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Essays on Public Contracts

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Introduction

Economic interactions involving the public sector have unique characteristics. While private firms maximize profits operating in a well-defined exogenous framework, the scope of governments' economic actions and objectives is much wider. Even when one restricts the analysis to a benevolent government, welfare maximizing encompasses considering externalities or effects in parties not directly involved in a particular transaction. In addition, governments are powerful entities and can sometimes define their own rules, which give them more occasions to behave opportunistically.

This thesis is composed by three chapters, each one addressing a policy question originated in the complex nature of public sector's objective function. In the first chapter, a policymaker intends to auction divisible goods among large and small bidders. Instead of being concerned only about maximizing the surplus of auction participants, he also wants to avoid having concentrated allocations. In the second chapter, I investigate the decision of providing incentives to public workers, when the government is concerned not only about its own efficiency, but also on how information rents in the public sector may affect the allocation of talents in the entire economy. Finally, the last chapter analyzes the problem of attracting investments in the natural resources sector when the government is privately informed about his ability to respect contracts.

In the first essay, the conflict of objectives of the policymaker is inspired in the framework regulating the process of auctioning spectrum. If the party selling the spectrum was a private firm, his objective would likely be to maximize the proceeds from the sale. In contrast, a regulator may rather prefer to maximize welfare, which includes not only the surplus of the auction participants but also the effects the spectrum allocation in consumers' surplus. For this reason, telecommunication's regulators may face a tension between assigning spectrum to those more willing to pay for it, and ending up with high concentration in the allocation of the spectrum, which might be against the benefit of consumers.[1] This conflict of objectives is explicitly pointed out by regulators as the foundation for using favoring devices to handicap large bidders, such as reserved quantities (set-asides or spectrum caps) or price discounts (biding credits). Although my work confirms that favoring small bidders can be an effective way of balancing both objectives when the policymaker is constrained to using standard auction mechanisms (Vickrey or uniform price), the analysis suggests that there are other dimensions that can contribute to the problem. Notably, I show that the auction format affects the way in which favoring devices should be used, and that each format adapts better to the goals of the policymaker depending on how he weights each of the objectives. The essay takes as departing point the fact that, under uniform pricing, bidders enjoy market power and have incentives to withhold demand. In order to avoid driving up their own price, bidders reduce the quantities that they demand. Larger bidders enjoy greater market power, and thus they have more incentives to withhold demand. In a Vickrey auction, bidders act as price takers and thus these incentives are absent. Hence, under uniform prices, allocated quantities are biased against large bidder with respect to Vickrey allocations. For this reason, the format choice is also affecting the objectives of the auctioneer in a way that can interact with favoring devices. A first conclusion of this chapter is that the choice of favoring devices is not independent of the format choice.

My first essay is the first work analyzing the interaction between favoring devices and auction formats. After establishing this interaction, I investigate whether there are grounds for the policymaker to choose among different "auction policies", *i.e.*, combinations of auction formats and

[1]This view on the objective of the policymaker is consistent with the aims of the Communication Act ruling the design of systems of competitive bidding for Spectrum in the US. Section 309(j) (3) of this Act describes the objectives of the design as "promoting the efficient and Intensive use of the Spectrum", but also "avoiding excessive concentration of licenses". Also, the UK regulator of Communications, OFCOM, has taken measures for encouraging entry and "(...) to guard against the risk of future competition problems arising from very asymmetric distributions of mobile spectrum". See OFCOM (2011).

favoring devices. I show that, in fact, for each weight that the policymaker assigns to surplus maximization and fairness of the outcome, auction policies can be ranked. Hence, the auction policy choice is always determined. If one analyzes actual auction policies in the light of my results, one can infer that outcome concentration is largely penalized by regulators. The use of uniform pricing together with quantity restrictions dominates among regulators' choices. In accordance with my results, this auction policy is only preferable when the concentration concern of the policymaker is sufficiently high.

Given that the conflict of objectives reveals to be a major concern for regulators, the first essay goes one step further and questions whether traditional auction policies are the best way of dealing with the conflict. By definition, group favoring can only affect intergroup allocations, but is not effective to control for unexpected size differences within groups. In contrast, my work highlights the good properties of exploiting bidders' market power within the auction, as a way of providing incentives to reduce demand to bidders according to their actual size, without the need of relying just in group prior information. Unfortunately, both with Vickrey or uniform pricing, the market power is exogenously given, and cannot adapt to different objective functions of the policymaker. My work provides a simple solution to this issue, and shows that an auction with transfers computed as a weighted average of each format allows the policymaker to control bidders' incentives to withhold demand and achieve first-best allocations.

The second essay of my thesis looks at the labor incentive policies that arise in the public and the private sector. The work is interested in answering the question: should the public and the private sector adopt the same incentive policies? Although there is a wide view on that the public sector tends to adopt lower powered policies, the reasons for this are not generally clear. The challenging part of answering this question is to construct a framework defining what would constitute equal incentive policies, since the private and the public sector produce different things and have different objectives. I tackle this issue by taking the structure of the most basic incentive problem: a worker whose probability of having an output y can be improved from $\underline{v}$ to $\bar{v}$ if he makes a hidden effort with private cost c. Assuming that the the three parameters $\bar{v}$, $\underline{v}$ and c are equal across sectors, the way we propose to compare the benefits from incentives is to find the threshold level of y for which incentives are profitable in each sector.

Textbook analysis of a moral hazard problem in a partial equilibrium framework shows that, for profit maximizing firms, the cost of incentives is the expected wage increase needed to make the worker's incentive constraint bind, *i.e.* the information rent.[2] However, what is the cost of providing information rents in a public sector run by a government whose objective is maximizing welfare? Answering this question from a partial equilibrium perspective is pointless. Funding a wage increase is about charging higher taxes, redistributing disposable rent. Understanding this cost is only possible if looking at the effect of the redistribution from a general equilibrium point of view. There are a number of ways in which this redistribution can be taken into account: tax distortions, pure redistributional aspects, differential weighting, allocations of talents. The usual way in which this problem has being dealt with is to take a partial equilibrium approach, and assume tax distortions or differential weighting.[3] Instead, I follow Acemoglu and Verdier (1998), and I consider the general equilibrium effects of information rents as the source of incentive costs. In line with these authors, I assume that innovation takes place in the private sector, and thus the agents' innovation talent does not affect the production of public goods. However, regardless of the sector in which a worker is employed, he can make a hidden effort of private cost c to improve the probabilities of having a high output from in $\triangle v$.

Ideally, a government would like to give incentives to their workers and also let the most innovative agents be allocated to the private sector. However, in order to achieve the first of these objectives, it needs to match the incentive wages (or efficiency wages) of the private sector, potentially attracting workers who are more productive in the private sector. This can be very

[2]Information rent in this chapter arises from limited liability and risk neutrality.

[3]For example, the classic work on procurement by Laffont and Tirole (1986) assumes that each dollar spent by the government costs society $1+\lambda$, with λ being an exogenous deadweight loss originated in tax distortions. Although for many purposes an exogenous cost does not affect the conclusions of the work, understanding how this cost is originated is crucial for my analysis.

costly in terms of welfare. Not only do highly innovative workers may end up allocated to jobs in which their innovation does not matter, but also they may force low talented agents to take positions in the private sector. In that case, it is likely that these agents will be less productive, firms will have fewer reasons for incentivizing them, and their low outside opportunity increases the size of the information rent needed to make them do the hidden effort. In fact, my work suggests that the government would like to avoid this second effect by enlarging its size: rather than pushing a low talent worker to the private sector, it would chose to employ the high and the low talent workers together. This phenomenon drives the government into a sort of inefficiency curse: either its workers do not make effort, or the government is excessively large.

The bottom-line of this analysis is that the decision of providing incentives by the government can potentially have large undesired general equilibrium costs. But most importantly, there is no reason a priori for thinking that the public sector can benefit from adopting the incentive policies of the private sector. This essay shows that the threshold level of output y that justifies using incentive contracts is sector-specific and can be higher or lower in either sector.

Finally, the third essay deals with the problem of contracting and attracting investments in natural resources industries in developing countries. The particular interest in this sector arises from the unique hazards for contracting in this framework. Except for the case of the largest OPEC countries, the price of crude oil is exogenous, and recent history has proved that it can be highly volatile and unpredictable. Developing countries often do not have the technological means to undertake oil projects by themselves, and can only benefit from their resources by delegating them to foreign investors. However, not only the prices are volatile, but also local political conditions, having governments that can go from one extreme to the other in the political compass. Although pro-business governments may respect contracts in a similar way than developed nations, nationalist governments may not be willing to let foreign investors cash huge profits in favorable price realizations. I capture this notion by assuming that the investor deals with a government that can be of two different types, each one facing a different cost of expropriation. The interest of the problem lies in that only one of the types is willing to respect a contract that allows the investors recover their sunk cost. However, the other type can also benefit from the transaction by pretending that he has a high level of commitment.

A negative result of the third essay is that governments don't have enough contracting elements to separate their offers in this situation, and signaling cannot arise in equilibrium. This necessarily introduces an additional risk for the investor, since he expects to be expropriated at least in some price realizations. These risks lead investors to apply high discounts to potential cash-flows in developing countries. However, my work emphasizes that, even though the need of an expropriation premium may be unavoidable, much can be done in order to minimize its size. A positive result of this essay is that contingent taxation can be used to reduce the probability of expropriation by making the contract credible for either type for as many states of nature as possible. The optimal contract resembles a debt contract with a "bonus" that is only paid when prices are sufficiently high. For most price realizations, the standard debt contract applies and is honored by both types of governments. Expropriation only occurs in price realizations in which the bonus is paid and the government is of the non credible type. I show that this contract has good properties in terms of the robustness of the out of equilibrium beliefs that support the equilibrium.

Finally, in the light of my results, I discuss contractual strategies to attract investors in the sector in a developing country. My main conclusion is that a contract that may be the best choice in a politically stable country is not a good choice in an unstable one. A same contract that may be judged as attractive for investors in a developed country may be highly discounted in a developing country for not being credible. A developing country is better off by exploiting the contract design in a way that reduces the impact of its political instability.

Chapter 1

The Fair Side of Uniform Price Auctions

1.1 Introduction

In practice, auctions often have specific devices to favor small bidders. Either for distributional or ex-post market structure concerns, the objective of the auction designer may exceed simple bidders' surplus or revenue maximization. In a context of multiple or divisible goods, this issue is especially interesting, since some bidders can be favored without the need of excluding others completely. Discussions on how to favor small bidders are generally focused on the specific devices, and more importantly, abstract from the auction format. In this work, we offer a new perspective to this discussion, and exploit the interactions between these devices and the market power that each auction format gives to bidders. Our results show that, even when restricting attention to Vickrey and uniform price auctions, the format choice can have a more relevant role to play than the favoring devices in case bidders enjoy sufficient market power (in the auction).

We model the sale of shares of goods or rights among large and small bidders. Both groups of bidders have quadratic utility functions over the total amount of shares purchased, but due to differences in valuation parameters, large bidders are willing to buy, in expectation, a greater share than small bidders for an equal price. Bidders compete by submitting demand functions according to the rules set by the auction designer or policymaker. The policymaker's objective is to maximize a function that weights total surplus but penalizes the concentration in the outcome. This view on the objective of the policymaker is consistent with the aims of the US Communication Act ruling the design of systems of competitive bidding for spectrum in the US. Section 309(j) (3) of this Act describes the objectives of the design as "promoting the efficient and intensive use of the spectrum", but also "avoiding excessive concentration of licenses".[1] Hence, the policy designer has some degree of conflict in his objective: on the one hand, he has a preference for egalitarian outcomes, but on the other one, he would like to allocate more goods to those who have a higher valuation. In order to maximize his objective function, the policymaker can use "favoring devices". Either he can choose to offer a fixed discount per unit to small bidders (bidding credits), or he can organize separate auctions for small and large bidders (set-asides). On top of deciding which and what amount of devices to use, he can choose among two auction formats: a uniform price and Vickrey auction, in the spirit of Vickrey (1961). In order to simplify our exposition, we call an "auction policy" the combination of an auction format and a favoring device.

This paper has two main objectives. The first one is to compare the results in terms of welfare of the commonly used auction policies. Second, it investigates if these or any other policies can implement the auction designer's objective optimally.

Our results show that the auction policies can be ranked in terms of welfare, and that the choice of optimal favoring devices is not independent of the auction format. Different auction policies adapt better to the weight that the auction designer assigns to surplus maximization and outcome equality. Unfortunately, our results also show that, even though favoring small bidders can improve expected welfare, it is generally not enough as a tool to allocate first-best quantities. In contrast, this work highlights the role of bidder's market power in discouraging large bidders to purchase large quantities. We propose an auction format that has as special cases a Vickrey and

[1] Similarly, in the United Kingdom, the Wireless Telegraphy Act of 1998 describes the matters to be taken into account in granting licenses as "the efficient use of electro-magnetic spectrum" as well as "promoting competition in the provision of telecommunication services".

a uniform price auction, which gives the policy designer flexibility on the market power he wants bidders to have in the auction. We prove that this format can implement the first-best allocations without the need of favoring specific groups of bidders.

This work is motivated in the recent experience of spectrum auctions. Spectrum auctions in Europe, such as 4G auctions in Switzerland, Netherlands and Austria adopted Vickrey pricing rules and quantity restrictions as auction policy.[2] In the US, the upcoming incentive auction is planning to adopt uniform prices and bidding credits have been recently adopted in the AWS-3 auction.[3] As a leitmotif, regulators treated the auction format and the favoring device as separate issues.[4] One of the main messages of this paper is that such decisions cannot be taken by separate, since the auction format by itself has an effect on the relative quantities allocated to small and large bidders and in the optimal intensity of use of favoring devices.

Our work builds on the well-known result of demand reduction equilibria in multi-unit uniform price auctions (Ausubel et al. (2014)). In a uniform price auction, bidders demanding large quantities have incentives to withhold demand in order to avoid driving up their own prices. Traditional analysis view these equilibria as potentially harmful for welfare as they can yield different allocations than the optimal theoretical benchmark from a Vickrey auction. In a Vickrey auction, bidders act as price takers, and thus their demands equal price to marginal utilities. In contrast, in our work, the allocation differences between a uniform price and a Vickrey auction can prove to be beneficial rather than harmful for welfare, in case the policymaker has sufficient distributional or equality concerns. A fundamental result of this paper is that uniform prices provide a natural bias against outcome concentration but yet assign more quantities to those valuing it the

[2]The Vickrey pricing rule adopted contained also core-adjustment mechanisms. The presence of such mechanism is irrelevant in our context, since all allocations studied here lie in the core.

[3]In the US the FCC has sometimes used both reserved spectrum and bidding credits at the same time. While reserved spectrum benefits small and medium bidders, bidding credits are only used to favor the smallest bidders. Our analysis considers only two class of bidders, and for this reason, using both mechanisms at the same time is not fruitful. However, the FCC has consistently adopted uniform pricing rules, if we understand clock and SMR auctions as different ascending versions of a uniform price auction. See Cramton (2002).

[4]Cramton et al. (2011) discuss methods for computing bidding credits without mentioning the auction format. Jehiel and Moldovanu (2003) also describe favoring devices without any connection to the format in which they are employed. For a description of the main features of these auctions, see Salant (2014).

most, which can be beneficial in settings where very asymmetric outcomes are not desirable. A high valuation bidder may refrain from demanding large quantities because every additional unit demanded drives up the price of its entire demand, and not just the marginal unit. Small bidders are less sensitive to this phenomenon (they have less market power), and hence the uniform price auction gives small bidders a natural advantage.

The fact that uniform price auction favors small bidders has been pointed out in numerous occasions (e.g. Cramton, 2003). Notably, in the non divisible 2-units case, bid shading appears only in the demand for the second unit (Noussair, 1995; Engelbrecht-Wiggans and Kahn, 1998). Perhaps the clearest formulation of this notion was presented in Ausubel et al. (2014):

> "Uniform pricing has several desirable properties, including: (i) it is easily understood in both static and dynamic forms; (ii) it is fair in the sense that the same price is paid by everyone; (iii) absent market power it is efficient and strategically simple ("you just bid what you think it's worth"); and (iv) the exercise of market power under uniform pricing favours smaller bidders. While the first three points are commonly made in practice, it is the fourth point that may decisively favour uniform pricing in many practical settings, including some spectrum and electricity markets".

Even though uniform pricing naturally has outcome equalizing forces, the demand reduction that it generates can be too large or too small. On the auction format choice side, a policymaker mostly concerned about surplus maximization would rather use a Vickrey auction and correct bidders bias slightly with favoring devices. In fact, the two auction formats appear to provide two extreme frameworks: one in which bidders do not have market power at all (Vickrey) and another one in which the market power is exogenously given. If one views the problem from this optic, favoring devices can serve to smooth the outcomes of these two extremes.

As a first set of results, we look at the optimal choices when the auction designer is constrained to choosing among these two auction formats. Our work shows that when the policymaker is mostly concerned about surplus maximization, he would choose to use a Vickrey auction and to favor small bidders with bidding credits. Bidding credits allow for more flexible outcomes than set-asides, as bidders valuation differential explain allocation differentials across groups. When the concerns about having a fair outcome grow, the policy designer prefers to use a uniform price auction with bidding credits. Finally, only when the welfare function applies large penalties to unequal outcomes set-asides become the optimal choice of favoring device. When this threshold is reached, the concerns about surplus maximization are small enough so that Vickrey auction is not an optimal format. Our results show that, for our welfare function, it only makes sense to use seat-asides in the context of a uniform price auction.

As a second result, we propose an auction format that allows the policy designer to have control of the market power that bidders have in the auction. This format incorporates a pricing rule that endogenizes the effect that a bidder demanding his k^{th} unit has on the price of its $k-1$ other units in a symmetric linear equilibrium. The format is equivalent to a Vickrey auction when no market power is assigned and can be set to be equivalent to a uniform price auction. But most importantly, it can control the the impact that bidders' demand has in their own prices, thereby affecting their incentives to reduce demand. The present work shows that this format can implement the optimal allocations without the need of any favoring device.

Our work relates to the literature on bidding credits, set-asides and devices to promote competition. Substantial contributions have been made regarding the single unit, non divisible good case, from a mechanism design approach (Pai and Vohra, 2012), and from an empirical point of view (Athey et al., 2013). Pre-existing analysis for multiple units goods is less general, and mostly focused on the effect of auction in ex-post market structure (e.g. Dana Jr and Spier, 1994; Jehiel and Moldovanu, 2003; Cramton et al., 2011). Our work provides the first systematic analysis of the effect of auction formats and devices to promote competition altogether, in a divisible good framework.

This work shares the interest with a large literature comparing the performance of formats of multiple units auctions in terms of revenue and welfare.[5] We differ from this literature in the way that we measure performance in our analysis. In this work, the policymaker is concerned not only about maximizing surplus of auction participants (bidders and auctioneer), but also about not having high concentration in the outcome.

A common issue with welfare analysis related to uniform price auctions is its multiplicity of equilibria, first noted by (Wilson, 1979). As it is commonly dealt with in linear demand models, we circumvent this issue by assuming an affine information structure and focusing in linear equilibrium (Weretka et al., 2009; Vives, 2010, 2011; Ausubel et al., 2014). Although we do not model it explicitly, our model is compatible with supply uncertainty in the spirit of Klemperer and Meyer (1989).

The work is organized as follows. Section 1.2 describes the framework and main assumptions of the model. Section 1.3 shows the results of the demand function games given each auction policy. Section 1.4 derives optimal levels of favoring devices and the optimal policy choice when the auction designer is constrained to commonly used auction policies. Finally, section 1.5 discusses first-best allocations and how to achieve them.

1.2 Setup

Bidders An amount Q of a divisible good is auctioned among n large bidders and m small bidders. The class of each bidder i is represented by the parameter $\theta_i \in \{\theta_s, \theta_l\}$ (large or small) and the vector $\Theta \in \{\theta_s, \theta_l\}^{n+m}$ is common knowledge across bidders and the policymaker. This means that the policymaker can identify, at every moment of the game, the class of each bidder. Bidders' utility function is quasilinear in money and quadratic with respect to the amount of goods purchased. The gross utility of an amount q_i purchased by a bidder i is given by the expression:

[5] For an analysis in the context of electricity markets, see von der Fehr and Harbord (1993) and Fabra et al. (2006). For the case of treasury auctions, see Hortacsu and McAdams (2010). For an analysis in a linear context similar to ours but with symmetric bidders, see Weretka et al. (2009). Kastl (2011) analyzes the case in which bidders are constrained to submitting discrete bid points.

$$U_i(q_i, v_i) = v_i q_i - \frac{b}{2}(q_i)^2$$

where the valuation parameter v_i is a random variable independently distributed according to $G(v_i \mid \theta_i) \sim [\underline{v}(\theta_i), \overline{v}(\theta_i)]$. By an abuse of notation, we refer to v_i as the random variable and its realizations indistinctly. The net utility that a bidder gets from purchasing a quantity q_i can be obtained by subtracting T_i, the payment that he has to make for the purchase, from $U_i(q_i, v_i)$.

We distinguish the class θ_i from the type v_i of the bidder, the former being commonly observably while the latter is private information. The functional forms given by the above expressions imply that small and large bidders differ in the intercept of their marginal utility, and that both groups are formed by symmetric bidders. Moreover, we call $Var(v_i \mid \theta_s) = \sigma_s$ and $Var(v_i \mid \theta_l) = \sigma_l$.

The linearity of marginal utility is standard in this framework and is the basis of tractability of our results. We assume that the quantity auctioned Q is constant, although our model is compatible with a positive sloped supply or uncertain shifts in the amount being auctioned, as in Klemperer and Meyer (1989). Bidders' framework is similar to the extensions to private values in Ausubel et al. (2014), but with the presence of two asymmetric groups of bidders.

Bidders are asymmetric in the sense that $E(v_i \mid \theta_l) = E(v_i \mid \theta_s) + k$, where $k > 0$. This condition is consistent with relatively inefficient entrants, perhaps due to initial market disadvantage, know-how or installation costs, or also consumers with a tighter wealth constraint. The asymmetries among the realization of bidders' valuation parameters (coming from either different realizations or different distributions) can potentially be the source of negative quantities or prices. Although this is a common feature in linear demand or supply models (e.g. Vives, 2010), in our framework valuations distributions can be restricted in a way that this never occurs in equilibrium.[6][7] Linear demand models behave well with intercept uncertainty but not with slope uncertainty, and thus it is fundamental that no player exits the auction for some realizations of the distributions. Although this may sound extreme, exit in spectrum auctions is rarely observed and equilibrium prices and quantities are positive.[8] However, bidders purchasing small quantities may be interpreted as unsuccessful entry, or near exit.[9]

Welfare The government, or the institution designing the auction, is concerned about the total surplus generated by the final allocations, but also assigns some weight on the equality of the outcome. There are a number of reasons for which the auction designer might not prefer to allocate solely based on surplus maximization. Large valuations may be associated with negative externalities affecting third parties that do not participate in the auction.[10] For example, very concentrated allocation of licenses or capacity may have adverse ex-post market effects,[11] or excessive use of

[6] A sufficient condition for positive quantities is that both $Qb+(-1+m+n)\underline{v}(\theta_s)-n\overline{v}(\theta_l)-(m-1)\overline{v}(\theta_s)>0$ and $Qb+(-1+m+n)\underline{v}(\theta_l)-m(\overline{v}(\theta_s)+k)-(n-1)\overline{v}(\theta_l)>0$ hold. Regarding positive prices, a sufficient condition is that $bQ\leq(n\underline{v}(\theta_s)+m\underline{v}(\theta_l))\frac{min\{m,n\}-2}{min\{m,n\}-1}$. For example, these conditions are met if $b=1$, $Q=10$, $n=m=3$ and bidders are uniformly distributed, with small bidders valuations being in the range $[3,4]$, and large bidders valuations in the range $[4,5]$.

[7] The way different authors deal with this issue is mixed. Some authors do not restrict the distributions but work in a context of symmetry and perfect information (Ausubel et al., 2014; Klemperer and Meyer, 1989). Vives (2011) allows for negative demand and prices. Levin and Skrzypacz (2014) restrict the domain of the valuations in a fashion similar to the one described in footnote 6.

[8] This is true especially in auctions without regional licenses, such as 4G auctions held Switzerland (2012), the Netherlands (2012) and Austria (2013). In these auctions, even though there are larger and smaller bidders, it is unlikely to expect a national wide participant to exit auction. In contrast, in auctions with regional licenses, exit by very small regional bidders is more likely. However, their role in the auction tends to be meaningless for large and medium players. Our model is consistent with this either if we consider very small regional bidders as atomless or as random traders.

[9] In the consultations for the joint award of the 800 MHz and 2.6 GHz bands, OFCOM considered that bidders obtaining small packages would not be able to become effective competitors. See, appendix 6 to OFCOM (2011).

[10] Borenstein (1988) discusses how allocating an operating license to the bidder that values it the most can be harmful for welfare.

[11] Hazlett and Muñoz (2009) found that spectrum auctions that favored concentration are associated with higher consumer prices. Similarly, Fabra et al. (2006) show that, in a context of procurement auctions, asymmetry in the capacities has an adverse effect on consumer prices. Modeling the ex-post use of capacities explicitly can be hard, and highly dependent on the assumptions on market competition. In particular, a bidder buying a large share of the capacities may affect other bidder's valuation, as well as consumers surplus. Introducing all these elements together in the context of a multi-unit auction can be hard, and perhaps only tractable in very simplistic formulations, which may not be realistic at all. For this reason, we instead use the standard framework of a multi-unit auction but reflecting the preference for non concentrated outcomes as an exogenous characteristic of the welfare function. Overall, this seems to be a more realistic way to capture policymakers objectives.

the lowest private cost technology may have negative welfare effects.[12] Similarly, in the case of consumer goods, part of the valuation differences may arise purely from wealth differences.

For a vector of allocated quantities $\mathbf{q} \in \mathbb{R}^{n+m}$, and for a given Q, we define the ex-ante welfare of the auction designer as:

$$W = E\left(\sum_{i=1}^{n+m} U_i\left(q_i, v_i\right) - \frac{\alpha}{2}\sum_{i=1}^{n+m}\left(q_i - \frac{Q}{m+n}\right)^2 \mid \Theta\right) + \varphi$$

where $E\left(\cdot \mid \Theta\right)$ denotes the expectation operator over the distributions $G\left(v_i \mid \theta_i\right)$, conditional on that bidders' classes are as in Θ. Moreover, $\alpha \in \mathbb{R}_+$ and $\varphi \in \mathbb{R}$. The welfare function increases with the expected surplus of bidders and decreases according to a quadratic loss function that penalizes the expected asymmetries in the outcome, as measured by the deviations of allocated quantities from the per-capita supply $\frac{Q}{m+n}$. One can understand this loss function as a non-normalized concentration index. In fact, the welfare function can be restated in terms of the well-known Herfindahl index:

$$W = E\left(\sum_{i=1}^{n+m} U_i\left(q_i, v_i\right) - \frac{\hat{\alpha}}{2} HHI\left(\mathbf{q}\right) \mid \Theta\right) + \hat{\varphi}$$

where $HHI\left(\mathbf{q}\right) = \sum_{i=1}^{n+m}\left(\frac{q_i}{Q}\right)^2$, $\hat{\alpha} = \alpha Q^2$ and $\hat{\varphi} = \varphi + \frac{Q^2}{m+n}$. We will work with the first formulation of this welfare function because it allows us to recover slightly simpler expressions. However, the results naturally extend to the second one if the supply is known with certainty, and results in terms of α can be restated in terms of $\hat{\alpha}$. Overall, the welfare function shows some degree of conflicting objectives, captured by the parameter α.

Finally, given that the main goal of this work is to analyze welfare differences among auction policies, the constant term φ does not play any meaningful role, and can be normalized to 0.

[12] This interpretation may be relevant for the electricity market, in which authorities may want to encourage the use of cleaner technologies with higher private cost, such as wind and solar generation.

Auction Formats We consider two popular auction formats that differ mainly in their pricing rules: uniform price and Vickrey auction. We exclude another popular format from our analysis, the discriminatory price auction, because of the analytical problems that it poses in the context of asymmetric bidders, which are inherited from first-price auctions.[13]

Both the uniform price and the Vickrey auctions that we analyze are consistent with ex-ante formulations in which bidders submit demand schedules. The behavior that we describe is also consistent with dynamic formulations of the auctions, if bidders do not condition their bids in rival's strategies.[14] For example, the Vickrey auction could be implemented as a clinching auction, as proposed by Ausubel (2004), and the uniform price auction as a clock auction.

In either format, bidders submit differentiable demand functions $q : P \to \mathbb{R}$, where P is the set of prices, that stipulate the amount of good demanded at each price. We impose the standard activity rule that $q(p) \leq q(p')$ if $p' > p$, meaning that demand should not be increasing with price. In either auction format, the auction price p^* is the highest price at which the condition $\sum_{i=1}^{n+m} q_i(p) = Q$ is met.[15]

Auction formats differ in how this price translates into bidders' payments. In a uniform price auction, a bidder who is awarded the quantity q_i^* should pay the auctioneer $q_i^* \times p^*$. In contrast, in a Vickrey auction, the payment is a more complex expression. Define $s_i(p) = Q - \sum_{-i} q_j(p)$ as the residual supply faced by bidder i at price p. Also, define $\hat{p}_i$ as the highest price at which $s_i(p) \leq 0$. Following Wilson (1979), the Vickrey payment is given by $T_i = q_i^* \times p^* - \int_{\hat{p}}^{p^*} s_i(p)\, dp$.

[13] Little is known about asymmetric pay-as-you-bid auctions. For non-divisible single unit bids with asymmetric bidders, see Maskin and Riley (2000). Weretka et al. (2009) discuss the technical difficulties faced in modeling pay as you bid auctions with heterogeneous bidders.

[14] Levin and Skrzypacz (2014) discuss this issue in more detail. In general, dynamic formats allow for contingent equilibria in which bidders condition their actions in rivals strategies. If this behavior is allowed, even Vickrey auctions suffer from severe multiplicity of equilibria.

[15] Given that our analysis concentrates in interior linear equilibrium, we avoid unnecessary technicalities in the definition of auction formats. More general definitions of Vickrey auction, which allow for non-differentiable demands, excess of supply, etc, are not relevant in our framework and would make readers' work more difficult.

Figure 1.1: Vickrey vs. Uniform Transfers

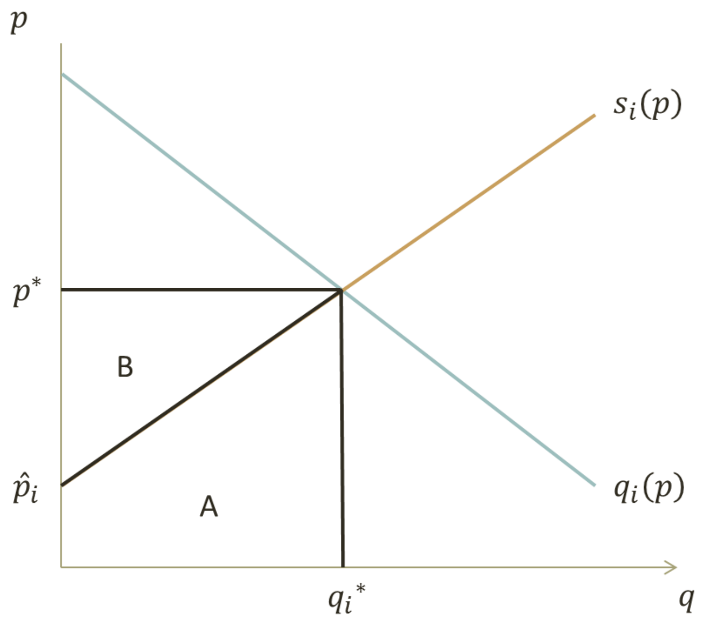

Figure 1.1 depicts the transfers that bidder i submitting the demand function $q_i(p)$ and facing a residual supply $s_i(p)$ pays under each format. If he is participating in a uniform price auction, the cost for q_i^* units is the sum of areas A and B. In contrast, in a Vickrey auction, his payment is only area A. Area B is the Vickrey discount. Note that in a Vickrey auction, an additional unit demanded does not affect the price of the infra-marginal units.

A naive look at figure 1.1 can push the reader to conclude that revenues are always higher in a uniform price auction. However, pricing differences can affect optimal strategies, and thus this conclusion might prove to be wrong.[16]

Devices to favor small bidders On top of choosing different formats, the auction designer can use devices to favor small bidders. We concentrate our efforts in the two most popular ones: bidding credits and set-asides. We consider an extreme version of a set-aside, in which the policymaker divides the quantity Q in portions Q_s and Q_l, to be separately sold to small and large bidders,

[16] In fact, Ausubel et al. (2014) prove in a linear framework slightly simpler than ours that expected revenues are higher in a Vickrey auction.

such that $Q_s + Q_l = Q$. In addition, the policymaker can use a bidding credit c, which, in its simplest form, is a fixed per unit discount or subsidy on the price. This is, given q units purchased, the transfer of a small bidders is reduced by $c \times q$. Throughout the text, we refer to these devices as "competition" or "favoring" devices.[17]

Timing The timing of the game is as follows:

1. The policymaker observes Θ and chooses an auction policy: an auction format, a favoring device and its level.
2. Nature draws v_i for each player.
3. Bidders observe the auction policy choice, their valuation parameters v_i, and submit demand schedules $q_i(p)$.
4. The auction is executed and payoffs are distributed.

Some clarifications regarding the timing. In stage 1, the policymaker chooses a combination of auction format and favoring device. The latter decision is not only a choice on the instrument but also on the intensity of use: the size of the bidding credit c or the quantities Q_s that are reserved for small bidders. Hence, a policy is an auction format $a \in \{a_v, a_u\}$ (Vickrey or uniform) and a favoring device $d \in \{c, Q_s\}$.

A perfect Bayesian equilibrium of the game is composed by a profile of demand functions $q_i^*(p, v_i \mid d, a, \theta_i)$ optimally chosen by each i for each auction policy, and an optimal auction policy choice d^*, a^*, given common beliefs about bidder's distributions and a vector of bidders classes Θ. Note that the policy designer does not have any private information, and thus bidders' prior beliefs

[17] We do not model another popular form of quantity restrictions: spectrum caps. However, binding spectrum caps may be understood as a more extreme version of set-asides, in which large bidders are allocated the binding quantities, and effective competition occurs only among small bidders. Hence, while set-asides suppress the intergroup competition, spectrum caps suppress both the intergroup and the intragroup competition among large bidders. However, in practice spectrum caps tend to be adopted when there is only one dominant or large bidder. In this case, set-asides and spectrum caps have the same effect on allocated quantities.

are not affected by the auction policy choice. As a consequence, we treat each auction sub-game as an independent demand function game.

1.3 Auction Equilibrium Analysis

We begin our analysis by characterizing the equilibrium outcomes of each policy choice. As it is standard in this framework, we focus our analysis in linear equilibrium of the form $q_i = \gamma_i v_i - \beta_i p + \delta_i$.[18] This does not mean that the strategy space is restricted to linear strategies, but rather that playing a linear strategy is optimal given that rivals are playing linear strategies. Also, in contrast to other related work, such as Vives (2011), we do not restrict our analysis to symmetric equilibria.

Our analysis is organized as follows. First we derive the equilibrium allocations for a given set of competition devices and auction format. Then, we compute optimal levels of bidding credits and set-asides. Finally we give conditions under which each policy is more convenient.

1.3.1 Bidding Credits

1.3.1.1 Vickrey Auction

In a Vickrey auction, well established results on dominance of truthful bidding apply to our framework. Bidders act as price takers, and it is a dominant strategy for them to submit a demand schedule that reflects their true preferences. Thus, their demand function equals the price to the marginal utility. For large bidders, this implies that, $q_i^* \left(p, v_i \mid c, a_v, \theta_l\right) = \frac{v_i}{b} - \frac{p}{b}$. In the case of small bidders, their bidding strategy is also affected by the bidding credits. For them, a marginal increase in their state by state demand generates the following utility:

[18] Ausubel et al. (2014) propose this as a reasonable equilibrium selection method for decreasing linear marginal demands.

$$v_i - bq_i = p - c.$$

Small bidders adjust their equilibrium strategies by the advantage that was given to them in the form of bidding credits: $q_i^* (p, v_i \mid c, a_v, \theta_s) = \frac{v_i + c}{b} - \frac{p}{b}$. In either case, bidding truthfully is optimal. In the case of the small bidders, the discount on the price can be interpreted as an increase in the valuation of the bidders. Optimality of linear strategies is inherited from linear marginal utility. In this framework, linear strategies are also a dominant strategy.

Lemma 1.1. *In a Vickrey auction with bidding credits there is a unique linear equilibrium. Indirect equilibrium quantities are given by:*

$$q_i (a_v, c, \theta_s) = \frac{Q}{(m+n)} + \frac{\left(v_i (n+m-1) - \sum_{-i} v_j + nc\right)}{b (m+n)}$$

$$q_i (a_v, c, \theta_l) = \frac{Q}{(m+n)} + \frac{\left(v_i (n+m-1) - \sum_{-i} v_j - mc\right)}{b (m+n)}$$

for small and large bidders respectively.

Proof. See Appendix. □

Lemma 1.1 describes the equilibrium allocations in a Vickrey auction with bidding credits. The functions $q_i (a_v, c, \theta_s)$ and $q_i (a_v, c, \theta_l)$ describe the indirect quantities allocated to each bidder in a linear equilibrium when the auction format is Vickrey and small bidders are favored by the bidding credit c. These quantities are obtained from solving the system of equations formed by the bidding strategy of each bidder and the market clearing condition. The first thing to note is that allocations are symmetric within groups. Second, as expected, bidding credits increase the amount of goods allocated to the small bidders.

1.3.1.2 Uniform Price Auction: Linear Equilibrium

In a uniform price auction, bidders' decision becomes more strategic, as their demand may affect the price of every unit purchased rather than just their marginal unit. Our analysis in this subsection is analogous to Ausubel et al. (2014), but extends it to asymmetric bidders and the presence of bidding credits.[19] We look at linear equilibria in which bidders play strategies of the following form $q_i^*(p, v_i \mid c, a_u, \theta_i) = \gamma_i v_i - \beta_i p + \delta_i$, although they consider any possible differentiable deviation from such behavior. Small and large bidders trade against an upward slopped residual supply with slope μ_i:[20]

$$v_i - bq_i - \mu_i q_i + c = p.$$

Large bidders face a similar condition, but without the bidding credit discount:

$$v_i - bq_i - \mu_i q_i = p.$$

Also, note that in an interior linear equilibrium, the slope of the residual supply faced by each bidder is given by:

$$\frac{\partial p(q_i, \cdot)}{\partial q_i} = \mu_i = \frac{1}{\sum_{-i} \beta_i}. \tag{1.1}$$

Again, here small bidders conditions reflect the advantage given by the bidding credit. The fact that we restrict the analysis to linear equilibrium makes the slope of the residual supply faced by each bidder constant. The presence of asymmetries may pose additional difficulties to bidders in this last point, if we were to think that for some realization of the parameters, small bidders could be driven out of the market. If this was the case, large bidders would face ex-ante uncertainty

[19] The analysis in this subsection is also closely related to Vives (2011). We consider more general distribution functions and equilibria set, but in our framework, signals are noiseless and values are uncorrelated.

[20] For a detailed explanation on why the problem can be tackled in this way, see proof of lemma 1.2.

about the slope. However, our assumptions in the setup section are enough to guarantee that interior solutions prevail for any realization of the vector of v_i.

Equation 1.1, together with first order conditions and the expression of the price, allow us to find a set of unique linear strategies characterized by:

$$\beta_i = \gamma_i = \frac{1}{b}\frac{(m+n-2)}{m+n-1}.$$

Groups of bidders differ in the intercept of their linear strategy. The intercept of small bidders' equilibrium strategy is:

$$\delta_i = (c)\,\beta_i$$

while large bidders intercept is:

$$\delta_i = 0$$

which means that bidders within each group play symmetric strategies and shade their demands.

Lemma 1.2. *In a uniform price auction with bidding credits and $n+m \geq 3$, there exists a unique linear equilibrium. Indirect equilibrium quantities are:*

$$q_i\left(a_u, c, \theta_s\right) = \frac{Q}{(m+n)} + \frac{n+m-2}{n+m-1}\frac{\left(v_i\left(n+m-1\right) - \sum_{-i} v_j + nc\right)}{b\left(m+n\right)}$$

$$q_i\left(a_u, c, \theta_l\right) = \frac{Q}{(m+n)} + \frac{n+m-2}{n+m-1}\frac{\left(v_i\left(n+m-1\right) - \sum_{-i} v_j - mc\right)}{b\left(m+n\right)}$$

for small and large bidders respectively.

Proof. The proof has the following structure. First, assuming linear strategies of rivals, each bidder solves its state-by-state problem. Then, we check that only one linear strategy is consistent with optimal behavior in more than two states. Next, we verify that an equilibrium exists if every player plays such linear strategy.

Our arguments starts by noting that, in each state $p = \frac{q_i + \sum_{-i} \gamma_j v_j + \sum_{-i} \delta_j - Q}{\sum_{-i} \beta_j}$ when rivals play linear strategies and the equilibrium is interior. This allows us to tackle the problem point-wise, and then reconstruct the bid function $q(p)$.

Each small bidder solves:

$$\max_{q_i} U_i(q_i) - \left[\frac{q_i + \sum_{-i} \gamma_j v_j + \sum_{-i} \delta_j - Q}{\sum_{-i} \beta_j} - c \right] q_i$$

where we replaced $p = \frac{q_i + \sum_{-i} \gamma_j - Q}{\sum_{-i} \beta_j}$. This gives the state by state first order conditions:

$$v_i - bq_i - \mu_i q_i + c = p$$

where $\mu_i = \frac{1}{\sum_{-i} \beta_j}$. An optimal demand function meets the condition:

$$q_i = \frac{v_i - p + c}{b + \mu_i}. \tag{1.2}$$

For uncertain values (three or more state contingent prices) of rivals' valuation parameter realization, the only linear strategy that meets equation 1.2 is given by $\beta_i = \gamma_i = \frac{1}{b + \mu_i}$, and $\delta_i = \beta_i c$. An analog analysis allows us to retrieve the conditions for large bidders: $\beta_i = \gamma_i = \frac{1}{b + \mu_i}$ and $\delta_i = 0$. The equilibrium β_i is determined by the system of $n + m$ equations:

$$\beta_i = \frac{1}{b + \frac{1}{\sum_{-i} \beta_j}}.$$

This system is equivalent to the one in Ausubel et al. (2014), and has as a unique result in $\mathbb{R}_+$:$\beta_i = \frac{1}{b} \frac{(m+n-2)}{m+n-1}$. We conclude that there exists a unique linear equilibrium if $m + n - 2 > 0$. Replacing the equilibrium strategies in equation 1.2 we obtain the quantities expressed in the proposition.

Finally, the second order conditions are met because $-(b + 2\mu_i) < 0$. $\square$

Lemma 1.2 presents the unique allocations arising in the linear equilibrium. The usual restriction on the number of bidders in linear models applies to our framework, and the equilibrium only exists with multiple bidders if they are greater than 2 in number.[21] The difference in the quantities allocated in the uniform and in the Vickrey auction arises from demand reduction. We do not comment much on demand reduction, as this has been largely discussed in the finance and uniform price auction literature. We just remind that this phenomenon arises from the fact that incremental quantities demanded increase the price of their entire demand, and not just the incremental quantity.

Although bidders are asymmetric, their marginal utility has a common slope, and thus the slope of the bid function with respect to the valuation is rotated equally in equilibrium. However, the intercept differentials together with a steepest demand curve explain allocations more centered towards the quantity $\frac{Q}{(m+n)}$ than in the Vickrey auction for the same level of bidding credits. A priori, this does not mean that allocations will in fact be less concentrated, because optimally computed bidding credits may be different for each type of auction.

The uncertainty on rivals' v_i plays a key role in the unicity of the linear equilibrium. Once the v_i are known, the strategies remain ex-post optimal. However, if bidders had perfect information at the moment of submitting their demand schedules, bidders could play with their slope and their intercept to meet the equilibrium quantities as described by Klemperer and Meyer (1989), and multiplicity of linear equilibrium would appear.[22]

[21] This limitation is common in linear demand models. See, for example Kyle (1989); Vives (2011, 2010). Milgrom (2004) has equilibrium for 2 players with known common knowledge of Q and bidders' valuations. Minimal uncertainty about Q or rivals valuation is sufficient to make this equilibrium disappear.

[22] Even though a bidders' choice of a different combination of slope and intercept that support the same outcome does not affect his outcome, it can have an effect on rivals' residual supply and prices. This feature is not exclusive from uniform price auctions and can also appear in dynamic Vickrey auctions. See Levin and Skrzypacz (2014).

1.3.2 Set-Asides

The analysis of set-asides is far less rich in our framework. Given a quantity Q_s reserved for small bidders, the sale is divided in two auctions in which bidders compete only with rivals of their own class. Although equilibrium prices and bid functions differ depending on whether the auction is Vickrey or uniform, there is a unique linear equilibrium in this framework in which bidders play symmetric strategies. The process of obtaining equilibrium quantities is the same as in the previous section, so we just describe the equilibrium outcomes. For a quantity Q_s reserved for small bidders, the indirect equilibrium quantities in a Vickrey auction are:

$$q_i\left(Q_s, a_v, \theta_s\right) = \frac{Q_s}{m} + \left(\frac{\left(v_i\left(m-1\right) - \sum_{-i} v_z\right)}{b\left(m\right)}\right) \tag{1.3}$$

$$q_i\left(Q_s, a_v, \theta_l\right) = \frac{Q - Q_s}{n} + \left(\frac{\left(v_j\left(n-1\right) - \sum_{-j} v_z\right)}{b\left(n\right)}\right) \tag{1.4}$$

where, in this case, small bidders are indexed by i and large bidders are indexed by j. As the bidder's classes are separated in different auctions, the indirect quantities allocated to each bidder only depend on the valuations of the bidders in their own kind. Similarly, in a uniform price auction with set-asides, quantities are respectively:

$$q_i\left(Q_s, a_u, \theta_s\right) = \frac{Q_s}{m} + \frac{m-2}{m-1}\left(\frac{\left(v_i\left(m-1\right) - \sum_{-i} v_z\right)}{b\left(m\right)}\right) \tag{1.5}$$

$$q_i\left(Q_s, a_u, \theta_l\right) = \frac{Q - Q_s}{n} + \frac{n-2}{n-1}\left(\frac{\left(v_j\left(n-1\right) - \sum_{-j} v_z\right)}{b\left(n\right)}\right). \tag{1.6}$$

Again, in a uniform price auctions, the allocated quantities are biased towards the average quantity reserved for each bidder's class with respect to the Vickrey auction. For the reasons described in lemma 1.2, the linear equilibrium exists only if $m > 2$ and $n > 2$. Hence, organizing a uniform price auction with set-asides is more demanding in terms of bidders' required participation.

1.4 Policy Choice

Having characterized equilibrium outcomes for each auction policy, we now discuss the optimal policy choice of a surplus maximizing policymaker with fairness concerns. The analysis is structured as follows: first we compute optimal levels of bidding credits and set-asides in each auction format, and then we analyze which format is more convenient. Hence, this section is devoted to the choice of $a \in \{a_v, a_u\}$ (Vickrey or uniform) and a favoring device $d \in \{c, Q_s\}$.

1.4.1 Optimal Bidding Credits

The equilibrium quantities described in lemmas 1.1 and 1.2 show that, in either auction format, the quantities that small bidders earn are positively affected by the bidding credit. The auction designer can compute optimal levels of c by solving the problem:

$$\max_{c} E\left(\sum_{i=1}^{m+n}\left(U_i\left(q_i\left(a,c,\theta_i\right),v_i\right)-\alpha\left(q_i\left(a,c,\theta_i\right)-\frac{Q}{m+n}\right)^2\right) \mid \Theta\right).$$

This is, given the indirect equilibrium quantities of each bidder class in each auction format, the policymaker chooses an amount of bidding credits to maximize the expected total welfare.

Proposition 1.1. *In a Vickrey auction, the optimal level of bidding credits is:* $c_v = \frac{\alpha}{b+\alpha}k$, *where* $k = \left(E\left(v_i \mid \theta_l\right) - E\left(v_i \mid \theta_s\right)\right)$. *In a uniform price auction in which bidders play the unique linear equilibrium, the optimal level of bidding credits is:* $c_u = c_v - \frac{b}{(b+\alpha)(m+n-2)}k$.

Proof. The proof is developed in the appendix. □

Proposition 1.1 describes the optimal level of bidding credits for each auction format. Bidding credits are increasing in the expected asymmetry between bidders and the weight that the auction designer assigns to the fairness of the outcome, α. The optimal level of bidding credits is different in a Vickrey and a uniform price auction. In fact, $c_u < c_v$ for any admissible value of the parameters. This result relates to the nature of allocations in each format. As explained before, bid shading in a uniform price auction affects more large bidders than small bidders. Hence, for an equal level of bidding credits, allocations are less concentrated in a uniform price auction. For this reason, in a Vickrey auction the policymaker decides to make more concessions to a small bidder. In fact, a priori, we cannot discard the case in which c_u is negative. A policymaker with $\alpha \to 0$, would view the uniform auction allocations as excessively inefficient, and would rather penalize small bidders to offset the effect of differential demand reduction.

1.4.2 Optimal Set-Asides

The problem of determining optimal set-asides is analog to the previous one. The policymaker solves:

$$\max_{Q_s} E\left(\sum_{i=1}^{m+n}\left(U_i\left(q_i\left(a,Q_s,\theta_i\right),v_i\right)-\alpha\left(q_i\left(a,Q_s,\theta_i\right)-\frac{Q}{m+n}\right)^2\right)\mid\Theta\right).$$

This problem gives as result a level of Q_s^* splitting Q in two quantities to be auctioned separately.

Proposition 1.2. *The optimal level of set-asides is* $Q_s^* = m\frac{Q}{(m+n)} - \frac{nm}{(m+n)(b+\alpha)}\left(E\left(v_i \mid \theta_l\right) - E\left(v_i \mid \theta_s\right)\right)$ *in both auction formats.*

Proof. We check first and second order conditions. Using the independence of bidder's distributions, the first order condition is:

$$\frac{\partial E\left(\cdot\right)}{\partial Q_s} = E\left(v_i - \frac{Q_s^*}{m}\left(1+\alpha\right)\mid\theta_s\right) - E\left(v_j - \frac{Q-Q_s^*}{n}\left(1+\alpha\right)\mid\theta_l\right) = 0.$$

The first order condition can be used to find the expression for Q_s^*:

$$Q_s^* = m\frac{Q}{m+n} - \frac{mn}{\left(m+n\right)\left(b+\alpha\right)}\left(E\left(v_i\mid\theta_l\right)-E\left(v_i\mid\theta_s\right)\right).$$

The second order condition is:

$$\frac{\partial^2 \left(W\left(\cdot\right) \mid \Theta\right)}{\partial Q_s^2} = -\frac{(m+n)(b+\alpha)}{mn} < 0.$$

Hence, the welfare function is globally concave in Q_s. □

Proposition 1.2 describes the optimal level of quantities reserved for small bidders. Q_s^* is determined in a way such that each small bidder receives, in expectation, the average per bidder supply $\frac{Q}{(m+n)}$ with a discount proportional to the expected efficiency differential between large and small bidders. Naturally, the size of such discount decreases with α. When $\alpha \to \infty$, reflecting a welfare function that only cares about fairness, the expected per bidder allocation becomes constant. On the other extreme, when $\alpha \to 0$ (or a surplus maximizing welfare function), small bidders expect, before learning their type v_i, the same allocation as in a pure Vickrey auction.

As opposed to the case of bidding credits, the optimal level of set-aside is independent of the auction format. The difference is explained on how each device promotes competition. Set-asides suppress the intergroup competition, and thus the differential bid shading of small and large bidders in a uniform price auction plays no role. On the contrary, bidding credits allow for intragroup and intergroup competition, and part of the favoring job is done by the auction format.

1.4.3 Auction Policy Choice

Having computed the optimal amount of favoring devices in each auction format, it remains to check under which circumstances one auction policy is preferred to others. The question is not obvious a priori, since optimally adjusted favoring devices may overcome strategic differences arising from auction formats. For example, even though the uniform price auction favors small bidders, the optimal level of bidding credits is smaller in this format than in the Vickrey auction.

Lemma 1.3. *When bidding credits and set-asides are optimally computed, the expected quantities allocated to each class of bidder are constant across policies.*

Proof. The statement requires to check that $E\left(q_i\left(a, d^*, \theta_i\right) \mid \Theta\right)$ is constant for any pair of auction format and optimally computed favoring device. This is done by replacing the optimally computed favoring devices in the indirect quantities of lemmas 1.1, 1.2 and equations 1.3-1.6, and using the distributional independence assumption to compute the conditional expectations. In either case, the expected quantities are:

$$E\left(q_i\left(\cdot, \theta_s\right) \mid \Theta\right) = \frac{Q}{m+n} - \frac{n\left(E\left(v_i \mid \theta_l\right) - E\left(v_i \mid \theta_s\right)\right)}{\left(m+n\right)\left(b+\alpha\right)},$$

$$E\left(q_i\left(\cdot, \theta_l\right) \mid \Theta\right) = \frac{Q}{m+n} + \frac{m\left(E\left(v_i \mid \theta_l\right) - E\left(v_i \mid \theta_s\right)\right)}{\left(m+n\right)\left(b+\alpha\right)}.$$

□

Lemma 1.3 contains a useful result for presenting the main proposition of this section. In expectation, allocations do not change with an auction format choice if the favoring device is optimally computed. Looking solely at this result could lead to the erroneous conclusion that the favoring devices are the only tool that is useful for the policymaker, and formats do not matter. However, in the rest of this section we show that this intuition is indeed wrong. Even though the allocations do not change in expectation, the auction format choice does affect the variance of the allocations, which has a significant effect on the welfare function.

The optimal policy choice is addressed by comparing the welfare function of the auction designer, having incorporated equilibrium outcomes and competition devices at its optimal levels. Naturally, the auction designer evaluates this in expectation, as he does not observe the realization of bidders' valuation parameters.

Proposition 1.3. *If $n > 2$ and $m > 2$, there exist thresholds $\underline{\alpha}\left(b, m, n\right) < \overline{\alpha}\left(b, m, n\right)$ such that:*

- *Vickrey auction with bidding credits is optimal if $\alpha \leq \underline{\alpha}\left(b, m, n\right)$.*
- *Uniform price auction with bidding credits is optimal if $\underline{\alpha}\left(b, m, n\right) \leq \alpha \leq \overline{\alpha}\left(b, m, n\right)$.*
- *Uniform price auction with set asides is optimal if $\alpha \geq \overline{\alpha}\left(b, m, n\right)$.*

Moreover, a Vickrey auction with set-asides is always dominated by at least one other policy choice.

Proof. See Appendix. □

Proposition 1.3 characterizes the optimal choice of the auction policy as a function of the degree of fairness concern of the auction designer. When α is small, the policymaker prefers a Vickrey auction with moderate sized bidding credits. For intermediate levels of α, the policymaker chooses a uniform price auction with bidding credits. In this case, intergroup competition is still allowed, but uniform pricing diminishes the effect of differences in the realization within the vector of v_i. Finally, when outcome concentration is a major concern for the policymaker, then set-asides in the context of a uniform price auction are the best choice. In this extreme, intergroup competition is suppressed and intragroup competition is limited by the effect of demand reduction.

In order to interpret these results, we first characterize the relation of the welfare function and the variability of the outcome. As hinted before, uniform price auction allocations are less sensitive to bidder's valuation differences, biasing the quantities towards the average supply per bidder. Following any change in the intercept of inverse demand function (bidders' private information), bidders face a steeper slope of the inverse residual supply (due to bid shading) and thus have a moderate effect on the outcome. A Vickrey auction, on the other hand, is more sensitive to bidders' valuations and it is a better option when surplus maximization is a primary concern. When $\alpha \leq \overline{\alpha}(b, m, n)$, bidding credits are preferred, but welfare is affected by the choice of the auction format.

When $\alpha \geq \overline{\alpha}(b, m, n)$, set-asides are a better way to allocate the goods. At this point, policymaker's preferences penalize outcome inequality in a degree sufficient that it is preferable to auction the goods separately between each class of bidders. By these means, the auction designer has a better control of the allocations, and washes away potential outcomes in which large bidders are awarded a high share. Even when the intergroup outcome variability is reduced with set-asides, intragroup variability persists. It may still be the case that one small bidder is awarded a share substantially high as compared to the other bidders. Hence, even with set-asides, the auction choice plays again a significant role. In fact, proposition 1.3 proves that the threshold for using set-asides is high enough so that a uniform price auction is always preferred once this threshold is reached.

Corollary 1.1. *Conditional on selecting an auction format, there exists a threshold level of* α *below which bidding credits are preferred, and above which set-asides are preferred. The threshold differs across formats.*

Proof. Using the notation in the proof of proposition 1.3, conditional on the format being a Vickrey auction, bidding credits are optimal if $\alpha < \alpha_2$, and set-asides are optimal otherwise. In contrast, conditional on using a uniform price auction, bidding credits are optimal if $\alpha < \alpha_5 = \overline{\alpha}(b, m, n)$, and set-asides are optimal otherwise. We conclude by noting that $\alpha_2 > \alpha_5$. □

Corollary 1.1 presents another reason expressing why the decision on favoring devices is not orthogonal to the auction format choice. For some values of α, conditional on using a given format, the choice between bidding credits and set-asides is opposite. In other words, there are levels of α for which bidding credits are the best choice if the auction uses a Vickrey pricing rule, while set-asides are optimal if the auctions uses a uniform pricing rule. Hence, not only the desirable amount of favoring is affected by the auction format, but also the choice among favoring devices.

The extent to which the thresholds in proposition 1.3 can be applied depends on that the conditions for equilibrium existence are met. In particular, the uniform price auction with set-asides has the hardest conditions, as it requires at least three large and three small bidders. In case this conditions are not met, direct allocation can be implemented to one of the groups, or a Vickrey auction with set-asides can as well be used as second-best solutions in one of either groups.[23] Naturally, this would require recomputing the threshold levels of α. Small number of bidders by group may be the explanation for the adoption of the otherwise dominated option of Vickrey auctions with quantity restrictions in 4G spectrum auctions in Europe.

[23] In the specific case in which the number of large bidders is small, another form of quantity restrictions may be more suitable, the so called "spectrum caps". See footnote 17.

1.5 Optimal Allocations

Although the results in the previous section are positive in that clear guidelines can be specified for selecting policies among the popular ones, an implied negative result is that none of them by itself is good enough to implement the optimal allocations. The focus on group aid of favoring devices oversees a fundamental aspect of the welfare function: it is the relation between quantities and values what matters. In this sense, the uniform price auction is appealing, since it allocates more to the bidders with higher valuations, but yet increasingly penalizes those who make large purchases. However, it proves insufficient to achieve the welfare goal, because it cannot adapt allocations to variations on the parameter α. In this section, we provide a general auction format combining elements of a uniform and a Vickrey auction that implements the optimal allocations of the policy designer if bidders play symmetric linear strategies.[24]

Proposition 1.4. *The first-best allocations are given by:* $q_i = \frac{Q}{m+n} + \frac{\left((m+n-1)v_i - \sum_{-i} v_j\right)}{(m+n)(b+\alpha)}$ *,*$\forall i$*.*

Proof. See Appendix. □

Proposition 1.4 shows the allocations that the policy designer would select if he could observe the vector of v_i. Comparing the allocations to those of lemma 1.1 with null bidding credits (pure Vickrey allocations), one can observe that they are biased towards $\frac{Q}{m+n}$, as it was the case in the uniform price auction. The bias arises from the penalty that the auction designer assigns to unequal allocations. It is clear from this expression that more general group-specific favoring devices will not be able to implement the first-best, as the optimal allocations involve intragroup distortions as well.

Interestingly, one can check that the pure versions of each auction format can implement first-best allocations for specific values of α. The fact that when $\alpha = 0$ the Vickrey auction implements the first-best quantity allocations is not surprising, and it is just a consequence that this format yields efficient allocations. However, it is also the case that when $\alpha = \frac{b}{-2+m+n}$ the uniform price

[24]We have not investigated yet if this holds for possibly asymmetric linear equilibrium.

auction implements the first-best. In this situation, the bid shading arising in uniform price auctions is just enough to control for the size differences among bidders.

For other values of α, none of the two options is good enough as a tool to implement the desired allocations. This happens because bid shading is too large or too small in each auction format. As a way to overcome this issue, we propose an auction format that generalizes Vickrey and uniform price auctions, and allows for flexible degrees of bid shading. In such auction, bidders payment for a market clearing price p^* is defined by the following formula:

$$T_i = q_i \times p^* - \rho \int_{\hat{p}}^{p^*} s_i(p)\, dp,$$

$$\text{with } \rho \leq 1.$$

One can note that the payment equals the Vickrey payment but with an additional parameter, ρ, that weights the Vickrey discount.[25] In fact, when $\rho = 1$ the auction corresponds to a Vickrey auction and when $\rho = 0$, it becomes a uniform price auction. It is easy to check that this payment rule has desirable properties, like for example that it is never negative if the price is positive, and that when $q_i = 0$, the payment is null. This payment rule can also be interpreted as a linear combination of a uniform and a Vickrey auction.

Proposition 1.5. *The first-best allocations can be implemented by means of an auction that weights the Vickrey discount by* $\rho = 1 - \alpha\frac{n+m-1}{b+\alpha}$ *if bidders play their unique linear symmetric equilibrium. When* $\alpha = 0$ *the auction corresponds to a Vickrey auction and when* $\alpha = \frac{b}{-2+m+n}$ *it coincides with a uniform price auction.*

Proof. We solve for the optimal quantities using a procedure similar to the one used in lemma 1.2. The framework is slightly simpler here, since we are only looking at symmetric equilibria. Although not proved yet, the author conjectures that the result holds without focusing on sym-

[25] See the auction definition section in the introduction.

metric equilibria. In each state, a marginal increase in quantities demanded generates bidders the following utility:

$$v_i - bq_i - \mu_i (1-\rho) q_i = p.$$

To obtain this last condition, we used the fact that when market clears, $s_i(p) = q_i$ and $\frac{\partial p}{\partial q_i} = \mu_i$.

In equilibrium, it holds for each bidder that: $\frac{v_i - p}{b+\mu_i(1-\rho)} = q_i$. Given linear strategies of rivals and the affine information structure, the unique linear strategy that implements the first order conditions is $\beta = \gamma = \frac{1}{b+\mu_i(1-\rho)}$ and $\delta = 0$. In a symmetric linear equilibrium, $\beta_i = \beta$, and thus $\mu_i = \mu = \frac{1}{\beta(n+m-1)}$. Putting these equations together, we find that $\beta = \frac{n+m-2+\rho}{b(n+m-1)}$.

Next, we use the market clearing conditions and the equilibrium strategies to compute the equilibrium allocations, which are given by the expression:

$$q_i = \frac{Q}{m+n} + \frac{(m+n-2+\rho)}{b(n+m-1)} \frac{\left((m+n-1)v_i - \sum_{-i} v_j\right)}{(m+n)}.$$

We check that when $\rho = 1 - \alpha\frac{n+m-1}{b+\alpha}$ the quantities coincide with the first-best quantities. Finally, second order conditions are met because $-b-\alpha < 0$. □

Proposition 1.5 shows that the generalized auction with $\rho = 1 - \alpha\frac{n+m-1}{b+\alpha}$ implements the first-best allocations. The key to this result is that the parameter ρ allows the policy designer to manipulate the amount of bid shading desired. It is interesting to note that bidders expected asymmetry in valuations plays no role in the auction proposed, as opposed to the procedure for determining optimal favoring devices. The proposed format links quantities and valuations in a much more powerful way for achieving the desired allocations.[26]

[26]It is worth noting that the optimal allocations can also be achieved by using a price penalty that increases quadratically with quantities, and adopting a Vickrey pricing rule. Such penalty has similar effects on bid shading as the ones described in this section. However, this solution can be cheated easily by bidders: large bidders can split into smaller ones and reduce the penalties. This critic is similar to the resale problem associated to bidding credits.

1.6 Conclusions

The results presented in this paper have simple and clear policy indications regarding the choice of favoring devices and auction formats in a multi-unit, divisible goods context. The first lesson is that these two choices cannot be made separately. In fact, we have proved that both the best favoring device and its optimal level can be different for each auction format.

The second lesson from this work is that each auction policy choice (combinations of auction format and favoring device) can be ranked in a simple way by a policymaker concerned not only about surplus maximization but also about not having too asymmetric outcomes. Binding set-asides are the best way to ensure a given allocation and are the best choice for a policymaker that is highly concerned about outcome concentration. Otherwise, bidding credits seem to be a better option, as they allow the allocation to adjust better with the information learned during the auction. However, our work shows that bidding credits in the context of a Vickrey auction allow for outcomes more sensitive to bidders' valuations than in the context of a uniform price auction. Hence, depending on the relative weight that a policymaker puts to surplus maximization, one or the other auction format can be the best choice. The predominance of quantity restrictions in spectrum auctions interpreted through our results suggests that outcome concentration is a major concern for telecommunication regulators.

The third lesson is that the role of bidder's market power in auctions deserves more attention in dealing with asymmetries. First because favoring devices are not independent of it, but most importantly, because it can be a very powerful tool to moderate the effect of valuation's asymmetries. In fact, we have shown that a using a linear combination of Vickrey and uniform payments, a policymaker can have a great degree of control on the market power it gives to bidders. In our setting, such mechanism proves to be more effective than using favoring devices.

Most generally this work gave guidelines for the adoption of auction policies in the context of the auction of a divisible good. Although not studied directly, the scope of our investigation can be extended to non divisible multi-unit auctions, as a discrete version of the auction we modeled.

Discreteness often introduce strategic incentives that have to be dealt with in a case by case way. If these issues are not fundamental, the lessons from our continuous approximation can be reasonably applied.

Finally, our framework does not endogenize the reasons for the policymaker to prefer other allocations over one that maximizes the surplus of the auction participants. We leave for future research this challenging task. We find particularly appealing the analysis in which too asymmetric results create a concentrated ex-post downstream market, as in Jehiel and Moldovanu (2000) but with multiple or divisible units. In such setting, uniform pricing can generate an interesting tradeoff: in order to exploit ex-post market power, bidders have to give up the use of their market power during the auction.

1.7 Chapter References

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Chapter 2

Incentive Contracts and The Allocation of Talents

2.1 Introduction

It is a well-established result in economics that property rights dilution is an obstacle for incentivizing agents. When workers are not full claimants of their production, managers have to rely on incentive contracts, which often involve information rents, to increase their effort. Property rights may not be diluted in small organizations, but governments and large private firms have to deal with this issue in similar terms. This work investigates the reasons for governments and firms to adopt different approaches to similar incentive problems. In particular, our theory relates the role of the government in controlling the allocation of talents in the economy with the prevalence of low powered incentives in the public sector.

We build a general equilibrium model in which the private and the public sector need to attract risk neutral workers to produce a private and a public good respectively. Although the output of the technologies used in each sector can be improved with workers' hidden effort, only the private sector's technology is sensible to the worker's entrepreneurial talent. Agents' limited liability forces private and public organizations to give information rents to induce effort. Standard profit maximizing leads private firms to provide incentives to workers whose expected production is above a threshold. Even though rents are not per se costly for a welfare maximizing government, providing incentives may attract too many workers to the public sector, thereby losing control of the allocation of talents. This tradeoff justifies the choice of an output threshold different than the one in the private sector for giving incentives to workers.

A fundamental aspect of this work is that the government, as opposed to the private sector, is concerned about the efficiency of the entire economy. From the point of view of a private firm, the decision of incentivizing agents is ruled by a standard moral hazard problem with limited liability in a partial equilibrium environment. It will only be profitable to induce effort if the expected increase in the worker's output offsets the wage increase associated with the cost of effort and the information rents needed to achieve incentive compatibility. In contrast, from the point of view of a welfare maximizing government, increasing wages is not per se costly. In particular, information rents can be viewed as mere transfers between agents with no effects in the welfare of the economy. However, the government may still be reluctant to pay high wages either if they have to be funded through taxation with negative distortions, for distributional reasons, or if those wages attract agents whose high entrepreneurial talent cannot be exploited in the public sector. As in Acemoglu and Verdier (1998), we concentrate on the third channel, and we assume that the government uses a non distortive and non discriminatory tax scheme (lump sum taxes), although we also discuss the robustness of our results with more complex tax schemes. Misallocation of talents is perceived as a social cost by the government through two different channels. First, a worker with high entrepreneurial talent can contribute more to the private than the public production. Second, agents with low entrepreneurial talent working in the private sector may not be productive enough to justify affording the cost of incentivizing them, and thus misallocation of talents can also trigger low powered incentives in the private sector. In this framework, the government's decision of providing incentives to public workers is not only ruled by the efficiency in the production of public goods, but also in the opportunity cost in terms of lost productivity in the private sector.

Our general equilibrium view of the incentive decisions in the public sector explains that the government will suffer from an "inefficiency curse". Even when it is optimal to incentivize public agents, the government chooses an excessively large size in order to mitigate the problem of misallocation of talents. Hence, our model predicts that either the production of public goods will not be efficient, or the government will be excessively large. However, in either case the society is better off with the existence of government, whose role in the production of public goods cannot be replaced by the private sector.

From a theoretical point of view, several authors provided explanations for the lack of incentive contracts in public organizations, as compared to private firms. Most of the work, however, concentrates on inherent differences on the informational framework or the contracting complexity that private and public organizations face. Tirole (1994) argues that difficulties to measure results reduce the scope for performance payments and that often bureaucrats' performance is harder to measure than that of private workers. Dixit (1997) considers that low powered incentives may arise from the large size of public organizations and from the multiprincipal nature of a government. This last argument is also present in Martimort (1996, 2006). Public workers tend to serve more masters than private ones, due to the fact that governments are complex organizations with different authorities living together. Lack of coordination between different masters may result in low powered incentives. The theories emphasizing obstacles to writing incentive contracts in the public sector can also interact with capture arguments. Tirole (1994) suggests that noisy performance measurement may promote incentive distortions to avoid collusion among members of an organization. Laffont and Tirole (1993) explain that such distortions can involve reducing the extent to which bureaucrats use potentially valuable private information to make decisions. On a different approach, Besley and Ghatak (2005) argue that low powered incentives arise from the mission oriented nature of the public sector production, as opposed to the profit oriented nature of the private sector.

Although these explanations seem satisfactory for numerous situations, the relative prevalence of incentive contracts faces a simple counterargument exposed by De Fraja (1993). Abstracting from the differences of the private and the public sector, a welfare maximizing government concerned about efficiency would be more inclined towards giving incentives to agents than a profit maximizing firm. After all, firms will only seek for efficiency if it is profitable whereas the government has efficiency as its main objective. If this argument held, then the prevalence of incentive contracts in the public sector would became a matter of weights: the different frameworks that make contracting different and less powered in the public sector would have more weight than the pro-efficiency bias of the public sector.

Our work shares with Corneo and Rob (2003) the interest on rethinking the reasons why a welfare maximizing government may choose lower incentives than a profit maximizing institution one, when they face a similar moral hazard problem. As opposed to their work, our results do not require the presence of cooperative actions and hold within a standard single task moral hazard problem, as long as incentive contracts require wage increases creating a sufficiently large misallocation of talents. In our model, the main characteristics of the transaction (conditional distribution of output and cost of effort) are equally observable in the private and the public sector. A key difference in our theory is the role that the government plays in sorting talents within a general equilibrium economy.

Our work relates to the literature on selection based efficiency wages initiated by Weiss (1980). We differ from this literature in that competition between firms is replaced by public sector and private sector competition, the former having welfare and talents allocation concerns rather than just profit maximization. In this sense, our model is close to Delfgaauw and Dur (2010), who argue that more talented managers tend to self select into the private sector. We depart from their work in that we emphasize the role of contracts on workers incentives, rather than assuming that workers earn their marginal product.

A central aspect of this paper is the tension between the allocation of talents and incentive contracts in a general equilibrium economy. Acemoglu and Verdier (1998, 2000) argue that misallocation of talents may be a cause for corruption to exist in governments. In their analysis, efficiency wages serve to fight corruption. A government trying to fight corruption too hard may attract too many talents to a task which does not allow them to deploy all their productivity. Our work shares with them the assumption that innovative talent is more relevant in the private than in the public sector, but we also model incentives the private sector, which allows us to compare the relative performance of public and private workers. This aspect of our work is closely related to Jaimovich and Rud (2014), who model both public and private sector labor markets. However, in our model low powered wages in the public sector can arise endogenously, rather than just being assumed.

The work is organized as follows. Section 2.2 presents the main components and assumptions of our model. Section 2.3 describes the first-best allocation and its relation to a framework without moral hazard. Section 2.4 contains the solution to the model with moral hazard and discusses the main findings of our theory. Finally, section 2.5 discusses the main policy implications of our theory.

2.2 Model

There are three actors in a the economy: government, large firms, and agents that can become workers or self-entrepreneurs. There are two goods, a private good and a pure public good. The government is a social welfare maximizing entity that employs agents to provide a public good funded by taxes. Firms are organizations that employ agents and own a technology that can boost agents' talent to produce the private goods. There is a continuum of risk neutral agents ordered by their entrepreneurial talent a, where $a \in A : [0, \bar{a}]$ and is privately observed by agents. This talent represents the ability of agents to innovate in the production of the private good. Agents can be either self-entrepreneurs or workers in public or private organizations.

Risk neutral agents derive utility from their consumption of the private good, the public good and their initial wealth. Agents in the economy benefit from the average amount of public good g and they consume from private sector production according to the wages they receive, the initial arbitrary shares of private firms they hold $s(a)$, their self production and the arbitrary initial wealth they hold $x(a)$. Their consumption of the private good is reduced by the amount of taxes they pay, and they derive direct disutility from making effort in the production process. The utility of each agent is the sum of the components listed above. Every transaction in the economy is measured in terms of the private good.

The objective of the government is to maximize social surplus expressed as the sum of individual utilities. In order to fund the public wages, the government collects a lump sum tax affecting every agent. For the sake of simplicity, we assume that the initial wealth of agents, $x(a)$, is sufficiently high to cover the per agent level of taxes t.

Technologies Every agent can choose to become self-entrepreneurs and use a technology that allows them to produce a units of the private good. If instead they decide to work in a private firm, they can use a different technology that is a function of the entrepreneurial talent of the agent: $f : A \to \mathbb{R}$, which is continuous and satisfies $f(0) = 0$, $f'(a) > 0$. In either case, the output they produce is dependent on their entrepreneurial talent. The government owns a technology that allows it to produce a constant output of the public good f_g from each worker it employs. We assume that the production of public goods is not related to the entrepreneurial talent of the workers. According to this view, the public good that the government produces is generated through well established procedures which require no entrepreneurial talent to produce.[1]Another way of interpreting this assumption is that the production of public goods is technologically non progressive, as defined by Baumol (1967), and hence the sector's production is mostly ruled by the

[1]Note that this assumption does not mean that government employees do not require any skill. One could interpret our model as one concerning a market for skilled workers that is dissociated from the one of unskilled workers, perhaps because the skills are easily observable or screened. With this interpretation, the government requires skilled workers but does not put particular value on the innovation capacity of the agent.

number of people it employes.

Even though the technology for producing the public good may be available to the private sector, the pure public nature of the good would make unprofitable its use. No agent would be willing exchange the private good for a public good, which he can already consume for free. For this reason, only the welfare maximizing government is interested in using such technology.

Either in the government or in the private sector, the realization of the output depends on the effort of the worker, $e \in \{0,1\}$. If the agent shirks ($e = 0$), the output is the one given by the production function with a probability $\underline{v}$ and 0 otherwise. If he makes a non-observable effort ($e = 1$) which involves a private cost c, the probability of achieving a productive output is increased to $\overline{v} = \underline{v} + \triangle v$, with $\triangle v > 0$. Sometimes through the text, we refer to these probabilities by the function $v : \{0,1\} \rightarrow \{\underline{v}, \overline{v}\}$. Summing up, an agent choosing $e = 1$ expects to produce $\overline{v}a$ as self-entrepreneur, $\overline{v}f(a)$ if employed in a private firm and $\overline{v}f_g$ if working for the government. Finally, we assume that, even though each agent faces uncertainty about his own output, there is no aggregate uncertainty about it conditional on knowing the fraction of agents making the high and the low level of effort.[2]

Assumption 2.1. $f_g > \frac{c}{\Delta v}$, $\overline{a} > \frac{c}{\Delta v}$.

Assumption 2.1 stipulates that the effort is valuable both in in the private and the public sector, at least for some agents.

Assumption 2.2. $f(a) > a$.

Assumption 2.2 refers to the relative productivity of agents taking different occupations within the private sector. In particular, it states that agents are more productive working in the context of a large private firm than as self-entrepreneurs.[3]

[2] Although this last assumption is not essential, it simplifies the exposition and allows us to avoid introducing additional balancing variables to meet government's budget constraint.

[3] This assumption could be replaced by the weaker $f(a) > a$ when $a < \frac{c}{\Delta v}$. With this alternative assumption, very innovative agents could be more productive as self-entrepreneurs than working in the context of a firm. Most

Assumption 2.3. $f\left(\frac{c}{\Delta v}\right) > f_g$.

Finally, assumption 2.3 refers to the relative productivity of agents across sectors. It implies that high entrepreneurial talent agents are more productive working in the production of private goods than in the production of public goods. Note that it may also be that $\overline{a} > f_g$, although this is not a necessary assumption for the construction of the model.

Agent's occupational choices and utility The government and the private firms attract agents by offering them a work contract that is signed ex-ante but can stipulate ex-post or contingent payoffs. A public contract is a wage $w_g : \{0, f_g\} \to \mathbb{R}$, and a private contract is a wage schedule $w_f : \{0, f[A]\} \to \mathbb{R}$, where $f[A]$ is the image set of the function f. Self-entrepreneurs are owners of their own production and they do not receive any wage. Through the text, we refer to agents that accept jobs in the public sector as "civil servants" or "public employees".

An agent with $a = z$ working as employee in the sector $i \in \{g, f\}$ receive the following utility function:

$$u(z) = E(w_i(\cdot) \mid e) - ce + s(z)\Pi(F) + x(z) + g - t.$$

Whereas, if he is employed as self-entrepreneur, he receives the following utility:

$$u(z) = v(e)z - ce + s(z)\Pi(F) + x(z) + g - t.$$

Call F the subset of agents employed in private firms, and G the subset employed in the government. We define $\Pi(F) = \int_F (v(e^*(a)) f(a) - E(w_i(\cdot) \mid e(a)^*))\, da$ as the aggregate profits of the private sector, while $g = \frac{1}{\overline{a}} \int_G v(e^*(a)) f_g da$ represents the per capita level of public good. The variable $e^*(a)$ represents the actual effort made by an agent with the talent a, considering his occupation.

of the results of our model are independent of this variant, but additional thresholds and a more complex allocation of talents make the exposition harder to follow. Earlier versions of our paper include the weaker assumption and are available on demand.

Finally, we impose that $\int_A s(a)\, da = 1$, meaning that shares represent splits of the private sector profits. Overall, the utility function of agents is linear in their consumption of the private good and the average amount of the public good. Each agent consumes an amount of the private good given by his self production, his wage, the shares of private firm profits he holds minus the taxes he pays, all of them measured in terms of the private good. On the contrary, every agent consumes the same amount of the public good.

Timing The timing is as follows:

1. The government offers a wage $w_g(\cdot)$ and chooses a size n.
2. Each worker observes his talent a, the offer and decides whether to apply for a government job, or wait for another offer. In case of excess of applications for public jobs, every application has equal chances of being accepted.
3. Firms offer a wage $w_f(\cdot)$ to remaining agents.
4. Remaining agents decide whether to work for firms or to become self-entrepreneurs.
5. Agents undertake a non observable action e.
6. Payoffs are distributed.

The timing deserves some explanation. In the worker's occupational choice problem, multiple job suppliers interact with a mass of workers. This brings the question about the relative bargaining power of each of the parties. This problem is per se a difficult theoretical issue that we would like to abstract from in order to focus on other insights. The timing we chose, together with the assumption that firms only make job offers to the residual workers, ensures that the full bargaining power is on the side of the employers and not the worker, which is a common ingredient in moral hazard problems. According to this approach, workers are paid according to their outside opportunity rather than their marginal product, as in Weiss (1980). In particular, alternative timings could bring the issue in an inappropriate way. For example, consider an alternative timing in which the government and the firms make job offers sequentially or simultaneously, and only after that agents decide where to work. This timing could trigger a wage competition between the government and firms that would give workers of certain entrepreneurial talent some bargaining power. However, one could also argue that the private sector is composed by several firms itself, and that they could compete for talents too which would not be reflected by modeling the sector as a whole. Hence, the competition between firms and the government by itself would not yield a reasonable characterization of the relative bargaining power of workers and employers. In addition, the timing we use in the paper is consistent with the one adopted by Jaimovich and Rud (2014) when dealing with a similar wage competition problem in a general equilibrium economy.

We can now define the equilibrium of the economy. In equilibrium, agents choose e^* optimally given their occupation and income (wage or self-production), firms are profit maximizers, the allocation of talents is determined by agents occupational choices at each stage and the government optimal policy choice is funded with tax collections.

2.3 First Best Allocation

We begin by looking at the best possible allocation that the economy could attain if a central planner controlled the allocation of talents and the effort that each agent is doing. Recall that S and F are the subsets of A that contain agents working as self employed and as employees in private firms respectively. Moreover, recall that n is the mass of agents employed in the government. The aggregate welfare of the economy is given by:

$$\int_A u(a)\,da = \int_F \left(v\left(e\left(f\left(a\right)\right)\right)f\left(a\right) - ce\left(f\left(a\right)\right)\right)da + \int_S \left(v\left(e\left(a\right)\right)a - ce\left(a\right)\right)da + n\left(v\left(e\left(f_g\right)\right)f_g - ce\left(f_g\right)\right).$$

Taxes, wages, profits and public goods, when added up altogether form the overall production of the economy, net of cost of effort. Although there are two goods produced in this economy, each of them generates an equivalent social value, given the linear nature of the utility function. Hence, fixing a level of production y, regardless of whether its generated by self-entrepreneurs, private firms or the public sector, it is optimal that agents make an effort if:

$$\Delta vy \geq c$$

This means that there is an equivalent threshold in either form of production deciding on the level of production that justifies bearing the cost of effort. The only other relevant decision for the planner is to decide how talents are allocated in the economy.

Proposition 2.1. *The optimal allocation of the economy is described as follows:*

1. *Agents with* $a \leq n^{fb}$ *where* $n^{fb} = f^{-1}(f_g)$ *work for the government,*
2. *Agents with* $a > n^{fb}$ *work in private firms.*

Moreover, every agents chooses $e = 1$*.*

Proof. Assumption 2.2 guarantees that the planner would never choose agents to be self-employed, since $f(a) > a$, meaning that there are always gains from employment. The planner can benefit from allocating the agents from lowest entrepreneurial talent to the government as long as $f(a) < f_g$. Assumption 2.3, monotonicity of $f(a)$ and continuity of $f(a)$ guarantee that there exists a such that $f(a) = f_g$, which in the proposition we call n^{fb}.

The rest of the agents have a productivity in the private sector $f(a) \geq f_g$. Assumption 2.1 implies that f_g is high enough to justify $e = 1$, since $f_g \geq \frac{c}{\triangle v}$. These two elements together imply that the planner would optimally choose that $e = 1$ for every agent. □

Proposition 2.1 describes the optimal allocation of the economy. In words, the planner would allocate the less innovative agents into the public sector, where their production does not depend on their entrepreneurial talent. Only agents sufficiently innovative would be more productive in the private sector. Assumption 2.2 implies that there exists a threshold in the level of entrepreneurial talent such that, for sure, an agent's productive contribution is higher in the private than in the public sector. This threshold determines the size of the government. The rest of the agents are more productive working in the context of a firm and no agent ends up self-employed.[4]

Regardless of where the planner allocates each particular agent, the option of being employable in the public sector sets a lower bound on their individual production: f_g. Given assumption 2.1, this level of production is high enough to justify $e = 1$. Note that the optimality of the high level of effort is a consequence of the optimal allocation of talents. A less efficient allocation of talents, in which agents with very little innovation talent are assigned to the private sector, would change the decision on the optimality of $e = 1$. In fact, if agents with $a < f^{-1}\left(\frac{c}{\triangle v}\right)$ work in the private sector, they are not able to recover the cost of effort with the increase in expected production.

Corollary 2.1. *If the effort is observable by employers, the government can implement the optimal allocation.*

Proof. Given that the effort is observable, the employers can offer wage schemes contingent on the effort. In particular, any wage schedule of the government that meets $E\left(w_g \mid e = 1\right) = n^{fb} + c$ and that penalizes low effort such that $E\left(w_g \mid e = 0\right) < n^{fb}$ is enough to attract only n^{fb} agents and make them do the high level of effort. This is possible because firms do not need to give information rents to workers either. In stage 3, firms profits' maximization implies a wage schedule meeting $E\left(w_f\left(f\left(a\right)\right) \mid e = 1\right) = a + c$ if $a > f^{-1}\left(\frac{c}{\triangle v}\right)$ and $E\left(w_f\left(f\left(a\right)\right) \mid e = 0\right) = a$ if $a < f^{-1}\left(\frac{c}{\triangle v}\right)$ and a penalty if the desired effort is not observed.[5] This wage schedule makes agent's outside opportunity at stage 2 equal to their productivity as self-entrepreneurs.

By these means, the government can effectively attract to the public sector as many agents as desired and give them incentives at the same time. □

[4]This result depends on the strict version of assumption 2.2 that we adopted in the paper. A simple change in this assumption could allow for optimality of self-employment at high levels of talent. See footnote 3. For a discussion on how entrepreneurial talent relates to employment, see Bianchi and Henrekson (2005).

[5]At this point, we do not develop the full argument of why profit maximization gives this result. However, section 2.4 contains a step by step analysis that applies to this simpler case.

Corollary 2.1 states that the government has enough tools to implement the first-best allocation when the effort is observable. Employers do not have to give information rents to induce effort, and the marginal worker can be paid on the basis of his outside opportunity as self-entrepreneur. The government can offer a uniform wage that matches the outside opportunity of worker $a = n^{fb}$ and give an extra compensation to cover the cost of effort when its observed. Self selection of agents does the rest of the work.

A fundamental aspect of the first-best case is that the three forms of production (government, self-entrepreneur, private firms) have an equal threshold for providing incentives to agents: the level of production having an expected increase sufficient to cover the cost of effort. We will next argue that this result does no longer hold when the effort is not observable.

2.4 Moral Hazard Rents and Talent Allocation

A main feature of our formulation is that e is not observable by the employer, and thus cannot be contracted upon. As it is well known in the information literature, a hidden action is not per se an obstacle for contracting. If agents are risk neutral and transfers are not constrained, an incentive scheme that induces effort can be implemented costlessly.[6] This kind of arrangement often involves having large negative transfers in some states of nature which are not unequivocally, only statistically, associated with misbehavior. These contracts are unrealistic in the framework of public and private contracting, and thus assumption 2.4 restricts the wage to be non negative:

Assumption 2.4. *For any output level* $y \in \mathbb{R}$, $w_i(y) \geq 0$, $i \in \{g, f\}$.

The presence of limited liability creates the need of giving information rents to induce effort when agents are not residual claimants of their production. In this section, we show that the way information rents translate into costs for private firms and the government is very different, and

[6] See, for example, Chapter 3 of Laffont and Martimort (2009).

can be the source of different incentive policies in each sector. We proceed to solve the problem by backward induction.

2.4.1 Agents' Incentives

Agents decision on making the costly effort can be affected by the incentives schemes that they are offered. Either if they are working on the public or in the private sector, the output that they can produce is limited to two values: 0 in case the technology fails and $f(a)$, f_g or a when they work in private, public or self employed sector respectively, and the technology is successful.

Self-Employed Agents Agents that choose to be self employed are residual claimants of their production. The expected level of output depends on the level of effort that they choose. In particular, choosing $e = 1$ is only profitable if:

$$\bar{v}a - c \geq \underline{v}a.$$

Assumption 2.1 guarantees that there exists a cutoff $a = \frac{c}{\triangle v}$ such that agents with lower entrepreneurial talent would not choose to make an effort. It is important to highlight that this threshold is equal to the first-best level, and this is exactly because self-entrepreneurs do not have to afford information rents to induce effort. Finally, note that initial wealth, taxes and private sector shares are irrelevant in this decision, as they are equal regardless the level of effort chosen.

Workers When agents act as workers either in the private or the public sector, they are not residual claimants of their production, and hence they are only interested in improving the production process by making a high effort if they are offered an incentive scheme. A key aspect here is that each agent's output is atomistic and does not affect the profits that agents receive as shares or as public goods. This dilution of agent's effort makes that the only thing that changes when the agent does not shirk is the probability of getting a wage associated with a high output.

The constraint on the wage schedule reduces the ability of employers to give incentives. Without loss of generality, we fix $w_i(0) = 0$, meaning that the wage associated with an output of 0 is the minimum possible. The choice of the agents' effort is ruled by a standard moral hazard incentive constraint with a limited liability. An agent employed in the sector i makes the costly effort if:

$$\overline{v}w_i(y) - c \geq \underline{v}w_i(y) \;\Rightarrow w_i(y) \geq \frac{c}{\Delta v}. \tag{2.1}$$

This constraint implies that effort cannot be induced with an expected wage lower than $\overline{v}\frac{c}{\Delta v}$ (incentive constraint).

Moreover, a worker is only going to accept a job if he expects to get a higher payment than working as self-entrepreneur. Hence, a necessary condition for a worker to apply for a job which offers him incentives is:[7]

$$w_i(y) \geq \frac{\underline{v}a + c}{\overline{v}}.$$

Lemma 2.1. *There exists a cutoff on the entrepreneurial talent of the agents such that when $a \geq \frac{c}{\Delta v}$, effort can be induced without information rent. If $a \leq \frac{c}{\Delta v}$, the information rent necessary to induce effort decreases in a.*

Proof. Assumption 2.1 guarantees that the cutoff exists. Given limited liability and that we look for the minimum wage inducing effort, a binding participation constraint implying that the agent gets 0 information rent would boil down to: $w(\overline{y}) = \frac{(a\underline{v}+c)}{\overline{v}}$. This wage is incentive compatible if: $a \geq \frac{c}{\Delta v}$.

If $a \leq \frac{c}{\Delta v}$, then the incentive constraint binds and the participation constraint ceases to do it. If so, the information rent is given by $\overline{v}\left(\frac{c}{\Delta v}\right) - c - \underline{v}a = \underline{v}\left(\left(\frac{c}{\Delta v}\right) - a\right)$, which is decreasing in a. □

Lemma 2.1 depicts an interesting issue relating the outside opportunity of an agent working

[7]Conditions are not sufficient in the case of public worker, who also have private sector offers as outside opportunity. This is discussed in detail later in the text.

as a self-entrepreneur and the cost providing him incentives if hired as a worker. An agent with entrepreneurial talent a would never accept a job offer if the expected wage net of cost of effort does not match or exceeds his outside opportunity as self-entrepreneur. This puts a lower bound in the expected wage, but given that agents are risk neutral, it does not constrain how the wage is structured in different states realizations of the output. The incentive constraint of a worker indicates that he will shirk unless the wage differential between the good and bad state of nature is large enough, specifically $\frac{c}{\Delta v}$. For large values of a, the expected wage needed for participation is high enough to promote incentives with a limited liability.

A consequence of lemma 2.1 is that agents that would choose $e = 1$ as self-entrepreneurs would not require an information rent to make effort as employees. The reason is that their outside opportunity is high enough so that the wage needed to attract them would allow creating an incentive scheme at no additional cost.[8]

On the contrary, workers with $a < \frac{c}{\Delta v}$ would choose $e = 0$ as self-entrepreneurs. However, this does not mean that an employer would not want him to make a high effort. According to assumptions 2.1 and 2.2, agents in this subset of talent are more productive either in firms or in the public sector than as self-entrepreneurs. The productive differential may be high enough to justify a different incentive policy.

2.4.2 Private Firms

Private firms observe the residual supply for jobs and decide on offers by maximizing their profits. Even though firms do not directly observe a when they make a job offer, the entrepreneurial talent of each agent can be inferred once the production is realized and thus contingent contracting can overcome the talent observability problems. The problem of private firms can be tackled in a pointwise way. According to lemma 2.1, the participation constraint of an agent with $a > \frac{c}{\Delta v}$ is

[8]This coincidence relies on the fact that the unfavorable output as self-entrepreneur and worker is equal. Say, for example that the minimum wage was $w > 0$, then the correspondence would be different but not in a way that would change the relevance of our results.

demanding enough to make the incentive constraint non binding. In this case, the decision of the firms is simply whether to hire agents or not.

On the contrary when $a \leq \frac{c}{\Delta v}$, providing incentives costs additional information rents and yields the following profits:

$$\bar{v}\left(f\left(a\right)-\frac{c}{\Delta v}\right).$$

Whereas if the firm decides not to give incentives, the profit of hiring an agent with entrepreneurial talent a is:

$$\underline{v}\left(f\left(a\right)-a\right).$$

In the latter case, incentives are not necessary and participation only requires that $w_f\left(f\left(a\right)\right) \geq a$. Providing incentives is a decision that involves weighting the benefits in terms of expected output increase against the information rents and additional participation costs.

Proposition 2.2. *Private firms offer a wage schedule $w_f\left(f\left(a\right)\right)$ to every unemployed agent. The schedule is incentive compatible for workers producing $f\left(a\right) \geq f\left(\tilde{a}\left(\cdot\right)\right)$, where $\tilde{a}\left(\cdot\right)$ is the unique solution to $f\left(a\right)=\frac{1}{\Delta v}\left(\bar{v}\frac{c}{\Delta v}-\underline{v}a\right)$, and satisfies the participation constraint of lower productivity workers. Every unemployed agent accepts the offer.*

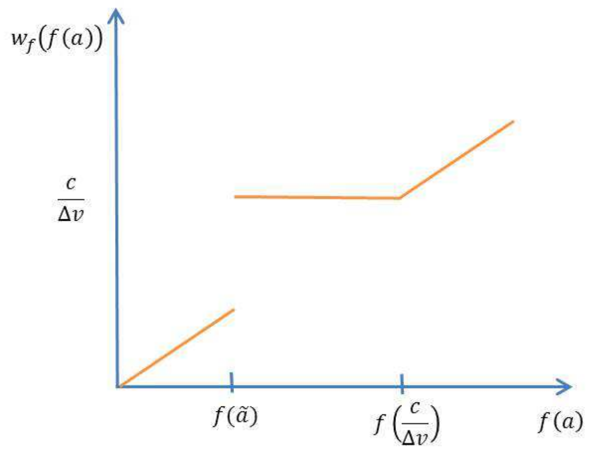
Proof. See Appendix □

Proposition 2.2 states that firms offer a wage schedule that is attractive enough to absorb every unemployed agent. From the point of view of a worker of productivity a, only two points of this wage schedule matter: $w_f\left(f\left(a\right)\right)$ and $w_f\left(0\right)$. The wage schedule induces incentives only to the most talented agents, in particular those whose entrepreneurial talent is higher than $\tilde{a}\left(\cdot\right)$, and thus producing above the threshold $f\left(\tilde{a}\left(\cdot\right)\right)$. This relates to lemma 2.1; the cost of providing incentives in non increasing in a. In fact, when the entrepreneurial talent of the agent is sufficiently large, his participation constraint (outside opportunity as self-entrepreneur) binds while the incentive constraint ceases to bind. Only workers whose entrepreneurial talent is on the range $\left[\tilde{a}(\cdot), \frac{c}{\Delta v}\right]$ expect to get an information rent as a reward additional to the one needed to attract them.

Finally, workers of low entrepreneurial talent do not find the wage schedule incentive compatible, and thus prefer to shirk after accepting the offer. Firms still find it profitable to contract with them, since $f(a) > a$. However, the cost of effort plus the information rents needed to induce effort are not outweighed by the expected increase in output. Hence, firms maximizing profits would rather not offer an incentive compatible wage for agents producing less than $f(\tilde{a}(\cdot))$.

Figure 2.1 depicts the private sector wage structure. For low levels of observed production the wage is only high enough to make agents participate. However, when the observed production is higher than $f(\tilde{a}(\cdot))$, the wage schedule jumps to the minimum incentive compatible levels: $\frac{c}{\Delta v}$. This wage level remains constant until production is $f\left(\frac{c}{\Delta v}\right)$. Production surpassing this threshold can only come from workers whose talent and outside opportunity are very high. Hence, above these levels the participation constraint is a more stringent requirement than the incentive constraint.

Figure 2.1: Private Sector Wage Schedule



Corollary 2.2. $f(\tilde{a}(\cdot)) > \frac{c}{\Delta v}$.

Finally, corollary 2.2 expresses that the level of production that justifies inducing effort is higher than the first-best level. This result is standard, and arises from the additional information rent that workers require in exchange for their effort.

2.4.3 The Choice of the Government

The government's choice is to fix a wage, a government size and lump sum taxes to maximize the expected social welfare. Workers decide whether to apply or not to a public job by considering their outside opportunity, which is their entrepreneurial talent and the job offer they may get in the private sector. In order to find the best decision for the government, we first consider the social welfare if the government does not offer incentives, and then we compare it with the social welfare when civil servants are incentivized.

Without loss of generality, we focus on offers in which $w_g(0) = 0$ and the expected wage required to attract agents is fully driven by $w_g(f_g)$. For a given expected wage level $w_g(f_g)$, the mass of applications for public jobs is given by:

$$\begin{cases} w_g(f_g) & \text{if } w_g(f_g) \leq \tilde{a}(\cdot) \\ \tilde{a}(\cdot) & \text{if } \tilde{a}(\cdot) < w_g(f_g) < \frac{c}{\Delta v} \\ w_g(f_g) & \text{if } w_g(f_g) \geq \frac{c}{\Delta v}. \end{cases} \tag{2.2}$$

The applications for public jobs do not respond to wage increases within the interval $\left[\tilde{a}(\cdot), \frac{c}{\Delta v}\right]$. Indeed, agents whose outside opportunity is larger than $\tilde{a}(\cdot)$ expect to earn incentive rents in the private sector, and thus would not apply to a public job unless the wage matches the one in the private sector. When the wage associated to a successful public technology reaches $w_g(f_g) = \frac{c}{\Delta v}$, agents within the range $a \in \left[\tilde{a}(\cdot), \underline{v}\left(\frac{c}{\Delta v}\right)\right]$ are indifferent between working for the private or public sector. As in Jaimovich and Rud (2014), we look at the equilibrium in which agents resolve their indifference by applying to the public job first, and in case they are rejected, they also apply to a private job. It must be noted that this is only a weak-best response for agents, since they have for granted that they will have such an offer if they just apply to a private job. However, one could easily make tiny modifications to the model structure in order to break this indifference. For example, if we add some frictional unemployment by assuming that a random small percentage of agents will not be able to apply for the private job and would have to become self-entrepreneurs, then every agent within this range would strictly prefer to make both job applications. Also, if agents had some public service motivation of the kind described in Delfgaauw and Dur (2010) in which agents perceive some utility when working for the government, one would also expect a similar response to job applications.

2.4.3.1 No incentives

When the government does not provide incentives, the wage is used just to attract workers. Given that incentive compatibility is not satisfied, the way the wage is structured (in terms of reward/penalty) has no meaningful role to play. The only relevant decisions are how high the expected wages are going to be and the size of the government.

Lemma 2.2. *A government restricted not to provide incentives chooses* $w(f_g) = n^{NI}$ *and* $t = \underline{v} n^{NI} \frac{n^{NI}}{\tilde{a}}$, *where* $n^{NI} = \min\{f^{-1}(f_g), \tilde{a}(\cdot)\}$.

Proof. See Appendix □

Lemma 2.2 characterizes a wage schedule that implements the best choice for the government among the ones that do not induce incentives in agents. Such wage is high enough to attract as many workers as the ones needed to fill the government size n^{NI}. A higher wage would produce a costly misallocation of talents, and a lower wage would not attract enough agents. The size n^{NI} equals the first-best size when it is possible to attract such mass of workers with a wage lower than the incentive compatibility levels. However, when $n^{FB} > \tilde{a}(\cdot)$, the government is not able to attract enough agents because of the discontinuity in their outside opportunity given by private sector offers, as described in equation 2.2. In this case, n^{NI} is determined by the corner solution $n^{NI} = \tilde{a}(\cdot)$. The welfare of the economy, when adopting a no-incentive policy, boils down to:

$$W_{NI} = n^{NI}(\underline{v}) f_g + \int_{n^{NI}}^{\tilde{a}(\cdot)} \underline{v} f(a)\, da + \int_{\tilde{a}(\cdot)}^{\overline{a}} (\overline{v} f(a) - c)\, da. \tag{2.3}$$

Finally, lump sum taxes can be recovered from the budget constraint of the government. A lump sum tax equal to $t = \underline{v} n^{NI} \frac{n^{NI}}{\overline{a}}$ is charged to each agent in order to cover the cost of public employees.

2.4.3.2 Efficient Government

We now look at the optimal labor policy when the government is constrained to providing incentives. On the bright side, the output of a civil servant is high enough so that the increase in the probability of success outweighs the cost of effort. On the other side, the government would have to pay information rents to induce effort in civil servants, which attract agents that would be more productive in the private sector. In order to simplify the notation, we define:

$$F(r) = \int_{0}^{\tilde{a}(\cdot)} \underline{v} f(a)\, da + \int_{\tilde{a}(\cdot)}^{r} (\overline{v} f(a) - c)\, da$$

as the total welfare generated by agents in the set $[0, r]$ if they all work in private firms.

Lemma 2.3. *If $\overline{v} f_g - c < \frac{F\left(\frac{c}{\Delta}\right)}{\frac{c}{\Delta}}$, the government is shut down ($n^I = 0$). If $\overline{v} f_g - c > \frac{F\left(\frac{c}{\Delta}\right)}{\frac{c}{\Delta}}$, then $w(f_g) = \frac{c}{\triangle v}$ and $n^I = \frac{c}{\Delta v}$. Taxes are given by $t = \overline{v} n^I \frac{n^I}{\overline{a}}$.*

Proof. See Appendix □

Lemma 2.3 characterizes an optimal wage schedule and government size that induces effort. The limited liability forces the government to set the wage at a minimum of $w(f_g) = \frac{c}{\triangle v}$ to induce effort. As a consequence, following equation 2.2, a mass of $\frac{c}{\triangle v}$ applies for the jobs. If the government wanted to fix a size smaller than this mass, there would inevitably be rationing. In this case, the government would lose control of the allocation of talents in the economy. Only n agents would be allocated a public job, out of a mass of $\frac{c}{\triangle v}$ agents with equal probability of being selected. From the point of view of the government, every additional agent employed in the public sector generates a social gain equal to $\overline{v}f_g - c$, but ceases to generate $\frac{F\left(\frac{c}{\Delta}\right)}{\frac{c}{\Delta}}$, the average welfare coming from private goods production of applicants. Hence, the choice of the government has a bang-bang solution. If the average private sector productivity is sufficiently high, the best option is to shut down the government, whereas, in the opposite case, the government would rather enlarge its size. In particular, it is verified that when $\frac{F\left(\frac{c}{\Delta}\right)}{\frac{c}{\Delta}} < \overline{v}f_g - c$, the government size is larger than in the first-best benchmark, $n^I > n^{FB}$. In other words, when the government decides to give incentives, it also enlarges the size of the government to avoid misallocation of talents.

Finally, it is worth mentioning that, even though job rationing plays a central role in the policy choice, there is no rationing in the equilibrium policy choice. When $\frac{F\left(\frac{c}{\Delta}\right)}{\frac{c}{\Delta}} < \overline{v}f_g - c$, the welfare function with incentives boils down to:

$$W_I = \frac{c}{\triangle v}\left(\overline{v}f_g - c\right) + \int_{\frac{c}{\triangle v}}^{\overline{a}} \left(\overline{v}f\left(a\right) - c\right) da. \tag{2.4}$$

Otherwise, the government shuts down and the welfare function represents only the private sector surplus, $F\left(\bar{a}\right)$.

2.4.3.3 Optimal Incentive Scheme for Public Workers

When deciding on the optimal incentive scheme for civil servants, the government weights the benefits and costs of providing incentives. We restrict attention to pure incentive schemes, in which the government gives all employees the same scheme. The costs arise from a suboptimal choice of government size as a response to the misallocation of talents. The benefits are efficiency gains in the production of public goods. The choice of the government is given by comparing W_I and W_{NI}. In particular, the government prefers not to give incentives if:

$$W_{NI} - W_I \geq 0.$$

Proposition 2.3. *There exists a threshold y_g, such that if $f_g \geq y_g$, incentives are optimal in the public sector, and if $f_g < y_g$, the optimal policy is not to give incentives to public workers. Moreover, $y_g > \frac{c}{\triangle v}$.*

Proof. The proof is structured as follows: we find upper and lower bounds for the difference $W_I - W_{NI}$, and we prove that the difference is increasing in f_g. When f_g is at its lower bound, the difference is negative, whereas when it is at the higher bound, it is positive. We conclude by applying the intermediate value theorem. The full proof is developed in the appendix. □

Proposition 2.3 relates the choice of the optimal incentive policy of the government with the production per public employee. The first thing to highlight is that, as in the case of private firms, moral hazard increases the production threshold for providing incentives. Recall that in the private information case, incentives were optimal if $f_g \geq \frac{c}{\triangle v}$. In turn, proposition 2.3 states that when $f_g = \frac{c}{\triangle v}$, the government would rather not provide incentives to the workers. Hence, the use of information rents also introduces additional costs to the government, and pushes up the production threshold.

Government shutdown is never optimal, since the provision of the public good is valuable even when the process is not very efficient. The decision of providing incentives weights the public sector efficiency with the average lost productivity from agents that would otherwise work in the public sector. The government chooses between a more productive and larger State and a less productive and smaller one that extracts fewer resources from the private sector. A key factor is that incentivizing agents causes the government to attract too many agents to the public sector, which it would rather hire than choosing a smaller government size and cause misallocation of talents. As a result, the government has to select among an excessively large size but with incentivized workers or a closer to optimal size government with low powered workers.

How does the allocation with moral hazard compare with the first-best? Both in the cases that the government decides to provide incentives and in the ones that it does not, there is efficiency loss. If the government does not incentivize agents, there is efficiency loss coming from a less productive public sector. In addition, if $n^{NI} = \tilde{a}(\cdot)$, the welfare is also reduced for having a smaller than optimal government. In contrast, when the government does provide incentives, welfare is affected by an excessively large government, which crowds out more valuable private sector production.

2.5 Policy Discussions

2.5.1 The Cost of Incentives in the Public and the Private Sector

One of the salient characteristics of the analysis above is that the way the public and the private sectors evaluate the costs of providing incentives can differ substantially. On the one hand, there is a very sharp distinction between in the cost of incentives for small companies (self-entrepreneurs) and large organizations (private firms and the government), for an ownership dilution argument in the spirit of Alchian and Demsetz (1972). Self-entrepreneurs are residual claimants of the effects of their effort, and thus they choose to exert it at the first-best level. On the contrary, in large organizations effort can be induced only in exchange of information rents, which increases the cost of incentives.

On the other hand, even when large organizations have to pay equal wages to induce effort, our results show that the interpretation of this cost can be substantially different from the general equilibrium point of view of the government and the partial equilibrium point of view of the firms. If one were to evaluate the decision of the public sector to provide incentive contracts from the optic of the private sector, one could conclude wrongly that the public sector is excessively inefficient or efficient. The underlying assumption in this reasoning is that the threshold for providing incentives is the same in either sector. In other words, it is tempting to infer that two jobs sharing the same triplet $\{y, c, \Delta v\}$ in the private and the public sector should face the same decision on incentives.

We now investigate the following question. For a given moral hazard problem characterized by the same pair $\{c, \Delta v\}$, are the thresholds for providing incentives, y, equal in the private and in the public sector?[9]

Suppose first that the effort is observable. In this case, whatever the form of production is (private sector, public sector and self-entrepreneur), the same triplet $\{y, c, \Delta v\}$ yields the same decision on whether to induce effort or not. As explained in section 2.3, the threshold is equivalent in either sector. The reason for this equivalence is that the private cost of effort is equally transferred to the principal in either case. Private firms would have to increase the wage of the agent to make him do the effort, and they would perceive this wage increase as a cost. On the other hand, the government would also perceive the cost of effort of the worker in its entirety but through a different channel. Higher wages are viewed as mere transfers among agents, and thus they do not impact the government's welfare. However, a benevolent government does perceive the private cost of effort as a social cost, and would only judge it necessary if outweighed by the the expected increase in public goods production $\triangle v f_g$. Summing up, when the effort is observable, the the production threshold under which the effort is valuable is the same in the private and in the public sector.

The answer is different when the effort is hidden information. It is trivial to point out that the incentive decision for self-employed and private employees are no longer equivalent. This is by no means a surprising result, and it is just a consequence of the information rents. Self employed agents do not have to deal with a moral hazard problem, while firms do.[10] The question is less obvious when comparing the decision in private firms and in the public sector. Proposition 2.3 gives conditions on when incentives are optimal in the public sector, but it does not compare these conditions with the ones prevailing in the private sector. In order to answer this question explicitly,

[9]This question involves comparing the production of two different goods in a non market economy. Given the linear form of utilities assumed and the utilitarian nature of the welfare function, public and private sector production quantities are comparable from the point of view the government. The social value of the a amount x of the private good is equal to the social value of an amount x of the public good.

[10]More specifically, for the same pair $\{c, \Delta v\}$, the level of production that justifies doing a high effort is $\frac{c}{\Delta v}$ for the self employed and $f(\tilde{a}(\cdot))$ for firms, with $f(\tilde{a}(\cdot)) > \frac{c}{\Delta v}$.

we add more structure to our model:

Assumption 2.5. $f(a) = ka$ *with* $k > 1$.

Assumption 2.5 states that private firms technology is linear in a. This assumption replaces assumption 2.2, as it is a narrower statement concerning the functional form of $f(a)$, and allows us to get explicit conditions on the incentives decision of the government.

Proposition 2.4. *The level of worker's production that justifies using an incentive contract is higher in the public than in the private sector if* $k \geq \tilde{k}$ *and lower if* $k < \tilde{k}$, *with* $\tilde{k} = \frac{1}{2}\left(1 + \sqrt{\frac{(\overline{v}+7\underline{v})}{\Delta v}}\right)$.

Proof. See appendix. □

Proposition 2.4 gives conditions under which the level of production justifying worker's effort is higher in the public than in the private sector. A remarkable aspect of this result is that the same triplet $\{y, c, \Delta v\}$ may result in a different decision on incentives in each sector. Hence, the trigger equivalence does not hold in this case unless by coincidence, as a consequence on the differences on how the information rents translates into costs for each sector. A key implication of our analysis is that, when effort is unobservable, the incentives decisions in each sector are different, and the comparison of private and public sector practices may be more complex than expected. In this sense, our work shares the word of caution with Moszoro and Spiller (2012) concerning the limits of comparing and importing public/private sector contracting practices.

2.5.2 Misallocation of Talents and the Size of the Government

In the baseline model presented so far, misallocation of talents arises from an excessively large public sector when the government decides to provide incentives. Although this may be understood as a poor allocation of talents, the labor market is still sorted in the appropriate way in equilibrium. Only the most innovative agents work in the private sector, but the threshold that separates each occupational choice is suboptimal from a welfare point of view.

However, there are a number of situations in which the problem of misallocation of talents can be larger when incentives are provided. Notably, if increasing the government size is not possible or too costly for reasons other than the allocation of talents, then the sorting problem would become more severe. Suppose, as in Acemoglu and Verdier (1998), that the maximum capacity of the government is fixed at $n < \frac{c}{\triangle v}$. In this case, the government cannot absorb all the agents applying for public jobs due to capacity restrictions, and a fraction $\frac{\frac{c}{\triangle v} - n}{\frac{c}{\triangle v}}$ will end up working in the private sector given a random rationing rule.[11] The misallocation problem becomes more severe because labor markets are not sorted in the appropriate way any more: very low entrepreneurial agents may end up working in the private sector. In fact, the proof of lemma 2.3 shows that without capacity constraints, the government is better off by increasing its size to $\frac{c}{\triangle v}$. This discussion may also shed light on usual policy recommendations, such as downsizing the State as a way to achieve efficiency. In our model, downsizing an efficient government to the first-best size turns out to be counterproductive.

In these situations, the government would become more reluctant to use incentive contracts, as the misallocation problem would be more severe. An inappropriate sorting is not only a bad thing for the allocation of talents, but it may also bring tax affordability issues. If very low entrepreneurial talent agents are allocated to the private sector, their wage or wealth may not be high enough to afford taxes in expectation. If so, it may not even be possible for the government to fund the wage increases necessary to induce incentives. This issue may be especially relevant in developing countries, in which agent's initial wealth is low.[12]

When the capacity of the government is not constrained, our model predicts that the decision of giving incentives is tied to choosing a larger size. Using our notation, $n^I > n^{NI}$. If our theory was

[11]Similar results would hold if the government's productivity was a decreasing function of the size n, or if taxes were associated with negative distortions in the economy.

[12]The issue of low wealth and allocation of talents may have other effects, as in Bianchi (2012). In that work, in developing countries where wealth is low, high entrepreneurial talent agents may not have the means to create their firms, which in turn lowers jobs demand and forces low talent agents to become self-entrepreneurs. This, together with other issues not taken into account in our framework such as corruption, makes our theory more suitable to developed than developing countries.

true, one should observe a positive correlation between the size of the government and its efficiency. This correlation, a by-product of our theory, can be the basis for an empirical assessment of the problem. Although this prediction may sound counter-intuitive, it is consistent with the evidence explored by La Porta et al. (1999). Using various and very complex measures of government performance, they systematically found a positive correlation between government performance and size. In spite of the fact that they found this correlation as something that appeared with different measures of efficiency and performance, they left open the question of its fundamental driver. Our work provides an explanation for this correlation, although it shares the word of caution regarding the policy inferences one could make from it. By no means our work implies that increasing the size of the government can be taken as a recipe for increasing efficiency. Rather than that, we pose that an excessively large government may be a consequence (not a cause) of choosing to give incentives to public workers.[13]

2.6 Extensions

2.6.1 Other forms of Taxation

We assumed so far that the government collects lump sum taxes to finance public wages. Although this simplifies the exposition, one could conjecture that more complex tax schemes can be used to have greater control of the allocation of talents and alleviate the cost of incentives for the government. In this section, we explore the feasibility of achieving a better allocation by using richer tax schemes.

We allow the government to charge taxes contingent on wages. This means that taxes are a function $t : \mathbb{R}_+ \to \mathbb{R}_+$ that map wage levels into tax levels. How can contingent taxes improve the allocation of talents? The government could attempt to use tax differentials to discourage certain

[13]This statement is better suited to developed countries. High levels of corruption have been associated with an increased government size as well, which may distort the overall conclusions. See, for example Niskanen (1971), Gelb et al. (1991) and Jaimovich and Rud (2014).

types of workers to apply for public jobs. In particular, for a size $n < \frac{c}{\triangle v}$, the government could try to use taxes to discourage applications from workers in the range $a \in \left[n, \frac{c}{\triangle v}\right]$, while still attracting workers in the range $[0, n]$.

Proposition 2.5. *Taxing wages cannot improve the allocation of talents if* $f_g \geq f(\tilde{a}(\cdot))$.

Proof. We analyze if the government can benefit through effectively setting $n < n^I = \frac{c}{\triangle v}$ and reducing the applications correspondingly. The allocation of talents can be improved only if some $a \in \left[n, \frac{c}{\triangle v}\right]$ is no longer interested in applying for a public job after considering wage-contingent taxes. We look at 2 cases:

Case 1: If $n > \tilde{a}(\cdot)$, the outside opportunity of workers of productivity within $\left[n, \frac{c}{\triangle v}\right]$ is the incentive compatible wages from the private sector:

$$\bar{v}(w_g - t(w_g)) - c < \overline{v}(w_f - t(w_f)) - c.$$

Note that incentive compatibility in the private sector means that $(w_f - t(w_f)) = \frac{c}{\triangle v}$. This contradicts the fact that the government is offering incentives to agents. The impossibility arises from the fact that workers within this range expect the same wage in both sectors, and cannot be screened.

Case 2: If $n < \tilde{a}(\cdot)$, the government could try to to discourage applications from workers in $[n, \tilde{a}(\cdot)]$. This would mean that for some a in this range:

$$\bar{v}(w_g - t(w_g)) - c < \underline{v}a - \underline{v}t(a).$$

Charging a very low tax on wages $[n, \tilde{a}(\cdot)]$ may have an impact on applications. Now, this change in the allocation of talents produces the welfare change of $\underline{v}f(a) - \overline{v}f_g + c$. This welfare change is negative if $\underline{v}f(\tilde{a}(\cdot)) < \overline{v}f_g - c$ which means that the allocation of talents is worse in this case. A sufficient condition for this is that $f_g \geq f(\tilde{a}(\cdot))$ □

Proposition 2.5 states that using wage contingent taxation cannot improve the allocation of talents if $f_g \geq f(\tilde{a}(\cdot))$. This means that this form of taxation is not useful if the productivity of public workers is comparable to levels that would justify incentive contracts in the private sector. The key intuition of this result is that taxes on wage cannot be used to screen out agents in the range $\left[\tilde{a}(\cdot), \frac{c}{\triangle v}\right]$, which are the ones generating a larger missalocation of talents. These agents would receive the incentive compatible level of wages in both sectors. Given that the private sector would fix the net wages at the minimum possible incentive compatible level, the government cannot achieve at the same time lower net wages and incentive compatibility. On the contrary, when $f_g \leq f(\tilde{a}(\cdot))$, the government may be able to obtain some efficiency gains by screening out applications from undesired agents with intermediate levels of talent. However the allocation of talents problem would remain, as agents in the range $\left[\tilde{a}(\cdot), \frac{c}{\triangle v}\right]$ would not be screened out in this case either. Moreover, the low productivity of the government when the condition $f_g \leq f(\tilde{a}(\cdot))$ is met would make unlikely that incentives prevail in the public sector.

2.7 Conclusions

This works is the first one to compare the incentives policies in the private and the public sector in a general equilibrium framework. Previous work has attributed the inefficiency of public contracting to the specific hazards that public contracts face. In contrast, our analysis shows that public contracts may be inefficient even when workers' hidden effort has equal characteristics in the production of public goods than in the private sector production. This result derives from the fact the the government may have only one instrument (wages) to control two conflicting variables: incentives and the allocation of talents. High wages are needed to induce incentives, but this can attract agents whose productivity is higher in the private sector. Our results suggest that public sector inefficiency may be a social cost that is necessary to afford in order to have a highly productive private sector.

A fundamental implication of this paper is that policymakers should be careful about importing private sector practices into the public sector. High powered incentives may not be optimal in the public sector, even in situations in which they are in the private sector. The way each sector internalizes the cost of information rents is substantially different, and thus it is natural that their optimal incentive policies diverge. In our work, this is reflected in that the production threshold that justifies the use of incentive policies is sector-specific, and they should not be assumed equal.

Another important derivation of this work is that governments will suffer an "inefficiency curse". Either they are not going to produce public goods in an efficient way, or they are going to be too large, thereby absorbing too many talents from the private sector. This may explain the inefficiency reputation that the government suffers from everywhere in the world, even in countries with relatively efficient production of public goods. This interpretation is consistent with the evidence provided by La Porta et al. (1999).

Finally, even though the government may be perceived as persistently inefficient, its role in the production of public goods remains fundamental and its existence is welfare improving. Our work suggests that policymakers should also be careful about downsizing governments to its first-best size, especially if the production of public goods is efficient. An excessively large government may be a way of avoiding a potentially more harmful misallocation of talents problem due to job rationing, when civil servants are effectively incentivized.

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Chapter 3

Oil Extraction Contracts and the Expropriation Premium

3.1 Introduction

Projects in the oil sector typically require large capital investments and know-how, which are generally not available locally in developing countries. Local governments delegate the projects to highly specialized international investors and benefit from the surplus that they can extract from them. When the welfare of foreign investors is not a concern of local governments, the investors willingness to participate in the project relies on the ability of the governments to respect contracts. There are two aspects that make contracting particularly difficult and atypical in this framework. First, random exogenous prices can generate abnormally high profits for the investor, and thus make expropriation attractive. Second, due to unstable political conditions in developing countries, investors may have to deal with governments of different degree of commitment.

We model the relation of an investor and a risk neutral government that is privately informed about its cost of expropriation. The government's decision to expropriate is related to the realization of international prices and its private cost of expropriation, which can be either high or low.

The investor would like to participate in the project when dealing with a high commitment (or high cost of expropriation) government, but would not be able to recover his investment if he is dealing with a low commitment one. In this framework, we look at the optimal contracting problem when the governments do not have enough contracting tools to separate themselves at the contract offering stage. Still, a project can be feasible when government types pool their offers at the cost of compensating the investor with an expropriation premium. We find that optimal contracts use price contingent taxation to make the investor participate and minimize the ex-ante probability of expropriation. This is achieved by setting high taxes to maintain the profits of the investor low enough to be credibly honored by both types of governments, and compensate the investor with higher profits in high prices states of nature. In the latter case, the investor is expropriated if he is dealing with a low commitment government, but he gets high profits otherwise. An optimal tax schedule can be implemented in a fairly simple way. Two different profits caps apply in low and high prices states of nature respectively: the investor is not taxed until his profits reach the cap, and government becomes a residual claimant of the profits exceeding the cap. Finally, we show that such optimal contract constitutes an equilibrium supported by reasonable off-path equilibrium beliefs (in the sense of Banks and Sobel (1987)), and that no other contract with strictly lower payoffs to each of the government types can be a part of an equilibrium supported by these beliefs.

Our work is motivated in the recent experience of expropriations of oil related assets in developing countries during a period of historically high international prices. Foreign investor's assets in developing countries are generally protected by bilateral investment treaties specifying that disputes should be resolved in foreign courts. However, these courts tend to have few enforcement tools in the host country, which limits the penalties that governments can face if they refuse to pay the compensation stipulated. Hence, even when courts rule a compensation equivalent to the expropriated contract, they may not be able to enforce it if the expropriated value is very large. For example, in the year 2007, Bolivia withdrew from the International Center for Settlement of investment Disputes (ICSID), the main court ruling international disputes, just before initiating large scale nationalizations in the oil sector. Similarly, Argentina refuses to pay adverse awards on past expropriations. Although in practice such decisions have effects on the ability of the country to get access to funds in international markets, the governments that expropriate may not be willing to make use of this possibility. For example, a government that its planning to live with its own means as a national project or that is already excluded from financial markets may face a significantly lower cost of expropriation. For this reason, political parties or government types may have different evaluation of costs of expropriation imposed by courts, even when the restrictions that they would face coincide.

This work provides new insights on how developing countries should attract investors. In contrast with what would prevail in developed countries with limited problems of credibility or political stability, offering generous deals is not the best way to ensure investor's participation. In fact, contracts in which government's take is low when the firms' profits are high are going to be largely discounted by investors who anticipate expropriation if the government is of low commitment type. Less generous contracts, which keep firms profits at levels at which both governments types can commit, generate a lower expropriation discount, and can be a more successful way to attract investors. In other words, our work suggests that minimizing the expropriation risk faced by the investor can be a more effective economic incentive than increasing their promised cash flows (by means of tax reductions). Even though participation is, indeed, limited by the degree of commitment of both types of government, we prove that credibility-oriented taxation can greatly affect the feasibility of projects.

The work is organized as follows. Section 3.2 reviews the existing literature related to this investigation. Section 3.3 describes the framework in detail and characterizes the solution to the symmetric information benchmark. Section 3.4 contains the main insights of this work, describing optimal and equilibrium contracts when the government is privately informed about its cost of expropriation. Finally, section 3.5 discusses strategies to attract investments involving high disbursements.

3.2 Related Economic Literature

This work relates to a growing literature that emerged after the recent wave of expropriations. From an empirical point of view, some authors studied the relation between of expropriations and the evolution of crude oil prices. For example, Boyarchenko (2007) finds that oil prices, together with other factors as exchange rate and the stock of long term liabilities, are key determinants of expropriations in the energy sector. Guriev et al. (2011) build and test a relational contracting model that predicts that expropriations, even though inefficient, can occur as equilibrium outcomes when oil prices are very high. They find evidence supporting that nationalizations tend to take place during price booms and in countries with weak institutions. Stroebel and van Benthem (2013) argue that, in practice, many expropriations occur because governments adopt linear rather than optimal contracts.

Our work relates to the literature on political risk associated with natural resources (Matsen et al., 2012), and more generally with populism (Acemoglu et al., 2012). This literature describes the misuse of resources for electoral motives with political uncertainty. In contrast, our investigation focuses on the effects on contract credibility that political uncertainty may have, and the degree to which this affects the feasibility of a project.

Our work is closely related to a number of authors looking at strategic responses to opportunistic expropriation. On the firm side, Thomas and Worrall (1994) find that an investor may decide to distort downwards the efficient path of investments so as to increase the trade-off faced by the government between short run temptation to expropriate and long run investments - attraction. Bohn and Deacon (2000) find evidence supporting that, when property rights are weak, investors slow down or increase the speed of exploitation depending on the kind of resource extraction industry. For example forests, which don't require large initial investments, are over exploited, while oil stocks remain under exploited. A similar reasoning has been recently pointed out by Aghion and Quesada (2010), who show that when investors anticipate a hold up problem, they invest less than optimally, which is a standard result of incentives theory. Our work abstracts from firm technological distortions, and concentrates on the interplay of different types of governments.

Closer to this paper are the works studying optimal contracting under the threat of expropriation. Engel and Fischer (2010) investigate which is the optimal contract that a government can offer to investors that face an exogenous risk of expropriation associated to the return on their projects. They find that profit capping contracts can be optimal if they can ensure the participation of the firm, due to the fact that they tend to reduce expropriation associated to a dead-weight loss. We differ from their work in that we endogenize the decision of expropriation, and we relate it with the dead weight loss that it causes. Wernerfelt and Zeckhauser (2010) extend the framework proposed by Engel and Fischer, and analyze the strategic responses that firms may adopt when investing in a country whose probability of expropriation is an increasing function of the profits. They show that the investors may take hidden action to choose suboptimal investments that reduce the dispersion and the expectation of profits. Similarly, investors may choose to delay the schedule of investments to prevent the ex-post expropriation problem.

Perhaps the closest work to ours is the recent paper by Stroebel and van Benthem (2013), who tackle both the determinants for expropriations and the optimal contracting problem. As in our work, they introduce asymmetric information on the government commitment level and describe how optimal contracts should be, focusing on risk allocation matters. Our approach differs from theirs in several aspects that make the results significantly different. Our main focus is to describe how a contract should be designed when one of the government types is not credible enough to attract the investor, even under symmetric information. This turns out to introduce discontinuities in the optimal tax schedule. Second, we fully characterize the optimal contract in the framework of an arbitrary continuous price distribution function, as oppose to their high-low price Bernoulli distribution. And finally, given that the contracting problem is one in which the principal has private information, we look at the problem from an equilibrium point of view, rather than just from a welfare point of view. Our work shows that the optimal contract constitutes an equilibrium supported by reasonable beliefs in the sense of Banks and Sobel (1987).

Finally, our work also relates to the literature of public contracting. This paper can be framed as a contracting response to government opportunism, described in Spiller (2008) as the fundamental risk of investors interacting with governments with limited commitment.

3.3 The Model

Consider a government that owns oil reserves but cannot extract them itself for technological reasons. A project can potentially produce q barrels of oil from those reserves, which we normalize to 1, that can later be sold in the international market at an exogenous price $p \sim F(p) : [\underline{p}, \overline{p}] \rightarrow [0, 1]$, a continuous cumulative function with probability distribution function $f(p)$ and expectation $E(p)$. The development of such project needs an ex ante investment, which costs I to an investor and $I_g > E(p)$ to the government, with $I < I_g$. Hence the only way this project can be feasible is if it is undertaken by international investors.

The government does not assign any weight to the investor's profits on its utility function, which is consistent with the idea that the investors are foreign. The only reason why the former would encourage investments is to extract part of the surplus from the latter. Moreover, we assume that the government is financially constrained, and cannot subsidize the project.

We assume that the government has all the bargaining power, and will make a take it or leave it offer to the investor. The offer consists on a tax schedule that can be potentially contingent on the price realization, $t(p) : p \rightarrow [0, p]$. The investor has an outside opportunity equal to zero, and undertakes the project only if he finds it profitable.

A key feature of this model is that the government takes an additional action if the project is done. After having observed the price, the government can expropriate if it considers that is profitable to do it. Expropriation consists on taking possession and control of the investments done by the investor.[1] Basically, the government will compare the tax revenues with the profits from

[1] A more general form expropriation can be thought as any unilateral change on the terms previously agreed. We could allow this possibility in our model without altering the results as long as the maximum expropriation

operating the company by itself, net of expropriation costs. The expropriation cost $c \in \{\underline{c}, \overline{c}\}$, with $\overline{c} > \underline{c}$, is private information of the government, and the investor assigns the prior probability α to the event that the government has a high expropriation cost and $1 - \alpha$ to the probability of $c = \underline{c}$. In addition, we assume that $\underline{c} < \underline{p}$, meaning that both types have some degree of commitment. If it expropriates, the government is able run the company. In addition, after observing an offer $t(p)$, the investor can update his beliefs and compute $\hat{\alpha}(t)$. Throughout the paper, we refer to type $\underline{c}$ as the "low cost", "low commitment" or "low credibility" government type. Similarly, through the text, we sometimes call "high cost", "high commitment" or simply "credible" government to the type with a cost $\overline{c}$.

The cost of expropriation c deserves special attention. We interpret this as the government's valuation of the maximum restrictions imposed by the international courts or other investors in the case of expropriation. For example, an international court may limit the access to financing from multilateral agencies, and private investors may require a higher premium to invest in the country. The degree to which this affects the government is directly linked to its financing needs and its commercial policy. A government that is planning to live with its own means may be less affected by such kind of penalties than one that is highly interconnected with the rest of the world. Hence, for the penalties that a government faces when expropriating, we could interpret $\overline{c}$ and $\underline{c}$ as two different functions of such penalties. Also, we do not rule out the possibility that c has an efficiency component, meaning that the government may have to face higher operating costs as compared to the investor. Although we believe that this may affect the magnitude of c, we do not see the efficiency component as a source of asymmetric information. In other words, we don't find any reason to claim that a government type can be more efficient than other operating an expropriated oil asset. Finally, we do not pose that the actual penalty that courts impose will always be constant, regardless the size of the expropriation. Rather, we view c as a measure of

costs that the government can face do not change. Minor changes in the contract could impose similar costs than those incurred if the government makes a radical expropriation of the property. Hence, the government would never choose to make small expropriations.

the maximum enforcements that third parties can impose on a government which is not respecting a contract. For example, if a government expropriates a property worth \$10 and a court can impose non pecuniary restrictions that can be at most worth \$5 for the government, even though the court's award may request a fair compensation of \$10 for the investor, the government would choose not to comply with it and face the non pecuniary restrictions.

The timing of the game is the following:

1. The government learns its cost of expropriation, and offers a tax schedule to the investor $t(p)$.

2. The investor chooses the probability of accepting $x \in [0,1]$ and making an investment I, given his beliefs $\hat{\alpha}(t(p))$ about the type of government.

3. The international price p is drawn from $f(p)$.

4. The government observes p and decides to expropriate with probability $e \in [0,1]$ and payoffs are realized.

Throughout the work, we sometimes alter slightly the structure of the game to refer to the simpler case in which the contract is offered before the government learns its private information. This alternative structure depicts the case in which the authorities change between the period when the investment is done and the one in which the oil is sold. This could happen, for example, if the investor is offered a contract before elections, but would sell the the oil once new authorities have been elected. In this case, the investor would be concerned about the commitment ability of the following government, rather than one in office offering the tax schedule. The analysis of this alternative structure is simpler because the party who makes the offer is not informed about its type at the contract offering stage.

3.3.1 Symmetric Information Benchmark

We begin by analyzing the result of the game with symmetric information, when there is a single, common knowledge c. There is still some uncertainty coming from the price realization, but both parties have the same individual beliefs about it.

In the last stage of the game, conditional on that the investor has accepted to participate, the government observes the price and decides whether to expropriate or not, by choosing e. Taking possession of the project and choosing $e = 1$ if $p - c > t(p)$ is a best response. In the rest of the cases, the government is weakly better of by respecting the contract and collecting $t(p)$. The investor anticipates this and only expects to receive his revenue net of taxes $(p - t(p))$ in the states of nature in which expropriation does not take place.[2]

For a given tax schedule $t(p)$, define the subsets $G(t)_- := \{p : p - c \leq t(p)\}$ and $G(t)_+ = \{p : p - c > t(p)\}$, such that $G(t)_- \cup G(t)_+ = [\underline{p}, \overline{p}]$. $G(t)_-$ is the subset of prices in which expropriation does not occur for a given tax schedule and cost of expropriation. The investor accepts the project as long as:

$$\pi(t(p), c) = \int_{G(t)_-} (p - t(p))\, dF(p) + \int_{G(t)_+} 0 dF(p) \geq I.$$

Lemma 3.1. *The profits schedule generated by a contract with expropriation can be reproduced by a contract without expropriation.*

Proof. See Appendix. □

The intuition for lemma 3.1 is quite simple. Take a contract which provides very high net profits to the investor in some realizations of p, such that expropriation occurs (and thus $G(t)_+ \neq \varnothing$). The investor would anticipate this, and would "discount" the profits expected for the contract, by considering that he would not get any proceeds from sales when he is expropriated, in the set $G(t)_+$. In turn, the government could offer a different contract that stipulated very high taxes

[2]In order to solve the game with symmetric information, we use the equilibrium concept of subgame perfection.

(and low profits) during such states of nature. If this was the case, the investor would expect to receive some small but positive profits while in the contract with expropriation he would expect to receive zero profits. Finally, the profits from the contract in which expropriation occurs could be reproduced by scaling down the profits in the rest of the states of nature.

Proposition 3.1. *A contract that induces investment can be implemented if* $I \leq I(c)$. *There is a set of optimal contracts characterized by* $T(p) = \{t(p) : E(p - t(p)) = I,\ p \geq t(p) \geq Max\{0, p - c\}\}$. *Expropriation does not occur in any state of nature if an optimal contract is implemented.*

Proof. At the contract offering stage, the government's problem is to maximize:

$$\underset{t(p)\in[0,p]}{Max}\ U(t(p), c) = \int_{G(t)_-} t(p)dF(p) + \int_{G(t)_+} (p - c)\, dF(p),$$

subject to the participation of the firm:

$$\int_{G(t)_-} (p - t(p))\, dF(p) \geq I.$$

Note that the objective of the government can be expressed as:

$$U(t(p), c) = E(p) - \int_{G(t)_-} (p - t(p))\, dF(p) - \int_{G(t)_+} cdF(p).$$

Assume by the moment that the contract is feasible, so that there exists $t(p)$ such that the investor wants to participate. Due to lemma 3.1, the government can avoid expropriation, which allows it to get rid of the possibility of paying the expropriation costs. The rent of the government is decreasing on the profits of the firm, so the participation constraint will bind. The optimal contract leaves utility $E(p) - I$ to the government, and no rent to the investor.

We now look at the issue of feasibility. Given that the c puts a limit on the credibility of the tax scheme, it affects the feasibility of a project with cost I. For each c, there is maximum level of profits that can be credibly given to an investor, and this is what we call $I(c)$:

$$I(c) = \int_{\underline{p}}^{\overline{p}} (p - Max\{p - c, 0\})\, dF(p) = \int_{\underline{p}}^{\overline{p}} (Min\{p, c\})\, dF(p)$$

Note that because of lemma 3.1, no contract with expropriation can give higher profits than one without expropriation. Finally, it is easy to see that no expropriation-proof contract can give higher profits to the investor since this contract is charging $t(p) = 0$ if $p \leq c$ and giving the maximum credible profit c if $p \geq c$. $\square$

Proposition 3.1 describes the set of optimal tax schedules when the cost of expropriation is common knowledge. Optimal contracts make the investor break even, and offer tax schedules which are credible for any price realization.[3]

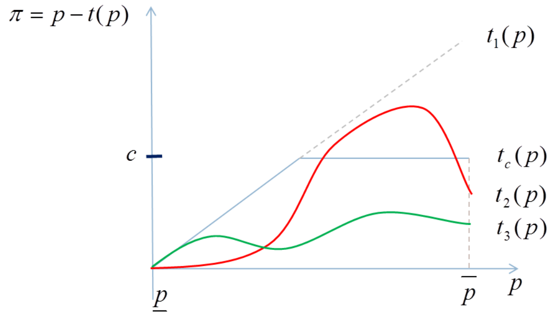
Figure 3.1 shows examples different tax schedules that can be offered to the investor: $t_1(p)$, $t_2(p)$, $t_3(p)$ and $t_c(p)$. In first place, $t_1(p)$ stipulates $t(p) = 0$ for each price, which implies no tax at all, and it is represented as a 45° line. Although naively this could seem attractive to the investors, it is actually a pretty bad deal for them. Whenever $p > c$, the government will not resist the temptation to expropriate, since the option value of it is very high as compared to the null tax collections.

Second, $t_2(p)$ (in red) is tax schedule that is not credible for intermediate realizations of prices, since $p - t(p) > c$. As a consequence of proposition 3.1, this tax schedule cannot be optimal.

Next, we have $t_3(p)$ and $t_c(p)$, two tax schedules that are credible for the entire domain of prices. The former leaves less profits to the investor, and thus it is a candidate optimal contract if the investment does not require very large disbursements. The tax schedule $t_c(p)$ has unique characteristics. It is the most generous tax schedule that can be credibly offered to an investor. It stipulates no taxes until the profits reach the credibility threshold c, and then caps the profits at c. The profits that the investor expect with this contract are $I(c)$, the upper bound of project feasibility.

[3]The optimal contract coincides with Engel and Fischer (2010)'s low demand case. The other cases are not compatible with our work, since expropriation occurs exogenously in their framework.

Figure 3.1: Contracts



The contract $I(c)$ dictates the maximum stream of profits that can be promised by a government with credibility c. This contract can be related to a debt contract in which the government promises the investor a fixed profit of c whenever its possible. However, if prices are lower than c, the investor will only receive the price. Hence, it can be understood as if the government enters into a debt contract with the investor, who funds and provides the technology for undertaking the project.

3.4 Asymmetric Information

We now look at the main purpose of this work, which is to describe contracts in the presence of asymmetric information. This case is interesting because is the situation that most of the investors face when they plan to invest in a country with a record of expropriations.

The first thing to note is that the resolution is not different from the symmetric information case if $I < I(\underline{c})$. If the project can be financed by means of a contract that is credible for both type of governments, then there is no scope for opportunistic behavior. The project can be done regardless the government type, and hence the asymmetry of information does not have any interesting effect. Similarly, if $I(\overline{c}) < I$, then the project is not feasible regardless the type of government in power.

The interesting case arises in the middle, when $I(\underline{c}) < I \leq I(\overline{c})$. In this case, only the government of high cost of expropriation is able to offer a deal that is attractive enough to make the investors undertake the project. However, the low cost government could pretend to be a credible government so as to get the project done, and later expropriate. This can be the source of information rents that will have as counterpart the need of providing more generous deals to the investors, a positive "expropriation premium". Hence, in this section we concentrate in this case, captured by the following assumption:

Assumption 3.1. $I\left(\underline{c}\right) < I \leq I\left(\overline{c}\right)$.

In order to facilitate the exposition, we define three mutually exclusive subsets of the set of prices relating to a tax scheme $t(p)$. Define:

$$\begin{cases} G(t)_{-} & p : p - t(p) \leq \underline{c} \\ \overline{G}(t) & p : \underline{c} < p - t(p) \leq \overline{c} \\ G(t)_{+} & p : \overline{c} < p - t(p). \end{cases}$$

The purpose of this subsets is to identify, for a given tax schedule, the price realizations in which each government type expropriates. The subset $G(t)_{-}$ represents those prices generating sufficiently low profits so that none of the two types of governments would expropriate. In second place, $\overline{G}(t)$ comprises the set of prices where profits are high enough to trigger expropriation from the low cost government, but they would not induce expropriation from the most credible government. Finally, we have $G(t)_{+}$, in which both governments expropriate.[4] Note that each tax schedule redefines these subsets, but the union of them always forms the set of possible prices.

We investigate the equilibrium contracts that may arise in this framework. We start by looking at the possibility of a separating equilibrium, in which each type of government offers a different contract. We then look at the optimal contract in a pooling situation. For the moment, the notion of equilibrium that we use is the Perfect Bayesian Equilibrium (PBE). In this context, a PBE is a composed by a commonly held distribution of beliefs $\hat{\alpha}\left(t\right)$ on the government type, and strategies

[4]Note that these subsets are associated with a choice of e by each government type which is a weak best response.

$(t^*(p), e^*|c)$, x^* of the government and the investor respectively, such that:

1. $t^*(p) \in \arg\max U(t(p), x^*(\hat{\alpha}(t)), e^*(p, t(p), c))$,
2. the investor chooses $x^* \in [0,1]$ as arg max $(\pi(t(p), \hat{\alpha}(t)) - I)x$,
3. the government chooses $e^* \in [0,1]$ as arg max $e(p-c) + (1-e)t(p)$,
4. $\hat{\alpha}(t)$ is computed using the Bayes rule whenever it is possible.

We simplify the notation by setting ex ante utility of the government in the following way: $U(t(p), x(\hat{\alpha}(t)), e^*(p, t(p), c)) = U(t(p), x(\hat{\alpha}(t)), c)$. Moreover, $\pi(t(p), \hat{\alpha}(t))$ are expected operating profits of the investor when he is offered the contract $t(p)$ and his beliefs are $\hat{\alpha}(t)$.

Proposition 3.2. *Under assumption 3.1, there does not exist a PBE with separation at the contract offering stage and in which the investor accepts at least one of the offers.*

Proof. Suppose it does. On the equilibrium path, the investor would apply Bayes rule and recognize with certainty the low credibility government. Thus, given assumption 3.1, he will assign probability 0 to accepting the contract, giving a payoff 0 to both parties. In turn, if the low cost government offers the same contract than the credible one, it will get:

$$U(t(p,\bar{c}), x=1, \underline{c}) = \int_{G(t)_-} t(p,\bar{c})dF(p) + \int_{\overline{G}(t)\cup G(t)_+} (p-\underline{c})\,dF(p) > 0,$$

where $t(p,\bar{c})$ is the contract offered by the credible government. This means that the low cost type has a profitable deviation. The existence of such positive deviation is guaranteed by the fact that the investor accepts the offer of the credible type of government, and by assumption 3.1 such offer cannot be credibly sustained by type $\underline{c}$. Hence, the second term of the expression above, which captures the proceeds from expropriation of the low type, is necessarily positive. The first term, which comprises the tax that the low type government would collect in the states of natures in which profits are low enough that it can commit not to expropriate, is non negative. □

Proposition 3.2 states that there cannot be an equilibrium in which each government type offers a different tax schedule at the initial stage. The investor is not interested in dealing with the government of low credibility, and assuming he would have been able to identify it at the contract offering stage, he would refuse to accept any offer from it. This sets a null opportunity cost of deviating from the equilibrium for the low credibility type and breaks incentive compatibility. The non credible government would always prefer to claim it is the credible type and obtain high proceeds from expropriation rather than not being able to contract with the foreign investor.

In order to explain the rationale of this result, it is useful to compare it with existing literature on the informed principal. Maskin and Tirole (1990, 1992) distinguish between the case of private and common values of an informed principal relation. In the private values case, the private information enters into the utility of the agent only through the behavior of the principal. On the contrary, in the common values case, the principal's information affects the agent's utility directly. From the agent's point of view at the contract offering stage, our framework does not fit well in any of those categories. On one hand, the agent is only concerned about the principal's behavior, which is clearly an element of a private value environment. On the other hand, keeping fixed the behavior of the principals at the contract offering stage (i.e., for a given tax schedule), the agent is in fact affected by the principal's private information. The key to understand this difference is that our framework is more complex than the standard informed principal environment: the game has an additional stage at which the private information becomes crucial. Overall, 3.2 is a consequence of the fact that the private information of the principal enters into the agent's utility only through the tax schedule.

3.4.1 Pooling Contracts and Expropriation Costs

When an investor faces a pooling contract interesting enough to make him participate, due to assumption 3.1 he knows that there is a risk of being expropriated. Expropriation is associated with two sorts of cost. On the one hand, expropriation involves a direct cost c which brings inefficiency to the transaction if it is incurred but could have been avoided. On the other hand, it generates a cost to the society if the government turns out to be of the high commitment type. Anticipating that the government could be of the low commitment type, the investor requires the credible government a high reward in order to undertake the project. We call expropriation premium to the difference between the profits that the most credible government has to give to the investor under asymmetric information and the profits that a government with the same cost of expropriation would need to promise under symmetric information. More precisely, for a given $t(p)$ offered by the high cost government and for beliefs $\hat{\alpha}(t)$, $1-\hat{\alpha}(t)$, we define the expropriation premium ρ as:

$$\rho = (1 - \hat{\alpha}(t)) \int_{\overline{G}(t)} (p - t(p))\, dF(p).$$

The interpretation of this premium is quite straightforward. When $p \in \overline{G}(t)$, one type of government expropriates and the other does not. Thus, the investor will discount the contractual values by $1-\hat{\alpha}(t)$, the probability of being expropriated according to his beliefs. The cost originated by this discount is completely borne by the credible government.

The expropriation premium captures the cost of information asymmetry and plays a key role in the equilibrium analysis. This premium can also be viewed as the negative externality that the non-credible government imposes on the high cost type. As can be seen from its expression, the magnitude of this cost is a function of the tax schedule that the the investor is offered in three different ways. First the tax schedule affects the profits of the investor that are subject to expropriation for a given price. Second, it defines the set $\overline{G}(t)$, the states of nature in which the premium arises. And finally, the tax schedule defines the beliefs on the type making the offer, which can have a dramatic effect on the size of the premium. If the credible government was able to choose a tax schedule which would generate beliefs $\hat{\alpha}(t) = 1$, then the expropriation premium would vanish. Although proposition 3.2 proves that this cannot happen in equilibrium, this can still be an interesting issue for out of equilibrium tax schedules and beliefs.

3.4.1.1 Optimal Contract

The presence of asymmetric information on the contract proposer side leads us to require out of equilibrium path beliefs. A first alternative would be to consider Perfect Bayesian Equilibrium (PBE), which would lead us to find an assessment of out of equilibrium beliefs and equilibrium strategies that are consistent with each other.

However, it is easy to see that PBE allows for infinite many pooling equilibria. In particular, consider an equilibrium given by a pooling contract $t^*(p)$ with a large expropriation premium and out of equilibrium beliefs $\hat{\alpha}(t') = 0$ for any $t'(p) \neq t^*(p)$. Under this specification, the investor would attribute to the non credible government any offer different from $t^*(p)$, and thus he would reject the offer with probability $x = 1$, giving 0 profits to all parties. Hence, as long as the expected profits $\pi(t^*(p), \hat{\alpha}(t^*)) \geq I$, he will accept the contract and this would be an equilibrium with participation.

Taking into account the multiplicity of equilibria, there are two interesting questions to be answered. On the one hand, we would like to know if one or some of these equilibria are preferable from a welfare point of view. On the other hand, we would like to know which of these allocations are robust to additional requirements on out of equilibrium beliefs, such as forward induction. In what remains of the paper we tackle both issues, and we focus on a simple way of contracting that has nice properties in the two dimensions.

Regarding the first of these issues, the definition of welfare is not obvious in this framework. On the one hand, one may view the low credibility government as a parasitical type who is only bringing inefficiency to the transaction, and thus there may be grounds for excluding it from welfare considerations. On the other hand, a contract designer concerned about the affected population would be inclined towards weighting this type as well, as this it may nevertheless represent the population.

We begin by taking the standard approach in the contract design by an informed party, and we look at the contract that a third party, who maximizes the expected welfare of the principals, would choose. However, we comment on the possibility that the contract designer wants to maximize only the welfare of the credible type. We assume that such third party is unbiased, meaning that he would not collude with any of the principals. Once we obtain this contract, we check if it constitutes an equilibrium of the informed principal game. We concentrate on pooling contracts given that we already know that no separating equilibrium exists.

This approach can also be interpreted as if the contract was designed by the principal before it receives its private information. If there are elections after the contract is signed but before the international price is realized, the authorities that contract with the investor are different from those that have the opportunity of expropriating. Given that neither the government that signs the contract nor the investor have private information about the cost of expropriation of the post elections authorities, the government would offer a contract that maximizes the expected welfare of the principals.

Lemma 3.2. *An optimal contract stipulates that the credible type never expropriates, minimizes the probability of expropriation by type* $\underline{c}$ *and makes the investor break even* $\pi\left(t^{*}\left(p\right),\alpha\right)=I$.

Proof. The problem of the contract designer is to choose $t\left(p\right)\geq 0$ to maximize:

$$\alpha U\left(t(p),x\left(\alpha\right),\overline{c}\right)+\left(1-\alpha\right)U\left(t(p),x\left(\alpha\right),\underline{c}\right)$$

and make the firm participate:

$$\pi\left(t\left(p\right),\alpha\right)=\int_{G(t)_{-}}\left(p-t\left(p\right)\right)dF(p)+\alpha\int_{\bar{G}(t)}\left(p-t\left(p\right)\right)dF(p)\geq I.$$

Note that the expected welfare function can be re expressed as:

$$E\left(p\right)-\pi\left(t\left(p\right),\alpha\right)-\left(1-\alpha\right)\int_{\bar{G}(t)}\underline{c}dF(p)-\int_{G(t)_{+}}\left(\alpha\overline{c}+\left(1-\alpha\right)\underline{c}\right)dF(p).$$

As in the symmetric information case, the last term can be taken to 0 without any cost, since the investor anticipates that he would be expropriated with probability 1 if $t(p) < p - \bar{c}$, and thus there are no gains from setting such a low tax. The third term represents a fixed cost weighted by the probability of expropriation of type $\underline{c}$. By assumption 3.1, we know that there does not exist any contract in which the firm participates and this term is 0. The problem of the planner boils down to minimizing the sum of $\pi(t(p), \alpha) + (1 - \alpha) \int_{\bar{G}(t)} \underline{c} dF(p)$. Note that $\pi(t(p), \alpha) > I$ cannot minimize this sum. The planner could simply set higher taxes, and make $\pi(t(p), \alpha) = I$ without increasing the second term. The problem reduces to finding $t(p)$ such that $\pi(t^*(p), \alpha) = I$ and $\int_{\bar{G}(t)} \underline{c} dF(p)$ is minimum. □

Lemma 3.2 provides a natural extension of the symmetric information case to the asymmetric one. There is no point of taxing too low when prices are high. The investor would simply not believe that the contract is honored in those states, and he would not count those promised profits in his investment decision. Assumption 3.1 states that the required investment is large enough so that it cannot be credibly contracted by the low type. Hence, any feasible contract requires the investor being compensated for the probability of being expropriated, or a positive expropriation premium. However, given that the contract designer weights the two types of government, only the expected expropriation cost $\underline{c}$ matters to him. When expropriation occurs, there is an additional social cost incurred, which if avoided it could report a higher welfare. Hence, an optimal contract stipulates a tax schedule that minimizes the probability of expropriation, among those that make the investor break even. By this means, the expropriation cost is incurred only when it is strictly necessary. The framework under consideration allows us to tell more about the shape of an optimal contract.

Proposition 3.3. *A project is feasible if $\underline{c} < \alpha\bar{c}$ and $I \leq I(\underline{c}, \bar{c})$. When these conditions are met, there exists a solution to the optimal contracting problem in which both government types offer $t^*(p) = Max\{0, p - \underline{c}\}$ for $p < p^*$ and $t^*(p) = Max\{0, p - \bar{c}\}$ for $p \geq p^*$, where p^* is the unique value in the set $\left[\frac{\underline{c}}{\alpha}, \bar{p}\right]$ making the investor break even. The investor accepts the offer, and expropriation occurs when the government is of type $\underline{c}$ and $p \geq p^*$.*

Proof. See Appendix. □

Proposition 3.3 is one of the main results of this paper. A project with cost I is feasible only if $\underline{c} < \alpha\overline{c}$. This condition means that the investor should be able to expect a substantial difference in profits if the government is of the credible type.[5] Otherwise, he will not be attracted to invest given assumption 3.1. As it was the case in the symmetric information benchmark, credibility issues limit the feasibility of projects, and thus there exist a maximum level of investment that can be achieved, which we call $I(\underline{c}, \overline{c})$. Naturally, with asymmetric information not only the cost of expropriation of the type in office matters, but also the cost of the other type. This last claim is a result of the pooling nature of feasible contracts.

Moreover, proposition 3.3 gives a functional form that solves the optimal contract problem when the project is feasible. The tax schedule initially caps the profits of the firm at $\underline{c}$, which ensures that no expropriation occurs for any of the types. The government does not collect any tax when prices are low (*i.e.*, lower than $\underline{c}$) and then taxes increase linearly so that the government extracts all the revenue exceeding $\underline{c}$. This cap applies for prices which are not very high; those lower than p^*. By fixing this cap, it avoids paying an expropriation costs for this level of prices, since both government types can commit to this level of profits. Unfortunately, assumption 3.1 makes it impossible that this cap applies to the entire domain of prices; there is no tax schedule that would make the investor participate and guarantee full commitment from the low cost government. In order to give the investor the expected profits he requires to undertake the project, the governments would offer the minimum taxes that are credible for type $\overline{c}$, but only when prices are higher than p^*. This means that when prices are higher than the threshold p^* taxes jump down, to the lowest possible level that make the contract credible for the high cost type. Hence, this contract is characterized by two profit caps implemented through contingent taxation, that apply in different

[5] Although this condition may sound extreme, it is consistent with Latin American politics experiences. Governments often shift from extreme open market postures to moderate to extreme levels of nationalism. In fact, not even the party to which a candidate belongs can be associated with a given posture. For example, presidents Menem and Kirchner in Argentina belonged to the same political party, but adopted opposite policies in terms of breaching contracts with foreign investors.

regions of the set of prices. When $p^* = \frac{\underline{c}}{\alpha}$, this contract allows the investor to recover the most expensive feasible project, $I(\underline{c}, \overline{c})$.

Why is this contract optimal? Lemma 3.2 states that the contract designer would like to minimize the probability of expropriation among all contracts that make the investor participate. The contract described in proposition 3.3 involves a credible profit stream for most states of nature, and higher profits when the prices are high. For most states of nature, the investor expects to receive the highest profits that both governments can commit to (*i.e.*, in the set $G(t^*)_-$). This reduces the need of fixing taxes that generate profits to which only type $\overline{c}$ can commit to, as compared to profits streams that involve higher taxes within the credible set of prices. Finally, when prices are higher than p^*, the contract described on proposition 3.3 make profits jump up to the maximum level that can be credibly contracted upon by type $\overline{c}$. This ensures that the probability of expropriation is not going to be larger than the minimum necessary. From the investor point of view, he would not expect to be expropriated in most states of nature. With probability $(1-\alpha)(1-F(p^*))$ he expects to be expropriated, but with probability $\alpha(1-F(p^*))$ he expects to receive the highest profits that the credible government can commit to. Even though $\alpha(1-F(p^*))$ may be small, the profits that the investor perceives if he is not expropriated would be large enough to make the offer attractive.

The contract in proposition 3.3 can be understood as debt contract including a bonus payment when prices are very high. This is, the contract involves a constant payment of $\underline{c}$ if the price is below p^*, which is increased to $\overline{c}$ when prices are above that threshold. However, in either case, the investor only receives the price if it is not high enough to cover the constant payment.

Finally, it is worth noting that the contract described in proposition 3.3 is not necessarily unique. For example, an alternative contract that divides the price regions in which the each cap of constant payment applies in another way may also be optimal. However, the existence of such alternative contract is not guaranteed. Moreover, every optimal contract divides the expected surplus among investor, credible and non credible government in the same way, giving the three parties the same expected payments. Given that other optimal contracts are qualitatively similar to the one we describe, throughout the rest of the text, we sometimes refer to the contract in proposition 3.3 as the optimal contract.

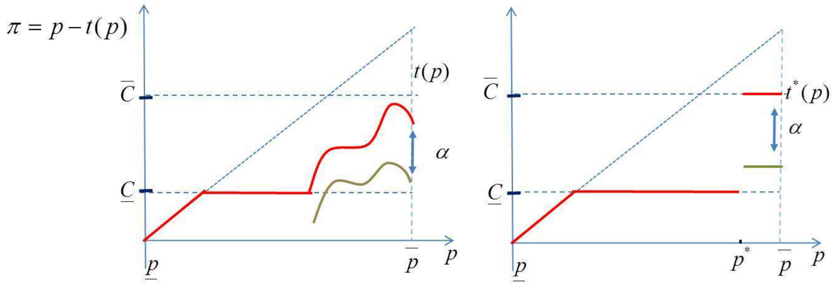
Corollary 3.1. *An optimal contract minimizes the expropriation premium born by the credible type among the tax schedules that make the investor break even with beliefs* α*.*

Proof. See Appendix □

The reader may be puzzled at first glance about how is it that this tax schedule relates to the expropriation premium. In particular, given that the expropriation premium is larger when the profits of the investor are larger within the set $\overline{G}(t)$, it may not be clear that taxes as low as possible are, in fact, what makes the premium minimal. Suppose, in turn, that for some subset of prices, taxes were $p - \underline{c} > t^*(p) > p - \overline{c}$, so that they would be higher than the minimum level that can be credibly offered by the high commitment type, but not high enough to make them credible to the type $\underline{c}$. As expropriation is a possibility with this tax schedule, the investor would discount the contractual profits by a factor of $1 - \alpha$, and hence contribute to the size of the expropriation premium. If this were the case, the contract designer could create another tax schedule which allows the investor to collect the same profits with a lower expropriation discount. This can be done by setting taxes at the minimum that both types of governments can commit to for a larger subset of prices, and then lower taxes to $\max\{0, p - \overline{c}\}$. This contract would reproduce the same profits, but a larger portion of them would be credible for both types of governments. Hence, this contract would necessarily have a lower expropriation premium and would thus be better choice for the credible type.

Corollary 3.1 is relevant because it relates the contract choice of an unbiased third party with that of a party only weighting the high type. In addition, if the credible government was to design the contract itself under the presumption that out of equilibrium offers would be interpreted of coming from the credible type with probability α, the contract choice would be equal. Type $\overline{c}$ would

Figure 3.2: Optimal Contract



find optimal the contract that minimizes the expropriation premium among those that make the investor break even. Corollary 3.1 implies that the solution to this problem is equivalent to the one that the unbiased third party would select. This link is going to be relevant in the equilibrium analysis.

Figure 3.2 depicts the main idea of proposition 3.3. Both figures contain tax schedules represented by a red line, $t(p)$ and $t^*(p)$. The one of the figure in the right is the contract described in proposition 3.3. The gray solid line shows how the investor perceives the portion of the contract belonging to $\overline{G}(t)$ and $\overline{G}(t^*)$ respectively. The difference between the 45 degrees line and the contract is the tax collection for each level of prices. As can be deduced by comparing both charts, the contract in the left appears to be more costly for the credible government, who would honor both contracts, but would collect lower taxes with the schedule $t(p)$. The low cost government is indifferent between both contracts, since they differ only the set of prices that it expropriates. The investor perceives both contracts as similar ones due to the fact that he applies a discount for the risk of being expropriated by type $\underline{c}$ and the contract $t(p)$ faces more of such risk. By setting collection at the minimum credible level for type $\overline{c}$, the $t^*(p)$ is able to extend the set of prices at which the scheme is credible, and thus minimize the premium that has to be given to the investor. A crucial fact for this to happen is that $\alpha\overline{c} - \underline{c} < 0$.

3.4.1.2 Equilibrium Analysis

As hinted before, the optimal contract is trivially a PBE of the informed principal game supported by pessimistic out of equilibrium beliefs. The investor would reject any alternative offer, as he would attribute them to the low credibility government. However, it would be interesting to know if the optimal contract survives beliefs refinements criteria that do not allow for other equilibria. This subsection is devoted to prove that that the optimal contract survives Divinity (Banks and Sobel, 1987), and no other contract with lower payoffs for the credible type of government survives it. Intuitively, this equilibrium concept requires that the investor tries to disentangle rationally who made an out of equilibrium offer, taking into account the incentives that each government had to do it. More precisely, when the investor faces an out of equilibrium offer, he analyzes which type is more eager to make such an offer, and thinks that the probability that the offer came from the most eager type is at least as large as the prior.[6]

A key intuition to understand the analysis in this section is that the low credibility government is less concerned about variations in the contract than the high credibility type. The latter is going to honor the contracts more often, and thus changes in the tax schedule can make a difference for him. On the contrary, if the variations in the contract refer to portions in which the low credibility type expropriates, he is going to be indifferent about the effect of such variations. Hence, there exists no contract which the credible type is willing to offer and the non credible is not. In essence, most of the contract variations are cheap talk for the low credibility government, whose only hope

[6] There are many belief based refinements of a Perfect Bayesian Equilibrium. However, there are two elements making our setting different from the standard, and thus making many of those refinements not suitable to our framework. First, most of the work is applied to and thought for selecting separating equilibrium, and are not directly compatible to our interest in pooling equilibria. Second, a large set of tax schedules for the non credible type are cheap talk, which distances our work from a signaling game. For example, the popular "intuitive criterion" by Cho and Kreps (1987) is not useful in our setting to discard equilibria, since it requires some degree of genuine conflict of interests in the choice of messages between types. However, this conflict does not exist our framework, as the low credibility type's own interest is to be confused with the high type, being this its only hope to participate in the transaction. Intuitively, we believe that the correct notion for refining beliefs in this context is to think that out of equilibrium messages that could potentially benefit (not equilibrium dominated) the credible type are not going to transmit information, since the cheap talk type is as willing to select this messages as it is to select the equilibrium message. This last notion is correctly captured by divinity.

is that his type is not revealed by his contract choice. Hence, any out of equilibrium offer that can potentially be good for the credible government, will also be good for the non credible government, and the investor will not update his prior beliefs upon observing it.

We now make a more formal treatment of the issue. Take an equilibrium tax schedule $t^*(p)$ and an alternative offer $t'(p)$. Call, if it exists, $\underline{x} \in [0,1]$ the minimum probability of accepting the offer, such that would make type $\underline{c}$ indifferent between the equilibrium and the alternative offer ($U(t'(p), \underline{x}, \underline{c}) = U(t^*(p), x^*, \underline{c})$). Similarly call $\overline{x} \in [0,1]$ the response of the investor such that $U(t'(p), \overline{x}, \overline{c}) = U(t^*(p), x^*, \overline{c})$. Divinity requires that when both thresholds exist, if $\underline{x} \geq \overline{x}$ then $\hat{\alpha}(t') \geq \alpha$ (and both inequalities are reversed if $\underline{x} \leq \overline{x}$). If only one does, meaning that the equilibrium dominates the alternative offer for one of types, the investor attributes the offer only to the type that could benefit from it. Thus, if only $\underline{x}$ exist, then $\hat{\alpha}(t') = 0$.

Assumption 3.2. *1) When both thresholds $\underline{x}$ and $\overline{x}$ exist, if $\underline{x} \geq \overline{x}$ then $\hat{\alpha}(t') = \alpha$ (and both inequalities are reversed if $\underline{x} \leq \overline{x}$). 2) If only $\underline{x}$ exist, then $\hat{\alpha}(t') = 0$. 3) If neither $\underline{x}$ nor $\overline{x}$ exist, any belief is admissible.*

The out of equilibrium beliefs in assumption 3.2 belong to the set of those admitted by Divinity. Strictly speaking, Divinity imposes that when both thresholds exist, if $\underline{x} \geq \overline{x}$ then $\hat{\alpha}(t') \geq \alpha$. In other words, it would make the investor revision of beliefs about type $\underline{c}$ not to decrease if he faces an offer which the non credible type would be more interested to make than the type $\overline{c}$. However, it is easy to see that no equilibrium would be supported by $\hat{\alpha}(t') > \alpha$, since this would allow out of equilibrium signaling. The credible government would be tempted to deviate by reproducing the equilibrium contract but with slightly higher taxes in the set of prices where type $\underline{c}$ does not expropriate. If such out of equilibrium contract was accepted, it would leave type $\underline{c}$ indifferent with the equilibrium, but would be strictly preferred by type $\overline{c}$. This would imply that $\underline{x} > \overline{x}$. Moreover, the more favorable beliefs $\hat{\alpha}(t') > \alpha$ would allow the credible government to find such contract and make the investor participate.

In addition, note that when neither $\underline{x}$ nor $\overline{x} \in [0,1]$ exist, this means that there is no relevant response from the investor that could make a deviation profitable for any of the type. Hence, a tax schedule that falls in part 3) of assumption 3.2, regardless the beliefs it induces in the investor, cannot constitute a deviation from an equilibrium.

Proposition 3.4. *Suppose the project is feasible. Then, there exists a pooling equilibrium supported by the beliefs in assumption 3.2 in which both government types offer the contract described in proposition 3.3. The investor accepts the offer, and expropriation occurs when the government is of type* $\underline{c}$ *and* $p \geq p^*$.

Proposition 3.4 states that offering the optimal contract is a part of an equilibrium supported by reasonable beliefs. From corollary 3.1 we know that $t^*(p)$ gives the maximum utility that $\overline{c}$ can have, among the contracts that make the investor participate with beliefs $\hat{\alpha}(t) = \alpha$. Hence, the credible government would not be tempted to deviate, but to contracts that are equilibrium dominated for type $\underline{c}$.[7] If such kind of deviation existed, the investor would guess that the offer was made by the credible government, and would be willing to give up information rents to trade under these conditions. Unfortunately, there is no such contract which outcome is dominated by the equilibrium one for type $\underline{c}$ and the credible type is interested in. If the optimal contract is executed, the non credible government expropriates when prices are high or gets taxes as low as it can commit to in low prices states of nature. Hence, any other tax schedule, if accepted by the investor leaves it indifferent. This rules out the possibility that the credible type can find out of equilibrium tax schedules in which it can signal itself.

Moreover, the non credible government does not have opportunities to deviate either. As hinted before, it would only be interested to deviate to a contract with higher taxes in low profits subset of prices. However, the investor would accept such a contract only if he is compensated with a higher expropriation premium entirely born by the credible type. Hence, tax schedules of this class are equilibrium dominated for the credible type, making the investor anticipate that only a non

[7] A tax schedule is equilibrium dominated for one government type if, when accepted by the investor, provides lower payoffs to that government than the equilibrium tax schedule.

credible type of government could offer such a contract. As a consequence, he would update his beliefs to $\hat{\alpha}(t) = 0$ and reject the contract, since he has no interest in dealing with type $\underline{c}$.

Corollary 3.2. *A tax schedule generating a higher expropriation premium than the optimal contract cannot be part of an equilibrium supported by the beliefs in assumption 3.2.*

Proof. Suppose it does. Then, the credible government would be willing to deviate to the optimal contract if $0 < \bar{x} \leq x < 1$. Regarding the non credible type, if he is worse off with the optimal contract, then the investor will attribute the offer to the credible government and accept it. In turn, if he is either better off or indifferent, then $0 < \underline{x} \leq x \leq 1$, and the investor will not update his prior. If the prior is not updated, this means that the credible government is willing to deviate from the equilibrium path, since he can offer the investor an alternative contract which he will accept with beliefs α and that involves a lower cost for him (lower expropriation premium). □

An important remark is that only equilibria in which the expropriation premium is minimal (as low as the one in the contract described in proposition 3.3) can constitute an equilibrium supported by the beliefs in assumption 3.2. In fact, corollary 3.2 states that any other tax schedule with a higher expropriation premium would make credible type of government deviate and offer a tax schedule as the one we described in proposition 3.3.

3.5 Attracting Investments Under the Threat of Expropriation

We have already shown that debt-like contract outperform other arrangements for maximizing the welfare of the credible government type and the weighted average welfare of both types of governments. We now discuss the question of project feasibility.

Suppose that, in the context of a asymmetry of information, a credible type of government would like to fund a very expensive project, such as a non traditional oil extraction development.

Call $t_{\bar{c}}(p)$ a contract that allows a profit stream $I(\bar{c})$ under symmetric information, the most generous contract that the credible type can commit to under symmetric information. It would be appealing to reason that offering such contract is the best option to ensure the participation of investors. In the extreme, if $\bar{c} > \bar{p}$, this contract would take the form of no taxes at all, leaving all the surplus of the project to the investor. Of course, under asymmetric information the investor would not expect to receive $I(\bar{c})$, as he would discount the profit stream accounting for the risk of dealing with type $\underline{c}$. Nevertheless, it could remain possible that the contract $t_{\bar{c}}(p)$ was the best way to attract investors.

Remark 3.1. $I(\underline{c}, \bar{c}) > \pi(t_{\bar{c}}(p), \alpha)$.

Remark 3.1 shows that there are feasible projects which will not make investors participate if they are offered the contract $t_{\bar{c}}(p)$.[8] This means that offering the most generous deal that the credible type can commit to, not only is not a good idea in terms of the surplus that the government can extract from it, but also in terms of the profits that the firm expects from this contract. Even though the profit schedule is indeed the highest type $\bar{c}$ can commit to, the investor will greatly discount that project for not being sustainable for most realizations of prices if the government turns out to be of type $\underline{c}$. In other words, the contract $t_{\bar{c}}(p)$ focuses too much on what the credible type can do, and does not exploit alternatives for minimizing the expropriation premium, which can generate gains for any participant of the transaction but the low credibility government.[9]

3.6 Conclusions

The recent failure of international courts to protect investors from governments that do not resist the temptation to expropriate when exogenous conditions are very favorable, suggested us to look

[8] Remark 3.1 is a direct consequence of the proof of proposition 3.3. In fact, $t_{\bar{c}}(p)$ can be characterized as a contract of the type described in proposition 3.3, with $p^* = \underline{p}$. The feasibility of projects is maximized at $p^* = \frac{c}{\alpha}$, which yields $I(\underline{c}, \bar{c})$.

[9] An exception to this remark would hold if one of the governments types lacked of any commitment capacity, *i.e.*, $\underline{c} = 0$. This rather unrealistic case is ruled out from our results by assumption. However, if this condition held, then $I(\underline{c}, \bar{c})$ would coincide with $\pi(t_{\bar{c}}(p), \alpha)$.

for alternatives in the lines proposed by Engel and Fischer (2010) and Stroebel and van Benthem (2013). Our work contributes to this problem by describing optimal contracts that can be offered to an investor which has to recover a sunk investment and faces price related business risk together with the risk of being expropriated. In a context in which only one of the types is credible enough to allow the investor recover his sunk cost, our results are negative in the sense that there is no hope, a priori, of signaling credibility, and thus transactions become costly, if feasible. On the other hand, our results are positive since we show that much can be done to improve welfare, even in a context in which different types pool their contract offers. The optimal contract is designed in a way that minimizes the probability of expropriation, and thus minimizes the impact of the low credibility government in the contract. It can be implemented in the form of two profit caps that apply below and after a price threshold respectively. Each of them makes the government residual claimant of the profits exceeding the cap. We also showed that this contract is a part of an equilibrium in the informed principal game, and it is supported by reasonable out of equilibrium beliefs.

Our line of investigation takes the problem of commitment as given and uses contract design to deal with it. An interesting alternative would be to investigate mechanisms to increase credibility. Although the game structure does not allow for signaling, it can still be possible to find ways of increasing the expropriations costs. Wernerfelt and Zeckhauser (2010) proposed interesting tools such encouraging the use of materials not available locally or using financial instruments that would bind in the case of expropriation. A key issue in designing such mechanism is to reduce the degree to which credibility depends on the penalties are imposed by a third party with limited enforcement tools.

3.7 Chapter References

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Appendix A

Proofs

A.1 Proofs of Chapter 1

This section contains proofs to some of the results in the main text. In order to facilitate the exposition, we introduce some notation. The function $W(a,d)$ represents the indirect welfare function arising from the policy choice composed by an auction format and a favoring device, evaluated at a given vector of v_i.

Proof of Lemma 1.1

The activity rules ensure that rivals are playing monotone strategies, and we initially assume market clearing (and we later check that this holds in equilibrium).[1] For any realization of rival's valuation, a small bidder's state by state first order conditions are:

$$v_i - bq_i + c = p.$$

Large bidders face a similar condition only that without bidding credits. In order to arrive at this condition, we used the fact that at any market clearing price, $s_i(p) = q_i$ and that $q_i \frac{\partial p(q_i, \cdot)}{\partial q_i}$

[1] More general proofs can be provided, although this one is enough for our linear environment.

cancels away when considering the Vickrey discount. Hence, for any realization of the bidder's valuation, truthful bidding is optimal. When bidders play their dominant strategy, linearity and monotonicity ensures that there is a single market clearing price. Finally, the indirect equilibrium quantities can be obtained by solving the system of equations formed by each bidders' first order conditions and the market clearing condition.

Second order conditions are met because $-b - \frac{\partial p(q_i, \cdot)}{\partial q_i} < 0$.

Proof of Proposition 1.1

Vickrey Auction:

The First order condition, using independence of the distributions, is:

$$\frac{\partial E\left(W\left(a_v, c\right) \mid \Theta\right)}{\partial c} = -c(b+\alpha) + \alpha\left(E\left(v_i \mid \theta_l\right) - E\left(v_i \mid \theta_s\right)\right) = 0.$$

With this condition, we recover the optimal level of bidding credits: $c_v = \frac{\alpha}{b+\alpha}\left(E\left(v_i \mid \theta_l\right) - E\left(v_i \mid \theta_s\right)\right)$.

Also, the second order conditions is $\frac{\partial^2 E(W(a_v,c)|\Theta)}{\partial c^2} = -\frac{(b+\alpha)mn}{b^2(m+n)} < 0$.

Uniform Price Auction:

The first order condition is:

$$\frac{\partial E\left(W\left(a_u, c\right) \mid \Theta\right)}{\partial c} = -b\left((m+n-2)\,c + E\left(v_i \mid \theta_l\right) - E\left(v_i \mid \theta_s\right)\right) + (m+n-2)\,\alpha\left(c - \left(E\left(v_i \mid \theta_l\right) - E\left(v_i \mid \theta_s\right)\right)\right) = 0.$$

This condition leads to the optimal level $c_u = \left(\frac{\alpha}{b+\alpha} - \frac{b}{(b+\alpha)(m+n-2)}\right)\left(E\left(v_i \mid \theta_l\right) - E\left(v_i \mid \theta_s\right)\right)$. The second order conditions is:

$$\frac{\partial^2 E\left(W\left(a_u, c\right) \mid Q, \Theta\right)}{\partial c^2} = -\frac{mn(-2+m+n)^2(b+\alpha)}{b^2(-1+m+n)^2(m+n)} < 0.$$

Proof of Proposition 1.3

The expressions of the welfare function evaluated at each of the optimal levels are cumbersome expressions. In order to simplify the exposition, we give the expression of the difference between welfare functions, which turn out to be much simpler expressions.

The structure of the proof is follows: we compare the four policy alternatives with each other and we find the conditions under which each policy is preferred. Claim A.1 presents a useful result for simplifying the computations.

Claim A.1. The welfare comparison between any two auction policies $W(a_z, d_z^*) - W(a_{z'}, d_{z'}^*)$ is reduced to the following difference:

$$\sum_{i=1}^{m+n} E\left(v_i\left(q_i\left(a_z, d_z^*, \theta_i\right) - q_i\left(a_{z'}, d_{z'}^*, \theta_i\right)\right) \mid \Theta\right) - (b+\alpha)\frac{1}{2}\left(Var\left(q_i\left(a_z, d_z^*, \theta_i \mid \Theta\right)\right) - Var\left(q_i\left(a_{z'}, d_{z'}^*, \theta_i\right) \mid \Theta\right)\right).$$

Proof. The first term of the sum $E\left(v_i\left(q_i\left(a_z, d_z^*, \theta_i\right) - q_i\left(a_{z'}, d_{z'}^*, \theta_i\right)\right) \mid \Theta\right)$, is straightforward and requires no particular proof. The second term uses the result in lemma 1.3. For a particular policy, the welfare is given by

$$W\left(a_z, d_z^*\right) = \sum_{i=1}^{m+n} E\left(v_i q_i\left(a_z, d_z^*, \theta_i\right) - \frac{b}{2} q_i\left(a_z, d_z^*, \theta_i\right)^2 - \frac{\alpha}{2}\left(q_i\left(a_z, d_z^*, \theta_i\right) - \frac{Q}{m+n}\right)^2 \mid \Theta\right).$$

Note that, by definition of the variance, $E\left(q_i\left(\cdot\right)^2 \mid \Theta\right) = Var\left(q_i\left(\cdot\right) \mid \Theta\right) + E\left(q_i\left(\cdot\right) \mid \Theta\right)^2$. By lemma 1.3, $E\left(q_i\left(a_z, d_z^*, \theta_i\right) \mid \Theta\right) = E\left(q_i\left(a_{z'}, d_{z'}^*, \theta_i\right) \mid \Theta\right)$, and hence the difference reduces to the differences in variances. Finally, note that $Var\left(q_i\left(\cdot\right) - \frac{Q}{m+n} \mid \Theta\right) = Var\left(q_i\left(\cdot\right) \mid \Theta\right)$, which concludes our argument. □

Next, table A.1 provides the variances of indirect quantities under each policy choice. In order to simplify the comparison, with $\beta_{uc} = \left(\frac{m+n-2}{m+n-1}\right)^2$ and $\lambda = \left(\frac{1}{(m+n)b}\right)^2$:

Table A.1: $Var\left(q_i\left(a_{z'}, d^*_{z'}, \theta_i\right) \mid \Theta\right)$

Policy	Large Bidders (θ_l)	Small Bidders (θ_s)
(a_v, c_v)	$\lambda\left((m+n-1)^2\sigma_l + (n-1)\sigma_l + \sigma_s m\right)$	$\lambda\left((m+n-1)^2\sigma_s + (m-1)\sigma_s + \sigma_l n\right)$
(a_u, c_u)	$\lambda\beta_{uc}\left((m+n-1)^2\sigma_l + (n-1)\sigma_l + \sigma_s m\right)$	$\lambda\beta_{uc}\left((m+n-1)^2\sigma_s + (m-1)\sigma_s + \sigma_l n\right)$
(a_v, Q_s)	$\left(\frac{1}{b}\right)^2\sigma_l\frac{(n-1)}{n}$	$\left(\frac{1}{b}\right)^2\sigma_s\frac{(m-1)}{m}$
(a_u, Q_s)	$\left(\frac{1}{b}\right)^2\sigma_l\frac{1}{n}\frac{(n-2)^2}{(n-1)}$	$\left(\frac{1}{b}\right)^2\sigma_s\frac{1}{m}\frac{(m-2)^2}{(m-1)}$

Similarly, Table A.2 completes information the needed to compute the expression in claim 1:

Table A.2: Claim A.1 Additional Information

Policies $(a_z, d^*_z), (a_{z'}, d^*_{z'})$	$\sum_{i=1}^{m+n} E\left(v_i\left(q_i\left(a_z, d^*_z, \theta_i\right) - q_i\left(a_{z'}, d^*_{z'}, \theta_i\right)\right) \mid \Theta\right)$
$(a_v, c_v), (a_u, c_u)$	$\frac{m\sigma_s + n\sigma_l}{b(m+n)}$
$(a_v, c_v), (a_v, Q_s)$	$\frac{m\sigma_l + n\sigma_s}{b(m+n)}$
$(a_v, c_v), (a_u, Q_s)$	$\frac{(2m+n.)\sigma_l + \sigma_s(2n+m)}{b(m+n)}$
$(a_u, c_u), (a_v, Q_s)$	$\frac{(m-n)}{b(m+n)}(\sigma_l - \sigma_s)$
$(a_u, c_u), (a_u, Q_s)$	$2\frac{m\sigma_l + n\sigma_s}{b(m+n)}$
$(a_v, Q_s), (a_u, Q_s)$	$\frac{\sigma_l + \sigma_s}{b}$

With the information on Tables A.1 and A.2 we can compute the expected difference in welfare of any two policies. Using this difference, we find the highest envelope of the welfare function for each value of α.

Claim A.2. There exists $\underline{\alpha}$ such that when $\alpha \leq \underline{\alpha}$, a Vickrey auction with bidding credits is the preferred policy.

Proof. The proof is obtained by comparing the welfare of (a_v, c_v) with each other policy alternative. Using the expression in claim A.1 and the values in Tables A.1 and A.2, we can find that: □

- $E\left(W\left(a_v, c_v\right) - W\left(a_u, c_u\right) \mid \Theta\right) > 0$ if and only if $\alpha < \frac{b}{-3+2m+2n} = \alpha_1$.
- $E\left(W\left(a_v, c_v\right) - W\left(a_v, Q_s\right) \mid \Theta\right) > 0$ if and only if $\alpha < b = \alpha_2$.
- $E\left(W\left(a_v, c_v\right) - W\left(a_u, Q_s\right) \mid \Theta\right) > 0$ if and only if:

$$\alpha < \frac{b\left(-1+m^2\right) n\sigma_l + bm\left(-1+n^2\right)\sigma_s}{(-1+m)\left(n(-3+2n)+m(-4+3n)\right)\sigma_l + (-1+n)\left(-4n+m\left(-3+2m+3n\right)\right)\sigma_s} = \alpha_3.$$

One can check that under the conditions in which the equilibrium exists in each policy,[2] α_1 is always the smallest of the three thresholds. Hence, when $\alpha < \alpha_1 = \underline{\alpha}$, it is optimal to have bidding credits in the context of a Vickrey auction.

Claim A.3. There exists $\overline{\alpha}$ such that when $\underline{\alpha} < \alpha \leq \overline{\alpha}$ a uniform price auction with bidding credits is the preferred policy.

Proof. The proof is obtained by comparing the welfare of (a_u, c_u) with each other alternative policy conditional on that $\underline{\alpha} < \alpha$.

In order find conditions for $E\left(W\left(a_u, c_u\right) - W\left(a_v, Q_s\right) \mid \Theta\right) > 0$, we first define α_4:

$$\alpha_4 = \frac{b(m+n)((-1+m)\sigma_l + (-1+n)\sigma_s)}{m^2(\sigma_l - 2\sigma_s) - m((1+n)\sigma_l + (-3+n)\sigma_s) + n((3-2n)\sigma_l + (-1+n)\sigma_s)}.$$

In this case, the denominator can be negative, in which case $E\left(W\left(a_u, c_u\right) - W\left(a_v, Q_s\right) \mid \Theta\right) > 0$ for any positive value of α, meaning that the uniform price auction with bidding credits dominates the Vickrey auction with set-asides. Otherwise, if $\alpha_4 > 0$, then the uniform price auction with bidding credits is preferred to a Vickrey auction with set-asides only if $\alpha < \alpha_4$.

Moreover, $E\left(W\left(a_u, c_u\right) - W\left(a_u, Q_s\right) \mid \Theta\right) > 0$ if and only if:

$$\alpha < \frac{bmn(m+n)\left((m-1)\sigma_l + (n-1)\sigma_s\right)}{(m-1)m\left(m\left(3n-4\right) + n\left(3n-8\right) + 4\right)\sigma_l + (n-1)n\left(n\left(3m-4\right) + m\left(3m-8\right) + 4\right)\sigma_s} = \alpha_5.$$

[2] The conditions are $m+n>2$ for (a_u, c_u) and $Min\left\{m, n\right\} > 2$ for (a_u, Q_s).

Note that, when an equilibrium in the set-asides auction exists, $\alpha_5 > 0$ because both m and n are greater than 2.

We can check that, when $\alpha_4 > 0$, then $\alpha_4 > \alpha_5$. In any case, it can also be checked that $\alpha_5 > \underline{\alpha}$. We conclude by noting that when $\underline{\alpha} < \alpha \leq \overline{\alpha} = \alpha_5$, uniform price with bidding credits is the preferred option. □

Claim A.4. When $\alpha > \overline{\alpha}$, a uniform price auction with set-asides is the preferred policy.

Proof. Claim A.3 checked that (a_u, Q_s) dominates (a_u, c_u) when $\alpha > \overline{\alpha}$. It remains to check whether if for some levels of α, (a_v, Q_s) is better than (a_u, Q_s).

$E\left(W\left(a_u, Q_s\right) - W\left(a_v, Q_s\right) \mid \Theta\right) > 0$ if and only if $\alpha > \frac{b((-1+m)\sigma_l + (-1+n)\sigma_s)}{(-1+m)(-3+2n)\sigma_l + (-3+2m)(-1+n)\sigma_s} = \alpha_6$. Intuitively, given the same level of optimal set-asides, the uniform price auction yields less intra-group outcome variation, and thus it is preferable when α is sufficiently high. Moreover, it can be checked that $\alpha_6 < \overline{\alpha}$, meaning that once the uniform price auction with bidding credits ceases to be the best option, α is high enough so that $E\left(W\left(a_u, Q_s\right) - W\left(a_v, Q_s\right) \mid \Theta\right) > 0$. □

Proof of Proposition 1.4

The problem of the auction designer is to find an allocation $\{q_i\}_{i=1}^{n+m}$ that maximizes welfare such that $\sum_{i=1}^{m+n} q_i - Q = 0$. The Lagrangian of the problem is:

$$\max_{\{q_i\}_{i=1}^{n+m}} \sum_{i=1}^{m+n} \left(U_i\left(q_i, v_i\right) - \alpha \left(q_i - \frac{Q}{m+n} \right)^2 \right) - \lambda \left(\sum_{i=1}^{m+n} q_i - Q \right).$$

The first order conditions are for each i:

$$v_i - bq_i - \alpha \left(q_i - \frac{Q}{m+n} \right) - \lambda = 0$$

where λ is a Lagrange Multiplier. This forms a system of $(n + m + 1)$ linear equations resulting in $q_i = \frac{Q}{m+n} + \frac{\left((m+n-1)v_i - \sum_{-i} v_j\right)}{(m+n)(b+\alpha)}$. In order to compute the second order conditions, we form the bordered hessian. The cross derivatives of the objective function form a $n+m$ diagonal matrix with $(-b-\alpha)$ in the diagonal. The bordered hessian is:

$$H = \begin{pmatrix} 0 & 1 & 1 & ... & 1 \\ 1 & -(b+\alpha) & 0 & ... & 0 \\ 1 & 0 & -(b+\alpha) & ... & ... \\ ... & ... & ... & ... & 0 \\ 1 & 0 & ... & 0 & -(b+\alpha) \end{pmatrix}.$$

One can check that it is indeed a global maximum, since the determinants of the principal minors alternate their sign: $H_2 = -1$,$H_3 = 2(b+\alpha)$,$H_i = (-1)^{i-1}(i-1)(b+\alpha)^{i-2}$.

A.2 Proofs of Chapter 2

Proof of Proposition 2.2

There are gains from employing an agent as long as $f(a) > a$, which is guaranteed by assumption 2.2. Hence, firms gain from ensuring participation of every agent. This can be achieved by fixing $w_f(f(a)) = f^{-1}(f(a)) = a$ to match the outside opportunity of workers.

Incentive compatibility: Providing incentives maximizes the profit of the firms when the condition $\bar{v}\left(f(a) - \frac{c}{\Delta v}\right) > \underline{v}(f(a) - a)$ is met. We argue that there exists a unique $\tilde{a}(\cdot)$ such that $\bar{v} \geq \underline{v}(f(a) - a)$ if $a > \tilde{a}(\cdot)$ and the inequality reverses otherwise.

Existence of $\tilde{a}(\cdot)$: By comparing the profit functions with and without inducing incentives, we find that it is optimal to give incentives when $f(a)\Delta v + \underline{v}a \geq \bar{v}\frac{c}{\Delta v}$. The function $f(a)\Delta v + \underline{v}a$ is continuous, increasing, starts from the origin and $f\left(\frac{c}{\Delta v}\right)\Delta v + \underline{v}\frac{c}{\Delta v} > \bar{v}\frac{c}{\Delta v}$ by assumption 2.2. Using the intermediate value theorem, we prove that there exists $\tilde{a}(\cdot)$ such that $f(a)\Delta v + \underline{v}a = \bar{v}\frac{c}{\Delta v}$ and the monotonicity of the function ensures that this value is unique.

When $a \geq \tilde{a}$, incentives are optimal and the minimum wage schedule that can induce them is

$w_f(f(a)) = Max\left\{\frac{c}{\Delta v}, a\right\}$. For levels of $\tilde{a}(\cdot) \leq a \leq \frac{c}{\Delta v}$, $w_f(\cdot) = \frac{c}{\Delta v}$ and when $a > \frac{c}{\Delta v}$, $w_f(\cdot) = a$. When $a \leq \tilde{a}$ providing incentives is not optimal, and thus the wage $w_f(\cdot) = a$ is the lowest one ensuring participation.

Proof of Lemma 2.2

We look at the maximum welfare that a government can achieve by fixing $w(f_g) < \frac{c}{\Delta v}$. With this limitation, given the application response to wages in equation 2.2 , the largest the government can be is $\tilde{a}(\cdot)$.

The proof has 3 steps. 1) Assuming that the wage is set to attract as many applicants as job vacancies, we prove that $n^{NI} = \min\{f^{-1}(f_g), \tilde{a}(\cdot)\}$. 2) We prove that the government does not benefit from setting $w(f_g) > n^{NI}$. 3) We find t such that the government's budget constraint is met.

1) Assume that $w(f_g) = n^{NI}$, so that the marginal applicant equals the government size. The problem of the government is to choose a size to maximize welfare subject to the restriction that $n^{NI} \leq \tilde{a}(\cdot)$.

$$\max_{n \in [0, \tilde{a}(\cdot)]} W_I = n(\overline{v} f_g - c) + \left(\int_n^{\tilde{a}(\cdot)} \underline{v} f(a)\, da + \int_{\tilde{a}(\cdot)}^{\bar{a}} (\overline{v} f(a) - c)\, da \right).$$

First order conditions are $f\left(n^{NI}\right) = f_g$ if $f^{-1}(f_g) < \tilde{a}(\cdot)$ or the corner solution $n^{NI} = \tilde{a}(\cdot)$ if $f(\tilde{a}(\cdot)) < f_g$. Second order conditions are met because $f'(a) > 0$.

2) Increasing the wage above n^{NI} triggers a misallocation of talents if $n^{NI} < \tilde{a}(\cdot)$. This implies a welfare decrease, since government's output would still be $n^{NI}(\underline{v}) f_g$, but would reduce private's sector productivity. In this case, a marginal increase in the wage makes an agent with private sector production $f\left(n^{NI}\right)$ apply for a public job, and given the rationing rule, an applicant with at most his own private production is rejected. This implies an expected decrease in welfare.

In turn, when $n^{NI} = \tilde{a}(\cdot)$, a marginal increase in the wage does not attract any additional agent, as long as the initial constraint $w(f_g) < \frac{c}{\Delta v}$ is met. Hence, this does not change the allocation in any meaningful way and cannot improve the productive process. Given the utilitarian form of the welfare function and the linear utilities, there are no potential gains from pure redistribution through public wages.

3) Taxes are set to meet the government's budget constraint. Considering that there is no aggregate uncertainty:

$$\bar{a}t = \underline{v}w(f_g)\,n^{NI} \Longrightarrow t = \underline{v}n^{NI}\frac{n^{NI}}{\bar{a}}.$$

Proof of Lemma 2.3

From the incentive constraint of agents (equation 2.1), we get that public wages should be at least $w(f_g) \geq \frac{c}{\triangle v}$. Hence, the minimum number of applicants that the government has in case it gives incentives is $\frac{c}{\triangle v}$. Our proof has 3 steps. The first one is to prove that the government does not want to have rationing if the wage is above $\frac{c}{\triangle v}$. The second one is to prove that the government does not want to fix a wage higher than $\frac{c}{\triangle v}$. Finally, we recover taxes from the government budget constraint.

Claim A.5. If $w(f_g) = r \geq \frac{c}{\triangle v}$, then $n^I = r$ if $\overline{v}f_g - c < F(r)$ and $n^I = 0$ otherwise.

The government chooses a size n to maximize:

$$\max_{n\in[0,r]} W_I = n(\overline{v}f_g - c) + \left(\int_0^{\tilde{a}(\cdot)} \underline{v}f(a)\,da + \int_{\tilde{a}(\cdot)}^{r} (\bar{v}f(a) - c)\,da\right)\frac{r-n}{r} + \int_r^{\overline{a}} (\bar{v}f(a) - c)\,da.$$

The first order conditions give two corner solutions depending on $(\overline{v}f_g - c) \lesseqgtr F(r)$. Note that given $f\left(\frac{c}{\triangle v}\right) > f_g > f(0)$ and $f'(a) > 0$, the functional form of $f(a)$ allows for the 2 corner solutions.

Claim A.6. If $\overline{v}f_g - c < F\left(\frac{c}{\triangle v}\right)$, the wage $w(f_g) = \frac{c}{\triangle v}$ is optimal.

Using the result in claim A.5, take $n^I = w(f_g)$. Consider a wage $w(f_g) = \frac{c}{\triangle v}$, and take the derivative of increasing the wage and the size of the government together:

$$\frac{dW}{dr}\mid_{r=\frac{c}{\triangle v}} = -\left(\overline{v}f\left(\frac{c}{\triangle v}\right) - c\right) + \left(\overline{v}f_g - c\right) < 0$$

because $f_g < f\left(\frac{c}{\triangle v}\right)$ by assumption 2.3.

Finally, taxes are set to meet the government's budget constraint:

$$\bar{a}t = \underline{v}w(f_g)\, n^I \Longrightarrow t = \bar{v}\frac{c}{\triangle v}\frac{\frac{c}{\triangle v}}{\bar{a}}.$$

This concludes the proof.

Proof of Proposition 2.3

The expression of the difference simplifies to:

$$W_I - W_{NI} = \frac{c}{\triangle v}\left(\overline{v}f_g - c\right) - \left(n^{NI}\left(\underline{v}\right) f_g + \int_{n^{NI}}^{\tilde{a}(\cdot)} \underline{v}f(a)\, da + \int_{\tilde{a}(\cdot)}^{\frac{c}{\triangle v}} \left(\overline{v}f(a) - c\right) da\right).$$

Monotonicity

The above expression is continuous with respect to f_g, but with a kink at the point $f_g = f(\tilde{a}(\cdot))$, since at this point n^{NI} changes in accordance with lemma 2.2. However, in any other point, the expression is smooth with positive derivative with respect to f_g :

$$\frac{d\left(W_I - W_{NI}\right)}{df_g}\Big|_{n^{NI}=\tilde{a}(\cdot)} = \frac{c}{\triangle v}\overline{v} - \tilde{a}(\cdot)\,\underline{v} > 0$$

$$\frac{d\left(W_I - W_{NI}\right)}{df_g}\Big|_{n^{NI}=f^{-1}(f_g)} = \frac{c}{\triangle v}\overline{v} - f^{-1}(f_g)\,\underline{v} > 0.$$

In either case, the inequalities hold because $\frac{c}{\triangle v} > n^{NI}$. Hence, the difference $W_I - W_{NI}$ is increasing with respect to f_g.

Lower Bound

Next, we evaluate f_g at its lower bound, $\left(\frac{c}{\triangle v}\right)$, given by assumption 2.1:

$$W_I - W_{NI}|_{f_g=\frac{c}{\triangle v}} = \left(\frac{c}{\triangle v} - n^{NI}\right)\left(\overline{v}\frac{c}{\triangle v} - c\right) - \left(\int_{n^{NI}}^{\tilde{a}(\cdot)} \underline{v} f(a)\, da + \int_{\tilde{a}(\cdot)}^{\frac{c}{\triangle v}} \left(\overline{v} f(a) - c\right) da\right).$$

Moreover, the above expression has as upper bound: $W_I - W_{NI}|_{f_g=\frac{c}{\triangle v}} < \left(\frac{c}{\triangle v} - \tilde{a}(\cdot)\right)\left(\overline{v}\frac{c}{\triangle v} - c\right) - \left(\int_{\tilde{a}(\cdot)}^{\frac{c}{\triangle v}} \left(\overline{v}\frac{c}{\triangle v} - c\right) da\right) - \int_{n^{NI}}^{\tilde{a}(\cdot)} \underline{v}\frac{c}{\triangle v} da < -\left(\int_{\tilde{a}(\cdot)}^{\frac{c}{\triangle v}} \left(\overline{v}\frac{c}{\triangle v} - c\right) da\right) < 0$. To get this result, we used the fact that $f(a)$ is increasing and $f\left(n^{NI}\right) > \frac{c}{\triangle v}$, because $n^{NI} \leq \tilde{a}(\cdot)$. We conclude that the welfare difference is negative at the lower bound of f_g, so not providing incentives is the optimal policy in such case.

Upper Bound

We evaluate f_g at its upper bound $f\left(\frac{c}{\triangle v}\right)$ given by assumption 2.3, and we compute the difference in welfare:

$$W_I - W_{NI}|_{f_g=f\left(\frac{c}{\triangle v}\right)} = \left(\frac{c}{\triangle v} - \tilde{a}(\cdot)\right)\left(\overline{v} f\left(\frac{c}{\triangle v}\right) - c\right) + \tilde{a}(\cdot)\left(\triangle v f\left(\frac{c}{\triangle v}\right) - c\right) - \left(\int_{\tilde{a}(\cdot)}^{\frac{c}{\triangle v}} \left(\overline{v} f(a) - c\right) da\right).$$

A lower bound for this expression is given by: $W_I - W_{NI}|_{f_g=f\left(\frac{c}{\triangle v}\right)} > \left(\frac{c}{\triangle v} - \tilde{a}(\cdot)\right)\left(\overline{v} f\left(\frac{c}{\triangle v}\right) - c\right) - \left(\int_{\tilde{a}(\cdot)}^{\frac{c}{\triangle v}} \left(\overline{v} f\left(\frac{c}{\triangle v}\right) - c\right) da\right) + \tilde{a}(\cdot)\left(\triangle v f\left(\frac{c}{\triangle v}\right) - c\right) = \tilde{a}(\cdot)\left(\triangle v f\left(\frac{c}{\triangle v}\right) - c\right) > 0$. Here, we used the fact that $f(a) > a$ and that it is increasing, so $\int_{\tilde{a}(\cdot)}^{\frac{c}{\triangle v}} \left(\overline{v} f\left(\frac{c}{\triangle v}\right) - c\right) da > \int_{\tilde{a}(\cdot)}^{\frac{c}{\triangle v}} \left(\overline{v} f(a) - c\right) da$.

Conclusion

We conclude by noting that, by the intermediate value theorem, there exists a level of y_g such that, if $f_g \geq y_g$, then $W_I - W_{NI} \geq 0$ with the same condition holding when the inequalities revert.

Proof of Proposition 2.4

The proof builds on the monotonicity and the bounds found in proposition 2.3. The difference $W_I - W_{NI}$ is evaluated $f_g = f(\tilde{a}(\cdot))$, and linearity of $f(a)$ allows to find conditions under which the difference is negative or positive. If it is negative, this implies that $y_g > f(\tilde{a}(\cdot))$, and if it is positive it implies that $y_g < f(\tilde{a}(\cdot))$. Overall, linearity allows to recover a new anchor over which one can apply the intermediate value theorem as in proposition 2.3.

When $f(a) = ka$, $\tilde{a}(\cdot) = \frac{c}{\triangle v}\frac{\overline{v}}{(\underline{v}+k\triangle v)}$. We use this as input for the calculations that follow.

As explained in proposition 2.3, the difference $W_I - W_{NI}$ is increasing in f_g. Evaluated at $f_g = f(\tilde{a}(\cdot))$ this welfare difference is:

$$W_I - W_{NI}|_{f_g=f(\tilde{a}(\cdot))} = -\frac{\bar{v}c^2(k-1)((k-1)k\triangle v - 2\underline{v})}{2k\triangle v(\underline{v}+k\triangle v)^2}.$$

Note that this is negative if $\frac{1}{2}k(k-1)\triangle v \geq \underline{v}$. The only root of the equation that verifies the assumption that $k > 1$, is $k = \frac{1}{2}\left(1+\sqrt{\frac{(\overline{v}+7\underline{v})}{\Delta v}}\right)$. At this level of k, both thresholds are equal. When $k > \frac{1}{2}\left(1+\sqrt{\frac{(\overline{v}+7\underline{v})}{\Delta v}}\right)$, the welfare difference is negative, and it becomes positive when the inequality reverses. Considering the bounds found in proposition 2.3 ($W_I - W_{NI}$ is positive at $f_g = f\left(\frac{c}{\triangle v}\right)$ and negative at $f_g = \frac{c}{\triangle v}$), and the monotonicity of the welfare difference, we use the expression $W_I - W_{NI}|_{f_g=f(\tilde{a}(\cdot))}$ as a new anchor for applying the intermediate value theorem. We conclude that the level of f_g that makes the government indifferent between using incentives or not, y_g, is $y_g < f(\tilde{a}(\cdot))$ if $k < \frac{1}{2}\left(1+\sqrt{\frac{(\overline{v}+7\underline{v})}{\Delta v}}\right)$ and $y_g > f(\tilde{a}(\cdot))$ if $k > \frac{1}{2}\left(1+\sqrt{\frac{(\overline{v}+7\underline{v})}{\Delta v}}\right)$.

A.3 Proofs of Chapter 3

Proof of Lemma 3.1

Take an arbitrary tax schedule, $\hat{t}(p)$, such that $G\left(\hat{t}\right)_+ \neq \varnothing$, and that with a positive probability expropriation occurs. This is, there exist a mass of prices in which $p - c > \hat{t}(p)$. Anticipating this, the firm's expected operating profits are:

$$\pi = \int_{G(\hat{t})_-} (p - t(p))\, dF(p) + \int_{G(\hat{t})_+} 0 dF(p).$$

Note that the government can offer a different schedule $t'(p)$, such that $G\left(t'\right)_+ = \varnothing$, and reproduce the same expected profits by simply increasing taxes in the set $\int_{G(\hat{t})_+}$. In particular, define $t'(p) = \hat{t}(p)$ if $p - c < \hat{t}(p)$, and $t'(p) = p$ otherwise. This tax schedule is equal to $\hat{t}(p)$ in the part in which it is credible, and leaves 0 profits to the firm in the rest of the contract. Hence, with $t'(p)$, the firm does not expect to be expropriated but expects the same profits as in the original contract.

Proof of Proposition 3.3

The structure of the proof is as follows. Denote $\tilde{\pi}(p^*)$ as the indirect profit schedule generated by a tax schedule as the one described in the statement of the proposition. In the proof of optimality, we show that the profit stream of any arbitrary tax schedule making the investor break even can be reproduced by a contract of the form:

$$\tilde{\pi}(p^*) = \int_0^{\underline{c}} p dF(p) + \int_{\underline{c}}^{p^*} \underline{c} dF(p) + \alpha \int_{p^*}^{\overline{p}} Min\left\{p, \overline{c}\right\} dF(p)$$

but with weakly lower expropriation premium. Next, we prove feasibility by finding $I\left(\underline{c}, \overline{c}\right)$. Finally, we find a lower bound for p^*.

Proof of Optimality

The argument of the proof is as follows: we take any arbitrary $t(p)$, and we argue that profit stream of the firm can be reproduced by contract of the form $\tilde{\pi}(y)$ with a weakly lower probability of expropriation.

We prove that any contract in which $t(p) > Max\{0, p - \overline{c}\}$ for a subset of prices included in $\overline{G}(t)$ or $t(p) > Max\{0, p - \underline{c}\}$ for $P \subset G(t)_{-}$ can be reproduced by a contract as the one stipulated in the proposition, with a weakly lower expropriation probability, and ensuring participation. Call $\tilde{t}(p)$ one of such contracts. Due to lemma 3.2, in order to be optimal the contract has to satisfy:

$$\int_{G(\tilde{t})_{-}} \left(p - \tilde{t}(p)\right) dF(p) + \alpha \int_{\overline{G}(\tilde{t})} \left(p - \tilde{t}(p)\right) dF(p) = I.$$

By continuity, there exists $\tilde{y} \in \left(\underline{p}, \overline{p}\right)$ such that $\int_{\underline{p}}^{\tilde{y}} Min\{p, \underline{c}\} dF(p) = \int_{G(\tilde{t})_{-}} \left(p - \tilde{t}(p)\right) dF(p)$. Because subsidies are not allowed, both sets $\left[\underline{p}, \tilde{y}\right]$ and $G\left(\tilde{t}\right)_{-}$ must contain the subset of prices $\left[\underline{p}, \underline{c}\right]$. Note that in this subset, $\widetilde{t}(p) \geq 0$ while the profit capping contract stipulates zero taxes. Similarly, for prices in $[\underline{c}, \tilde{y}]$, the profit cap taxes are $p - t(p) = \underline{c}$, while for the tax schedule $\widetilde{t}(p)$, $p - \widetilde{t}(p) \leq \underline{c}$ in the subset $G\left(\tilde{t}\right)_{-} \setminus \left[\underline{p}, \underline{c}\right]$. These two things imply that $\int_{\underline{p}}^{\tilde{y}} dF(p) = F(\tilde{y}) \leq \int_{G(\tilde{t})_{-}} dF(p)$.

Consider an alternative contract that extends the low profit cap until the price level $\hat{y} \in [\tilde{y}, \overline{p}]$ such that $F(\hat{y}) = \int_{G(\tilde{t})_{-}} dF(p)$ (so both sets are equiprobable) and applies the high profit cap $p - t(p) = Min\{p, \overline{c}\}$ for the set of prices $[\hat{y}, \overline{p}]$. The two complementary sets of prices would also have the same probability measure: $1 - F(\hat{y}) = \int_{\overline{G}(\tilde{t})} dF(p)$, but the profits of the profit capping contract are weakly higher than those of $\tilde{t}(p)$. This happens because taxes are the minimum possible such that they belong to the set of prices in which only type $\underline{c}$ expropriates, and also the subset $[\hat{y}, \overline{p}]$ contains prices at least as large as those in the set $G\left(\tilde{t}\right)_{-}$. This implies that $\int_{\hat{y}}^{\overline{p}} Min\{p, \overline{c}\} dF(p) \geq \int_{\overline{G}(\tilde{t})} \left(p - \tilde{t}(p)\right) dF(p)$.

Together all these imply that $\tilde{\pi}(\hat{y}) = \int_{\underline{p}}^{\hat{y}} Min\{p, \underline{c}\} dF(p) + \alpha \int_{\hat{y}}^{\overline{p}} Min\{p, \overline{c}\} dF(p) \geq I$. Given that when $\tilde{\pi}(\overline{p}) = I(\underline{c}) < I$ (by assumption 3.1), the intermediate value theorem suffices to find $p^* \in [\hat{y}, \overline{p}]$ such that $\tilde{\pi}(p^*) = I$. In case there is more than one p^* such that $\pi(p^*) = I$, take the highest valued solution. We conclude by noting that $1 - F(p^*) \leq \int_{\overline{G}(\tilde{t})} dF(p)$, and hence the probability of expropriation is lower.

Existence

The optimality part of the proof states that any contract allowing participation can be reproduced by a contract of the form $\tilde{\pi}(p^*)$. Hence, $I(\underline{c}, \overline{c})$ can also be expressed in this terms and can be found by maximizing $\tilde{\pi}(p^*)$ with respect to p^*. Next, we prove that, when $\underline{c} - \alpha\overline{c} < 0$, then $I(\underline{c}, \overline{c}) > I(\underline{c})$, and thus there exists levels of I for which the project is feasible. Once we have this result, we can apply the intermediate value theorem to state the existence of such contract for any value of $I < I(\underline{c}, \overline{c})$, since $I(\underline{c}, \overline{c}) \geq I > I(\underline{c})$.

The problem is then to maximize $\tilde{\pi}(y)$ with respect to y. The candidate for first order conditions are:

$$\frac{d\tilde{\pi}(y)}{dy} = \underline{c} - \alpha y = 0 \text{ if } y < \overline{c} \text{ or,}$$

$$\frac{d\tilde{\pi}(y)}{dy} = \underline{c} - \alpha\overline{c} = 0 \text{ if } y \geq \overline{c}.$$

Suppose $\underline{c} - \alpha\overline{c} \geq 0$. Then, a contract fixing $y = \bar{p}$ would allow a profit at least as large as the one with $y = \overline{c}$. However this contract would coincide with $I(\underline{c})$ that is not feasible by assumption 3.1. Hence, it can only be that $\underline{c} - \alpha\overline{c} < 0$. However, if this is the case, one can verify that $\frac{d\tilde{\pi}(y)}{dy} = \underline{c} - \alpha\overline{c} < 0$, and thus it cannot be that $y \geq \overline{c}$.

This leaves us with only 1 candidate solution: $\underline{c} - \alpha y = 0$, yielding the optimal $y = \frac{\underline{c}}{\alpha}$. Second order conditions are met because $-\alpha f\left(\frac{\underline{c}}{\alpha}\right) < 0$.

We conclude by defining $I(\underline{c}, \overline{c}) = \int_0^{\underline{c}} p dF(p) + \int_{\underline{c}}^{\frac{\underline{c}}{\alpha}} \underline{c} dF(p) + \alpha \int_{\frac{\underline{c}}{\alpha}}^{\overline{p}} Min\{p, \overline{c}\} dF(p)$ and proving that $I(\underline{c}) < I(\underline{c}, \bar{c})$ if $\underline{c} - \alpha\bar{c} < 0$. When the later is met, the difference in profit streams between both contracts becomes:

$$I(\underline{c}, \bar{c}) - I(\underline{c}) = \int_{\frac{\underline{c}}{\alpha}}^{\bar{c}} (\alpha p - \underline{c})\, dF(p) + \int_{\frac{\underline{c}}{\alpha}}^{\bar{c}} (\alpha\bar{c} - \underline{c})\, dF(p) > 0.$$

Lower bound for p^*

From the proof of optimality, we know that a solution to the optimal contracting problem can be found by taking the highest valued p^* solving $\tilde{\pi}(p^*) = I$. We argue that such value belongs to $\left[\frac{\underline{c}}{\alpha}, \bar{p}\right]$.

The proof of existence shows that $\tilde{\pi}(p^*)$ reaches a maximum value at $\tilde{\pi}\left(p^* = \frac{\underline{c}}{\alpha}\right)$ when the project is feasible. Assuming that the the conditions for feasibility are met $(c - \alpha\bar{c} < 0)$, it can be verified that $\frac{d\tilde{\pi}(p^*)}{dp^*} < 0$ if $p^* > \frac{\underline{c}}{\alpha}$. This means that the function $\pi(p^*)$ is monotonously decreasing in the set $\left[\frac{\underline{c}}{\alpha}, \bar{p}\right]$.

Finally, assumption 3.1 guarantees that no project is feasible at $\tilde{\pi}(p^* = \bar{p})$, so by the intermediate value theorem and the monotonicity result, there exists a unique $p^* \in \left[\frac{\underline{c}}{\alpha}, \bar{p}\right]$ such that $\tilde{\pi}(p^*) = I$ if the project is feasible. Note that other $p^* < \frac{\underline{c}}{\alpha}$ may also solve $\tilde{\pi}(p^*) = I$. However, this cannot be the optimal contract, since it would have a larger probability of expropriation than the unique and higher valued solution belonging to $\left[\frac{\underline{c}}{\alpha}, \bar{p}\right]$.

Proof of Corollary 3.1

An optimal contract minimizes $\int_{\bar{G}(t)} dF(p)$ subject to participation from the investor with beliefs $\hat{\alpha}(t) = \alpha$. Equivalently, it maximizes $1 - \int_{\bar{G}(t)} dF(p) = \int_{G(t)_-} dF(p)$. Given that $Min\{p, \underline{c}\}$ gives the maximum profits for every $p \in G(t)_-$, then an optimal contract also maximizes $\int_{G(t)_-} (p - t(p))\, dF(p)$, among the tax schedules that make him participate with the beliefs under consideration. Considering that $\int_{G(t)_-} (p - t(p))\, dF(p) + \alpha \int_{\bar{G}(t)} (p - t(p))\, dF(p) = I$, this is equivalent to minimizing $\int_{\bar{G}(t)} (p - t(p))\, dF(p)$.

Proof of Proposition 3.4

The utility function of the government of type $\bar{c}$, conditional on that the investor accepts the contract can be expressed as:

$$U\left(t(p), x\left(\hat{\alpha}\left(t\right)\right)=1, \bar{c}\right)=E(p)-I-\left(1-\hat{\alpha}\left(t\right)\right)\int_{\bar{G}(t)}\left(p-t\left(p\right)\right)dF(p)-\varphi\left(\hat{\alpha}\left(t\right)\right)-\int_{G(t)_{+}}\bar{c}dF(p)$$

where $\varphi\left(\hat{\alpha}\left(t\right)\right)=\pi\left(\cdot\right)-I$ is the rent given to the investor. So it would prefer contracts that have low rents given to the investor, and in which it does not expropriate. The utility function of type $\underline{c}$, conditional on that the investor accepts is:

$$U\left(t(p), x\left(\hat{\alpha}\left(t\right)\right)=1, \underline{c}\right)=E(p)-\int_{G(t)_{-}}\left(p-t\left(p\right)\right)dF(p)-\int_{\bar{G}(t)\cup G(t)_{+}}\underline{c}dF(p)).$$

Given that in many states of nature this type expropriates, its utility is only sensitive to the tax scheme in low profit states of natures. Conditional on acceptance, it would strictly prefer an alternative tax schedule to the optimal only when $t(p)>Max\left\{0,p-\underline{c}\right\}$ for $P\subset G\left(t\right)_{-}$ with positive probability measure. Otherwise, it is indifferent among the contracts. In either case $\underline{x}\in(0,1]$. Note that there is no contract making it worse off if accepted, because $t\left(p\right)$ cannot be negative, and in the proposed contract taxes are the minimum possible within $G\left(t\right)_{-}$.

Type $\bar{c}$ would only prefer an alternative contract if the total rent given to the investor is strictly lower. If this is not satisfied, $\hat{\alpha}\left(t\right)=0$ and the investor rejects the contract. If this condition is met then $\hat{\alpha}\left(t\right)=\alpha$ and the investor would not accept the contract either, since given corollary 3.1, the optimal contract minimizes the rent given to the investor among all the contracts that ensure participation.

Finally, we need to check that type $\underline{c}$ would not be tempted to deviate to a contract that generates beliefs $\hat{\alpha}\left(t\right)=\alpha$ on the investor. It has to be that the total rent given to the investor equals the one in the optimal contract (otherwise, the investor would not accept it or the out of eq. beliefs would be different). Also, to provide incentives to deviate to $\underline{c}$; $t(p) > Max\{0, p - \underline{c}\}$ for $P \subset G(t)_-$ with positive probability measure. However, by the proof of proposition 3.3, we know that the optimal contract has a lower probability of expropriation than any of such contracts, and from the proof of corollary 3.1, we know that a higher probability of expropriation is associated with a higher rent required by the investor. Hence, it cannot be the case that $\overline{c}$ is indifferent, and thus $\hat{\alpha}(t) = 0$.