Optimal Delegated Search with Adverse Selection and Moral Hazard*

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Abstract

The paper studies a model of delegated search. The distribution of search revenues is unknown to the principal and has to be elicited from the agent in order to design the optimal search policy. At the same time, the search process is unobservable, requiring search to be self-enforcing. The two information asymmetries are mutually enforcing each other; if one is relaxed, delegated search is efficient. With both asymmetries prevailing simultaneously, search is almost surely inefficient (it is stopped too early). Second-best remuneration is shown to optimally utilize a menu of simple bonus contracts. In contrast to standard adverse selection problems, indirect nonlinear tariffs are strictly dominated.

Keywords: adverse selection, bonus contracts, delegated search, moral hazard, optimal stopping.

JEL Classification: D82, D83, D86, C72.

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1 Introduction

Searching is an important aspect of many agency relationships. To name a few, recruiting agencies are hired to search for job candidates; real estate agents are contracted to search for prospective tenants or housing; and insurance brokers are employed to attract new clients. More generally, many forms of problem-oriented thinking require searching for ideas or solutions. This includes research centers searching for new product ideas, advocates thinking about strategies to defend their clients, and business consultancies (or managers) looking for profitable business strategies.

This paper analyzes optimal searching when it is delegated to an agent. The model is based on the standard (single-agent) search model, in which a problem-solver sequentially samples "solutions" from a time-invariant distribution until she is satisfied (McCall, 1970; Mortensen, 1970). Departing from the standard search model, I study optimal search when the revenues are not collected by the problem-solver but by a distinct principal.

I consider two information asymmetries governing the relationship between the problem-solving agent and the principal. First, motivated by the aforementioned examples, I model the agent as an expert who has an *ex ante* informational advantage over the principal in assessing the prospects of searching. For instance, recruiting agencies are likely to be better informed about the chances of finding qualified candidates than their clients; real estate agents are likely better in assessing the likelihood that a house sells at a certain price compared to house owners; *et cetera*. In an effort to capture this notion of asymmetry, I assume that payoffs x are sampled from a time-invariant but state-dependent distribution $F(x|\theta)$, where θ is privately known by the agent. Second, I assume that the search process itself cannot be observed (or verified) by the principal. This second asymmetry reflects that many search routines are either intrinsically unobservable (e.g., thinking for ideas), or are hard to be verified due to their soft and easily manipulatable nature (e.g., sampling a *genuine* buyer).

In this search environment the precise configuration of information frictions is crucial to the delegated search process. If either of the two asymmetries occurs in isolation, then the efficient benchmark can be sustained under delegation. This holds true independent of liability constraints or the risk attitude of the agent. If, however, both asymmetries prevail simultaneously, then each acts as a catalyst to the other one, and search is almost surely inefficient (it is stopped too early).[1]

A natural question then is: how should one design the contractual arrangements to achieve second-best optimality?

Confronted with both asymmetries, the challenge is to bring the agent to reveal the optimal search policy (which depends on θ) and, at the same time, to induce her to also search according to the revealed policy. It turns out that the second-best optimum can be implemented via a menu of simple bonus contracts. Each contract pays a fixed bonus when a previously specified target is reached, and nothing otherwise. Other information about the realized outcome is optimally ignored.

Underlying this result is the endogenous nature of the search environment as perceived by the *agent*. Contracts that are more sensitive to the outcome of the search process than bonus contracts are shown to increase the agent's temptation to underreport the optimal search policy when she has a stochastic advantage of finding "high" outcomes. Paying a fixed bonus conditional on achieving a certain search target minimizes this temptation, while preserving incentives to implement the revealed search policy. The same logic also rules out indirect tariffs that only condition on the realized search revenues, since they necessarily increase the sensitivity of the compensation scheme.

This result provides a novel angle to the common practice of using bonus schemes rather than fully state-contingent schedules to set incentives (e.g., Moynahan, 1980; Churchill, Ford and Walker, 1993). It thereby complements a small literature that seeks to explain why real world compensation schemes are often simpler than standard theories would suggest.[2] In particular, Herweg, Müller and Weinschenk (2010) have recently demonstrated that bonus schemes are optimal if agents are averse to losses relative to an expectation-based reference point.[3]

More generally, the paper relates to a large literature focusing on the delegation of certain tasks subject to contracting constraints. The delegation of search has recently been explored by Lewis and Ottaviani (2008) and Lewis (2012).[4] Lewis and Ottaviani,

[1] In a broad sense, this is similar to how liability constraints and risk-aversion unleash the moral hazard in standard moral hazard settings (Holmstrom, 1979; Innes, 1990).

[2] Seminal examples include Holmstrom and Milgrom (1987), Townsend (1979) and Innes (1990) rationalizing linear compensations schemes and standard debt contracts.

[3] See also Park (1995), Kim (1997), Demougin and Fluet (1998) and Oyer (2000) showing that bonus schemes are "knife-edge" optimal under limited liability if agents are exactly risk-neutral, while they are generally suboptimal if agents are risk-averse to only the slightest degree (Jewitt, Kadan and Swinkels, 2008).

[4] For an overview of the (non-delegated) search literature, see Mortensen (1986) and Rogerson,

however, study search over a long-term horizon, using techniques from the dynamic moral hazard literature (Toxvaerd, 2006; Sannikov, 2008). In these environments search revenues are decreasing in time and the central challenge is to induce the agent to search at the right *speed*.

In this paper, in contrast, search is assumed to take place during a comparatively short span of time and the main challenge is to learn (and induce) the preferred stopping rule. For the principal, the difficulty thus lies in disentangling an *ex ante* poor distribution of search revenues from a poorly chosen search policy. At a more technical level, this aspect closely relates to principal-agent models with joint moral hazard and adverse selection (see Gottlieb and Moreira, 2013 and Faynzilberg and Kumar, 2000 for general treatments, and Bolton and Dewatripont, 2005, ch. 6 for a survey of applications). Similar to, e.g., Laffont and Tirole (1986, 1993) and Mirrlees (1971), dealing jointly with the two asymmetries becomes ultimately tractable here, because the second-best optimal dealing with adverse selection turns out to be also an adequate mean to optimally address the moral hazard.

On the empirical side, the efficiency of search agencies has been studied in the context of the real estate industry. In line with the findings in this paper, the literature documents that search spells are inefficiently short and sales prices are inefficiently low (Levitt and Syverson, 2008; Rutherford, Springer and Yavas, 2005).[5]

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 provides the first-best benchmark and shows how it can be implemented under delegation if only one of the two asymmetries is active. Section 4 analyzes the solution to the model with both information asymmetries, and Section 5 concludes. All proofs are confined to the appendix.

Shimer and Wright (2005). Multi-agent variations include Albrecht, Anderson and Vroman (2010) and Compte and Jehiel (2010) looking at non-homogeneous search committees that jointly decide over the continuation of a search process.

[5]Specifically, Levitt and Syverson (2008) compare home sales in which real estate agents are hired to when an agent sells his own home, finding that agent-owned homes sell on average for 3.7 percent more than other homes and stay on the market for 9.5 days longer. Further stratifying their sample by the local heterogeneity of the housing stock, they find that these gaps are increasing in heterogeneity (which makes it harder for house owners to learn about likely sales prices from prior transactions). Levitt and Syverson interpret this as evidence for the importance of prior information asymmetries. In a similar study, Rutherford, Springer and Yavas (2005) find that agent-owned homes sell on average for 4.5 percent more than other homes.

2 A simple model of delegated search

There are two parties, a principal and an agent. Both parties are risk-neutral and have unlimited access to cash.[6] The principal hires the agent to operate a search technology that yields a monetary outcome $x \in X = [0, B]$. The agent samples outcomes at constant (non-monetary) costs $c > 0$ from a twice differentiable cumulative distribution function $F(x|\theta)$, where θ is an exogenous state with support $\Theta = [\underline{\theta}, \bar{\theta}]$. The prior cumulative distribution function of θ is common knowledge, is denoted by P, and has a differentiable density p such that $p(\theta) > 0$ for all $\theta \in \Theta$. Each time the agent samples an outcome, she can either stop search and select any previously sampled outcome, or continue searching.[7] If she selects an outcome, the principal collects its monetary value, the agent receives her remuneration, and the game ends. The outside option from not selecting any outcome and from not contracting is normalized to zero for both parties. Without loss of generality, I restrict attention to the case where, in the absence of information asymmetries, searching is profitable in all states ($\mathbb{E}\{x|\theta\} \geq c$ for all $\theta \in \Theta$).

I consider two information asymmetries.

Assumption A1 (Adverse Selection)**.** *The state θ is privately revealed to the agent before she contracts with the principal. The principal knows the set of potential states Θ and their distribution $P(\theta)$.*

Assumption A2 (Moral Hazard)**.** *Search by the agent and the sampled selection of outcomes cannot be observed by the principal. The value of the selected outcome is observable and verifiable.*

[6] Both assumptions can be relaxed without affecting the results. Maintaining them helps simplifying notation, avoids dealing with (irrelevant) corner solutions, and highlights that any inefficiency emerging in this search environment is unrelated to limited liability and risk-sharing.

[7] Two comments are in order. First, there will be no recall under the optimal mechanism, so that the assumption of perfect recall is without loss of generality. Second, to resolve some indeterminacies, I assume throughout that the agent continues searching if indifferent. If the agent would stop instead, delegated search under the optimal mechanism remains the same, but the supremum of all mechanism implementing it would not be attained exactly, since the principal would need to leave a marginal rent to the agent in certain states where the agent receives zero rents under the supremum mechanism.

3 Benchmark cases

For reference, I first describe the full information (first-best) benchmark and examine the cases where only one of the two information asymmetries prevails.

3.1 Full information

Under full information the principal reaps the (joint) surplus and implements the search policy that maximizes it. This is merely the standard search model. I skip the derivation and simply state the result.[8] For details, see e.g. McCall (1970).

Proposition 1. *In the first best, the agent searches as long as for all previously sampled outcomes it holds that $x \leq \bar{x}^{FB}(\theta)$. Otherwise she stops search and selects the last-sampled outcome. The first-best stopping rule, $\bar{x}^{FB} : \Theta \to X$, is uniquely defined by*

$$c = \int_{\bar{x}^{FB}(\theta)}^{B} (x' - \bar{x}^{FB}(\theta))\,\mathrm{d}F(x'|\theta). \tag{1}$$

Under full information, the problem is separated across states. Conditional on θ, search continues until the agent samples a solution of at least value $\bar{x}^{FB}(\theta)$. The optimal "stopping rule" $\bar{x}^{FB}(\theta)$ is hereby chosen to equate the marginal expected benefit of finding a better outcome than $\bar{x}^{FB}(\theta)$ (the right-hand side of equation (1)) with the marginal (social) cost of continuing search c.

3.2 Only adverse selection

Consider now the case where the principal is able to observe (and verify) the sampled selection of outcomes, and only faces uncertainty from not knowing the state θ (Assumption A1 holds but not A2). In this case, the first-best search policies can be implemented by exactly compensating the agent for her search costs. Because this makes her payoffs effectively independent of the pursued search policy, she is indifferent and finds it (weakly) optimal to adopt the first-best policy. I state the precise result in the following.

[8] See Lemma 1 below for a more general proof that comprises the first-best problem by letting $T(x) = x$. For payment schemes implementing the first best, see, e.g., Propositions 2 and 3 below.

Proposition 2. *Suppose Assumption A1 holds, but the principal is able to verify the sampled selection of outcomes. Then the first-best search policies can be implemented by specifying a transfer $T(N)$ from the principal to the agent, where $T(N) = N\,c$, and N is the number of outcomes in the final sample.*

Proof Sketch. Under the proposed contract T, the agent breaks even independently of her search behavior, preventing any profitable deviation. Hence, the contract trivially implements the first-best solution where the agent accepts the contract and pursues first-best search policies, and the principal reaps all the surplus. *Q.E.D.*

With only adverse selection, the principal is able to construct a contract, in which the agent's private knowledge about the state θ is not payoff-relevant to her. The agent is therefore willing to reveal the state without any explicit incentives. Critical to this contract is that the principal is able to verify the sampled selection of outcomes, allowing him to assess the actual search costs of the agent. This is precisely what is prevented by Assumption A2. Under moral hazard, the principal can only form an expectation about how often the agent sampled before selecting an outcome, preventing him from differentiating a poor distribution of outcomes (caused by θ) from an early termination of search by the agent. In this sense, Assumption A2 "unleashes" Assumption A1 by rendering the agent's private knowledge of θ necessarily payoff-relevant (for any non-trivial contracting).

3.3 Only moral hazard

A similar conclusion holds regarding the flipside scenario where Assumption A2 holds but not Assumption A1. Again the first-best search policies can be implemented via a simple contractual arrangement. Perhaps the most obvious approach is to make the agent the residual claimant, as it is then in her own interest to maximize the joint surplus.[9] In view of the subsequent analysis, it is, however, useful to observe that the agent does not need to have full claim on the realized outcome x to efficiently

[9] Specifically, the first-best can be implemented by specifying a transfer $T(x)$ from the principal to the agent, where $T(x) = -\bar{x}^{FB}(\theta) + x$. Here the lump-sum transfer $\bar{x}^{FB}(\theta)$ equals the first-best expected surplus (conditional on θ), so that the principal reaps all the surplus while the agent becomes residual claimant and implements the efficient search. If both asymmetries co-exist, this arrangement is not feasible, since with θ unknown the set of states where a (then necessarily unconditional) lump-sum transfer is accepted by the agent will be subject to adverse selection in the original sense (Akerlof, 1970).

implement a particular search policy. Generally, the agent will adopt some stopping rule $\bar{x}$ given any contractual arrangement that generates a marginal remuneration which exceeds c for all $x \leq \bar{x}$, and which is smaller than c for all $x > \bar{x}$. Accordingly, there are infinitely many remuneration schemes that implement $\bar{x}$ efficiently. For instance, consider a bonus arrangement of the following form.

Definition. Let τ be a nonrandom constant. Then a contract T is called a *bonus contract* when it is of the following form:

$$T(x) = \begin{cases} 0 & \text{if } x \leq \bar{x} \\ \tau & \text{if } x > \bar{x}. \end{cases}$$

With τ set sufficiently high, a bonus contract will clearly implement $\bar{x}$. Moreover, because payments are zero for all $x \leq \bar{x}$, the agent's (marginal) net benefit of searching, $[1 - F(\bar{x}|\theta)]\tau - c$, will be nonnegative if and only if her expected utility from contracting exceeds her outside option. Bonus contracts are therefore (weakly) "cheapest" in implementing a particular stopping rule (subject to the agent accepting the contract).[10] Letting $\bar{x} = \bar{x}^{FB}(\theta)$ then gives the following result.

Proposition 3. *Suppose Assumption A2 holds, but the principal learns θ prior to contracting the agent. Then first-best search policies can be implemented by utilizing a bonus contract with $\bar{x} = \bar{x}^{FB}(\theta)$ and $\tau = [1 - F(\bar{x}|\theta)]^{-1}\, c$.*

Proof Sketch. Under the proposed contract, the agent's expected benefits of searching are zero as long as $x \leq \bar{x}$ and become negative for all $x > \bar{x}$. Hence the contract implements first-best search. Moreover, the agent receives zero expected benefits. Hence she accepts the contract since she breaks even, and the principal reaps all the surplus. *Q.E.D.*

Again, this arrangement is not feasible if both asymmetries co-exist. The reason is that with θ unknown the first-best search policy $\bar{x}^{FB}(\theta)$ will be private information of the agent, which creates incentives to strategically misreport the state as will be seen below. Accordingly, adverse selection triggers the moral hazard problem in the search

[10] While this reasoning also applies to other remuneration schemes, bonus contracts are unique in that they do not impose risk or negative payments on the agent, making them robust to limited liability or risk-sharing concerns. In the next section, I will show that bonus contracts also minimize incentives to misreport the state in the presence of both moral hazard and adverse selection, generating an additional rational for compensating search by bonus schemes.

environment similar to how risk aversion and limited liability unleash moral hazard in traditional principal-agent settings.

4 Adverse selection and moral hazard

I now turn to the case where both asymmetries co-exist. The challenge for the principal then becomes to design an incentive scheme that brings the agent to reveal her knowledge of the state θ and, at the same time, induces her to search according to the search policies that the principal finds optimal given θ.

Let a contract be a (possibly state-contingent) mapping $T_\theta : X \to \mathbb{R}$, which specifies, for every outcome $x \in X$, a transfer from the principal to the agent. Under Assumption A2 it is clear that all incentives to search have to be self-enforcing given T_θ. Taking into account the mapping from T_θ to search policies, the principal's objective is to maximize expected search revenues net of transfers. By the revelation principle, a solution to this problem may be obtained via a direct revelation mechanism in which the agent truthfully reports the state θ, and for each θ is assigned a contract T_θ. The principal's problem is then to find the optimal set of contracts $\{T_\theta\}_{\theta\in\Theta}$.

I approach this problem as follows. Since any contract T_θ effectively designs a search problem from the perspective of the agent, I first characterize the agent's optimal search policy for an arbitrary contract. With this implementability constraint in hand, I then examine the optimization problem of the principal and obtain some defining properties of the optimal menu. In particular, I establish that bonus contracts minimize overall agency rents from both moral hazard and adverse selection and, therefore, continue to be optimal in the presence of adverse selection. After simplifying the problem accordingly, I lastly solve for the optimal menu $\{T_\theta\}_{\theta\in\Theta}$ and derive the optimal search policies.

4.1 Implementability constraints

Once the agent has chosen a contract $T_{\tilde{\theta}}$ from the menu offered to her, sequential rationality requires that she pursues the search policy which is then optimal for her. Since the agent is effectively facing a search problem over the transfers $T_{\tilde{\theta}}(x)$ specified by the chosen contract, delegated search is characterized by the solution to this search problem. The following lemma states the solution.

Lemma 1. *An agent with distribution θ and contract $T_{\tilde{\theta}}$ searches as long as for all previously sampled outcomes it holds that $T_{\tilde{\theta}}(x) \leq \bar{T}_{\tilde{\theta}}(\theta)$. Otherwise she stops search and selects the last-sampled outcome. Whenever $\int T_{\tilde{\theta}}(x')\,\mathrm{d}F(x'|\theta) \geq c$, the stopping rule, $\bar{T}_{\tilde{\theta}} : \Theta \to X$, is uniquely defined by*

$$c = \int \left(\left\{ T_{\tilde{\theta}}(x') - \bar{T}_{\tilde{\theta}}(\theta) \right\} \cdot \psi_{\tilde{\theta}}(x', \bar{T}_{\tilde{\theta}}(\theta)) \right) \mathrm{d}F(x'|\theta)\,, \tag{2}$$

where $\psi_{\tilde{\theta}} : X \times \mathbb{R} \to \{0,1\}$ is an indicator function, such that

$$\psi_{\tilde{\theta}}(x, \bar{T}_{\tilde{\theta}}(\theta)) = \begin{cases} 0 & \text{if } T_{\tilde{\theta}}(x) \leq \bar{T}_{\tilde{\theta}}(\theta), \text{ and} \\ 1 & \text{if } T_{\tilde{\theta}}(x) > \bar{T}_{\tilde{\theta}}(\theta). \end{cases}$$

Otherwise the agent does not search at all.

Similar to the first-best case, the optimal stopping rule $\bar{T}_{\tilde{\theta}}$ equates the marginal cost of searching c with the marginal expected benefits from finding a better outcome. However, in contrast to the first best, the value of searching from the perspective of the agent is now defined by $T(x)$ rather than x. For what is coming next, it will be useful to formulate the solution to the agent's problem in terms of outcomes $x \in X$.[11] To ensure that $T_{\tilde{\theta}}(x)$ maps back into a unique solution in X, I therefore impose the following restriction.

Assumption A3. *Contracts are monotonically increasing, i.e. $T_\theta(x') \leq T_\theta(x'')$ for all $(x', x'', \theta) \in \{X^2 \times \Theta \,|\, x' \leq x''\}$.*

It is well known that this assumption can be rationalized by the possibility of free disposal; i.e., the ability of the agent to freely downscale any realized outcome x.[12] Under Assumption A3, inverting $\bar{T}_{\tilde{\theta}}$ then immediately defines a stopping rule in X, given by,

$$\bar{x}(T_{\tilde{\theta}}, \theta) = \max_x \left\{ x : T_{\tilde{\theta}}(x) \leq \bar{T}_{\tilde{\theta}}(\theta) \right\}. \tag{3}$$

[11] This guarantees that the Spence-Mirrlees property holds with respect to any change in the payment scheme $\mathrm{d}T$. See Footnote 14 for details.

[12] To see this note that with free disposal the agent can guarantee herself a payoff of $T^*_\theta(x) \equiv \max_{x' \in [0,x]}\{T_\theta(x')\}$. Hence, w.l.o.g., one could replace T_θ by $\hat{T}_\theta$, which for all x, pays $\hat{T}_\theta(x) = T^*_\theta(x)$, whereas it can be easily verified that $\hat{T}_\theta$ is indeed increasing in x.

The following proposition formulates the resulting implementability constraints by defining $\bar{x}(T_{\tilde{\theta}}, \theta)$ directly as a function of $T_{\tilde{\theta}}$ (eliminating the intermediate dependence on $\bar{T}_{\tilde{\theta}}$).

Proposition 4. *Suppose Assumptions A2 and A3 hold. Let $\mathcal{M}$ be the space of monotonically increasing functions $X \to \mathbb{R}$. Then search is determined by a function $\bar{x} : \{(T, \theta) \in \mathcal{M} \times \Theta : \int T(x') \, \mathrm{d}F(x'|\theta) \geq c\} \to X$, which specifies, for a contract $T_{\tilde{\theta}} \in \mathcal{M}$ and a state $\theta \in \Theta$, a number $\bar{x}(T_{\tilde{\theta}}, \theta)$, such that the agent searches as long as for all previously sampled outcomes it holds that $x \leq \bar{x}(T_{\tilde{\theta}}, \theta)$. Otherwise she stops search and selects the last-sampled outcome. The stopping rule $\bar{x}$ is uniquely defined by the following inequalities.*

$$c \leq \int_{\hat{x}}^{B} \Big(T_{\tilde{\theta}}(x') - T_{\tilde{\theta}}(\hat{x})\Big) \, \mathrm{d}F(x'|\theta) \quad \textit{for all } \hat{x} \leq \bar{x}(T_{\tilde{\theta}}, \theta) \tag{4a}$$

$$c > \int_{\hat{x}}^{B} \Big(T_{\tilde{\theta}}(x') - T_{\tilde{\theta}}(\hat{x})\Big) \, \mathrm{d}F(x'|\theta) \quad \textit{for all } \hat{x} > \bar{x}(T_{\tilde{\theta}}, \theta)\,. \tag{4b}$$

For all $(T_{\tilde{\theta}}, \theta) \in \mathcal{M} \times \Theta$ outside the domain of $\bar{x}$ the agent does not search at all.

4.2 Contract properties

I am now ready to characterize the problem from the perspective of the principal. The optimal menu of contracts $\{T_\theta\}_{\theta \in \Theta}$—if it exists[13]—is given by the solution to the following maximization problem:

$$\max_{\{T_\theta\}_{\theta \in \Theta}} \left\{ \int_{\theta \in \Theta} \int_{\bar{x}(T_\theta, \theta)}^{B} \left(\frac{x' - T_\theta(x')}{\bar{F}(\bar{x}(T_\theta, \theta)|\theta)} \right) \mathrm{d}F(x'|\theta) \, \mathrm{d}P(\theta) \right\}$$

subject to the constraints,

$$\frac{1}{\bar{F}(\bar{x}(T_\theta, \theta)|\theta)} \left[\int_{\bar{x}(T_\theta, \theta)}^{B} T_\theta(x') \, \mathrm{d}F(x'|\theta) - c \right] \geq 0 \tag{IR_θ}$$

[13]Existence may fail because the supremum may not be achieved exactly. This may happen for two reasons. First, the objective is not continuous in T_θ at points where the agent stops searching ($\int T_\theta(x') \, \mathrm{d}F(x'|\theta) = c$), and where $\bar{T}_\theta(\theta)$ is attained on an interval $[\underline{x}, \bar{x}]$. Second, $T_\theta(x)$ may be potentially unbounded. In the subsequent analysis I will impose additional structure that allows me to derive an optimal mapping from intended search policies $\{\bar{x}(\theta)\}_{\theta \in \Theta}$ to contracts. This then allows me to transform the principal's problem into a continuous maximization problem in $\{\bar{x}(\theta)\}_{\theta \in \Theta}$ subject to a compact set of constraints, guaranteeing the existence of a well-defined solution. (Feasibility of the constraints is not an issue here, since $T_\theta(x) = c$ for all $\theta \in \Theta$ trivially fulfills all constraints.)

$$\frac{1}{\bar{F}(\bar{x}(T_\theta,\theta)|\theta)}\left[\int_{\bar{x}(T_\theta,\theta)}^{B} T_\theta(x')\,\mathrm{d}F(x'|\theta)-c\right]$$
$$\geq \frac{1}{\bar{F}(\bar{x}(T_{\tilde{\theta}},\theta)|\theta)}\left[\int_{\bar{x}(T_{\tilde{\theta}},\theta)}^{B} T_{\tilde{\theta}}(x')\,\mathrm{d}F(x'|\theta)-c\right] \quad (IC_{\theta,\tilde{\theta}})$$

for all $(\theta,\tilde{\theta})\in\Theta^2$, where $\bar{F}\equiv 1-F$, and where $\bar{x}(T_{\tilde{\theta}},\theta)$ is characterized by

$$c \leq \int_{\hat{x}}^{B}\Big(T_{\tilde{\theta}}(x')-T_{\tilde{\theta}}(\hat{x})\Big)\,\mathrm{d}F(x'|\theta) \quad \text{for all } \hat{x}\leq\bar{x}(T_{\tilde{\theta}},\theta) \qquad (SP^{-}_{\theta,\tilde{\theta}})$$
$$c > \int_{\hat{x}}^{B}\Big(T_{\tilde{\theta}}(x')-T_{\tilde{\theta}}(\hat{x})\Big)\,\mathrm{d}F(x'|\theta) \quad \text{for all } \hat{x}>\bar{x}(T_{\tilde{\theta}},\theta)\,, \qquad (SP^{+}_{\theta,\tilde{\theta}})$$

whenever $\int T_{\tilde{\theta}}(x')\,\mathrm{d}F(x'|\theta)\geq c$.

The objective of the principal here is to maximize his expected payoff subject to three kinds of constraints. First, constraints (IR_θ) require that it must be individually rational for the agent in state θ to accept contract T_θ, rather then choosing her outside option. Second, constraints $(IC_{\theta,\tilde{\theta}})$ require that it must be optimal for the agent in state θ to truthfully reveal the state to the principal by choosing T_θ from the menu of all contracts $\{T_{\tilde{\theta}}\}_{\tilde{\theta}\in\Theta}$. These constraints stem from the principal not knowing the state θ. The third set of constraints reflect the requirement to also incentivize search by the agent. As follows from Proposition 4, $(SP^{-}_{\theta,\tilde{\theta}})$ and $(SP^{+}_{\theta,\tilde{\theta}})$ pin down the stopping rule $\bar{x}(T_{\tilde{\theta}},\theta)$ implemented in state θ under contract $T_{\tilde{\theta}}$.

Before proceeding to the solution, let me impose some structure on the distribution of outcomes $F(x|\theta)$ and states $P(\theta)$. Let $H\equiv\partial\bar{F}^{-1}/\partial x$, and let subscripts of H denote partial derivatives. Then:

Assumption A4. *$H_\theta\leq 0$, and $H_{\theta\theta}\geq H_\theta^2/H$.*

Assumption A5. *$HH_{x\theta}\leq H_xH_\theta$.*

Assumption A6. $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{p(\theta)}{1-P(\theta)}\right)\geq 0$.

The first part of Assumption A4 introduces a stochastic ordering over distributions in θ. A sufficient condition for H to be decreasing is the commonly used monotone likelihood ratio condition. Intuitively, the imposed ordering in H requires that at any point of search, continuing search will yield higher outcomes—in the sense of first-order stochastic dominance—in state θ'' than in state $\theta'<\theta''$. At a technical level, this guarantees that the Spence-Mirrlees property holds in a stochastic sense.[14] The second part of Assumption A4 strengthens the ordering, such that H is convexly increasing in θ (at a sufficient rate[15]). Intuitively, this requires that the benefit of being in a better state than θ is (sufficiently) decreasing in θ. Assumption A5 is of more technical nature, ensuring that the objective function of the principal is concave.[16] Finally, Assumption A6 is standard in many mechanism design applications, meaning that the likelihood to be in a better state than θ is decreasing in θ. This keeps results clean by ensuring the existence of an interior solution.

Parametric distributions for F consistent with these assumptions exist, for instance, within the Beta family and the (generalized) family of Pareto distributions (see the end of the next subsection for a particular simple example).[17] A sufficient condition for Assumption A6 to hold is that the likelihood $p(\theta)$ is weakly decreasing in θ (e.g., θ being uniform).

I am now ready to show that the principal optimally designs a menu of contracts which is exclusively comprised of bonus contracts as defined in Section 3.3. I begin by establishing a lower bound on the utility of the agent (and hence the transfers) under any mechanism employed by the principal that implements a given menu of stopping rules. Subsequently, I then show that bonus contracts attain this bound.

Let $\bar{x}(\theta)$ (with one argument) be a shortcut for the stopping rule $\bar{x}(T_\theta, \theta)$ adopted in state θ when the agent chooses the intended contract T_θ, and let

$$U(\theta) \equiv \frac{1}{\bar{F}(\bar{x}(\theta)|\theta)} \left[\int_{\bar{x}(\theta)}^{B} T_\theta(x') \, \mathrm{d}F(x'|\theta) - c \right]$$

[14]More precisely, Assumption A4 implies that the agent's indifference curves between *expected* transfers and different stopping rules cross only once over different states. To see this, let $T_\theta^e \equiv \mathbb{E}\{T(x) \,|\, x \geq \bar{x}, \theta\}$ denote the expected transfers to the agent in state θ with a given contract T, and let $u_\theta(T_\theta^e, \bar{x}) \equiv T_\theta^e - c/\bar{F}(\bar{x}|\theta)$ denote the expected utility of the agent when pursuing stopping rule $\bar{x}$. Then the single crossing property holds, if for any $(\theta, \theta') \in \{\Theta^2 | \theta > \theta'\}$ it holds that $-(\partial u_\theta/\partial \bar{x})/(\partial u_\theta/\partial T_\theta^e) \leq -(\partial u_{\theta'}/\partial \bar{x})/(\partial u_{\theta'}/\partial T_{\theta'}^e)$, which simplifies to $H(x|\theta) \leq H(x|\theta')$. In conjunction with Assumption A3 this then ensures that indifference curves are indeed single-crossing for any differential $\mathrm{d}T$, since for any monotonic contract it holds that $\mathrm{d}T_\theta^e \geq \mathrm{d}T_{\theta'}^e$.

[15]In many cases, convexity of H in θ suffices. Specifically, when $p(\theta)/(1 - P(\theta))$ is increasing at a sufficiently high rate, or when the marginal benefit of search is increasing in θ, it suffices that $H_{\theta\theta} \geq 0$. For details see the proof of Proposition 7.

[16]Clearly, a sufficient condition for this to hold is that $H_x \leq 0$ and $H_{x\theta} \leq 0$.

[17]Given certain regularity conditions that ensure that a first-best solution exists, the analysis in this paper also seamlessly extends to the case where $B \to \infty$, permitting distributions for F with half-bounded supports (e.g., the exponential distribution with $\bar{F}(x|\theta) = e^{-x/\theta}$ for $\underline{\theta} > 0$).

denote the utility of the agent under the intended contract. Then:

Proposition 5. *Suppose Assumptions A1–A4 hold. Then for any menu of contracts $\{T_\theta\}_{\theta\in\Theta}$ that implements $\bar{x}$,*

$$U(\theta) \geq \underline{U}(\theta, \bar{x}) \equiv \int_{\underline{\theta}}^{\theta} -\frac{\partial}{\partial\tilde{\theta}}\left(\frac{c}{\bar{F}(\bar{x}(\tilde{\theta})|\tilde{\theta})}\right) \mathrm{d}\tilde{\theta}.$$

Intuitively, $\underline{U}$ is a lower bound on the information rent that the agent can guarantee herself by misreporting the state. Depending on the contractual form used to implement $\bar{x}$, it might be necessary to grant the agent additional benefits in order to prevent her from misreporting θ or to incentivize her to pursue the intended search policy. This is because expected payments under contract T_θ and stopping rule $\bar{x}(T_\theta, \tilde{\theta})$ may vary across different states, which has to be taken into account to discourage misreporting of θ, and to incentivize searching. The lower bound $\underline{U}$ defines the information rent when all additional benefits due to changes in expected payments are set to zero and the agent receives no moral hazard rents.

From Proposition 5, it follows that any solution to the principal's problem is bounded above by the expected surplus net of $\underline{U}$:

Corollary 1. *Expected profits of the principal are bounded above by*

$$\bar{V} = \sup_{\bar{x}}\left\{\int_{\theta\in\Theta}\left(\frac{1}{\bar{F}(\bar{x}(\theta)|\theta)}\left[\int_{\bar{x}(\theta)}^{B} x' \,\mathrm{d}F(x'|\theta) - c\right] - \underline{U}(\theta,\bar{x})\right)\mathrm{d}P(\theta)\right\}.$$

Equipped with Corollary 1, I show the following result.

Proposition 6. *Suppose Assumptions A1–A4 hold. Then for all nondecreasing $\bar{x}$, $\underline{U}$ is attained by a menu of bonus contracts with bonus payment $\tau(\theta) = [\bar{F}(\bar{x}(\theta)|\theta)]^{-1}c + \underline{U}(\theta, x)$. Moreover, if there exists a nondecreasing $\bar{x}^*$ that attains $\bar{V}$, then $\bar{V}$ can be attained by a menu of bonus contracts that implements $\bar{x}^*$.*

The proposition establishes that bonus contracts minimize the agency rents $U(\theta)$ reaped by the agent. That is, taking into account all constraints stemming from both moral hazard and adverse selection, bonus contracts are an optimal mechanism to implement any (nondecreasing) search policy $\bar{x}$ (the condition that $\bar{x}$ must be nondecreasing is shown below to be irrelevant).

In Section 3.3, I have already discussed how bonus contracts minimize the moral hazard rents that accrue from incentivizing the agent to pursue the intended search policy (conditionally on the principal knowing the search policy that he likes to implement).

To develop an intuition why bonus contracts also minimize the information rents due to adverse selection, suppose the principal wants to implement the stopping rule $\bar{x}(\theta)$ in state θ. In order to incentivize the agent to continue search for all $x \leq \bar{x}(\theta)$, he needs to provide her with a certain expected benefit of finding $x > \bar{x}(\theta)$. Let $\hat{\tau}(\theta)$ denote the expected payment necessary to implement this benefit.[18] Then it must hold that $\int_{\bar{x}(\theta)}^{B} T_\theta(x')\,\mathrm{d}F(x'|\theta) \geq \hat{\tau}(\theta)$. As already noted in Section 3.3 the precise shape of T_θ on $[\bar{x}(\theta), B]$ is, however, irrelevant for the purpose of incentivizing the agent in state θ. The shape of T_θ on $[\bar{x}(\theta), B]$ can therefore be freely used to reduce the agent's temptation of misreporting the state θ. As is typical for adverse selection problems, the relevant temptation in this context is to underreport the state, giving the agent in all states better than θ a *stochastic advantage* in finding high outcomes relative to state θ. Because of this stochastic advantage, any schedule T_θ that is strictly increasing on $[\bar{x}(\theta), B]$ yields an expected return that is strictly higher than $\hat{\tau}(\theta)$ in all states better than θ. By paying a fixed remuneration, bonus contracts eliminate this (additional) premium associated with underreporting the state and thus minimize the agent's temptation to misreport θ.[19]

4.3 Optimal search policies

I now solve for the second-best search policies. From Proposition 6 it follows that if there exists a nondecreasing $\bar{x}^*$ that attains $\bar{V}$, then a menu of bonus contracts implementing $\bar{x}^*$ is also a solution to the principal's problem stated at the beginning of Section 4.2. The following proposition establishes that this is the case and states the solution.[20]

[18]Generally $\hat{\tau}(\theta)$ depends on $\sup\{T_\theta(x) : x \in [0, \bar{x}(\theta)]\}$ in conjunction with $(SP^{\pm}_{\theta,\theta})$. For small values of the former term it is pinned down by (IR_θ) instead.

[19]There still remains a second type of rent associated with underreporting the state that accrues from having smaller expected costs of pursuing a particular search policy. Because of this second rent, $\underline{U}$ is generally nonzero and delegated search will be inefficient.

[20]Note how the steps leading to finding $\bar{V}$ effectively amount to a transformation of the original problem of finding a menu of optimal functionals (i.e, contracts) into a problem of finding a menu of optimal stopping rules $\bar{x}$. This suggests that as long as there exists *some* optimal mapping from search policies to contracts, knowledge of this mapping allows substituting out contracts by search policies and simplifies the problem accordingly. See Faynzilberg and Kumar (2000) for a general treatment of a similar decomposition (conditioning on indirect utilities rather than policies) and for conditions when such a decomposition is feasible.

Proposition 7. *Suppose Assumptions A1–A6 hold. Then there exists a unique, non-decreasing* $\bar{x}^*$ *that attains* $\bar{V}$*. In particular, for some (nonempty)* $\Phi \subseteq \Theta$*, search is "sequential" with* $\bar{x}^*(\theta) > 0$ *for all* $\theta \in \Phi$*, characterized by*

$$c + D_\theta(\bar{x}^*(\theta)) = \int_{\bar{x}^*(\theta)}^{B} (x' - \bar{x}^*(\theta))\, \mathrm{d}F(x'|\theta) \tag{5}$$

with

$$D_\theta(x) = \begin{cases} 0 & \text{if } x = 0 \\ -\dfrac{1 - P(\theta)}{p(\theta)} \dfrac{\partial H(x|\theta)}{\partial \theta} \dfrac{c}{H(x|\theta)} & \text{if } x > 0\,. \end{cases} \tag{6}$$

For all $\theta \notin \Phi$*, search is "nonsequential", with* $\bar{x}^*(\theta) = 0$.[21]

Comparing equation (5) to its first-best counterpart (1), the marginal cost of delegated search is inflated by an agency term D_θ. Here D_θ reflects the cost of learning the optimal search policy: Under delegation, increasing $\bar{x}(\theta)$ not only increases the expected search costs, but also makes it more tempting for the agent to misreport the search policy in states $\theta' \in \{\theta' \in \Theta : \theta' > \theta\}$. To offset for this additional temptation, the principal needs to pay the agent a (higher) rent $U(\theta')$ in all states θ', making it (in expectations) more expensive to search in state θ.

Since the benefits of search are the same for delegated and nondelegated search (the right-hand sides of (5) and (1)), it follows:

Corollary 2. *Suppose Assumptions A1–A6 hold. Then delegated search is almost surely inefficient:* $\bar{x}^{SB}(\theta) < \bar{x}^{FB}(\theta)$ *for all* $\theta \in [0, B)$*.*

Proposition 7 distinguishes two cases. First, for all $\theta \in \Phi$, second-best search policies continue to be directed towards some target $\bar{x}(\theta)$, but the target is generally set too low (search stops too early). Second, for $\theta \notin \Phi$, sequential search is not profitable at all (if delegated), and the principal simply asks the agent to sample a single outcome and to unconditionally select it as final.

Figure 1 illustrates the two cases. Delegated search is nonsequential ($\theta \notin \Phi$) if D_θ increases the left-hand side of (5) beyond the right-hand side for all $\bar{x} > 0$, either because it is unprofitable to search from an *ex post* perspective (taking into account the

[21] That is, the agent samples a single outcome, which she unconditionally selects.

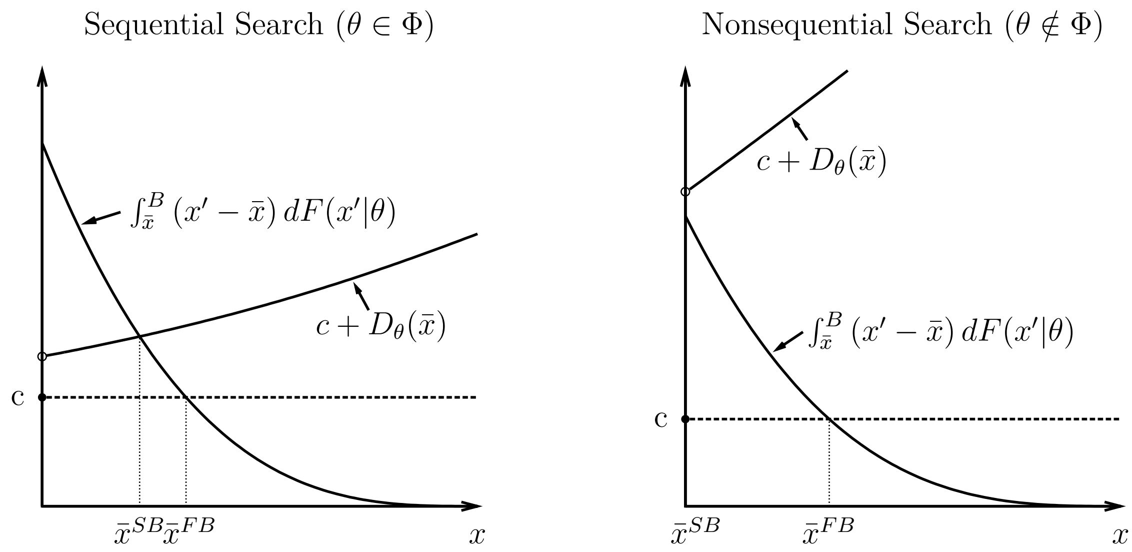

Figure 1: Second-best search.

agency rents paid in θ), or because it is sufficiently unlikely to be in state θ, such that it is not worth the increase in rents in more likely states from an *ex ante* perspective. Because the agent is equally good at securing $x > 0$ in all states, there are no rents from deviating to $\bar{x} = 0$ that have to be compensated. Together with monotonicity of $\bar{x}$ this implies $D_\theta(0) = 0$ for all θ. Nonsequential search is therefore preferred over no search whenever implementing $\bar{x} > 0$ is too costly.

A sufficient condition for $\theta \in \Phi$ is $c + D_\theta(\bar{x}) < \mathbb{E}(x|\theta)$ for a marginal $\bar{x}$:

$$c + \lim_{x \searrow 0} D_\theta(x) < \int x \,\mathrm{d}F(x|\theta). \tag{7}$$

Using that agency rents $U(\theta')$ are increasing in $x(\theta)$ for all $\theta' > \theta$, the condition can be shown to be also necessary.

Proposition 8. *Suppose Assumptions A1–A6 hold. Then $\theta \in \Phi$ if and only if θ fulfills condition* (7)*.*

In particular, since $\bar{x}(\theta)$ is increasing, it follows that Φ has the following "monotonicity" property.

Corollary 3. *Let $\theta'' > \theta'$. Then it holds that (i) if $\theta' \in \Phi$, then $\theta'' \in \Phi$; and (ii) if $\theta'' \notin \Phi$, then $\theta' \notin \Phi$.*

For an example, consider the case where θ is uniform on $[\frac{1}{10}, 4]$ and $\bar{F}(x|\theta) = (1-x)^{1/\theta}$. Figure 2 displays the optimal search policies as a function of θ. The example

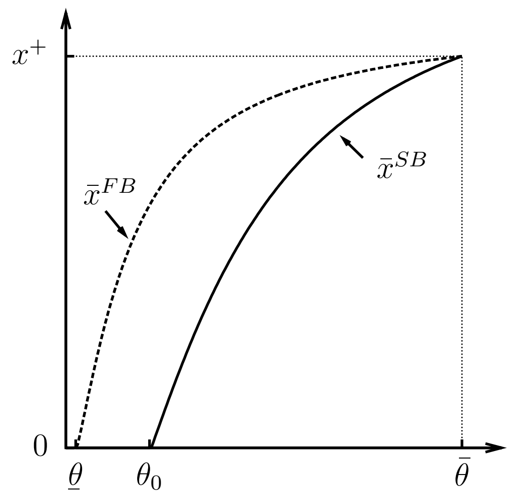

Figure 2: Optimal search policies.

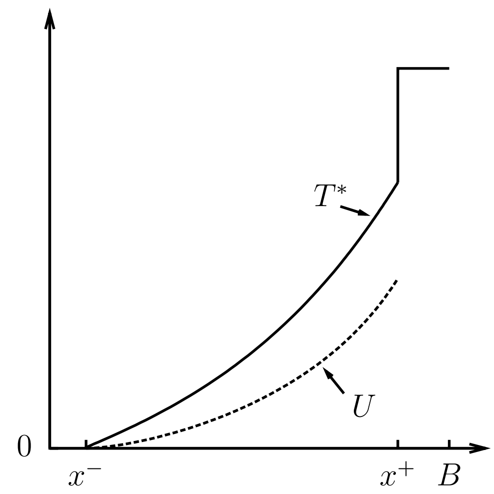

Figure 3: Indirect tariff.

is chosen so that $\underline{\theta}$ is the lowest value of θ for which search is (first-best) profitable ($\bar{x}^{FB}(\underline{\theta}) \approx 0$). In state $\bar{\theta}$, delegated search is efficient (indicated by $x^+ \approx 0.81$). For all $\theta < \bar{\theta}$, second-best search stops too early compared to the first best, and is nonsequential on $[\underline{\theta}, \theta_0]$, where $\theta_0 \equiv \inf \Phi$.

4.4 Indirect tariffs

The second-best bonus scheme is arguably a particular simple scheme among the class of direct mechanisms. An interesting question is, whether there also exists a simple indirect mechanism that implements the second best. In particular, does there exists a nonlinear (possibly discontinuous) tariff $T^* : X \to \mathbb{R}$ that implements the second best?

The answer is no. In contrast to pure adverse selection problems, any tariff T^* that implements the optimal search policies $\bar{x}^*$ is strictly more costly then the direct bonus scheme.

Proposition 9. *Suppose Assumptions A1–A6 hold. Let T^* define the tariff that implements the second-best search policies $\bar{x}^*$ at lowest expected costs. Then T^* exists, and expected transfers from the principal to the agent are strictly higher than $\tau(\theta)$ for all $\theta > \inf \Phi$.*

Underlying this inefficiency result is that the implementability conditions $(SP^-_{\theta,\tilde{\theta}})$ and $(SP^+_{\theta,\tilde{\theta}})$ deplete most degrees of freedom in designing T^*. Specifically, T^* must satisfy the following integral equation:

$$\bar{F}(x|q(x))T^*(x) + c = \int_x^B T^*(y)\,\mathrm{d}F(y|q(x)) \quad \text{for all } x \in (x^-, x^+), \tag{8}$$

where $x^- \equiv \bar{x}(\inf \Phi)$, $x^+ \equiv \bar{x}(\bar{\theta})$, and $q(x) \equiv \bar{x}^{-1}(x)$ (for details, see the proof in the appendix). This leaves only the shape of T^* on $[x^+, B]$ as a means to replicate the second-best compensation scheme. It turns out that this degrees of freedom do not suffice to fulfill $(IC_{\theta,\tilde{\theta}})$ and (IR_θ) at the (expected) second-best costs.

To build an intuition, note that $T(\bar{x}(\theta))$ defines the indirect utility of the agent in state θ, since for any $x = \bar{x}(\theta)$ she must be indifferent whether or not to continue searching. The key insight is that any solution to (8) is necessarily steeper than $U(q(x))$ (see Figure 3 for an illustration). Hence, satisfying individual rationality in state $\inf \Phi$ necessarily increases the rents in all other states beyond their second-best level. The reason is closely related to the optimality of bonus contracts. Because T^* must be strictly increasing on $[x^-, x^+]$ in order to implement the (continuous) mapping $\theta \mapsto \bar{x}^*(\theta)$, the agent can generate the deviation premium that bonus contracts had eliminated by choosing $\bar{x}(\theta')$ in state $\theta > \theta'$. In order to nevertheless implement $\bar{x}^*$, benefits of continued search have to compensate this premium, reflected in the steeper slope of T^* (defining the agent's utility under T^*) compared to U.

5 Summary

I have studied a model of delegated search under varying assumptions about what can be observed by the principal. If the principal can observe either the search process or shares the same information as the agent regarding its prospects, then delegated search is demonstrated to be efficient. If, however, the relation between the principal and the agent is governed by both imperfect monitoring of search and *ex ante* uncertainty about its prospects, these sources of uncertainty exacerbate each other, and search is found to be stopped too early. In the presence of this inefficiency, utilizing a menu of bonus contracts is shown to be second-best. The scheme strictly dominates any nonlinear (indirect) tariff.

Of course, the precise configuration of compensation schemes is often driven by more complex considerations than captured by (any) simple model. Nevertheless it may be worth to examine how the second-best optimal remuneration fits with some of the examples given in the introduction. Specifically, remuneration for recruiting agencies and real estate agents often takes the form of linear commission fees.[22] In light of the preceding analysis, a possible interpretation is that search in these professions is primarily targeted towards some non-monetary criterion (e.g., finding a qualified employee, a "nice" house, or a calm and responsible tenant). When search is hence conducted within a particular price-segment (or, similarly, when prices are fixed in advance as it is common for salaries and rents), then any initial consultation on that price-segment essentially amounts to an indirect mechanism where the agent announces a state and is assigned a bonus scheme.[23] In this sense, it turns out that remuneration in these industries can be indeed interpreted to be broadly in line with the optimal schemes found above.

Beyond the specific context of search agencies, the analysis may also illuminate the delegation of certain non-routine problems. Solving non-routine problems often requires investigating potential solutions that in the process may turn out unsatisfactory and require further thinking until a sufficiently promising solution strategy is conceived. Delegating such tasks resembles many aspects of the environment considered in this paper. For instance, managers are expected to come up with good business plans, consultants are hired to search for solutions to pending problems, and advocates need to find good strategies to defend their clients.

With such a more general interpretation of the model in mind, the optimal utilization of bonus contracts may further help explaining the widespread usage of such bonus schemes whenever moral hazard and adverse selection are jointly relevant. Regarding the adopted notion of moral hazard, I conjecture that searching for solutions in non-routine tasks is often intrinsically unobservable, in particular when the search is of cognitive nature. Adverse selection regarding the optimal search policy, in contrast, is expected to increase in relevance with the expertise of the agent. I therefore suspect bonus contracts to be particularly relevant when tasks are both non-routine and require specialized skills. Similarly, for less specialized tasks the experience of tenured agents may serve as an alternative source for prior information asymmetries.

[22] Contracts with recruiting agencies (in particular, contingency and retained recruiters) typically pay a flat fee upon completion or a percentage of the first year's salary. The following discussion focuses on the latter case; the former being clearly in line with bonus schemes.

[23] While indirect tariffs are shown to be strictly dominated, it is worth noticing that this does not rule out other kinds of indirect mechanisms.

A Mathematical Appendix

A.1 Proof of Lemma 1

Let $V(x|\theta)$ denote the indirect utility of the agent in state θ after sampling x. Then:

$$V(x|\theta) = \max\left\{T_{\tilde{\theta}}(x),\, -c + \int V(x'|\theta)\,\mathrm{d}F(x'|\theta)\right\}, \tag{9}$$

whereas outcome x is selected as final whenever the associated transfer $T_{\tilde{\theta}}(x)$ exceeds the expected utility from continuing search. Since this expected utility is independent of x, it holds that the agent selects outcome x whenever $T_{\tilde{\theta}}(x) > \bar{T}_{\tilde{\theta}}(\theta)$, where $\bar{T}_{\tilde{\theta}}(\theta) = -c + \int V(x'|\theta)\,\mathrm{d}F(x'|\theta)$. Let $\psi_{\tilde{\theta}} : X \times \mathbb{R} \to \{0,1\}$ be an indicator, such that

$$\psi_{\tilde{\theta}}(x, \bar{T}_{\tilde{\theta}}(\theta)) = \begin{cases} 0 & \text{if } T_{\tilde{\theta}}(x) \leq \bar{T}_{\tilde{\theta}}(\theta)\text{ , and} \\ 1 & \text{if } T_{\tilde{\theta}}(x) > \bar{T}_{\tilde{\theta}}(\theta)\,. \end{cases} \tag{10}$$

Then, using (9), I can rewrite $\bar{T}_{\tilde{\theta}}(\theta)$ as

$$\bar{T}_{\tilde{\theta}}(\theta) = -c + \int \Big(\big(1 - \psi_{\tilde{\theta}}(x', \bar{T}_{\tilde{\theta}}(\theta))\big)\,\bar{T}_{\tilde{\theta}}(\theta) + \psi_{\tilde{\theta}}(x', \bar{T}_{\tilde{\theta}}(\theta))\,T_{\tilde{\theta}}(x')\Big)\,\mathrm{d}F(x'|\theta), \tag{11}$$

or

$$\begin{aligned} \bar{T}_{\tilde{\theta}}(\theta)\left(1 - \int \big(1 - \psi_{\tilde{\theta}}(x', \bar{T}_{\tilde{\theta}}(\theta))\big)\,\mathrm{d}F(x'|\theta)\right) \\ = -c + \int \psi_{\tilde{\theta}}(x', \bar{T}_{\tilde{\theta}}(\theta))\,T_{\tilde{\theta}}(x')\,\mathrm{d}F(x'|\theta), \end{aligned} \tag{12}$$

or

$$c = \int \psi_{\tilde{\theta}}(x', \bar{T}_{\tilde{\theta}}(\theta))\,T_{\tilde{\theta}}(x')\,\mathrm{d}F(x'|\theta) - \bar{T}_{\tilde{\theta}}(\theta)\int \psi_{\tilde{\theta}}(x', \bar{T}_{\tilde{\theta}}(\theta))\,\mathrm{d}F(x'|\theta) \tag{13}$$

$$= \int \psi_{\tilde{\theta}}(x', \bar{T}_{\tilde{\theta}}(\theta))\Big(T_{\tilde{\theta}}(x') - \bar{T}_{\tilde{\theta}}(\theta)\Big)\,\mathrm{d}F(x'|\theta). \tag{14}$$

Because any increase in $\bar{T}_{\tilde{\theta}}(\theta)$ weakly decreases $\psi_{\tilde{\theta}}(x', \bar{T}_{\tilde{\theta}}(\theta))$, the RHS of (14) is strictly decreasing in $\bar{T}_{\tilde{\theta}}(\theta)$. Hence, if there exists a solution to (14), it is unique. Moreover, the RHS of (14) is zero for $\sup_x T_{\tilde{\theta}}(x)$. Hence, a unique solution to (14) exists whenever $\int T(x)\,\mathrm{d}F(x|\theta) \geq c$. Otherwise, marginal costs of searching always exceed the marginal benefits, and the agent trivially abstains from search.

A.2 Proof of Proposition 4

From Lemma 1 it follows that if $\bar{T}_{\tilde{\theta}}(\theta)$ exists, then setting $T_{\tilde{\theta}}(\hat{x}) = \bar{T}_{\tilde{\theta}}(\theta)$ satisfies (4a) with equality. Moreover, the proof of Lemma 1 implies that whenever the agent does not abstain from search, she adopts an "interior" stopping rule: $T_{\tilde{\theta}}(x') \leq \bar{T}_{\tilde{\theta}}(\theta) \leq T_{\tilde{\theta}}(x'')$ for some $(x', x'') \in X^2$, $x' < x''$. Hence the stopping rule $\bar{x}(T_{\tilde{\theta}}, \theta)$ defined by (4a) and (4b) exists and is unique. It remains to be checked that search implied by $\bar{x}(T_{\tilde{\theta}}, \theta)$ coincides with search implied by Lemma 1. For $T_{\tilde{\theta}}$ strictly increasing and continuous around $T_{\tilde{\theta}} = \bar{T}_{\tilde{\theta}}(\theta)$, we have that $\bar{x}(T_{\tilde{\theta}}, \theta) = T_{\tilde{\theta}}^{-1}(\bar{T}_{\tilde{\theta}}(\theta)) \in X$ uniquely exists, and obviously coincides with the value defined by (4a) and (4b). To verify the remaining cases, suppose that $\bar{T}_{\tilde{\theta}}(\theta)$ is not attained by $T_{\tilde{\theta}}(x)$ on X. Then from Lemma 1 the unique stopping rule is given by the point of discontinuity where $\lim_{x \nearrow \bar{x}(T_{\tilde{\theta}}, \theta)} T_{\tilde{\theta}}(x) < \bar{T}_{\tilde{\theta}}(\theta)$ and $\lim_{x \searrow \bar{x}(T_{\tilde{\theta}}, \theta)} T_{\tilde{\theta}}(x) > \bar{T}_{\tilde{\theta}}(\theta)$, which is precisely the value assigned by (4a) and (4b). Finally, suppose that $\bar{T}_{\tilde{\theta}}(\theta)$ is attained on an interval $[\underline{x}, \bar{x}]$. Then from Lemma 1, the agent continues search for all $x \leq \bar{x}$ and stops search for $x > \bar{x}$.[24] Thus $\bar{x}(T_{\tilde{\theta}}, \theta) = \bar{x}$, identical to the rule given by (4a) and (4b).

A.3 Proof of Proposition 5

Consider an arbitrary menu of contracts $\{T_\theta\}_{\theta \in \Theta}$, let $u(T, \bar{x}, \theta)$ be the utility of the agent in state θ when she chooses contract T and search policy $\bar{x}$. Then utility under the intended contract is given by $U(\theta) = u(T_\theta, \bar{x}(T_\theta, \theta), \theta)$. Continuity of F in θ implies continuity of U, since otherwise at any point of discontinuity θ', $(IC_{\theta'-\epsilon, \theta'+\epsilon})$ cannot hold for both $\epsilon \searrow 0$ and $\epsilon \nearrow 0$. Moreover, $(IC_{\theta, \tilde{\theta}})$ together with Assumptions A3 and A4 trivially implies that U is nondecreasing and, hence, differentiable a.e. (given that U is bounded below by (IR_θ) and above by standard profit maximization arguments).

By sequential rationality, $u(T_{\tilde{\theta}}, \bar{x}(T_{\tilde{\theta}}, \theta), \theta) \geq u(T_{\tilde{\theta}}, \bar{x}(T_{\tilde{\theta}}, \tilde{\theta}), \theta)$, and therefore a necessary condition for $(IC_{\theta, \tilde{\theta}})$ to hold is that

$$U(\theta) = u(T_\theta, \bar{x}(T_\theta, \theta), \theta) \geq u(T_{\tilde{\theta}}, \bar{x}(T_{\tilde{\theta}}, \tilde{\theta}), \theta) \quad \text{for all } \tilde{\theta} \in \Theta\,.$$

[24]While the agent is indifferent whether to continue search or not for all $x \in [\underline{x}, \bar{x}]$, Lemma 1 assigns a unique stopping rule due to the assumption that the agent continues search when indifferent (cf. Footnote 7).

Hence $\tilde{\theta} = \theta$ maximizes the RHS of the inequality, with $U(\theta)$ also being the value function of $\max_{\tilde{\theta}} u(T_{\tilde{\theta}}, \bar{x}(T_{\tilde{\theta}}, \tilde{\theta}), \theta)$. Milgrom and Segal's (2002, Theorem 1) version of the envelope theorem implies

$$\frac{\mathrm{d}U}{\mathrm{d}\theta} = \left\{ \frac{\partial}{\partial \theta} \left(\frac{\int_{\bar{x}(T_{\tilde{\theta}}, \tilde{\theta})}^{B} T_{\tilde{\theta}}(x') \,\mathrm{d}F(x'|\theta)}{\bar{F}(\bar{x}(T_{\tilde{\theta}}, \tilde{\theta})|\theta)} \right) - \frac{\partial}{\partial \theta} \left(\frac{c}{\bar{F}(\bar{x}(T_{\tilde{\theta}}, \tilde{\theta})|\theta)} \right) \right\} \Bigg|_{\tilde{\theta}=\theta} \tag{15}$$

wherever $\mathrm{d}U/\mathrm{d}\theta$ exists. By Assumptions A3 and A4 the first term in (15) is positive. Hence,

$$\frac{\mathrm{d}U}{\mathrm{d}\theta} \geq -\frac{\partial}{\partial \theta} \left(\frac{c}{\bar{F}(\bar{x}(T_{\theta}, \theta)|\theta)} \right) \quad \text{where it exists.} \tag{16}$$

By continuity and differentiability a.e. of U, it can be represented as an integral of its derivative. Hence,

$$U(\theta) \geq \int_{\underline{\theta}}^{\theta} -\frac{\partial}{\partial \tilde{\theta}} \left(\frac{c}{\bar{F}(\bar{x}(T_{\tilde{\theta}}, \tilde{\theta})|\tilde{\theta})} \right) \mathrm{d}\tilde{\theta} + U(\underline{\theta}), \tag{17}$$

where $U(\underline{\theta}) \geq 0$ by (IR_{θ}).

A.4 Proof of Proposition 6

Fix some menu of intended stopping rules $\bar{x}$, and consider a menu of bonus $\{T_{\theta}\}_{\theta \in \Theta}$ contracts with $\tau(\theta) = [\bar{F}(\bar{x}(\theta)|\theta)]^{-1} c + \underline{U}(\theta, \bar{x})$. By construction,

$$U(\theta) = \tau(\theta) - [\bar{F}(\bar{x}(\theta)|\theta)]^{-1} c = \underline{U}(\theta, x). \tag{18}$$

Hence, to prove the claim, I have to show that $\{T_{\theta}\}_{\theta \in \Theta}$ implements $\bar{x}$, or (equivalently) that $\{T_{\theta}\}_{\theta \in \Theta}$ is consistent with (IR_{θ}), $(IC_{\theta,\tilde{\theta}})$, $(SP^{-}_{\theta,\tilde{\theta}})$, and $(SP^{+}_{\theta,\tilde{\theta}})$.

Clearly, individual rationality holds since $\underline{U}(\theta, \bar{x}) \geq 0$ for all $\bar{x}$ and θ.

Consider the moral hazard constraints next. From Proposition 4 it follows that an agent with bonus contract $T_{\tilde{\theta}} = (\bar{x}(\tilde{\theta}), \tau(\tilde{\theta}))$ chooses a stopping rule $\bar{x}(T_{\tilde{\theta}}, \theta) = \bar{x}(\tilde{\theta})$ if $\tau(\tilde{\theta}) \geq [\bar{F}(\bar{x}(\tilde{\theta})|\theta)]^{-1} c$, and does not search at all otherwise. Letting $\tilde{\theta} = \theta$, it follows that the adopted search policy $\bar{x}(T_{\theta}, \theta)$ under the intended contract indeed equals the intended stopping rule $\bar{x}(\theta)$ for all $\theta \in \Theta$.

Finally, to show consistency with $(IC_{\theta,\tilde{\theta}})$, let

$$u(\theta,\tilde{\theta}) \equiv \max\{0, -[\bar{F}(\bar{x}(\tilde{\theta})|\theta)]^{-1}c + \tau(\tilde{\theta})\} \tag{19}$$

denote the agent's indirect utility in state θ when she chooses bonus contract $T_{\tilde{\theta}} = (\bar{x}(\tilde{\theta}), \tau(\tilde{\theta}))$. Since $U(\theta) \geq 0$, $(IC_{\theta,\tilde{\theta}})$ clearly holds whenever $u(\theta,\tilde{\theta}) = 0$. To prevent any deviation by the agent, it hence suffices to show that for all $\theta \in \Theta$,

$$-[\bar{F}(\bar{x}(\tilde{\theta})|\theta)]^{-1}c + \tau(\tilde{\theta}) \tag{20}$$

is maximized by $\tilde{\theta} = \theta$.

Consider local deviations first. For the agent in state θ to not locally deviate, it is sufficient that the first order condition to (20),

$$-H(\bar{x}(\tilde{\theta})|\theta)\frac{\mathrm{d}\bar{x}(\tilde{\theta})}{\mathrm{d}\tilde{\theta}}\,c + \frac{\mathrm{d}\tau(\tilde{\theta})}{\mathrm{d}\tilde{\theta}} = 0 \quad \text{for } \tilde{\theta} = \theta, \tag{21}$$

and the corresponding second order condition,

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{\theta}}\left(-H(\bar{x}(\tilde{\theta})|\theta)\frac{\mathrm{d}\bar{x}(\tilde{\theta})}{\mathrm{d}\tilde{\theta}}\,c + \frac{\mathrm{d}\tau(\tilde{\theta})}{\mathrm{d}\tilde{\theta}}\right) \leq 0 \quad \text{for } \tilde{\theta} = \theta, \tag{22}$$

hold for all $\theta \in \Theta$. Differentiating $\tau(\tilde{\theta}) = [\bar{F}(\bar{x}(\tilde{\theta})|\tilde{\theta})]^{-1}c + \underline{U}(\tilde{\theta},\bar{x})$ with respect to $\tilde{\theta}$ yields:

$$\begin{aligned}\frac{\mathrm{d}\tau(\tilde{\theta})}{\mathrm{d}\tilde{\theta}} &= H(\bar{x}(\tilde{\theta})|\theta)\frac{\mathrm{d}\bar{x}(\tilde{\theta})}{\mathrm{d}\tilde{\theta}}\,c + \frac{\partial}{\partial\tilde{\theta}}\left(\frac{c}{\bar{F}(\bar{x}(\tilde{\theta})|\tilde{\theta})}\right) + \frac{\partial\underline{U}(\tilde{\theta},\bar{x})}{\partial\tilde{\theta}} && (23)\\ &= H(\bar{x}(\tilde{\theta})|\theta)\frac{\mathrm{d}\bar{x}(\tilde{\theta})}{\mathrm{d}\tilde{\theta}}\,c, && (24)\end{aligned}$$

so that (21) holds for all $\theta \in \Theta$. Moreover, since we have just shown that (21) holds for all $\theta \in \Theta$, it is an identity in θ, and thus

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{\theta}}\left(-H(\bar{x}(\tilde{\theta})|\theta)\frac{\mathrm{d}\bar{x}(\tilde{\theta})}{\mathrm{d}\tilde{\theta}}\,c + \frac{\mathrm{d}\tau(\tilde{\theta})}{\mathrm{d}\tilde{\theta}}\right) - \frac{\mathrm{d}}{\mathrm{d}\theta}\left(H(\bar{x}(\tilde{\theta})|\theta)\frac{\mathrm{d}\bar{x}(\tilde{\theta})}{\mathrm{d}\tilde{\theta}}\,c\right) = 0 \quad \text{for } \tilde{\theta} = \theta. \tag{25}$$

Assumption A4 implies that the second term (including the minus sign) is nonnegative if $\mathrm{d}\bar{x}/\mathrm{d}\theta \geq 0$. Hence, a sufficient condition for the first term (and, hence, for (22)) to be nonpositive is that $\bar{x}$ is nondecreasing. Hence, the second order condition holds for all θ under the assumptions of the proposition.

To conclude the proof, I argue that (21) is also sufficient to prevent the agent from deviating globally. Suppose to the contrary that for some $\theta \in \Theta$, $\tilde{\theta} = \theta$ does not maximize (20), i.e. $u(\theta, \tilde{\theta}) - u(\theta, \theta) > 0$ for some $(\theta, \tilde{\theta}) \in \Theta^2$, or by the fundamental theorem of calculus,

$$\int_{\theta}^{\tilde{\theta}} \left(-H(\bar{x}(\theta')|\theta) \frac{\mathrm{d}\bar{x}(\theta')}{\mathrm{d}\theta'} c + \frac{\mathrm{d}\tau(\theta')}{\mathrm{d}\theta'} \right) \mathrm{d}\theta' > 0. \tag{26}$$

Suppose $\tilde{\theta} > \theta$. Then, Assumption A4 implies that $H(\bar{x}(\tilde{\theta})|\tilde{\theta}) \leq H(\bar{x}(\tilde{\theta})|\theta)$, and therefore (26) implies

$$\int_{\theta}^{\tilde{\theta}} \left(-H(\bar{x}(\theta')|\theta') \frac{\mathrm{d}\bar{x}(\theta')}{\mathrm{d}\theta'} c + \frac{\mathrm{d}\tau(\theta')}{\mathrm{d}\theta'} \right) \mathrm{d}\theta' > 0 \tag{27}$$

since $\mathrm{d}\bar{x}/\mathrm{d}\theta \geq 0$. However, equation (21) implies that the integrand in (27) is equal to 0 for all θ', contradicting that for any $\tilde{\theta} > \theta$, contract $T_{\tilde{\theta}}$ is preferred over T_θ. The same logic establishes a contradiction for the case where $\tilde{\theta} < \theta$.

A.5 Proof of Proposition 7

From Corollary 1, the principal's objective function is

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(\int_{\bar{x}(\theta)}^{B} \frac{x'}{\bar{F}(\bar{x}(\theta)|\theta)} dF(x'|\theta) - \frac{c}{\bar{F}(\bar{x}(\theta)|\theta)} + \int_{\underline{\theta}}^{\theta} \frac{\partial}{\partial \tilde{\theta}} \left(\frac{c}{\bar{F}(\bar{x}(\tilde{\theta})|\tilde{\theta})} \right) \mathrm{d}\tilde{\theta} \right) \mathrm{d}P(\theta), \tag{28}$$

or, after an integration by parts,

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(\int_{\bar{x}(\theta)}^{B} \frac{x'}{\bar{F}(\bar{x}(\theta)|\theta)} dF(x'|\theta) - \frac{c}{\bar{F}(\bar{x}(\theta)|\theta)} + \frac{1 - P(\theta)}{p(\theta)} \frac{\partial}{\partial \theta} \left(\frac{c}{\bar{F}(\bar{x}(\theta)|\theta)} \right) \right) \mathrm{d}P(\theta). \tag{29}$$

Differentiating point-wise with respect to $\bar{x}(\theta)$, and rearranging, the first-order conditions satisfy

$$c + D_\theta(\bar{x}(\theta)) = \int_{\bar{x}(\theta)}^{B} (x' - \bar{x}(\theta))\,\mathrm{d}F(x'|\theta), \tag{30}$$

where function $D_\theta : X \to \mathbb{R}_+$ is defined by,

$$D_\theta(x) = -\frac{1-P(\theta)}{p(\theta)} \frac{\partial H(x|\theta)}{\partial \theta} \frac{c}{H(x|\theta)}\,. \tag{31}$$

Differentiating (29) twice and substituting (30), it is straightforward to see that Assumption A5 implies that (29) is concave at any $\bar{x}(\theta)$ that satisfies (30). Hence (29) is globally quasi-concave in $\bar{x}$ for all θ, so that the second-order conditions are satisfied, the maximizer $\bar{x}^*$ is unique, and $\bar{x}^*$ attains the supremum $\bar{V}$.

By Assumption A4, the RHS of (30) is increasing in θ for a given $\bar{x}$. Hence, a sufficient condition for $\bar{x}^*$ to be nondecreasing is that $D_\theta(\bar{x}(\theta))$ is nonincreasing in θ. By Assumptions A4 and A6 this is true since H is sufficiently convex and $p/(1-P)$ is increasing in θ.[25]

Because $x \in [0, B]$, the solution can be "truncated" without loss of generality, whenever $\bar{x}^*(\theta) < 0$ or $\bar{x}^*(\theta) > B$. Because for $\bar{x}(\theta) = B$ benefits of search (the RHS of (30)) are equal to 0, corner solutions may at most be given by $\bar{x}(\theta) = 0$. This is the case whenever the left-hand side of (30) exceeds the right-hand side for all values of $\bar{x}(\theta) \in [0, B]$.

So far, I ignored the possibility of the principal implementing no search at all for some $\theta \in \Theta$. From quasi-concavity of (29) and given that $\bar{x}^*$ is increasing, it suffices to inspect $\bar{x}(\theta) = 0$ to determine whether this might be the case. Note that the last term in (29) drops out for $\bar{x} = 0$ since $\bar{F}(0|\theta) = 1$ for all θ. Hence, for $\bar{x} = 0$ the problem collapses to the first-best problem where per assumption $\mathbb{E}(x|\theta) \geq c$ for all $\theta \in \Theta$, so that "nonsequential" search is always preferred over no search.

[25] Alternatively, $\bar{x}$ would also be increasing whenever $p/(1-P)$ is increasing at a sufficiently high rate, or when the RHS of (30) is sufficiently increasing in θ (e.g., it would be sufficient that the marginal benefit of search is increasing in θ; i.e., $\frac{\partial^2}{\partial \bar{x} \partial \theta}\mathbb{E}\{x|x \geq \bar{x}, \theta\} \geq 0$). In either case, the second part of Assumption A4 could be relaxed accordingly as outlined in Footnote 15.

A.6 Proof of Proposition 8

From Proposition 7 marginal costs of searching are given by $c + D_\theta(\bar{x})$. Differentiating with respect to $\bar{x}$ yields

$$-\frac{1-P}{P}\left(\frac{\partial^2 H}{\partial x \partial \theta}\frac{1}{H} - \frac{\partial H}{\partial \theta}\frac{\partial H}{\partial x}\frac{1}{H^2}\right) c \geq 0\,, \tag{32}$$

by Assumptions A4 and A5. Moreover, marginal benefits are trivially decreasing in $\bar{x}$. Thus a necessary and sufficient condition for $\bar{x}(\theta) > 0$ is that for $\hat{x} \searrow 0$ sequential search is beneficial:

$$\lim_{\hat{x}\searrow 0}\left\{\int_{\hat{x}}^{B}(x'-\hat{x})\,\mathrm{d}F(x'|\theta) - c - D_\theta(\hat{x})\right\} > 0\,, \tag{33}$$

or

$$c + \lim_{x\searrow 0} D_\theta(x) < \int x\,\mathrm{d}F(x|\theta). \tag{34}$$

A.7 Proof of Proposition 9

Existence Consider existence of T^* first. By continuity of $\bar{x}$, a necessary and sufficient condition for T^* to implement $\{\bar{x}(\theta)\}$ is that the equivalent to $(SP^{\pm}_{\theta,\theta})$ holds with equality for all θ in $(\theta_0, \bar{\theta})$, where $\theta_0 \equiv \inf \Phi$, or equivalently:

$$c = \int_x^B (T(y) - T(x))\,\mathrm{d}F(y|q(x)), \tag{35}$$

or

$$T(x) = -\frac{c}{\bar{F}(x|q(x)} + \frac{1}{\bar{F}(x|q(x))}\int_x^B T(y)\,\mathrm{d}F(y|q(x)), \tag{36}$$

for all $x \in (x^-, x^+)$, $x^- \equiv \bar{x}(\theta_0)$, $x^+ \equiv \bar{x}(\bar{\theta})$, and where $q(x) \equiv \bar{x}^{-1}(x)$.[26] Separate T^* into T^- defined on $[0, x^+]$ and T^+ defined on $[x^+, B]$, and fix some T^+. Then T^- is

[26] Here I assume without loss of generality that $\partial H/\partial\theta < 0$, so that $\bar{x}$ is invertible. If it were not, I could simply define a new variable that treats all instances of θ where H is constant as a single state and carry out the following analysis with respect to that variable.

given by the functional $\mathcal{T}$,

$$(\mathcal{T}f)(x) = g(x) + \frac{1}{\bar{F}(x|q(x))} \int_x^{x^+} f(y)\,\mathrm{d}F(y|q(x)), \tag{37}$$

where

$$g(x) = \frac{1}{\bar{F}(x|q(x))} \left(-c + \int_{x^+}^{B} T^+(y)\,\mathrm{d}F(y|q(x)) \right). \tag{38}$$

Inspecting $\mathcal{T}$, it is clearly increasing in f. Moreover, for any constant k,

$$\mathcal{T}(f+k)(x) = (Tf)(x) + \frac{F(x^+|q(x)) - F(x|q(x))}{1 - F(x|q(x))}\,k.$$

By (1), $x^+ \in (0,1)$, such that the term multiplying k is in $(0,1)$. Hence, $\mathcal{T}$ satisfies Blackwell's sufficient conditions to be a contraction, establishing existence of a unique T^- for a given T^+.

Having taken care of implementing $\{\bar{x}(\theta)\}$, the only other constraints to address are individual rationality. Since $\mathcal{T}$ is increasing in T^+, individual rationality can be guaranteed by setting T^+ accordingly. We conclude that there exists a T^* implementing the second best search policies and leave it to the reader to formally establish the shape of T^+ that defines the cost-minimizing tariff. (The answer is: $T^+(x) = \mathit{const}$ for all $x \in [x^+, B]$, where *const* is set such that $T^-(x^-) = 0$.)

Inefficiency Suppose there exists a tariff T that implements $\{\bar{x}(\theta)\}$ at the same costs as in the second best in all states $\theta \in \Theta$. Since both parties are risk neutral, this implies that $U(\theta)$ corresponds to the second-best rents for all θ. Contradicting the existence of such a T, I first show that T necessarily violates individual rationality for $\theta \to \theta_0$. Subsequently I then argue that restoring individual rationality for θ_0 requires increasing rents for all $\theta \in \Phi$ above their second-best level.

Let $U(\theta)$ denote the second-best rents as given by Propositions 5 and 6, and suppose that T implements $U(\theta)$ for all θ. Then from (36), $T(x) = U(q(x))$. Hence, implementability requires

$$\bar{F}(x|q(x))\tilde{U}(q(x)) = \\ -c + \int_{x^+}^{B} T(y)\,\mathrm{d}F(y|q(x)) + \int_{x}^{x^+} \int_{\underline{\theta}}^{q(x)} m(\tilde{\theta})\,\mathrm{d}\tilde{\theta}\,\mathrm{d}F(y|q(x)), \quad (39)$$

for all $x \in (x^-, x^+)$, where $\tilde{U}(\theta)$ is the actual utility implemented by T, and

$$m(\theta) = -\frac{\partial}{\partial\theta}\left(\frac{c}{\bar{F}(\bar{x}(\theta)|\theta}\right). \quad (40)$$

After an integration by parts, a change in variables, and a collecting of terms, (39) becomes

$$\bar{F}(x|q(x))\tilde{U}(q(x)) = \\ -c + \int_{x^+}^{B} T(y)\,\mathrm{d}F(y|q(x)) - F(x|q(x)) \int_{\underline{\theta}}^{q(x)} m(\tilde{\theta})\,\mathrm{d}\tilde{\theta} \\ + F(x^+|q(x)) \int_{\underline{\theta}}^{\bar{\theta}} m(\tilde{\theta})\,\mathrm{d}\tilde{\theta} - \int_{q(x)}^{\bar{\theta}} m(\tilde{\theta}) F(\bar{x}(\tilde{\theta})|q(x))\,\mathrm{d}\tilde{\theta}, \quad (41)$$

or, after another change in variables, and substituting again for $U(\theta)$,

$$\bar{F}(\bar{x}(\theta)|\theta)\tilde{U}(\theta) = -c + \int_{x^+}^{B} T(y)\,\mathrm{d}F(y|\theta) - F(\bar{x}(\theta)|\theta)U(\theta) \\ + F(x^+|\theta)U(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} m(\tilde{\theta}) F(\bar{x}(\tilde{\theta})|\theta)\,\mathrm{d}\tilde{\theta}. \quad (42)$$

Consider the first two terms on the right-hand side. By Assumption A4, $\tilde{U}(\theta)$ is maximized for all $\theta < \bar{\theta}$, subject to $U(\bar{\theta})$ being fixed, by setting T^+ constant. Hence,

$$\int_{x^+}^{B} T(y)\,\mathrm{d}F(y|\theta) - c \le \frac{\bar{F}(x^+|\theta)}{\bar{F}(x^+|\bar{\theta})} \int_{x^+}^{B} T(y)\,\mathrm{d}F(y|\bar{\theta}) - c \;= \\ \bar{F}(x^+|\theta)\left(U(\bar{\theta}) + \frac{c}{\bar{F}(x^+|\bar{\theta})}\right) - c. \quad (43)$$

Substituting in (42) yields

$$\begin{aligned}
&\bar{F}(\bar{x}(\theta)|\theta)\tilde{U}(\theta) \leq \\
&-F(\bar{x}(\theta)|\theta)U(\theta) + U(\bar{\theta}) - c\left(1 - \frac{\bar{F}(x^+|\theta)}{\bar{F}(x^+|\bar{\theta})}\right) - \int_{\theta}^{\bar{\theta}} m(\tilde{\theta})F(\bar{x}(\tilde{\theta})|\theta)\,\mathrm{d}\tilde{\theta} = \\
&\qquad \bar{F}(\bar{x}(\theta)|\theta)U(\theta) - c\left(1 - \frac{\bar{F}(x^+|\theta)}{\bar{F}(x^+|\bar{\theta})}\right) + \int_{\theta}^{\bar{\theta}} m(\tilde{\theta})\bar{F}(\bar{x}(\tilde{\theta})|\theta)\,\mathrm{d}\tilde{\theta}.
\end{aligned} \tag{44}$$

By Assumption A4, $\partial H(x|\tilde{\theta})/\partial\tilde{\theta} = \partial^2[\bar{F}(x|\tilde{\theta})]^{-1}/\partial x\partial\tilde{\theta} \leq 0$. Moreover, $\bar{F}(x|\theta)$ is also (strictly) decreasing in x. Hence, the last term on the right-hand side satisfies

$$-c\int_{\theta}^{\bar{\theta}} \frac{\partial}{\partial\tilde{\theta}}\left(\frac{\bar{F}(\bar{x}(\tilde{\theta})|\theta)}{\bar{F}(\bar{x}(\tilde{\theta})|\tilde{\theta})}\right)\mathrm{d}\tilde{\theta} < -c\int_{\theta}^{\bar{\theta}} \frac{\partial}{\partial\tilde{\theta}}\left(\frac{\bar{F}(x^+|\theta)}{\bar{F}(x^+|\tilde{\theta})}\right)\mathrm{d}\tilde{\theta} = -c\left[\frac{\bar{F}(x^+|\theta)}{\bar{F}(x^+|\tilde{\theta})}\right]_{\tilde{\theta}=\theta}^{\bar{\theta}}. \tag{45}$$

Substituting in (44), the last two terms on the right-hand side cancel out. Hence,

$$\tilde{U}(\theta) < U(\theta) \quad \text{for all } \theta \in (\theta_0, \bar{\theta}), \tag{46}$$

contradicting $\tilde{U}(\theta) = U(\theta)$. In particular, it holds that for $\theta \to \theta_0$, $\tilde{U}(\theta_0) < 0$ since $U(\theta_0) = 0$. From the existence proof above, it is clear that the only degree of freedom to restore individual rationality under the constraint of implementability consists in raising T^+. Under Assumption A4 any change $\mathrm{d}T^+$ implies $\mathrm{d}U(\theta) > \mathrm{d}U(\theta_0)$ for all $\theta > \theta_0$. Moreover, since $\bar{x}$ is increasing in θ, the difference between the left-hand side to the right-hand side in (45) is decreasing in θ. Hence, any T^+ that sets $\tilde{U}(\theta_0) = 0$ necessarily sets $\tilde{U}(\theta) > U(\theta)$ for all $\theta > \theta_0$.

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