anarchy with Eleutheristic Property Rights

What remains to be proved is that the redistribution of property rights described above results in an equilibrium. The "too rich" player 1 must compare the loss of output entailed by reducing his initial stock of productive resources against the cost of a fight, including that involved in diverting productive resources for that. This is dealt with in proposition 5 below.

Proposition 5: Peaceful anarchy is preferred to the eruption of violence, even if it requires an initial redistribution of property rights, if and only if:

$$\mu \geq \frac{\alpha_1}{\alpha_2}\left(1-\frac{(1-\lambda_1)\omega^*(\alpha_1 N_1+\alpha_2 N_2)}{p^*(p^*+\omega^*)((1-\lambda_1)\alpha_1 N_1 - B_1)}\right). \tag{42}$$

Proof: From (39), (3), and (6), it is easily derived that:

$$\tilde{N}_1 = \frac{p^*(\alpha_2\,\mu-\alpha_1)N_1+(1+\omega^*)B_1}{p^*([\xi(1-\lambda_1)-1]\alpha_1+\alpha_2\,\mu)}. \tag{43}$$

Substituting this back in (40) allows to write:

$$U_1 = \frac{(\alpha_2\,\mu-\alpha_1)(1-\lambda_1)\alpha_1 N_1+(1-\lambda_1)\alpha_1 B_1(1+\omega^*)/p^*}{\xi(1-\lambda_1)\alpha_1+\alpha_2\,\mu-\alpha_1}. \tag{44}$$

It is easily checked that this is larger than B_1 if (42) holds. **QED**

Notice first that, because player 1 is by assumption the "too rich" one, we have: $\xi(1-\lambda_1)>1$ and $(1-\lambda_1)\alpha_1 N_1 > B_1$, which is useful for signing the different terms above. Condition (42) is only relevant if its right-hand side is positive. Otherwise, violence can never occur. It decomposes the determinants of the lower bound on μ as the product of two terms. The one on the left-hand side captures a relative productivity effect: the efficiency of the property right transfer must be higher, the less productive is the beneficiary (unless it is irrelevant). Then a large transfer is required to have even a small redistributive effect, and this reduces the attractiveness of such a move. The term on the right-hand side is decreasing in the decisiveness parameter, via ω^*: the larger the latter, the larger the resource cost which can be avoided by opting for peace rather than war. It is increasing in $(1-\lambda_1)\alpha_1 N_1 - B_1$, which is a measure of the extent by which player 1 is "too rich", i.e. a measure of the "sacrifice" that he must make to secure peace. Hence, this right-hand term is a composite index of the cost and benefit of securing peace. A striking aspect of proposition 5 is that there are cases where peace is always preferred to violence, irrespective of the value of μ, even after taking due account of the required redistribution of property rights, if the right-hand side of (42) is negative. Even the mere destruction of productive resources, i.e. performing an old fashioned sacrifice, is enough for ensuring peace in such a case. This is partly due to the fact that the destruction of productive resources reduces both the need to transfer income, with the credibility problem that this raises, and the potential benefit of violence, for both players. An interesting analogy can be made between the beneficial role of the sacrifice analyzed here and the fact that people in Grossman (2002) prefer paying taxes for feeding the ruling elite to the state of violent anarchy. However, this should not be pushed too far, as Grossman's king provides a public good by ensuring that predation is deterred. Moreover, the sacrifice solution is ruled out if the right-hand side of (42) is strictly positive. Then, a too low value of μ might preclude the emergence of a peaceful anarchy equilibrium, and make fighting unavoidable.

Therefore, this section establishes in fact a variant of the Coase theorem, under the threat of fighting. The latter creates a special type of externality, described in the previous section. Proposition 5 states that if the players can re-allocate their property rights among themselves to a large enough extent before the beginning of the game, violence can be averted in a state of anarchy, even if the contenders have a limited level of credibility in their commitment to deliver a transfer later in the game. If the latter is too small for the "too rich" player to be able to promise credibly a transfer in return for not engaging in a fight, then he will give up unconditionally some productive resources before the beginning of the game, rather than run the risk of war. This section thus provides some foundation for an eleutheristic allocation of property rights, which is freely decided by unreliable equals, for securing peace. It comes closer to a true state of anarchy than the original setting used by Hirshleifer (1995), as the players thus get rid of the influence of the implicit third party involved in assuming an exogenous initial allocation of resources. Then, peace prevails, as established in proposition 5.

As a corollary, the result of the present section points in the direction of the imperfections of property rights as the root cause of violence. The crucial point is whether or not property rights on productive resources can be given unconditionally by the "too rich" player to the "too poor" one. In some cases, where the gift of some resource by player 1 would not benefit at all player 2 , then even the mere destruction of these assets is enough to secure peace. However, it is clear that a higher value of μ is preferable from a social point of view, as the loss incurred by the "too rich" player is then compensated by the gain of the "too poor" one, at least to some extent. Proposition 5 shows that this variant of the Coase theorem holds even when there are substantial transactions costs, here captured by the "melting ice" assumption, whereby a part of the resources given up by player 1 does not benefit player 2. However, there might be a upper limit on the level of these transactions costs beyond which violence is the preferred equilibrium.

The next section extends the model to the case of n > 2 players, with a view to characterize further the links between anarchy, commitment, and peace. This is done after making several simplifying assumptions.

5. Extension of the Peaceful Gift Exchange Model to n > 2 Players

For the sake of tractability, a number of simplifying assumptions are made. First, there is a number n of players, which are identical in all respects except for credibility. They have the same initial endowment N, and the same productivity, normalized to 1. As in Skaperdas's model of warlord competition (Skaperdas, 2002), the players are located on a circle, and can only interact with their neighbors, be it for fighting or for bargaining. The analysis of the fighting equilibrium presented below differs from Skaperdas's by the assumed random matching process. The players are indexed by $i \in \{1,...,n\}$, with the convention that $n+i=i$. The players' expected payoff in case of fighting is determined first, before looking at the condition under which a gift-exchange equilibrium can secure peace.

Fighting

The fighting technology is as follows. When the players have spared some resources for fighting, they have to meet for doing so. Ex ante co-ordination is ruled out, and a random matching process is assumed instead. It works as follows. (i) With probability π, player *i* moves towards his higher-numbered neighbor $i+1$, which he meets with probability $1-\pi$, i.e. if the latter has not moved in the same direction, towards his own higher-numbered neighbor. If such a match occurs, with probability $\pi(1-\pi)$, the two players fight, and *i* wins with probability $p(F_i/F_{i+1})$, which is an increasing and concave function as assumed in the previous sections. He then captures the whole output of the two players $N_i + N_{i+1} - F_i - F_{i+1}$. Otherwise, he looses everything, and consumes nothing. With the complementary probability π, he does not meet anybody for fighting, and he keeps his remaining income $N - F_i$, while the game stops here for him. (ii) With probability $1-\pi$, he moves towards his lower-numbered neighbor, which he meets with probability π, i.e. if the latter has moved towards his own higher-numbered neighbor. If the match occurs, *i* wins with probability $1-p(F_{i-1}/F_i)$. He then captures the total output owned by the two players together i.e. $N_{i-1} + N_i - (F_{i-1} + F_i)$. Otherwise, he looses everything, and the fighting ends. The indices used in this paragraph are meant to help the reader grasp the assumed matching process, and some of them can in fact be dropped, as the symmetry assumption announced above entails that $N_{i-1} = N_i = N_{i+1} = N, \forall i$.

Given this fighting technology, and the symmetry of the problem, one can prove the following proposition, where $p'(1)$ denotes the derivative of $p(.)$ when the two sides engage the same level of forces in the fight.

Proposition 6: In case of violent anarchy, the symmetric Nash equilibrium is such that:

$$F_i = F^* = \frac{\rho^* N}{1+\rho^*}, \forall i, \tag{45}$$

and:

$$B_i = B^* = \frac{N}{1+\rho^*}, \forall i, \tag{46}$$

where:

$$\rho^* = 4\pi(1-\pi)p'(1) \geq 0, \forall i. \tag{47}$$

Proof: Given the matching process described above, each player can only fight, at most, with one of his neighbors. Therefore, agent *i*'s expected gain in case of violent anarchy is:

$$B_i = \max_{F_i} \pi\left((1-\pi)p\left(\frac{F_i}{F_{i+1}}\right)Z_{i,1} + \pi(N-F_i)\right) + (1-\pi)\left(\pi\left(1-p\left(\frac{F_{i-1}}{F_i}\right)\right)Z_{i,2} + (1-\pi)(N-F_i)\right), \tag{48}$$

where:

$$Z_{i,1} = 2N - F_i - F_{i+1}, \tag{49}$$

and:

$$Z_{i,2} = 2N - F_{i-1} - F_i. \tag{50}$$

Therefore, the first-order condition may be written as:

$$F_i = \pi(1-\pi)\left(p'\left(\frac{F_i}{F_{i+1}}\right)\frac{F_i}{F_{i+1}}Z_{i,1} + p'\left(\frac{F_{i-1}}{F_i}\right)\frac{F_{i-1}}{F_i}Z_{i,2}\right). \tag{51}$$

Then, the symmetric Nash equilibrium requires:

$$F_{i-1} = F_i = F_{i+1} = F^*. \tag{52}$$

Substituting from (52) in (51), and rearranging the terms, yields (45), and (46), using the definition of $B^* = N - F^*$. **QED**

Notice by looking at (47) that ρ^* is an increasing function of $p'(1)$, which is in turn directly related to Hirshleifer's "decisiveness parameter". Hence, (45) entails that each player engages more resources in fighting, the larger is the latter parameter. It also follows from (46) that the expected payoff from the fight is lower, the more decisive are the forces engaged in fighting. In other words, this model reproduces the familiar result already discussed in section 2 above.

Equation (46) in proposition 6 provides the outside option for the bargaining in which each player engages with his two neighbors.

Bargaining

Assume that each agent bargains in turn with his two neighbors, taking as given the gifts that he gets from- and gives to his other neighbor. Denote g_i^+ the gift from *i* to his higher-numbered neighbor, and g_i^- his gift to his lower-numbered one. For the sake of keeping the symmetry of the problem, all the players are assumed to have the same bargaining power, as in the original Nash bargaining solution. Then, for every *i*, the exchange of gifts must maximize:

$$\left(N-\left(\lambda_i g_i^- - \lambda_{i-1} g_{i-1}^+\right)-\left(\lambda_i g_i^+ - \lambda_{i+1} g_{i+1}^-\right)-B^*\right)\left(N-\left(\lambda_{i+1} g_{i+1}^- - \lambda_i g_i^+\right)-\left(\lambda_{i+1} g_{i+1}^+ - \lambda_{i+2} g_{i+2}^-\right)-B^*\right) \quad (53)$$

Now, assume that the distribution of λ_i is such that the latter is bounded from below, with $\lambda_i \geq \lambda_L, \forall i$. Then we can prove the following proposition:

Proposition 7: There exists a peaceful anarchy equilibrium with:

$$g_i^+ = \frac{\upsilon^+}{\lambda_i} \text{ and } g_i^- = \frac{\upsilon^-}{\lambda_i}, \upsilon^+ > 0, \upsilon^- > 0, \forall i \in \{1, ..., n\}, \quad (54)$$

if:

$$\lambda_L \geq \frac{\upsilon^+ + \upsilon^-}{N}. \quad (55)$$

Proof: The solution to the maximization of (53) must solve:

$$\left(\lambda_i\, g_i^- - \lambda_{i-1}\, g_{i-1}^+\right)+\left(\lambda_i\, g_i^+ - \lambda_{i+1}\, g_{i+1}^-\right)=\left(\lambda_{i+1}\, g_{i+1}^- - \lambda_i\, g_i^+\right)+\left(\lambda_{i+1}\, g_{i+1}^+ - \lambda_{i+2}\, g_{i+2}^-\right). \quad (56)$$

By substituting from (54) into (56) for all *i*, one can check that the latter holds. Moreover, if (55) holds, then the following inequality holds:

$$N \geq \frac{\upsilon^+ + \upsilon^-}{\lambda_L} \geq g_i^+ + g_i^- \quad (57)$$

for all *i*. It follows that the budget constraint holds for each player. **QED**

Notice that (55) is a sufficient condition, and not a necessary one. The main point for it is its simplicity. However, because υ^{+} and υ^{-} are largely arbitrary, subject to the non-negativity constraint, then (55) does not seem very stringent. It says that, in the simple setting analyzed here, a simple gift-exchange equilibrium can secure peace in a state of anarchy provided the least credible of the players is credible enough to promise transfers with a positive expected value. The value of the transfers is largely arbitrary, and can be chosen so as to accommodate even very low values of this minimal level of credibility.

6. Conclusion

The model discussed above reconciles anarchy and peace. In this framework, unarmed peace prevails without the intervention of a government, under the condition that people have access to a commitment technology, which makes their promises to deliver a transfer credible enough, given the distribution of productive resources. In some cases, when credibility is too low to ensure that peace prevails among equals, the gift of some productive resource before production takes place can help to relax the credibility constraint, and secure peace. Then, the anarchy equilibrium can secure peace, provided the transfer of property rights over some productive resources can be performed effectively, even if this implies some cost. The resulting distribution of property rights among equals is called eleutheristic, as it results from the correction of the exogenous initial allocation of endowments by symmetrically positioned players, who are equal in formal freedom. Even the sacrifice of some of these resources, i.e. their destruction by one player without any benefit for the other player, may be sufficient for securing peace in this model, in some cases. Hence, the latter predicts that the absence of a government triggers the eruption of violence only if the players cannot commit credibly enough to transfer some income, and are not able to transfer effectively enough property rights over productive resources. This approach revives a tradition that dates back at least to Thucydides, in ancient Greece, who blamed the eruption of the Peloponnesian war on the emergence of a condition of civil strife, which he called *stasis*. The latter refers to a set of symptoms indicating an internal disturbance in both individuals and states. He emphasizes the overall disregard for genuinely Hellenic customs, codes of morality, and civic loyalty (Price, 2001).

While the analysis presented here is formally conducted in terms of individual players, it can easily be interpreted as a metaphor that can be used in different contexts. In particular, it seems appropriate in the field of international relations, with a view to explain why independent countries can co-exist without being at war with their neighbors all the time, despite the absence of a powerful third party. Then, the roots of the ability to commit credibly to deliver some transfers need to be sought not only in the moral background of the rulers, but more importantly in the institutional frameworks that restrain their powers in their own countries, as suggested by the reference to Kant given in the introduction. In this field, the word "transfer" must also be understood as a metaphor, and its meaning should be extended to encompass many other types of concessions or agreements between states, which potentially impose a cost on one of the parties and a benefit on the other one. Beside aid flows and other forms of transfers, this may include preferential trade agreements (e.g. the E.U.), privileged mining rights, fishing rights, etc. In the real world, we observe a much larger number of civil wars than of international wars, at least since World War 2 (Gleditsch *et al.*, 2002). A potential interpretation of this fact is that the Lockean horizontal social contract analyzed here, among symmetrically positioned states, is more effective at securing peace than the Hobbesian vertical social contract, between a government and its people. May be governments are more credible in their promises to one another, than they are to their people. This shows a potentially fruitful avenue for further research, aiming at identifying the mechanisms that enhance the credibility of governments in their dealings with one another, on the one hand, and with their people, on the other hand.

This model has some limitations, which show the way to further research. In particular, war takes place after production in this model. This limits the catch from war to the value of output. In a dynamic framework, wars of conquest, whereby one side could capture some of the productive resources of the other side, with a view to get more output later, would have to be taken into account in the analysis. A link with the theory of empire building, as explored by Findlay (1996), thus seems to be a worthwhile avenue to explore.

Similarly, the extension to the case of $n > 2$ players is here performed in a very simplified setting, where the players are located along a circle, and can only fight or bargain with their neighbors. The players are identical in all respects, except for the credibility of their promises. A more general setting would have to allow for more complex interactions, involving possibly coalitions of several players, as well as to allow for other differences among players. This falls clearly outside the scope of the present paper, and has to be left for future research.

References

Akerlof, G. A. (1982): "Labor Contracts as Partial Gift Exchange", *Quarterly Journal of Economics* 97 (November), 543-569.

Akerlof; G. A. (1983): "Loyalty Filters", *American Economic Review* 73 (March), 54-63.

Anbarci, N., S. Skaperdas and C. Syropoulos (2002): "Comparing Bargaining Solutions in the Shadow of Conflict: How Norms against Threat Can Have Real Effects", *Journal of Economic Theory*, 106 (1), 1-16.

Azam, J.-P. (1995): "How to Pay for the Peace ? A Theoretical Framework with References to African Countries." *Public Choice* 83 (April), 173-84.

Azam J.-P. (2001): "The Redistributive State and Conflicts in Africa." *Journal of Peace Research* 38 (July) , 429-44.

Azam, J.-P. (2002): "Looting and Conflict between Ethno-Regional Groups : Lessons for State Formation in Africa." *Journal of Conflict Resolution*, 46 (1) (February), 131-153.

Barbieri, K. and G. Schneider (1999): "Globalization and Peace: Assessing New Directions in the Study of Trade and Conflict", *Journal of Peace Research* 36 (4), 387-404.

Barro, R. J. and R. M. McCleary (2002): "Religion and Political Economy in an International Panel", Unpublished, Harvard University: Cambridge.

Basu, K. (2000): *Prelude to Political Economy*, Oxford University Press: Oxford.

Charlier, C. (1996): "The Notion of Prudence in Smith's *Theory of Moral Sentiments*", *History of Economic Ideas* 4 (1-2), 271-297.

Derrida, J. (1995): *The Gift of Death* (translated by D. Wills), The University of Chicago Press: Chicago and London.

Ekelund, R. B., R. F. Hébert and R. D. Tollison (2002): "An Economic Analysis of the Protestant Reformation", *Journal of Political Economy* 110, 646-671.

Esteban, J. and D. Ray (1999): "Conflict and Distribution", *Journal of Economic Theory*, 87 (2), 379-415.

Findlay, R. (1996): "Towards a Model of Territorial Expansion and the Limits of Empire", in M. R. Garfinkel and S. Skaperdas (eds.): *The Political Economy of Conflict and Appropriation*, 41-56, Cambridge University Press: Cambridge.

Girard, R. (1979): *Violence and the Sacred* (Translated by P. Gregory), Johns Hopkins University Press: Baltimore.

Gleditsch, N. P., P. Wallensteen, M. Eriksson, M. Sollenberg and H. Strand (2002): "Armed Conflict 1946-2000: A New Dataset", *Journal of Peace Research* 39 (5) (September).

Grossman H.I. (1991): "A General Equilibrium Model of Insurrections." *American Economic Review* 81 (September), 912-21.

Grossman, H. I. (2002): ""Make us a King": Anarchy, Predation, and the State." *European Journal of Political Economy* 18, 31-46.

Grossman H.I., and Kim, M. (1995): "Swords or Plowshares ? A Theory of the Security of Claims to Property", *Journal of Political Economy* 103 (December), 1275-88.

Hirshleifer, J. (1991): "The Paradox of Power", *Economics and Politics* 3, 177-200.

Hirshleifer, J. (1995): "Anarchy and Its Breakdown", *Journal of Political Economy* 103, 26-52.

Kalai, E. and M. Smorodinsky (1975): "Other Solutions to Nash's Bargaining Problem", *Econometrica* 43, 513-518.

Kant, I. (1983): *Perpetual Peace, and Other Essays on Politics, History and Morals* (Edited by T. Humphreys) (Original 1795), Hackett: New York.

Kolm, S.-C. (1996): *Modern Theories of Justice*, MIT Press: Cambridge.

Laffont, J.- J. (1975): "Macroeconomic Constraints, Economic Efficiency and Ethics: An Introduction to Kantian Economics", *Economica* 42, 430-437.

Neary H.M. (1997): "A Comparison of Rent-Seeking Models and Economic Models of Conflict", *Public Choice* 93 (2), 373-88.

Nietszche, F. (1999): *On the Genealogy of Morals* (translated by D. Smith) (Original 1887), Oxford World's Classics, Oxford University Press: Oxford.

North, D.C., Weingast. B.R. (1989): "Constitutions and Commitment: The Evolution of Institutions Governing Public Choice in Seventeenth-Century England", *Journal of Economic History* 69, 803-32.

Price, J. J. (2001): *Thucydides and Internal War*, Cambridge University Press: Cambridge.

Reno, W. (1998): *Warlord Politics and African States*, Lynne Rienner Publishers: Boulder and London.

Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model", *Econometrica* 50, 207-211.

Sen, A. K. (1976): "Rational Fools: A Critique of the Behavioral Foundations of Economic Theory", *Philosophy and Public Affairs* 6, 317-344.

Skaperdas S. (1992): "Cooperation, Conflict, and Power in the Absence of Property Rights." *American Economic Review* 82 (September), 720-39.

Skaperdas, S. (1996): "Contest Success Functions", *Economic Theory* 7, 283-90.

Skaperdas, S. (2002): "Warlord Competition", *Journal of Peace Research* 39 (4), 435-446.

Touré, A. and Y. Konaté (1990): *Sacrifices dans la ville*, Editions Douga: Abidjan.