Risk Aversion and Planning Horizon*

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ABSTRACT: Number of empirical studies seem to reject the additive separability of preferences that is assumed in most theoretical models of the life cycle. We show that, when additive separability is abandoned, and interactions between consumptions at different dates are taken into account, an interesting relation emerges between risk aversion and length of planning horizon. Specifically, we show that when consumptions at different dates are specific substitutes, risk aversion increases with horizon length. This may explain the surprising empirical finding that individuals seem to increase the share of wealth held in risky assets as they become older.

KEY WORDS: RISK AVERSION, LIFE CYCLE, NON SEPARABLE PREFERENCES
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1 Introduction

For many economic issues, such as for the design of social security or the management of pension funds, it is fundamental to know how the readiness of an individual to take financial risks may change as he/she gets older.

Such a question has been intensively addressed in the economic literature. In the standard additive model with CRRA preferences, Merton (1969) and Samuelson (1969) find that it is optimal to invest in risky assets a fraction of wealth that is independent of age. Samuelson (1989) contains a clear explanation of this result, which contradicts conventional wisdom. Later contributions try to find ways to get out of this surprising result. Bodie, Merton and Samuelson (1992) incorporate endogenous labor supply into the initial Merton-Samuelson model. They find that if labor supply is less flexible at older ages than at younger ages then relative risk aversion increases with age. Another important contribution is that of Gollier and Zeckhauser (2002), who show that because of the dynamic aspect of portfolio choice problems, the conclusion of Samuelson does not extend to all separably additive preferences.

The present paper contributes to this literature by examining the role of non separability of preferences. The assumption that preferences are separably additive has been consistently rejected by empirical evidence (see Muellbauer, 1988 and Carrasco, Labeaga and López-Salido, 2004, for example). It remains nonetheless extensively used, essentially because it is very convenient. This assumption is however crucial for discussing the impact of horizon length on attitudes towards financial risks. In fact, we show that, out of the additively separable case, individuals' relative risk aversion will generally change along the life cycle, independently of any age, wealth or time inconsistency effects. Moreover we relate the life cycle variations of relative risk aversion to standard measures of complementarity and substitutability of consumptions occurring at different dates. Roughly speaking, we find that if consumptions at different dates are specific substitutes, then relative risk aversion indices decrease along the life cycle, while they increase if consumptions at different dates are specific complements.

Our results provide therefore a simple explanation of why risk aversion may change along the life cycle: individual preferences may simply be non additively separable. Moreover, our results also indicate that relaxing this assumption of additive separability is not the source of an insuperable complexity. Even when it proves difficult or impossible to explicitly solve for intertemporal consumption-portfolio choices, it is nevertheless possible to infer how relative risk aversion varies along the life cycle by looking at intertemporal budget shares, Frisch's cross price elasticities and single period indexes of risk aversion.

The remainder of the paper is organized as follows. In Section 2 we briefly review the empirical literature on the relation between risk aversion on the one hand and age or horizon length on the other hand. In Section 3, we study, as an illustrative example, a simple model of portfolio choice with non-separable preferences. Section 4 defines a natural measure of intertemporal risk aversion and shows how this measure is related to portfolio choice. In Section 5, we consider the case of preferences that are separable but not necessarily additive in order to stress the impact of additivity on risk aversion. In Section 6, we examine the general case where both separability and additivity are relaxed. The empirical implications of our results are discussed in Section 7. Section 8 concludes.

2 Empirical findings

Before getting into theoretical considerations, we may wonder whether empirical studies suggest any relation between horizon length (or age) and risk aversion. The most direct way to assess intertemporal risk aversion of individuals is to elicit these individuals' preferences over lotteries on their lifetime income. However, it is rather difficult to observe actual situations where individuals have indeed to choose between several lotteries on their lifetime income. This is why most empirical papers base their estimates on virtual experiments. For example, Barsky et al (1997) find that the relation between relative risk aversion and age has an inverse U shape. Their sample is however restricted to people that are older than 50. By contrast, Guiso and Paiella (2001) find a positive relation between risk aversion and age. These results should nonetheless be interpreted with caution. They are based on hypothetical choices, and not on actual behavior. Also, they are cross-sectional studies and, therefore, do not allow to control for cohort effects.

Another approach is to look at the share of wealth held in risky assets, and to see how it changes along the life cycle. There are many studies that follow this track, including two recent books providing international comparisons (Guiso, Haliassos, and Jappelli, 2002a and 2002b), and a longitudinal study by Ameriks and Zeldes (2001). The cross sectional studies that are reported in Guiso, Haliassos, and Jappelli (2002a and 2002b) provide mixed evidence on how the share of wealth held in risky assets varies with age. In most cases, no significant relation between age and the share of risky assets is found. A U shape relation is found in the Netherlands and a weak positive relation is found in the USA. Longitudinal studies are more seldom. The well documented study of Ameriks and Zeldes (2001) concludes that, when controlling for cohort effects, there is, in the USA, a strong positive relation between the share of financial portfolios held in risky assets and age.

It should be stressed that the empirical studies reported in Guiso, Haliassos, and Jappelli (2002a and 2002b) and that of Ameriks and Zeldes (2001) analyze the share of financial or non-human wealth held in risky assets. None of them reports the share of total wealth (including human wealth) held in risky assets, which would be the relevant information to assess individual relative risk aversion, as it is made clear in Bodie, Merton and Samuelson (1992). The share of total wealth held in risky assets is related to the share of financial wealth held in risky assets by the following simple relation:

$$\frac{\text{Risky assets}}{\text{Human Wealth+Financial Wealth}} = \left(\frac{\text{Risky assets}}{\text{Financial Wealth}}\right)\left(\frac{1}{1+\frac{\text{Human Wealth}}{\text{Financial Wealth}}}\right)$$

Since the ratio $\frac{\text{Human Wealth}}{\text{Financial Wealth}}$ tends to decline along the life cycle, studies that focus on the ratio $\frac{\text{Risky assets}}{\text{Financial Wealth}}$ tend to underestimate the (positive) slope of the relation between age and the degree of aggregate risk taking.

Thus, the absence of relation between portfolio composition and age found in cross-sectional studies as well as the positive relation between portfolio's risk and age, found in the longitudinal study of Ameriks and Zeldes, would be consistent with a positive relation between relative risk tolerance and age, once human wealth is taken into account. Such a relation cannot be attributed to life cycle variations in wealth, for at least two reasons. First, most studies do control for individual wealth. Second, it is generally found that relative risk aversion decreases with wealth, while wealth tends to decline at the end of the life cycle. Therefore relative risk aversion should increase with age, while the opposite is found in the data. The present paper shows that non separability of preferences may provide an explanation for these findings.

3 Portfolio choice with non separable preferences: an illustrative example

In order to illustrate the impact of non separability of preferences on intertemporal variations of individuals' risk aversion, we consider in this section a very simple consumption-portfolio model. Individuals live 2 periods and have a lifetime von Neumann-Morgenstern utility function given by:

$$U(C_1, C_2) = f(u(C_1) + u(C_2)),$$

where $u(C) = \ln C$ is the instantaneous utility function[4] and $f(A) = \frac{1-e^{-kA}}{k}$.

The coefficient k measures interactions between consumptions are different dates:

$$\frac{\partial^2 U}{\partial C_1 \partial C_2}(C_1, C_2) = -\frac{k}{C_1 C_2} e^{-k(u(C_1)+u(C_2))}.$$

Thus consumptions at dates 1 and 2 are want independent[5] (separable preferences) if $k = 0$, specific substitutes[6] if $k < 0$, and specific complements if $k > 0$.

[4]To neutralize age-effects, we assume that the instantaneous utility is independent of age.

[5]When $k = 0$, we adopt the usual convention that $f(A) = A$.

[6]See a precise definition in Section 6.

At date 0, the individual invests a fraction θ_0 of her initial wealth W_0 in a risky asset, the rest being invested in a riskless asset. At date 1, she gets wealth W_1 out of which she chooses to consume C_1. Her remaining wealth is $W_1 - C_1$. Then she invests a fraction θ_1 of her remaining wealth in the same risky asset. At date 2, she consumes her final wealth W_2 . We denote by $\tilde{R}_0$ and $\tilde{R}_1$ the (random) returns of the risky asset in periods 0 and 1, supposed to be i.i.d. The riskless return is normalized to zero. We assume that the risky asset has a positive expected return R. The budget constraints are given by:

$$
\begin{aligned}
W_1 &= W_0(1+\theta_0\tilde{R}_0), && (1)\\
C_2 &= (W_1 - C_1)(1+\theta_1\tilde{R}_1). && (2)
\end{aligned}
$$

We focus on the evolution of the share of risky assets along the life cycle, i.e. whether $\theta_0 > \theta_1$ or not.

Notice first that in the separable case ($k = 0$), this share is constant: $\theta_0 = \theta_1$. Indeed at date 1, the individual chooses θ_1 to maximize $E\left[\ln(1+\theta_1\tilde{R}_1)\right]$. θ_1 does not depend on $(W_1 - C_1)$ because $u(C) = \ln C$ is CRRA. C_1 is chosen to maximize $\ln C_1 + \ln(W_1 - C_1)$ thus leading to

$$C_1 = W_1 - C_1 = \frac{1}{2}W_0(1+\theta_0\tilde{R}_0)$$

By backward induction, θ_0 is chosen at date 0 in order to maximize

$$E(\ln C_1 + \ln(W_1 - C_1)) = \text{ constant } + E\ln(1+\theta_0\tilde{R}_0).$$

Thus if $\tilde{R}_0$ and $\tilde{R}_1$ are identically distributed, θ_0 and θ_1 coincide: The share of risky assets in the portfolio of the individual is constant across the life cycle. Of course this is due, in part, to the fact that we have neutralized age effects and wealth effects. We claim that this is also due to intertemporal separability. To see this, consider now the non separable case ($k \neq 0$):

$$U(C_1, C_2) = \frac{1 - \exp^{-k(\ln C_1 + \ln C_2)}}{k} = \frac{1 - C_1^{-k} C_2^{-k}}{k}.$$

At date 1, the individual chooses θ_1 to maximize $EU(C_1, (W_1 - C_1)(1 + \theta_1 \tilde{R}_1))$. Again, the optimal θ_1 is independent of C_1 and W_1 because instantaneous utility u is CRRA (no wealth effect):

$$\theta_1 = \arg\max \left[-E(1 + \theta_1 \tilde{R}_1)^{-k} \right]. \tag{3}$$

Like in the separable case, C_1 is chosen to maximize $\ln C_1 + \ln(W_1 - C_1)$, leading to $C_1 = \frac{1}{2} W_1$, but the choice of θ_0 is changed. The objective function becomes

$$-E\left[C_1^{-k} C_2^{-k}\right],$$

which is proportional to

$$-E\left[(1 + \theta_0 \tilde{R}_0)^{-2k}\right]. \tag{4}$$

We obtain an expression similar to (3), but with a different exponent. This is because the risk on the portfolio chosen at $t = 0$ impacts two consumptions levels C_1 and C_2, whereas θ_1 only impacts C_2. When preferences are not separable, this changes the portfolio decision.

Specifically, in our example, it is easy to see that when $k > 0$, $\theta_0 < \theta_1$, which means that the individual takes more risk at date 1. Indeed this is an easy consequence of the following comparative statics property (proved in Appendix 1):

Lemma 1 *: Let*

$$\theta^*(k) = \arg\max \frac{1 - E[(1 + \theta \tilde{R})^{-k}]}{k}.$$

Then θ^* decreases in k.

When R, the expected return on the risky asset, tends to zero (while its variance σ^2 stays constant) a simple approximation of θ_0 and θ_1 can be derived. Indeed, a second order Taylor expansion of (3) shows that, when $R \to 0$ we have:

$$\theta_0 \simeq \frac{1}{(1+2k)} \frac{R}{\sigma^2} \text{ and } \theta_1 \simeq \frac{1}{(1+k)} \frac{R}{\sigma^2}. \tag{5}$$

As we will see in the following, this approximation could have been obtained from risk aversion considerations without solving the portfolio choice problem: the coefficients that appear before $\frac{R}{\sigma^2}$ in the above expressions are the intertemporal risk tolerance indices of the individual, a measure that we define in the next section.

4 Intertemporal risk aversion

The above example shows that relaxing the assumption of separability may significantly affect the relation between age and financial strategies. But so far, it is difficult to tell what drives the result. Is it a particularity of our simple model of portfolio choice, or does it reflect a fundamental aspect of non-additive preferences? The latter hypothesis is actually the right one. As we explain below, interactions between consumptions at different dates in individual's preferences are a key determinant of the relation between intertemporal risk aversion and horizon length. Moreover, since financial strategies are closely related to intertemporal risk aversion, we also find that portfolio choice depends on horizon length when preferences are not separable. However optimal portfolio selection with non separable preferences is a formidable computational problem, with no hope for a closed form solution, except in very peculiar cases So our strategy will be to define an intertemporal measure of risk aversion in a neighborhood of a deterministic consumption profile (the risk tolerance index at age n), and to study how this index varies along the life cycle. Then we will show that this index allows us to obtain a good approximation of the share of risky assets in the portfolios chosen by individuals of different ages, at least when the excess return of the risky asset is small.

Before introducing theoretical considerations on intertemporal risk aversion, a natural question arises: how can we compare risk aversions of individuals having different horizons? Individuals with different horizons have indeed preferences over different consumption sets. Comparative risk aversion was originally developed by Arrow (1971) and Pratt (1964) for preferences over a single commodity. It was extended by Kihlstrom and Mirman (1974) to the case where people consume several goods but have the same ordinal preferences. Clearly, this cannot be applied to individuals who consume over different numbers of periods. Karni (1979, 1983) suggested different approaches to multivariate comparative risk aversion but, again, no clear comparison can be obtained when applying these approaches to individuals who care for different goods. Thus, strictly speaking there is no theoretical foundation for comparing risk aversions of individuals with different horizon lengths.

A possibility, however, consists in comparing the degree of risk aversion of their indirect utility functions. Again, there are various options. The comparison can bear on relative or on absolute risk aversion. Also, since individuals of different ages may have different wealths, it is not clear whether we should we control for wealth variations or not. We define below an index of risk tolerance and explains that it is a natural measure for analyzing how the individuals' attitude towards risk varies along the life cycle.

Consider indeed individuals with a lifetime utility function $U(C_1, C_2, ..., C_N)$, and assume that they are time consistent. Formally speaking that means that an agent of age n with past consumptions $(C_1^*, C_2^*, ..., C_{n-1}^*)$ has preferences over $(C_n, ..., C_N)$ represented by the utility function

$$U_n(C_n, ..., C_N) = U(C_1^*, C_2^*, ..., C_{n-1}^*, C_n, ..., C_N)$$

The price of the composite good consumed in period i is denoted p_i. In absence of uncertainty, individuals with initial wealth W initially choose the consumption path $(C_1^*, C_2^*, .., C_N^*)$ that maximizes $U(C_1, ..., C_N)$ under the budget constraint $\sum_{i=1}^{N} p_i C_i = W$. At any age n, the remaining wealth is $W_n = W - \sum_{i=1}^{n-1} p_i C_i^*$ and individuals choose $(C_n, ...C_N)$ in order to maximize $U_n(C_n, ...C_N)$ under the budget constraint $\sum_{i=n}^{N} p_i C_i = W_n$. The time consistency assumption implies that the solution to the maximization program at age n is given by $(C_n^*, .., C_N^*)$ and therefore that individuals stick to their initial choices.

Definition 1 *The (global) risk tolerance index at age* n*, along consumption path* $C^* = (C_1^*, \dots, C_N^*)$*, is defined as:*

$$T_n\left(C_1^*, ..., C_N^*\right) = -\frac{V_n'(W_n)}{W_n V_n''(W_n)}$$

where $W_n = W - \sum_{i=1}^{n-1} p_i C_i^*$ *is the wealth held at age* n *and* $V_n(.)$ *is the value function of an individual of age* n *with utility function* U*:*

$$(\mathcal{P}_n) \begin{cases} V_n(W_n) = \max_{C_n, ... C_N} U(C_1^*, ..., C_{n-1}^*, C_n, ...C_N) \\ \sum_{i=n}^{N} p_i C_i = W_n \end{cases}, \tag{6}$$

To illustrate why the above index of risk tolerance is informative about how attitudes towards risk change along the life cycle, we consider two cases where some marginal uncertainty is added to the deterministic setting described above.

For the first illustration, imagine that at age n the individual is offered a choice between giving up a share α_n of his wealth (leaving him with wealth $(1-\alpha_n)W_n$) or going through a fair lottery that provides him with wealth $(1+\varepsilon)W_n$ or $(1-\varepsilon)W_n$ with equal probabilities. Now ask what is the share $\alpha_n(\varepsilon)$ that leaves the individual indifferent between the two alternatives. This is similar to computing a risk premium in one dimensional analysis. It is easy to show that:

$$\alpha_n(\varepsilon) = \frac{\varepsilon^2}{2T_n(C^*)} + o(\varepsilon^2).$$

This formula means that in a first approximation, the relative risk premium for a lottery on the individual's wealth at age n is inversely proportional to the (relative) tolerance index at age n, in conformity with the classical analysis à la Arrow-Pratt.

The second illustration consists in introducing a risky asset in the economy and looking at the limit behavior of portfolio choices at different ages when the return on the risky asset tends to zero, so that the fraction of wealth held in risky assets is small. We thus extend our model of Section 3 to the N period case. The return of the riskless asset is still assumed to be zero, but there is no loss of generality here since we assume that consumptions at different dates may have different prices. Individuals have an initial wealth W_0. At date 0 individuals choose θ_0, the fraction of W_0 that is invested in the risky asset. The return on the risky asset is R_0 which provides them, at date 1, with wealth $W_1 = W_0(1+\theta_0 R_0)$. Then individuals choose C_1, and the fraction θ_1 of their remaining wealth $W_1 - p_1 C_1$ invested in the risky asset. The return on the risky asset is R_1, which provides them at date 2 with a wealth $W_2 = (W_1 - p_1 C_1)(1 + \theta_1 R_1)$. The consumption C_2 is chosen, and so on, till period N, where individuals end up consuming all their wealth.

Assume that the risky returns R_i are i.i.d. with $E(R_i) = R$ and $var(R_i) = \sigma^2$. Denote by C^* the consumption path that is chosen when there is no risky asset (or when $R = 0$).

Lemma 2 *When R is close to zero, the share of wealth invested in the risky asset at date n is given by:*

$$\theta_n(R) = \frac{R}{\sigma^2} T_{n+1}(C^*) + o(R)$$

Proof. Let us denote by $V_n(C_1^*, ..., C_{n-1}^*, W_n)$ the indirect utility function at age n. By definition:

$$V_n(C_1^*, ..., C_{n-1}^*, W_n) = \max_{\theta_n, C_n} E[V_{n+1}(C_1^*, ..., C_n, (W_n - p_n C_n)(1 + \theta_n R_n))].$$

The first order condition of this problem with respect to θ_n gives:

$$E[(W_n - p_n C_n) R_n . V'_{n+1}(C_1^*, ..., C_n, (W_n - C_n)(1 + \theta_n R_n))] = 0$$

where the derivative is taken with respect to the last argument. When $E[R_n]$ and thus θ_n is small, a Taylor expansion gives, after simplifying by $(W_n - p_n C_n)$:

$$E[R_n] V'_{n+1}(W_n - p_n C_n) + \theta_n E\left[R_n^2\right](W_n - p_n C_n) V''_{n+1}(W_n - p_n C_n) \sim 0.$$

Since $E(R_n) = R$ is small, we can replace $E(R_n^2)$ by σ^2 and we obtain the desired result. ■

The share of wealth invested in the risky asset at date n is therefore proportional, in a first order approximation, to the risk tolerance index $T_{n+1}(C^*)$. We now study how $T_n(C^*)$ changes with n . This will give us a first approximation of how the optimal financial strategy of individuals varies along the life cycle.

5 Risk aversion with separable but non necessarily additive preferences

To stress the role played by the additivity assumption found in most studies, we consider in this section the simplest extension of the separably additive model. The (ordinal) assumption of separability of preferences is maintained[7], but we do not assume that the von Neumann-Morgenstern utility function is additive. From Gorman (1968) we know that separability implies that the lifetime von Neumann-Morgenstern utility function is of the

[7]That means that the indifference curves between consumption at two different periods do not depend on the consumption at other periods.

form:

$$U(C_1, \ldots, C_N) = f\left(\sum_{i=1}^{N} u_i(C_i)\right).$$

The function $f(\cdot)$ and the instantaneous utility functions $u_i(\cdot)$ are assumed to be twice continuously differentiable and to have positive first order derivatives. The shape of f captures the interactions between consumptions at different dates: nil if f is linear, complementarities if f is convex, substitutabilities if f is concave.

Proposition 1 *In the separably additive case (i.e. when f is linear), the risk tolerance index at date n along the consumption path $C^* = (C_1^*, \ldots, C_N^*)$ is a weighted sum of instantaneous risk tolerance indices:*

$$T_n(C^*) = \sum_{i=n}^{N} \alpha_i^n t_i(C_i^*), \tag{7}$$

where $\alpha_i^n = \frac{p_i C_i^*}{\sum_{j=n}^{N} p_j C_j^*}$ is the share of (remaining) intertemporal budget spent at date i and $t_i(C_i^*) = -\frac{u_i'(C_i^*)}{C_i^* u_i''(C_i^*)}$ is the instantaneous index of relative risk tolerance at date i.

This is a standard result. We do not provide a proof here, since this result is a particular case of Proposition 3, stated in Section 5, and proven in the Appendix.

Formula (7) shows well the different reasons why the risk tolerance index may vary along the life cycle. This may be because the functions $t_i(.)$, that measure instantaneous risk tolerance, change with age. We would have then "age effects". This would be the case, for example, if for some psychological reasons, older people prove to me more or less risk averse than younger individuals with respect to instantaneous consumption. Another possibility is that the functions $t_i(.)$ are all identical (no age effects) but that they are not constant in C^* and that consumption changes along the life cycle. Then we would have "wealth effects". However, since the weights α_i^n in formula (7) sum to one, it is clear that besides these age and wealth effects, there is no other element that may lead risk aversion to change along the life cycle:

if $t_i(C_i^*)$ is independent of i, then $T_n(C^*)$ is independent of n and there are no horizon effects.

In the particular case where instantaneous utility functions are all identical (no age effect) and CRRA (so that there is no wealth effect), relative risk tolerance is constant over the life cycle. This explains why there is no relation between horizon length and relative risk aversion in Merton-Samuelson's model (Merton, 1969, Samuelson, 1969).

From now on, we focus on the impact of relaxing additive separability on risk tolerance. To do this, we neutralize age effects by assuming that the instantaneous utility functions are identical across dates, up to a time preference factor, ($u_i \equiv \delta_i u$ with $\delta_i > 0$). We also neutralize wealth effects by considering stationary consumption paths. Proposition 2 shows that when f is non linear, risk tolerance indices vary along the life cycle:

Proposition 2 *Along any stationary consumption path* $(C^*, C^*, \ldots, C^*)$*, the sequence of risk tolerance indices* $T_1, \ldots, T_N$ *is increasing if* $f'' < 0$*, decreasing if* $f'' > 0$*, and constant if* $f'' \equiv 0$*.*

Proof. In order to sustain a stationary consumption path, prices must be proportional to δ_i. We normalize them so that $\sum_{i \geq n} p_i = 1$. Then $W = C$ and V_n is explicit

$$V_n(W) = f\left[\sum_{i<n} \delta_i u(C^*) + \left(\sum_{i \geq n} \delta_i\right) u(W)\right]. \tag{8}$$

Thus, we can immediately find V_n':

$$V_n'(W) = f'(A)\left(\sum_{i \geq n} \delta_i\right) u'(W),$$

where A denotes the term between brackets in (8), computed at the stationary consumption path $(W, W, \ldots, W)$ (notice that A is independent of n).

Similarly:

$$V_n''(W) = f'(A)\left(\sum_{i\geq n}\delta_i\right)u''(W) + f''(A)\left(\sum_{i\geq n}\delta_i\right)^2 u^{'2}(W).$$

Thus:

$$R_n = \frac{1}{T_n} = -\frac{C^*u''(C^*)}{u'(C^*)} - C^*\frac{f''(A)}{f'(A)}\left(\sum_{i\geq n}\delta_i\right)u'(C^*). \tag{9}$$

When f is linear ($f'' = 0$), risk aversion is constant along any stationary consumption path $(C^*, C^*, \ldots, C^*)$ and equal to the static risk aversion index $-C^*\frac{u''}{u'}(C^*)$. However when $f'' \neq 0$, there is a correcting term, which is positive and decreasing in n when $f'' < 0$ (but negative and increasing in n when $f'' > 0$). ■

It remains to extend the analysis to the case where preferences are neither additive nor separable. This is the object of the next section.

6 The impact of consumption interactions on risk aversion

The previous section made it clear that relaxing the assumption of additive separability may lead to revise significantly the relation between horizon length and risk aversion. It would however be excessively optimistic to say that empirical studies have so far clearly established how consumptions at different moments in time interact in consumers preferences. Most papers that challenged the additivity assumption have proposed particular extensions of the additively separable model and tested whether such extensions fit the data better. This is for example the case of papers on “habit formation”, who extend the standard additive model by allowing the marginal utility of current consumption to depend on past consumption (see Muellbauer 1988 and Dynan, 2000, for example). However the choice of these extensions is guided by intuitive arguments, or by technical reasons, rather than imposed by empirical evidence. The additive model is probably unrealistic but there is no less doubt about the validity of these simple extensions, as well as about the validity of the separable but non additive model that we studied in the previous section.

For that reason, it appeared important to us to derive results that do not rely on any particular specification. In the following we thus consider the general case where preferences are represented by a general, concave, twice continuously differentiable von Neumann-Morgenstern utility function:

$$U(C) = U(C_1, C_2, ..., C_N),$$

without making any further assumption. With this general formulation, we have to resort to the fundamental concepts of utility theory to describe individuals preferences. As we are interested in the cardinal properties of the utility function, we will naturally refer to the seminal contributions of Frisch (1959) and Houthakker (1960) and use their vocabulary:

Definition 2 *Consumptions at dates i and j are specific substitutes if and only if $[D^2U]^{-1}_{ij} > 0$. They are specific complements if and only if $[D^2U]^{-1}_{ij} < 0$, and "want independent" if and only if $[D^2U]^{-1}_{ij} = 0$.*

With intertemporally separable preferences, all consumptions at different periods are "want independent" since D^2U (and thus $[D^2U]^{-1}$) are diagonal (or block diagonal if several goods are consumed at each period).

Definition 3 *The coefficient of specific substitutability between consumptions at dates i and j (for a consumption profile C) is given by:*

$$\kappa_{ij}(C) = \frac{u_i u_j [D^2U]^{-1}_{ij}}{C_i u_i + C_j u_j} \tag{10}$$

where $u_i = \frac{\partial U}{\partial C_i}$ for $i = (1, \ldots, N)$.

This coefficient is positive if consumptions at dates i and j are specific substitutes and negative if they are specific complements. It is related to the notion of want elasticity of consumption at date i with respect to consumption at date j introduced by Frisch (1959):

$$x_{ij} \equiv \frac{u_j}{C_i}[D^2U]^{-1}_{ij}, \quad \text{for} \quad i \neq j.$$

We prefer, however, to use the κ_{ij}s rather than of the x_{ij}s, because they show better the symmetry of our results (the κ_{ij}s are symmetric, while the x_{ij}s are not).

The following result gives a general formula linking risk tolerance indices along the life cycle and coefficients of specific substitutability between consumptions at different dates (the κ_{ij}s, as defined in (10)). Our formula is valid when interactions are small but non negligible, i.e. when $\kappa \equiv \max_{i\neq j}|\kappa_{ij}|$ is small but not zero.

Proposition 3 *When interactions between consumptions at different dates are small but not negligible, the relative risk tolerance index at age n (along any consumption path) can be approximated by a weighted sum of instantaneous risk tolerance indices plus a correcting term. This correcting term is negative when consumptions at different dates are specific substitutes. More specifically, the relative risk tolerance at age n is given by:*

$$T_n(C^*) = \sum_{i\geq n} \alpha_i^n t_i(C^*) - \sum_{\substack{i,j\geq n \\ i\neq j}} (\alpha_i^n + \alpha_j^n)\kappa_{ij} + \kappa o(\kappa), \tag{11}$$

where $t_i(C^*) = -\frac{\frac{\partial U}{\partial C_i}(C^*)}{C_i^* \frac{\partial^2 U}{\partial C_i^2}(C^*)}$ *is the instantaneous risk tolerance index at date i,* $\alpha_i^n = \frac{C_i^* \frac{\partial U}{\partial C_i}(C^*)}{\sum_{j\geq n} C_j^* \frac{\partial U}{\partial C_j}(C^*)}$ *is the budget share spent at date i (relative to the budget to be spent in the remaining periods of life),* $\kappa = \max_{i\neq j}|\kappa_{ij}|$ *and* $\frac{o(\kappa)}{\kappa} \to 0$ *when* $\kappa \to 0$.

Proposition 3, which is proven in Appendix 2, allows to measure the bias introduced by neglecting intertemporal interactions. When U is additively separable, (i.e. $U(C) = \sum_i u_i(C_i)$) all the κ_{ij}s are zero and the intertemporal risk tolerance index $T_n(C^*)$ reduces to a weighted sum of instantaneous indices as stated in Proposition 1. However when consumptions at different dates are specific substitutes ($\kappa_{ij} > 0$) but such interactions remain small (κ small), the relative risk tolerance at age n is decreased by a factor that roughly equals a weighted sum of coefficients of specific substitutability between consumptions at different dates. The adjustment on risk tolerance is therefore negative when consumptions at different dates are specific substitutes and positive when consumptions at different dates are specific complements. More generally when some goods are specific complements to themselves (at other dates) but others are specific substitutes, the sign of the bias is given by the sum of these coefficients, weighted by the budget shares.

The size of the correcting term that accounts for the non separability of preferences varies with horizon length. Indeed this term, given by:

$$-\sum_{\substack{i,j \geq n \\ i \neq j}} (\alpha_i^n + \alpha_j^n)\kappa_{ij} \tag{12}$$

is a sum restricted to indices i and j that are equal or greater than current age. There are $\frac{1}{2}(N-n)(N-n+1)$ terms in that sum. However, the relative budget shares, α_i^n, are on average lower when the horizon length is large, since $\sum_{i \geq n} \alpha_i^n = 1$ by definition. Roughly speaking (that is if we omit the variations in the κ_{ij} and in the α_i^n) there are $\frac{1}{2}(N-n)(N-n+1)$ terms of size $\frac{2\kappa}{(N-n+1)}$ in the sum (12), which gives a term of size $\kappa(N-n)$. Hence, the correcting term increases (in absolute value) with the strength of the interaction between consumptions at different dates and with horizon length. The bias due to the assumption of separable additivity is therefore typically larger for younger individuals, who still have many periods to live, than for older individuals.

The reason why complementarity or substitutability of consumptions at different dates affects intertemporal risk tolerance is rather intuitive. For individuals who smooth consumption along the life cycle, a negative shock on wealth at date n will translate into negative shocks on consumption in all the remaining periods of life. Inversely a positive shock on wealth will generate positive shocks on consumption. The point to stress is that, whether the shock on wealth is positive or negative, it generates a sequence of shocks on instantaneous consumptions that are positively correlated. Risk aversion with respect to wealth is therefore akin to risk aversion with respect to positively correlated risks on instantaneous consumptions. If consumptions at different periods are neither complement nor substitute, as in the additive model, the fact that the risks on instantaneous consumptions are positively correlated does not matter. However, as soon as there are substitutabilities or complementarities between consumptions at different periods, the positive correlation does matter: it increases the degree of risk aversion when consumptions at different periods are substitutes and decreases it when they are complements. That explains the sign of the correcting term in (11). The magnitude of the correction depends on the number of correlations at play, and is therefore increasing (in absolute value) with the number of remaining periods of life. This explains the horizon effect that we obtain.

7 Are consumption at different ages specific substitutes or specific complements ?

Equation (11) shows that the relation between relative risk aversion and horizon length depends on the sign of the coefficients of specific substitutability. Thus it appears important to see whether there are any empirical findings or theoretical arguments that suggest a particular sign for the κ_{ij}s.

To our knowledge, the only paper to provide estimates of cross "want elasticities" is Browning[8] (1991). For parsimony reasons, Browning considers that such elasticities are non zero only for expenditures in adjacent time periods (expenditures at date t only interact with expenditures at dates $t-1, t$ and $t+1$). He finds that such interactions are small but non negligible. Most types of expenditures seem to be want independent but expenditures on durables are found (as expected) to be specific substitutes with themselves in adjacent periods[9].

[8] There is however an empirical literature on the estimation of Frisch intertemporal demand functions initiated by the important study of labor supply by MaCurdy (1981).

[9] Browning also finds that fuel is a specific complement with itself but the coefficient is

Hayashi (1985) also provides some support for the presumption that consumptions at different moments in time are substitutes. Although Hayashi does not estimate "want elasticities", his findings indicate that changes in consumption are strongly negatively autocorrelated. Hayashi attributes such a result to the "durability of consumption". This is in fact another way to express that consumptions at different moments in time are substitutes.

Theoretical arguments can also be given as to why consumptions at different dates can be specific substitutes. This has to do with the notion of "temporal risk aversion" or "intertemporal correlation aversion" introduced by Richard (1975).

Consider for example 2 dates ($n = 1, 2$) and 2 intertemporal lotteries:

$$L_1 = \begin{cases} (C_1, C_2) \\ \\ (c_1, c_2) \end{cases} \text{and } L_2 = \begin{cases} (C_1, c_2) \\ \\ (c_1, C_2) \end{cases},$$

both with equal probabilities $1/2, 1/2$. Assume that $c_1 < C_1$ and $c_2 < C_2$. An individual with separable preferences (i.e. $U(C_1, C_2) = U_1(C_1) + U_2(C_2)$) is indifferent between L_1 and L_2, since both lotteries give the same sum of expected utilities $\frac{1}{2}[U_1(c_1)+U_1(C_1)]+\frac{1}{2}[U_2(c_2)+U_2(C_2)]$. But this is generally not the case when preferences are not additively separable. Now, we say that an individual is "averse to intertemporal correlation" if he prefers L_2 to L_1. Intuitively, he prefers to have some of the worst and some of the best, rather than to take a chance on all of the worst or all of the best. Such a pattern happens when $\frac{\partial^2 U}{\partial C_1 \partial C_2} < 0$ and thus when goods 1 and 2 are specific substitutes.

In the N period model, it is no longer true that aversion to intertemporal correlation and positive specific substitutability are equivalent properties. However, the equivalency holds locally when we consider weak interactions. Indeed, from Lemma 3 in the appendix, we know that when interactions are weak

$$[D^2U]^{-1}_{ij} \simeq -\frac{\frac{\partial^2 U}{\partial C_i \partial C_j}}{\frac{\partial^2 U}{(\partial C_i)^2}\frac{\partial^2 U}{(\partial C_j)^2}} \tag{13}$$

smaller.

Thus it is the case that when interactions are weak, preferences that exhibit aversion to intertemporal correlation also exhibit positive specific substitutability.

Another argument can be made by comparing relative risk aversion and intertemporal elasticity of substitution. It is well known that the standard life-cycle model with additive preferences and isoelastic instantaneous utility functions implies that relative risk aversion equals the inverse of intertemporal elasticity of substitution. The most popular way to break this unpleasant relation between two apparently distinct concepts is to relax the von Neumann-Morgenstern axioms and follow the theory developed by Kreps and Porteus (1978) on temporal lotteries (see for example Epstein and Zin, 1989, Weil, 1990, Farmer, 1990, or Campbell, 1993). However the relation can also be broken while remaining within the standard von Neumann-Morgenstern framework on atemporal lotteries. Actually, within the von Neumann-Morgenstern framework, (local) relative risk aversions always equal the inverse of the (local) intertemporal elasticity of substitution if and only if the utility function is additively separable (see Bommier, 2003). Moreover the difference between (local) relative risk aversion and (local) intertemporal elasticity of substitution is precisely determined by aversion to intertemporal correlation. Aversion to intertemporal correlation and specific substitutability being closely related (at least when interactions are weak) the coefficients of specific substitutability that we consider in the present paper can be related to the difference between local measures of intertemporal elasticity of substitution and local indices of relative risk aversion. By definition, the elasticity of substitution between consumptions at dates i and j (holding consumption in other periods constant) is given by[10]:

$$\sigma_{ij} = \frac{\frac{1}{C_i \frac{\partial U}{\partial c_i}} + \frac{1}{C_j \frac{\partial U}{\partial c_j}}}{-\frac{\frac{\partial^2 U}{(\partial C_i)^2}}{\left(\frac{\partial U}{\partial C_i}\right)^2} + 2\frac{\frac{\partial^2 U}{\partial C_i \partial C_j}}{\frac{\partial U}{\partial C_i}\frac{\partial U}{\partial C_j}} - \frac{\frac{\partial^2 U}{(\partial C_j)^2}}{\left(\frac{\partial U}{\partial C_j}\right)^2}}$$

[10]Note that with separable additive preferences and an isoelastic instantaneous utility function the σ_{ij}s equal a constant, the so-called "intertemporal elasticity of substitution".

It follows simply from (10) and (13) that, in a first order approximation, the coefficients of specific substitutability are also given by:

$$\kappa_{ij} \simeq \frac{1}{2r_i r_j}\left[\frac{\alpha_j}{\alpha_i+\alpha_j}(r_i - \frac{1}{\sigma_{ij}}) + \frac{\alpha_i}{\alpha_i+\alpha_j}(r_j - \frac{1}{\sigma_{ij}})\right]$$

where $r_i = -\frac{C_i \frac{\partial^2 U}{(\partial C_i)^2}}{\frac{\partial U}{\partial C_i}}$ is the relative risk aversion index with respect to consumption in period i and α_i the budget share spent in period i. Thus, if all local coefficients of relative risk aversion are greater than the inverse of intertemporal elasticity of substitution between any two periods then preferences exhibit positive specific substitutability. Empirical measures of local relative risk aversion indexes and intertemporal elasticities of substitution could then be used to determine whether consumptions at different dates are specific complements or specific substitutes, and ultimately to elucidate the relation between horizon length and (global) risk aversion[11]. Unfortunately, both risk aversion and intertemporal elasticities of substitution are particularly difficult to measure and the empirical literature remain inconclusive about the sign and the magnitude of these differences.

To conclude this section on intertemporal interactions, we discuss the relation between specific interactions and habit formation. For technical reasons, most papers on habit formation assume that the intertemporal utility function has some form of additive structure. Namely, the N period utility

[11] Equation (11) can actually be rewritten:

$$T_n(C^*) = \sum_{i\geq n} \alpha_i^n t_i(C^*) - \sum_{\substack{i,j\geq n\\ i\neq j}} \frac{\alpha_j^n (r_i - \frac{1}{\sigma_{ij}})}{r_i r_j} + \kappa o(\kappa).$$

Therefore, it is clear that the correcting term that appears in equation (11) and accounts for interactions between consumption at different dates can also be written, in a first order approximation, as a weighted sum of differences between local relative risk aversion indices and the inverse of local intertemporal elasticities of substitution. In particular, when all local relative risk aversion indices are greater than inverse elasticities of substitution, the correction is negative. Moreover, for the same arguments as we gave after equation (12), the magnitude of the correction typically increases with horizon length.

function is generally specified as the sum of instantaneous utility functions:

$$U^H(C_1, ...C_N) = \sum_{i=1}^{N} V_i(C_i, X_i) \tag{14}$$

where X_i, the stock of habits in period i, is positively related to consumptions in the previous periods. It is also assumed that the cross derivatives $\frac{\partial^2 V_i}{\partial C_i \partial X_i}$ are positive. As a consequence, for any $i \neq j$, $\frac{\partial^2 U^H}{\partial C_i C_j} > 0$ and preferences exhibit negative intertemporal correlation aversion. The coefficients of specific substitutability are therefore negative. Thus, it appears that most papers on habit formation do assume that consumptions at different dates are specific complements, which implies that there is a positive relationship between risk tolerance and horizon length. However, this is only the case because these papers rely on the additive structure of (14). For a general (non additively separable) specification, there is no systematic relation between habit formation and specific substitutability. Habit formation (as defined in Becker and Murphy, 1988) is equivalent to the notion of adjacent complementarity introduced by Ryder and Heal (1973). Preferences are said to exhibit adjacent complementarity if $\frac{\partial}{\partial C_i}(\frac{\frac{\partial U}{\partial C_{i+1}}}{\frac{\partial U}{\partial C_{i+2}}}) > 0$ for all $i \leq N-2$ (the marginal rate of substitution between present and future consumption increases with past consumption). This is an ordinal notion, that is preserved under any increasing transformation. In other words, if a utility function U exhibits adjacent complementarity then any monotonic transformation $\widetilde{U} = f(U)$ will also exhibit adjacent complementarity. However for f sufficiently concave, consumptions at different dates become specific substitutes. Preferences involving habit formation can therefore exhibit specific complementarity as well as specific substitutability.

8 Concluding remarks

We have shown in this paper that interactions between consumptions at different dates could generate variations of relative risk aversion along the life cycle, even if tastes do not vary with age and wealth effects are controlled for. More specifically, Proposition 2 has shown that when the von Neumann-Morgenstern utility of an individual is a concave transformation of an additively separable function, relative risk aversion decreases with age along any stationary consumption path. Proposition 3 extends this result to a more general form of interactions and to non stationary consumption paths. It provides an evaluation of the bias introduced by the separability assumption in the estimation of intertemporal risk aversion. This bias is approximately equal to minus the sum of specific substitutability coefficients, weighted by budget shares. The bias is typically larger (in absolute values) for young individuals, who still have many periods to live, than for older ones.

Our results can be used in different ways. We can apply them to models that assume simple specifications for the utility function. Take for example an exponential transformation of a sum of CRRA utilities:

$$U(C) = -\frac{1}{k}\exp(-k\sum_{i=1}^{N}\frac{C_i^{1-\gamma}-1}{1-\gamma}) \tag{15}$$

where k is positive. A simple application of Formula (9) immediately gives the relative risk aversion coefficient R_n of an individual of age n along any constant consumption path:

$$R_n = \frac{1}{T_n} = \gamma + k(N-n+1)C^{1-\gamma}. \tag{16}$$

As expected, R_n decreases with n, since $k > 0$.

The utility function that we used for our illustrative example in Section 3 is obtained for $\gamma = 1$ and $N = 2$. From (16) we obtain $T_1 = \frac{1}{1+2k}$ and $T_2 = \frac{1}{1+k}$ which, combined with Lemma 2, leads to the result obtained in Section 3 (see equation 5). We have thus found two ways to obtain the same results. The first way, that we followed in Section 3, consists in providing an explicit solution to the portfolio choice problem and deriving some of its properties. This is indisputably the most popular approach in the Finance literature. The second way consists in looking at marginal properties of the utility function, and in particular at our measure of intertemporal risk tolerance indices. The first method has an obvious advantage: it works even when the share of risky assets is relatively large, and portfolio risks are not small. But it has also a major drawback: it can only work when it is possible to derive a closed form solution to the portfolio choice problem. Needless to say, the number of specifications for which such closed form solutions are available is very limited. The literature has naturally focused on these particular specifications, but they have no reason to fit observed behavior particularly well.

The alternate route that we have followed in this paper does not suffer from such technical constraints. Actually our results make it possible to derive estimates of how risk aversion varies with age, even if we only have a limited and local knowledge on individuals preferences. Consider for example formula (11):

$$T_n(C^*) \simeq \sum_{i \geq n} \alpha_i^n t_i(C^*) - \sum_{\substack{i,j \geq n \\ i \neq j}} (\alpha_i^n + \alpha_j^n) \kappa_{ij}$$

Budget shares α_i^n are usually relatively well observed. The other ingredients needed to obtain intertemporal risk tolerance indices as a function of age are local estimates of the instantaneous risk tolerance indices, $t_i(C^*)$, and the coefficients of specific substitutability, κ_{ij}. Imagine, for example, that we observe that all the budget shares are equal. Also assume that, at the optimal consumption path, there is no variation in the instantaneous indices of relative risk tolerance along the life cycle ($t_i(C^*) = \frac{1}{\gamma}$), and the coefficients of specific substitutability are of the form $\kappa_{ij} = \kappa\rho^{|i-j|-1}$. The parameter κ gives then the strength of the interactions while ρ determines their shape (specific substitution decreases with time distance if $\rho < 1$ and increases with time distance if $\rho > 1$). In such a case, formula (11) leads to:

$$\begin{aligned} T_n(C^*) &\simeq \tfrac{1}{\gamma} - \tfrac{2\kappa}{(N-n+1)} \textstyle\sum_{\substack{i,j\geq n \\ i\neq j}} \rho^{|i-j|-1} \\ &= \tfrac{1}{\gamma} - \tfrac{4\kappa}{(N-n+1)} \left[\tfrac{\rho(\rho^{N-n}-1)+(N-n)(1-\rho)}{(1-\rho)^2} \right] \end{aligned} \tag{17}$$

This relation between relative risk tolerance and horizon length is shown in Figure 1. The corresponding picture for the relative risk aversion is found in Figure 2. In particular, we observe that the relation between relative risk tolerance and horizon length is convex if specific substitutability decreases with time distance, and concave if specific substitutability increases with time distance.

An unresolved issue is whether considerations on risk aversion suffice to provide a relatively good approximation of the life cycle financial strategy of individuals. We have shown that this is the case when the share of risky assets are small, but one may wonder whether it remains true when agents take non infinitesimal risks. In this latter case, individuals' wealth follows a random path and the dynamic aspects of the problem that are underlined in Gollier and Zeckhauser (2002) must be considered. In particular it matters whether risk tolerance indices are convex or concave with respect to wealth. Whether these considerations are likely to generate larger effects than those discussed in this paper is hard to tell. One can reasonably think however that the fundamental properties of preferences that drives the result of Gollier and Zeckhauser (which are related to the fourth derivative of the utility function) will be more difficult to test empirically than the complementarities or substitutabilities that we have discussed, which depend on second derivatives only. In particular, the impact of non separability that we analyze in this paper is already present when instantaneous preferences are CRRA, whereas the phenomenon studied by Gollier and Zeckhauser (2002) would vanish in this case.

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APPENDIX 1: Proof of Lemma 1

$$\theta^*(k) = \arg\max \varphi(\theta, k),$$

where

$$\varphi(\theta, k) = \frac{1 - E[1 + \theta\tilde{R}]^{-k}}{k}.$$

To establish that $\theta^*(\cdot)$ is decreasing, it is enough to show that $\frac{\partial^2\varphi}{\partial\theta\partial k}(\theta^*(k), k) < 0$ (single crossing property).

Indeed

$$\frac{\partial\varphi}{\partial\theta} = E\left[\tilde{R}(1 + \theta\tilde{R})^{-i-1}\right]$$

and

$$\frac{\partial^2\varphi}{\partial k\partial\theta} = -E\left[\tilde{R}\ln(1 + \theta\tilde{R})(1 + \theta\tilde{R})^{-k-1}\right].$$

Now for all $\theta > 0$ and all $\tilde{R}$, $\tilde{R}\ln(1 + \theta\tilde{R}) > 0$. Thus $\frac{\partial^2\varphi}{\partial k\partial\theta} < 0$. The fact that $\theta^*(k) > 0$ comes from our assumption that $E[\tilde{R}] > 0$ (since $\frac{\partial\varphi}{\partial\theta}(0, k) = E[\tilde{R}]$).

APPENDIX 2: Proof of Proposition 3

It relies on two simple ingredients :

- a formula due to Hanoch (1977) that relates $T(C)$, the intertemporal risk tolerance index along a consumption path C to the matrix $(D^2U)^{-1}(C)$ and the utility gradient $\nabla U(C)$:

$$T(C) = \frac{{}^t\nabla U(D^2U)^{-1}\nabla U}{t_{C.\nabla U}}$$

- a linear algebra lemma about the inverse of non singular matrices that are almost diagonal:

Lemma 3 *Consider a matrix* $M = (m_{ij})$ *with* $m_{ii} \neq 0$ *for all* i *and note* $m = sup_{i \neq j}|m_{ij}|$*. Then, when* m *is small enough ,* M *is non singular and the* $i, j - th$ *elements of* M^{-1} *are given by:*

$$\begin{aligned} [M^{-1}]_{ii} &= \frac{1}{m_{ii}} + mo(m) \\ [M^{-1}]_{ij} &= -\frac{m_{ji}}{m_{ii}m_{jj}} + mo(m) \ \textit{if } i \neq j \end{aligned}$$

where $\frac{o(m)}{m} \to 0$ *when* $m \to 0$*.*

Proof. Take M non singular, with $m = Sup_{i \neq j}|m_{ij}|$ close to zero and define $\varphi_{ij}(M) = [M^{-1}]_{ij}$, the generic term of M^{-1}. $\varphi_{ij}(M)$ is given explicitly by the classical formula:

$$\varphi_{ij}(M) = \frac{(-1)^{i+j} \det(M_{ji})}{\det(M)}, \tag{A1}$$

where $\det(A)$ denotes the determinant of any square matrix A and M_{ij} is the submatrix obtained by deleting the $i - th$ row and the $j - th$ column of M. Define $\Delta = Diag(M)$, the matrix obtained from M by deleting off-diagonal terms. Since φ_{ij} is differentiable on its domain (we note $D\varphi_{ij}$ its derivative) we can write a Taylor expansion around Δ, that is valid for m small:

$$\varphi_{ij}(M) = \varphi_{ij}(\Delta) + D\varphi_{ij}(\Delta)(M - \Delta) + mo(m),$$

where $\frac{o(m)}{m} \to 0$ when $m \to 0$.

Since $\varphi_{ii}(\Delta) = (m_{ii})^{-1}$ and $\varphi_{ij}(\Delta) = 0$ for $i \neq j$, Lemma 3 is proven if we can establish that $D\varphi_{ii}(\Delta)(M - \Delta) = 0$ and $D\varphi_{ij}(\Delta)(M - \Delta) = \frac{(-1)^{i+j} m_{ji}}{m_{ii}m_{jj}}$ for $i \neq j$. To do so, let us first compute the partial derivatives of φ_{ij} by differentiating (A1) with respect to m_{kl} (for arbitrary k, l). We find:

$$\frac{\partial \varphi_{ij}}{\partial m_{kl}}(\Delta) = \frac{(-1)^{i+j}}{\det(\Delta)} \frac{\partial[\det(M_{ji})]}{\partial m_{kl}}|_{M=\Delta} - (-1)^{i+j} \frac{\det(\Delta_{ji})}{\det^2(\Delta)} \frac{\partial[\det(M)]}{\partial m_{kl}}|_{M=\Delta}.$$

Now

$$\begin{aligned}\frac{\partial[\det(M_{ji})]}{\partial m_{kl}}|_{M=\Delta} &= (-1)^{i+j-1}\frac{\det(\Delta)}{m_{ii}m_{jj}} \text{ if } k=i, l=j \\ &= 0 \text{ otherwise,}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial[\det(M)]}{\partial m_{kl}} &= \frac{\det(\Delta)}{m_{kk}} \text{ if } k=l, \\ &= 0 \text{ otherwise.}\end{aligned}$$

Since $(M-\Delta)_{kl} = m_{kl}$ if $k \neq l$ and zero otherwise,

$$D\varphi_{ij}(\Delta)(M-\Delta) = \sum_{k\neq l} m_{kl}\left(\frac{-1}{m_{ii}m_{jj}}\mathbb{1}_{l=j,\,k=i}\right).$$

Thus we have established the desired result:

$$\begin{aligned}D\varphi_{ij}(\Delta)(M-\Delta) &= 0 \text{ if } i=j \\ &= -\frac{m_{ji}}{m_{ii}m_{jj}} \text{ if } i\neq j.\end{aligned}$$

■

Proof of Proposition 3

For any past consumptions $(C_1^*, ..., C_{n-1}^*)$ we define

$$U_n(C_n, ..., C_N) = U(C_1^*, ..., C_{n-1}^*, C_n, ..., C_N)$$

Using Hanoch's formula (see Hanoch 1977, p. 416) in developed form:

$$T_n(C) = -\sum_{i=n}^{N} \frac{[D^2U_n]^{-1}_{ii} u_i^2}{{}^t c.\nabla U_n} - \sum_{\substack{i \neq j \\ n \leq i,j \leq N}} \frac{[D^2U_n]^{-1}_{ij} u_i u_j}{{}^t c.\nabla U_n}. \tag{A2}$$

where $u_i = \frac{\partial U_n}{\partial C_i} = \frac{\partial U}{\partial C_i}$. Recall the expressions of the relative budget shares $\alpha_i^n = \frac{C_i u_i}{{}^t c.\nabla U_n}$, specific substitutability coefficients $\kappa_{ij} = \frac{[D^2U]^{-1}_{ij} u_i u_j}{c_i u_i + c_j u_j}$ and instantaneous risk tolerance coefficients $t_i = -\frac{u_i}{C_i \frac{\partial^2 U}{\partial C_i^2}}$. Lemma 3 shows that when $\kappa = \max_{i \neq j} |\kappa_{ij}|$ is small, $(D^2U_n)^{-1}_{ii} = (\frac{\partial^2 U_n}{\partial C_i^2})^{-1} + \kappa o(\kappa) = (\frac{\partial^2 U}{\partial C_i^2})^{-1} + \kappa o(\kappa)$ and thus:

$$-\frac{[D^2U_n]^{-1}_{ii} u_i^2}{{}^t c.\nabla U_n} = \alpha_i^n t_i + \kappa o(\kappa).$$

Moreover, from Lemma 3, for $i \neq j$ we have $[D^2U_n]^{-1}_{ij} = [D^2U]^{-1}_{ij} + \kappa o(\kappa)$ and therefore:

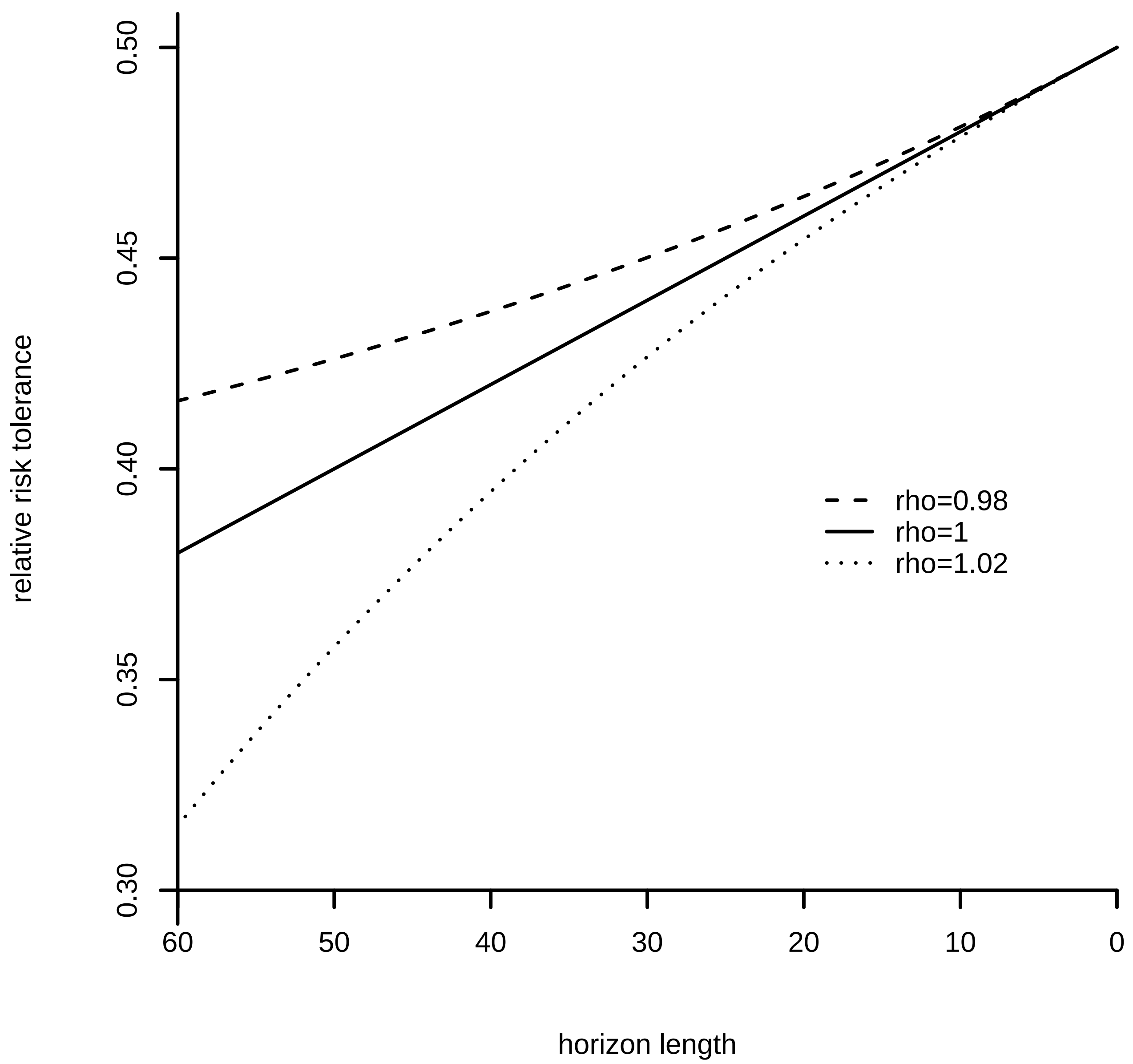
$$\frac{[D^2U_n]^{-1}_{ij} u_i u_j}{{}^t c.\nabla U_n} = (\alpha_i^n + \alpha_j^n)\kappa_{ij} + \kappa o(\kappa)$$

Thus (A2) can be written:

$$T_n(C) = \sum_{i=n}^{N} \alpha_i^n t_i - \sum_{\substack{i \neq j \\ n \leq i,j \leq N}} (\alpha_i^n + \alpha_j^n)\kappa_{ij} + \kappa o(\kappa),$$

and the proof of Proposition 3 is complete ■

Figure 1: Relative risk tolerance according to horizon length



Estimation form equation (10) with sigma=0.001 and gamma=2

Figure 2: Relative risk aversion according to horizon length