Carbon Sequestration, Economic Policies and Growth

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Abstract

The possibility of capturing and sequestering some fraction of the CO_2 emissions arising from fossil fuel combustion, often labeled as carbon capture and storage (CCS), is drawing an increasing amount of attention in the business and academic communities. We present here a model of endogenous growth in which the use of a non-renewable resource in production yields flows of pollution whose accumulated stock negatively affects welfare. A CCS technology allows, via some effort, for the partial reduction of CO_2 emissions in the atmosphere.

We characterize the social optimum and how the availability of the CCS technology affects it, and we study the decentralized economy's trajectories. We then analyze economic policies. We first characterize the first-best policy. We derive the expression of the Pigovian carbon tax, and we give a full interpretation of its level, which is unique. We then study the impacts of three different second-best policies: a carbon tax, a subsidy to sequestered carbon, and a subsidy to labor in CCS. The first two tools foster CCS activity; so does the third, but only if it is coupled with one of the other two. While the tax postpones resource extraction, the two subsidies accelerate it – possibly yielding a rise in short-term CO_2 emissions. The effects on growth are more complex. If the weight of the CCS sector in the economy is high, the tax will generally be detrimental to output growth, while the subsidies can foster it in the long-term. Finally, the carbon tax has a negative impact on the output level in the short-term, contrary to the subsidies.

Keywords: carbon capture and storage (CCS), endogenous growth, polluting non-renewable resources, carbon tax, subsidy to CCS.

JEL classification: O3, Q3

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1 Introduction

The exploitation of fossil resources raises two concerns. The first one is scarcity, because fossil resources are exhaustible by nature. The second one is related to the greenhouse gases (GHG) emissions associated with their combustion. Numerous models deal with this double issue; some of them in the context of partial equilibrium (e.g. Sinclair, 1992; Withagen, 1994; Ulph and Ulph, 1994; Hoel and Kverndokk, 1996; Farzin and Tahvonen, 1996; or Tahvonen, 1997) and others within general equilibrium growth frameworks (Stollery, 1998; Schou, 2000, 2002; Groth and Schou, 2007; or Grimaud and Rouge, 2008). A common feature of those papers lies in the fact that reducing carbon emissions necessarily means extracting less resource. Indeed, a systematic link between resource extraction and polluting emissions, in the form of a simple functional relation (e.g. linear), is generally made. In terms of economic policy, it is therefore equivalent to tax either the pollution stream or the resource use itself. Nevertheless, it is now well known that abatement technologies that allow the reduction of emissions for a given amount of extracted resource exist. In particular, the possibility of capturing and sequestering some fraction of the carbon embedded in fossil fuels, whether this capture occurs pre- or post-combustion, has recently caught a lot of attention. This has been reinforced by its recent demonstrated viability (for an overview, see IPCC special report, 2005). This process, often referred to as carbon capture and storage or carbon capture and sequestration (CCS), consists in separating carbon from hydrogen in the pre-combustion process or in separating carbon dioxide from other flux gases in the post-combustion process in an energy production plant. Once captured, the CO_2 is injected into a reservoir[1] for long-term storage. So, the availability of CCS technologies means that the simple relation between resource extraction and carbon emissions is partially broken.

Here we consider the availability of such an abatement technology in the context of a theoretical general equilibrium model with endogenous growth and a polluting exhaustible resource. We study how the socially optimal trajectories of the economy are modified by the availability of the CCS option, and how the first-best outcome can be restored in the decentralized economy. We also study the impact of three different second-best policies: a carbon tax, a subsidy to sequestered carbon and a subsidy to labor in the CCS activity. Endogenous growth allows us in particular to analyze the effects of the availability of the CCS technology and the economic policy tools on growth, along the transition path and at the steady-state.

[1] The sequestration reservoirs include depleted oil and gas fields, depleted coal mines, or deep saline aquifers. Those various deposits differ in their respective capacities, their costs of access or their effectiveness in storing the carbon permanently.

Numerous uncertainties still surround the sizable deployment of carbon capture technologies, especially with regard to the ecological consequences of massive carbon injection. The social acceptance of this abatement technique is also uncertain - for a survey on these issues, see for instance Jepma and Hauck (2011). Nevertheless, this technological option has become promising for the fossil energy extractive industry. For instance, Grimaud et al. (2011) show in an empirical model that, insofar as the right climate policy is implemented - that is, a carbon tax in their model -, the percentage of carbon sequestered can exceed 50%.

We develop a Romer-type endogenous growth model in which the production of final good requires the input of an extracted resource, whose stock is available in limited quantities. This resource use generates polluting emissions, which we take to be CO_2 emissions, whose flow in turn adds to the pre-existing stock of the pollutant - which features partial natural decay. Finally, this stock enters the utility function as an argument and thus allows gauging how pollution accumulation negatively affects welfare. We then consider that a CCS technology is available. Via some effort, it allows for the partial reduction of the level of CO_2 release. We thus distinguish between the total potential CO_2 emission associated to one unit of fossil resource (henceforth referred to as total carbon content per unit of resource) and the effective emissions, i.e. the fraction that remains after CO_2 removal. Note that we do not account for geological CO_2 leakage - on this issue, see for instance van der Zwaan and Gerlagh (2009). The implication in terms of climate change policy is then straightforward: the first best outcome can only be restored by taxing pollution, i.e., emissions remaining after sequestration, and not by taxing the resource itself[2]. However, for various reasons, it is likely that the tax cannot be set at its Pigovian level in the real world. Hence, we study second-best policies: a second-best tax on effective carbon emissions, a subsidy to sequestered carbon, and a direct subsidy to labor used in CCS activity. In this second-best world, such complementary policies can be Pareto-improving. This analysis constitues the main contribution of our paper. We show that it is important to understand how these policies affect the time profile of the total price paid by resource users. This time profile determines the resource extraction path, and hence, indirectly, the path of CCS activity, carbon emissions, R&D and output.

We first depict the socially optimal trajectories of the economy, and we study how such trajectories are affected by the availability of the CCS technology. Then we fully characterize the trajectories of the decentralized economy, and we derive the expression of the Pigovian levels

[2] Here we assume that the regulator is able to fully measure the greenhouse gases emissions. This may not be systematically the case: while emission data is fairly reliable in industrialized countries, collecting accurate data on industrial activities from developing regions and deducting the emissions may prove more difficult.

and growth rates of the economic policy tools, and we give a full interpretation of them. In the general case, at the social optimum as well as in the decentralized equilibrium, the economy is always in transition; we nevertheless obtain closed-form solutions. This allows us to study the impacts of the three different types of second-best economic policies.

A strand of literature tackles the question of CCS within calibrated empirical models - see for instance Edenhofer et al. (2005); Gerlagh and van der Zwaan (2006); van der Zwaan and Gerlagh (2009); Golombek et al. (2011); Grimaud et al. (2011); or Kalkuhl et al. (2012). The focus of our paper is on the theoretical side of the issue. Several authors have studied the links between carbon abatement, optimal climate policy and technical change in theoretical models. In particular, Goulder and Mathai (2000) show that the presence of induced technical change generally lowers the time profile of optimal carbon taxes. Moreover, efforts in $R\&D$ shift part of the abatement from the present to the future. In a similar framework, Gerlagh et al. (2008) study the link between innovation and abatement policies under certain assumptions, in particular, the fact that patents can have a finite lifetime. In these studies, the authors use partial equilibrium frameworks in which baseline CO_2 emissions are exogenous, and final (or effective) carbon emissions are endogenous as there is an abatement activity with dedicated technical progress. Hoel and Jensen (2010) show, in a two period model, that if the climate policy is imperfect - that is, if it can only be implemented in the second period -, cost reductions are more desirable in the CCS than in the renewable sector in particular because they postpone resource extraction.

Many recent contributions take into account the availability of a CCS technology. Most of them are placed in the context of partial equilibrium frameworks: see for instance Lafforgue et al. (2008), Narita (2009), Amigues et al. (2011) or Rickels (2011). These papers mainly focus on socially optimal issues, and in particular they study the optimal time profile of carbon sequestration. Lontzek and Rickels (2008) and Ayong le Kama et al. (2009) study the same questions, but they also consider a decentralized economy. However, they do not study the impact of economic policies on the decentralized equilibrium. Most of these papers consider a carbon ceiling; in this case, Lafforgue et al. show that CCS is implemented only when the ceiling is reached. When the CCS cost function is convex however, as in Rickels, it is optimal to sequester carbon before the ceiling. Similarly, the CCS activity has to start from the short run when there is no ceiling but a damage function, as in Ayong le Kama et al. Finally, technical progress is not explicitly considered in these studies.

Our main results can be summarized as follows[3]. At the social optimum, the greatest effort in abatement should happen today[4] - this contrasts with the result of Lafforgue et al. (2008) presented in the previous paragraph. We also show that the availability of the CCS technology modifies the socially optimal trajectories of the economy. It speeds up the optimal pace of resource extraction, as it relaxes the environmental constraint. While it diminishes polluting emissions in the long run, it fosters them in the short run when the rise in resource extraction, and thus of potential emissions, is less than proportionally compensated by the CCS activity. Lastly, the availability of such a technology is detrimental to the socially optimal growth of output as a result of the acceleration in resource extraction combined with a negative effect on R&D effort.

Due to the availability of CCS technology, the Pigovian carbon tax is unique, which contrasts with the standard result obtained in a context without abatement, as in Dasgupta and Heal (1979), Sinclair (1992), Groth and Schou (2007) or Grimaud and Rouge (2008) for instance. In these models, there are an infinity of optimal taxes which have the same dynamics, but differ in their levels. Here, the tax level matters and especially allows for setting the optimal abatement effort level. The optimal carbon tax is equal to both the sum of discounted social costs of one unit of carbon and the cost of sequestering this unit. We study its properties, and we show that it is an increasing function of time.

The second-best carbon tax fosters CCS activity and postpones resource extraction, as well as polluting emissions. Here, as polluting emissions stem from the use of non-renewable resources, if no carbon abatement technology was available, a more stringent environmental policy would generally enhance economic growth, since it leads to postponing resource extraction (see for instance Groth and Schou, 2007, or Grimaud and Rouge, 2008). When CCS technology becomes available, we show that this result is not always true: the impact of this climate policy on output growth is more complex. It depends on the relative strengths of its effects on extraction and R&D. Basically, if the weight of the CCS sector in the economy is high, the tax is likely to be detrimental to growth. However, the level of output in the short term is unambiguously reduced because of the effect on resource extraction.

The subsidy to sequestered carbon is a perfect substitute to the carbon tax with regard to its

[3] We often resort to the distinction between short and long-term. In a Hotelling world, where the whole stock of resource is asymptotically exhausted, any increase (resp. decrease) in resource extraction at date t generates changes for all subsequent dates. The short-term refers to the period during which resource extraction is also increased (resp. decreased), that is, the current period and its neighborhood (i.e, the first generations), whereas the long-term refers to the period during which resource use is consequently decreased (resp. increased), that is, the distant future.

[4] We show that this result can be slightly altered if one expects a high rate of technical progress in the CCS technology.

impact on CCS activity. However, the effects of the two policy instruments on resource use are opposite: with the subsidy, extraction is faster. Therefore, a kind of (weak) green paradox can occur here in the sense that this environmental or green policy can foster short-term emissions (on the issue of green paradox in other contexts, see e.g. Sinn, 2008, Gerlagh, 2011, or van der Ploeg and Withagen, 2012). This happens when the higher resource use overcomes the abatement impact of the CCS activity. The effects of both policies on output can also be opposite. In the short run, the subsidy is unambiguously bad for growth. In the longer term, the overall impact depends, as before, on the relative strengths of the effects on resource extraction and R&D. If the weight of the CCS sector in the economy is high, it can foster growth in the long term, contrary to the carbon tax. Moreover, we show that, unlike the tax, the subsidy prompts greater output level in the early periods. This last point should be considered when taking into account public acceptance issues. Indeed, this subsidy could be seen as a good complementary tool to a second-best carbon tax since it alleviates the burden of the climate policy in the short term.

Another result is that the subsidy to labor in CCS alone does not trigger any CCS activity. This tool has an effect only when it is used jointly with a carbon tax or a subsidy to sequestered carbon. In this case, it also stimulates CCS activity. However, its impact on the dynamics of resource extraction, carbon emissions and the level and growth of output are similar to those of the subsidy to sequestered carbon, and thus they can be opposite to the effects of the carbon tax.

The remainder of the paper is organized as follows. We present the model and we portray the social optimum in section 2. We characterize the equilibrium in the decentralized economy in section 3, and we study the first-best economic policy and the impact of the second-best policies in section 4. Finally, we conclude in section 5.

2 Model and welfare

2.1 The model

At each date $t \in [0, +\infty)$, the final output is produced using the range of available intermediate goods, labor and a flow of resource. The production function is

$$Y_t = \left(\int_0^{A_t} x_{it}^{\alpha} di \right) L_{Yt}^{\beta} R_t^{\gamma}, \ \alpha + \beta + \gamma = 1, \tag{1}$$

where x_{it} is the amount of intermediate good i, L_{Yt} the quantity of labor employed in the production sector, and R_t is the flow of non-renewable resource. A_t is a technological index which measures the range of available innovations. The production of innovations writes

$$\dot{A}_t = \delta L_{At} A_t, \ \delta > 0, \tag{2}$$

where L_{At} is the amount of labor devoted to research, and δ is a constant characterizing the efficiency of $R\&D$ activity.

To each available innovation is associated an intermediate good produced from the final output:

$$x_{it} = y_{it}, \ \ i \in [0, A_t]. \tag{3}$$

The non-renewable resource is extracted from an initial finite stock S_0. At each date t, a flow $-\dot{S}_t$ is extracted. This implies the following standard law of motion:

$$\dot{S}_t = -R_t. \tag{4}$$

There are no extraction costs, as it is the case in most endogenous growth models with polluting non-renewable resources (see for instance Schou, 2000, 2002 or Groth and Schou, 2007)[5].

Pollution is generated by the use of the non-renewable natural resource within the production process. In case of no abatement, the pollution flow would be a linear function of resource use: hR_t, where $h > 0$. In this way, hR_t can be seen as the carbon content of resource extraction or, equivalently, as maximum potential pollution at time t. Nevertheless, firms can abate part of this carbon so that the actual emitted flow of pollution is

$$P_t = hR_t - Q_t, \tag{5}$$

where Q_t is the amount of carbon that is removed from the potential emission flow. The ratio P_t/hR_t thus represents the effective emissions per unit of carbon content at date t, that is, the quantity of carbon actually emitted in the atmosphere relative to the carbon that would be emitted with the same extraction level but without the CCS option. Q_t/hR_t is the rate of sequestration, that is, the amount of sequestered carbon relative to the total carbon content of

[5] Our main results are obtained in the case of no extraction costs. This allows to avoid heavy computational complexity. For general optimal solutions in the presence of extraction costs à la André and Smulders (2004) in a model with no abatement, see for instance Grimaud and Rouge (2008). Using data on the prices of fossil fuels over the last century, Gaudet (2007) shows that, despite high volatility, these prices remained approximatively constant, or at most weakly increased.

the flow R_t of extracted resource.

We assume that Q_t is produced from two inputs, the pollution content hR_t and dedicated labor L_{Qt}, according to the following Cobb-Douglas abatement technology[6][7]:

$$Q_t = (hR_t)^{\eta} L_{Qt}^{1-\eta},\ 0 < \eta < 1, \text{ if } L_{Qt} < hR_t \tag{6}$$

and

$$Q_t = hR_t, \text{ if } L_{Qt} \geq hR_t,$$

that is, the pollution flow is fully abated as soon as $L_{Qt} = hR_t$[8]. The ratio L_{Qt}/hR_t represents the CCS effort, i.e. the amount of labor devoted to this activity, per unit of carbon content.

The Cobb-Douglas form allows simple analytical developments. For any given hR_t, the total cost of labor, $L_{Qt} = Q_t^{1/(1-\eta)}(hR_t)^{-\eta/(1-\eta)}$, is an increasing and convex function of Q_t. The marginal and average labor costs, respectively $\partial L_{Qt}/\partial Q_t = [1/(1-\eta)]\, Q_t^{\eta/(1-\eta)}(hR_t)^{-\eta/(1-\eta)}$ and $L_{Qt}/Q_t = Q_t^{\eta/(1-\eta)}(hR_t)^{-\eta/(1-\eta)}$, are also increasing functions of Q_t. Given any quantity of potentially emitted carbon hR_t, it is the effort in terms of labor only that enables pollution abatement. Introducing capital in the technology (6) would certainly improve the model by making it more realistic - the same applies for production functions (1) or (2). However, this would add a fourth state variable, which would prevent us from obtaining closed-form solutions in the transition toward the steady-state.

Similar technologies can be found in Stokey (1998), Copeland and Taylor (2004) or Aghion and Howitt (1998). In the latter, output is an increasing function of a technological index, and the pollution flow is an increasing function of the output level and of this index. However, there are two main differences here. First, pollution is a by-product of the resource use, and not of output; second, pollution can be abated by using more labor - and not through a different technological index. Finally, note that, for the sake of simplicity, we do not consider storage constraints[9].

[6] More generally, one could have considered the technology $Q_t = (hR_t)^{\eta}(\xi L_{Qt})^{1-\eta}$, $0 < \eta < 1$, if $L_{Qt} < hR_t/\xi$ and $Q_t = hR_t$, if $L_{Qt} \geq hR_t/\xi$, with $\xi > 0$. Here, we normalize ξ at one.

[7] Note that, contrary to Goulder and Mathai (2000) or Gerlagh et al. (2008) for instance, we do not consider technical progress in abatement. Of course, such assumption would be more realistic. For instance, one can consider the function $Q_t = (hR_t)^{\eta}(A_{Qt}L_{Qt})^{1-\eta}$, where A_{Qt} grows over time at exogenous rate. However, in this endogenous growth framework, it would make our computations much more complex; in particular, it prevents us from getting a closed-form solution for R_t. We nevertheless show how such formulation can generalize certain results of our paper later in the text.

[8] In the following sections, we make an assumption on parameters so that this corner solution never occurs.

[9] At the local scale, such constraints can be important, especially when transportation costs are non negligible (on this issue, see for instance Lafforgue et al., 2008, in a partial equilibrium framework). Here, we implicitly assume that carbon sinks are large enough to store any stock of CO_2.

The flow of pollution P_t adds to the existing stock Z_t. We assume $Z_t = Z_0 + \int_0^t P_s e^{\theta(s-t)} d_s$, with $Z_0 > 0$, and θ is the (supposed constant) positive rate of natural decay. This gives the following law of motion[10]

$$\dot{Z}_t = P_t + \theta(Z_0 - Z_t). \tag{7}$$

Production flow Y_t is used for consumption (C_t) and for the production of intermediate goods:

$$Y_t = C_t + \int_0^{A_t} y_{it} di. \tag{8}$$

Population is assumed constant, normalized at one, and each individual is endowed with one unit of labor. Thus we have:

$$1 = L_{Yt} + L_{At} + L_{Qt}. \tag{9}$$

So, the key trade-offs in this model are characterized through the allocation of labor between the three activities that are production, research and sequestration.

The household's instantaneous utility function depends on both consumption, C_t, and the stock of CO_2, Z_t. The intertemporal utility function is:

$$U = \int_0^{+\infty} \left[\ln C_t - \omega Z_t\right] e^{-\rho t} dt, \;\; \rho > 0 \text{ and } \omega \geq 0. \tag{10}$$

The separability of the utility function allows to simplify the computations. This means that, though the impact of a change in the pollution stock on the marginal utility of consumption could be considered positive or negative, we take it as nil. Concerning environmental preferences, as Goulder and Mathai (2000) say, the damage function can be regarded as convex or concave. Here, we simply consider a linear relationship.

2.2 Welfare analysis

2.2.1 Social optimum

We present the socially optimal trajectories of the economy. The social planner maximizes $U = \int_0^{+\infty} (\ln C - \omega Z) e^{-\rho t} dt$ subject to (1)-(4) and (7)-(9). The planned economy is always in transition; however, we obtain closed-form solutions. All computations and results are given in Appendix 1, where we fully depict the socially optimal transition time-paths of all variables. In this section, we focus only on the most relevant trade-offs. Hereafter, we denote by $g_{Xt} = \dot{X}_t /$

[10] Such formulation is standard in the literature. As Goulder and Mathai (2000) or Gerlagh et al. (2008) point out, the dynamics of the stock of carbon are more complex in reality. We could consider a non constant rate of decay, for instance. However, such formulation would make the model much less tractable.

X_t the growth rate of any variable X_t, and by X_t^o its socially optimal level.

We obtain

$$L_{Qt}^o = \left[\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right]^{1/\eta} hR_t^o. \tag{11}$$

Thus the ratio L_{Qt}^o/hR_t^o is constant, which implies Q_t^o/hR_t^o and P_t^o/hR_t^o constant. We now explain where this result comes from - we will see that, in particular, it depends on the functional forms we have chosen for CCS, carbon accumulation in the atmosphere and utility. Recall first that, in this model, the main trade-offs are characterized through the allocation of labor between its three competing uses: production (L_Y), research (L_A) and CCS (L_Q). The social planner allocates labor between these sectors so that a marginal increase of this input in any sector yields the same variation of intertemporal utility. On the one hand, $\Delta L_{Qt}^o = 1$ yields an increase in Q_t and thus a decrease in pollution P_t: $\Delta P_t = -\Delta Q_t = -(1-\eta)(hR_t/L_{Qt})^\eta$, from (5) and (6). ΔP_t is a linear function of $(hR_t/L_{Qt})^\eta$; it stems from the fact that the CCS technology (6) is Cobb-Douglas, homogeneous of degree 1, which implies that the partial derivatives are homogeneous of degree 0 and thus only depend on the inputs ratio. By assumption, we have $Z_t = Z_0 + \int_0^t P_s e^{\theta(s-t)} d_s$, thus $\Delta Z_v = \Delta P_t e^{\theta(t-v)} = -(1-\eta)(hR_t/L_{Qt})^\eta e^{\theta(t-v)}$ for all $v \geq t$. Finally, using (10), we have $\Delta_1 U_t = -\int_t^{+\infty} \omega \Delta Z_v e^{-\rho(v-t)} dv = [\omega(1-\eta)(\rho+\theta)]/(hR_t/L_{Qt})^\eta$. The fact that $\Delta_1 U_t$ is a linear function of $(hR_t/L_{Qt})^\eta$ stems from the fact that the rate of CO_2 decay is constant and utility is separable and linear in Z_t. On the other hand, we show in Appendix 1 (section i) that $\Delta L_{At}^o = 1$ yields $\Delta_2 U_t = \delta/\rho$. By equalizing $\Delta_1 U_t$ and $\Delta_2 U_t$, one gets equation (11).

We also have

$$R_t^o = \frac{\gamma}{\chi_0 e^{\rho t} + B}, \tag{12}$$

in which $\chi_0 = B/(e^{\frac{B\rho S_0}{\gamma}} - 1)$ and $B = \frac{(1-\alpha)\omega h}{\rho+\theta}\left[1 - \eta\left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right]$. Moreover, we get

$$g_{Rt}^o = g_{L_{Qt}}^o = g_{Q_t}^o = g_{P_t}^o = \frac{-\rho}{1 + (e^{\frac{B\rho S_0}{\gamma}} - 1)e^{-\rho t}}. \tag{13}$$

As shown in Appendix 1 (section iv), since $B > 0$, $g_{Rt}^o < 0$ for all t. L_{Qt}^o/hR_t^o being constant, this means that L_{Qt}^o decreases over time. In other words, the important effort in CCS should occur today: the social planner places the strongest sequestration efforts in the short run and progressively diminishes them over time.

Finally, the socially growth rate of the economy is

$$g_{Yt}^{o} = g_{At}^{o} + (\gamma/(1-\alpha))g_{Rt}^{o}. \tag{14}$$

Remark: Suppose now that there is technical progress in the CCS function. For instance, assume $Q_t = (hR_t)^{\eta}(A_{Qt}L_{Qt})^{1-\eta}$, where A_{Qt} grows over time at exogenous rate. In this case, equation (11) becomes $L_{Qt}^{o}/hR_t^{o} = [\rho\omega(1-\eta)/\delta(\rho+\theta)]^{1/\eta}\, A_{Qt}^{(1-\eta)/\eta}$, and one gets $Q_t^{o}/hR_t^{o} = [\rho\omega(1-\eta)/\delta(\rho+\theta)]^{(1-\eta)/\eta}\, A_{Qt}^{(1-\eta)^2/\eta}$. Then, if technical progress in CCS is high, L_{Qt}^{o} and Q_t^{o} increase over time: our result can be reversed. In other words, the socially optimal effort in CCS can increase over time over some intervals of time. As mentioned above, our basic model does not feature technical progress in CCS because it would make our computations much more complex and would prevent us from getting a closed-form solution for R_t.

Finally, the economy asymptotically tends to a steady-state which corresponds to the state the economy would immediately jump to if environmental preferences were nil ($\omega = 0$).

2.2.2 Impact of CCS on the socially optimal trajectories

In order to study the impact of carbon abatement on the socially optimal paths, we consider the social optimum in the case where the CCS technology is not available. We denote by $X_t^{o\varnothing}$ the optimal level of any variable X_t in this case - X_t^{o} still standing for the optimal value in the CCS case. We provide the optimal levels and growth rates in the no-CCS case in Appendix 2.

We now compare the optimal growth rates of resource extraction in the two cases. We obtain the following inequality:

$$g_{Rt}^{o} < g_{Rt}^{o\varnothing}.$$

The literature has shown that the laissez-faire resource extraction is too fast (see for instance Withagen, 1994), and thus that g_R is too low. Here, this inequality shows that if a CCS technology is available, the optimal extraction is faster than in the absence of such technology, and thus less restrictive. In other words, CCS allows to partially relax the environmental constraint; the sacrifice made in the early periods is reduced.

The impact of CCS on the optimal pollution paths is less obvious. We first consider the near term. Two opposite effects drive the pollution path. First, $hR_t^{o\varnothing} < hR_t^{o}$, that is, potential emissions are fostered. Indeed, since resource extraction is increased, carbon emissions tend to rise as well. At the same time, CCS activity tends to reduce pollution. Thus, according to the relative strengths of these two effects, the introduction of a CCS technology can entail either a rise or a fall in the socially optimal polluting emissions in the short run. In the case where the positive effect on extraction dominates the impact of the CCS activity, carbon emissions are stimulated[11]. In the long term, CCS unambiguously induces lower emissions. Indeed, we have shown that extraction decreases; thus, whatever the amount of sequestered carbon, pollution decreases.

We now turn to the effect of CCS on optimal growth. First, $L_{Qt}^{o\varnothing}$ and $Q_t^{o\varnothing}$ are obviously nil. Moreover, $L_Y^o = L_Y^{o\varnothing} = \beta\rho/\delta(1-\alpha)$ (see equation (40) in Appendix 1, and Appendix 2). This implies $L_{At}^o < L_{At}^{o\varnothing}$: the amount of labor devoted to $R\&D$ is lower in the "CCS case" as CCS is a third competing use for labor. So there is a first effect on research which is detrimental to growth - such mechanism also occurs in growth models with renewable resources, as in Smulders and Gradus (1996) for instance. Here, it is reinforced by an additional mechanism which we have presented above: resource extraction is faster ($g_{Rt}^o < g_{Rt}^{o\varnothing}$). Thus, we have the following inequality: $g_{Yt}^o = \delta L_{At}^o + (\gamma/(1-\alpha))g_{Rt}^o < g_{Yt}^{o\varnothing} = \delta L_{At}^{o\varnothing} + (\gamma/(1-\alpha))g_{Rt}^{o\varnothing}$. In other words, CCS is detrimental to economic growth, because of the lower effort in $R\&D$ and the acceleration of resource extraction.

Finally, CCS fosters consumption levels in the early stages. Indeed, we have seen that the amount of labor in production remains unchanged by the introduction of the CCS technology, and that resource extraction is increased in the near term. If we consider a sufficiently short period of time during which the reduced growth of knowledge does not offset these two effects, then the production level is fostered. Hence, the optimal short-run consumption levels are greater in an economy with CCS.

3 Decentralized Economy

We now study the equilibrium trajectories of the decentralized economy, which will enable us to study the impacts of climate policies in the following section. Since we study a Romer model, there are two first basic distortions: the standard public good character of knowledge and the monopolistic structure of the intermediate sector. Moreover, a third distortion arises from polluting emissions whose accumulated stock harms welfare. In order to correct these distortions, we introduce three economic policy tools: a unit subsidy to the use of intermediate goods, a research subsidy, and a tax on polluting emissions. Note that the climate policy does not consist of a tax on the polluting resource, as in Groth and Schou (2007) or Grimaud and Rouge (2008).

[11] This result can be related to Goulder and Mathai (2000), in which a more efficient abatement technology leads to higher pollution levels in the short-run. However, baseline emissions are exogenous in their model, and pollution rises because abatement falls.

Indeed, the basic externality is polluting emissions and, as an abatement technology is available, a tax on these emissions and a tax on the polluting resource are no longer equivalent. The latter tax would only modify the extraction path, and would have no impact on CCS activity. Conversely, as we will show below, the tax on carbon emissions has two main effects: it leads to postponing extraction (as in models without abatement) and it yields incentives to produce optimal efforts in carbon abatement at each date t.

For many reasons, e.g. lack of international political consensus, the Pigovian level of the carbon tax is not always implementable. Hence we consider two additional economic policy tools aimed at -partially- compensating, in a second-best world, the fact that the carbon tax cannot be set at its first-best level. The first is a subsidy to sequestered carbon [12]. The second is a subsidy to labor devoted to CCS, which can be considered as observable as sequestered carbon.

3.1 Agents' behavior

The price of the final good is normalized at one, and w_t, p_{it}, p_{Rt}, and r_t are, respectively, the wage, the price of intermediate good i, the price of the non-renewable resource, and the interest rate on a perfect financial market.

3.1.1 Household

The representative household maximizes (10) subject to her budget constraint $\dot{b}_t = r_t b_t + w_t + \pi_t - C_t + T_t$, where b_t is her total wealth, π_t represents total profits - including the resource rent $p_{Rt} R_t$ - in the economy and T_t is a lump-sum subsidy (or tax). One gets the following standard Ramsey-Keynes condition:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{15}$$

3.1.2 Non-renewable resource sector

On the competitive natural resource market, the maximization of the profit function

$\int_t^{+\infty} p_{Rs} R_s e^{-\int_t^s r_u du} ds$, subject to $\dot{S}_s = -R_s$, $S_s \geq 0$, $R_s \geq 0$, $s \geq t$, yields the standard equilibrium "Hotelling rule":

$$\frac{\dot{p}_{Rt}}{p_{Rt}} = r_t. \tag{16}$$

As usual, the transversality condition is $\lim_{t \to +\infty} S_t = 0$.

[12] We thank an anonymous referee for suggesting this tool.

3.1.3 Final sector

The final sector maximizes the following profit function:

$$\pi_{Yt} = \left(\int_0^{A_t} x_{it}^{\alpha} di\right) L_{Yt}^{\beta} R_t^{\gamma} - \int_0^{A_t} p_{it}(1-s)x_{it}di - w_t L_{Yt} - w_t(1-\varphi)L_{Qt} - p_{Rt}R_t \quad (17)$$
$$-\tau_t h(R_t - h^{\eta-1}R_t^{\eta}L_{Qt}^{1-\eta}) + \lambda_t(hR_t)^{\eta}L_{Qt}^{1-\eta}.$$

s and φ are constant rates of subsidy to the use of intermediate goods and labor in the CCS activity, respectively. τ_t is a unit tax on polluting emissions P_t (i.e., $hR_t - (hR_t)^{\eta}L_{Qt}^{1-\eta}$) and λ_t is a subsidy to sequestered carbon Q_t. The first-order conditions of this program are:

$$\frac{\partial \pi_{Yt}}{\partial x_{it}} = \alpha x_{it}^{\alpha-1} L_{Yt}^{\beta} R_t^{\gamma} - p_{it}(1-s) = 0, \text{ for all } i \quad (18)$$

$$\frac{\partial \pi_{Yt}}{\partial L_{Yt}} = \beta Y_t / L_{Yt} - w_t = 0, \quad (19)$$

$$\frac{\partial \pi_{Yt}}{\partial R_t} = \gamma Y_t / R_t - p_{Rt} - \tau_t h(1 - \eta h^{\eta-1}R_t^{\eta-1}L_{Qt}^{1-\eta}) + \lambda_t \eta h^{\eta} R_t{}^{\eta-1} L_{Qt}^{1-\eta} = 0, \quad (20)$$

$$\text{and } \frac{\partial \pi_{Yt}}{\partial L_{Qt}} = -(1-\varphi)w_t + (\tau_t + \lambda_t)(1-\eta)h^{\eta}R_t^{\eta}L_{Qt}^{-\eta} = 0. \quad (21)$$

This last condition highlights the fact that the carbon tax τ and the subsidy to sequestered carbon λ have similar effects on the effort put into the CCS activity. We develop this point later in section 4.2.

In this study, it is useful to identify the "total" price paid by the final sector for the resource. We denote it by $\tilde{p}_R$. Looking at the profit function (17), one can see that it is composed of three elements: the price paid to resource owners (p_{Rt}), the tax paid on carbon emissions ($\tau_t h(1 - (L_{Qt}/hR_t)^{1-\eta}$) and the subsidy to sequestered carbon ($\lambda_t h(L_{Qt}/hR_t)^{1-\eta}$). Following equations (17), we have

$$\tilde{p}_{Rt} = p_{Rt}\left[1 + \frac{\tau_t h}{p_{Rt}} - \frac{(\tau_t + \lambda_t)h}{p_{Rt}}\left(\frac{L_{Qt}}{hR_t}\right)^{1-\eta}\right] \equiv p_{Rt}M_t. \quad (22)$$

3.1.4 Intermediate and research sectors

Innovations are protected by infinitely lived patents. This gives rise to a monopoly position in the intermediate sector. The profit function of the i^{th} monopolist is $\pi_i^m = (p_i - 1)x_i(p_i)$, where $x_i(p_i)$ is the demand for intermediate good i by the final sector (see (18)). Hence, the price chosen by the monopolist is

$$p_{it} \equiv p = 1/\alpha, \text{ for all } i. \tag{23}$$

As a result, quantities and profits are symmetric. One gets

$$x_{it} \equiv x_t = \left(\frac{\alpha^2 L_{Yt}^{\beta} R_t^{\gamma}}{1-s} \right)^{1/(1-\alpha)} \tag{24}$$

and

$$\pi_{it}^m \equiv \pi_t^m = \frac{1-\alpha}{\alpha} x_t. \tag{25}$$

The market value of a patent is $V_t = \int_t^{+\infty} (\pi_s^m + \sigma_s) e^{-\int_t^s r_u du} ds$, where σ_s is a subsidy to research aimed at correcting the standard distortion caused by the intertemporal spillovers[13]. Differentiating this equation with respect to time gives

$$r_t = \frac{\dot{V}_t}{V_t} + \frac{\pi_t^m + \sigma_t}{V_t}, \tag{26}$$

which states that bonds and patents have the same rate of return in equilibrium.

The profit function of the research sector is $\pi_t^{RD} = V_t \delta A_t L_{At} - w_t L_{At}$. Free-entry in this sector leads to the standard zero-profit condition:

$$V_t = \frac{w_t}{\delta A_t}. \tag{27}$$

3.1.5 Government

The government's budget constraint comprises: the carbon tax ($\tau_t Q_t = \tau_t \left[hR_t - (hR_t)^{\eta} L_{Qt}^{1-\eta} \right]$), the subsidy to the use of intermediate goods ($\int_0^{A_t} s p_{it} x_{it} di = A_t s x_t / \alpha$), the subsidy to research (σ_t), the subsidy to labor in CCS ($\varphi w_t L_{Qt}$), the subsidy to sequestered carbon ($\lambda_t Q_t = \lambda_t (hR_t)^{\eta} L_{Qt}^{1-\eta}$) and the lump-sum subsidy (or tax) T_t (see section 3.1.1). Assuming that it is balanced at each date t, it writes: $\tau_t \left[hR_t - (hR_t)^{\eta} L_{Qt}^{1-\eta} \right] - A_t s x_t / \alpha - \sigma_t - \varphi w_t L_{Qt} - \lambda_t (hR_t)^{\eta} L_{Qt}^{1-\eta} - T_t = 0$ for all t.

3.2 Equilibrium

The preceding first-order conditions enable us to determine the equilibrium in the decentralized economy, that is, the set of quantities, prices and growth rates at each date expressed as functions

[13] Note that Barro and Sala-i-Martin (2003), for instance, consider a direct subsidy to labor in research; our assumption alleviates computational complexity in the present context of polluting non-renewable resources and abatement.

of the economic policy tools (s, σ, τ, φ and λ). Here also, the economy is always in transition. We fully characterize the decentralized economy in Appendix 3, where we provide all equilibrium levels and growth rates. In this section, we focus on the variables that are relevant to our analysis.

As we mentioned above, the three basic distortions concern research and polluting emissions. Recall that, in the present model, there is no directed technical change[14], in particular in the abatement technology. We do not study the links between the climate policy and research subsidies - for such analysis in a partial equilibrium framework, see for instance Goulder and Mathai (2000) or Gerlagh et al. (2008). Thus, in order to focus on the climate policy, we assume here that research is optimally funded. In other words, both subsidies, s and σ, are set at their optimal levels, that is $s = 1 - \alpha$, and $\sigma_t = V_t g_{At}$ (proof: see Appendix 3, section i).

3.2.1 Equilibrium with no climate policy

We first consider the case in which no climate policy is implemented: $\tau_t = \lambda_t = 0$ at each date. The economy immediately jumps to its steady-state, where the amount of labor devoted to CCS is nil (see equation (28)): $L_{Qt} = 0$, which means that no carbon is abated ($Q_t = 0$). This, in turn, implies that the total potential emission is released in the atmosphere, i.e. $P_t = hR_t$. Moreover, since labor used in the production of the final good (L_Y) is constant, labor devoted to the research sector ($L_A = 1 - L_Y$) is also constant[15]. The flow of extraction at date t is $R_t = \rho S_0 e^{-\rho t}$. This implies $g_R = -\rho$ for all t. Finally, the growth rate of output, g_Y, is equal to $\delta - \rho$, as in more general endogenous growth models with non-polluting non-renewable resources. This steady-state is obviously identical to the first-best steady-state when environmental preferences are nil, that is, when $\omega = 0$ (see section 2.2.1).

3.2.2 Equilibrium with climate policies

Now, we consider the equilibrium in presence of the climate policy tools. For obvious reasons, it is impossible to study all types of carbon tax and subsidy profiles. We will then limit our analysis to specific types. We show in the next section (section 4.1) that the first-best carbon tax is a linear function of Y. Moreover, studying the class of economic policy tools growing at the same rate as output allows to fully characterize the equilibrium, and in particular to obtain

[14] For an endogenous growth model with a stock of pollution and directed technical change, see for instance Grimaud and Rouge (2008) or Acemoglu et al. (2011).

[15] This property stems from an arbitrage condition in the allocation of labor between production and research activities. A similar trade-off occurs at the social optimum; we give a more detailed analysis in Appendix 1 (i).

a closed-form solution for resource extraction. We thus focus on a climate policy such that $\tau_t = a_1 Y_t$ and $\lambda_t = a_2 Y_t$, where a_1 and a_2 are positive constants.

The main findings are the following. Labor in final good production, L_Y, is constant over time, and L_{Qt}, the effort in CCS, is given by

$$L_{Qt} = \left[\frac{(\tau_t + \lambda_t)\rho(1-\eta)}{(1-\varphi)\delta(1-\alpha)Y_t} \right]^{1/\eta} hR_t. \tag{28}$$

Here, we assume $0 \leq (\tau_t + \lambda_t)/Y_t \leq (1-\varphi)\delta(1-\alpha)/\rho(1-\eta)$ in order to avoid the corner solution in which the whole carbon content of R_t is abated at any time. The flow of resource extraction is given by

$$R_t = \frac{\gamma}{\psi_0 e^{\rho t} + G}, \tag{29}$$

where $\psi_0 = G/(e^{\frac{G\rho S_0}{\gamma}} - 1)$ and $G = \frac{h\tau_t}{Y_t} - \eta h \left(\frac{\rho(1-\eta)}{(1-\varphi)\delta(1-\alpha)}\right)^{\frac{1-\eta}{\eta}} \left(\frac{\tau_t + \lambda_t}{Y_t}\right)^{\frac{1}{\eta}}$ (see Appendix 3 (iii)). Since τ_t/Y_t and λ_t/Y_t are constant, G is constant. The growth rate of resource extraction is

$$g_{Rt} = \frac{-\rho}{1 + (e^{\frac{G\rho S_0}{\gamma}} - 1)e^{-\rho t}}. \tag{30}$$

g_{Rt} is negative and asymptotically converges toward its long-run level $-\rho$. Along the transition, one can see that if $G > 0$, then g_{Rt} is higher than its asymptotic value, while if $G < 0$, it is lower. The value of G depends on the relative values of τ_t and λ_t.

Since the effort in CCS (L_{Qt}), abated carbon (Q_t) and pollution (P_t) are linear functions of R_t, they also decrease over time[16]. Since L_{Qt} decreases over time, labor devoted to research, L_{At}, increases over time and converges to the constant level $1 - L_Y = 1 - \beta\rho/\delta(1-\alpha)$ as time goes to infinity. Note that the relations between the CCS effort, sequestration, pollution and extraction depend on the economic policy tools - see later the effects of policies.

The growth rate of output is given by

$$g_{Yt} = g_{At} + (\gamma/(1-\alpha))g_{Rt}. \tag{31}$$

Since $g_{At} = \delta L_{At}$ (see equation (2)), g_{At} increases over time and tends to $\delta - \beta\rho/(1-\alpha)$. Simultaneously, g_{Rt} tends to its limit $-\rho$ (see equation (30)). Thus, in the long run, g_{Yt} tends to $\delta - \rho$, which we can consider positive. This is a fairly standard expression of long-run output

[16] If we consider technical progress in the CCS function, for instance $Q_t = (hR_t)^\eta (A_{Qt}L_{Qt})^{1-\eta}$, where A_{Qt} grows over time at exogenous rate, then (28) becomes $L_{Qt}/hR_t = [\tau_t\rho(1-\eta)/(1-\varphi)\delta(1-\alpha)Y_t]^{1/\eta} A_{Qt}^{(1-\eta)/\eta}$. L_{Qt}/hR_t and Q_t/hR_t are then increasing functions of time, which is more consistent with Grimaud et al. (2011), for instance. In this case, even if R_t decreases over time, Q_t can be increasing over some intervals.

growth in a model with non-renewable resources - see for instance Stiglitz (1974) in an exogenous growth model without pollution. Along the transition path, however, g_{At} is lower than its long-run level and g_{Rt} can be lower or higher according to the relative values of the carbon tax and the subsidy to sequestered carbon - see comments below equation (30). Thus, over some intervals of time, output growth can be positive or negative. We provide further elements on the impact of the economic policy tools on output growth in the next section.

4 Economic policies

4.1 First best climate policy

We now characterize the first-best policy. Recall that there are three basic market failures in this economy. Since we have set the research subsidies at their optimal levels, only the environmental distortion remains. Hence, in order to implement the first-best, one just needs to set the carbon tax at its Pigovian level. Obviously, there is no need here for the CCS tools, that is, the subsidies to sequestered carbon and to labor in CCS. Thus we set $\lambda = \varphi = 0$ in this section.

Proposition 1 *At each date t, $\tau_t^o = \frac{\omega(1-\alpha)}{\rho+\theta} Y_t$ is the level of the carbon tax that implements the socially optimal path. This tax is unique, and it is generally an increasing function of time.*

Proof. Comparing the optimal levels of the variables to their levels in the decentralized equilibrium, for instance L_{Qt}^o/hR_t^o (11) and L_{Qt}/hR_t (28), yields τ_t^o. Then, one can easily check that all the other variables in the decentralized economy are at their socially optimal levels. ■

First, note that $\tau_t^o = (-\zeta_t)e^{\rho t}(1-\alpha)Y_t$, where ζ_t is the co-state variable associated to Z_t, the stock of carbon, in the social planner program (see Appendix 1, equation (41)). This socially optimal tax can be linked to the ones obtained in partial equilibrium frameworks: see for instance Hoel and Kverndokk (1996, equation (17)), Goulder and Mathai (2000, equation (13)) or Gerlagh et al. (2008, equation (18)). However, in our context, this tax exhibits specific properties, which we comment below.

Economic interpretation of the first-best carbon tax

One can show that, if we use the non-specified expression of the utility function, $U(C_t, Z_t)$, the optimal tax is equal to $\frac{-1}{U_C}\int_t^{+\infty} U_Z e^{-(\rho+\theta)(s-t)}ds > 0$, since $U_Z < 0$[17]. Thus the optimal tax is the product of two terms. The first, $1/U_C$, is the amount of consumption good that

[17] Indeed, using (10), we have $1/U_C = C_t = (1-\alpha)Y_t$, and $-\int_t^{+\infty} U_Z e^{-(\rho+\theta)(s-t)}ds = \frac{\omega}{\rho+\theta}$: we get the result given in Proposition 1.

compensates a unit change in utility. The second, $-\int_t^{+\infty} U_Z e^{-(\rho+\theta)(s-t)}ds$, is the expression of the optimal tax in terms of utility, that is, the sum of discounted social costs of one unit of carbon emitted at date t, for all (present and future) times. Hence, τ_t^o is the sum of discounted social costs of one unit of carbon measured in terms of final good.

Note that the tax level matters here. Indeed, when abatement technology is available, the social planner has to give the right incentive in terms of social costs of pollution to firms, so as to induce the optimal effort in abatement. Thus, the optimal tax has to be equal to $\partial Y_t/\partial Q_t \equiv (\partial Y_t/\partial L_{Yt})/(\partial Q_t/\partial L_{Qt})$, which is the cost for firms of sequestering one unit of carbon[18] - indeed, increasing CCS leads to a decrease in output through a labor transfer from the final good sector to the CCS one. Since $\partial Y_t/\partial L_{Yt} = \beta Y_t/L_{Yt}$ and $\partial Q_t/\partial L_{Qt} = (1-\eta)Q_t/L_{Qt}$, using the optimal values given in Appendix 1 (section vii), we get τ_t^o as expressed in the proposition. This sharply contrasts with the standard result of the literature without abatement which states that the tax level generally does not matter (see Dasgupta and Heal, 1979; Sinclair, 1992; Groth and Schou, 2007; or Grimaud and Rouge, 2008 for instance). In this context, there are an infinity of optimal taxes which have the same dynamics, but differ in their levels. Here, in a model featuring CCS, we have shown that the socially optimal tax is unique.

Main properties of the first-best carbon tax

We have seen that the optimal tax level is $\frac{-1}{U_C}\int_t^{+\infty} U_Z e^{-(\rho+\theta)(s-t)}ds$. The term $-\int_t^{+\infty} U_Z e^{-(\rho+\theta)(s-t)}ds$ is the optimal tax expressed in terms of utility, and it is equal to $\omega/(\rho+\theta)$. For obvious reasons, it is increasing in environmental preferences, ω, and decreasing in the psychological discount rate ρ and the rate of natural CO_2 decay θ. Moreover, since we use a separable utility function with a constant marginal disutility of the stock of CO_2 ω, and since ρ and θ are constant, the tax is constant under this form.

However, when the optimal tax is measured in terms of final good, its growth rate is equal to the growth rate of output. This also comes from the utility function we have chosen: the utility of consumption is logarithmic, as stated in equation (10). $1/U_C$ is equal to $(1-\alpha)Y_t$. So, the optimal tax grows at the same rate as output - which is generally positive. The economic intuition behind this property is the following. If $g_{Yt} > 0$, the marginal utility of consumption decreases over time. Thus, the amount of final good that compensates the household for the emission of one unit of carbon increases over time. Observe that the Pigovian tax is increasing even if utility is a linear function of Z_t. A convex functional form would probably reinforce this result - see for instance the discussion on this issue in Goulder and Mathai (2000, p.34). This

[18] Goulder and Mathai (2000) provide a similar expression in a partial equilibrium context with exogenous baseline emissions, that is, exogenous total carbon content in our framework (see equation (11) in their paper).

confirms what is obtained by Grimaud et al. (2011) in a calibrated model, in the absence of carbon ceiling. Finally, since $g_\tau^o = g_Y^o$, the Ramsey-Keynes condition (15) implies $g_\tau^o = r - \rho < r$; in other words, this policy will postpone resource extraction - see Dasgupta et al. (1981) on this issue.

Remark: Ex-post interpretation of the increasing unit carbon tax.

In many growth models with climate change (see for instance Sinclair, 1992, Groth and Schou, 2007, or Grimaud and Rouge, 2008), the socially optimal policy instrument consists of a decreasing ad-valorem tax on resource use - which is equivalent to a tax on carbon emissions if there is no abatement. Here we have shown that the optimal tool is an increasing unit tax on carbon emissions. Both results can be linked. Indeed, the optimal carbon tax, which leads the decentralized economy to postpone resource extraction, can be interpreted *ex-post* as a decreasing ad valorem tax on the resource. When the optimal tax is implemented, the total (i.e., including the price of the resource and the carbon tax) unit price paid by users for the resource increases less fast than the unit price perceived by owners of the resource -whose growth rate is equal to the interest rate. That is why extraction is postponed. Ex-post, this has the same effect as a decreasing ad valorem tax. Indeed, we have seen that the total price paid by firms for the resource is $\tilde{p}_{Rt}$ (see 22). Using (28) and $\tau_t^o = \omega(1-\alpha)Y_t/(\rho+\theta)$ (see proposition 1), $\tilde{p}_{Rt}$ is equal to $p_{Rt}\left[1+\left(1-\left(\frac{\omega\rho(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right)\frac{\omega(1-\alpha)hY_t}{(\rho+\theta)p_{Rt}}\right]$ at the first-best. Thus, $\tilde{p}_{Rt}$ can be written as $p_{Rt}(1+\xi_t)$, where ξ_t can be interpreted as an ad valorem tax on the resource. Since $g_{Yt} = r_t - \rho$ and $g_{p_{Rt}} = r_t$, the ratio Y_t/p_{Rt} decreases over time and so does the ad valorem tax.

4.2 Impact of second-best economic policies

We suppose here that the Pigovian level of the carbon tax - stated in proposition 1- cannot be achieved by the policy maker, and that it can only be lower than this level at each date t. As mentioned above, many reasons could explain this situation, such as a lack of international political consensus. In such a case, additional policies could prove useful. We thus study the impact of second-best policies: a carbon tax τ_t inferior to τ_t^o [19], the subsidy to sequestered carbon λ_t and the subsidy to labor in CCS φ. When they are not necessary, we drop time subscripts for notational convenience.

[19] We include the carbon tax in the second-best tools when it is below its first-best level in order to simplify our presentation.

4.2.1 Carbon tax

We first consider the impact of the carbon tax τ. We have shown in section 4.1 that the first-best level of the carbon tax is strictly positive; we thus study here the impact of setting a positive carbon tax in an economy where there is none, or, equivalently, of increasing the tax level if one is already in place. We implicitly assume that, in either case, the tax level is below its first-best level. This means that the studied policy is always Pareto-improving - even if, as we shall see, it can be detrimental for the environment in the short run. As mentioned above, we focus on policies such that the ratio τ/Y is constant. The effects of these policies on the equilibrium trajectories of the economy are described in the following proposition.

Proposition 2 *An increase in τ/Y has the following impact on the economy:*

(i) On CCS activity: The effort by unit of carbon content (L_Q/hR) and the instantaneous rate of carbon sequestration (Q/hR) increase. Effective pollution by unit of carbon content (P/hR) decreases.

(ii) On the dynamics of resource extraction, carbon emissions and CCS: Resource extraction, carbon emissions, the effort in CCS as well as CCS activity itself are postponed, i.e.: g_R, g_P, g_{L_Q} and g_Q increase.

(iii) On the level and growth of output: If the weight of the CCS sector in the whole economy is high, the tax is likely to be detrimental to growth. Since extraction is postponed, the output level is unambiguously lowered by this policy in the short run.

Proof. See Appendix 3 (section v). ■

Result (i) is due to the fact that the carbon tax makes CCS activity become more profitable. That is why the amount of labor by unit of carbon content (see equation (28)) increases. Therefore, the instantaneous rate of CCS also increases. Simultaneously, effective pollution by unit of carbon content decreases.

The carbon tax also modifies the dynamics of resource extraction - result (ii). One can easily see that $G \geq 0$ if and only if $\lambda_t \leq \eta^{-\eta}\ [\rho(1-\eta)/Y_t(1-\varphi)\delta(1-\alpha)]^{\eta-1}\,\tau_t^{\eta} - \tau_t$ - see equation (29). In particular, if $\tau_t > 0$ and $\lambda_t = 0$, then $G > 0$, that is, with the carbon tax alone, the growth rate of resource extraction is higher than its value in the absence of climate policy, $-\rho$. More generally, it is straightforward that $\partial G/\partial(\tau_t/Y_t)$ is positive (see Appendix 3, section v). Thus, $\partial g_{Rt}/\partial(\tau_t/Y_t)$ is positive: the carbon tax postpones resource extraction. The key transmission channel is the resource price. To understand it, consider the total price paid by the final sector for the resource, $\tilde{p}_{Rt}$ (see (22) and (28)). The growth rate of $\tilde{p}_R$ is equal to

$g_{p_R} + g_M$. According to the sign of g_M, the total price of the resource grows more or less rapidly than p_R. τ/p_R can be rewritten as $(\tau/Y).(Y/p_R)$. Since τ/Y is constant, and $g_{Y/p_R} = -\rho$ (see equations (15) and (16)), we have $g_{\tau/p_R} = g_Y - g_{p_R} = -\rho$. Similarly, $g_{\lambda/p_R} = -\rho$. Hence, we obtain $g_M = \rho(1-M)/M$.

In order to isolate the effects of the carbon tax on the dynamics of the total price of the resource, we assume here that $\lambda_t = 0$. In this case, $M = 1 + \frac{\tau h}{p_R}\left[1 - \left(\frac{\tau\rho(1-\eta)}{Y(1-\varphi)\delta(1-\alpha)}\right)^{(1-\eta)/\eta}\right]$. If $\tau/Y > 0$, using the fact that $0 \leq \tau_t/Y_t \leq (1-\varphi)\delta(1-\alpha)/\rho(1-\eta)$ - see section 3.2.2 -, it is straightforward that $M > 1$; thus, g_M is negative. In other words, as soon as $\tau/Y > 0$, we have $\tilde{p}_{Rt} > p_{Rt}$ with $g_{\tilde{p}_{Rt}} < r_t$, for all t. So, when the policy maker implements a climate policy, the total price of the resource paid by its user, $\tilde{p}_R$, is higher but it grows less fast; in other words, the increase in the total price paid by the resource user between two given dates gets lower. Hence, the instantaneous resource use flows are reallocated over time: less resource is extracted today, and more tomorrow. Resource extraction is thus postponed, that is, g_R increases. Q, L_Q and P being proportional to R, their dynamics are affected in the same way.

We now comment the effects of this climate policy on the growth and level of output (result iii). The impact on output growth is less straightforward. Equation (31) shows that two different effects drive this impact: the effect of the tax on knowledge growth (g_A) and the effect of the tax on the growth rate of resource extraction (g_R). In the absence of CCS technology, the effect on research is nil, since the allocation of labor between production and research is unchanged over time (see equation (42) in Appendix 3). In this case, since the climate policy postpones resource extraction, i.e., g_R increases, it unambiguously promotes output growth[20]. If a carbon abatement technology is available however, the effect on the research effort can play a key role since there is now a third competing use for labor. Using equations (2) and (9), we can see that this effect is the opposite to $\partial L_Q/\partial(\tau/Y)$ - since, here also, L_Y is constant. We have shown that $L_Q = [\tau\rho(1-\eta)/\delta(1-\alpha)Y]^{1/\eta}\, hR$ (equation (28)). An increase in τ/Y has a positive impact on the term between brackets: for a given level of extraction, the tax increases the price of carbon used in the CCS process, which prompts a rise in L_Q. However, as we have already seen, it also has a negative impact on R in the short run and a positive impact in the long run. Hence, the carbon tax has a negative impact on knowledge growth in the long run, but the short-run effect is ambiguous. Since this tax has a positive impact on g_R, the overall effect on output growth

[20] This contradicts the general finding of models in which pollution is a by-product of production or capital, and does not result from the use of non-renewable resources. In most of these models, when no specific assumptions on, say, returns to scale on the abatement technology or the external effects of environmental quality on productivity are made, Gradus and Smulders (1993), or Grimaud (1999), show that there is a trade-off between environmental quality and economic growth. For a survey on this question, see for instance Ricci (2007).

is ambiguous. Finally, we can conclude that, when a costly CCS technology is available, the impact of the carbon tax on growth depends on the strength of its - possibly negative - impact on knowledge growth. This means that it depends on the weight of the CCS activity in the whole economy. If this weight is high, that is, if L_Q is relatively high, the carbon tax is likely to be bad for growth.

As shown above, output level is given by $Y = \alpha^{\alpha/(1-\alpha)} A L_Y^{\beta/(1-\alpha)} R^{\gamma/(1-\alpha)}$. In the short run, the postponement of resource extraction drives the economy to less resource use, which negatively impacts output. In sum, the carbon tax is detrimental for output and consumption in the early times.

4.2.2 Subsidy to sequestered carbon

Here we consider the effects of the subsidy to sequestered carbon λ_t. As previously mentioned, we restrict our analysis to subsidies that grow at the same rate as output, that is, λ_t/Y_t is constant. We study the impact of setting a positive subsidy in an economy where there is none, or, equivalently, of increasing the level of the subsidy if it is already implemented. The effects of such policies on the equilibrium trajectories of the economy are described in the following proposition.

Proposition 3 *An increase in λ/Y has the following impact on the economy:*

(i) On CCS activity: Like the carbon tax, the subsidy stimulates CCS activity. The effects of the subsidy on the effort by unit of carbon content (L_Q/hR), the rate of carbon sequestration (Q/hR), and effective pollution by unit of carbon content (P/hR) are identical to the effects of the carbon tax - see result (i) in proposition 2.

(ii) On the dynamics of resource extraction, carbon emissions and CCS: The effects on resource extraction, carbon emissions, the effort in CCS and CCS activity itself are opposite to the effects of the carbon tax - see result (ii) in proposition 2 -, that is, g_R, g_P, g_{L_Q} and g_Q decrease.

(iii) On the level and growth of output: In the short run, the subsidy is unambiguously bad for growth. In the longer term, if the weight of the CCS sector in the whole economy is high, the subsidy can promote growth. Since resource extraction is faster, the level of output is unambiguously increased by this policy in the short run.

Proof. See Appendix 3 (section vi). ■

The subsidy to sequestered carbon, as the carbon tax, makes CCS activity more profitable. Its impact on L_Q/hR, Q/hR and P/hR is summarized in equation (28). Here, λ_t and τ_t are perfect substitutes since they appear in a sum. Henceforth, their impacts are clearly identical - results (i) in propositions 2 and 3. Moreover, if $\tau_t < \tau_t^o$ - and, in particular, if $\tau_t = 0$ -, the policy makers can always set $\lambda_t = \tau_t^o - \tau_t$ and restore the socially optimal L_Q/hR, Q/hR and P/hR. Nonetheless, the levels L_Q, Q and P will not be socially optimal since, as we show below, this level of subsidy does not entail a socially optimal resource extraction.

The impact of the subsidy on resource extraction is opposite to the impact of the carbon tax. We have seen in section 4.2.1 that $G < 0$ if and only if $\lambda_t > \eta^{-\eta} \left[\rho(1-\eta)/Y_t(1-\varphi)\delta(1-\alpha)\right]^{\eta-1} \tau_t^{\eta} - \tau_t$. In particular, if $\lambda_t > 0$ and $\tau_t = 0$, then $G < 0$, which means that, with the subsidy alone, the extraction growth rate is lower than its value in the absence of climate policy, $-\rho$. More generally, it is straightforward that $\partial G/\partial(\lambda_t/Y_t)$ is negative; for this reason, $\partial g_{Rt}/\partial(\lambda_t/Y_t)$ is negative. In other words, the subsidy to sequestered carbon accelerates resource extraction. To understand this, we study how the resource price is affected by this policy. Recall that the total price paid for the resource by the final sector is given by (22) and (28). We consider the symmetric case of 4.2.1, that is, in order to study the effects of the subsidy on the dynamics of this price, we assume that the carbon tax is nil: $\tau_t = 0$. In this case, $M = 1 - \frac{\lambda h}{p_R}\left(\frac{\lambda\rho(1-\eta)}{Y(1-\varphi)\delta(1-\alpha)}\right)^{(1-\eta)/\eta}$. Here $M < 1$, which means that $g_M > 0$. In other words, the subsidy lowers the level of the price paid for the resource but it makes it grow faster: $\tilde{p}_{Rt} < p_{Rt}$ and $g_{\tilde{p}_{Rt}} > r_t$ for all t. This entails a reallocation of instantaneous resource uses over time so that more resource is extracted today and less tomorrow, that is, resource extraction is accelerated: g_R decreases - result (ii) in proposition 3.

Q, L_Q and P being proportional to R, their dynamics are affected in the same way. This means that a type of green paradox can occur here. Indeed, following Sinn (2008) and subsequent contributions like Gerlagh (2011) and Van der Ploeg and Withagen (2012), a (weak) green paradox occurs when climate policies induce a more rapid extraction of fossil fuels, thus fostering short-term emissions. Here, two opposite effects drive short-term effective emissions: more resource is extracted, and more carbon is sequestered. If the former effect overcomes the latter, then the subsidy to CCS yields higher carbon emissions in the short run.

We now turn to the impact of this policy on output (results iii). Recall that $g_Y = g_A + (\gamma/(1-\alpha))g_R$ (see equation (31)). As proved above, the subsidy entails a decrease in g_R. If this effect is strong, and thus dominates any possible positive impact on research, output growth declines. More generally, we have shown in the preceding subsection that g_A is a decreasing function of L_Q. The impact of the subsidy on L_Q is given by equation (28). We can see that the increase in λ_t/Y_t has a positive effect on the term between brackets. Moreover, we have stated that the effect on R is positive in the short run and negative in the long run. Therefore, the impact of the subsidy on knowledge growth is negative in the short run and ambiguous in the long run. Hence, the overall effect of the subsidy to sequestered carbon on output growth is unambiguously negative in the short run. In the long run, the subsidy is detrimental to growth if the weight of the CCS activity in the economy is low (i.e. a relatively low L_Q). Conversely, if this weight is relatively high, the subsidy can have a negative or a positive impact on growth. In this last case, the impact of the subsidy is opposite to the impact of the carbon tax.

The effect on output level in the short run is obviously also opposite to the effect of the tax: more resource is used in the early stages, which tends to increase output and consumption in the short run.

More generally, the interest of this policy is twofold. First, for many reasons - in particular, political consensus issues -, the first-best level of the carbon tax is not likely to be reached. Then, one can think that a complementary tool, such as this subsidy to sequestered carbon, may help to Pareto improve the trajectories of the economy. Second, we have seen that the impacts of both policies are sometimes opposed, in particular on production and consumption in the short run. Whereas the carbon tax entails a decrease in output, the subsidy fosters it. In other words, the latter tool helps to reduce the burden of the climate policy in the earlier periods. Then, one can think that mixing both policies may favor public acceptance when the regulator has to implement her policy scheme.

Remark: We have seen that the carbon tax and the subsidy to sequestered carbon have opposite effects on the time profile of resource extraction: the tax postpones it while the subsidy accelerates it. When these two policies are simultaneously applied, the effect of the tax on extraction conflicts with that of the subsidy. However, we are able to characterize the overall effect on the growth rate of resource extraction of any couple $(\tau_t; \lambda_t)$. Indeed, as stated in the comments below proposition 3, resource extraction is postponed (resp. accelerated) if and only if λ_t is lower (resp. higher) than $\eta^{-\eta}\ [\rho(1-\eta)/Y_t(1-\varphi)\delta(1-\alpha)]^{\eta-1}\, \tau_t^{\eta} - \tau_t$.

4.2.3 Subsidy to labor in CCS

We finally consider the subsidy to labor in CCS, φ. The main effects of this tool are summarized in the following proposition.

Proposition 4 *The subsidy to labor in CCS φ has no impact on the economy if there is no complementary climate policy, that is, if $\tau_t = 0$ and $\lambda_t = 0$.*

If $\tau > 0$ and/or $\lambda > 0$, φ has the following impact on the economy:

(i) On CCS activity: the effects are similar to the effects of the carbon tax and the subsidy to sequestered carbon - see results (i) in propositions 2 and 3.

(ii) On the dynamics of resource extraction, carbon emissions and CCS, and (iii) on the level and growth of output: the effects are similar to the effects of the subsidy to sequestered carbon - see results (ii) and (iii) in proposition 3.

Proof. Differentiating equations (28)-(30), and (44), (45), (48) and (49) in Appendix 3 with respect to φ yields the results. ■

First, in the absence of the carbon tax (τ_t) and the subsidy to sequestered carbon (λ_t), a subsidy to labor in CCS has no effect on the economy. This can be observed in equations (28) and (30). One can see that, if $\tau = \lambda = 0$, then $L_Q = 0$, that is, there is no CCS activity, and $g_R = -\rho$ since $G = 0$. Indeed, carbon - sequestered or emitted - is not priced, and thus CCS activity is not profitable.

When $\tau_t > 0$ and/or $\lambda_t > 0$, the subsidy to labor does have an impact. First, the profitability of CCS resulting from the implementation of the carbon tax and/or the subsidy to sequestered carbon is strengthened, thus L_{Qt}/hR_t and Q_t/hR_t increase, and, consequently, P_t/hR_t decreases - result (i) in proposition 4. Second, the subsidy accelerates resource extraction. To understand this, we need to analyze how this tool affects the dynamics of the total price paid by the resource user. By using equation (22), we get $\partial M/\partial\varphi < 0$ and thus $\partial g_M/\partial\varphi > 0$. Thus, the subsidy to labor accelerates the growth of the total price of the resource, and thus accelerates its extraction. If $\tau_t > 0$ and $\lambda_t = 0$, we have seen in section 4.2.1 that $\tilde{p}_{Rt}$ grows less fast than p_{Rt}. Hence the subsidy to labor in CCS goes against the effect of the carbon tax on resource extraction. If $\tau_t = 0$ and $\lambda_t > 0$, as stated in section 4.2.2, $\tilde{p}_{Rt}$ grows faster than p_{Rt}; so φ strengthens the effect of the subsidy to sequestered carbon. This obviously means that, as the subsidy to sequestered carbon, the subsidy to labor in CCS can yield a green paradox.

When this policy is implemented together with a carbon tax τ_t and/or a subsidy to sequestered carbon λ_t, it affects output levels and growth in the same way as λ_t since their impact on knowledge accumulation and resource extraction are alike. For the same reasons, this policy can be seen as a good complement to a carbon tax since, while strengthening its impact on CCS activity, it can favor its social acceptance by alleviating its burden at the early stages.

5 Conclusion

We have developed an endogenous growth model with climate change that features a CCS technology. Such abatement technology allows, for a given use of fossil fuel, endogenizing CO_2 emissions.

We have fully depicted the socially optimal outcome of this economy and we have shown that the greatest effort in CCS should happen today. Moreover, the availability of a CCS technology can yield a rise in CO_2 emissions in the short run since it speeds up the pace of resource extraction, which can offset the CCS activity. We have computed the first-best carbon tax, which is unique and generally increasing over time.

We have fully characterized the decentralized economy's trajectories and, when the Pigovian carbon tax cannot be implemented, we have studied three types of second-best economic policies. The first one is a standard unit tax on carbon emissions. The second and the third are subsidies to sequestered carbon and the effort in CCS, respectively, which can both favor the public acceptance of the carbon tax.

All three tools foster CCS activity. However, the second subsidy has an effect only when it is coupled with one of the other two. The carbon tax postpones resource extraction whereas the two subsidies accelerate it, which means that they can yield a green paradox in the form of a rise in short-term GHG emissions. The effects on growth are more complex: when the CCS sector is important, the carbon tax is generally detrimental to output growth, while the two subsidies can foster growth in the long term. Finally, the carbon tax has a negative impact on output level in the short term, contrary to the subsidies.

The decarbonization of the economy and the switch to renewable or non-fossil fuel based energy remains necessary (Gerlagh, 2006). In order to keep the model tractable, the availability of a clean and renewable energy source has not been introduced. This so-called backstop would not drastically alter the qualitative properties of our results. Nevertheless, it would be interesting to study the impact of the CCS option on the adoption timing of these alternative sources of energy. We can infer that the possibility to sequester carbon emissions would delay the introduction of renewable energy.

Appendix

Appendix 1: Welfare

We drop time subscripts for notational convenience. The social planner maximizes $U = \int_0^{+\infty} (\ln C - \omega Z) e^{-\rho t} dt$ subject to (1)-(4) and (7)-(9). We assume that $[\rho\omega(1-\eta)/\delta(\rho+\theta)]^{1/\eta} < 1$ (see equation (11)) in order to avoid a corner solution in which carbon emissions are fully abated, i.e. $L_Q = hR$. Thus, it is unnecessary to incorporate a Kuhn-Tucker condition for $L_Q \leq hR$. The Hamiltonian of the program is

$$\begin{aligned} H &= (\ln C - \omega Z)e^{-\rho t} + \mu\delta A(1 - L_Y - L_Q) - \nu R + \zeta\left[h(R - h^{\eta-1}R^{\eta}L_Q^{1-\eta}) + \theta(Z_0 - Z)\right] \\ &\quad + \chi\left[(\int_0^A x_i^{\alpha} di) L_Y^{\beta} R^{\gamma} - C - \int_0^A x_i di\right], \end{aligned}$$

where μ, ν, ζ and χ are the co-state variables. The first order conditions $\partial H/\partial C = 0$ and $\partial H/\partial x_i = 0$ yield

$$e^{-\rho t}/C - \chi = 0, \tag{32}$$

$$\text{and } \alpha x_i^{\alpha-1} L_Y^{\beta} R^{\gamma} - 1 = 0, \text{ for all } i. \tag{33}$$

Note that this implies $x_i = x$, for all i. $\partial H/\partial L_Y = 0$, $\partial H/\partial L_Q = 0$ and $\partial H/\partial R = 0$ yield

$$-\mu\delta A + \chi\beta Y/L_Y = 0, \tag{34}$$

$$-\mu\delta A - \zeta h^{\eta}(1-\eta)R^{\eta}L_Q^{-\eta} = 0, \tag{35}$$

$$\text{and } \zeta h(1 - \eta h^{\eta-1}R^{\eta-1}L_Q^{1-\eta}) + \chi\gamma Y/R - \nu = 0. \tag{36}$$

Moreover, $\partial H/\partial A = -\dot{\mu}$, $\partial H/\partial S = -\dot{\nu}$, and $\partial H/\partial Z = -\dot{\zeta}$ yield

$$-\dot{\mu} = \mu\delta L_A + \chi(x^{\alpha}L_Y^{\beta}R^{\gamma} - x), \tag{37}$$

$$-\dot{\nu} = 0, \tag{38}$$

$$\text{and } -\dot{\zeta} = -\omega e^{-\rho t} - \zeta\theta. \tag{39}$$

i) Computation of L_Y.

(33) can be rewritten as $Y = Ax/\alpha$. Since $Y = C + Ax$, one gets $C = (1-\alpha)Y$.

Dividing both hand sides of (37) by μ gives $-g_\mu = \delta L_A + (x^\alpha L_Y^\beta R^\gamma - x)\chi/\mu$. The term between brackets can be rewritten as $Y/A - \alpha Y/A$, which is equal to $(1-\alpha)Y/A$. Moreover, from (34), we have $\chi/\mu = \delta A L_Y/\beta Y$ and $g_\mu + g_A = g_\chi + g_Y - g_{L_Y}$. Since (32) yields $g_\chi = -\rho - g_C = -\rho - g_Y$, one gets $-g_\mu = g_A + \rho + g_{L_Y}$. Plugging these results in the first expression of $-g_\mu$, we obtain the following Bernoulli differential equation: $\dot{L}_Y = (\delta(1-\alpha)/\beta)L_Y^2 - \rho L_Y$. In order to transform this equation into a first-order linear differential equation, we consider the new variable $z = 1/L_Y$, which implies $\dot{z} = -\dot{L}_Y/L_Y^2$. The differential equation becomes $\dot{z} = \rho z - \delta(1-\alpha)/\beta$, whose solution is $z = e^{\rho t}\left[z_0 - \delta(1-\alpha)/\beta\rho\right] + \delta(1-\alpha)/\beta\rho$. Replacing z by $1/L_Y$ leads to $L_Y = \frac{1}{e^{\rho t}[1/L_{Y0} - \delta(1-\alpha)/\beta\rho] + \delta(1-\alpha)/\beta\rho}$. Using the transversality condition $\lim\limits_{t \longrightarrow +\infty} \mu A = 0$, one can show that L_Y immediately jumps to its steady-state level:

$$L_Y = \beta\rho/\delta(1-\alpha). \tag{40}$$

Indeed, using (34) it turns out that the transversality condition is only satisfied when $L_Y = L_{Y0} = \beta\rho/\delta(1-\alpha)$.

The optimal level of L_Y results from an arbitrage in the allocation of labor between production and research activities. The heuristic argument is the following. Suppose a marginal increase of labor in production, $\Delta L_{Yt} = 1$, at date t. This leads to an increase in production and thus in consumption: $\Delta Y_t = \Delta C_t = \beta Y_t/L_{Yt}$. Since $C_t = (1-\alpha)Y_t$, one gets $\Delta C_t = \beta C_t/(1-\alpha)L_{Yt}$, which yields the following increase in utility: $\Delta U_t = \Delta C_t/C_t = \beta/(1-\alpha)L_{Yt}$. Assume now $\Delta L_{At} = 1$, at date t. This leads to an increase in knowledge, ΔA_s, and thus in net production: $\Delta Y_s = (\partial Y_s/\partial A_s - x_s)\Delta A_s$, for all $s \geq t$. Since $\partial Y_s/\partial A_s = Y_s/A_s$, and $x_s = \alpha Y_s/A_s$, one gets $\Delta Y_s = (1-\alpha)Y_s \Delta As/A_s$. Moreover, $A_s = A_0 e^{\int_0^s \delta L_{Au} du}$, thus $dA_s = A_s \delta dL_{At} = \delta A_s$, for all $s \geq t$. This yields $\Delta Y_s = \delta(1-\alpha)Y_s$. Since $\Delta Y_s = \Delta C_s$ and $C_s = (1-\alpha)Y_s$, one gets $\Delta C_s = \delta C_s$. The increase in the instantaneous utility at s is thus δ. Finally, since $\int_0^{+\infty} e^{-\rho t} dt = 1/\rho$, we see from (10) that the increase in the intertemporal utility is equal to δ/ρ. Equating both increases in the intertemporal utility leads to $L_Y = \beta\rho/\delta(1-\alpha)$.

ii) Computation of ζ.

The solution for equation (39) is $\zeta = e^{\theta t}(\int_0^t \omega e^{-(\rho+\theta)s} ds + \zeta_0)$. Moreover, the transversality condition associated to Z writes

$\lim\limits_{t \longrightarrow +\infty} \zeta Z = \lim\limits_{t \longrightarrow +\infty} e^{\theta t}\left[\int_0^t \omega e^{-(\rho+\theta)s} ds + \zeta_0\right]\left[X_0 + \int_0^t P_s e^{\theta(s-t)} ds\right] = 0.$

We obtain $\zeta_0 = \int_0^{+\infty} (-\omega) e^{-(\rho+\theta)s} ds$, which gives $\zeta = e^{\theta t} \int_t^{+\infty} (-\omega) e^{-(\rho+\theta)s} ds = e^{-\rho t} \int_t^{+\infty} (-\omega) e^{-(\rho+\theta)(s-t)} d$

$= e^{-\rho t}\int_0^{+\infty}(-\omega)e^{-(\rho+\theta)u}du$. Finally, we get

$$\zeta = -\omega e^{-\rho t}/(\rho+\theta). \tag{41}$$

ζ is the discounted value at $t = 0$ of the social cost of one unit of carbon emitted at date t, expressed in terms of utility. This expression can be linked to the value of the optimal carbon tax at date t, measured in final good, in proposition 1: $\tau^o = [\omega(1-\alpha)/(\rho+\theta)]\, Y = -\zeta e^{\rho t}(1-\alpha)Y$.

iii) Computation of L_Q.

Using (41), (35) becomes $-\mu\delta A + \omega e^{-\rho t}h^{\eta}(1-\eta)R^{\eta}L_Q^{-\eta}/(\rho+\theta) = 0$. Using (32), (34) and (40), we get $\mu\delta A = \delta e^{-\rho t}/\rho$. Plugging this result into the preceding one, we get (11).

iv) Computation of R.

Using (36), (41) and (11), we obtain $R = \frac{\gamma}{\chi_0 e^{\rho t}+B}$, in which $B = \frac{(1-\alpha)\omega h}{\rho+\theta}\left[1-\eta\left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right]$. Since we have assumed $[\rho\omega(1-\eta)/\delta(\rho+\theta)]^{1/\eta} < 1$ at the beginning of this appendix, then $B > 0$.

We compute χ_0 using the constraint $\int_0^{+\infty} R_t dt = S_0$. We have $S_0 = \int_0^{+\infty}\frac{\gamma}{\chi_0 e^{\rho t}+B}dt = \int_0^{+\infty}\frac{\gamma e^{-\rho t}}{\chi_0+Be^{-\rho t}}dt$. Consider the new variable $u = \chi_0 + Be^{-\rho t}$, which gives $du = -\rho B e^{-\rho t}dt$. We have $S_0 = \int_{\chi_0+B}^{\chi_0}\frac{-\gamma}{\rho B}\frac{du}{u} = \frac{-\gamma}{\rho B}\ln\left(\frac{\chi_0}{\chi_0+B}\right)$. From this equation, one obtains $\chi_0 = B/(e^{\frac{B\rho S_0}{\gamma}}-1)$. Finally, we get $g^o_{Rt} = \frac{-\rho}{1+(e^{B\rho S_0/\gamma}-1)e^{-\rho t}}$.

v) Computation of Q and P.

Plugging (11) into $Q = (hR)^{\eta}L_Q^{1-\eta}$, one gets $Q = \left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta} hR$.

Then, using $P = hR - Q$, we have $P = \left[1-\left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right]hR$.

vi) Computation of x.

(1) can be rewritten as $Y = (Ax)x^{\alpha-1}L_Y^{\beta}R^{\gamma}$. Since $Ax = \alpha Y$ and using (40), we get $x = \alpha^{1/(1-\alpha)}(\beta\rho/\delta(1-\alpha))^{\beta/(1-\alpha)}R^{\gamma/(1-\alpha)}$.

vii) Computation of growth rates.

The growth rates directly follow from the log-differentiation of the preceding results.

In summary, one gets:

$$L^o_Y = \beta\rho/\delta(1-\alpha),$$

$$L^o_{Qt} = \left[\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right]^{1/\eta} hR^o_t,$$

$$L^o_{At} = 1 - L^o_Y - L^o_{Qt},$$

$$R_t^o = \frac{\gamma}{\chi_0 e^{\rho t} + B},$$

where $\chi_0 = B/(e^{\frac{B\rho S_0}{\gamma}} - 1)$ and $B = \frac{(1-\alpha)\omega h}{\rho+\theta}\left[1 - \eta\left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right]$,

$$Q_t^o = \left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta} hR_t^o,$$

$$P_t^o = \left[1 - \left(\frac{\rho\omega(1-\eta)}{\delta(\rho+\theta)}\right)^{(1-\eta)/\eta}\right] hR_t^o,$$

$$g_{At}^o = \delta L_{At}^o,$$

$$g_{Rt}^o = g_{L_{Qt}}^o = g_{Q_t}^o = g_{P_t}^o = \frac{-\rho}{1 + (e^{\frac{B\rho S_0}{\gamma}} - 1)e^{-\rho t}},$$

$$g_{Yt}^o = g_{At}^o + (\gamma/(1-\alpha))g_{Rt}^o.$$

Appendix 2: Welfare in the no-CCS case

When no CCS technology is available, maximizing welfare leads to the following results (recall that we denote by $X_t^{o\varnothing}$ the optimal level of any variable X_t in this case):

$L_Y^{o\varnothing} = \beta\rho/\delta(1-\alpha)$, $L_A^{o\varnothing} = 1 - \beta\rho/\delta(1-\alpha)$, $R_t^{o\varnothing} = \frac{\gamma}{\chi_0^{\varnothing} e^{\rho t} + B^{\varnothing}}$, $g_R^{o\varnothing} = \frac{-\rho}{1+(e^{\frac{B^{\varnothing}\rho S_0}{\gamma}} - 1)e^{-\rho t}}$, $g_A^{o\varnothing} = \delta L_A^{o\varnothing}$, $g_Y^{o\varnothing} = \delta L_A^{o\varnothing} + (\gamma/(1-\alpha))g_R^{o\varnothing}$, where $\chi_0^{\varnothing} = \frac{B^{\varnothing}}{e^{(B^{\varnothing}\rho S_0/\gamma)} - 1}$ and $B^{\varnothing} = (1-\alpha)\omega h/(\rho+\theta)$.

Since $B < B^{\varnothing}$, we have $g_{Rt}^o < g_{Rt}^{o\varnothing}$.

Appendix 3: Equilibrium in the decentralized economy

Here also, we drop time subscripts for notational convenience.

i) Computation of L_Y

In this paper, we focus on climate policy and its impacts on the economy. Hence we assume that research is optimally funded; in other words, we assume that both subsidies to research, s and σ, are set at their optimal levels. As in the standard case, the optimal level for the subsidy to the demand for intermediate goods, s, is $1-\alpha$. This can be shown as follows. Equation (3) shows that the marginal cost of x_i is equal to 1. Thus, the socially optimal price paid by the final sector, $p_i(1-s)$, must be equal to 1. From (23), the monopoly price is $p_i = 1/\alpha$ (> 1). Hence, we have $(1-s)/\alpha = 1$, that is, $s = 1-\alpha$.

The optimal value of the subsidy to research σ is obtained in what follows.

Equation (18), in which $p_i(1-s) = 1$ (from (23)), can be rewritten $Y = Ax/\alpha$. Since $Y = C + Ax$, one gets $C = (1-\alpha)Y$, as it is the case at the social optimum.

From (15) and (26), we have $r = \rho + g_C = g_V + \frac{\pi^m + \sigma}{V}$, where $g_C = g_Y$.

From (27) and (19), after log-differentiation, we get $g_V = g_w - g_A = g_Y - g_{L_Y} - g_A$. Moreover, from (19), (25) and (27), we obtain $\pi^m/V = \delta(1-\alpha)AxL_Y/\alpha\beta Y$; since $Ax = \alpha Y$, we get $\pi^m/V = \delta(1-\alpha)L_Y/\beta$. Plugging these two results into the expression of r given above yields $\rho = -g_{L_Y} - g_A + \delta(1-\alpha)L_Y/\beta + \sigma/V$. It is now obvious that, if $\sigma/V = g_A = \delta L_A$, the previous equation becomes the following Bernoulli differential equation $\dot{L}_Y = (\delta(1-\alpha)/\beta)L_Y^2 - \rho L_Y$. This equation is identical to the equation obtained in Appendix 1 (section i). We thus solve it in the same way, only this time we use the transversality condition of the household's program. One can show that L_Y immediately jumps to its steady-state level:

$$L_Y = \beta\rho/\delta(1-\alpha). \tag{42}$$

ii) Computation of L_Q, Q and P.

From (19), (21) and (42), we have $Y\delta(1-\alpha)/\rho = (\tau+\lambda)(1-\eta)(hR/L_Q)^\eta/(1-\varphi)$. This yields

$$L_Q = \left[\frac{(\tau+\lambda)\rho(1-\eta)}{(1-\varphi)\delta(1-\alpha)Y}\right]^{1/\eta} hR. \tag{43}$$

Plugging (43) into (6), we get

$$Q = \left[\frac{(\tau+\lambda)\rho(1-\eta)}{(1-\varphi)\delta(1-\alpha)Y}\right]^{(1-\eta)/\eta} hR. \tag{44}$$

Finally, (44) and (5) yield

$$P = \left[1 - \left(\frac{(\tau+\lambda)\rho(1-\eta)}{(1-\varphi)\delta(1-\alpha)Y}\right)^{(1-\eta)/\eta}\right] hR. \tag{45}$$

iii) Computation of R.

Basically, R is obtained from (20). In order to express R as a function of time and of the climate policy, we need to rewrite three elements of this equation. First, L_Q/hR is obtained from (43). Secondly, using (15) in which $g_C = g_Y$, we get $Y = Y_0 e^{\int_0^t (r_u - \rho)du}$. Finally, from (16), we have $p_R = p_{R0}e^{\int_0^t r_u du}$. Plugging these three results into (20) yields $R = \frac{\gamma}{(p_{R0}/Y_0)e^{\rho t} + G}$, where $G = \frac{h\tau}{Y} - \eta h \left(\frac{\rho(1-\eta)}{(1-\varphi)\delta(1-\alpha)}\right)^{\frac{1-\eta}{\eta}} \left(\frac{\tau+\lambda}{Y}\right)^{\frac{1}{\eta}}$. Since τ/Y and λ/Y are assumed constant, G is constant. Hence using the condition $\int_0^{+\infty} R_t dt = S_0$ and the method used in Appendix 1 (section iv), one gets:

$$R = \frac{\gamma}{\psi_0 e^{\rho t} + G}, \tag{46}$$

where $\psi_0 = G/(e^{\frac{G\rho S_0}{\gamma}} - 1)$.

iv) Computation of the rates of growth.

The growth rates directly follow from the log-differentiation of the preceding results. We obtain

$$g_A = \delta \left[1 - \frac{\beta\rho}{\delta(1-\alpha)} - \left[\frac{(\tau+\lambda)\rho(1-\eta)}{(1-\varphi)\delta(1-\alpha)Y} \right]^{1/\eta} hR \right], \tag{47}$$

$$\text{and } g_R = g_{L_Q} = g_Q = g_P = \frac{-\rho}{1 + (e^{\frac{G\rho S_0}{\gamma}} - 1)e^{-\rho t}}. \tag{48}$$

Finally, we know that $x_i \equiv x$ (see equation (24)) and $Y = Ax/\alpha$. Replacing x by its value in (1) gives $Y = \alpha^{\alpha/(1-\alpha)} A L_Y^{\beta/(1-\alpha)} R^{\gamma/(1-\alpha)}$. Thus we have

$$g_Y = g_A + (\gamma/(1-\alpha))g_R. \tag{49}$$

v) Impact of carbon tax.

In order to analyze the impact of a change in τ/Y on the economy, one has to study its impact on G. One gets: $(1/h)\frac{\partial G}{\partial(\tau/Y)} = 1 - \left(\frac{(\tau_t+\lambda_t)\rho(1-\eta)}{Y_t(1-\varphi)\delta(1-\alpha)}\right)^{(1-\eta)/\eta}$, which is positive, since we assume $0 \leq (\tau_t + \lambda_t)/Y_t \leq (1-\varphi)\delta(1-\alpha)/\rho(1-\eta)$. Then, using equations (28)-(30), and (44), (45), (48) and (49), the results described in proposition 2 follow.

vi) Impact of the subsidy to sequestered carbon.

Here, we have $\frac{\partial G}{\partial(\lambda/Y)} < 0$. As in subsection v), the results described in proposition 3 follow.

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