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Abstract

To prevent climate change, three options are currently considered: improve the energy conversion efficiency of primary energy sources, develop carbon free alternatives to polluting fossil fuels, abate potential emissions before they are released inside the atmosphere. We study the optimal mix and timing of these three mitigation options in a stylized dynamic model. Useful energy can come from two sources: a non-renewable fossil fuel resource and a carbon free renewable resource. The extraction cost of the non-renewable resource is an increasing function of past cumulated extraction. The conversion efficiency rates of crude energy into useful energy are open to choice but higher conversion performances are also more costly to achieve. In addition the economy can choose to abate some fraction of its potential emissions and an higher abatement rate incurs higher costs. The society objective is to maintain below some mandated level, or carbon cap, the atmospheric carbon concentration. In the interesting case where the economy would be actually constrained by the cap, at least temporarily, we show the following. The optimal path is a sequence of four time regimes: a 'pre-ceiling' regime before the economy is actually constrained by the cap, a 'ceiling' regime at the cap, a 'post-ceiling' regime below the cap and a final regime of exclusive exploitation of renewable resources. If the abatement option has ever to be used, it should be around the beginning time of the ceiling regime, first at an increasing rate and next at a decreasing rate. The efficiency performance from any source steadily improves with the exception of a time phase under the ceiling regime when it is constant. Renewables take progressively a larger share of the energy mix but their exploitation may be delayed significantly. Carbon emissions drop down continuously although not sufficiently to prevent carbon accumulation up to the cap during the pre-ceiling regime.

Keywords: energy efficiency; carbon pollution; non-renewable resources; renewable resources; abatement.

JEL classifications: Q00, Q32, Q43, Q54.

1 Introduction

Following the first oil shock of the seventies, policymakers have expressed a strong concern for energy efficiency improvements. Numerous administrations, plans and projects have been created and designed to alleviate the effects of high energy prices on welfare. If the economic relevance of such policy initiatives has never been put into question, the same cannot be said of their actual impacts. Many studies have shown that even given positive incentives, households or firms may not adopt more energy efficient devices.[1]

However, whatever the alleged resistance of economic agents to become more energy efficient, there is clear evidence that the overall energy efficiency of industrialized economies has constantly risen in the last decades, as an effect of dedicated policies or the mere working of market forces and the respective dynamics of costs and benefits. The following figure illustrates the past and projected trend of US energy efficiency expressed in energy consumption per GDP unit. It shows that between 1950 and 2010, primary energy consumption per $ GDP has dropped more than a half and is expected to continue to decrease in the next decades.

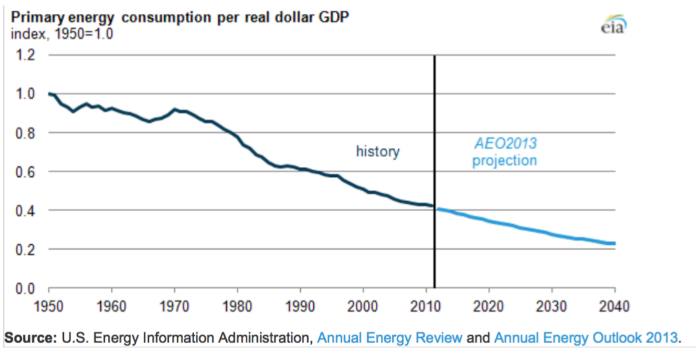

Figure 1: **US Past and Projected Energy Efficiency Trend.**

[1]The U.S. government has created a dedicated agency for energy efficiency, the American Council for an Energy Efficient Economy which publishes regularly reports on this topic. Similar institutions exist in UK, France or Germany. For the energy efficient devices adoption debate, see Brown (2004), Mc Kinsey (2009) and Allcott and Greenstone (2012). For global estimates of energy efficiency trends, see Schafer (2005).

The fear of oil shocks has been replaced today by the concern for climate change. Fossil fuels accounts for 87% of primary energy consumption at the world scale and fossil fuel burning is the largest source of GHG emissions. Raising energy efficiency of fossil fuel exploitation stands as a top priority in carbon emissions mitigation. This is not the only option however.

Three main ways toward 'green' energy systems are today on top of the desk. The first one is energy efficiency improvements, resulting in less carbon emissions release in the atmosphere per unit of GDP. The second one is the substitution of fossil fuels use by other carbon free primary energy sources, renewable or not: nuclear power, hydropower, wind and solar power, biofuels. The transformation of these primary energy sources into useful energy raises also energy efficiency problems. As an example of such difficulties, it is currently pleaded in the energy debate that renewables like wind or solar being intermittent sources with low conversion performances, the economic scope of these options should be rather limited. The third option is 'end-of-pipe' abatement of emissions and geological carbon capture and storage (CCS) is currently viewed as the most promising technique in that respect. However, being still in infancy, the deployment of such abatement techniques also raises strong efficiency concerns before becoming economically relevant.

For self-evident reasons, a lot of attention has been devoted to the coordination failures faced by today governments trying to cope with a global externality like climate change. But even in a fully cooperative world, the design of an efficient policy agenda able to curb the current trend of GHG emissions is not en easy task. One must take care of the temporal heterogeneity between policies, usually cast in the short run, and climate dynamics, a set of complex evolutions extending well ahead the next centuries. Moreover, available options to mitigate climate change interact themselves both in the scale and time dimensions and thus should not be assessed in isolation, or on the sole basis of their relative profitability at some given time. Such 'profitabilities', whatever might be their definitions, are jointly determined by the relative competitiveness prospects of the mitigation options, themselves depending on the climate dynamics and the climate policy agenda.

In this context, we ask the following questions. How the various mitigation options previously outlined should be combined ? What should be the right agenda of options in terms of relative scale and priorities? What will be the consequences of fossil fuel scarcity on the optimal mix and timing of mitigation options?

A lot of applied studies have explored the economic 'greening' issue. Many of them have relied on numerical simulation models at various space and time scales. As an example, the following figure shows projections from the International Energy Agency about the desirable mix of different 'green' options under an atmospheric carbon concentration stabilization target in line with the $+2^0C$ objective:[2]

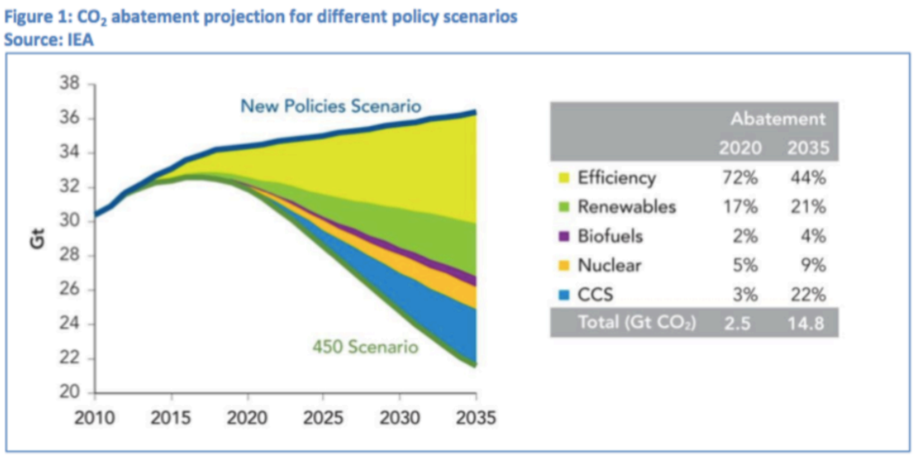

Figure 2: **Mitigation Efforts to Meet the 450 ppmV Target.**

As usual, it is hard to assess the reliability and the underlying economic rationale for such figures. They suggest however that efficiency gains should play a leading role in the mitigation efforts to meet the temperature rise stabilization goal. But if the substitution options and the abatement options have received considerable attention in the theoretical economic literature on climate change management, this is much less true for energy efficiency dynamics. Most models assume given and fixed efficiency rates for different energy sources or introduce exogenous trends of efficiency improvements.[3]

[2] IEA/OECD (2013).

[3] On the theoretical side, main original contributions are Tahvonen (1991), Tahvonen and Kuuluvainen (1991), Farzin and Tahvonen (1996), Withagen (1994), Tahvonen and Withagen (1996), Toman and Withagen (2000). For more recent contributions see Golosov *et al.* (2014), Van der Ploeg and Withagen (2014). The applied literature has intensively used IAM's to assess climate policy options. Prominent contributions are Gerlagh, Van der Zwaan (2006), Stern (2007), Nordhaus (2008).

Efficiency gains being usually explained by technical progress, the issue is hence accommodated inside the large body of literature devoted to technological advances and innovations. This is in particular the case for all the recent works on so-called 'green' R&D. In this literature, innovation improves inputs efficiencies, in particular those of primary energy sources. But economic efficiency is a slightly different concept than energy efficiency as used in many empirical studies. One objective of this paper is to build a bridge between these two concepts.[4]

To address these questions, we develop a stylized model summarizing the main ingredients of the problem. Useful energy can be obtained from two primary sources: a polluting non-renewable resource and a carbon-free renewable resource. The exploitation of the non-renewable resource incurs extraction costs increasing with past extraction. The conversion rates of primary energy from any source into useful energy may be adjusted over time, but more efficient energy conversion devices are more costly to manage. The global economy can also engage in emissions abatement, higher abatement rates being also more costly to achieve.

As in Chakravorty *et al.* (2006), we assume that the economy wants to maintain the atmospheric carbon concentration below a mandated level. Under this stabilization constraint, we study how the economy should manage the three options at its disposal to mitigate carbon emissions: raise the energy efficiency performance from any source, use more clean renewables or abate some fraction of the potential carbon emissions before they are released in the atmosphere.

In the interesting case where the economy would be at least temporarily constrained by the carbon stock mandate, we show that if the abatement option has ever to be used, it must do so around the beginning of the time phase under the carbon cap and be resumed strictly before the economy can escape the constraint, a consequence of the time increasing costs of the polluting non-renewable resource. The abatement rate should first increase before the economy is constrained by the cap, second be constantly decreasing

[4]The issue of technical change has quite naturally attracted a lot of attention in the theoretical and empirical literature. See for example Manne and Richels (2004), Gerlagh (2004), Edenhofer *et al.* (2005), Grimaud *et al.* (2011), Hassler *et al.* (2012), Sorrell (2014). Recent contributions in the macro growth literature are Acemoglu *et al*, (2011, 2015), Smulders *et al.*, (2011), Van der Meijden and Smulders, (2014).

during the constrained time period, and last be nil strictly before the end of the constrained period.

Furthermore, the energy conversion performance from any source should steadily improve with the exception of the last phase of the constrained period without abatement, a phase during which the economy should maintain at constant levels energy conversion efficiencies from any source. The renewable option rises progressively at the expense of the polluting non renewable option. The combination of efficiency gains, substitution toward renewables and abatement induces, with a stationary useful energy demand, a permanent drop down of polluting emissions anyway insufficient in a first stage to prevent the progressive accumulation of carbon into the atmosphere.

The rising costs of extraction of the non renewable resource imply that the resource exploitation should be resumed in finite time, some fraction of the initial resource endowment remaining permanently stored underground. Then the economy relies only on carbon-free renewables to provide its energy needs. The time increasing cost of fossil fuels extraction also induces a permanent rise of the useful energy price until the complete transition toward green energy, with the exception of the no abatement phase when constrained by the cap, during which the energy price should be constant.

Section 2 presents our model of energy use and production. The optimality problem faced by the society is laid down in Section 3. We focus on the interesting case of an economy actually constrained by the atmospheric carbon stock size, at least during some time period, a period we call the ceiling regime. Section 4 describes the optimal behavior of the economy under the ceiling regime. Section 5 performs the same job for the unconstrained time phases before the end of fossil fuels exploitation. The main features of the optimal policies are presented in Section 6 and next discussed in detail in Section 7. The last Section 8 concludes.

2 The model

We consider a model similar to Amigues and Moreaux (2015) in which final or useful energy can be produced by exploiting two primary sources, a non-renewable one, potentially polluting (coal), and a clean renewable one (solar).

Gross surplus

Let $q(t)$ denote the instantaneous consumption rate of final energy at time t and $u\left(q(t)\right)$ the associated instantaneous gross surplus. The gross surplus function $u(.)$ satisfies the following standard assumptions:[5]

Assumption A. 1 $u : (0, \infty) \to \mathbb{R}_+$ *is twice continuously differentiable, strictly increasing,* $u'(q) > 0$*, strictly concave,* $u''(q) < 0$*, and satisfies the first Inada condition* $u'(0^+) = +\infty$*.*

Alternatively we denote by p the marginal gross surplus u' and by $p^d(q)$ the marginal gross surplus function, or inverse demand function, $p^d(q) \equiv u'(q)$. The inverse of $u'(q)$, the direct demand function, is denoted by $q^d(p)$.

Producing useful energy from coal

To produce useful energy from coal requires first, to extract the underground coal, next to transform the extracted coal into useful energy, and last possibly to abate the potential pollution generated by the transformation.

Let $X(t)$ be the available stock of underground coal at time t, measured in energy units, X^0 be the initial endowment, $X^0 = X(0)$, and $x(t)$ the instantaneous extraction rate: $\dot{X}(t) = -x(t)$.

The unitary extraction cost depends upon the grade X under exploitation. Let $a(X)$ denote this unitary cost so that the total extraction cost at time t amounts to $a\left(X(t)\right)x(t)$. The extraction cost function $a(.)$ satisfies the following usual assumptions:[6]

[5] For any function $f(x)$ defined on $X \subseteq \mathbb{R}$ and for any x_c, $x_c \in \bar{X}$ where $\bar{X}$ is the closure of X, we denote by $f(x_c^+)$ and $f(x_c^-)$ respectively the limits $\lim_{x \downarrow x_c} f(x)$ and $\lim_{x \uparrow x_c} f(x)$ when such limits exist.

[6] Assumption A.2 results from discounted costs minimization and a ranking of grades by decreasing order of unitary extraction costs. With a constant discount rate, minimizing the sum of the discounted extraction costs implies that the grades must be exploited by increasing order of unitary costs. See Heal (1979) and Hanson (1980).

Assumption A. 2 *$a : (0, X^0] \to \mathbb{R}_+$ is twice continuously differentiable on $(0, X^0)$, strictly decreasing, $a'(X) < 0$, strictly convex, $a''(X) > 0$, with $a(0^+) = +\infty$.*

Together with farther assumptions on the solar energy costs, the assumption $a(0^+) = +\infty$ implies that some part of the initial coal endowment, X^0, will be kept underground.

Let η_x be this fraction of the extracted coal energy content converted into useful energy by the coal transformation industry. Getting more useful energy from a given quantity of coal requires more costly industrial processes. We denote by $b(\eta_x)$ the constant unitary processing cost of the coal input, equal to the marginal cost, given the chosen efficiency index η_x. The constant unitary cost of the output, q_x, also equal to the marginal cost, amounts to $b(\eta_x)/\eta_x$. We assume that $b(\eta_x)/\eta_x$ is increasing, hence also $b(\eta_x)$.[7]

Assumption A. 3
- *$b : [0, 1) \to \mathbb{R}_+$ is twice continuously differentiable on $(0, 1)$, strictly increasing, $b'(\eta_x) > 0$, strictly convex, $b''(\eta_x) > 0$, with $b(0^+) = 0$, $b'(0^+) > 0$, and $b(1^-) = +\infty$ and $b'(1^-) = +\infty$.*
- *The unit processing cost of useful coal energy (and so the marginal cost) is a strictly increasing function of η_x: $b'(\eta_x) > b(\eta_x)/\eta_x$ and $\lim_{\eta_x \downarrow 0} b(\eta_x)/\eta_x > 0$.*

Producing useful energy from coal requires other inputs, hence a strictly positive marginal cost of useful energy at 0^+: $\lim_{\eta_x \downarrow 0} b(\eta_x)/\eta_x > 0$. The assumptions $b(1^-) = +\infty$ and $b'(1^-) = +\infty$ are a simple way to formalize that a complete energy conversion ($\eta_x = 1$) is not physically possible.[8]

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[7]Differentiating the unitary cost yields:

$$\frac{d}{d\eta_x}\frac{b(\eta_x)}{\eta_x} = \frac{1}{\eta_x}\left[b'(\eta_x) - \frac{b(\eta_x)}{\eta_x}\right] .$$

Hence $b'(\eta_x) > 0$ is necessary for $d\left(b(\eta_x)/\eta_x\right)/d\eta_x > 0$.

[8]Other formulations of this impossibility are examined in Section 8 resulting in more complex optimal paths. However the main results of the paper can be obtained under this simpler assumption.

A by-product of burning coal to produce useful energy is a potential emission of pollution in the atmosphere. Let ζ be the unitary pollution content of coal so that the potential flow of pollution would amount to $\zeta x(t)$ when the transformation industry uses $x(t)$ units of coal. However, the industry can abate some part of the potential emissions, hence an actual flow smaller than the potential one. Let $\eta_z(t)$ be the fraction of potential emissions which is captured and sequestered before flowing into the atmosphere[9] and $(1-\eta_z(t))\,\zeta x(t)$ be the actual pollution flow which feeds at time t the atmospheric pollution stock denoted by $Z(t)$.

The atmospheric pollution stock self-regenerates at a proportional rate α assumed to be constant to simplify.[10] Thus the dynamics of $Z(t)$ is given by

$$\dot{Z}(t) = (1-\eta_z(t))\,\zeta x(t) - \alpha Z(t)\ .$$

The atmospheric pollution stock is constrained to be kept at most equal to some cap, or ceiling, $\bar{Z}$, as in Chakravorty, Magné and Moreaux (2006) to prevent excessive damages. We denote by Z^0 the pollution stock inherited from the past, $Z(0) = Z^0$. In order that the model makes sense, we assume that $Z^0 < \bar{Z}$.[11]

Abating potential pollution is costly. Let $g(\eta_z)$ be the constant unitary processing cost of the potential pollution flow ζx necessary to abate the fraction η_z of the flow, hence a total cost $g(\eta_z)\zeta x$ to abate $\eta_z\zeta x$ and a constant unitary cost $g(\eta_z)/\eta_z$ per unit of abated pollution, equal to the marginal cost in the same units. Higher abatement rates η_z are technically more difficult to bring into operation, hence more costly.

[9]Capturing and sequestering the potential CO_2 flow resulting from the exploitation of fossil fuels can be constrained by the limited capacities of the stockpiling sites. See Lafforgue *et al.* (2008.a, 2008.b) for models taking into account limited sequestration capacities and the final comments in Section 8 on this issue in the present model context.

[10]See Forster (1975), Farzin (1996), Tahvonen and Salo (1996), Tahvonen and Withagen (1996) and Toman and Withagen (2000) for theoretical models in which α depends on Z. Some self-regeneration processes give rise to non-convex problems for which the first order conditions are not sufficient to characterize the optimal programs. Such processes would induce the same difficulties in the present model.

[11]The other standard formulation is to assume that the damages increase with the pollution stock. See Moreaux and Withagen (2015) for a recent study of the optimal abatement policy in such a case but without the efficiency improvement option. An atmospheric carbon concentration control policy is here seen as equivalent to a temperature rise stabilization (e.g. the $+2^0C$ objective). Actually, the matter may be quite more complicated, see Weitzman (2010), Mason and Wilmot (2015).

Assumption A. 4 • *$g : [0, 1) \to \mathbb{R}_+$ is twice continuously differentiable on $(0, 1)$, strictly increasing, $g'(\eta_z) > 0$, strictly convex, $g''(\eta_z) > 0$, with $g(0) = 0$, $g'(0^+) > 0$, and $g(1^-) = +\infty$ and $g'(1^-) = +\infty$.*

- *The unit cost of abated pollution (and so the marginal cost) is a strictly increasing function of η_z: $g'(\eta_z) > g(\eta_z)/\eta_z$ and $\lim_{\eta_z \downarrow 0} g(\eta_z)/\eta_z > 0$.*

Abating requires some inputs, hence $\lim_{\eta_z \downarrow 0} g(\eta_z)/\eta_z > 0$. Again $g(1^-) = +\infty$ and $g'(1^-) > 0$ mean that full abatement is not feasible.

Solar energy

Let y^n be the natural flow of primary solar energy and η_y be the fraction which is transformed into useful energy. The production of solar useful energy is thus $q_y = \eta_y y^n$. Producing solar energy is a clean process generating no pollution. Denote by $c(\eta_y)$ the average processing cost of the solar input. The total cost incurred to produce $q_y = \eta_y y^n$ amounts to $c(\eta_y)y^n$. The average cost per unit of useful solar energy is equal to $c(\eta_y)/\eta_y$ and is an increasing function of η_y.

Assumption A. 5 • *$c : [0, 1) \to \mathbb{R}_+$ is twice continuously differentiable on $(0, 1)$, strictly increasing, $c'(\eta_y) > 0$, strictly convex, $c''(\eta_y) > 0$, with $c(0) = 0$, $c'(0^+) > 0$, and $c(1^-) = +\infty$ and $c'(1^-) = +\infty$.*

- *The unit processing cost of production of solar energy (and so the marginal cost) is a strictly increasing function of η_y: $c'(\eta_y) > c(\eta_y)/\eta_y$ and $\lim_{\eta_y \downarrow 0} c(\eta_y)/\eta_y > 0$.*

The rationale for the properties of the cost function $c(.)$ is the same than the rationale of the properties of the other cost functions $b(.)$ and $g(.)$.

Total useful energy consumption and production

Once the transformation costs $b(\eta_x)x$ and $c(\eta_y)y^n$ have been incurred, the useful coal and solar energies are perfect substitutes for the end-users, whatever the abatement rate of the emissions of the coal useful energy production industry. Thus, assuming that the production of useful energy is never stored for later consumption, we may define the total useful energy consumption, q, as the sum of the coal and solar useful energy productions: $q \equiv q_x + q_y$.

Discounting and social welfare

The social rate of discount, ρ, is assumed to be positive and constant throughout time. The social welfare is the sum of discounted net surpluses provided that the constraint on the pollution stock be satisfied.

3 Optimality conditions

The optimality problem can be framed in terms of x, η_x, η_y and η_z. An optimal path $\left\{x^*(t), \eta_x^*(t), \eta_y^*(t), \eta_z^*(t)\right\}_{t=0}^{\infty}$ is a solution of the following problem $(O.P.)$:[12]

$$
\begin{aligned}
(O.P.) \quad \max_{x,\eta_x,\eta_y,\eta_z} \quad & \int_0^\infty \{u\left(\eta_x(t)x(t) + \eta_y(t)y^n\right) - a\left(X(t)\right)x(t) - b\left(\eta_x(t)\right)x(t) \\
& \quad -c\left(\eta_y(t)\right)y^n - g\left(\eta_z(t)\right)\zeta x(t)\} e^{-\rho t} dt \qquad (3.1) \\
s.t. \quad & \dot{X}(t) = -x(t) \quad , \quad X(0) = X^0 > 0 \text{ given} \qquad (3.2) \\
& \dot{Z}(t) = \left(1 - \eta_z(t)\right)\zeta x(t) - \alpha Z(t) \quad , \quad Z(0) = Z^0 < \bar{Z} \text{ given} \\
& \text{and } \bar{Z} - Z(t) \geq 0 \qquad (3.3) \\
& x(t) \geq 0 \ , \ \eta_x(t) \geq 0 \ , \ \eta_y(t) \geq 0 \text{ and } \eta_z(t) \geq 0 \ . \qquad (3.4)
\end{aligned}
$$

Let λ_X and $-\lambda_Z$ be the costate variables associated to X and Z respectively, then the current value Hamiltonian, $\mathcal{H}$, reads:[13]

$$
\begin{aligned}
\mathcal{H} &= u\left(\eta_x x + \eta_y y^n\right) - a(X)x - b(\eta_x)x - c(\eta_y)y^n - g(\eta_z)\zeta x \\
&\quad -\lambda_X x - \lambda_Z\left[(1-\eta_z)\zeta x - \alpha Z\right] \ .
\end{aligned}
$$

Denote by ν the Lagrange multiplier associated to the ceiling constraint, $\bar{Z} - Z \geq 0$, and by $\gamma' s$ the multipliers associated to the non negativity

[12]The constraints $X(t) \geq 0$, $\eta_x(t) \leq 1$, $\eta_y(t) \leq 1$ and $\eta_z(t) \leq 1$ are never active under the assumptions A.1 - A.5 and are neglected.

[13]The time variable is omitted when not necessary.

constraints (3.4). Let $\mathcal{L}$ be the Lagrangian of the problem $(O.P.)$:

$$\mathcal{L} = \mathcal{H} + \nu \left[\bar{Z} - Z\right] + \gamma_x x + \gamma_{\eta x}\eta_x + \gamma_{\eta y}\eta_y + \gamma_{\eta z}\eta_z \ .$$

The first order conditions are:

$$\begin{aligned}
&\text{- w.r.t. } x &:& \ u'(\eta_x x + \eta_y y^n)\eta_x = a(X) + \lambda_X + b(\eta_x) \\
& & & \qquad + \zeta\left[g(\eta_z) + (1-\eta_z)\lambda_Z\right] - \gamma_x & (3.5)\\
&\text{- w.r.t. } \eta_x &:& \ u'(\eta_x x + \eta_y y^n)x = b'(\eta_x)x - \gamma_{\eta x} & (3.6)\\
&\text{- w.r.t. } \eta_z &:& \ \lambda_Z \zeta x = g'(\eta_z)\zeta x - \gamma_{\eta z} & (3.7)\\
&\text{- w.r.t. } \eta_y &:& \ u'(\eta_x x + \eta_y y^n)y^n = c'(\eta_y)y^n - \gamma_{\eta y} \ , & (3.8)
\end{aligned}$$

together with the usual complementary slackness conditions.

When time differentiable at time t, the paths of the costate variables satisfy:

$$\dot{\lambda}_X(t) = \rho\lambda_X(t) + a'\left(X(t)\right)x(t) \qquad (3.9)$$

$$\begin{aligned}
\dot{\lambda}_Z(t) &= (\rho+\alpha)\lambda_Z(t) - \nu(t) \ , \\
&\nu(t) \geq 0 \ , \ \bar{Z} - Z(t) \geq 0 \text{ and } \nu(t)\left[\bar{Z} - Z(t)\right] = 0 \ . \qquad (3.10)
\end{aligned}$$

Last, the transversality condition at infinity is:

$$\lim_{t\uparrow\infty} e^{-\rho t}\left[\lambda_X(t)X(t) + \lambda_Z(t)Z(t)\right] = 0 \ . \qquad (3.11)$$

Condition (3.5) states that, at any time, when $x > 0$ hence $\gamma_x = 0$, the marginal surplus generated by an additional unit of extracted coal when this additional unit is transformed into η_x additional units of useful energy, that is $u'\eta_x$, must be equal to the full marginal cost of transforming this additional unit of extracted coal into useful energy given the efficiency rate η_x and the abatement rate η_z, that is the sum of:

- The full marginal extraction cost of coal $a(X) + \lambda_X$, the monetary unitary extraction cost $a(X)$ augmented by the shadow marginal value of the grade X, λ_X;

- The marginal cost of transformation of one unit of extracted coal into η_x units of useful energy, $b(\eta_x)$;

- The full marginal cost of the potential pollution ζ generated by this additional unit when the fraction η_z is abated, that is the monetary marginal abatement cost $g(\eta_z)\zeta$ augmented by the shadow marginal cost of the non abated potential pollution $(1-\eta_z)\zeta\lambda_Z$.

Condition (3.6) states that, at any time, the marginal surplus generated by a slight increase $d\eta_x > 0$ of the transformation rate of the extracted coal into useful energy, $u' \cdot d\eta_x \cdot x$, must be equal to its marginal cost, $b'(\eta_x)d\eta_x \cdot x$. Note that when coal is exploited (3.6) reads more simply:

$$u'(\eta_x x + \eta_y y^n) \quad = \quad b'(\eta_x) \ . \tag{3.12}$$

Condition (3.8) for the solar energy production may be interpreted in the same way as (3.6) for coal transformation. When solar energy is exploited, (3.8) reads simply:

$$u'(\eta_x x + \eta_y y^n) \quad = \quad c'(\eta_y) \ . \tag{3.13}$$

Condition (3.7) states that at any time, the marginal cost of abatement generated by a slight increase $d\eta_z > 0$ of the abatement rate, $g'(\eta_z)d\eta_z \cdot \zeta x$, must be equal to the marginal decrease of the shadow penalty induced by the abatement rate increase, $\lambda_Z \cdot d\eta_z \cdot \zeta x$. When pollution is abated, (3.7) reads more simply:

$$\lambda_Z \quad = \quad g'(\eta_z) \ . \tag{3.14}$$

Full marginal cost of the extracted coal

Let us denote by $\mu(t)$ the full marginal cost of the extracted coal: $\mu(t) \equiv a\,(X(t)) + \lambda_X(t)$, the unitary extraction cost $a\,(X(t))$ of the grade $X(t)$ exploited at time t augmented by the unitary mining rent of the corresponding grade, $\lambda_X(t)$. Equation (3.9) suggests that the mining rent component could not be monotonic since $a'\,(X(t))$ is negative. However $\mu(t)$ is increasing up to the time at which ends coal exploitation. Time differentiating $\mu(t)$ and substituting the r.h.s. of (3.9) for $\dot{\lambda}_X(t)$, we get, denoting by $\bar{t}_x$ the closing time of coal exploitation:

$$\dot{\mu}(t) \quad = \quad \dot{a}\,(X(t)) + \dot{\lambda}_X(t) = \rho\lambda_X(t) > 0 \ , \ t < \bar{t}_x \ . \tag{3.15}$$

Marginal shadow cost of the pollution cap constraint

Let $\underline{t}_Z$ and $\bar{t}_Z$ be the times at which the ceiling constraint, $\bar{Z} - Z \geq 0$, respectively begins and ends to be active. Before $\underline{t}_Z$, $Z < \bar{Z}$ hence $\nu = 0$ and $\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z$ by (3.10). The shadow marginal cost of the polluting emissions increases at the proportional rate $\rho + \alpha$, a standard result:[14]

$$\lambda_Z(t) \; = \; \lambda_Z^0 e^{(\rho+\alpha)t} \; , \quad \text{where } \lambda_Z^0 \equiv \lambda_Z(0) \; , \; t < \underline{t}_Z \; . \tag{3.16}$$

After $\bar{t}_Z$, since the ceiling constraint is no more active and forever, then $\lambda_Z = 0$:

$$\lambda_Z(t) \; = \; 0 \; , \; t \geq \bar{t}_Z \; . \tag{3.17}$$

The main difficulty is to determine what happens within the time period during which the ceiling constraint is active. We call the ceiling regime this time period. Because $\nu(t) > 0$ when the ceiling constraint binds, then (3.10) suggests that $\lambda_Z(t)$ is not necessarily a monotonous function of time under the ceiling regime.

4 Dynamics under the ceiling regime

In this section we reformulate the problem that faces the society at each point of time under the ceiling regime as a static problem in which the full marginal cost of coal for the mining industry, μ, is taken for given and we determine how vary the variables of the model as functions of μ. Since we know that $\dot{\mu}(t) > 0$ by (3.15) we obtain how move all the model variables when at the ceiling. These moves are strongly contrasted according to it is optimal to abate or not.

4.1 A new formulation of the optimality problem when at the ceiling

When the ceiling constraint binds then $Z(t) = \bar{Z}$ hence $\dot{Z}(t) = (1 - \eta_z(t))\,\zeta x(t) - \alpha\bar{Z} = 0$ so that $x(t) = \bar{x}\,(\eta_z(t)) \equiv \alpha\bar{Z}/\zeta\,(1 - \eta_z(t))$. To determine its

[14]See for example Chakravorty *et al.* (2006).

useful energy consumption, the society must choose the optimal efficiency-abatement mix of its coal consumption and the optimal coal-solar mix of its primary resources exploitation. The best way to illustrate the logic of these arbitrages and their dynamics during the period at the ceiling, assuming that such a period exists along the optimal path, is to explicit the constraint on coal consumption in the instantaneous choice problem. Let (OPc) be the problem to be solved at any time of the ceiling period:[15]

$$
\begin{array}{rl}
(OPc) \max\limits_{x,\eta_x,\eta_y,\eta_z} & u(\eta_x x + \eta_y y^n) - \mu x - b(\eta_x)x - g(\eta_z)\zeta x - c(\eta_y)y^n \\
s.t. & \bar{x}(\eta_z) - x \geq 0 \ , \ \eta_z \geq 0 \ , \ \eta_y \geq 0 \ .
\end{array}
$$

Denote by $\bar{\gamma}_x$ the multiplier associated to the constraint on the amount of coal to be processed in the useful coal energy transformation industry and denote by $\mathcal{L}_c$ the current value Lagrangian of the (OPc) problem:

$$
\begin{aligned}
\mathcal{L}_c = \ & u(\eta_x x + \eta_y y^n) - \mu x - b(\eta_x)x - g(\eta_z)\zeta x - c(\eta_y)y^n \\
& + \bar{\gamma}_x \left[\bar{x}(\eta_z) - x\right] + \gamma_{\eta z}\eta_z + \gamma_{\eta y}\eta_y \ .
\end{aligned}
$$

The first order conditions are:

$$
\begin{aligned}
&\text{- w.r.t. } x && : \ u'(\eta_x x + \eta_y y^n)\eta_x = \mu + b(\eta_x) + g(\eta_z)\zeta + \bar{\gamma}_x && (4.1)\\
&\text{- w.r.t. } \eta_x && : \ u'(\eta_x x + \eta_y y^n)x = b'(\eta_x)x && (4.2)\\
&\text{- w.r.t. } \eta_z && : \ g'(\eta_z)\zeta x = \bar{\gamma}_x \bar{x}'(\eta_z) + \gamma_{\eta z} && (4.3)\\
&\text{- w.r.t. } \eta_y && : \ u'(\eta_x x + \eta_y y^n)y^n = c'(\eta_y)y^n - \gamma_{\eta y} \ , && (4.4)
\end{aligned}
$$

together with the usual complementary slackness conditions.

The condition (4.2) is the condition (3.12) of the preceding formulation since $x > 0$ and $\eta_x > 0$ when the ceiling constraint is tight, and the condition (4.4) is the condition (3.8). The other conditions, (4.1) and (4.3), are the new formulations of (3.5) and (3.7) respectively. For (4.1) there appears $\bar{\gamma}_x$ for $\zeta(1-\eta_z)\lambda_Z$ in (3.5) since now λ_Z is absent from the picture and for (4.3) $\bar{\gamma}_x \bar{x}'(\eta_z)$ is substituted for $\lambda_Z \zeta x$, for the same reason. Note that in (4.1)-(4.4), $x = \bar{x}(\eta_z)$ since the ceiling constraint is assumed to be active hence, in (4.1) and (4.3), $\bar{\gamma}_x > 0$.

[15] Since we assume that the ceiling constraint is tight then $x > 0$ and $\eta_x > 0$, hence the corresponding non negativity constraints are omitted.

4.2 Marginal cost of useful energy

Arbitraging between transformation efficiency and pollution abatement in the coal sector

Assume that the efficiency rate in the solar sector is fixed to some given level $\bar{\eta}_y$ (possibly $\bar{\eta}_y = 0$). To get more useful energy the society can resort to two extreme policies:

a- either increase the efficiency rate η_x without changing the abatement rate η_z nor the extraction rate so that $\bar{x}(\eta_z)$ is kept constant;

b- or increase $\bar{x}(\eta_z)$ by increasing the abatement rate η_z and simultaneously the extraction rate.

The society has to balance higher transformation costs permitting to save the non renewable coal primary resource stock against higher abatement and extraction costs and a forever decrease of its primary resource coal stock which could be used to satisfy its future energy needs. To determine the marginal costs of both options we must first determine the respective $d\eta_x$ and $d\eta_y$ which permit to obtain the same dq_x, given that $q_x = \eta_x \bar{x}(\eta_z)$, hence:

a. for the efficiency option, since η_z is kept constant, we have:

$$q_x = \eta_x \bar{x}(\eta_z) \implies d\eta_x = \frac{1}{\bar{x}(\eta_z)} dq_x = \frac{\zeta(1-\eta_z)}{\alpha \bar{Z}} dq_x \ ; \tag{4.5}$$

b. for the abatement option, since η_x is kept constant:

$$dq_x = \eta_x dx \ , \ dx = \bar{x}'(\eta_z) d\eta_z \text{ and } \bar{x}'(\eta_z) = \frac{\alpha \bar{Z}}{\zeta(1-\eta_z)^2} \ ; \tag{4.6}$$

hence:

$$d\eta_z = \frac{\zeta(1-\eta_z)^2}{\eta_x \alpha \bar{Z}} dq_x \quad \text{and} \quad dx = \frac{1}{\eta_x} dq_x \ . \tag{4.7}$$

Thus the marginal costs of a given $dq_x > 0$ amounts to:

a- with the first option since $b(\eta_x)\bar{x}(\eta_z)$ is the only cost component which increases and since $d\eta_x = dq_x/\bar{x}(\eta_z)$ through (4.5):

$$\text{Marginal cost of } dq_x \quad = \quad b'(\eta_x)\bar{x}(\eta_z)d\eta_x = b'(\eta_x)dq_x \ ; \qquad (4.8)$$

b- with the second option the marginal cost is the sum of two components:

- first the extraction, processing and abatement cost of the increase of extracted coal dx, that is:

$$[\mu + b(\eta_x) + g(\eta_z)\zeta]\, dx \quad = \quad \frac{1}{\eta_x}\,[\mu + b(\eta_x) + g(\eta_z)\zeta]\, dq_x \ ,$$

  since $dx = dq_x/\eta_x$ through (4.7), and:

- second, the abatement cost increases, that is, given that $\bar{x}(\eta_z) = \alpha\bar{Z}/\zeta(1-\eta_z)$ while $d\eta_z = \zeta(1-\eta_z)^2/\eta_x\alpha\bar{Z}$ by (4.7), then:

$$g'(\eta_z)\zeta\bar{x}(\eta_z)d\eta_z \quad = \quad \frac{1}{\eta_x}\,[g'(\eta_z)\zeta(1-\eta_z)]\, dq_x \ .$$

Summing up the two components, we get the following expression of the marginal cost *via* the second option:

$$\text{Marginal cost of } dq_x \quad = \quad \frac{1}{\eta_x}\,[\mu + b(\eta_x) + g(\eta_z)\zeta + g'(\eta_z)\zeta(1-\eta_z)]\, dq_x \ . \qquad (4.9)$$

In order that the coal sector be optimally managed when both options are exploited, we should have, according to (4.8) and (4.9):

$$b'(\eta_x) \quad = \quad \frac{1}{\eta_x}\,[\mu + b(\eta_x) + g(\eta_z)\zeta + g'(\eta_z)\zeta(1-\eta_z)] \ . \qquad (4.10)$$

This is precisely what imply the optimality conditions (4.1), (4.2) and (4.3) when some fraction of the potential emissions has to be abated within the ceiling period, because then:

- according to (4.2) : $b'(\eta_x) = u'(\eta_x\bar{x}(\eta_z) + \bar{\eta}_y y^n)$

- according to (4.3) with $\gamma_{\eta z} = 0$ since $\eta_z > 0$ and $x = \bar{x}(\eta_z)$, then $\bar{\gamma}_x = g'(\eta_z)\zeta(1 - \eta_z)$, so that (4.10) may be rewritten as:

$$u'(\eta_x \bar{x}(\eta_z) + \bar{\eta}_y y^n) \quad = \quad \mu + b(\eta_x) + g(\eta_z)\zeta + \bar{\gamma}_x \ .$$

that is the f.o.c. (4.1).

Arbitraging between coal and solar energies

Assume now that the efficiency rate of the solar sector η_y is not fixed at some given level $\bar{\eta}_y$ as in the preceding paragraph but must be chosen optimally together with the extraction rate and the efficiency-abatement mix in the coal sector. Assume that $\eta_y > 0$ and consider an increase $dq_y > 0$ of the useful solar energy consumption. To obtain such an increase dq_y the efficiency rate of the solar energy industry η_y must be increased by some $d\eta_y$:

$$q_y = \eta_y y^n \implies d\eta_y = \frac{1}{y^n} dq_y \ . \tag{4.11}$$

Since the only cost of the solar energy industry is the transformation cost $c(\eta_y)y^n$, then from (4.11):

$$\text{Marginal cost of } dq_y \quad = \quad c'(\eta_y) y^n d\eta_y = c'(\eta_y) dq_y \ . \tag{4.12}$$

When the solar industry is active, $\eta_y > 0$ and $\gamma_{\eta y} = 0$, so that the first order condition (4.4) may be more simply rewritten as: $u'(\eta_x x + \eta_y y^n) = c'(\eta_y)$. Thus the marginal cost of dq_y amounts to $u'(\eta_x x + \eta_y y^n) dq_y$. As seen in the preceding paragraph the marginal cost of a variation $dq_x = dq_y$ would amount to $b'(\eta_x) dq_x = u'(\eta_x x + \eta_y y^n) dq_x$. Hence, if solar is exploited when the ceiling constraint is active, the marginal cost of useful solar energy must be equal to the marginal cost of useful coal energy.

4.3 Dynamics of the useful energy production during the phases at the ceiling

When the ceiling constraint binds, the energy transformation industries may be in either one of the four following states:

- the two states of exploiting coal and only coal while abating of not some fraction of its potential pollution;
- the two states of exploiting both coal and solar while abating or not some fraction of its potential emissions.

The dynamics of these regimes are strongly contrasted. The key difference is that the efficiency rates η_x and η_y have to be kept constant when the abatement option is too costly to be exploited, that is when $\eta_z = 0$, while they must change through time together with the abatement rate η_z when abating is optimal, that is when $\eta_z > 0$.

To determine these dynamics, the strategy is to express the different variables η_x, η_y, η_z and $\bar{\gamma}_x$ as functions of μ and determine the signs of their derivatives. Since we know that $\dot{\mu}(t) = \rho\lambda_X(t) > 0$ by (3.15), the signs of the time derivatives $\dot{\eta}_x(t)$, $\dot{\eta}_y(t)$, $\dot{\eta}_z(t)$ and $\dot{\bar{\gamma}}_x(t)$ are the same as the signs of the derivatives with respect to μ.

The no-abatement phases

To illustrate this solving strategy, let us consider the case in which, when the ceiling constraint binds, both coal and solar energies have to be exploited. Then (4.1), (4.2) and (4.4) may be rewritten as follows, taking into account that b' may be substituted to u' in (4.1) and (4.4) thanks to (4.2) according to $u' = b'$:

$$\begin{aligned} b'(\eta_x)\eta_x - b(\eta_x) - \bar{\gamma}_x &= \mu && (4.13) \\ u'\left(\eta_x \frac{\alpha \bar{Z}}{\zeta} + \eta_y y^n\right) - b'(\eta_x) &= 0 && (4.14) \\ b'(\eta_x) - c'(\eta_y) &= 0\ . && (4.15) \end{aligned}$$

Differentiating with respect to η_x, η_y, $\bar{\gamma}_x$ and μ, we get, after simplification, the following system:

$$\begin{bmatrix} b''\eta_x & 0 & -1 \\ u''\dfrac{\alpha\bar{Z}}{\zeta} - b'' & u''y^n & 0 \\ b'' & -c'' & 0 \end{bmatrix} \begin{bmatrix} d\eta_x \\ d\eta_y \\ d\bar{\gamma}_x \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} d\mu\ . \qquad (4.16)$$

Easy calculations show that:

$$\frac{d\eta_x}{d\mu} = 0 \ , \ \frac{d\eta_y}{d\mu} = 0 \quad \text{and} \quad \frac{d\bar{\gamma}_x}{d\mu} = -1 \ . \tag{4.17}$$

It results, since the extraction rate $x = \alpha\bar{Z}/\zeta$ is constant, that $dq/d\mu = 0$ hence $dp/d\mu = 0$. Furthermore, since the $\bar{\gamma}_x$ of the present formulation is equal to the $\zeta\lambda_Z$ of the Section 3 formulation, we conclude that:

$$\frac{d\lambda_Z}{d\mu} \quad = \quad \frac{1}{\zeta}\frac{d\bar{\gamma}_x}{d\mu} < 0 \ . \tag{4.18}$$

If only coal is exploited when at the ceiling and abating is too costly, then the system reduces to the two equations (4.13) and (4.14) with $\eta_x\alpha\bar{Z}/\zeta$ as the more simple argument of u', determining the two variables η_x and $\bar{\gamma}_x$. Straightforward calculations show that (4.17) and (4.18) still hold. We conclude as follows:

Proposition P. 1 *Within a phase at the ceiling during which it is optimal to not abate the potential pollution flow:*

a. The transformation rate of extracted coal into useful energy, $\eta_x(t)$, must be kept constant; if furthermore it is optimal to simultaneously exploit the solar resource, then its transformation rate into useful energy, $\eta_y(t)$, must also be kept constant.

b. The consumption of useful energy, $q(t)$, is constant hence also its price, $p(t)$.

c. The shadow marginal cost of the pollution emissions, $\lambda_Z(t)$, decreases.

The abatement phases

Assume that it is optimal to abate some fraction of the potential pollution emissions and that both coal and solar must be exploited. Then (4.1)-(4.4) can be rewritten as follows, substituting b' to u' in (4.1) as in the preceding paragraph:

$$\begin{aligned} b'(\eta_x)\eta_x - b(\eta_x) - g(\eta_z)\zeta - \bar{\gamma}_x &= \mu & (4.19)\\ u'\left(\eta_x \frac{\alpha \bar{Z}}{\zeta(1-\eta_z)} + \eta_y y^n\right) - b'(\eta_x) &= 0 & (4.20)\\ g'(\eta_z)(1-\eta_z)\zeta - \bar{\gamma}_x &= 0 & (4.21)\\ b'(\eta_x) - c'(\eta_y) &= 0\ . & (4.22) \end{aligned}$$

Differentiating with respect to η_x, η_z, η_y, $\bar{\gamma}_x$ and μ we get, after some substitutions detailed in appendix A.1 , a reduced system in $d\eta_x$, $d\eta_z$ and $d\mu$:

$$\begin{bmatrix} b''\eta_x & -g''(1-\eta_z)\zeta \\ \dfrac{\alpha\bar{Z}}{\zeta(1-\eta_z)^2} + \dfrac{b''}{c''}y^n - \dfrac{b''}{u''} & \dfrac{\eta_x\alpha\bar{Z}}{\zeta(1-\eta_z)^2} \end{bmatrix} \begin{bmatrix} d\eta_x \\ d\eta_z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} d\mu\ . \tag{4.23}$$

Simple calculations show that:

$$\frac{d\eta_x}{d\mu} > 0 \quad \text{and} \quad \frac{d\eta_z}{d\mu} < 0\ . \tag{4.24}$$

From (4.20): $u''dq = b''d\eta_x$, and from (4.22): $b''d\eta_x = c''d\eta_y$, hence:

$$\frac{dq}{d\mu} = \frac{b''}{u''}\frac{d\eta_x}{d\mu} < 0 \quad \text{and} \quad \frac{d\eta_y}{d\mu} = \frac{b''}{c''}\frac{d\eta_x}{d\mu} > 0\ . \tag{4.25}$$

From $q = q_x + q_y$, $dq_y/d\mu > 0$ and $d\eta_x/d\mu > 0$, we deduce that:

$$\frac{dq_x}{d\mu} < 0 \quad \text{and} \quad \frac{dx}{d\mu} < 0\ . \tag{4.26}$$

Last, since $d\eta_z/d\mu < 0$ and by (3.7): $\lambda_Z = g'(\eta_z)$, we obtain:

$$\frac{d\lambda_Z}{d\mu} = g''\frac{d\eta_z}{d\mu} < 0\ . \tag{4.27}$$

Similar results concerning the signs of the derivatives of η_x, η_z, x, q and λ_Z also hold when only coal is exploited. The following proposition derives the main implications of the above calculations on the dynamics of the variables:

Proposition P. 2 *Within a phase at the ceiling during which it is optimal to abate some fraction of the potential pollution flow:*

a. *The transformation rate of the extracted coal into useful energy, $\eta_x(t)$, increases. If furthermore it is optimal to simultaneously exploit the solar energy, then its transformation rate into useful energy, $\eta_y(t)$, also increases.*

b. *The abatement rate of the potential pollution flow, $\eta_z(t)$, decreases hence the coal input of the coal transformation industry, $\bar{x}(\eta_z(t))$, also decreases. Although the efficiency rate $\eta_x(t)$ increases, the production of coal useful energy, $q_x(t)$, decreases and although the production of useful solar energy, $q_y(t)$, increases, the production of useful energy, $q(t)$, decreases and its price, $p(t)$, increases.*

c. *The shadow marginal cost of the pollution emissions, $\lambda_Z(t)$, decreases.*

4.4 Types of periods at the ceiling

Since at the end of the period at the ceiling, $\lambda_Z(\bar{t}_Z) = 0$ and since whatever the type of phase, with or without abatement, $\lambda_Z(t)$ must decrease when positive, there may exist only two types of periods during the time phase at the ceiling, $[\underline{t}_Z, \bar{t}_Z]$:

a. either $\lambda_Z(\underline{t}_Z) \leq g'(0^+)$, hence $\lambda_Z(t) < g'(0)$ for all $t \in (\underline{t}_Z, \bar{t}_Z]$. The shadow marginal cost of emissions is too small to justify the abatement of whatever fraction of the potential emission flow and the period at the ceiling reduces to the unique no-abatement phase detailed in Proposition 1.

b. or $\lambda_Z(\underline{t}_Z) > g'(0^+)$ and there exists an intermediate date $\bar{t}_a$, $\underline{t}_Z < \bar{t}_a < \bar{t}_Z$, at which $\lambda_Z(\bar{t}_a) = g'(0^+)$ and the ceiling period is made of two phases. During the first phase, $[\underline{t}_Z, \bar{t}_a)$, it is optimal to abate since $\lambda_Z(t) > g'(0^+)$ and the dynamics of the energy sector within the phase is detailed in Proposition 2. This first phase is followed by a second one, $[\bar{t}_a, \bar{t}_Z]$, without abatement since now $g'(0^+) \geq \lambda_Z(t)$, with constant production and transformation rates as detailed in Proposition 1.

For the first type of period the solar energy can be exploited or not, but whatever the case, the exploitation regime of this energy does not change during the period. For the second type it may happen that the solar exploitation begins during the first phase. This is the case when $p(\underline{t}_Z) < c'(0^+) < p(\bar{t}_a)$. Then t_y, the time at which begins the exploitation of solar energy is this time at which $p(t) = c(0^+)$ and the solar energy is also exploited during the second phase. When $p(\bar{t}_a) \leq c'(0^+)$, solar energy is never exploited within the ceiling period.

5 Dynamics of the pre-ceiling and post-ceiling periods

Consider first the pre-ceiling period. Several states of the energy industries may appear. Let us assume that it is optimal to exploit both types of energy and to abate. Let us start from (3.6) written as, after substitution of b' for u':

$$b'\left(\eta_x(t)\right)\eta_x(t) \quad = \quad \mu(t) + b\left(\eta_x(t)\right) + \zeta\left[g\left(\eta_z(t)\right) + \left(1 - \eta_z(t)\right)\lambda_Z(t)\right] \ . \tag{5.1}$$

Note that time differentiating $g\left(\eta_z(t)\right) + \left(1 - \eta_z(t)\right)\lambda_Z(t)$ and substituting $g'\left(\eta_z(t)\right)$ for $\lambda_Z(t)$ (from (3.7)), we get:

$$\frac{d}{dt}\left\{g\left(\eta_z(t)\right) + \left(1 - \eta_z(t)\right)\lambda_Z(t)\right\} \quad = \quad \left(1 - \eta_z(t)\right)\dot{\lambda}_Z(t) \ , \tag{5.2}$$

so that, time differentiating (5.1) results in:

$$\dot{\eta}_x(t) = \frac{\dot{\mu}(t) + \zeta\left(1 - \eta_z(t)\right)\dot{\lambda}_Z(t)}{b''\left(\eta_x(t)\right)\eta_x(t)} > 0 \implies \dot{\eta}_y(t) = \frac{b''\left(\eta_x(t)\right)}{c''\left(\eta_y(t)\right)}\dot{\eta}_x(t) > 0 \ . \tag{5.3}$$

Let us restore $u'\left(q(t)\right)$ in the l.h.s. of (5.1), time differentiate once again and take (5.2) into account. We get:

$$\dot{q}(t) = \frac{\dot{\mu}(t) + b'\left(\eta_x(t)\right)\dot{\eta}_x(t) + \zeta\left(1 - \eta_z(t)\right)\dot{\lambda}_Z(t)}{u''\left(q(t)\right)} < 0 \implies \dot{p}(t) > 0 \ . \tag{5.4}$$

From (5.3) and (5.4) we obtain:

$$\dot{q}_x(t) = \dot{q}(t) - \dot{q}_y(t) < 0 \quad \text{and} \quad \dot{x}(t) = \frac{\dot{q}(t)}{\eta_x(t)} - \frac{q(t)\dot{\eta}_x(t)}{\eta_x^2(t)} < 0 \ . \tag{5.5}$$

Last, from (3.7) and (5.5):

$$\dot{\eta}_z(t) = \frac{\dot{\lambda}_Z(t)}{g''(\eta_z(t)} > 0 \quad \text{and} \quad \frac{d}{dt}\left\{\zeta\left(1 - \eta_z(t)\right)x(t)\right\} < 0 \ . \tag{5.6}$$

It is easy to check that all the time derivatives have the same signs in the other possible states of the energy industry during the pre-ceiling period when only coal is exploited with or without abatement, or when both coal and solar are jointly exploited but without abatement. Also the same signs hold for $\dot{\eta}_x(t)$ and $\dot{\eta}_y(t)$ in the post-ceiling phase during which $\lambda_Z(t) = 0$ and $\eta_z(t) = 0$ before the end of coal exploitation.[16] To conclude:

Proposition P. 3 *During both the pre-ceiling period and the post-ceiling period preceding the end of coal exploitation:*

a. *The transformation rate of the extracted coal into useful energy, $\eta_x(t)$, increases. If the solar energy is exploited, its transformation rate, $\eta_y(t)$, also increases.*

b. *The coal input of the coal transformation industry, $x(t)$, decreases and although the transformation rate $\eta_x(t)$ increases, the useful coal energy output $q_x(t)$ decreases. Although the production of useful solar energy, $q_y(t)$, increases, the total production of useful energy, $q(t)$, decreases.*

During the pre-ceiling period:

c. *When it is optimal to abate some fraction of the potential pollution flow, then the abated fraction, $\eta_z(t)$, increases, the flow of pollution released in the atmosphere, $(1 - \eta_z(t))\,\zeta x(t)$, decreases and the shadow marginal cost of the released emissions, $\lambda_Z(t)$, increases.*

[16] The necessary existence of such a phase in proved in Proposition 4.

6 Optimal paths

We first sum up the general characteristics of all optimal paths before describing the two main broad types of paths.

6.1 Structural characteristics of the optimal paths

There may exist several types of optimal paths. First the ceiling constraint may bind or not. Next, when it binds, it may be optimal to abate or not. It may be also optimal to begin to exploit solar energy before the period at the ceiling, within the period or only after the period. When it is optimal to abate and to use the solar energy before the period at the ceiling, which one must be started first?

The main structural characteristics of all possible optimal paths are subsumed in the following Proposition 4 where $\underline{t}_a$ and $\bar{t}_a$ denote respectively the dates at which begins and ends the abatement of some part of the potential pollution flow.

Proposition P. 4 *Along the optimal path:*

a. *If it is optimal to abate, then the ceiling constraint must bind during some time interval. It is never optimal to abate to merely avoid a temporary constraint on coal exploitation.*

b. *When it is optimal to abate during some time period:*

   b.1. *The time period during which it is optimal to abate is a period $(\underline{t}_a, \bar{t}_a)$ during which it is never optimal to stop abating.*

   b.2. *Abatement must begin before the cap on the extracted coal input of the coal transformation industry, endogenously determined as $\bar{x}\left(\eta_z(\underline{t}_Z)\right) = \alpha\bar{Z}/\zeta\left(1-\eta_z(\underline{t}_Z)\right)$, begins to restrict its use: $\underline{t}_a < \underline{t}_Z$.*

   b.3. *Abatement must end within the period at the ceiling: $\underline{t}_Z < \bar{t}_a < \bar{t}_Z$.*

b.4 There must exist a post-ceiling phase $(\bar{t}_Z, \bar{t}_x)$, $\bar{t}_Z < \bar{t}_x$, during which the coal exploitation decreases from $x(\bar{t}_Z) = \alpha\bar{Z}/\zeta$ at the end of the ceiling regime, down to $x(\bar{t}_x) = 0$ at the end of the period of coal extraction.

c. Whatever the type of optimal path, with or without abatement, the flow of non abated potential pollution feeding the atmospheric carbon stock decreases up to the end of coal exploitation, excepted within the period at the ceiling when such a period exists, during which it is constant and equal to $\alpha\bar{Z}$.

d. Whatever the type of optimal path with or without a ceiling period:

d.1. For any given unitary extraction cost function of the different grades $a(X)$, the last grade of coal having to be exploited, denoted by $\tilde{X}$, depends upon the unitary transformation cost function $c(\eta_y)$ of the solar industry.

d.2. At the time $\bar{t}_x$ at which ends the coal exploitation, the efficiency rates in both coal and solar industries, $\eta_x(t)$ and $\eta_y(t)$, attain their maximum, denoted by $\tilde{\eta}_x$ and $\tilde{\eta}_y$ respectively.

d.3. When t tends toward $\bar{t}_x$, the time derivative of the useful energy price, $\dot{p}(t)$, tends smoothly down to 0, and:

d.4. The rate of coal extraction, $x(t)$, the efficiency rates $\eta_x(t)$ and $\eta_y(t)$, the productions of useful energies, $q_x(t)$ and $q_y(t)$, all tend smoothly toward 0, $\tilde{\eta}_x$, $\tilde{\eta}_y$, 0 and $\tilde{q}_y$ respectively, where $\tilde{q}_y$ is the constant production rate of useful solar energy when the solar industry is the only supplier of useful energy.

Proof: **a.** Assume that pollution is abated during some time interval and that $\sup\{Z(t), t \geq 0\} < \bar{Z}$. It would be possible to reduce slightly the abatement rate while maintaining the inequality, hence the non optimality of the path.

b. Let us first show that $\lambda_Z(\underline{t}_Z) > g'(0^+)$ if it is ever optimal to abate. Assume first that $\lambda_Z(\underline{t}_Z^-) \leq g'(0^+)$, then by (3.16), $\lambda_Z(t) = \lambda_z(\underline{t}_Z^-)e^{-(\rho+\alpha)(\underline{t}_Z - t)} < g'(0^+)$ for all $t \in [0, \underline{t}_Z)$ hence it is never optimal to abate before $\underline{t}_Z$.

Assume next that $\lambda_Z(\underline{t}_Z^+) < g'(0^+)$. Then, since $\lambda_Z(t)$ always decreases during the period at the ceiling, according to Propositions 1 and 2, we would have $\lambda_Z(t) < g'(0^+)$ for all $t \in [\underline{t}_Z, \bar{t}_Z]$ and it would not be optimal to abate during the period at the ceiling. Last under ou strict concavity/convexity assumptions, $\lambda_Z(t)$ is continuous (but not necessarily differentiable), hence $\lambda_Z(\underline{t}_Z^-) = \lambda_Z(\underline{t}_Z^+) = \lambda_Z(\underline{t}_Z)$ and we conclude that $\lambda_Z(\underline{t}_Z) \le g'(0^+)$ implies that it is never optimal to abate.

Now when $\lambda_Z(\underline{t}_Z) > g'(0^+)$, then, since $\lambda_Z(t)$ is continuous:

- because $\lambda_Z(t) = \lambda_{Z0}e^{(\rho+\alpha)t}$ before $\underline{t}_Z$, there exists θ_1: $0 < \theta_1 \le \underline{t}_Z$, such that $\lambda_Z(t) > g'(0^+)$ for all $t \in (\underline{t}_Z - \theta_1, \underline{t}_Z]$ and $\lambda_Z(t) < g'(0^+)$ for all $t < \underline{t}_Z - \theta_1$ if $\theta_1 < \underline{t}_Z$.
- because $\lambda_Z(t)$ decreases down to 0 at $\bar{t}_Z$, there exists θ_2: $0 < \theta_2 < \bar{t}_Z - \underline{t}_Z$ such that $\lambda_Z(t) > g'(0^+)$ for all $t \in [\underline{t}_Z, \underline{t}_Z + \theta_2)$ and $\lambda_Z(t) < g'(0^+)$ for all $t > \underline{t}_Z + \theta_2$. Thus abatement begins at $\underline{t}_a = \underline{t}_Z - \theta_1 < \underline{t}_Z$ and ends at $\bar{t}_a = \underline{t}_Z - \theta_2 < \bar{t}_Z$.

The last point to prove is the necessary existence of a post-ceiling phase $(\bar{t}_Z, \bar{t}_a)$. Let us denote by $\tilde{\eta}_y$ the efficiency rate of the solar industry once the exploitation of coal is closed. This is the value of η_y which solves $\max u(\eta_y y^n) - c(\eta_y)y^n$, hence the solution of the f.o.c.: $u'(\eta_y y^n)\eta_y = c'(\eta_y)$. Assume that $\bar{t}_Z = \bar{t}_x \equiv \bar{t}$. Since $u'(.)$ is time continuous then we must have $\eta_y(\bar{t}^-) = \eta_y(\bar{t}^+) = \tilde{\eta}_y$, hence $q_y(\bar{t}^-) = q_y(\bar{t}^+) \equiv \tilde{q}_y = \tilde{\eta}_y y^n$. On the other hand, the simultaneous use of the two resources requires that $c'(\eta_y) = b'(\eta_x)$ before $\bar{t}$, from which we conclude that $\eta_x(\bar{t}^-) > 0$. However, since $x(\bar{t}^-) \ge \alpha\bar{Z}/\zeta$ and hence $q_x(\bar{t}_x^-) \ge \eta_x(\bar{t}^-)\alpha\bar{Z}/\zeta$, we should have $q(\bar{t}^+) - q(\bar{t}^-) = \tilde{q}_y - q_x(\bar{t}^-) - q_y(\bar{t}^-) \le -\eta_x(\bar{t}^-)\alpha\bar{Z}/\zeta < 0$, implying a downward jump of $q(t)$ at time $\bar{t}_x$, and thus a positive jump of the useful energy price, $p(t)$, at the same time, which cannot be optimal under our convexity assumption on the gross surplus function.

c. We have shown in Proposition 3 that outside the period at the ceiling, the exploitation rate of coal, $x(t)$, decreases, hence:

- without abatement, the releases within the atmosphere $\zeta x(t)$ decrease,
- with abatement during the pre-ceiling period, as pointed out in Proposition 3, the abatement rate $\eta_z(t)$ increases so that the releases,

$(1-\eta_z(t))\,\zeta x(t)$ decrease.

d. At time $\bar{t}_x$, coal extraction ends and solar energy becomes the only supplier of useful energy. The efficiency rate in the solar sector must solve $u'(\eta_y y^n) = c'(\eta_y)$ (c.f. (3.13)). Let us denote by $\tilde{\eta}_y$ the solution. Then the useful energy production amounts forever to $\tilde{q} = \tilde{q}_y \equiv \tilde{\eta}_y y^n$ and the useful energy price is $\tilde{p} = u'(\tilde{q})$.

The useful energy price $\tilde{p}$ determines the last grade of coal to be exploited. For this grade, the mining rent λ_X is nil and, at time $\bar{t}_x$, the shadow marginal cost of pollution is also nil, hence $\eta_z(\bar{t}_x^-) = 0$. According to (3.6), $\eta_x(\bar{t}_x^-)$ must solve $\tilde{p} = b'(\eta_x)$. Denote by $\tilde{\eta}_x$ the solution and substitute in (3.5) to get $\tilde{p}\tilde{\eta}_x = a(X) + b(\tilde{\eta}_x)$. Then $\tilde{X}$, the solution, is the last grade to be exploited, ending the proof of the claim (d.1.) of the proposition.

During the last phase before closing coal extraction, both coal and solar resources are exploited and according to Proposition 3, both $\eta_x(t)$ and $\eta_y(t)$ increase, thus they attain their maximum at $\bar{t}_x$ when $\eta_x(\bar{t}_x) = \tilde{\eta}_x$ and $\eta_y(\bar{t}_x) = \tilde{\eta}_y$, hence the claim ($d$.2.) of the proposition.

Next, during that last phase, the useful energy price $p(t)$ satisfies (3.5) with $\lambda_Z(t) = 0$ and $\eta_z(t) = 0$, thus:

$$p(t)\eta_x(t) \quad = \quad \mu(t) + b(\eta_x(t)) \ .$$

Time differentiating and taking (3.6) and (3.15) into account results in:

$$\dot{p}(t)\eta_x(t) \quad = \quad \rho\lambda_X(t) \ .$$

Hence:

$$\lim_{t\uparrow\bar{t}_x} \dot{p}(t)\eta_x(t) = \tilde{\eta}_x \lim_{t\uparrow\bar{t}_x} \dot{p}(t) = \rho \lim_{t\uparrow\bar{t}_x} \lambda_X(t) = 0 \ ,$$

so that:

$$\lim_{t\uparrow\bar{t}_x} \dot{p}(t) \quad = \quad 0 \ ,$$

that is the claim (d.3.) of the proposition.

Thus $p(t)$ tends smoothly toward $\tilde{p}$ while $q(t)$ tends smoothly toward $\tilde{q}$, hence $\eta_x(t)$ and $\eta_y(t)$ tend smoothly toward $\tilde{\eta}_x$ and $\tilde{\eta}_y$ respectively, $q_y(t)$ tends smoothly toward $\tilde{q}_y$, hence $q_x(t)$ tends smoothly toward 0 hence also $x(t)$, that is the claim (d.4.) of the proposition. ■

6.2 Optimal paths

Any optimal path, either unconstrained or constrained, may be described as a fundamental sequence of coal exploitation regimes, the possible paths differing by the regime during which begins the exploitation of solar energy. Unconstrained paths and constrained ones without abatement are detailed in Amigues and Moreaux (2015). Hence we characterize only those paths along which it is optimal to abate and where thus, in light of the claim [a] of the Proposition 4, the ceiling constraint necessarily binds. The following Proposition 5 is a quasi-immediate corollary and/or a summing up of the preceding Propositions.

Proposition P. 5 *Assume that the ceiling constraint binds along the optimal path and it is optimal to abate, then:*

a- *There exist five successive fundamental regimes of coal exploitation: Two pre-ceiling regimes, first without abatement and next with abatement, the first one possibly reduced to zero, followed by a regime at the ceiling first with abatement and next without abatement, and last a post-ceiling phase, without abatement, up to the time $\bar{t}_x$ at which ends coal exploitation.*

b- *Excepted under the regime at the ceiling without abatement, the coal extraction rate, $x(t)$, decreases, down to 0 at $\bar{t}_x$, the efficiency rate of coal transformation into useful energy, $\eta_x(t)$, increases but the useful coal energy production, $q_x(t)$, decreases down to 0 at $\bar{t}_x$. When at the ceiling without abatement, the coal extraction rate is constant, $x(t) = \bar{x}(0) = \alpha\bar{Z}/\zeta$, the coal transformation rate is constant, $\eta_x(t) = \bar{\eta}_x$, hence also the production of useful coal energy, $q_x(t) = \bar{q}_x = \bar{\eta}_x\bar{x}(0)$.*

c- *The abatement rate of the potential emission flow, $\eta_z(t)$, increases during the pre-ceiling phase with abatement, attains a maximum at the beginning of the first ceiling phase with abatement, and next decreases down to 0 before the end of the period at the ceiling. However, the flow of non-abated emissions feeding the atmospheric pollution stock decreases under both the pre-ceiling and post-ceiling regimes, and is constant when at the ceiling. The pollution stock first increases under* *the pre-ceiling regimes, is constant at its maximum $\bar{Z}$ under the ceiling regimes and last decreases forever down to 0 at infinity.*

d- *The solar energy exploitation may begin under any regime depending upon its cost, including at $t = 0$, excepted under the ceiling regime without abatement, and must begin before the end of coal exploitation. Once the solar exploitation starts, its transformation rate, $\eta_y(t)$, increases, excepted during the period at the ceiling without abatement when its exploitation begins earlier. Under this regime, when it is exploited, its transformation rate is constant, $\eta_y(t) = \bar{\eta}_y$. Thus the production of useful solar energy, $q_y(t)$, increases once started, excepted when at the ceiling without abatement where it is constant, and tends to its post-coal stationary level, $\tilde{q}_y = \tilde{\eta}_y y^n$.*

e- *Excepted during the regime at the ceiling without abatement, under which it is constant, the production of useful energy, $q(t) = q_x(t)+q_y(t)$, decreases, down to $\tilde{q} = \tilde{q}_y$, at the end of coal exploitation, and its price, $p(t) = u'(q(t))$, moves in the opposite direction up to $\tilde{p} = u'(\tilde{q}_y)$ at $\bar{t}_x$.*

7 Discussion

The Proposition 5 shows that the optimal paths may fall into two main categories: scenarios with or without abatement of the polluting emissions. In this section we focus on the scenarios with abatement, the other type of optimal paths being discussed in detail in Amigues and Moreaux, (2015). To facilitate the discussion, it is useful to introduce the gross marginal added value function of the coal transformation industry.

Gross marginal added value function of the coal transformation industry

Let us denote by A the gross marginal value that is generated by the processing of one more unit of extracted coal in the coal transformation industry. By gross, we mean that we neglect the effect on the pollution stock, hence $A = u'(q)\eta_x - b(\eta_x)$, where $u'(q)\eta_x$ is the gross marginal surplus and $b(\eta_x)$ is the marginal processing cost. As long as coal is exploited, $u'(q) = b'(\eta_x)$ by (3.6) and A may be expressed as a function of η_x only: $A(\eta_x) = b'(\eta_x)\eta_x - b(\eta_x)$, an increasing function since $A'(\eta_x) = b''(\eta_x)\eta_x > 0$.

Next remark that $u'(q)$ is the implicit price p of useful energy and, from (3.6) again, we may define η_x as an increasing function of p. Denote by $\eta_x^e(p)$ this function: $d\eta_x^e/dp = b''(\eta_x^e(p)) > 0$. Last let $V(p) \equiv A(\eta_x^e(p))$ be the gross marginal added value as a function of p, also an increasing function: $V'(p) = (b''(\eta_x^e(p)))^2 \, \eta_x^e(p) > 0$.

Optimal scenarios with abatement

In qualitative terms, the optimal paths with abatement mainly differ by the type of regime during which solar energy becomes competitive, knowing that solar energy production must always begin strictly before the end of coal energy exploitation. The paths of useful energy price and shadow marginal cost of pollution illustrated in Figure 1 correspond to an optimal scenario with abatement in which solar energy production begins before the economy starts to abate its polluting emissions. The associated production paths of useful energies are illustrated in Figure 2. The Figure 3 shows the paths of emissions feeding the atmospheric pollution stock and the time profile of abated potential emissions while the time dynamics of the atmospheric pollution stock is illustrated in Figure 4.

Figure 1 about here

Figure 2 about here

Figure 3 about here

Figure 4 about here

The features of the useful energy implicit price path profile pictured in Figure 1 allows describing the main characteristics of the optimal path with abatement. Excepted during the no abatement phase under the ceiling regime, the useful energy price constantly rises until the end of coal exploitation. However this overall trend is explained differently under the ceiling regime and outside this regime.

Phases outside the ceiling regime without abatement

Before the beginning of the abatement phase, the optimality condition (3.5) writes:

$$V(p(t)) \;=\; \mu(t) + \zeta\lambda_Z(t)\ , \tag{7.1}$$

where $\mu(t)$ is the full marginal cost in the coal extraction sector and $\zeta\lambda_Z(t)$ is the shadow marginal cost of emissions generated by the burning of the extracted coal. The increasing scarcity of coal and its increasing extraction cost imply a continuous increase of $\mu(t)$ over time and $\lambda_Z(t)$ also increases as long the pollution stock, $Z(t)$, has not yet reached its upper limit $\bar{Z}$. Since $V(p)$ is an increasing function of p, the useful energy price, $p(t)$, should also increase. Efficiency requiring that $p = b'(\eta_x)$, the economy reacts to the increasing trend of the useful energy price by improving the transformation rate, $\eta_x(t)$, of extracted coal into useful energy. As a consequence, the unit processing cost of coal useful energy, $b(\eta_x(t))/\eta_x(t)$ increases through time under the assumption A.3. The same applies during the post-ceiling phase preceding the end of coal exploitation since (3.5) writes $V(p(t)) = \mu(t)$ and $\mu(t)$ increases. Hence, during any time phase without abatement and outside the ceiling regime, the phases $[0, \underline{t}_a)$ and $(\bar{t}_Z, \bar{t}_x)$ in the figures, the price of useful energy increases through time, together with the unit coal processing cost.

In the scenario under consideration, $p(0) < c'(0^+)$: solar energy production is not initially competitive. On the other hand, $\lambda_Z(0) < g'(0^+)$, implying that the abatement option is also not competitive. The economy thus relies initially only on coal energy production. Because the price of useful energy, $p(t)$, increases and coal energy is the sole supplier of the energy demand, $q_x(t) = q(t)$, the supply of useful coal energy should decrease through time. Since the transformation rate $\eta_x(t)$ improves and $q_x(t) = \eta_x(t)x(t)$, the economy reduces the coal input consumption rate, $x(t)$. Hence the polluting emission rate, $\zeta x(t)$, falls over time, although not sufficiently to prevent the positive accumulation of carbon into the atmosphere: $Z(t)$ increases.

Once the energy price has sufficiently increased, solar energy becomes competitive at the time t_y such that $p(t_y) = c'(0^+)$. At time t_y, $\lambda_Z(t_y) < g'(0^+)$ in the present scenario and thus the abatement of emissions is not yet competitive. Then begins a phase of simultaneous exploitation of the coal and solar energy sources. Optimality requires that the marginal costs of the two energies be equalized: $b'(\eta_x) = c'(\eta_y)$. Since the transformation rate of coal energy remains time increasing because of the continuous rise of the net shadow cost of coal exploitation, the transformation rate of solar energy should also increase once the exploitation of this energy source begins. The useful energy price being time increasing, the economy reduces in parallel the aggregate supply of useful energy $q(t)$. This means a reduction of the coal exploitation rate, $x(t)$, by an even higher amount because of the improvement of both energy sources transformation rates, $\eta_x(t)$ and $\eta_y(t)$. Hence, the carbon emission rate, $\zeta x(t)$, decreases through time, while the carbon stock continues to increase. Although the relative competitiveness of the two energies are equalized at the margin, the decline of the coal exploitation rate induces an increase of the share of solar energy in the energy mix, $q_x(t) = \eta_x(t)x(t)$ being time decreasing whereas $q_y(t) = \eta_y(t)y^n$ increases.

Phase outside the ceiling regime with abatement

Turn to the abatement phase preceding the ceiling regime. The abatement optimization condition (3.7) reads more simply $\lambda_Z = g'(\eta_z)$, hence a positive relationship between η_z and λ_Z that we denote by $\eta_z^e(\lambda_Z)$: $d\eta_z^e/d\lambda_Z = 1/g''(\eta_z^e(\lambda_Z)) > 0$. Since the pollution opportunity cost $\lambda_Z(t)$ increases up to the time at which the ceiling constraint binds, the economy should abate higher and higher fractions of its potential emissions during this phase.

Let $G(\lambda_Z)$ be the marginal cost of potential emissions due to coal burning, given that the fraction which is abated is optimized:[17]

$$G(\lambda_Z) \equiv g(\eta_z^e(\lambda_Z)) + (1 - \eta_z^e(\lambda_Z))\lambda_Z \quad , \quad \text{with } G'(\lambda_Z) = 1 - \eta_z^e(\lambda_Z) > 0 \ . \tag{7.2}$$

Now the relation (3.5) reads

$$V(p(t)) = \mu(t) + \zeta G(\lambda_Z(t)) \ . \tag{7.3}$$

The two forces at work in the r.h.s. of (7.3) are the same than these at work in the r.h.s. of (7.1): Both $\mu(t)$ and $\lambda_Z(t)$ increase through time and so does $G(\lambda_Z(t))$ since $G'(\lambda_Z) > 0$. Hence, like in the preceding phases, the price of useful energy should also increase through time. The rise of the energy

[17] Since $\lambda_Z = g'(\eta_z)$, then

$$\frac{dG(\lambda_Z)}{d\lambda_Z} = (g'(\eta_z^e(\lambda_Z)) - \lambda_Z)\frac{d\eta_z^e(\lambda_Z)}{d\lambda_Z} + (1 - \eta_z^e(\lambda_Z)) = 1 - \eta_z^e(\lambda_Z) \ .$$

price induces parallel evolutions of the coal transformation rate, $\eta_x(t)$, and the solar energy transformation rate, $\eta_y(t)$.

Throughout the first abatement phase preceding the beginning of the ceiling regime, the two energy sources transformation rates and the abatement rate move in the same direction. All improve to slow down the process of carbon accumulation in the atmosphere. However the net emission flow $\zeta\eta_z^e(\lambda_Z(t))\,x(t)$ is still larger than the natural self-regeneration flow of the pollution stock, $\alpha Z(t)$, so that $Z(t)$ still increases up to $\bar{Z}$ at the end of the phase.

The ceiling regime

The abatement dynamics undergoes a significant change when entering the ceiling regime. During the abatement phase under the ceiling regime, the allowed coal extraction rate is an increasing function of the abatement rate, $\bar{x}(\eta_z)$. On the other hand, it is easily checked that the relation (3.6) defines a negative relationship between η_x and η_z.[18] Since $\lambda_Z = g'(\eta_z)$, the pollution opportunity cost evolves in the same direction as η_z and thus in the reverse direction as η_x during the abatement period under the ceiling regime. On the other hand, it is immediately checked that (3.5) defines η_x as an increasing function of μ.[19] Since $\mu(t)$ increases through time because of the increasing scarcity of coal, then $\eta_x(t)$ should also increase. The rise of the coal transformation rate has two implications. Firstly, it implies through (3.6) that the useful energy price should continue to increase during the abatement phase under the ceiling regime. To this rise of the useful energy price is also associated an increase of the solar energy transformation rate.

[18]Differentiating (3.6) while taking into account that (3.6) and (3.8) imply that $b''(\eta_x)d\eta_x = c''(\eta_y)d\eta_y$ yields:

$$\underbrace{\left[\bar{x}(\eta_z) + \frac{b''}{c''}y^n - \frac{b''}{u''}\right]}_{(>0)} d\eta_x \quad = \quad -\eta_x \underbrace{\frac{d\bar{x}(\eta_z)}{d\eta_z}}_{(>0)} d\eta_z \ .$$

[19]Differentiating (3.5): $A(\eta_x) = \mu + \zeta G(\lambda_Z)$ yields:

$$\underbrace{\left[A'(\eta_x) - \zeta G'(\lambda_Z) g'(\eta_z)\frac{d\eta_z}{d\eta_x}\right]}_{(>0)} d\eta_x \quad = \quad d\mu \ .$$

Secondly, the increase of $\eta_x(t)$ implies that $\eta_z(t)$ should decrease together with the opportunity cost of the carbon pollution stock, $\lambda_Z(t)$. The progressive fall of the abatement rate induces a parallel decrease of $\bar{x}(\eta_z(t))$, the coal consumption rate.

Roughly speaking, the beginning of the ceiling regime is the 'worst time' for the economy. It must wait the whole length of the ceiling regime before getting a chance to escape the constraint. Discounting makes the opportunity cost of the pollution constraint the largest in present terms. To sweeten as much as possible this moment, the economy performs abatement at its highest level. This allows coal consumption to override what would be the maximum amount allowed by the natural regeneration rate alone, absent any abatement effort. In parallel, the economy goes on improving the energy transformation rates of both energy sources. This results into a continued substitution from fossil energy toward renewable energy. Even if the useful energy price rises during the phase, it remains below the price corresponding to no abatement when at the ceiling.

Before the beginning of the ceiling regime, the dynamics of λ_Z, commanded by (3.10), stands as largely independent from the details of the energy scenario. Since the rise of the coal shadow cost induces an increase of the coal energy transformation rate and the increase of $\lambda_Z(t)$ induces a parallel increase of the abatement effort, the coal efficiency improvement option and the abatement option look superficially complementary. What reveals the abatement phase under the ceiling regime is that these two options are actually competing each other along the lines of the arbitrage argument presented in subsection 4.2. At the beginning of the ceiling period, the main justification for the abatement of emissions is that it allows burning more coal. However, the increasing scarcity of coal deteriorates progressively the competitiveness of the abatement option with respect to the energy efficiency improvement option combined with the deployment of the carbon free energy option, inducing a progressive decline of the abatement rate until $\lambda_Z(t)$ is returned back to the level $g'(0^+)$ and the abatement of emissions cancels out.

Such a dynamic process comes to an halt after the abatement phase under the ceiling regime. Since the allowed consumption rate of coal to be burnt is now fixed at the level $\bar{x}(0)$, the useful energy price remains constant at the level $\bar{p}$ (see Figure 1).[20] Implied by the time stationarity of the energy price is the constancy of the energy transformation rates from both the coal and the solar source. Coming close to the end of the ceiling regime, the economy should stop the abatement of carbon emissions and the improvement of the energy transformation rates. Hence, the share of solar energy in the total energy mix is stabilized at a constant level.

Last phase of coal exploitation

However this situation can only be transitory. The rising scarcity of coal finally forces the economy out of the ceiling regime. Then begins a period of joint exploitation of the two energy sources until the end of coal exploitation. During this time period, the coal consumption rate, $x(t)$, decreases down toward 0. The carbon pollution stock, $Z(t)$, decreases, the economy being not anymore constrained by the carbon ceiling. Since the useful energy price increases again, the economy improves the transformation rates of both energy sources. Solar energy production rises again at the expense of coal energy.

At the end of this phase, the transformation rate of solar energy is determined by (3.8): $c'(\eta_y) = u'(\eta_y y^n)$. Let $\tilde{\eta}_y$ denote this transformation rate. To this rate is associated a final transformation rate of coal energy at time $\bar{t}_x$, $\tilde{\eta}_x$, itself solution of $b'(\eta_x) = c'(\tilde{\eta}_y)$, and a final price level of useful energy $\tilde{p} = c'(\tilde{\eta}_y)$. At the end of coal exploitation, the scarcity rent of the coal resource must be nil, $\lambda_X(\bar{t}_x) = 0$. Then (3.5): $\tilde{p}\tilde{\eta}_x = a(X(\bar{t}_x)) + b(\tilde{\eta}_x)$, defines the final grade of extracted coal, $\tilde{X} = X(\bar{t}_x)$. It is worth pointing out that this final grade depends only on the economic conditions characterizing the pure solar exploitation regime and not of the energy transition features toward this regime.

[20] It is easily observed that (3.6) defines a unique constant level of η_x and thus of η_y when $x = \bar{x}(0)$. Differentiating (3.6) yields:

$$\left[\bar{x}(0) + \frac{b''}{c''}y^n - \frac{b''}{u''}\right] d\eta_x \quad = \quad 0\ .$$

8 Concluding remarks

The present paper has explored the optimal mix of three mitigation options alleviating climate change: energy efficiency gains, substitution of fossil fuels by carbon free renewables and abatement of potential emissions. This work may be extended in several directions.

A well known limit of the abatement option is the disposability of favorable carbon storage sites. This may be accommodated inside our framework as such. Denote by $\bar{S}$ the initially available carbon storage capacity and by $S(t)$ the remaining capacity at time t. Then the dynamics of carbon storage is defined as $\dot{S}(t) = -\eta_z(t)\zeta x(t)$. Consider the most interesting case of a binding storage capacity. Denote by $\lambda_S(t)$ the shadow cost of carbon storage scarcity.

With respect to the optimality conditions of the original problem, the conditions (3.6) and (3.8) stay unmodified. Assuming $x > 0$ and $\eta_z > 0$, the conditions (3.5) and (3.7) become:

$$\begin{aligned} u'(\eta_x x + \eta_y y^n)\eta_x &= a(X) + \lambda_X + b(\eta_x) \\ &\quad + \zeta\left[g(\eta_z) + (1-\eta_z)\lambda_Z\right] + \lambda_S\zeta\eta_z \qquad (3.5)' \\ \lambda_Z &= g'(\eta_z) + \lambda_S \; . \qquad (3.7)' \end{aligned}$$

The new condition (3.5)' states that the shadow marginal cost of coal exploitation is increased by a term corresponding to the marginal cost of carbon storage into limited capacity reservoirs. The condition (3.7)' states that the marginal benefit of carbon storage, that is the avoided shadow cost of atmospheric carbon, has to be equalized to the full marginal cost of carbon capture, itself the sum of the abatement unitary cost and the unitary scarcity rent of storage sites. Remark that there is no more any simple link between the abatement rate, η_z, and the shadow cost of carbon in the atmosphere, λ_Z.

Consider first the ceiling regime. Focus on the interesting case where the carbon storage capacity would not be exhausted before the beginning of this regime and where abatement, even under the storage capacity constraint, remains optimal at least at the beginning of the ceiling regime. Since (3.6), (3.8) are unmodified, we know that η_x and η_z should move in opposite directions. Let $\hat{\eta}_z(\eta_x)$ denote the implicit function so defined with $d\hat{\eta}_x/d\eta_z < 0$. Next, add and subtract $\zeta\lambda_S$ in (3.5)' while using (3.6) and (3.7)' to get:

$$\begin{aligned} A(\eta_x) &= a(X) + \lambda_X + \zeta\lambda_S + \zeta\left[g(\eta_z) + (1-\eta_z)(\lambda_Z - \lambda_S)\right] \\ &= \mu + \zeta\lambda_S + \zeta\left[g(\eta_z) + (1-\eta_z)g'(\eta_z)\right] \equiv \mu' + G'(\eta_z)\ . \end{aligned}$$

On the one hand $\mu'(t) \equiv \mu(t) + \zeta\lambda_S(t)$ is a time increasing function since $\mu(t)$ increases throughout time while $\dot{\lambda}_S(t) = \rho\lambda_S(t) > 0$. On the other hand, since $dG'(\eta_z)/d\eta_z = (1-\eta_z)g''(\eta_z) > 0$, $A(\eta_x) - G(\hat{\eta}_z(\eta_x)) = \mu'$ defines η_x as an increasing function of μ'. Thus $\eta_x(t)$ increases while $\eta_z(t)$ decreases, implying that the ceiling regime is a sequence of a first phase with abatement followed by a no abatement phase, as in the original model. The carbon storage capacity is exhausted at the time at which η_z has decreased down to 0, that is $S(\bar{t}_a) = 0$. Most qualitative features of the model remain valid with scarce carbon storage facilities. Since $\eta_x(t)$ increases, $p(t) = b'(\eta_x(t))$ also increases while $\eta_y(t)$ increases if the solar resource is exploited. Thus $q(t)$ drops while $q_y(t)$ increases, implying that $q_x(t)$ decreases and thus also $x(t)$.

The only difference is that now, λ_Z may increase during the abatement phase if λ_S is sufficiently high. Producing useful energy from coal involves three resources. The first resource is the coal endowment stored underground, the second one is the carbon storage capacity of the atmosphere, the third one the carbon sinks resource. For any unit of available underground coal, the economy must balance between three options: first, keep this coal unit underground and use instead renewable energy, second, extract the coal unit, burn it and release its carbon content into the atmosphere, thus bear the induced pollution shadow cost but benefit from natural dilution or last, store the carbon content of the unit underground forever and incur the scarcity shadow cost of carbon storage. With small carbon storage capacities, the second option can outweigh the third one, inducing a possible rise over time of the shadow cost of atmospheric carbon. Note that the dynamics of λ_Z being dependent on the shapes of the cost functions, $b(\eta_x)$, $c(\eta_y)$ and $g(\eta_z)$, the evolution of the pollution shadow cost may be non monotonous during the abatement phase under the ceiling regime.

Turn to the abatement phase before the ceiling regime. It is immediately checked that $\eta_z(t)$ should increase, thus $A(\eta_x) = \mu' + G'(\eta_z)$ implies in turn that $A(\eta_x)$ should increase and thus also η_x.[21] It results that the dynamics of

[21] Time differentiating (3.7)' gets: $g''(\eta_z)\dot{\eta}_z = \dot{\lambda}_Z - \dot{\lambda}_S$. Taking (3.10) into account

p, q, η_y, q_x, q_y and x are qualitatively the same as in the original model during the abatement phase preceding the ceiling regime. Outside the abatement phase, the qualitative features of the optimal path in the original model are also preserved.

Under our assumptions, it is possible, even if not necessarily optimal, to perform any energy transformation rate between 0 and 1, the same applying to the abatement rate. Assume to the contrary that the technically feasible conversion rates or the abatement rate are constrained by upper bounds strictly lower than 1, $\hat{\eta}_i < 1$, $i = x, y, z$. The main qualitative dynamics of the original model stay unmodified although other optimal scenarios may arise. For example, assume that $\hat{\eta}_x < \hat{\eta}_y$ and that the coal energy conversion constraint becomes binding during the last phase of coal exploitation. Then appears a new time phase when the coal conversion performance remains fixed at the level $\hat{\eta}_x$ while the conversion rate of solar energy continues to increase. This phase ends when $\eta_y = \hat{\eta}_y$ after which the two conversion rates stay at their maximum levels until the transition toward the pure solar economy.

The same type of constraint could apply to abatement possibilities. Denote by $\hat{\gamma}_{\eta z}$ the Lagrange multiplier associated to the constraint $\eta_z \leq \hat{\eta}_z$, then the f.o.c. (3.7) is modified as follows during an active abatement phase:

$$\lambda_Z \;=\; g'(\eta_z) + \hat{\gamma}_{\eta z}/\zeta x \;\geq\; g'(\eta_z) \; .$$

If the abatement rate constraint binds, it must do so before the beginning of the ceiling regime. The abatement phase is now a sequence of three subphases. First the abatement rate rises until $\eta_z = \hat{\eta}_z$. Then the abatement rate stays constant while λ_Z continues to increase until the beginning of the ceiling regime. After $\underline{t}_Z$, the shadow cost of carbon, λ_Z, decreases. The coal extraction rate is maintained at the constant rate $\bar{x}(\hat{\eta}_z)$. If solar energy is used in combination with coal energy, its energy conversion performance increases together with the useful energy price. This induces in turn a rise of the coal energy conversion rate η_x. But if solar energy is not yet competitive, coal being the sole supplier of useful energy at the beginning of the ceiling regime, the useful energy price will be constant and so the coal energy transformation rate. When λ_Z has decreased down to the level $g'(\hat{\eta}_z)$, the economy is no

during an unconstrained phase, $\dot{\lambda}_Z - \dot{\lambda}_S = \rho(\lambda_Z - \lambda_S) + \alpha\lambda_Z = \rho g'(\eta_z) + \alpha\lambda_Z > 0$. Thus $\dot{\eta}_z > 0$.

more constrained by limited abatement possibilities. Next, the abatement rate decreases together with the shadow cost of carbon until $\lambda_Z = g'(0^+)$. Of course, the constraints on conversion rates and the abatement rate can combine themselves to generate even more complex optimal paths.

For the sake of simplicity, we have assumed a fixed supply of renewable crude energy in the form of a constant available flow, y^n, of this energy. Hence substitution from coal toward solar energy relies only on the improvement of the transformation rate of this energy source into useful energy. This approach may be generalized to substitution policies increasing the potential renewable energy supply. Such policies may be constrained by physical limitations in terms of favorable sites for solar or wind energy generation for example. More complex trade-offs can also arise. Renewable energy production may be in competition for space with other economic activities, the competition between biofuel production and food production being a well known example of such situations, worth a specific study.

Our work emphasizes that energy efficiency improvements may result from fundamental trade-offs between mitigation options under fossil fuel scarcity and a global carbon stabilization objective. Innovation and technical progress are usually offered as main explanations for the historical trend of energy efficiency. Introducing technological progress in our framework requires a careful distinction between incremental and drastic technical advances, that is possible technological revolutions in energy generation. We leave this problem for future research.

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Appendix

A.1 Construction of the system (4.23)

Differentiating the condition (4.22) yields a relationship between $d\eta_x$ and $d\eta_y$:

$$d\eta_y \;=\; \frac{b''(\eta_x)}{c''(\eta_y)}d\eta_x \; .$$

Inserting the expression of $\bar{\gamma}_x$ resulting from (4.21) into (4.19), we get:

$$b'(\eta_x)\eta_x - b(\eta_x) - g(\eta_z)\zeta - g'(\eta_z)(1-\eta_z)\zeta \;=\; \mu \; .$$

Differentiating this relation results in:[22]

$$\begin{aligned} & b''\eta_x d\eta_x + b'd\eta_x - b'd\eta_x - g'\zeta d\eta_z - g''(1-\eta_z)\zeta d\eta_z + g'\zeta d\eta_z \\ = \; & b''\eta_x d\eta_x - g''(1-\eta_z)\zeta d\eta_z \;=\; d\mu \; . \end{aligned} \tag{A.1.1}$$

Differentiating the relation (4.2), while substituting for $d\eta_y$ its expression as a function of $d\eta_x$, we obtain:

$$\begin{aligned} & u''\left[\frac{\alpha\bar{Z}}{\zeta(1-\eta_z)}d\eta_x + \eta_x\frac{\alpha\bar{Z}}{\zeta(1-\eta_z)}d\eta_z + y^n\frac{b''}{c''}d\eta_x\right] - b''d\eta_x \\ = \; & u''\left[\frac{\alpha\bar{Z}}{\zeta(1-\eta_z)} + \frac{b''}{c''}y^n - \frac{b''}{u''}\right]d\eta_x + u''\eta_x\frac{\alpha\bar{Z}}{\zeta(1-\eta_z)}d\eta_z = 0 \; . \end{aligned} \tag{A.1.2}$$

Expressing in matrix form the relations (A.1.1) and (A.1.2), we get the system (4.23).

[22] We drop the arguments of the functions when this causes no confusion.

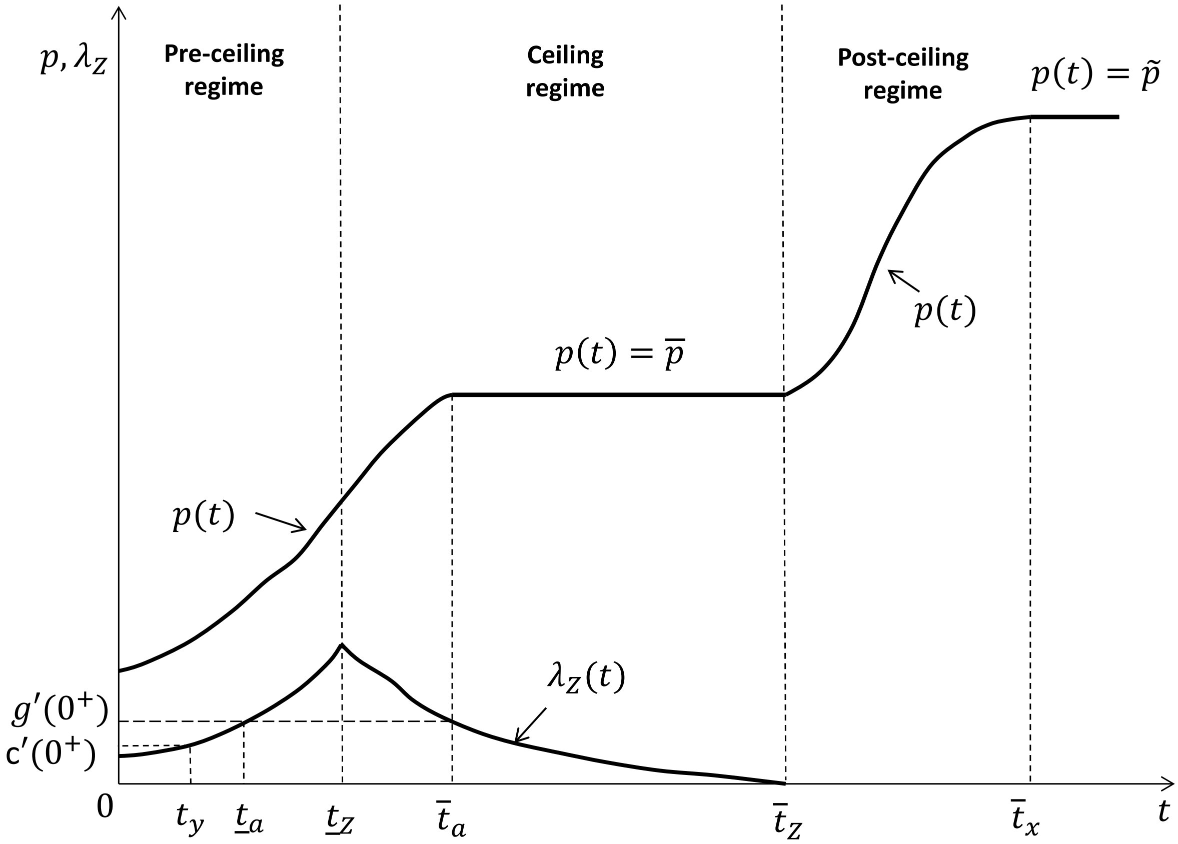

Figure 1: **Optimal Path of the Useful Energy Price and the Marginal Shadow Cost of the Pollution Stock.**

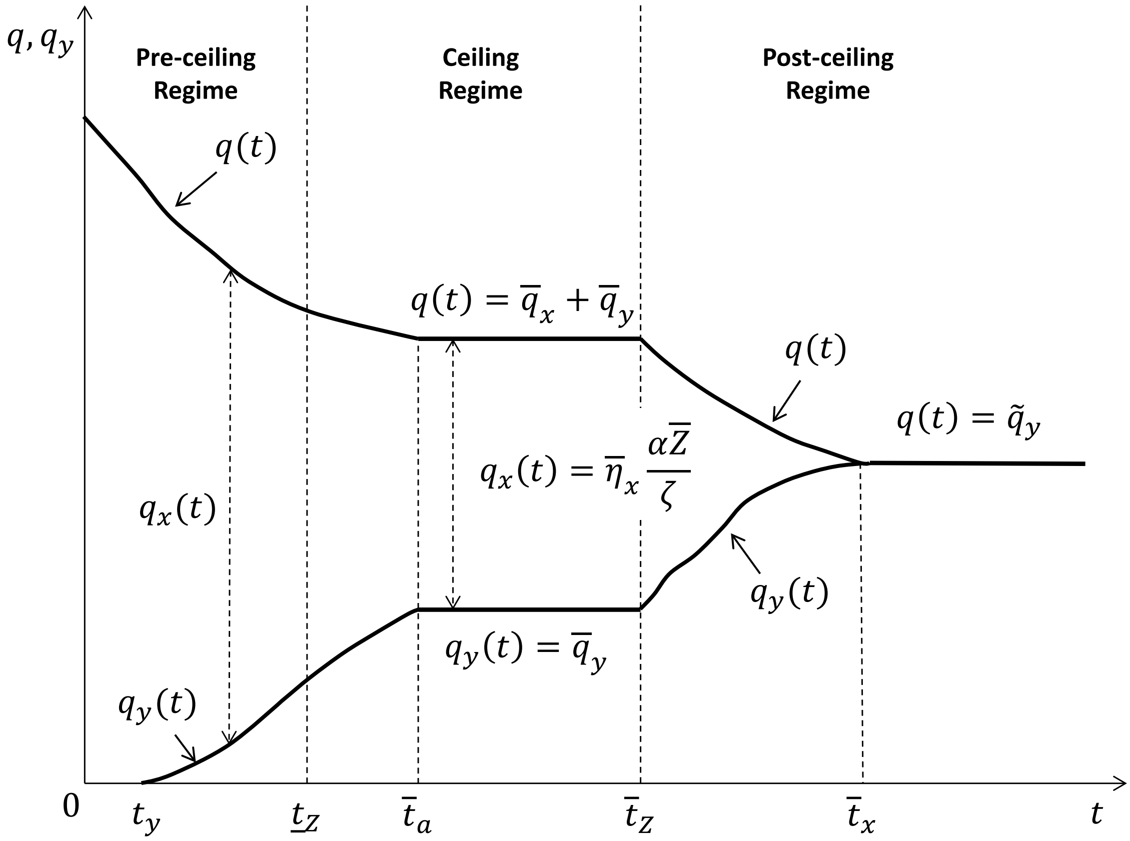

Figure 2: **Useful Energy Production Rates.**

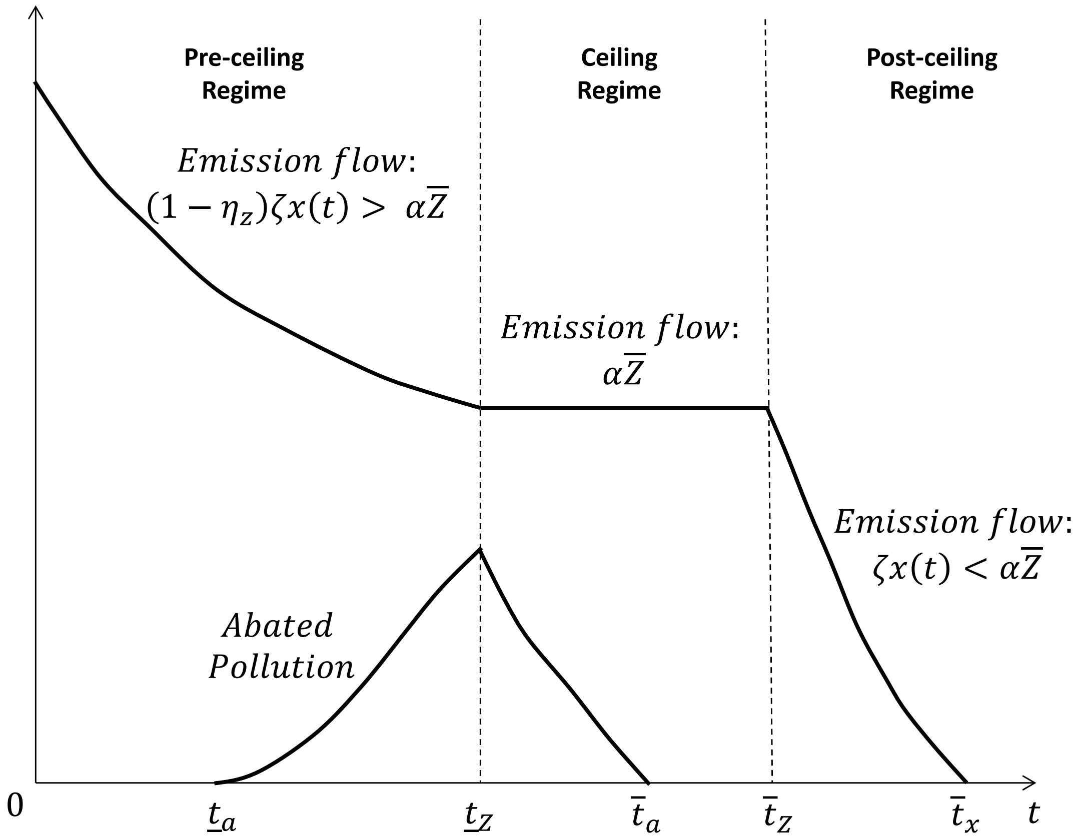

Figure 3: **Emissions and Abated Flows Paths.**

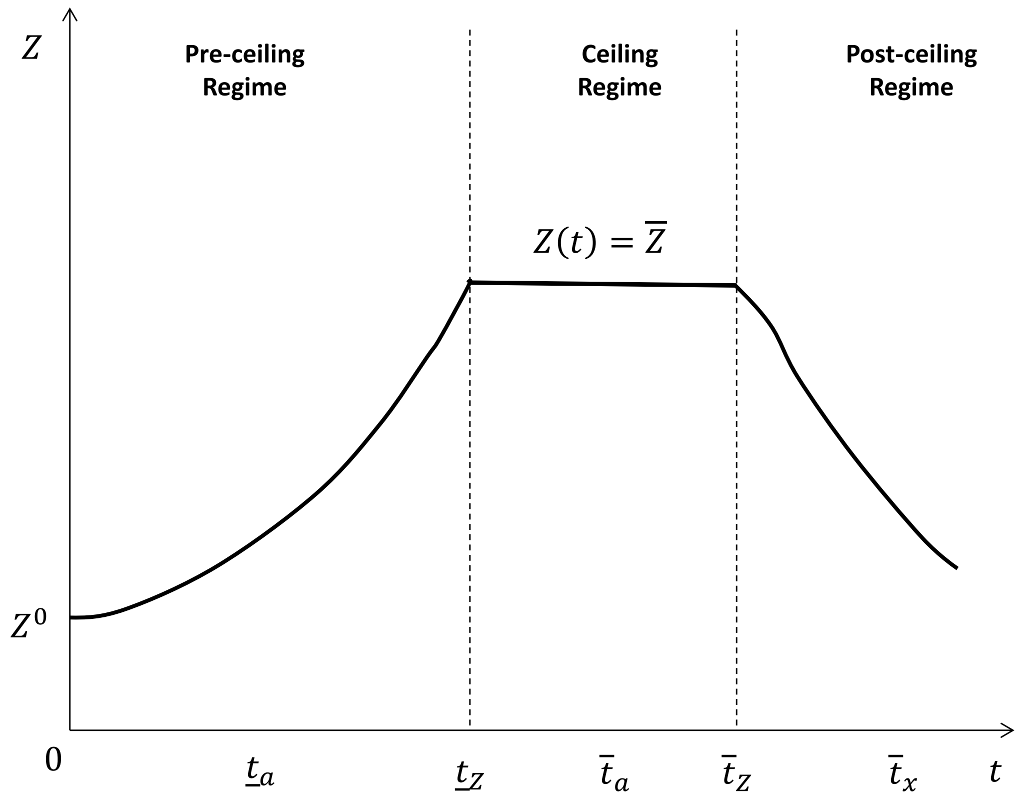

Figure 4: **Pollution Stock Path.**