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“On Competing Mechanisms under Exclusive Competition”

Andrea Attar, Eloisa Campioni and Gwenaël Piaser

Toulouse School of Economics

On Competing Mechanisms under Exclusive Competition

Andrea Attar* Eloisa Campioni† Gwenaël Piaser‡

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Abstract

We study games in which several principals design incentive schemes in the presence of privately informed agents. Competition is exclusive: each agent can participate with at most one principal, and principal-agents corporations are isolated. We analyze the role of standard incentive compatible mechanisms in these contexts. First, we provide a clarifying example showing how incentive compatible mechanisms fail to completely characterize equilibrium outcomes even if we restrict to pure strategy equilibria. Second, we show that truth-telling equilibria are robust against unilateral deviations toward arbitrary mechanisms. We then consider the single agent case and exhibit sufficient conditions for the validity of the revelation principle.

Keywords: Competing Mechanisms, Exclusive Competition, Incomplete Information.
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*Toulouse School of Economics (CRM-IDEI) and DEF Università di Roma Tor Vergata, Via Columbia 2 - 00133 Roma (Italy); andrea.attar@tse-fr.eu

†DEF Università di Roma Tor Vergata, Via Columbia 2 - 00133 Roma (Italy); eloisa.campioni@uniroma2.it

‡IPAG Business School, 27 rue de l'université - 75007 Paris (France); gwenael.piaser@ipag.fr

CORRESPONDING AUTHOR : Andrea Attar, Toulouse School of Economics, 21 allee de Brienne, 31000 Toulouse (France).

1 Introduction

We study competing mechanism games of incomplete information in which competition is exclusive: contracts incorporate exclusivity clauses and they are fully enforced. Thus, upon observing her private information and the publicly observable mechanisms, each agent chooses to participate with at most one principal. Given all participation decisions, each principal-agents corporation is isolated: the payoff to any member of a given corporation does not depend on any decision taken outside the corporation. Final allocations are determined by the contracts that principals independently sign with agents.

Exclusive competition is at the hearth of most theoretical analyses of markets in which agents hold some private information. This assumption plays a fundamental role in competitive screening (Rothschild and Stiglitz (1976)), competitive search (Guerrieri et al. (2010)) and competing auctions (McAfee (1993)) models, among many others. At the same time, letting firms compete over exclusive contracts is regarded a convenient way to represent full observability of agents' trades in the extension of general equilibrium theory to private information economies (Prescott and Townsend (1984), Bisin and Gottardi (2006)).

Despite the prominent role of these economic applications, we still lack a general characterization of equilibrium mechanisms in exclusive competition settings. Indeed, the approaches above share the restriction to standard incentive compatible mechanisms: principals commit to incentive schemes such that each agent finds it optimal to truthfully report her private information to the principal she negotiates with, as long as the others do so. However, few, if any, theoretical arguments have been developed to support this choice. If competition is exclusive, to what extent one can safely restrict attention to incentive compatible mechanisms? The present paper contributes to answer this question. Specifically, we analyze the role of the incentive compatible mechanisms introduced in the traditional, single-principal, mechanism design (Myerson (1979) and Myerson (1982)) in competing mechanism games under exclusive competition. We provide two main results.

First, we show that the restriction to incentive compatible mechanisms involves a loss of generality. Indeed, we exhibit a *pure strategy* equilibrium outcome of a game in which principals post indirect mechanisms that cannot be reproduced by incentive compatible ones. This is supported by agents' randomizing over messages and participation following every principal's deviation, given the indirect mechanisms posted by his competitors. All such randomizations serve the role of threats for principals. It turns out that incentive compatible mechanisms are not rich enough to reproduce the same threats and at least one principal can profitably deviate. The result suggests that standard models of exclusive competition may fail to provide a full characterization of equilibrium outcomes. This failure, however, only arises with multiple agents. In the particular case in which principals compete for a single agent, we show that incentive compatible mechanisms retain full power if the way in which the agent breaks her ties is properly considered. Every equilibrium outcome which survives to principals' unilateral deviations, irrespective of the continuation equilibrium selected by the agent, can also be characterized by incentive compatible mechanisms. Although being demanding, this restriction is tight as we illustrate by means of an example.

Second, we investigate whether an equilibrium outcome supported by incentive compatible mechanisms survives if a principal is allowed to deviate to general mechanisms. This amounts to analyze the implications of enlarging the strategy space of a single principal, holding fixed the behavior of his rivals. We show that any pure strategy equilibrium in which agents truthfully report their private information survives if a principal deviates to any indirect mechanism. This provides a rationale for the restriction to standard incentive compatible mechanisms postulated in economic models of exclusive competition.

1.1 Incentive compatibility and exclusive competition: an overview

We analyze competing mechanism games of incomplete information in which agents can participate with at most one principal and their preferences are such that, given participation decisions, each principal-agents corporation is isolated. If competition is non-exclusive, several game-theoretic examples[1] show that incentive compatible mechanisms may fail to provide a full characterization of equilibrium outcomes. The result, however, crucially exploits the fact that each agent participates with many principals at a time. If agents can participate with at most one principal, a potential loss of generality associated to incentive compatible mechanisms is documented in the literature on competition between principal-agents corporations, building on the original example in Myerson (1982). This literature models situations in which, although each agent is restricted to be part of only one corporation, competition is affected by an externality between corporations: the payoff to an agent of a given corporation is determined by market conditions, which depend on the behavior of all agents, including those belonging to the other corporations.[2] Martimort (1996) is the first to show how this externality can be responsible for a failure of incentive compatible mechanisms to be a general characterization device. A similar insight, though in a more abstract setting, is provided by Epstein and Peters (1999) who construct equilibria in which principals correlate agents' decisions through messages in a way that cannot be reproduced by incentive compatible mechanisms.[3] To achieve the correlation, they exploit the assumption that agents in one corporation send non-trivial messages to the principals of other corporations. In the absence of these externalities, are incentive compatible mechanisms restrictive? We show the existence of equilibrium outcomes that cannot be supported by such mechanisms even if every player's payoff is only affected by the decisions of the members of her corporation. Such a failure of the revelation principle may be relevant in the light of strategic analyses of markets with incomplete information as discussed in Section 2.2. If principals compete for a single agent, intuition suggests that one can follow the general logic in Myerson (1982) and rely on incentive compatible mechanisms. To the best of our knowledge, we are the first to provide a formal proof of this result.

[1] See, for instance, Peters (2001) and Martimort and Stole (2002).

[2] See, for example, the analysis of competition among manufacturers when their retailers are privately informed of market conditions. In these contexts, different works investigate the link between pre-commitment effects and renegotiation (Caillaud et al. (1995)), the rationale behind alternative forms of vertical restraints (Gal-Or (1991)), the welfare implications of information sharing (Pagnozzi and Piccolo (2013)). See Rey and Vergé (2008) for a survey of recent results.

[3] See Appendix A in Epstein and Peters (1999).

Another key issue for equilibrium characterization is whether incentive compatible mechanisms may allow to characterize any best reply of a single principal to a given profile of mechanisms posted by his opponents. In a framework of exclusive competition and many agents, Peck (1997) exhibits a negative result. Given the mixed strategy of one principal, his opponent can post a mechanism inducing a continuation equilibrium which cannot be reproduced by requiring that agents behave truthfully. This suggests that incentive compatible mechanisms may fail to characterize some of the principals' best replies. We identify a role for these mechanisms under exclusive competition.[4] Indeed, our Proposition 1 shows that truth-telling equilibria in which principals play pure strategies are robust to deviations towards indirect mechanisms. This in turn clarifies that Peck (1997)'s result fundamentally relies on principals playing mixed strategies. More generally, throughout the paper, and in line with standard approaches to competing mechanism games, principals are not allowed to condition their mechanisms on those of their competitors. The recent work of Szentes (2015) is one of the first attempts to analyze the implications of allowing for contractible contracts in exclusive competition settings. He shows that equilibrium outcomes supported by "non contingent" mechanisms are robust to the introduction of contingent contracts, although the latter typically induce equilibrium indeterminacy. In the light of his first result, one could therefore reinterpret our Proposition 1 to argue that truth-telling equilibria provide a useful reference point also under very general contracting assumptions.

The paper is organized as follows: Section 2 develops a general model of competing mechanisms under exclusive competition and illustrates its main economic applications. Section 3 analyzes the multiple agent setting, and Section 4 the single agent one.

2 The model

2.1 Competing mechanisms under exclusive competition

We refer to a scenario in which several principals (indexed by $j \in \mathcal{J} = \{1, \ldots, J\}$) contract with several agents (indexed by $i \in I = \{1, \ldots, I\}$). Each agent i has private information about her type $\omega^i \in \Omega^i$ and $\omega = \{\omega^1, \cdots, \omega^I\} \in \Omega = \underset{i \in I}{\times} \Omega^i$ is the array of agents' types, which is a random variable with distribution F.

Each principal j may choose an action $x_j \in X_j$. Agents only take participation decisions, with $a^i_j \in \{Y, N\}$ being the decision of agent i to participate with principal j, in which $\{N\}$ stands for not participating. We take $v_j : X \times A \times \Omega \to \mathbb{R}_+$ and $u^i : X \times A \times \Omega \to \mathbb{R}_+$ to be the payoff to principal j and to agent i, respectively, with $X = \underset{j \in \mathcal{J}}{\times} X_j$ and $A = \underset{i \in I}{\times} A^i$. For a given array of agents' types ω, of actions $a = \left(a^1, a^2, \ldots, a^I\right)$ and of principals' decisions $x = (x_1, x_2, \ldots, x_J)$, the payoffs to agent i and to principal j are $u^i(x, a, \omega)$ and $v_j(x, a, \omega)$, respectively.

Each principal perfectly observes the set of agents who participate with him. Communication is one-sided: each agent i may send a private message $m^i_j \in M^i_j$ to principal j. We let

[4] Under non-exclusive competition, Han (2007), Attar et al. (2010) and Attar et al. (2013) show that this result requires demanding sufficient conditions. In particular, under incomplete information, Attar et al. (2013) show that equilibria supported by incentive compatible mechanisms do not satisfy such a robustness requirement, in general. They therefore introduce a class of two-sided communication mechanisms to obtain a characterization of principals' best replies.

each set M_j^i be sufficiently rich to include the element $\{\emptyset\}$ corresponding to the information "agent i does not communicate with principal j", and to satisfy the standard size restriction $\sharp M_j^i > \sharp\Omega^i$ for every i and j. Principal j takes his decisions contingent on the array of messages m_j he receives, with $m_j = \left(m_j^1, m_j^2, \ldots, m_j^I\right) \in M_j = \underset{i \in I}{\times} M_j^i$, and on the participation choices of the agents. Formally, we say that a mechanism proposed by principal j is the measurable mapping $\gamma_j : M_j \times \{Y, N\}^{\sharp I} \to \Delta(X_j)$. We take Γ_j to be the set of mechanisms available to principal j and denote $\Gamma = \underset{j \in \mathcal{J}}{\times} \Gamma_j$. All relevant sets are taken to be compact and measurable with respect to the topology of weak convergence. The competing mechanism game relative to Γ begins when principals publicly and simultaneously commit to mechanisms. Given the posted mechanisms $(\gamma_1, \gamma_2, \ldots, \gamma_J) \in \Gamma$ and their privately observed types, agents simultaneously take a participation and communication decision with respect to every principal. In this incomplete information game, a (mixed) strategy for principal j is a randomization $\delta_j \in \Delta(\Gamma_j)$, and $\delta = (\delta_1, \cdots, \delta_J) \in \Delta(\Gamma)$ is a profile of mixed strategies for principals.

In line with economic applications, we model exclusive competition as a restriction on agents' preferences and on their strategy spaces. If competition is exclusive, for a given participation choice, the payoff to any agent i who participates with some principal j does not depend on the decisions of the principals she is not participating with, nor on the agents' participations with principal j's opponents. That is, agents' preferences satisfy the following:

Assumption S-u *Let $I_j = \{i \in I : a_j^i = Y\}$. For every $j \in \mathcal{J}$, $\omega \in \Omega$, $h \in I_j$, $x_j \in X_j$ and I_j:*

$$u^h\left(x_j, x_{-j}, a, \omega\right) = u^h\left(x_j, x'_{-j}, a', \omega\right),$$

for each $x_{-j}, x'_{-j} \in X_{-j}$ and for each a, $a' \in A$ such that $a_j^i = a_j^{i\prime} = Y$ for every $i \in I_j$.

A strategy for each agent i associates to every profile of posted mechanisms a joint participation and communication decision. In a pure strategy, every agent participates with at most one principal and sends a non-degenerate message only to the principal she participates with. We let $S^i = \left\{s^i \in M^i \times A^i : m_j^i = \emptyset \text{ iff } a_j^i = \{N\}\right\}$ be the strategy set for agent i, with $A^i = \left\{a^i = \left(a_1^i, \ldots, a_J^i\right) \in \{Y, N\}^{\sharp \mathcal{J}} : a_j^i = \{Y\} \text{ for at most one } j\right\}$ and $M^i = \underset{j \in \mathcal{J}}{\times} M_j^i$ representing the sets of participation and communication decision, respectively. A strategy for agent i is then the measurable mapping $\lambda^i : \Gamma \times \Omega^i \to \Delta\left(S^i\right)$. Every $\lambda = \left(\lambda^1, \lambda^2, \ldots, \lambda^I\right)$ induces a probability distribution over principals' decisions. Given $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_J)$, we let $\beta_j^\lambda \in \Delta(X_j)$ be the distribution over principal j's decisions induced by λ, and let $\beta^\lambda = \underset{j \in \mathcal{J}}{\times} \beta_j^\lambda$. The expected payoff to each type ω^i of agent i is:

$$U^i\left(\gamma, \lambda, \omega^i\right) = \int_{\Omega^{-i}} \int_X u^i\left(x, a, \omega^i, \omega^{-i}\right) F\left(\omega^{-i} | \omega^i\right) d\beta^\lambda \, d\omega^{-i},$$

with $F\left(\omega^{-i} | \omega^i\right)$ being the conditional probability of ω^{-i} given ω^i. The expected payoff to principal j when he plays δ_j against his opponents' strategies δ_{-j} is:

$$V_j\left(\delta_j, \delta_{-j}, \lambda\right) = \int_{\Gamma_j} \int_{\Gamma_{-j}} \int_\Omega \int_X v_j(x, a, \omega) \, F(\omega) \, d\beta^\lambda \, d\omega \, d\delta_{-j}(\gamma_{-j}) \, d\delta_j(\gamma_j).$$

The strategies (δ,λ) constitute an equilibrium relative to $\Delta(\Gamma)$ if λ is a continuation equilibrium for every γ in the support of δ and if, given δ_{-j} and λ, for every $j \in \mathcal{J}$: $\delta_j \in \underset{\delta'_j \in \Delta(\Gamma_j)}{argmax}\, V_j\left(\delta'_j,\delta_{-j},\lambda\right)$.

A mechanism available to principal j is *direct* if agents can only communicate their types to principal j, i.e. if $M^i_j = \Omega^i \cup \{\emptyset\}$ for every i, with $\{\emptyset\}$ representing no communication. We denote a direct mechanism for principal j as $\tilde{\gamma}_j : \underset{i\in I}{\times}\left(\Omega^i \cup \{\emptyset\}\right) \times \{Y,N\}^{\sharp I} \to \Delta(X_j)$ and the set of direct mechanisms as $\Gamma^D_j \subseteq \Gamma_j$. We let G^Γ be the competing mechanism game induced by a given Γ, and G^D the game in which principals are restricted to direct mechanisms. As in Myerson (1982), a direct mechanism is *incentive compatible* from the point of view of principal j if, given the mechanisms offered by the other principals, it induces a continuation equilibrium in which agents truthfully reveal their types to him. A direct mechanism $\tilde{\gamma}_j$ can therefore be incentive compatible for a given array $\tilde{\gamma}_{-j}$, but not for some other $\tilde{\gamma}'_{-j} \neq \tilde{\gamma}_{-j}$. An equilibrium is *truth-telling* if every principal posts an incentive compatible mechanism and agents truthfully reveal their private information to the principals they participate with, whenever this constitutes an equilibrium in their continuation game.

2.2 Applications

Our general model encompasses standard economic applications of exclusive competition with incomplete information, as illustrated below.

Competitive Insurance In their canonical analysis, Rothschild and Stiglitz (1976) study strategic competition between insurance companies for the exclusive right to serve a customer. The customer faces a binary risk on her endowment $w \in \{w_L, w_H\}$, with probabilities $(p, 1-p)$ that constitute her private information. Her (expected) payoff is $pu(w_L + d_L) + (1-p)u(w_H + d_H)$, with $(d_L, d_H) \in \mathbb{R}^2$ being the state-contingent transfers issued by the company she trades with, which guarantees that Assumption S-u is satisfied. Insurers are restricted to post incentive compatible (deterministic) mechanisms. The model of this section can hence be interpreted in terms of competitive insurance by letting $I = 1$, $\omega \equiv (p, 1-p)$ and $\tilde{\gamma}_j : \Omega \times \{Y,N\} \to \mathbb{R}^2$.

Competitive Search The model above has been subsequently extended to analyze the interaction between pricing and trading probability. Specifically, Inderst and Wambach (2002) study a competitive market in which privately informed consumers apply to firms who face capacity constraints, and may therefore end up being rationed. The competitive search literature endogeneizes rationing as a byproduct of search frictions and bilateral matching (Guerrieri et al. (2010)). Under competitive search, principals simultaneously post mechanisms and each agent applies to at most one of them. The ratio between the number of principals posting a given mechanism γ and the number of agents applying for it determines the rationing probability $\mu(\gamma)$, which every agent takes as given. If an agent is not rationed, she gets her exogenous reservation utility. For a given application decision, each agent's payoff therefore only depends on the mechanism she applies for and on the corresponding rationing probability. Our Assumption S-u is hence satisfied if we interpret an application decision as our participation choice. On the principals' side, attention is restricted to incentive compatible mechanisms that are contingent on agents' reported types and, possibly, on their application decisions.

Competing Auctions In a seminal paper, McAfee (1993) analyzes sellers who compete over auctions when buyers' valuation constitute their private information. Peters (1997) shows that in a decentralized market with many sellers and buyers, second-price auctions arise as equilibrium mechanisms. In these settings, sellers simultaneously and anonymously post their reservation prices and buyers choose at most one auction to participate in. A seller and the buyers who participate in his auction form an isolated corporation, hence Assumption S-u is satisfied. In addition, sellers are restricted to post direct mechanisms, asking each buyer $i \in I$ to report her valuation $v^i \in [0,1]$. A strategy for seller j is a mechanism $\tilde{\gamma}_j : |I_j| \times [0,1]^{|I_j|} \to \mathbb{R}$, where $I_j \subseteq I$ is the set of buyers that participate in auction j. A pure strategy for buyer i is a mapping $\lambda^i : \Gamma_1 \times ... \times \Gamma_J \times [0,1] \to A^i \times [0,1] \times \mathbb{R}_+$, with Γ_j being set of second-price auctions for $j \in J$. Given her participation decision, it is always a dominant strategy for each of the buyers to truthfully report ther private valuations. The model of this section therefore adapts to the competing auctions settings of Peters (1997), Peters and Severinov (1997), Burguet and Sakovics (1999), Viràg (2010), Han (2014), Peck (2015).

Despite its large domain of applicability, our model does not incorporate the effects of introducing exclusive contracts in contexts in which agents' participation choices are unrestricted. This last feature indeed implies that such settings (see for instance O'Brien and Shaffer (1997) and Bernheim and Whinston (1998)) do not typically satisfy Assumption S-u.

3 The multi-agent case

This section analyzes the role of incentive compatible mechanisms in games with several agents.

3.1 Failure of the revelation principle

We first show a negative result: there exist *pure strategy* equilibrium outcomes of G^Γ that cannot be supported in any truth-telling equilibrium of G^D. The result is established in a complete information context, in which agents' preferences satisfy Assumption S-u, and there is no direct externality between principals. It provides an instance of the failure of the revelation principle in multi-agent games of exclusive competition.

Example 1 *Let $I = J = 2$ and $\Omega^1 = \Omega^2 = \{\omega\}$. In addition, take $X_1 = \{x_{11}, x_{12}\}$ and $X_2 = \{x_{21}, x_{22}\}$ to be the decision sets of principal 1 ($P1$) and 2 ($P2$). Let $A^1 = A^2 = \{YN, NY, NN\}$, with YN denoting the agent's decision to accept the offer of $P1$ and reject that of $P2$. Payoffs are represented in the matrix below, in which agents decide in the external box and principals decide in the internal 2x2 cells. Each array represents the payoffs to $P1$, $P2$, agent 1 ($A1$) and agent 2 ($A2$), respectively.*

		YN			NY			NN	
		x_{21}	x_{22}		x_{21}	x_{22}		x_{21}	x_{22}
YN	x_{11}	$(0,0,5,12)$	$(0,0,5,12)$	x_{11}	$(0,8,5,12)$	$(0,10,5,8)$	x_{11}	$(0,0,5,0)$	$(0,0,5,0)$
	x_{12}	$(0,0,10,4)$	$(0,0,10,4)$	x_{12}	$(0,8,10,12)$	$(0,10,10,8)$	x_{12}	$(0,0,10,0)$	$(0,0,10,0)$
		x_{21}	x_{22}		x_{21}	x_{22}		x_{21}	x_{22}
NY	x_{11}	$(0,0,2,10)$	$(0,0,1,10)$	x_{11}	$(0,0,2,12)$	$(0,0,1,8)$	x_{11}	$(0,0,2,0)$	$(0,0,1,0)$
	x_{12}	$(0,0,2,5)$	$(0,0,1,5)$	x_{12}	$(0,0,2,12)$	$(0,0,1,8)$	x_{12}	$(0,0,2,0)$	$(0,0,1,0)$
		x_{21}	x_{22}		x_{21}	x_{22}		x_{21}	x_{22}
NN	x_{11}	$(0,0,0,10)$	$(0,0,0,10)$	x_{11}	$(0,8,0,12)$	$(0,10,0,8)$	x_{11}	$(0,0,0,0)$	$(0,0,0,0)$
	x_{12}	$(0,0,0,5)$	$(0,0,0,5)$	x_{12}	$(0,8,0,12)$	$(0,10,0,8)$	x_{12}	$(0,0,0,0)$	$(0,0,0,0)$

Consider the game G^{Γ} in which $M^1_j = M^2_j = \{s,s'\}$ for $j = 1,2$ and the array of mechanisms (γ_1,γ_2) such that:

$$\gamma_1(m_1,a_1) = \begin{cases} x_{12} & \textit{if at least one agent participates and only one } s \textit{ is received} \\ x_{11} & \textit{otherwise} \end{cases}$$

$$\gamma_2(m_2,a_2) = \begin{cases} x_{22} & \textit{if only one agent participates sending } s \\ & \textit{or if both agents participate and A1 sends } s' \\ x_{21} & \textit{otherwise} \end{cases}$$

These mechanisms are part of an equilibrium in G^{Γ}. Indeed, they induce the following continuation game between agents:

	YNs	YNs'	NYs	NYs'	NN
YNs	$(5,12)$	$(10,4)$	$(10,8)$	$(10,12)$	$(10,0)$
YNs'	$(10,4)$	$(5,12)$	$(5,8)$	$(5,12)$	$(5,0)$
NYs	$(1,5)$	$(1,10)$	$(2,12)$	$(2,12)$	$(1,0)$
NYs'	$(2,5)$	$(2,10)$	$(1,8)$	$(1,8)$	$(2,0)$
NN	$(0,5)$	$(0,10)$	$(0,8)$	$(0,12)$	$(0,0)$

in which YNs represents the decision to accept the proposal of $P1$ and to report him the message s.[5] *The game admits only one equilibrium: $A1$ playing YNs and $A2$ playing NYs'. The corresponding decisions are (x_{12},x_{21}) which induce the outcome $(0,8,10,12)$. Since $P1$'s payoff is constantly equal to zero, posting γ_1 is optimal for him. We only need to prove that P2 has no profitable deviations.*

Suppose first that he deviates to a stochastic take-it or leave-it offer. This corresponds to a probability distribution over (x_{21},x_{22}), which we denote $(\alpha,1-\alpha)$. Following such deviation, the continuation game among agents is:

[5] For the sake of notation, we choose not to represent the empty messages of the agents' that are implied by a rejection decision, N.

	YNs	YNs'	NY	NN
YNs	$(5,12)$	$(10,4)$	$(10,8+4\alpha)$	$(10,0)$
YNs'	$(10,4)$	$(5,12)$	$(5,8+4\alpha)$	$(5,0)$
NY	$(1+\alpha,5)$	$(1+\alpha,10)$	$(1+\alpha,8+4\alpha)$	$(1+\alpha,0)$
NN	$(0,5)$	$(0,10)$	$(0,8+4\alpha)$	$(0,0)$

Since NY and NN are strictly dominated for $A1$, and NN is strictly dominated for $A2$, the game reduces to:

	YNs	YNs'	NY
YNs	$(5,12)$	$(10,4)$	$(10,8+4\alpha)$
YNs'	$(10,4)$	$(5,12)$	$(5,8+4\alpha)$

This game has only one equilibrium, in which $A1$ randomizes over (YNs,YNs') with probabilities $\left(\frac{1+\alpha}{2},\frac{1-\alpha}{2}\right)$ and $A2$ randomizes over (YNs,NY) with probabilities $\left(\frac{1}{2},\frac{1}{2}\right)$. The payoff to P2 is $\frac{1}{2}(10-2\alpha)<8$ for every $\alpha\in[0,1)$. Any such deviation is hence unprofitable.

Consider now a deviation towards a general stochastic mechanism γ_2' that associates a probability distribution over (x_{21},x_{22}) to every array of agents' messages and participation choices:

$$
\begin{aligned}
\gamma_2'(s,\emptyset,Y,N) &= (\alpha_1,1-\alpha_1), & \gamma_2'(s,s',Y,Y) &= (\alpha_5,1-\alpha_5),\\
\gamma_2'(s',\emptyset,Y,N) &= (\alpha_2,1-\alpha_2), & \gamma_2'(s,s,Y,Y) &= (\alpha_6,1-\alpha_6),\\
\gamma_2'(\emptyset,s,N,Y) &= (\alpha_3,1-\alpha_3), & \gamma_2'(s',s,Y,Y) &= (\alpha_7,1-\alpha_7),\\
\gamma_2'(\emptyset,s',N,Y) &= (\alpha_4,1-\alpha_4), & \gamma_2'(s',s',Y,Y) &= (\alpha_8,1-\alpha_8),
\end{aligned}
$$

with α_k denoting the probability of x_{21} for $k=1,2,...,8$. The agents' continuation game is:

	YNs	YNs'	NYs	NYs'	NN
YNs	$(5,12)$	$(10,4)$	$(10,8+4\alpha_3)$	$(10,8+4\alpha_4)$	$(10,0)$
YNs'	$(10,4)$	$(5,12)$	$(5,8+4\alpha_3)$	$(5,8+4\alpha_4)$	$(5,0)$
NYs	$(1+\alpha_1,5)$	$(1+\alpha_1,10)$	$(1+\alpha_6,8+4\alpha_6)$	$(1+\alpha_5,8+4\alpha_5)$	$(1+\alpha_1,0)$
NYs'	$(1+\alpha_2,5)$	$(1+\alpha_2,10)$	$(1+\alpha_7,8+4\alpha_7)$	$(1+\alpha_8,8+4\alpha_8)$	$(1+\alpha_2,0)$
NN	$(0,5)$	$(0,10)$	$(0,8+4\alpha)$	$(0,8+4\alpha)$	$(0,0)$

which, by iterated elimination of strictly dominated strategies, reduces to:

	YNs	YNs'	NYs	NYs'
YNs	$(5,12)$	$(10,4)$	$(10,8+4\alpha_3)$	$(10,8+4\alpha_4)$
YNs'	$(10,4)$	$(5,12)$	$(5,8+4\alpha_3)$	$(5,8+4\alpha_4)$

Once again, this game exhibits a unique mixed strategy equilibrium, which yields to P2 a payoff strictly lower than 8.

Consider now the game G^D. A direct mechanism for $P1$ maps the agents' participation choices into lotteries on X_1 and can be represented as follows:

$$
\begin{aligned}
\tilde{\gamma}_1(Y,N) &= (\delta_1,1-\delta_1), & \tilde{\gamma}_1(Y,Y) &= (\delta_3,1-\delta_3),\\
\tilde{\gamma}_1(N,Y) &= (\delta_2,1-\delta_2), & \tilde{\gamma}_1(N,N) &= (\delta_4,1-\delta_4),
\end{aligned}
$$

with δ_k denoting the probability of x_{11} for $k=1,2,3,4$. Observe that to support the outcome $(0,8,10,12)$ at equilibrium, A1 must participate with P1 with probability one, and $\tilde{\gamma}_1(Y,.)$ must *select x_{12} with probability one. That is,* $\delta_1 = \delta_3 = 0$. *Suppose then that P2 chooses the lottery* $(\alpha, 1-\alpha)$ *over* x_{21} *and* x_{22}*, irrespective of the agents' participation choices. The agents' continuation game is:*

	YN	NY	NN
YN	$(10,4)$	$(10,8+4\alpha)$	$(10,0)$
NY	$(1+\alpha,5+5\delta_2)$	$(1+\alpha,8+4\alpha)$	$(1+\alpha,0)$
NN	$(0,5+5\delta_4)$	$(0,8+4\alpha)$	$(0,0)$

which, by iterated elimination of strictly dominated strategies, admits only one Nash equilibrium: A1 choosing YN, and A2 choosing NY. The corresponding payoff to P2 is $10-2\alpha > 8$ *for every* $\alpha \in [0,1)$.

In the example every principal plays a pure strategy at equilibrium and, following every principal's deviation, agents coordinate on a (unique) mixed strategy equilibrium in the continuation game. Each agent's randomization over participation and communication decisions therefore serves the role of a threat. Indeed, direct mechanisms turn out not to be flexible enough to reproduce all these threats, leaving room for the existence of profitable deviations for some principals.

3.2 Incentive compatible mechanisms and robust equilibria

The previous analysis shows that incentive compatible mechanisms fail to sustain all possible equilibrium outcomes even in standard exclusive competition settings. Yet, an important question from the viewpoint of economic applications is whether outcomes supported by incentive compatible mechanisms survive to a principal deviating towards general indirect mechanisms. A positive answer to this question would provide some foundation for the standard restriction to incentive compatible mechanisms made in economic applications.

Proposition 1 *Let* $\left(\tilde{\gamma}, \tilde{\lambda}\right)$ *be a pure strategy truth-telling equilibrium in the game* G^D*. Then, under Assumption S-u, the corresponding outcome can be supported in a pure strategy equilibrium of any communication game* G^{Γ}.

Proof. Consider the game G^D. Let $\tilde{V}_j$ and $\tilde{U}^i$ be the equilibrium payoffs for every principal $j \in \mathcal{J}$ and every agent $i \in I$ supported by the incentive compatible mechanisms $\tilde{\gamma} = (\tilde{\gamma}_j, \tilde{\gamma}_{-j})$ and by the agents' truth-telling strategies $\tilde{\lambda} = \left(\tilde{\lambda}^i, \tilde{\lambda}^{-i}\right)$. The proof is developed by contradiction.

We fix an arbitrary game G^{Γ} and extend the continuation equilibrium $\tilde{\lambda}$ in G^D to all profile of mechanisms in G^{Γ} as follows. First, we let

$$\lambda^i\left(\gamma, \omega^i\right) = \tilde{\lambda}^i\left(\gamma, \omega^i\right) \quad \forall i, \forall \omega^i \text{ and } \forall \gamma \in \Gamma^D \subseteq \Gamma. \tag{1}$$

That is, for every mechanism in Γ^D agents take the same participation and communication decisions that they were taking at the original equilibrium of G^D, in the "enlarged" game in which all mechanisms are feasible. Next, for all $\gamma \in \Gamma \setminus \Gamma^D$, we let agents select any continuation equilibrium. Denote $\lambda = (\lambda^i, \lambda^{-i})$ the corresponding profile of agents' strategies.

Assume now that principal j has a profitable deviation $\gamma_j' \in \Gamma_j$. Then, it must be that

$$V_j\left(\gamma_j',\tilde{\gamma}_{-j},\lambda\left(\gamma_j',\tilde{\gamma}_{-j}\right)\right) \equiv V_j' > \tilde{V}_j,$$

in which $\lambda\left(\gamma_j',\tilde{\gamma}_{-j}\right)$ is the array of agents' communication and participation behaviours at the continuation equilibrium induced by $\left(\gamma_j',\tilde{\gamma}_{-j}\right)$. Necessarily, $\gamma_j' \notin \Gamma_j^D$ otherwise $\left(\tilde{\gamma},\tilde{\lambda}\right)$ would not be an equilibrium in G^D. For any such $\gamma_j' \in \Gamma_j \setminus \Gamma_j^D$, we construct an equivalent incentive compatible mechanism for principal j yielding exactly the payoff V_j'. To start with, observe that for each type ω^i of each agent i, $\lambda^i\left(\gamma_j',\tilde{\gamma}_{-j},\omega^i\right) \in \Delta\left(S^i\right)$ represents the vector of joint probability distributions over communication and participation induced by $\left(\gamma_j',\tilde{\gamma}_{-j}\right)$. Its j-th element can be written as

$$\lambda_j^i\left(\gamma_j',\tilde{\gamma}_{-j},\omega^i\right) = \mu_j^i\left(m_j^i\left|a_j^i,\gamma_j',\tilde{\gamma}_{-j},\omega^i\right.\right) \times \pi_j^i\left(a_j^i|\gamma_j',\tilde{\gamma}_{-j},\omega^i\right),$$

in which $\mu_j^i\left(.\left|a_j^i,\gamma_j',\tilde{\gamma}_{-j},\omega^i\right.\right)$ is the conditional probability distribution over messages and $\pi_j^i\left(.\left|\gamma_j',\tilde{\gamma}_{-j},\omega^i\right.\right)$ the probability distribution over participation with principal j. We now construct the direct mechanism $\tilde{\gamma}_j'$ as follows: for every $a_j = (a_j^1,\ldots,a_j^I)$ and for every ω,

$$\tilde{\gamma}_j'(\omega,a_j) = \int_{M_j}\left(\prod_{i\in I}\mu_j^i\left(m_j^i\left|a_j^i,\gamma_j',\tilde{\gamma}_{-j},\omega^i\right.\right)\right)\gamma_j'(m_j,a_j)\,dm_j. \tag{2}$$

The mechanism $\tilde{\gamma}_j'$ determines a probability distribution over principal j's decisions which incorporates agents' equilibrium strategies over communication, for every vector of participation decisions a_j.[6] Given Assumption S-u, the mechanism chosen by principal j does not affect the communication behavior of those agents who do not participate with him. Thus, as long as every agent i who participates with principal j behaves truthfully, the mechanism $\tilde{\gamma}_j'$ reproduces the same probability distribution over principal j's decisions induced by the equilibrium strategy λ for a given a_j. It follows from Myerson (1982) that, given the mechanisms $\left(\tilde{\gamma}_j',\tilde{\gamma}_{-j}\right)$, it is a best reply for each agent to truthfully reveal her type to principal j, when she participates with him.

Considering agents' participation, we show that $\pi_j^i(.|\gamma_j',\tilde{\gamma}_{-j},\omega^i)$ is part of agent i equilibrium behaviour. By definition of π_j^i, it must be that

$$\int_{a^{-i}}\int_{a^i} U^i\left(\gamma_j',\tilde{\gamma}_{-j},a,\omega^i\right)\pi^i\left(a^i|\gamma_j',\tilde{\gamma}_{-j},\omega^i\right)\pi^{-i}\left(a^{-i}|\gamma_j',\tilde{\gamma}_{-j},\omega^{-i}\right) \geq$$
$$\geq \int_{a^{-i}}\int_{a^i} U^i\left(\gamma_j',\tilde{\gamma}_{-j},a,\omega^i\right)\pi'^i\left(a^i|\gamma_j',\tilde{\gamma}_{-j},\omega^i\right)\pi^{-i}\left(a^{-i}|\gamma_j',\tilde{\gamma}_{-j},\omega^{-i}\right)$$

[6] For those a_j^i which are not in the support of λ^i, we let μ_j^i select an arbitrary message in M_j^i.

for all $\pi'^{i}(.|.)$, given $\pi^{-i}(.|.)$, $(\gamma'_j, \tilde{\gamma}_{-j})$ and ω. Following (2), we get

$$\int_{a^{-i}} \int_{a^{i}} U^{i}\left(\tilde{\gamma}'_j, \tilde{\gamma}_{-j}, a, \omega^{i}\right) \pi^{i}\left(a^{i}|\gamma'_j, \tilde{\gamma}_{-j}, \omega^{i}\right) \pi^{-i}\left(a^{-i}|\gamma'_j, \tilde{\gamma}_{-j}, \omega^{-i}\right) \geq$$
$$\geq \int_{a^{-i}} \int_{a^{i}} U^{i}\left(\tilde{\gamma}'_j, \tilde{\gamma}_{-j}, a, \omega^{i}\right) \pi'^{i}\left(a^{i}|\gamma'_j, \tilde{\gamma}_{-j}, \omega^{i}\right) \pi^{-i}\left(a^{-i}|\gamma'_j, \tilde{\gamma}_{-j}, \omega^{-i}\right).$$

That is, $\left(\pi^{i}, \pi^{-i}\right)$ are part of a continuation equilibrium in G^{D}. Given (1) and (2), from the view point of principal j,

$$V_j\left(\tilde{\gamma}'_j, \tilde{\gamma}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}'_j, \tilde{\gamma}_{-j}\right)\right) = V'_j > \tilde{V}_j,$$

which contradicts that $\left(\tilde{\gamma}, \tilde{\lambda}\right)$ is an equilibrium in G^{D}. ■

The proposition provides an intuitive robustness result for truth-telling equilibrium outcomes in settings with multiple agents. Two features are key. The first is Assumption S-u. If this assumption is violated, then each principal can make a strategic use of the messages sent by his agents to his competitors, and reproducing such complex interactions may not be possible with incentive compatible mechanisms.[7] The second is that principals play pure strategies at equilibrium. Were principals using mixed strategies, agents may hold some relevant information before taking their participation and communication decisions, based on the observation of the realized lottery posted by the principals. Hence, following Peck (1997), indirect communication may be useful for principals to profitably extract this information, in which case incentive compatible mechanisms would fail to characterize principals' best replies.

4 The single agent case

We now consider games with a single agent and show that incentive compatible mechanisms can be interpreted as canonical, once the potential multiplicity of equilibria in the agent's continuation game is taken into account. That is, for every game G^{Γ} we identify a set of equilibrium outcomes that can be supported by incentive compatible mechanisms: the set of strongly robust equilibrium outcomes of G^{Γ}. A *strongly robust equilibrium* is a Perfect Bayesian equilibrium in which, regardless of the continuation equilibrium selected by agents, no principal has a profitable deviation.[8] Formally, the strategies (δ, λ) constitute a strongly robust equilibrium relative to Γ if for every $j \in \mathcal{J}$,

$$V_j(\delta_j, \delta_{-j}, \lambda) \geq V_j(\delta'_j, \delta_{-j}, \lambda') \qquad \forall \delta'_j \in \Delta(\Gamma_j), \forall \lambda' \neq \lambda,$$

with λ' being a continuation equilibrium relative to $\left(\delta'_j, \delta_{-j}\right)$. In a strongly robust equilibrium, everything happens as if every principal has the power to "select" his preferred continuation

[7] This would de facto reintroduce externalities on agents' payoffs whose role is analyzed in the general setting of Attar et al. (2013). In this case, any principals' deviation to an indirect mechanism can be reproduced by an incentive compatible one only if the latter incorporates two-sided communication, as they show in Example 3.

[8] This notion is introduced in Peters (2001), and extended to multiple agent settings in Han (2007).

equilibrium both on- and off-the-equilibrium path. Although this notion appears to be demanding, it cannot be easily dispensed with. The following example exhibits a non-strongly robust equilibrium outcome that cannot be supported in a truth-telling equilibrium.

Example 2 *Let $I = 1$, $J = 2$ and $\Omega = \{\omega^1, \omega^2\}$. The decision set of Principal 1 (P1) is $X_1 = \{x_{11}, x_{12}, x_{13}\}$ and that of Principal 2 (P2) is $X_2 = \{x_{21}, x_{22}\}$. The agent's types are equally likely. If the agent's type is ω^1, the payoff matrices are:*

	If she chooses YN	
	x_{21}	x_{22}
x_{11}	$(-5,0,5)$	$(-5,0,5)$
x_{12}	$(6,0,4)$	$(6,0,4)$
x_{13}	$(5,0,5)$	$(5,0,5)$

	If she chooses NY	
	x_{21}	x_{22}
x_{11}	$(0,1,3)$	$(0,2,3)$
x_{12}	$(0,1,3)$	$(0,2,3)$
x_{13}	$(0,1,3)$	$(0,2,3)$

in which each cell represents the payoff of $P1$, that of $P2$ and that of the agent, respectively. If the agent's type is ω^2, the payoff matrices are:

	If she chooses YN	
	x_{21}	x_{22}
x_{11}	$(5,0,5)$	$(5,0,5)$
x_{12}	$(0,0,4)$	$(0,0,4)$
x_{13}	$(-5,0,5)$	$(-5,0,5)$

	If she chooses NY	
	x_{21}	x_{22}
x_{11}	$(0,1,4)$	$(0,2,4)$
x_{12}	$(0,1,4)$	$(0,2,4)$
x_{13}	$(0,1,4)$	$(0,2,4)$

If the agent rejects all offers, every player gets a payoff of zero. Assumption S-u is satisfied. Observe that for ω^1 it is a dominant strategy to accept $P1$'s proposal.

Consider a game G^Γ with $M_1 = M_2 = \{s, s', \emptyset\}$, we show that $(3,1,4)$ can be supported as a non-strongly-robust equilibrium outcome. To do so, let P1 post $\gamma_1 = \{x_{12}\}$ and P2 post $\gamma_2 = \{x_{22}\}$ irrespective of the message sent by the agent and of her participation decision. Let type-ω^1 strategy $\lambda(., \omega^1)$ be to participate with $P1$ for every (γ_1, γ_2) and to send the message

$$m_1(\gamma_1, \gamma_2, \omega^1) = \begin{cases} \text{any } m_1 \in \{s, s'\} & \text{if } \gamma_1(s) = \gamma_1(s') \\ \text{any } m_1 \in \{s, s'\} \,:\, \gamma_1(m_1) \in \{x_{11}, x_{13}\} & \text{otherwise.} \end{cases}$$

Assume that type-ω^2 strategy $\lambda(., \omega^2)$ is such that she participates with P2 only if $\gamma_1(s) = \gamma_1(s') = \{x_{12}\}$. In all other cases, she participates with $P1$ sending him the message $m_1(\gamma_1, \gamma_2, \omega^2) = \{m_1 \in \{s, s'\} : \gamma_1(m_1) \in \{x_{11}, x_{13}\}\}$. Clearly, λ constitutes a best reply to the offers (x_{12}, x_{22}). The strategies $(\gamma_1, \gamma_2, \lambda)$ support the outcome $(3,1,4)$. We now show that they form an equilibrium. For $P1$ to have a profitable deviation, he must attract both types and induce them to trade different contracts. Now, following any of such deviations, λ prescribes to both types to participate with him and to send him the same message. Hence, the deviation cannot be profitable. For P2 to have a profitable deviation, he should attract both types, which is impossible given λ. The outcome $(3,1,4)$ is therefore supported at equilibrium with different types participating with different principals. The equilibrium is not strongly robust: suppose that $P1$ deviates to the mechanism γ_1' such that $\gamma_1'(s) = x_{11}$ and $\gamma_1'(s') = x_{13}$ for every participation decision of the agent.

Following this deviation, there is a continuation equilibrium in which both types participate with him, with ω^1 sending s' and ω^2 sending s and P1 getting a payoff of 5. The agent's equilibrium strategy, however, guarantees that such a continuation equilibrium is not selected.

Consider now the game G^D. We show that $(3,1,4)$ cannot be supported in any truth-telling equilibrium. Indeed, for $(3,1,4)$ to be such an equilibrium outcome, it should be that type ω^2 participates with P2, and the equilibrium mechanism of P2 must satisfy $\tilde{\gamma}_2(\omega^2) = x_{22}$. Suppose now that P1 posts the mechanism $\tilde{\gamma}'$ such that $\tilde{\gamma}'_1(\omega^1) = x_{13}$ and $\tilde{\gamma}'_1(\omega^2) = x_{11}$, for every participation decision of the agent. Both ω^1 and ω^2 then participate with P1 with probability one. Since the agent's equilibrium strategy has to be truthful, $\tilde{\gamma}'_1$ yields P1 a payoff of 5, which constitutes a profitable deviation.

The result crucially relies on how the agent's indifferences are resolved in each of the two games. In G^Γ, following the deviation to γ'_1 she chooses the most harmful alternative for P1, which sustains $(3,1,4)$ at equilibrium. In G^D, there is a truth-telling equilibrium supported by the incentive compatible mechanism $\tilde{\gamma}'_1$ which guarantees to P1 a payoff of 5 with the agent being honest. The example develops an intuition similar to that provided by Myerson (1982) to show a possible non-existence of equilibria in competing mechanism games. We take a different perspective emphasizing that additional outcomes may be supported in equilibria that fail to be truth-telling.

We finally prove that every strongly robust equilibrium outcome can be characterized by restricting principals to incentive compatible mechanisms.

Proposition 2 *Under Assumption S-u and $I = 1$, every strongly robust equilibrium outcome of any game G^Γ is a truth-telling equilibrium outcome of the game G^D.*

Proof. Consider an arbitrary game G^Γ and suppose there is a strongly-robust equilibrium outcome supported by the strategies (δ,λ). Denote V_j and U the equilibrium payoffs for every principal $j \in \mathcal{J}$ and for the agent, respectively. For a given realization of the principals' mixed strategies (γ_j,γ_{-j}), and for a given type ω, Assumption S-u implies that the set of optimal messages that ω sends to principal j when participating with him only depends on γ_j. We denote $\hat{M}_j(\gamma_j,\omega)$ such set. The proof is organized in two steps.

1. We construct a profile of probability distributions $(\tilde{\delta}_j,\tilde{\delta}_{-j})$ over incentive compatible mechanisms and a truth-telling strategy $\tilde{\lambda}$ that induce the outcomes $(V_j)_{j\in\mathcal{J}}$ and U.

Take an array (γ_j,γ_{-j}) in the support of $\delta = (\delta_j,\delta_{-j})$, a principal j and a type ω. We let

$$\begin{aligned}\tilde{\gamma}_j(\omega,a) &= \pi_j(a_j = Y|\gamma_j,\gamma_{-j},\omega)\,\gamma_j(\mu_j(m_j|a_j = Y,\gamma_j,\gamma_{-j},\omega)) + \\ &\quad +(1-\pi_j(a_j = Y|\gamma_j,\gamma_{-j},\omega))\,\gamma_j(\hat{m}_j(\gamma_j,\omega))\end{aligned} \tag{3}$$

for each a. To grasp the logic of equation (3), consider the case in which type ω participates with principal j at equilibrium, i.e. $\pi_j(a_j = Y|\gamma_j,\gamma_{-j},\omega) = 1$. Given the equilibrium communication strategy $\mu_j(m_j|a_j = Y,\gamma_j,\gamma_{-j},\omega)$, $\tilde{\gamma}_j(\omega,a)$ is constructed to reproduce the probability distribution on principal j's decisions induced on γ_j by λ. Conversely, if $\pi_j(a_j = Y|\gamma_j,\gamma_{-j},\omega) = 0$, $\tilde{\gamma}_j(\omega,a)$ reproduces any decision induced on γ_j by an arbitrary optimal message $\hat{m}_j(\gamma_j,\omega) \in \hat{M}_j(\gamma_j,\omega)$.

Iterating this procedure for every ω, we get a direct mechanism $\tilde{\gamma}_j$ for each principal j. We construct such an array of direct mechanisms for every (γ_j, γ_{-j}) in the support of (δ_j, δ_{-j}). Finally, given δ_{-j}, there exists a mixed strategy $\tilde{\delta}_j$ that would replicate the equilibrium probability distribution of principal j if the agent behaves truthfully. The mechanisms $(\tilde{\delta}_j, \tilde{\delta}_{-j})$ are such that the agent has access to the same payoffs induced by (δ_j, δ_{-j}). It then follows from Myerson (1982) that it is possible to specify the equilibrium strategy $\tilde{\lambda}$ in such a way that types are revealed truthfully, participation decisions coincide with those specified by λ, and the payoffs $(V_j)_{j\in\mathcal{J}}$ and U are attained.

2. We show that if $(\tilde{\delta}, \tilde{\lambda})$ is not an equilibrium in G^D, then (δ, λ) is not strongly robust in G^Γ.

If $\left(\tilde{\delta}, \tilde{\lambda}\right)$ is not an equilibrium, there exists a principal j and an incentive compatible mechanism $\tilde{\gamma}_j'$ such that:

$$V_j\left(\tilde{\gamma}_j', \tilde{\delta}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}\right)\right) > V_j\left(\tilde{\delta}_j, \tilde{\delta}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}_j, \tilde{\gamma}_{-j}\right)\right) = V_j,$$

with $\tilde{\lambda}(\tilde{\gamma}_j', \tilde{\gamma}_{-j})$ representing the agent's decision over participation and communication induced by principal j's deviation, for a given realization $\tilde{\gamma}_{-j}$ of the joint lottery $\tilde{\delta}_{-j}$. By construction of $\tilde{\lambda}$, it is a best reply for the agent to truthfully report her type to the principal she participates with. Let us now look back at G^Γ. Since $\sharp M_j \geq \sharp\Omega$ for every j, there exists an invertible surjective mapping $\phi_j : M_j \to \Omega \cup \{\emptyset\}$ with $\phi_j(\emptyset) = \emptyset$ for every principal j. Consider the indirect mechanism γ_j' such that $\gamma_j'(m_j) = \tilde{\gamma}_j'(\phi_j(m_j))$ for all $m_j \in M_j$.

There exists a strategy profile $\lambda' = (m', \pi')$, such that the agent uses the same probabilities over participation as in $\tilde{\lambda}$, and sends messages which induce the same decisions that are available with $\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}\right)$. Let this be

$$\begin{cases} \pi_j' = \tilde{\pi}_j\left(a_j = Y | \tilde{\gamma}_j', \tilde{\gamma}_{-j}, \omega\right) \\ m_j' \text{ be any element of } (\phi)_j^{-1}(\omega) \end{cases}$$

for each ω, for each $\tilde{\gamma}_{-j}$ in the support of $\tilde{\delta}_{-j}$, and for each j. Hence, given γ_j' and any γ_{-j} in the support of δ_{-j}, it is a best reply for the agent to play $\lambda' = (\pi_j', m_j')_{j\in J}$. It follows that, for every γ_{-j} in the support of δ_{-j}, the payoff to principal j is the same that he would get by deviating to $\tilde{\gamma}_j'$, for every $\tilde{\gamma}_{-j}$ in the support of $\tilde{\delta}_{-j}$, i.e.

$$V_j\left(\gamma_j', \gamma_{-j}, \lambda'\left(\gamma_j', \gamma_{-j}\right)\right) = V_j\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}\right)\right)$$

Therefore, the expected payoff for principal j satisfies

$$V_j\left(\gamma_j', \delta_{-j}, \lambda'\left(\gamma_j', \gamma_{-j}\right)\right) = V_j\left(\tilde{\gamma}_j', \tilde{\delta}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}\right)\right) > V_j,$$

which contradicts that V_j is supported in a strongly robust equilibrium of G^Γ. ■

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On Competing Mechanisms under Exclusive Competition

Andrea Attar* Eloisa Campioni† Gwenaël Piaser‡

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Abstract

We study games in which several principals design incentive schemes in the presence of privately informed agents. Competition is exclusive: each agent can participate with at most one principal, and principal-agents corporations are isolated. We analyze the role of standard incentive compatible mechanisms in these contexts. First, we provide a clarifying example showing how incentive compatible mechanisms fail to completely characterize equilibrium outcomes even if we restrict to pure strategy equilibria. Second, we show that truth-telling equilibria are robust against unilateral deviations toward arbitrary mechanisms. We then consider the single agent case and exhibit sufficient conditions for the validity of the revelation principle.

Keywords: Competing Mechanisms, Exclusive Competition, Incomplete Information.
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*Toulouse School of Economics (CRM-IDEI) and DEF Università di Roma Tor Vergata, Via Columbia 2 - 00133 Roma (Italy); andrea.attar@tse-fr.eu

†DEF Università di Roma Tor Vergata, Via Columbia 2 - 00133 Roma (Italy); eloisa.campioni@uniroma2.it

‡IPAG Business School, 27 rue de l'université - 75007 Paris (France); gwenael.piaser@ipag.fr

CORRESPONDING AUTHOR : Andrea Attar, Toulouse School of Economics, 21 allee de Brienne, 31000 Toulouse (France).

1 Introduction

We study competing mechanism games of incomplete information in which competition is exclusive: contracts incorporate exclusivity clauses and they are fully enforced. Thus, upon observing her private information and the publicly observable mechanisms, each agent chooses to participate with at most one principal. Given all participation decisions, each principal-agents corporation is isolated: the payoff to any member of a given corporation does not depend on any decision taken outside the corporation. Final allocations are determined by the contracts that principals independently sign with agents.

Exclusive competition is at the hearth of most theoretical analyses of markets in which agents hold some private information. This assumption plays a fundamental role in competitive screening (Rothschild and Stiglitz (1976)), competitive search (Guerrieri et al. (2010)) and competing auctions (McAfee (1993)) models, among many others. At the same time, letting firms compete over exclusive contracts is regarded a convenient way to represent full observability of agents' trades in the extension of general equilibrium theory to private information economies (Prescott and Townsend (1984), Bisin and Gottardi (2006)).

Despite the prominent role of these economic applications, we still lack a general characterization of equilibrium mechanisms in exclusive competition settings. Indeed, the approaches above share the restriction to standard incentive compatible mechanisms: principals commit to incentive schemes such that each agent finds it optimal to truthfully report her private information to the principal she negotiates with, as long as the others do so. However, few, if any, theoretical arguments have been developed to support this choice. If competition is exclusive, to what extent one can safely restrict attention to incentive compatible mechanisms? The present paper contributes to answer this question. Specifically, we analyze the role of the incentive compatible mechanisms introduced in the traditional, single-principal, mechanism design (Myerson (1979) and Myerson (1982)) in competing mechanism games under exclusive competition. We provide two main results.

First, we show that the restriction to incentive compatible mechanisms involves a loss of generality. Indeed, we exhibit a *pure strategy* equilibrium outcome of a game in which principals post indirect mechanisms that cannot be reproduced by incentive compatible ones. This is supported by agents' randomizing over messages and participation following every principal's deviation, given the indirect mechanisms posted by his competitors. All such randomizations serve the role of threats for principals. It turns out that incentive compatible mechanisms are not rich enough to reproduce the same threats and at least one principal can profitably deviate. The result suggests that standard models of exclusive competition may fail to provide a full characterization of equilibrium outcomes. This failure, however, only arises with multiple agents. In the particular case in which principals compete for a single agent, we show that incentive compatible mechanisms retain full power if the way in which the agent breaks her ties is properly considered. Every equilibrium outcome which survives to principals' unilateral deviations, irrespective of the continuation equilibrium selected by the agent, can also be characterized by incentive compatible mechanisms. Although being demanding, this restriction is tight as we illustrate by means of an example.

Second, we investigate whether an equilibrium outcome supported by incentive compatible mechanisms survives if a principal is allowed to deviate to general mechanisms. This amounts to analyze the implications of enlarging the strategy space of a single principal, holding fixed the behavior of his rivals. We show that any pure strategy equilibrium in which agents truthfully report their private information survives if a principal deviates to any indirect mechanism. This provides a rationale for the restriction to standard incentive compatible mechanisms postulated in economic models of exclusive competition.

1.1 Incentive compatibility and exclusive competition: an overview

We analyze competing mechanism games of incomplete information in which agents can participate with at most one principal and their preferences are such that, given participation decisions, each principal-agents corporation is isolated. If competition is non-exclusive, several game-theoretic examples[1] show that incentive compatible mechanisms may fail to provide a full characterization of equilibrium outcomes. The result, however, crucially exploits the fact that each agent participates with many principals at a time. If agents can participate with at most one principal, a potential loss of generality associated to incentive compatible mechanisms is documented in the literature on competition between principal-agents corporations, building on the original example in Myerson (1982). This literature models situations in which, although each agent is restricted to be part of only one corporation, competition is affected by an externality between corporations: the payoff to an agent of a given corporation is determined by market conditions, which depend on the behavior of all agents, including those belonging to the other corporations.[2] Martimort (1996) is the first to show how this externality can be responsible for a failure of incentive compatible mechanisms to be a general characterization device. A similar insight, though in a more abstract setting, is provided by Epstein and Peters (1999) who construct equilibria in which principals correlate agents' decisions through messages in a way that cannot be reproduced by incentive compatible mechanisms.[3] To achieve the correlation, they exploit the assumption that agents in one corporation send non-trivial messages to the principals of other corporations. In the absence of these externalities, are incentive compatible mechanisms restrictive? We show the existence of equilibrium outcomes that cannot be supported by such mechanisms even if every player's payoff is only affected by the decisions of the members of her corporation. Such a failure of the revelation principle may be relevant in the light of strategic analyses of markets with incomplete information as discussed in Section 2.2. If principals compete for a single agent, intuition suggests that one can follow the general logic in Myerson (1982) and rely on incentive compatible mechanisms. To the best of our knowledge, we are the first to provide a formal proof of this result.

[1]See, for instance, Peters (2001) and Martimort and Stole (2002).

[2]See, for example, the analysis of competition among manufacturers when their retailers are privately informed of market conditions. In these contexts, different works investigate the link between pre-commitment effects and renegotiation (Caillaud et al. (1995)), the rationale behind alternative forms of vertical restraints (Gal-Or (1991)), the welfare implications of information sharing (Pagnozzi and Piccolo (2013)). See Rey and Vergé (2008) for a survey of recent results.

[3]See Appendix A in Epstein and Peters (1999).

Another key issue for equilibrium characterization is whether incentive compatible mechanisms may allow to characterize any best reply of a single principal to a given profile of mechanisms posted by his opponents. In a framework of exclusive competition and many agents, Peck (1997) exhibits a negative result. Given the mixed strategy of one principal, his opponent can post a mechanism inducing a continuation equilibrium which cannot be reproduced by requiring that agents behave truthfully. This suggests that incentive compatible mechanisms may fail to characterize some of the principals' best replies. We identify a role for these mechanisms under exclusive competition.[4] Indeed, our Proposition 1 shows that truth-telling equilibria in which principals play pure strategies are robust to deviations towards indirect mechanisms. This in turn clarifies that Peck (1997)'s result fundamentally relies on principals playing mixed strategies. More generally, throughout the paper, and in line with standard approaches to competing mechanism games, principals are not allowed to condition their mechanisms on those of their competitors. The recent work of Szentes (2015) is one of the first attempts to analyze the implications of allowing for contractible contracts in exclusive competition settings. He shows that equilibrium outcomes supported by "non contingent" mechanisms are robust to the introduction of contingent contracts, although the latter typically induce equilibrium indeterminacy. In the light of his first result, one could therefore reinterpret our Proposition 1 to argue that truth-telling equilibria provide a useful reference point also under very general contracting assumptions.

The paper is organized as follows: Section 2 develops a general model of competing mechanisms under exclusive competition and illustrates its main economic applications. Section 3 analyzes the multiple agent setting, and Section 4 the single agent one.

2 The model

2.1 Competing mechanisms under exclusive competition

We refer to a scenario in which several principals (indexed by $j \in \mathcal{J} = \{1, \dots, J\}$) contract with several agents (indexed by $i \in I = \{1, \dots, I\}$). Each agent i has private information about her type $\omega^i \in \Omega^i$ and $\omega = \{\omega^1, \cdots, \omega^I\} \in \Omega = \underset{i \in I}{\times} \Omega^i$ is the array of agents' types, which is a random variable with distribution F.

Each principal j may choose an action $x_j \in X_j$. Agents only take participation decisions, with $a^i_j \in \{Y, N\}$ being the decision of agent i to participate with principal j, in which $\{N\}$ stands for not participating. We take $v_j : X \times A \times \Omega \to \mathbb{R}_+$ and $u^i : X \times A \times \Omega \to \mathbb{R}_+$ to be the payoff to principal j and to agent i, respectively, with $X = \underset{j \in \mathcal{J}}{\times} X_j$ and $A = \underset{i \in I}{\times} A^i$. For a given array of agents' types ω, of actions $a = \left(a^1, a^2, \dots, a^I\right)$ and of principals' decisions $x = (x_1, x_2, \dots, x_J)$, the payoffs to agent i and to principal j are $u^i(x, a, \omega)$ and $v_j(x, a, \omega)$, respectively.

Each principal perfectly observes the set of agents who participate with him. Communication is one-sided: each agent i may send a private message $m^i_j \in M^i_j$ to principal j. We let

[4] Under non-exclusive competition, Han (2007), Attar et al. (2010) and Attar et al. (2013) show that this result requires demanding sufficient conditions. In particular, under incomplete information, Attar et al. (2013) show that equilibria supported by incentive compatible mechanisms do not satisfy such a robustness requirement, in general. They therefore introduce a class of two-sided communication mechanisms to obtain a characterization of principals' best replies.

each set M^i_j be sufficiently rich to include the element $\{\emptyset\}$ corresponding to the information "agent i does not communicate with principal j", and to satisfy the standard size restriction $\sharp M^i_j > \sharp\Omega^i$ for every i and j. Principal j takes his decisions contingent on the array of messages m_j he receives, with $m_j = \left(m^1_j, m^2_j, \ldots, m^I_j\right) \in M_j = \underset{i \in I}{\times} M^i_j$, and on the participation choices of the agents. Formally, we say that a mechanism proposed by principal j is the measurable mapping $\gamma_j : M_j \times \{Y, N\}^{\sharp I} \to \Delta(X_j)$. We take Γ_j to be the set of mechanisms available to principal j and denote $\Gamma = \underset{j \in \mathcal{J}}{\times} \Gamma_j$. All relevant sets are taken to be compact and measurable with respect to the topology of weak convergence. The competing mechanism game relative to Γ begins when principals publicly and simultaneously commit to mechanisms. Given the posted mechanisms $(\gamma_1, \gamma_2, \ldots, \gamma_J) \in \Gamma$ and their privately observed types, agents simultaneously take a participation and communication decision with respect to every principal. In this incomplete information game, a (mixed) strategy for principal j is a randomization $\delta_j \in \Delta(\Gamma_j)$, and $\delta = (\delta_1, \cdots, \delta_J) \in \Delta(\Gamma)$ is a profile of mixed strategies for principals.

In line with economic applications, we model exclusive competition as a restriction on agents' preferences and on their strategy spaces. If competition is exclusive, for a given participation choice, the payoff to any agent i who participates with some principal j does not depend on the decisions of the principals she is not participating with, nor on the agents' participations with principal j's opponents. That is, agents' preferences satisfy the following:

Assumption S-u *Let $I_j = \{i \in I : a^i_j = Y\}$. For every $j \in \mathcal{J}$, $\omega \in \Omega$, $h \in I_j$, $x_j \in X_j$ and I_j:*

$$u^h(x_j, x_{-j}, a, \omega) = u^h\left(x_j, x'_{-j}, a', \omega\right),$$

for each $x_{-j}, x'_{-j} \in X_{-j}$ and for each a, $a' \in A$ such that $a^i_j = a'^i_j = Y$ for every $i \in I_j$.

A strategy for each agent i associates to every profile of posted mechanisms a joint participation and communication decision. In a pure strategy, every agent participates with at most one principal and sends a non-degenerate message only to the principal she participates with. We let $S^i = \left\{s^i \in M^i \times A^i : m^i_j = \emptyset \text{ iff } a^i_j = \{N\}\right\}$ be the strategy set for agent i, with $A^i = \left\{a^i = \left(a^i_1, \ldots, a^i_J\right) \in \{Y, N\}^{\sharp\mathcal{J}} : a^i_j = \{Y\} \text{ for at most one} j\right\}$ and $M^i = \underset{j \in \mathcal{J}}{\times} M^i_j$ representing the sets of participation and communication decision, respectively. A strategy for agent i is then the measurable mapping $\lambda^i : \Gamma \times \Omega^i \to \Delta\left(S^i\right)$. Every $\lambda = \left(\lambda^1, \lambda^2, \ldots, \lambda^I\right)$ induces a probability distribution over principals' decisions. Given $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_J)$, we let $\beta^\lambda_j \in \Delta(X_j)$ be the distribution over principal j's decisions induced by λ, and let $\beta^\lambda = \underset{j \in \mathcal{J}}{\times} \beta^\lambda_j$. The expected payoff to each type ω^i of agent i is:

$$U^i\left(\gamma, \lambda, \omega^i\right) = \int_{\Omega^{-i}} \int_X u^i\left(x, a, \omega^i, \omega^{-i}\right) F\left(\omega^{-i} | \omega^i\right) d\beta^\lambda \, d\omega^{-i},$$

with $F\left(\omega^{-i} | \omega^i\right)$ being the conditional probability of ω^{-i} given ω^i. The expected payoff to principal j when he plays δ_j against his opponents' strategies δ_{-j} is:

$$V_j(\delta_j, \delta_{-j}, \lambda) = \int_{\Gamma_j} \int_{\Gamma_{-j}} \int_\Omega \int_X v_j(x, a, \omega) F(\omega) \, d\beta^\lambda \, d\omega \, d\delta_{-j}(\gamma_{-j}) \, d\delta_j(\gamma_j).$$

The strategies (δ, λ) constitute an equilibrium relative to $\Delta(\Gamma)$ if λ is a continuation equilibrium for every γ in the support of δ and if, given δ_{-j} and λ, for every $j \in \mathcal{J}$: $\delta_j \in \underset{\delta'_j \in \Delta(\Gamma_j)}{argmax} V_j \left(\delta'_j, \delta_{-j}, \lambda\right)$.

A mechanism available to principal j is *direct* if agents can only communicate their types to principal j, i.e. if $M^i_j = \Omega^i \cup \{\emptyset\}$ for every i, with $\{\emptyset\}$ representing no communication. We denote a direct mechanism for principal j as $\tilde{\gamma}_j : \underset{i \in I}{\times} \left(\Omega^i \cup \{\emptyset\}\right) \times \{Y, N\}^{\sharp I} \to \Delta\left(X_j\right)$ and the set of direct mechanisms as $\Gamma^D_j \subseteq \Gamma_j$. We let G^Γ be the competing mechanism game induced by a given Γ, and G^D the game in which principals are restricted to direct mechanisms. As in Myerson (1982), a direct mechanism is *incentive compatible* from the point of view of principal j if, given the mechanisms offered by the other principals, it induces a continuation equilibrium in which agents truthfully reveal their types to him. A direct mechanism $\tilde{\gamma}_j$ can therefore be incentive compatible for a given array $\tilde{\gamma}_{-j}$, but not for some other $\tilde{\gamma}'_{-j} \neq \tilde{\gamma}_{-j}$. An equilibrium is *truth-telling* if every principal posts an incentive compatible mechanism and agents truthfully reveal their private information to the principals they participate with, whenever this constitutes an equilibrium in their continuation game.

2.2 Applications

Our general model encompasses standard economic applications of exclusive competition with incomplete information, as illustrated below.

Competitive Insurance In their canonical analysis, Rothschild and Stiglitz (1976) study strategic competition between insurance companies for the exclusive right to serve a customer. The customer faces a binary risk on her endowment $w \in \{w_L, w_H\}$, with probabilities $(p, 1-p)$ that constitute her private information. Her (expected) payoff is $pu(w_L + d_L) + (1-p)u(w_H + d_H)$, with $(d_L, d_H) \in \mathbb{R}^2$ being the state-contingent transfers issued by the company she trades with, which guarantees that Assumption S-u is satisfied. Insurers are restricted to post incentive compatible (deterministic) mechanisms. The model of this section can hence be interpreted in terms of competitive insurance by letting $I = 1$, $\omega \equiv (p, 1-p)$ and $\tilde{\gamma}_j : \Omega \times \{Y, N\} \to \mathbb{R}^2$.

Competitive Search The model above has been subsequently extended to analyze the interaction between pricing and trading probability. Specifically, Inderst and Wambach (2002) study a competitive market in which privately informed consumers apply to firms who face capacity constraints, and may therefore end up being rationed. The competitive search literature endogeneizes rationing as a byproduct of search frictions and bilateral matching (Guerrieri et al. (2010)). Under competitive search, principals simultaneously post mechanisms and each agent applies to at most one of them. The ratio between the number of principals posting a given mechanism γ and the number of agents applying for it determines the rationing probability $\mu(\gamma)$, which every agent takes as given. If an agent is not rationed, she gets her exogenous reservation utility. For a given application decision, each agent's payoff therefore only depends on the mechanism she applies for and on the corresponding rationing probability. Our Assumption S-u is hence satisfied if we interpret an application decision as our participation choice. On the principals' side, attention is restricted to incentive compatible mechanisms that are contingent on agents' reported types and, possibly, on their application decisions.

Competing Auctions In a seminal paper, McAfee (1993) analyzes sellers who compete over auctions when buyers' valuation constitute their private information. Peters (1997) shows that in a decentralized market with many sellers and buyers, second-price auctions arise as equilibrium mechanisms. In these settings, sellers simultaneously and anonymously post their reservation prices and buyers choose at most one auction to participate in. A seller and the buyers who participate in his auction form an isolated corporation, hence Assumption S-u is satisfied. In addition, sellers are restricted to post direct mechanisms, asking each buyer $i \in I$ to report her valuation $v^i \in [0,1]$. A strategy for seller j is a mechanism $\tilde{\gamma}_j : \left|I_j\right| \times [0,1]^{\left|I_j\right|} \to \mathbb{R}$, where $I_j \subseteq I$ is the set of buyers that participate in auction j. A pure strategy for buyer i is a mapping $\lambda^i : \Gamma_1 \times ... \times \Gamma_J \times [0,1] \to A^i \times [0,1] \times \mathbb{R}_+$, with Γ_j being set of second-price auctions for $j \in J$. Given her participation decision, it is always a dominant strategy for each of the buyers to truthfully report ther private valuations. The model of this section therefore adapts to the competing auctions settings of Peters (1997), Peters and Severinov (1997), Burguet and Sakovics (1999), Viràg (2010), Han (2014), Peck (2015).

Despite its large domain of applicability, our model does not incorporate the effects of introducing exclusive contracts in contexts in which agents' participation choices are unrestricted. This last feature indeed implies that such settings (see for instance O'Brien and Shaffer (1997) and Bernheim and Whinston (1998)) do not typically satisfy Assumption S-u.

3 The multi-agent case

This section analyzes the role of incentive compatible mechanisms in games with several agents.

3.1 Failure of the revelation principle

We first show a negative result: there exist *pure strategy* equilibrium outcomes of G^Γ that cannot be supported in any truth-telling equilibrium of G^D. The result is established in a complete information context, in which agents' preferences satisfy Assumption S-u, and there is no direct externality between principals. It provides an instance of the failure of the revelation principle in multi-agent games of exclusive competition.

Example 1 *Let $I = J = 2$ and $\Omega^1 = \Omega^2 = \{\omega\}$. In addition, take $X_1 = \{x_{11}, x_{12}\}$ and $X_2 = \{x_{21}, x_{22}\}$ to be the decision sets of principal 1 ($P1$) and 2 ($P2$). Let $A^1 = A^2 = \{YN, NY, NN\}$, with YN denoting the agent's decision to accept the offer of $P1$ and reject that of $P2$. Payoffs are represented in the matrix below, in which agents decide in the external box and principals decide in the internal 2x2 cells. Each array represents the payoffs to $P1$, $P2$, agent 1 ($A1$) and agent 2 ($A2$), respectively.*

		YN		NY		NN	
		x_{21}	x_{22}	x_{21}	x_{22}	x_{21}	x_{22}
YN	x_{11}	$(0,0,5,12)$	$(0,0,5,12)$	$(0,8,5,12)$	$(0,10,5,8)$	$(0,0,5,0)$	$(0,0,5,0)$
	x_{12}	$(0,0,10,4)$	$(0,0,10,4)$	$(0,8,10,12)$	$(0,10,10,8)$	$(0,0,10,0)$	$(0,0,10,0)$
		x_{21}	x_{22}	x_{21}	x_{22}	x_{21}	x_{22}
NY	x_{11}	$(0,0,2,10)$	$(0,0,1,10)$	$(0,0,2,12)$	$(0,0,1,8)$	$(0,0,2,0)$	$(0,0,1,0)$
	x_{12}	$(0,0,2,5)$	$(0,0,1,5)$	$(0,0,2,12)$	$(0,0,1,8)$	$(0,0,2,0)$	$(0,0,1,0)$
		x_{21}	x_{22}	x_{21}	x_{22}	x_{21}	x_{22}
NN	x_{11}	$(0,0,0,10)$	$(0,0,0,10)$	$(0,8,0,12)$	$(0,10,0,8)$	$(0,0,0,0)$	$(0,0,0,0)$
	x_{12}	$(0,0,0,5)$	$(0,0,0,5)$	$(0,8,0,12)$	$(0,10,0,8)$	$(0,0,0,0)$	$(0,0,0,0)$

Consider the game G^{Γ} in which $M_j^1 = M_j^2 = \{s, s'\}$ for $j = 1, 2$ and the array of mechanisms (γ_1, γ_2) such that:

$$\gamma_1(m_1, a_1) = \begin{cases} x_{12} & \textit{if at least one agent participates and only one } s \textit{ is received} \\ x_{11} & \textit{otherwise} \end{cases}$$

$$\gamma_2(m_2, a_2) = \begin{cases} x_{22} & \textit{if only one agent participates sending } s \\ & \textit{or if both agents participate and A1 sends } s' \\ x_{21} & \textit{otherwise} \end{cases}$$

These mechanisms are part of an equilibrium in G^{Γ}. Indeed, they induce the following continuation game between agents:

	YNs	YNs'	NYs	NYs'	NN
YNs	$(5,12)$	$(10,4)$	$(10,8)$	$(10,12)$	$(10,0)$
YNs'	$(10,4)$	$(5,12)$	$(5,8)$	$(5,12)$	$(5,0)$
NYs	$(1,5)$	$(1,10)$	$(2,12)$	$(2,12)$	$(1,0)$
NYs'	$(2,5)$	$(2,10)$	$(1,8)$	$(1,8)$	$(2,0)$
NN	$(0,5)$	$(0,10)$	$(0,8)$	$(0,12)$	$(0,0)$

in which YNs represents the decision to accept the proposal of $P1$ and to report him the message s.[5] *The game admits only one equilibrium: $A1$ playing YNs and $A2$ playing NYs'. The corresponding decisions are (x_{12}, x_{21}) which induce the outcome $(0, 8, 10, 12)$. Since $P1$'s payoff is constantly equal to zero, posting γ_1 is optimal for him. We only need to prove that $P2$ has no profitable deviations.*

Suppose first that he deviates to a stochastic take-it or leave-it offer. This corresponds to a probability distribution over (x_{21}, x_{22}), which we denote $(\alpha, 1-\alpha)$. Following such deviation, the continuation game among agents is:

[5] For the sake of notation, we choose not to represent the empty messages of the agents' that are implied by a rejection decision, N.

	YNs	YNs'	NY	NN
YNs	$(5,12)$	$(10,4)$	$(10,8+4\alpha)$	$(10,0)$
YNs'	$(10,4)$	$(5,12)$	$(5,8+4\alpha)$	$(5,0)$
NY	$(1+\alpha,5)$	$(1+\alpha,10)$	$(1+\alpha,8+4\alpha)$	$(1+\alpha,0)$
NN	$(0,5)$	$(0,10)$	$(0,8+4\alpha)$	$(0,0)$

Since NY and NN are strictly dominated for A1*, and NN is strictly dominated for A*2*, the game reduces to:*

	YNs	YNs'	NY
YNs	$(5,12)$	$(10,4)$	$(10,8+4\alpha)$
YNs'	$(10,4)$	$(5,12)$	$(5,8+4\alpha)$

This game has only one equilibrium, in which A1 *randomizes over* (YNs,YNs') *with probabilities* $\left(\frac{1+\alpha}{2},\frac{1-\alpha}{2}\right)$ *and A*2 *randomizes over* (YNs,NY) *with probabilities* $\left(\frac{1}{2},\frac{1}{2}\right)$*. The payoff to P*$2$ *is* $\frac{1}{2}(10-2\alpha)<8$ *for every* $\alpha\in[0,1)$*. Any such deviation is hence unprofitable.*

Consider now a deviation towards a general stochastic mechanism γ_2' *that associates a probability distribution over* (x_{21},x_{22}) *to every array of agents' messages and participation choices:*

$$\begin{array}{lll} \gamma_2'(s,\emptyset,Y,N) & = & (\alpha_1,1-\alpha_1), \\ \gamma_2'(s',\emptyset,Y,N) & = & (\alpha_2,1-\alpha_2), \\ \gamma_2'(\emptyset,s,N,Y) & = & (\alpha_3,1-\alpha_3), \\ \gamma_2'(\emptyset,s',N,Y) & = & (\alpha_4,1-\alpha_4), \end{array} \qquad \begin{array}{lll} \gamma_2'(s,s',Y,Y) & = & (\alpha_5,1-\alpha_5), \\ \gamma_2'(s,s,Y,Y) & = & (\alpha_6,1-\alpha_6), \\ \gamma_2'(s',s,Y,Y) & = & (\alpha_7,1-\alpha_7), \\ \gamma_2'(s',s',Y,Y) & = & (\alpha_8,1-\alpha_8), \end{array}$$

with α_k *denoting the probability of* x_{21} *for* $k=1,2,...,8$*. The agents' continuation game is:*

	YNs	YNs'	NYs	NYs'	NN
YNs	$(5,12)$	$(10,4)$	$(10,8+4\alpha_3)$	$(10,8+4\alpha_4)$	$(10,0)$
YNs'	$(10,4)$	$(5,12)$	$(5,8+4\alpha_3)$	$(5,8+4\alpha_4)$	$(5,0)$
NYs	$(1+\alpha_1,5)$	$(1+\alpha_1,10)$	$(1+\alpha_6,8+4\alpha_6)$	$(1+\alpha_5,8+4\alpha_5)$	$(1+\alpha_1,0)$
NYs'	$(1+\alpha_2,5)$	$(1+\alpha_2,10)$	$(1+\alpha_7,8+4\alpha_7)$	$(1+\alpha_8,8+4\alpha_8)$	$(1+\alpha_2,0)$
NN	$(0,5)$	$(0,10)$	$(0,8+4\alpha)$	$(0,8+4\alpha)$	$(0,0)$

which, by iterated elimination of strictly dominated strategies, reduces to:

	YNs	YNs'	NYs	NYs'
YNs	$(5,12)$	$(10,4)$	$(10,8+4\alpha_3)$	$(10,8+4\alpha_4)$
YNs'	$(10,4)$	$(5,12)$	$(5,8+4\alpha_3)$	$(5,8+4\alpha_4)$

Once again, this game exhibits a unique mixed strategy equilibrium, which yields to P2 *a payoff strictly lower than* 8*.*

Consider now the game G^D*. A direct mechanism for* $P1$ *maps the agents' participation choices into lotteries on* X_1 *and can be represented as follows:*

$$\begin{array}{lll} \tilde{\gamma}_1(Y,N) & = & (\delta_1,1-\delta_1), \\ \tilde{\gamma}_1(N,Y) & = & (\delta_2,1-\delta_2), \end{array} \qquad \begin{array}{lll} \tilde{\gamma}_1(Y,Y) & = & (\delta_3,1-\delta_3), \\ \tilde{\gamma}_1(N,N) & = & (\delta_4,1-\delta_4), \end{array}$$

with δ_k *denoting the probability of* x_{11} *for* $k=1,2,3,4$*. Observe that to support the outcome* $(0,8,10,12)$ *at equilibrium, A1 must participate with P1 with probability one, and* $\tilde{\gamma}_1(Y,.)$ *must* *select x_{12} with probability one. That is, $\delta_1 = \delta_3 = 0$. Suppose then that P2 chooses the lottery $(\alpha, 1-\alpha)$ over x_{21} and x_{22}, irrespective of the agents' participation choices. The agents' continuation game is:*

	YN	NY	NN
YN	$(10,4)$	$(10,8+4\alpha)$	$(10,0)$
NY	$(1+\alpha,5+5\delta_2)$	$(1+\alpha,8+4\alpha)$	$(1+\alpha,0)$
NN	$(0,5+5\delta_4)$	$(0,8+4\alpha)$	$(0,0)$

which, by iterated elimination of strictly dominated strategies, admits only one Nash equilibrium: A1 choosing YN, and A2 choosing NY. The corresponding payoff to P2 is $10-2\alpha > 8$ for every $\alpha \in [0,1)$.

In the example every principal plays a pure strategy at equilibrium and, following every principal's deviation, agents coordinate on a (unique) mixed strategy equilibrium in the continuation game. Each agent's randomization over participation and communication decisions therefore serves the role of a threat. Indeed, direct mechanisms turn out not to be flexible enough to reproduce all these threats, leaving room for the existence of profitable deviations for some principals.

3.2 Incentive compatible mechanisms and robust equilibria

The previous analysis shows that incentive compatible mechanisms fail to sustain all possible equilibrium outcomes even in standard exclusive competition settings. Yet, an important question from the viewpoint of economic applications is whether outcomes supported by incentive compatible mechanisms survive to a principal deviating towards general indirect mechanisms. A positive answer to this question would provide some foundation for the standard restriction to incentive compatible mechanisms made in economic applications.

Proposition 1 *Let $\left(\tilde{\gamma}, \tilde{\lambda}\right)$ be a pure strategy truth-telling equilibrium in the game G^D. Then, under Assumption S-u, the corresponding outcome can be supported in a pure strategy equilibrium of any communication game G^Γ.*

Proof. Consider the game G^D. Let $\tilde{V}_j$ and $\tilde{U}^i$ be the equilibrium payoffs for every principal $j \in \mathcal{J}$ and every agent $i \in I$ supported by the incentive compatible mechanisms $\tilde{\gamma} = (\tilde{\gamma}_j, \tilde{\gamma}_{-j})$ and by the agents' truth-telling strategies $\tilde{\lambda} = \left(\tilde{\lambda}^i, \tilde{\lambda}^{-i}\right)$. The proof is developed by contradiction.

We fix an arbitrary game G^Γ and extend the continuation equilibrium $\tilde{\lambda}$ in G^D to all profile of mechanisms in G^Γ as follows. First, we let

$$\lambda^i\left(\gamma, \omega^i\right) = \tilde{\lambda}^i\left(\gamma, \omega^i\right) \quad \forall i, \forall \omega^i \text{ and } \forall \gamma \in \Gamma^D \subseteq \Gamma. \tag{1}$$

That is, for every mechanism in Γ^D agents take the same participation and communication decisions that they were taking at the original equilibrium of G^D, in the "enlarged" game in which all mechanisms are feasible. Next, for all $\gamma \in \Gamma \setminus \Gamma^D$, we let agents select any continuation equilibrium. Denote $\lambda = (\lambda^i, \lambda^{-i})$ the corresponding profile of agents' strategies.

Assume now that principal j has a profitable deviation $\gamma_j' \in \Gamma_j$. Then, it must be that

$$V_j\left(\gamma_j', \tilde{\gamma}_{-j}, \lambda\left(\gamma_j', \tilde{\gamma}_{-j}\right)\right) \equiv V_j' > \tilde{V}_j,$$

in which $\lambda\left(\gamma_j', \tilde{\gamma}_{-j}\right)$ is the array of agents' communication and participation behaviours at the continuation equilibrium induced by $\left(\gamma_j', \tilde{\gamma}_{-j}\right)$. Necessarily, $\gamma_j' \notin \Gamma_j^D$ otherwise $\left(\tilde{\gamma}, \tilde{\lambda}\right)$ would not be an equilibrium in G^D. For any such $\gamma_j' \in \Gamma_j \setminus \Gamma_j^D$, we construct an equivalent incentive compatible mechanism for principal j yielding exactly the payoff V_j'. To start with, observe that for each type ω^i of each agent i, $\lambda^i\left(\gamma_j', \tilde{\gamma}_{-j}, \omega^i\right) \in \Delta\left(S^i\right)$ represents the vector of joint probability distributions over communication and participation induced by $\left(\gamma_j', \tilde{\gamma}_{-j}\right)$. Its j-th element can be written as

$$\lambda_j^i\left(\gamma_j', \tilde{\gamma}_{-j}, \omega^i\right) = \mu_j^i\left(m_j^i \left| a_j^i, \gamma_j', \tilde{\gamma}_{-j}, \omega^i\right.\right) \times \pi_j^i\left(a_j^i | \gamma_j', \tilde{\gamma}_{-j}, \omega^i\right),$$

in which $\mu_j^i\left(. \left| a_j^i, \gamma_j', \tilde{\gamma}_{-j}, \omega^i\right.\right)$ is the conditional probability distribution over messages and $\pi_j^i\left(. \left| \gamma_j', \tilde{\gamma}_{-j}, \omega^i\right.\right)$ the probability distribution over participation with principal j. We now construct the direct mechanism $\tilde{\gamma}_j'$ as follows: for every $a_j = (a_j^1, \ldots, a_j^I)$ and for every ω,

$$\tilde{\gamma}_j'(\omega, a_j) = \int_{M_j} \left(\prod_{i \in I} \mu_j^i\left(m_j^i \left| a_j^i, \gamma_j', \tilde{\gamma}_{-j}, \omega^i\right.\right)\right) \gamma_j'(m_j, a_j)\, dm_j. \tag{2}$$

The mechanism $\tilde{\gamma}_j'$ determines a probability distribution over principal j's decisions which incorporates agents' equilibrium strategies over communication, for every vector of participation decisions a_j.[6] Given Assumption S-u, the mechanism chosen by principal j does not affect the communication behavior of those agents who do not participate with him. Thus, as long as every agent i who participates with principal j behaves truthfully, the mechanism $\tilde{\gamma}_j'$ reproduces the same probability distribution over principal j's decisions induced by the equilibrium strategy λ for a given a_j. It follows from Myerson (1982) that, given the mechanisms $\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}\right)$, it is a best reply for each agent to truthfully reveal her type to principal j, when she participates with him.

Considering agents' participation, we show that $\pi_j^i(.|\gamma_j', \tilde{\gamma}_{-j}, \omega^i)$ is part of agent i equilibrium behaviour. By definition of π_j^i, it must be that

$$\int_{a^{-i}} \int_{a^i} U^i\left(\gamma_j', \tilde{\gamma}_{-j}, a, \omega^i\right) \pi^i\left(a^i | \gamma_j', \tilde{\gamma}_{-j}, \omega^i\right) \pi^{-i}\left(a^{-i} | \gamma_j', \tilde{\gamma}_{-j}, \omega^{-i}\right) \geq$$
$$\geq \int_{a^{-i}} \int_{a^i} U^i\left(\gamma_j', \tilde{\gamma}_{-j}, a, \omega^i\right) \pi'^i\left(a^i | \gamma_j', \tilde{\gamma}_{-j}, \omega^i\right) \pi^{-i}\left(a^{-i} | \gamma_j', \tilde{\gamma}_{-j}, \omega^{-i}\right)$$

[6] For those a_j^i which are not in the support of λ^i, we let μ_j^i select an arbitrary message in M_j^i.

for all $\pi'^{i}(.|.)$, given $\pi^{-i}(.|.)$, $(\gamma'_j, \tilde{\gamma}_{-j})$ and ω. Following (2), we get

$$\int_{a^{-i}} \int_{a^i} U^i\left(\tilde{\gamma}'_j, \tilde{\gamma}_{-j}, a, \omega^i\right) \pi^i\left(a^i|\gamma'_j, \tilde{\gamma}_{-j}, \omega^i\right) \pi^{-i}\left(a^{-i}|\gamma'_j, \tilde{\gamma}_{-j}, \omega^{-i}\right) \geq$$
$$\geq \int_{a^{-i}} \int_{a^i} U^i\left(\tilde{\gamma}'_j, \tilde{\gamma}_{-j}, a, \omega^i\right) \pi'^{i}\left(a^i|\gamma'_j, \tilde{\gamma}_{-j}, \omega^i\right) \pi^{-i}\left(a^{-i}|\gamma'_j, \tilde{\gamma}_{-j}, \omega^{-i}\right).$$

That is, $\left(\pi^i, \pi^{-i}\right)$ are part of a continuation equilibrium in G^D. Given (1) and (2), from the view point of principal j,

$$V_j\left(\tilde{\gamma}'_j, \tilde{\gamma}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}'_j, \tilde{\gamma}_{-j}\right)\right) = V'_j > \tilde{V}_j,$$

which contradicts that $\left(\tilde{\gamma}, \tilde{\lambda}\right)$ is an equilibrium in G^D. ■

The proposition provides an intuitive robustness result for truth-telling equilibrium outcomes in settings with multiple agents. Two features are key. The first is Assumption S-u. If this assumption is violated, then each principal can make a strategic use of the messages sent by his agents to his competitors, and reproducing such complex interactions may not be possible with incentive compatible mechanisms.[7] The second is that principals play pure strategies at equilibrium. Were principals using mixed strategies, agents may hold some relevant information before taking their participation and communication decisions, based on the observation of the realized lottery posted by the principals. Hence, following Peck (1997), indirect communication may be useful for principals to profitably extract this information, in which case incentive compatible mechanisms would fail to characterize principals' best replies.

4 The single agent case

We now consider games with a single agent and show that incentive compatible mechanisms can be interpreted as canonical, once the potential multiplicity of equilibria in the agent's continuation game is taken into account. That is, for every game G^Γ we identify a set of equilibrium outcomes that can be supported by incentive compatible mechanisms: the set of strongly robust equilibrium outcomes of G^Γ. A *strongly robust equilibrium* is a Perfect Bayesian equilibrium in which, regardless of the continuation equilibrium selected by agents, no principal has a profitable deviation.[8] Formally, the strategies (δ, λ) constitute a strongly robust equilibrium relative to Γ if for every $j \in \mathcal{J}$,

$$V_j(\delta_j, \delta_{-j}, \lambda) \geq V_j(\delta'_j, \delta_{-j}, \lambda') \qquad \forall \delta'_j \in \Delta(\Gamma_j), \forall \lambda' \neq \lambda,$$

with λ' being a continuation equilibrium relative to $\left(\delta'_j, \delta_{-j}\right)$. In a strongly robust equilibrium, everything happens as if every principal has the power to "select" his preferred continuation

[7]This would de facto reintroduce externalities on agents' payoffs whose role is analyzed in the general setting of Attar et al. (2013). In this case, any principals' deviation to an indirect mechanism can be reproduced by an incentive compatible one only if the latter incorporates two-sided communication, as they show in Example 3.

[8]This notion is introduced in Peters (2001), and extended to multiple agent settings in Han (2007).

equilibrium both on- and off-the-equilibrium path. Although this notion appears to be demanding, it cannot be easily dispensed with. The following example exhibits a non-strongly robust equilibrium outcome that cannot be supported in a truth-telling equilibrium.

Example 2 *Let $I = 1$, $J = 2$ and $\Omega = \{\omega^1, \omega^2\}$. The decision set of Principal 1 (P1) is $X_1 = \{x_{11}, x_{12}, x_{13}\}$ and that of Principal 2 (P2) is $X_2 = \{x_{21}, x_{22}\}$. The agent's types are equally likely. If the agent's type is ω^1, the payoff matrices are:*

	If she chooses YN x_{21}	x_{22}
x_{11}	$(-5,0,5)$	$(-5,0,5)$
x_{12}	$(6,0,4)$	$(6,0,4)$
x_{13}	$(5,0,5)$	$(5,0,5)$

	If she chooses NY x_{21}	x_{22}
x_{11}	$(0,1,3)$	$(0,2,3)$
x_{12}	$(0,1,3)$	$(0,2,3)$
x_{13}	$(0,1,3)$	$(0,2,3)$

in which each cell represents the payoff of $P1$, that of $P2$ and that of the agent, respectively. If the agent's type is ω^2, the payoff matrices are:

	If she chooses YN x_{21}	x_{22}
x_{11}	$(5,0,5)$	$(5,0,5)$
x_{12}	$(0,0,4)$	$(0,0,4)$
x_{13}	$(-5,0,5)$	$(-5,0,5)$

	If she chooses NY x_{21}	x_{22}
x_{11}	$(0,1,4)$	$(0,2,4)$
x_{12}	$(0,1,4)$	$(0,2,4)$
x_{13}	$(0,1,4)$	$(0,2,4)$

If the agent rejects all offers, every player gets a payoff of zero. Assumption S-u is satisfied. Observe that for ω^1 it is a dominant strategy to accept $P1$'s proposal.

Consider a game G^Γ with $M_1 = M_2 = \{s, s', \emptyset\}$, we show that $(3,1,4)$ can be supported as a non-strongly-robust equilibrium outcome. To do so, let P1 post $\gamma_1 = \{x_{12}\}$ and P2 post $\gamma_2 = \{x_{22}\}$ irrespective of the message sent by the agent and of her participation decision. Let type-ω^1 strategy $\lambda(., \omega^1)$ be to participate with $P1$ for every (γ_1, γ_2) and to send the message

$$m_1(\gamma_1, \gamma_2, \omega^1) = \begin{cases} \text{any } m_1 \in \{s, s'\} & \text{if } \gamma_1(s) = \gamma_1(s') \\ \text{any } m_1 \in \{s, s'\} \;:\; \gamma_1(m_1) \in \{x_{11}, x_{13}\} & \text{otherwise.} \end{cases}$$

Assume that type-ω^2 strategy $\lambda(., \omega^2)$ is such that she participates with P2 only if $\gamma_1(s) = \gamma_1(s') = \{x_{12}\}$. In all other cases, she participates with $P1$ sending him the message $m_1(\gamma_1, \gamma_2, \omega^2) = \{m_1 \in \{s, s'\} : \gamma_1(m_1) \in \{x_{11}, x_{13}\}\}$. Clearly, λ constitutes a best reply to the offers (x_{12}, x_{22}). The strategies $(\gamma_1, \gamma_2, \lambda)$ support the outcome $(3,1,4)$. We now show that they form an equilibrium. For $P1$ to have a profitable deviation, he must attract both types and induce them to trade different contracts. Now, following any of such deviations, λ prescribes to both types to participate with him and to send him the same message. Hence, the deviation cannot be profitable. For $P2$ to have a profitable deviation, he should attract both types, which is impossible given λ. The outcome $(3,1,4)$ is therefore supported at equilibrium with different types participating with different principals. The equilibrium is not strongly robust: suppose that $P1$ deviates to the mechanism γ_1' such that $\gamma_1'(s) = x_{11}$ and $\gamma_1'(s') = x_{13}$ for every participation decision of the agent.

Following this deviation, there is a continuation equilibrium in which both types participate with him, with ω^1 *sending* s' *and* ω^2 *sending* s *and P1 getting a payoff of 5. The agent's equilibrium strategy, however, guarantees that such a continuation equilibrium is not selected.*

Consider now the game G^D*. We show that* $(3,1,4)$ *cannot be supported in any truth-telling equilibrium. Indeed, for* $(3,1,4)$ *to be such an equilibrium outcome, it should be that type* ω^2 *participates with P2, and the equilibrium mechanism of P2 must satisfy* $\tilde{\gamma}_2\left(\omega^2\right) = x_{22}$*. Suppose now that P1 posts the mechanism* $\tilde{\gamma}'$ *such that* $\tilde{\gamma}_1'\left(\omega^1\right) = x_{13}$ *and* $\tilde{\gamma}_1'\left(\omega^2\right) = x_{11}$*, for every participation decision of the agent. Both* ω^1 *and* ω^2 *then participate with P1 with probability one. Since the agent's equilibrium strategy has to be truthful,* $\tilde{\gamma}_1'$ *yields P1 a payoff of 5, which constitutes a profitable deviation.*

The result crucially relies on how the agent's indifferences are resolved in each of the two games. In G^Γ, following the deviation to γ_1' she chooses the most harmful alternative for P1, which sustains $(3,1,4)$ at equilibrium. In G^D, there is a truth-telling equilibrium supported by the incentive compatible mechanism $\tilde{\gamma}_1'$ which guarantees to P1 a payoff of 5 with the agent being honest. The example develops an intuition similar to that provided by Myerson (1982) to show a possible non-existence of equilibria in competing mechanism games. We take a different perspective emphasizing that additional outcomes may be supported in equilibria that fail to be truth-telling.

We finally prove that every strongly robust equilibrium outcome can be characterized by restricting principals to incentive compatible mechanisms.

Proposition 2 *Under Assumption S-u and* $I = 1$*, every strongly robust equilibrium outcome of any game* G^Γ *is a truth-telling equilibrium outcome of the game* G^D*.*

Proof. Consider an arbitrary game G^Γ and suppose there is a strongly-robust equilibrium outcome supported by the strategies (δ,λ). Denote V_j and U the equilibrium payoffs for every principal $j \in \mathcal{J}$ and for the agent, respectively. For a given realization of the principals' mixed strategies (γ_j,γ_{-j}), and for a given type ω, Assumption S-u implies that the set of optimal messages that ω sends to principal j when participating with him only depends on γ_j. We denote $\hat{M}_j(\gamma_j,\omega)$ such set. The proof is organized in two steps.

1. We construct a profile of probability distributions $(\tilde{\delta}_j,\tilde{\delta}_{-j})$ over incentive compatible mechanisms and a truth-telling strategy $\tilde{\lambda}$ that induce the outcomes $(V_j)_{j\in\mathcal{J}}$ and U.

Take an array (γ_j,γ_{-j}) in the support of $\delta = (\delta_j,\delta_{-j})$, a principal j and a type ω. We let

$$\begin{aligned}\tilde{\gamma}_j(\omega,a) \quad &= \pi_j(a_j = Y|\gamma_j,\gamma_{-j},\omega)\,\gamma_j(\mu_j(m_j|a_j = Y,\gamma_j,\gamma_{-j},\omega)) + \\ &+(1-\pi_j(a_j = Y|\gamma_j,\gamma_{-j},\omega))\,\gamma_j(\hat{m}_j(\gamma_j,\omega))\end{aligned} \tag{3}$$

for each a. To grasp the logic of equation (3), consider the case in which type ω participates with principal j at equilibrium, i.e. $\pi_j(a_j = Y|\gamma_j,\gamma_{-j},\omega) = 1$. Given the equilibrium communication strategy $\mu_j(m_j|a_j = Y,\gamma_j,\gamma_{-j},\omega)$, $\tilde{\gamma}_j(\omega,a)$ is constructed to reproduce the probability distribution on principal j's decisions induced on γ_j by λ. Conversely, if $\pi_j(a_j = Y|\gamma_j,\gamma_{-j},\omega) = 0$, $\tilde{\gamma}_j(\omega,a)$ reproduces any decision induced on γ_j by an arbitrary optimal message $\hat{m}_j(\gamma_j,\omega) \in \hat{M}_j(\gamma_j,\omega)$.

Iterating this procedure for every ω, we get a direct mechanism $\tilde{\gamma}_j$ for each principal j. We construct such an array of direct mechanisms for every (γ_j, γ_{-j}) in the support of (δ_j, δ_{-j}). Finally, given δ_{-j}, there exists a mixed strategy $\tilde{\delta}_j$ that would replicate the equilibrium probability distribution of principal j if the agent behaves truthfully. The mechanisms $(\tilde{\delta}_j, \tilde{\delta}_{-j})$ are such that the agent has access to the same payoffs induced by (δ_j, δ_{-j}). It then follows from Myerson (1982) that it is possible to specify the equilibrium strategy $\tilde{\lambda}$ in such a way that types are revealed truthfully, participation decisions coincide with those specified by λ, and the payoffs $(V_j)_{j\in\mathcal{J}}$ and U are attained.

2. We show that if $(\tilde{\delta}, \tilde{\lambda})$ is not an equilibrium in G^D, then (δ, λ) is not strongly robust in G^Γ.

If $\left(\tilde{\delta}, \tilde{\lambda}\right)$ is not an equilibrium, there exists a principal j and an incentive compatible mechanism $\tilde{\gamma}_j'$ such that:

$$V_j\left(\tilde{\gamma}_j', \tilde{\delta}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}\right)\right) > V_j\left(\tilde{\delta}_j, \tilde{\delta}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}_j, \tilde{\gamma}_{-j}\right)\right) = V_j,$$

with $\tilde{\lambda}(\tilde{\gamma}_j', \tilde{\gamma}_{-j})$ representing the agent's decision over participation and communication induced by principal j's deviation, for a given realization $\tilde{\gamma}_{-j}$ of the joint lottery $\tilde{\delta}_{-j}$. By construction of $\tilde{\lambda}$, it is a best reply for the agent to truthfully report her type to the principal she participates with. Let us now look back at G^Γ. Since $\sharp M_j \geq \sharp\Omega$ for every j, there exists an invertible surjective mapping $\phi_j : M_j \to \Omega \cup \{\emptyset\}$ with $\phi_j(\emptyset) = \emptyset$ for every principal j. Consider the indirect mechanism γ_j' such that $\gamma_j'(m_j) = \tilde{\gamma}_j'(\phi_j(m_j))$ for all $m_j \in M_j$.

There exists a strategy profile $\lambda' = (m', \pi')$, such that the agent uses the same probabilities over participation as in $\tilde{\lambda}$, and sends messages which induce the same decisions that are available with $\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}\right)$. Let this be

$$\begin{cases} \pi_j' = \tilde{\pi}_j\left(a_j = Y | \tilde{\gamma}_j', \tilde{\gamma}_{-j}, \omega\right) \\ m_j' \text{ be any element of } (\phi)_j^{-1}(\omega) \end{cases}$$

for each ω, for each $\tilde{\gamma}_{-j}$ in the support of $\tilde{\delta}_{-j}$, and for each j. Hence, given γ_j' and any γ_{-j} in the support of δ_{-j}, it is a best reply for the agent to play $\lambda' = (\pi_j', m_j')_{j\in J}$. It follows that, for every γ_{-j} in the support of δ_{-j}, the payoff to principal j is the same that he would get by deviating to $\tilde{\gamma}_j'$, for every $\tilde{\gamma}_{-j}$ in the support of $\tilde{\delta}_{-j}$, i.e.

$$V_j\left(\gamma_j', \gamma_{-j}, \lambda'\left(\gamma_j', \gamma_{-j}\right)\right) = V_j\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}\right)\right)$$

Therefore, the expected payoff for principal j satisfies

$$V_j\left(\gamma_j', \delta_{-j}, \lambda'\left(\gamma_j', \gamma_{-j}\right)\right) = V_j\left(\tilde{\gamma}_j', \tilde{\delta}_{-j}, \tilde{\lambda}\left(\tilde{\gamma}_j', \tilde{\gamma}_{-j}\right)\right) > V_j,$$

which contradicts that V_j is supported in a strongly robust equilibrium of G^Γ. ■

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