Renewable Portfolio Standards and implicit tax-subsidy schemes: Structural differences induced by quantity and proportional mandates

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Abstract

Renewable Portfolio Standards (RPS) are often used to promote renewable energy and to foster substitution for fossil energy sources. In practice, the minimal level of renewable energy to be supplied can be defined either as a ratio of the energy-mix (proportional mandate) or as an independent quantitative target (quantity mandate). The objective of this paper is to compare the consequences of the two types of quantitative mandates in terms of energy prices, support economic policies and carbon emissions. We thus extend the Chakravorty et al. (2006) model by considering, in addition to the carbon cap constraint, a RPS constraint (alternatively, a quantity or a proportional mandate). Our main results are the following. Independently of any carbon taxation, a quantity mandate requires a single subsidy on renewable energy to be enforced whereas a proportional mandate is equivalent to a tax-subsidy scheme that is revenue-neutral. Whatever its type, the mandate lowers the energy price and the social objective function, and it delays the date at which the carbon cap constraint is binding. If the two types of mandates are such that they yield the same social objective value then, the quantity mandate implies a lower energy price, a larger subsidy on renewable energy and a smaller fossil fuel tax (including the carbon tax) than the proportional mandate.

Keywords: Renewable Portfolio Standard; Carbon stabilization cap; Fossil energy; Renewable energy; Tax-subsidy scheme.

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1 Introduction

Including power generation, transport and industry, the energy sector accounts for 65% of total greenhouse gases emissions in 2006 (Stern, 2007). More particularly, the largest source of carbon emissions is due to electricity generation and heat, accounting for 41% of emissions worldwide according to the International Energy Agency (IEA, 2008). In this context, substituting renewable energy sources for fossil energy sources has, at least, two advantages. First, it relaxes the availability constraint on the fossil non-renewable resource stocks. Second, since renewable energy sources are the most often carbon-free, such a substitution constitutes a key technological option to reduce carbon emissions in the energy sector and to satisfy some mitigation targets.[1]

The main break to a massive development of technologies based on renewable energy sources is their cost as compared with traditional fossil fuels.[2] Since they are more costly, renewable energy sources currently represent only a small portion of the energy portfolio, that is 19% including hydropower (IEA, 2008). However, as underlined by Popp et al. (2011), while the costs of these technologies are higher than other fuels, they have also been falling. On the one hand, competitiveness of renewable energies may grow since the traditional fossil energies are expected to be more expensive in the future because of their scarcity. On the other hand, the fall in their costs which is observed since the 1980s (in particular for solar PV) is mainly due to a technical change effect, both through learning processes and R&D activities. The gap between renewable and fossil energies is then expected to be gradually reduced, but this will take time. Consequently, specific policies to promote renewable energy technologies are needed to accelerate the energy transition (Kalkhul et al., 2011).

Many renewable energy promoting policies take the form of quantitative mandates or, equivalently of Renewable Portfolio Standards (RPS). This regulation requires the increased production of energy from renewable energy sources and aims at reaching some given target by a pre-definite future date. Apart from its contribution to climate policy (through lower carbon emissions per unit of energy), the main motive of RPS is the security

[1] Another mitigation option would consist in maintaining the fossil fuel consumption and reducing its carbon print by using some abatement devices, such as CCS or air capture (see Lafforgue et al., 2008, or Amigues et al., 2011).

[2] For wind and geothermal technologies, the extra-cost of generation is near \$0.05/kWh in the most favorable locations, and larger elsewhere. This extra-cost rises to \$0.20-0.30/kWh for solar PV (IEA, 2006).

of energy supply since it allows for decreasing the dependence on foreign fossil energy deliveries (Boeters and Koornneef, 2011).

Technically, the type of obligation placed on the energy supplier can be twofold since the associated mandate can be defined either in quantity or in proportion. In practice, both types of mandates have been unclearly adopted. In United States, the 2005 Energy Policy Act established a Renewable Fuel Standard (RFS) that mandates specific quantity targets for biofuel use (36 billion gallons by 2022). For electricity, US Renewable Electricity Standards (RES) policies are implemented at the federal level. Reviewing state-by-state RES programs, it is clear that the norm is to differentiate support by technology type, but without any harmonization about the type of mandates. Hence, the Michigan's target is to produce 300MW of renewable electricity by 2013 and 600MW by 2015 whereas Colorado committed to reach 30% of renewable electricity by 2020 for instance. Even at the international level, RPS are not harmonized. China adopted in 2009 a renewable energy target aiming at producing 500GW of renewable electricity by 2020 (300 from hydro, 150 from wind, 30 from biomass and 20 from solar PV). In 2007, the EU Directive on Energy Production from Renewable Sources committed itself to target of 20% of renewable energy in the total EU energy consumption by 2020, and of 33% of renewable electricity.

Even if the quantitative objectives induced by quantity and proportional mandates can seem to be equivalent, differences in their direct economic consequences, both in quantities and prices, are not harmless. These two approaches contrast at least in two points. The first difference concerns the definition and the implementation of the critical support economic policies induced by each type of mandate. A quantity mandate requires a single instrument to be enforced: a subsidy for renewable energy use (Vedenov and Wetzstein, 2008, Galinato and Yoder, 2011).[3] In contrast, a proportional mandate requires a combination of fossil fuel taxes and renewable energy subsidies that are revenue neutral (Lapan and Moschini, 2011). This tax-subsidy scheme is equivalent to a feed-in-tariff: a tax on fossil energy is used to cross-finance a subsidy on renewable energy and thus yields an income-neutral policy for the government (Kalkhuhl et al. 2011). Taking into account these differences, the energy consumer-price may vary according to the type of mandate.

The second difference lies in the dynamic properties on the induced energy-mix. Whereas non-renewable and renewable energy uses are disconnected under a quantity mandate, they are linked with under proportional device. In particular, if the proportional target is con-

[3]In the case of biofuels in the US, Vedenov and Wetzstein (2008) remark that, historically, these subsidies are funded from general tax funds, mostly from income and labor taxes, and not from fuel tax revenus.

stant over time, as long as renewable energy is still more expensive than the non-renewable one, the energy producer will just supply the minimal amount of renewable required to satisfy the mandate. This may lead to a situation where both primary energy consumptions are declining over time, the fossil energy because of the standard Hotelling scarcity effect, and the renewable one because of its proportionality link with the fossil energy use.

The economic literature on RPS mentioned above is quite recent and analyzes separately these two kinds of mandates, mainly through simulated models. To our knowledge, there does not exist any comparative study of quantity and proportional mandates. Nevertheless, such a comparative analysis would be useful, in a policy harmonization objective, to answer to some crucial questions such as: i) Which type of mandate is the cheaper for the energy user? For the energy producer? For the social planner? What are their impacts on the fossil fuel consumption path (and then on carbon emissions)? The objective of our paper is to provide answers to these questions.

We use the Chakravorty et al. (2006) model to determine the optimal exploitation time-paths of two primary energy sources that are perfect substitutes, a fossil energy and a renewable one. These optimal paths are considered along with the two following features. First, the cumulative atmospheric pollution stock is set not to exceed some critical threshold. Second, the renewable energy use is constrained by a mandate which is constant over time and defined ab-initio. This mandate can be defined either in quantity or proportionally. We do not discuss about the normative properties of the carbon cap and the RPS. We simply assumed that they are recommended by an independent expert and that they must be taken as given by the agents. Then, the policy-maker must implement a carbon tax to enforce the carbon bank constraint, and, alternatively, a single subsidy on renewable energy use to enforce the quantity mandate, or a simultaneous tax-subsidy scheme on the energy-mix to enforce the proportional mandate.

The main results of the paper are the following. i) Whatever the type of mandate, the optimal subsidy on renewable energy is declining over time. ii) Under a proportional mandate, an additional penalty on fossil energy, proportional to the subsidy, must be added to the carbon tax. The resulting global tax on fossil fuel, which combines this penalty with the traditional carbon tax, can exhibit unusual non-monotonous time paces as long as the atmospheric carbon cap is not reached. iii) Whatever the type of mandate, a RPS lowers the energy price, reduces the social objective function and delays the date at which the carbon cap constraint is binding. iv) If the two types of mandates are such that they yield the same social objective value then, with an inelastic energy demand function and as long as the carbon cap is not reached, a quantity mandate implies a lower energy price, a larger subsidy on renewable energy and a smaller fossil fuel tax (including the carbon tax) than a proportional mandate.

The paper is organized as follows. Section 2 presents the model and lays down the social planner program under each type of mandate. In section 3, we derive the optimal solution under the quantity mandate. In particular, we characterize the content and the time pace of the optimal energy-mix, the energy price trajectory and the policy-mix allowing to sustain this optimal path. The case of a proportional mandate is examined in section 4. In section 5, we develop a comparative analysis of the two types of mandates in the case where the energy demand function is inelastic. Finally, we briefly conclude in section 6.

2 Model and notations

We consider a stationary economy in which the final consumption good is a bundle of energy services, which can be produced from two primary natural resources: A carbon-emitting non-renewable resource, oil, and a carbon-free renewable resource, solar.

Fossil resource

Let us denote by X^0 the initial oil endowment of the economy, by $X(t)$ the remaining part of this endowment at time t, and by $x(t)$ the instantaneous consumption flow of oil, so that:

$$\dot{X}(t) = -x(t), \quad \text{with } X(0) = X^0,\ X(t) \geq 0 \tag{1}$$

$$x(t) \geq 0 \tag{2}$$

The average delivery cost of oil c_x is assumed to be constant and, absent any sunk cost, equal to the marginal cost. This cost includes the extraction cost of the resource, the cost of industrial processing (crude oil refining) and the transportation cost. To keep matter as simple as possible, we also assume that no oil is lost during the delivery process. Equivalently, the oil stock $X(t)$ may be understood as measured in ready-for-use units.

Carbon emissions and atmospheric carbon stock

Let $Z(t)$ be the stock of carbon within the atmosphere at time t, and Z^0 be the initial stock. The atmospheric carbon stock is fed by carbon emission flows resulting from oil burning. We denote by ζ the quantity of carbon by unit of oil which is released into the atmosphere after combustion. The atmospheric carbon stock is assumed to be self-regenerating at some constant proportional rate α, $\alpha > 0$, so that the dynamics of $Z(t)$ results into:

$$\dot{Z}(t) = \zeta x(t) - \alpha Z(t), \quad \text{with } Z(0) = Z^0 \geq 0 \tag{3}$$

We assume that a carbon cap policy is prescribed to prevent any catastrophic damages which would be infinitely costly for the society. This policy consists in forcing the atmospheric carbon stock to not overshoot some critical level $\bar{Z}$, $\bar{Z} > Z^0$, resulting in the following constraint upon the state variable $Z(t)$:

$$\bar{Z} - Z(t) \geq 0 \tag{4}$$

To satisfy the constraint when the atmospheric carbon stock reaches its critical level, the oil consumption $x(t)$ must be at most equal to $\bar{x} = \alpha\bar{Z}/\zeta$, the oil consumption rate generating an emission flow that is balanced by natural regeneration.

Carbon-free renewable resource

The alternative energy source is supplied by the carbon-free renewable resource, the solar energy. Let $y(t)$ be the solar energy instantaneous consumption rate and c_y its average delivery cost. Because c_x and c_y both include all the costs necessary to deliver ready-for-use energy services to the potential users, both resources may be seen as perfect substitutes for the users, so that we may define the instantaneous aggregate energy consumption rate as $q(t) = x(t) + y(t)$, provided that the costs c_x and c_y are incurred.

The average cost c_y is assumed to be constant and higher than c_x. We also assume that the natural flow of available solar energy, denoted by y^n, is large enough so that no rent has ever to be imputed for its use. Denoting by $\tilde{y}$ the solar energy consumption rate it would be optimal to consume at the marginal cost c_y, we thus assume $y^n > \tilde{y}$.

Renewable Portfolio Standards (RPS)

The renewable energy promoting policy takes the form of a RPS and we alternatively consider the cases of a quantity mandate and a proportional mandate. Each type of mandate implies a distinguishing additional constraint on the energy-mix. For the sake of simplicity, we restrict the analysis to the case where the two types of constraints must be satisfied *ab initio*.[4]

- **Quantity mandate:**

  This first type of RPS requires that some constant minimal amount of renewable energy $\underline{y}$, $\underline{y} > 0$, has to be supplied at any point of time, which results in the following constraint:

$$y(t) - \underline{y} \geq 0 \tag{5}$$

  We assume that $\underline{y} < \tilde{y}$, so that the mandate is smaller than the solar consumption rate when oil is exhausted.

- **Proportional mandate:**

  The proportional mandate requires that some constant minimal proportion σ, $\sigma \in (0, 1)$, of the energy services have to be supplied by the solar energy: $y(t) \geq \sigma[x(t) + y(t)]$. Equivalently, defining θ as the ratio $\sigma/(1-\sigma)$ of energy supply from renewable source to energy supply from fossil source, this constraint may be rewritten as:

$$y(t) - \theta x(t) \geq 0 \tag{6}$$

Gross surplus

The instantaneous gross surplus derived from the instantaneous energy consumption rate $q(t)$ is given by some function $u(q)$ satisfying the following standard assumptions. Function $u(.)$, $u : \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{C}^2$, strictly increasing, strictly concave and verifies the Inada conditions: $\lim_{q \downarrow 0} u'(q) = +\infty$ and $\lim_{q \uparrow +\infty} u'(q) = 0$. We denote by $p(q) = u'(q)$ the marginal gross surplus function, that is the energy consumer price, and by $q^d(p) = p^{-1}(p)$ its inverse, that is the direct demand function.

We also assume that the solar marginal cost c_y is high enough so that solar energy is not competitive without any specific promoting policy during any phase at the ceiling. Denoting by $\bar{p}$ the energy consumer price at the ceiling when energy services are only supplied by oil, that is $\bar{p} = u'(\bar{x})$, then: $\bar{p} < c_y$.

In the last section, we will restrict the above general framework to the case of an inelastic demand function over the price range within which the energy price must evolve along any

[4] More detailed scenarios in which some targeting is introduced by assuming that the RPS constraint must satisfied at some later date can also be derived. They are available upon request.

optimal path. The lowest and highest benchmarks of this price will be determined by the supply side of the model. This simplification will allow to provide a comparison of the two types of mandates without changing the main analytical properties of the optimal paths.

Social discount rate and program of the social planner

We assume that the instantaneous social discount rate, denoted by ρ, is constant and strictly positive, $\rho > 0$. The program of the social planner consists in determining the trajectories of $x(t)$ and $y(t)$ that maximize the sum of the discounted net surplus:

$$\max_{\{x(t),y(t)\}} \int_0^\infty \{u\,[x(t)+y(t)] - c_x x(t) - c_y y(t)\}\, e^{-\rho t} dt$$

subject to (1)-(4) and, alternatively, to (5) or (6).

Let us denote by λ_X the costate variable of the state variable X, by λ_Z minus the costate variable of Z, by η the Lagrange multiplier associated with the ceiling constraint on Z, by ν the Lagrange multiplier associated with the constraint on the solar energy (quantity or proportional mandate) and by γ's the Lagrange multipliers associated with the non-negativity constraints on the command variables.

3 Optimal solution under the quantity mandate

3.1 Optimal conditions

Under the nominal quantity mandate, resulting in constraint (5) on the solar energy use, the current valued Lagrangian $\mathcal{L}^{qm}$ of the optimal program writes as:

$$\begin{aligned}\mathcal{L}^{qm}(t) &= u[x(t)+y(t)] - c_x x(t) - c_y y(t) - \lambda_X(t)x(t) - \lambda_Z(t)[\zeta x(t) - \alpha Z(t)] \\ &\quad + \eta(t)[\bar{Z} - Z(t)] + \nu(t)[y(t) - \underline{y}] + \gamma_x(t)x(t)\end{aligned}$$

The first-order conditions relative to the command and to the state variables are:

$$\frac{\partial \mathcal{L}^{qm}}{\partial x} = 0 \quad\Rightarrow\quad u'[x(t)+y(t)] = c_x + \lambda_X(t) + \zeta\lambda_Z(t) - \gamma_x(t) \tag{7}$$

$$\frac{\partial \mathcal{L}^{qm}}{\partial y} = 0 \quad\Rightarrow\quad u'[x(t)+y(t)] = c_y - \nu(t) \tag{8}$$

$$\dot{\lambda}_X = \rho\lambda_X - \frac{\partial \mathcal{L}^{qm}}{\partial X} \quad\Rightarrow\quad \dot{\lambda}_X(t) = \rho\lambda_X(t) \tag{9}$$

$$\dot{\lambda}_Z = \rho\lambda_Z + \frac{\partial \mathcal{L}^{qm}}{\partial Z} \quad\Rightarrow\quad \dot{\lambda}_Z(t) = (\rho+\alpha)\lambda_Z(t) - \eta(t) \tag{10}$$

The associated complementary slackness conditions are:

$$\gamma_x(t) \geq 0, \quad x(t) \geq 0 \quad \text{and} \quad \gamma_x(t)x(t) = 0 \tag{11}$$

$$\nu(t) \geq 0, \quad y(t) - \underline{y} \geq 0 \quad \text{and} \quad \nu(t)[y(t) - \underline{y}] = 0 \tag{12}$$

$$\eta(t) \geq 0, \quad \bar{Z} - Z(t) \geq 0 \quad \text{and} \quad \eta(t)[\bar{Z} - Z(t)] \geq 0 \tag{13}$$

Last, the transversality conditions write as:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) = 0 \tag{14}$$

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) = 0 \tag{15}$$

Due to the number of inequality constraints, all captured by the complementary slackness conditions (11)-(13), the resulting number of possibilities to be analyzed is quite large. The following preliminary remarks help to eliminate many of them and to directly draw the optimal solution.[5]

Remark 1

The shadow marginal value $\lambda_X(t)$ of the stock of oil, or mining rent, must grow at the social rate of discount ρ. From (9), we get: $\lambda_X(t) = \lambda_{X0}e^{\rho t}$, with $\lambda_{X0} = \lambda_X(0)$. Thus the transversality condition (14) reduces to $\lambda_{X0}\lim_{t\uparrow\infty} X(t) = 0$. If oil has some positive initial value, $\lambda_{X0} > 0$, then it must be exhausted along the optimal path.

Remark 2

Concerning the shadow marginal cost of the atmospheric carbon stock $\lambda_Z(t)$, note that, as long as the atmospheric carbon cap is not attained yet, we must have $\eta(t) = 0$ from (13) and then (10) reduces to $\dot{\lambda}_Z(t) = (\rho + \alpha)\lambda_Z(t)$. Once the ceiling is definitively left, this shadow cost is nil since the economy is no more facing any stabilization constraint. Thus, denoting respectively by $\underline{t}_Z$ and $\bar{t}_Z$ the date at which the ceiling constraint is beginning to be active and the latest date at which $Z(t) = \bar{Z}$, we may have:

$$t < \underline{t}_Z \quad \Rightarrow \quad \lambda_Z(t) = \lambda_{Z0}e^{(\rho+\alpha)t}, \ \lambda_{Z0} = \lambda_Z(0)$$

$$t \geq \bar{t}_Z \quad \Rightarrow \quad \lambda_Z(t) = 0$$

[5]The multiplicity of solutions is a recurrent problem in this strand of literature already pointed out by Tahvonen (1997).

Remark 3

The full marginal cost of oil, as given by the right-hand-side of (7), includes the delivery cost c_x, the resource rent $\lambda_X(t)$ and the environmental shadow cost $\zeta\lambda_Z(t)$ in order to enforce the ceiling constraint. The right-hand-side of (8) gives the full marginal cost of solar energy, which writes as the delivery cost c_y diminished by a "grant" $\nu(t)$ to enforce the constraint on the energy-mix. From (7) and (8), as long as the two primary energy sources are simultaneously used and since they are perfect substitutes, these two cost expressions must be equal to the marginal gross surplus of consuming energy: $c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_Z(t) = c_y - \nu(t) = u'[x(t) + y(t)]$.

During a phase of simultaneous oil and solar consumption before the ceiling is attained, since both $\lambda_X(t)$ and $\lambda_Z(t)$ are increasing (see remarks 1 and 2), we must have $\nu(t) > 0$ so that $y(t) = \underline{y}$. The same conclusion can be drawn for a simultaneous use of both types of energy after the ceiling is definitively left since $\lambda_Z(t)$ is nil and $\lambda_X(t)$ is still growing over time. Finally, considering the case of a simultaneous use of oil and solar energy during a phase at the ceiling, we must have: $u'(\bar{x} + y(t)) = c_y - \nu(t)$. If $y(t) > \underline{y}$, then $\nu(t) = 0$ and we would get $u'(\bar{x} + y(t)) = c_y$, which is clearly not possible since $u'(\bar{x} + y(t)) < u'(\bar{x}) < c_y$ by assumption. We conclude that, as long as oil is not exhausted yet, the supplying flow of solar energy must be equal to the quantity mandate $\underline{y}$.

Remark 4

Since oil is cheaper than solar, minimizing the sum of discounted cost flows implies that this energy may be used as much as possible. As a result, the energy-mix consumption policy consists in two regimes: i) a first period $[0, \bar{t}_x)$ during which both primary energy sources are simultaneously used, the amount of solar energy being fixed to $\underline{y}$ and the fossil resource being exhausted at time $\bar{t}_x$ and ii) a second period $[\bar{t}_x, \infty)$ during which the energy consumption is only supplied by the solar energy flow $y(t) = \tilde{y} = q^d(c_y) > \underline{y}$.

The different phases characterizing the optimal trajectory thus differ depending on whether the ceiling constraint is binding or not and depending on whether the two primary energies are simultaneously used or not. We get four successive phases, separated by dates $\underline{t}_Z$, $\bar{t}_Z$, and $\bar{t}_x$, with $\underline{t}_Z < \bar{t}_Z \leq \bar{t}_x$.

3.2 Description of a typical optimal path

The solution is similar to the optimal path derived in Chakravorty et al. (2006) for a constant structure of costs and a stationary demand, except that the solar energy use is now constrained by the RPS policy to be at least equal to $\underline{y}$. This implies that an incentive subsidy $\nu(t)$ on the solar energy use must be enforced as long as the full marginal cost of the energy-mix remains lower than the trigger price c_y, that is as long as solar energy is not competitive yet. This subsidy is optimally set such that the respective full marginal costs of using each type of primary resource are equal. The resulting optimal solution is a four-phase path as illustrated in figure 1 in the case where the initial oil endowment is large enough to trigger the binding of the ceiling constraint. The entire characterization of this solution depends upon the five variables λ_{X0}, λ_{Z0}, $\underline{t}_Z$, $\bar{t}_Z$ and $\bar{t}_x$ whose determination is detailed in Appendix A.1.

[Figure 1 here]

The starting phase, for $t \in [0, \underline{t}_Z)$, takes place before the atmospheric carbon stabilization cap is reached. During this phase, both energy sources are simultaneously used and the energy consumer price is equal to the full marginal cost of using oil: $p(t) = c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$. This price is growing over time at a rate which is larger than the social discount rate ρ. Since solar energy consumption is constant and equals to $\underline{y}$, oil consumption writes as the remaining part of the energy demand: $x(t) = [q^d(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}) - \underline{y}]$. This phase ends at time $\underline{t}_Z$, when the energy price equals $u'(\bar{x} + \underline{y})$ or, equivalently, when the atmospheric carbon stock attains the ceiling level $\bar{Z}$.

The second phase, for $t \in [\underline{t}_Z, \bar{t}_Z)$, is a phase during which the atmospheric carbon stock is constrained by the ceiling: $Z(t) = \bar{Z}$. Minimizing the energy production cost means that oil must be used as far as possible: $x(t) = \bar{x}$. The solar energy consumption still amounts to $\underline{y}$ and the energy price is thus constant: $p(t) = u'(\bar{x} + \underline{y})$. This implies that the environmental cost of carbon emissions, $\zeta\lambda_Z(t) = u'(\bar{x} + \underline{y}) - c_x - \lambda_{X0}e^{\rho t}$, is decreasing: $\zeta\dot{\lambda}_Z(t) = -\rho\lambda_{X0}e^{\rho t} < 0$. The phase ends at time $\bar{t}_Z$ when this cost is nil and when, simultaneously, the atmospheric carbon stock starts to fall down below $\bar{Z}$.

The third phase, for $t \in [\bar{t}_Z, \bar{t}_x)$, is a phase during which the environmental constraint is not active anymore and the fossil resource extraction runs until exhaustion. The energy consumer price is $p(t) = c_x + \lambda_{X0} e^{\rho t}$ and the energy-mix consists of $x(t) = [q^d(c_x + \lambda_{X0} e^{\rho t}) - \underline{y}]$ units of oil and $y(t) = \underline{y}$ units of solar. The phase ends at time $\bar{t}_x$, when the energy price reaches c_y. At this point of time, the fossil resource must be exhausted and the solar energy consumption must be equal to $q^d(c_y) = \tilde{y}$. Consequently, the oil consumption trajectory makes a downward jump from $(\tilde{y} - \underline{y})$ to 0 and the solar consumption trajectory, an upward jump from $\underline{y}$ to $\tilde{y}$. These discontinuities occur because of the constant structure of costs and RPS which is assumed in the model.

The last phase, for $t \in [\bar{t}_x, \infty)$, is a pure solar energy consumption regime with a constant energy price c_y and a corresponding energy consumption $\tilde{y}$. Since solar is becoming competitive, no specific subsidy is required anymore.

3.3 The optimal policy-mix

As discussed in the previous subsection, a second level of distortion, in addition to the environmental externality, is introduced by the RPS policy which imposes, in the case of a quantity mandate, a minimal amount of solar energy consumption. The policy-mix induced by these two constraints, the atmospheric carbon cap and the mandate, is characterized in the following proposition.

Proposition 1 *Under the nominal quantity mandate, the policy-mix allowing to sustain the optimal path consists of:*

1. *A unitary subsidy $\nu(t)$ on solar energy consumption, whose time pace is given by:*

$$\nu(t) = \begin{cases} c_y - [c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}] & t \in [0, \underline{t}_Z) \\ c_y - u'(\bar{x} + \underline{y}) & t \in [\underline{t}_Z, \bar{t}_Z) \\ c_y - \big(c_x + \lambda_{X0} e^{\rho t}\big) & t \in [\bar{t}_Z, \bar{t}_x) \\ 0 & t \in [\bar{t}_x, \infty) \end{cases} \tag{16}$$

2. *A unitary carbon tax $\zeta \lambda_Z(t)$ on oil consumption, whose time pace is given by:*

$$\zeta \lambda_Z(t) = \begin{cases} \zeta \lambda_{Z0} e^{(\rho+\alpha)t} & t \in [0, \underline{t}_Z) \\ u'(\bar{x} + \underline{y}) - \big(c_x + \lambda_{X0} e^{\rho t}\big) & t \in [\underline{t}_Z, \bar{t}_Z) \\ 0 & t \in [\bar{t}_Z, \infty) \end{cases} \tag{17}$$

As long as the solar energy is not competitive yet, the unitary subsidy $\nu(t)$ to promote this type of energy, which is given by (16), simply writes as the difference between the energy price level c_y at which solar becomes competitive and the current energy price, as shown in figure 1. Since this price is always increasing over time and since c_y is constant, the optimal subsidy rate must be non-increasing. At the end of the third phase, when the fossil resource is fully exhausted, solar energy becomes competitive even without promoting policy and the subsidy is nil.

As usual in dynamic climate models in which the environmental damages are captured by a ceiling constraint on the atmospheric carbon stock, the optimal carbon tax, given by (17), corresponds to the shadow marginal cost of the carbon stock. Graphically (see figure 1), this tax is measured by the gap between the current energy consumer price path and the current producer price path $p(t) = c_x + \lambda_{X0}e^{\rho t}$, or, equivalently, the marginal delivery cost of oil augmented by the resource rent. Its shape is similar to the one obtained in Chakravorty et al. (2006). As long as the ceiling constraint is not binding, the carbon tax is increasing at a rate which is larger than ρ in order to take into account the natural regeneration of the atmosphere at a constant rate α. It is next decreasing during the phase at the ceiling and finally, it falls to zero once the ceiling is definitively left.

The optimal time profiles of $\nu(t)$ and $\zeta\lambda_Z(t)$ are depicted in figure 2.

[Figure 2 here]

4 Optimal solution under the proportional mandate

4.1 Optimal conditions

Under the proportional mandate, constraint (5) on the energy-mix composition must be replaced by (6) and the current valued Lagrangian of the optimal program is:

$$\begin{aligned}\mathcal{L}^{pm}(t) &= u[x(t)+y(t)] - c_x x(t) - c_y y(t) - \lambda_X(t)x(t) - \lambda_Z(t)[\zeta x(t) - \alpha Z(t)] \\ &\quad + \eta(t)[\bar{Z} - Z(t)] + \nu(t)[y(t) - \theta x(t)] + \gamma_x(t)x(t)\end{aligned}$$

The first-order conditions (7) and (8) become, respectively:

$$u'[x(t)+y(t)] = c_x + \lambda_X(t) + \zeta\lambda_Z(t) + \theta\nu(t) - \gamma_x(t) \tag{18}$$

$$u'[x(t)+y(t)] = c_y - \nu(t) \tag{19}$$

and the complementary slackness condition (12) is replaced by:

$$\nu(t) \geq 0, \quad y(t) - \theta x(t) \geq 0 \quad \text{and} \quad \nu(t)[y(t) - \theta x(t)] = 0 \tag{20}$$

The other conditions remain unchanged, as well as remarks 1 and 2.

As compared with the quantity mandate case, the main change induced by the proportional mandate is that the constraint on the energy-mix now also relies on oil consumption. Remark 3 is thus modified in the following terms: i) The full marginal cost of oil, as given by the right-hand-side of (18), includes an additional "penalty" component $\theta\nu(t)$ to promote solar energy, in addition to the other usual terms. ii) This penalty writes as the Lagrange multiplier associated with the RPS constraint, multiplied by the relative weight θ of solar energy in the energy-mix. From (19), the full marginal cost of solar must be diminished by a unitary subsidy $\nu(t)$, as in the quantity mandate case. In other words, using both types of primary energy, which implies the equality of their full marginal costs, requires to simultaneously subsidize solar at rate $\nu(t)$ as long as this energy source is not competitive yet, and to tax oil at rate $\theta\nu(t)$, irrespective of the usual carbon tax required to enforce the constraint on the atmospheric carbon accumulation.

Next, it can be shown that the RPS constraint (6) must be binding until oil exhaustion. The proof calls for the same arguments as in the quantity mandate case. Hence, as long as the two primary energy resources are simultaneously used, solar energy must be proportional to oil, $y(t) = \theta x(t)$, and, from (18) and (19), we must have $\nu(t) = (1-\sigma)\{c_y - [c_x + \lambda_X(t) + \zeta\lambda_Z(t)]\}$.

Finally, note that, as in Lapan and Moschini (2011), the above tax-subsidy scheme is revenue-neutral at each point of time. By construction, for any t, the tax burden $\theta\nu(t)x(t)$ must balance the amount of subsidy $\nu(t)y(t)$ since $y(t) = \theta x(t)$. However, this does not mean that tax and subsidy rates coincide. Actually they are equal if and only if $\theta = 1$ or, equivalently, if $\sigma = 50\%$.

4.2 Description of a typical optimal path

A first phase A typical optimal path is a four-phase path as illustrated in figure 3 in the case where the reserves of oil are sufficiently large to guarantee that the atmospheric carbon cap is attained. The details of its entire characterization are provided in Appendix A.2.

[Figure 3 here]

During the first phase, for $t \in [0, \underline{t}_Z)$, the ceiling constraint is not binding yet and the two primary energy sources are simultaneously used. The energy price $p(t)$ writes as the sum of two terms. The first one is the marginal cost σc_y of σ% of the energy-mix supplied by solar and the second one, the marginal cost $(1-\sigma)[c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}]$ of $(1-\sigma)$% of the energy-mix supplied by oil. Given this price $p(t)$, the energy-mix content consists of $\sigma q^d(p(t))$ units of solar energy and $(1-\sigma)q^d(p(t))$ units of oil. Since the energy price is growing over time, the total energy consumption must be declining, meaning that oil and solar energy are both decreasing during this phase. A declining oil extraction path is a standard result in the Hotelling literature, when a decreasing renewable resource consumption is less usual. It is in fact a direct implication of the specific RPS which is under consideration here and which links proportionally the consumption of the two types of primary energy sources **(insérer ici détails arbitrage)**.

The second phase, for $t \in [\underline{t}_Z, \bar{t}_Z)$, is a phase at the ceiling during which the energy price is constant: $p(t) = u'(\bar{x}/(1-\sigma))$. Oil and solar consumptions are then also constant and amount, respectively, to $\bar{x}$ and $\theta\bar{x}$.

During the third phase, for $t \in [\bar{t}_Z, \bar{t}_x)$, the ceiling constraint is not active anymore and the fossil resource must be fully exhausted at the end of the phase. The energy price is $p(t) = \sigma c_y + (1-\sigma)[c_x + \lambda_{X0}e^{\rho t}]$. As in the first phase, this price reads as the sum of the marginal cost σc_y of σ% of the energy-mix supplied by solar and the marginal cost $(1-\sigma)(c_x + \lambda_{X0}e^{\rho t})$ of $(1-\sigma)$% of the energy-mix supplied by oil. The energy-mix consists of $(1-\sigma)q^d(p(t))$ units of oil and $\sigma q^d(p(t))$ units of solar and these consumption trajectories are both decreasing through time.

Last, the fourth phase, for $t \in [\bar{t}_x, \infty)$, is the same type of pure solar energy consumption regime as described in the quantity mandate case.

4.3 The optimal policy-mix

The policy-mix required to implement the optimal path as described above is characterized in the following proposition.

Proposition 2 *Under the proportional mandate, the policy-mix allowing to sustain the optimal path consists of:*

1. A subsidy rate $\nu(t)$ on solar energy consumption:

$$\nu(t) = \begin{cases} (1-\sigma)\left\{c_y - \left[c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}\right]\right\} & t \in [0, \underline{t}_Z) \\ c_y - u'\left(\frac{\bar{x}}{1-\sigma}\right) & t \in [\underline{t}_Z, \bar{t}_Z) \\ (1-\sigma)\left[c_y - \left(c_x + \lambda_{X0}e^{\rho t}\right)\right] & t \in [\bar{t}_Z, \bar{t}_x) \\ 0 & t \in [\bar{t}_x, \infty) \end{cases} \tag{21}$$

2. A global tax rate $\tau(t)$ on fossil energy use:

$$\tau(t) = \begin{cases} \sigma\left[c_y - (c_x + \lambda_{X0}e^{\rho t})\right] + (1-\sigma)\zeta\lambda_{Z0}e^{(\rho+\alpha)t} & t \in [0, \underline{t}_Z) \\ u'\left(\frac{\bar{x}}{1-\sigma}\right) - (c_x + \lambda_{X0}e^{\rho t}) & t \in [\underline{t}_Z, \bar{t}_Z) \\ \sigma\left[c_y - (c_x + \lambda_{X0}e^{\rho t})\right] & t \in [\bar{t}_Z, \bar{t}_x) \\ 0 & t \in [\bar{t}_x, \infty) \end{cases} \tag{22}$$

As in the quantity mandate case, the unitary solar subsidy $\nu(t)$, as given by (21), can be graphically measured by the gap between c_y and the current energy price level $p(t)$ (see Figure 3). Its trajectory exhibits the same time pace as the one depicted by figure 2.

The fossil energy tax $\tau(t)$ expressed in (22) writes as the sum of two tax rates. The first one is the usual carbon tax $\zeta\lambda_Z(t)$ whose time profile is given by:

$$\zeta\lambda_Z(t) = \begin{cases} \zeta\lambda_{Z0}e^{(\rho+\alpha)t} & t \in [0, \underline{t}_Z) \\ \frac{1}{(1-\sigma)}\left[u'\left(\frac{\bar{x}}{1-\sigma}\right) - \sigma c_y - (1-\sigma)(c_x + \lambda_{X0}e^{\rho t})\right] & t \in [\underline{t}_Z, \bar{t}_Z) \\ 0 & t \in [\bar{t}_Z, \infty) \end{cases} \tag{23}$$

Its shape is similar to the one obtained in the quantity mandate case and illustrated in figure 2. The second component is the additional penalty $\theta\nu(t)$ on oil consumption required to promote solar energy. The resulting global tax is then $\tau(t) = \zeta\lambda_Z(t) + \theta\nu(t)$. It can be graphically identified in figure 3 as the gap between the current energy consumer price $p(t)$ and the oil producer price $c_x + \lambda_{X0}e^{\rho t}$.

From (22), the fossil energy tax formally writes as a linear combination of an increasing term, $\zeta\lambda_Z(t)$ and a decreasing one, $\nu(t)$. Without any ambiguity, it is decreasing through time once the ceiling has been reached, i.e. for $t \geq \underline{t}_Z$, since $\zeta\lambda_Z(t)$ and $\nu(t)$ are both decreasing during this time interval. However, the dynamics of $\tau(t)$ is not clearly determined for $t \in [0, \underline{t}_Z)$ and it depends upon the relative magnitude of variation of its two components. During this interval of time, the differentiation with respect to time of (22) yields $\dot{\tau}(t) = -\rho\sigma\lambda_{X0}e^{\rho t} + (\rho+\alpha)(1-\sigma)\zeta\lambda_{Z0}e^{(\rho+\alpha)t}$. Consequently, the following cases have to be considered.

Proposition 3 *The dynamics of the fossil energy tax obeys to the following rule:*

1. *If $\theta\rho\lambda_{X0} \leq (\rho+\alpha)\zeta\lambda_{Z0}$, the growth effect of the carbon tax overrides the decline of the penalty rate during the first phase. The tax is then first increasing for $t \in [0, \underline{t}_Z)$, next declining for $t \in [\underline{t}_Z, \infty)$.*

2. *If $\theta\rho\lambda_{X0} > (\rho+\alpha)\zeta\lambda_{Z0}$, then there exists a date $\tilde{t}$, $\tilde{t} = \frac{1}{\alpha}\ln\left[\frac{\theta\rho\lambda_{X0}}{(\rho+\alpha)\zeta\lambda_{Z0}}\right] > 0$, such that $\dot{\tau}(\tilde{t}) = 0$. Hence:*

   (a) *if $\tilde{t} < \underline{t}_Z$, the global tax $\tau(t)$ is non-monotonously evolving, being first decreasing for $t \in [0, \tilde{t})$, next increasing for $t \in [\tilde{t}, \underline{t}_Z)$ and finally declining again for $t \in [\underline{t}_Z, \infty)$.*

   (b) *if $\tilde{t} \geq \underline{t}_Z$ then, for any t, the tax is always decreasing through time.*

These possible time profiles of $\tau(t)$ are illustrated in figure 4 **(ajouter commentaires)**.

[Figure 4 here]

5 Comparison of the two mandates when the demand function is inelastic

In this section, we assume that the energy demand is inelastic within the price range $[c_x, c_y]$, i.e. the price range within which the energy price must evolve along any optimal trajectory. This restriction will allow us to develop a comparative analysis of the quantity and proportional mandates. Moreover, as it will be shown, this assumption does not change the analytical properties of the optimal paths. We denote by $\bar{q}$ the energy flow having to be delivered at any point of time within this price range in order to satisfy the demand. Given that the energy production is fixed, the social planner program may be reduced to a simple cost-minimization problem:

$$\min_{\{x(t),y(t)\}} \int_0^\infty [c_x x(t) + c_y y(t)]\, e^{-\rho t} dt$$

subject to constraints (1)-(4), alternatively to (5) or (6) and to:

$$x(t) + y(t) \geq \bar{q} \tag{24}$$

We denote by γ_q the Lagrange multiplier associated with (24) and by V the optimal value of the social planner program, that is the discounted sum of energy consumption costs. We keep the same notations for the other costate variables.

5.1 The quantity mandate case

Under the quantity mandate and under the inelastic demand assumption, the first-order conditions (7) and (8) become, respectively:

$$\gamma_q(t) = c_x + \lambda_X(t) + \zeta\lambda_Z(t) - \gamma_x(t) \tag{25}$$

$$\gamma_q(t) = c_y - \nu(t) \tag{26}$$

together with the following complementary slackness condition:

$$\gamma_q(t) \geq 0, \quad x(t) + y(t) - \bar{q} \geq 0 \quad \text{and} \quad \gamma_q(t)[x(t) + y(t) - \bar{q}] = 0 \tag{27}$$

The first-order conditions relative to the state variables, the other complementary slackness conditions and the transversality conditions are those of section 3. In (25) and (26), the Lagrange multiplier $\gamma_q(t)$ plays the role of the energy consumer price, the pendant of u' in the elastic demand case. The interpretation of all the others multipliers does not change.

In order to focus directly on the most relevant case, we assume that $\underline{y}+\bar{x} < \bar{q}$, implying both $\bar{x} < \bar{q}$ and $\underline{y} < \bar{q}$. This assumption guarantees that oil has to be continuously used until exhaustion and that the carbon stabilization cap $\bar{Z}$ is reached in finite time. Since the objective of the social planner is to minimize costs, it can be easily shown that the inequality constraint (24) must be always binding so that the energy-mix $x(t) + y(t)$ must supply the totality $\bar{q}$ of the demand at any point of time.

As compared with the elastic demand case, the date at which the ceiling constraint becomes no more active coincides with the date at which the fossil resource is exhausted, $\bar{t}_Z = \bar{t}_x$, thus reducing to three the number of phases of the optimal path. The expression of the energy price is simplified in accordance and we get $p(t) = c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ during the first phase before the ceiling, i.e. for $t \in [0, \underline{t}_Z)$, and $p(t) = c_y$ during the two next phases, i.e. for $t = \underline{t}_Z$ onwards. Since the total energy to be supplied is constant, the composition of the energy-mix is also constant within each of these three phases. The oil consumption is discontinuously decreasing, making a downward jump from $(\bar{q} - \bar{y})$ to $\bar{x}$ between the first and the second phase, and a downward jump from $\bar{x}$ to 0 between the second and the third phase. Conversely, the solar energy consumption first jumps from $\underline{y}$

up to $(\bar{q}-\bar{x})$ and next, from $(\bar{q}-\bar{x})$ up to $\bar{q}$. A typical optimal path is depicted by figure 5.

[Figure 5 here]

As in the elastic demand case, the policy-mix allowing to sustain this optimal path is composed by a carbon tax $\zeta\lambda_Z(t)$ on the oil consumption and a subsidy $\nu(t)$ on solar energy use, as given by:

$$\zeta\lambda_Z(t) = \begin{cases} \zeta\lambda_{Z0}e^{(\rho+\alpha)t} & t \in [0, \underline{t}_Z) \\ c_y - \left(c_x + \lambda_{X0}e^{\rho t}\right) & t \in [\underline{t}_Z, \bar{t}_Z) \\ 0 & t \in [\bar{t}_Z, \infty) \end{cases} \tag{28}$$

$$\nu(t) = \begin{cases} c_y - \left[c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}\right] & t \in [0, \underline{t}_Z) \\ 0 & t \in [\underline{t}_Z, \infty) \end{cases} \tag{29}$$

The four variables $\underline{t}_Z$, $\bar{t}_Z$, λ_{X0} and λ_{Z0} are the solutions of a four-equation system as detailed in Appendix A.1. The discounted sum of total energy expenditures, that is the optimal value Φ of the objective function, is:

$$V = \frac{1}{\rho}\left[c_x(\bar{q}-\underline{y}) + c_y\underline{y} + (c_y - c_x)(\bar{q}-\bar{x}-\underline{y})e^{-\rho\underline{t}_Z} + (c_y - c_x)\bar{x}e^{-\rho\bar{t}_Z}\right] \tag{30}$$

Finally, the effect of a change in the mandate level $\underline{y}$ on the optimal path is summarized in the following proposition.

Proposition 4 *An increase (respectively a decrease) in $\underline{y}$ results in an increase (resp. a decrease) in $\underline{t}_Z$, $\bar{t}_Z$ and V, and a decrease (resp. an increase) in $(\bar{t}_Z - \underline{t}_Z)$, λ_{X0} and λ_{Z0}.*

Proof: see Appendix A.3. ■

Imposing a larger minimal amount of solar in the energy-mix reduces the consumption of oil during the first phase before the ceiling since $x = \bar{q} - \underline{y}$ with an inelastic demand. On the one hand, since during the next phase at the ceiling the oil consumption does not depend on $\underline{y}$, this diminution in the first period means that oil is less scarce and that the date at which the initial reserves are exhausted is postponed. These results are illustrated in figure 5 by $\lambda'_{X0} < \lambda_{X0}$ and $\bar{t}'_Z > \bar{t}_Z$. On the other hand, the reduction of the oil consumption during the first phase implies less carbon emissions. Consequently, the initial shadow cost of the pollution stock is reduced and the date at which the ceiling is reached is postponed, as shown in figure 5 by $\lambda'_{Z0} < \lambda_{Z0}$ and $\underline{t}'_Z > \underline{t}_Z$. Remark that, in this case, the length $[\bar{t}_Z - \underline{t}_Z]$ of the phase at the ceiling is shortened. Since both λ_{X0} and λ_{Z0} decline, the energy price trajectory shifts downward during the first phase[6]. Since the marginal cost c_y of solar, that is the price at which this energy becomes competitive, does not depend on the mandate level, the unitary solar subsidy must increase and, simultaneously, the carbon tax is diminished. Finally, the discounted overall impact of an increase in $\underline{y}$ can be assessed by looking at the effect on V. We obtain that an increase in the imposed amount of solar tends to increase the discounted sum of total cost of using the two kinds of resource, thus implying a decrease in the social welfare. Again, with an inelastic demand function, increasing the solar consumption means decreasing by an equivalent amount the oil consumption. Due to the difference in the marginal costs of each resource, this increases the discounted sum V of total costs.

5.2 Proportional mandate

With a proportional mandate, the development of the social planner problem in the inelastic case obeys to the same rules as with a quantity mandate, except that the constraint on the energy-mix (5) is replaced by (6). Deriving the optimal solution requires to consider the following additional assumption: $\bar{x} < (1-\sigma)\bar{q}$. We thus obtain again a three-phase optimal path. The only difference with the quantity mandate case lies in the energy price trajectory which is followed during the first phase. As in the general elastic demand case, this price is a linear combination between the full marginal cost of the solar energy and the full marginal cost of oil, respectively weighed by σ and $(1-\sigma)$: $p(t) = \sigma c_y + (1-\sigma)\left[c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}\right]$. From $t = \underline{t}_Z$ onwards, this price is constant and equal to the marginal cost of the solar energy: $p(t) = c_y$. Consequently, the consumption of each primary energy resource is also constant. Oil consumption first amounts to $(1-\sigma)\bar{q}$ during the first phase before the ceiling, next to $\bar{x}$ during the second phase at the ceiling and finally to 0 when the ceiling is definitively left. The solar consumption is this complement part that allows for satisfying the demand function: $y(t) = \bar{q} - x(t)$.

The corresponding environmental policy tools mixes a subsidy $\nu(t)$ on the solar energy

[6] This result is obtained through the assumptions of perfect substitution between the two primary energy sources, and with constant marginal delivery cost. In a static model without any environmental constraint, but with elastic energy supply curves, Fischer (2010) shows that a RPS lowers energy prices depending on the elasticity of energy supply from renewable energy sources relative to nonrenewable ones and the effective stringency of the RPS target.

use and a tax $\tau(t)$ on oil consumption, which includes the tax $\zeta\lambda_Z(t)$ on carbon emissions and the penalty $\theta\nu(t)$:

$$\nu(t) = \begin{cases} (1-\sigma)\left\{c_y - [c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}]\right\} & t \in [0, \underline{t}_Z) \\ 0 & t \in [\underline{t}_Z, \infty) \end{cases} \tag{31}$$

$$\tau(t) = \begin{cases} \zeta\lambda_{Z0}e^{(\rho+\alpha)t} + \theta\nu(t) & t \in [0, \underline{t}_Z) \\ \zeta\lambda_Z(t) = c_y - \left(c_x + \lambda_{X0}e^{\rho t}\right) & t \in [\underline{t}_Z, \bar{t}_Z) \\ 0 & t \in [\bar{t}_Z, \infty) \end{cases} \tag{32}$$

The dynamic properties of these instruments, as discussed in section 4 in the general case, are preserved by the inelasticity of the demand function.

Last, the four variables $\underline{t}_Z$, $\bar{t}_Z$, λ_{X0} and λ_{Z0}, are characterized in Appendix A.2, and the optimal value of the social planner program is:

$$V = \frac{1}{\rho}\left\{c_x(1-\sigma)\bar{q} + c_y\sigma\bar{q} + (c_y - c_x)[(1-\sigma)\bar{q} - \bar{x}]e^{-\rho\underline{t}_Z} + (c_y - c_x)\bar{x}e^{-\rho\bar{t}_Z}\right\} \tag{33}$$

It can easily be shown that the optimal path is modified by a change in σ in the same terms as in the quantity mandate case. Hence, results of Proposition 3 as well as the discussion that followed, hold whatever the type of RPS which is implemented.

5.3 Comparison of the two systems

Since the two typical optimal paths described above are parameterized by the level of the two specific types of mandate, comparing them requires to exogenously select a particular couple $\{\underline{y}, \sigma\}$. We propose to identify the relationship between these two parameters such that the two corresponding programs have the same objective function values. In what follows, we index by i, $i = \{pm, qm\}$, the type of promoting policy which is considered, proportional versus quantity mandate. It can be easily shown that, if $\underline{y} = \sigma\bar{q}$, i.e. if the mandate advocated by a quantity RPS policy is set to be equal to the amount of solar energy resulting from the proportional RPS policy that is $\sigma\%$ of the total energy demand $\bar{q}$, then the two sets of variables (46)-(49) and (62)-(65) coincide. From (30) and (33), the two corresponding programs thus have the same value:

$$\underline{y} = \sigma\bar{q} \Rightarrow \begin{cases} \underline{t}_Z^{qm} &= \underline{t}_Z^{pm} \\ \bar{t}_Z^{qm} &= \bar{t}_Z^{pm} \\ \lambda_{X0}^{qm} &= \lambda_{X0}^{pm} \\ \lambda_{Z0}^{qm} &= \lambda_{Z0}^{pm} \end{cases} \Rightarrow V^{qm} = V^{pm}$$

With an inelastic demand function, since the consumer price of the energy-mix is constant and equal to c_y from $\underline{t}_Z$ onward for each type of mandate system, these two cases contrast only during the first phase $[0, \underline{t}_Z)$. During this phase, the key distinguishing features between the two types of policies are summarized in the following proposition:

Proposition 5 *If $\underline{y} = \sigma\bar{q}$ so that $\Phi^{qm} = \Phi^{pm}$, then for $t \in [0, \underline{t}_Z)$:*

1. *The price of the energy-mix with a proportional mandate is larger than the price with a quantity mandate as long as the ceiling is not reached: $p^{pm}(t) > p^{qm}(t)$.*

2. *The trajectories of the carbon taxes are the same: $\lambda_Z^{pm}(t) = \lambda_Z^{qm}(t)$.*

3. *A proportional mandate induces a lower subsidy rate on solar energy than a quantity mandate: $\theta\nu^{pm}(t) < \nu^{qm}(t)$*

4. *A proportional mandate requires a larger global tax rate on oil consumption than a quantity mandate: $\tau^{pm}(t) > \lambda_Z^{qm}(t)$.*

Proof: We have $p^{pm}(t) = \sigma c_y + (1-\sigma)[c_x + \lambda_{X0}e^{\rho t} + \lambda_{Z0}e^{(\rho+\alpha)t}] = \sigma c_y + (1-\sigma)p^{qm}(t)$, resulting in $\dot{p}^{pm}(t) = (1-\sigma)\dot{p}^{qm}(t) < \dot{p}^{qm}(t)$. Since $\underline{t}_Z^{pm} = \underline{t}_Z^{qm}$, the two price paths reach the same level c_y at the same time. The price $p^{qm}(t)$ being strictly increasing and growing faster than the price $p(t)^{pm}$, this later must be always larger than the former, which gives the proof of result 1. Result 2 directly comes from the equality between λ_{Z0}^{qm} and λ_{Z0}^{pm}. Results 3 and 4 are an implication of result 1 by observing that $\nu^i(t) = c_y - p^i(t)$ and that $\tau^i(t) = p^i(t) - (c_x + \lambda_{X0}e^{\rho t})$, for $i = \{pm, qm\}$. ■

These results are illustrated in figure 6.

[Figure 6 here]

Starting from the benchmark case where $\underline{y} = \sigma\bar{q}$, we can use the results of Proposition 3 to investigate the other cases, that is to underline the effects of an unilateral change in $\underline{y}$. Consequently, if $\underline{y} > \sigma\bar{q}$, we would have: $\underline{t}_Z^{qm} > \underline{t}_Z^{pm}$, $\bar{t}_Z^{qm} > \bar{t}_Z^{pm}$, $\lambda_{X0}^{qm} < \lambda_{X0}^{pm}$, $\lambda_{Z0}^{qm} < \lambda_{Z0}^{pm}$ and $\Phi^{qm} > \Phi_{id}^{pm}$ (and the opposite if $\underline{y} < \sigma\bar{q}$). Hence, increasing the quantity mandate $\underline{y}$ with respect to the proportional objective σ mainly reinforces the results of Proposition 4. It lowers the price of the energy-mix under the quantity mandate by increasing the solar subsidy and reducing the carbon tax, thus enlarging the gap with the respective *pm* values. In the opposite case, when the quantity mandate is reduced as compared with the proportional policy, the qm energy price is increased and becomes larger than the pm price, at least at the end of the phase. However, since such parameter changes will also affect the timing of each phase, it is difficult to rank the policy instruments as it was previously done.

6 Conclusion

We have extended the Chakravorty et al. (2006) model to compare two kinds of RPS that only differ in their definition basis. The minimal level of renewable energy supply to be achieved was defined either as a ratio of the energy-mix (proportional mandate) or as an independent quantitative target (quantity mandate). We found the following results. i) Whatever the type of mandate, the optimal subsidy on renewable energy is declining over time. ii) Under a proportional mandate, an additional penalty on fossil energy, proportional to the subsidy, must be added to the carbon tax. The resulting global tax on fossil fuel can exhibit unusual non-monotonous time paces as long as the atmospheric carbon cap is not reached. iii) Whatever the type of the mandate, a RPS lowers the energy price, reduces the social objective function and delays the date at which the carbon cap constraint is binding. iv) If the two types of mandates are such that they yield the same social objective value then, with an inelastic energy demand function and as long as the carbon cap is not reached: a quantity mandate implies a lower energy price, a larger subsidy on renewable energy and a smaller fossil fuel tax (including the carbon tax) than a proportional mandate.

We have not discussed about the normative properties of the carbon cap and the RPS. We simply assumed that they come from some recommendations by an independent expert and that they must be taken as given by the agents. However, one can ask the question of the relative efficiency of these two climate policy instruments (see Fischer and Preonas, 2010). An obvious extension of this paper would be to derive the second-best optimal mandate time pace in the case where a carbon cap policy is not achievable. We would thus be able to compare it with the optimal solution under a carbon cap constraint and to conclude about its efficiency both in terms of social welfare and environmental performance. We let these developments for future works.

Appendix

A.1 Full characterization of the optimal solution: The case of a quantity mandate

The optimal solution is parameterized by λ_{X0}, λ_{Z0}, $\underline{t}_Z$, $\bar{t}_Z$ and $\bar{t}_x$. With a general elastic demand function, these five variables are given as the solution of the following system:

$$\begin{aligned}
\int_0^{\underline{t}_Z} \left\{ q^d \left[c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t} \right] - \underline{y} \right\} dt + (\bar{t}_Z - \underline{t}_Z)\bar{x} & \\
+ \int_{\bar{t}_Z}^{\bar{t}_x} \left[q^d \left(c_x + \lambda_{X0} e^{\rho t} \right) - \underline{y} \right] dt &= X^0 \quad (34) \\
Z^0 + \zeta \int_0^{\underline{t}_Z} \left\{ q^d \left[c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t} \right] - \underline{y} \right\} e^{\alpha t} dt &= \bar{Z} e^{\alpha \underline{t}_Z} \quad (35) \\
c_x + \lambda_{X0} e^{\rho \underline{t}_Z} + \zeta \lambda_{Z0} e^{(\rho+\alpha)\underline{t}_Z} &= u'(\bar{x} + \underline{y}) \quad (36) \\
c_x + \lambda_{X0} e^{\rho \bar{t}_Z} &= u'(\bar{x} + \underline{y}) \quad (37) \\
c_x + \lambda_{X0} e^{\rho \bar{t}_x} &= c_y \quad (38)
\end{aligned}$$

Equations (34) and (35) means, respectively, that the non-renewable resource stock X^0 must be exhausted at time $\bar{t}_x$ and that the atmospheric carbon stock must reach the ceiling at time $\underline{t}_Z$, i.e. $Z(\underline{t}_Z) = \bar{Z}$. Equations (36)-(38) insure continuity of the energy price path, and then of the energy consumption path, at time $\underline{t}_Z$, $\bar{t}_Z$ and $\bar{t}_x$ respectively. From (37) and (38), we can deduce the following optimal dates:

$$\begin{aligned}
\bar{t}_Z &= \frac{1}{\rho} \ln \left(\frac{u'(\bar{x} + \underline{y}) - c_x}{\lambda_{X0}} \right) \quad (39) \\
\bar{t}_x &= \frac{1}{\rho} \ln \left(\frac{c_y - c_x}{\lambda_{X0}} \right) \quad (40)
\end{aligned}$$

Such a solution exists if $\lambda_{X0} < \bar{p} - c_x$ and if $\lambda_{X0} < c_y - c_x$. Since c_y is assumed to be larger than $u'(\bar{x})$, with $u'(\bar{x}) > u'(\bar{x} + \underline{y})$, the existence condition of the optimal solution writes:

$$\lambda_{X0} < u'(\bar{x} + \underline{y}) - c_x < c_y - c_x \quad (41)$$

With an inelastic demand function, $\bar{t}_Z = \bar{t}_x$ and the above system reduces to the following four-equation system:

$$\begin{aligned}
(\bar{q} - \underline{y})\underline{t}_Z + \bar{x}(\bar{t}_Z - \underline{t}_Z) &= X^0 \quad (42) \\
Z^0 + \frac{\zeta(\bar{q} - \underline{y})}{\alpha} \left(e^{\alpha \underline{t}_Z} - 1 \right) &= \bar{Z} e^{\alpha \underline{t}_Z} \quad (43) \\
c_x + \lambda_{X0} e^{\rho \underline{t}_Z} + \zeta \lambda_{Z0} e^{(\rho+\alpha)\underline{t}_Z} &= c_y \quad (44) \\
c_x + \lambda_{X0} e^{\rho \bar{t}_Z} &= c_y \quad (45)
\end{aligned}$$

Contrary to the general elastic demand case, the restriction to an inelastic demand function allows us to solve this system:

$$\underline{t}_Z = \frac{1}{\alpha}\ln\left[\frac{\zeta(\bar{q}-\underline{y})-\alpha Z_0}{\zeta(\bar{q}-\underline{y})-\alpha\bar{Z}}\right] \tag{46}$$

$$\bar{t}_Z = \frac{\zeta X_0-[\zeta(\bar{q}-\underline{y})-\alpha\bar{Z}]\underline{t}_Z}{\alpha\bar{Z}} \tag{47}$$

$$\lambda_{X0} = (c_y-c_x)e^{-\rho\bar{t}_Z} \tag{48}$$

$$\lambda_{Z0} = \frac{(c_y-c_x)}{\zeta}\left(e^{-\rho\underline{t}_Z}-e^{-\rho\bar{t}_Z}\right)e^{-\alpha\underline{t}_Z} \tag{49}$$

A.2 Full characterization of the optimal solution: The case of a proportional mandate

The five variables λ_{X0}, λ_{Z0}, $\underline{t}_Z$, $\bar{t}_Z$ and $\bar{t}_x$ are given as the solution of the following five equations system:

$$\begin{aligned}(1-\sigma)\int_0^{\underline{t}_Z} q^d\left\{\sigma c_y+(1-\sigma)[c_x+\lambda_{X0}e^{\rho t}+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}]\right\}dt&\\ +(\bar{t}_Z-\underline{t}_Z)\bar{x}+(1-\sigma)\int_{\bar{t}_Z}^{\bar{t}_x} q^d[\sigma c_y+(1-\sigma)(c_x+\lambda_{X0}e^{\rho t})]dt &= X^0\end{aligned} \tag{50}$$

$$\begin{aligned}\zeta(1-\sigma)\int_0^{\underline{t}_Z} q^d\left\{\sigma c_y+(1-\sigma)[c_x+\lambda_{X0}e^{\rho t}+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}]\right\}e^{\alpha t}dt&\\ +Z^0 &= \bar{Z}e^{\alpha\underline{t}_Z}\end{aligned} \tag{51}$$

$$\sigma c_y+(1-\sigma)[c_x+\lambda_{X0}e^{\rho\underline{t}_Z}+\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_Z}] = u'\left(\frac{\bar{x}}{1-\sigma}\right) \tag{52}$$

$$\sigma c_y+(1-\sigma)\left(c_x+\lambda_{X0}e^{\rho\bar{t}_Z}\right) = u'\left(\frac{\bar{x}}{1-\sigma}\right) \tag{53}$$

$$c_x+\lambda_{X0}e^{\rho\bar{t}_x} = c_y \tag{54}$$

from which we deduce the following optimal dates:

$$\bar{t}_Z = \frac{1}{\rho}\ln\left[\frac{u'\left(\frac{\bar{x}}{1-\sigma}\right)-\sigma c_y-(1-\sigma)c_x}{(1-\sigma)\lambda_{X0}}\right] \tag{55}$$

$$\bar{t}_x = \frac{1}{\rho}\ln\left[\frac{c_y-c_x}{\lambda_{X0}}\right] \tag{56}$$

Those expressions implicitly require the following existence condition:

$$\lambda_{X0} < \frac{u'\left(\frac{\bar{x}}{1-\sigma}\right)-\sigma c_y-(1-\sigma)c_x}{1-\sigma} < c_y-c_x \tag{57}$$

With an inelastic demand function, the above system becomes:

$$
\begin{aligned}
(1-\sigma)\bar{q}\underline{t}_Z + \bar{x}(\bar{t}_Z - \underline{t}_Z) &= X^0 && (58)\\
\frac{\zeta(1-\sigma)\bar{q}}{\alpha}\left(e^{\alpha\underline{t}_Z} - 1\right) + Z^0 &= \bar{Z}e^{\alpha\underline{t}_Z} && (59)\\
\sigma c_y + (1-\sigma)[c_x + \lambda_{X0}e^{\rho\underline{t}_Z} + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_Z}] &= c_y && (60)\\
c_x + \lambda_{X0}e^{\rho\bar{t}_Z} &= c_y && (61)
\end{aligned}
$$

and its solution is given by:

$$
\begin{aligned}
\underline{t}_Z &= \frac{1}{\alpha}\ln\left[\frac{\zeta(1-\sigma)\bar{q} - \alpha Z_0}{\zeta(1-\sigma)\bar{q} - \alpha\bar{Z}}\right] && (62)\\
\bar{t}_Z &= \frac{\zeta X_0 - [\zeta(1-\sigma)\bar{q} - \alpha\bar{Z}]\underline{t}_Z}{\alpha\bar{Z}} && (63)\\
\lambda_{X0} &= (c_y - c_x)e^{-\rho\bar{t}_Z} && (64)\\
\lambda_{Z0} &= \frac{(c_y - c_x)}{\zeta}\left(e^{-\rho\underline{t}_Z} - e^{-\rho\bar{t}_Z}\right)e^{-\alpha\underline{t}_Z} && (65)
\end{aligned}
$$

A.3 Proof of proposition 3

The proof is done from the partial derivative of the system (42)-(45) with respect to $\underline{y}$.

- First, from (43), we have:

$$\zeta(\bar{q}-\underline{y})e^{\alpha \underline{t}_Z}\frac{\partial \underline{t}_Z}{\partial \underline{y}} - \frac{\zeta}{\alpha}(e^{\alpha \underline{t}_Z}-1) = \alpha \bar{Z} e^{\alpha \underline{t}_Z}\frac{\partial \underline{t}_Z}{\partial \underline{y}} \quad \Rightarrow \quad \frac{\partial \underline{t}_Z}{\partial \underline{y}} = \frac{(1-e^{-\alpha \underline{t}_Z})}{\alpha(\bar{q}-\underline{y}-\bar{x})} > 0 \tag{66}$$

- Second, the partial derivative of (42) is:

$$-\underline{t}_Z + (\bar{q}-\underline{y}-\bar{x})\frac{\partial \underline{t}_Z}{\partial \underline{y}} + \bar{x}\frac{\partial \bar{t}_Z}{\partial \underline{y}} = 0$$

  Using (66), it comes:

$$\bar{x}\frac{\partial \bar{t}_Z}{\partial \underline{y}} = \underline{t}_Z - \frac{(1-e^{-\alpha \underline{t}_Z})}{\alpha} \tag{67}$$

  Since the function $f(z) = z-(1-e^{-\alpha z})/\alpha$ is such that $f(0) = 0$ and $f'(z) = 1-e^{-\alpha z} > 0$ for any $z > 0$, then $f(z) > 0$ implying $\partial \bar{t}_Z/\partial \underline{y} > 0$.

- Third, from (66) and (67), we have:

$$\begin{aligned} \frac{\partial(\bar{t}_Z-\underline{t}_Z)}{\partial \underline{y}} &= \frac{1}{\bar{x}}\left[\underline{t}_Z - \left(\frac{\bar{q}-\underline{y}}{\bar{q}-\underline{y}-\bar{x}}\right)\frac{(1-e^{-\alpha \underline{t}_Z})}{\alpha}\right] \\ &< \frac{1}{\bar{x}}\left[\underline{t}_Z - \left(\frac{\bar{q}-\underline{y}-\alpha Z_0/\zeta}{\bar{q}-\underline{y}-\bar{x}}\right)\frac{(1-e^{-\alpha \underline{t}_Z})}{\alpha}\right] \end{aligned} \tag{68}$$

  Let us define $\beta_1 = \bar{q}-\underline{y}-\bar{x}$ and $\beta_2 = \bar{q}-\underline{y}-\alpha Z_0/\zeta$, with $\beta_2 > \beta_1$ since $Z_0 < \bar{Z} = \zeta\bar{x}/\alpha$. Then, from (46), $\underline{t}_Z$ can be rewritten as $\ln(\beta_2/\beta_1)/\alpha$ and, after simplification, inequality (68) becomes:

$$\frac{\partial(\bar{t}_Z-\underline{t}_Z)}{\partial \underline{y}} < \frac{1}{\alpha\bar{x}}\left[\ln\frac{\beta_2}{\beta_1} - \left(\frac{\beta_2}{\beta_1}\right) + 1\right] \tag{69}$$

  Defining the function $g(z) = \ln(z) - z + 1$, with $z > 1$, we have $\lim_{z\downarrow 1} g(z) = 0$, $\lim_{z\uparrow+\infty} g(z) = -\infty$, $g'(z) = (1-z)/z < 0$ and then $g(z) < 0$ for any $z > 1$. Then the right-hand-side of equation (69) is negative and $\partial(\bar{t}_Z - \underline{t}_Z)/\partial\underline{y}$ is also proved to be negative.

- Next, from (45), we obtain:

$$\frac{\partial \lambda_{X0}}{\partial \underline{y}} = -\rho\lambda_{X0}\frac{\partial \bar{t}_Z}{\partial \underline{y}} < 0 \tag{70}$$

- The partial derivative of (44) yields:

$$\zeta e^{\alpha \underline{t}_Z}\frac{\partial \lambda_{Z0}}{\partial \underline{y}} = -\frac{\partial \lambda_{X0}}{\partial \underline{y}} - \left[\rho\lambda_{X0} + (\rho+\alpha)\zeta\lambda_{Z0}e^{\alpha \underline{t}_Z}\right]\frac{\partial \underline{t}_Z}{\partial \underline{y}}$$

Using (66), (67) and (70), and rearranging some terms, this last expression can be rewritten as:

$$\alpha e^{\alpha \underline{t}_Z} \frac{\partial \lambda_{Z0}}{\partial \underline{y}} = \frac{(\rho+\alpha)\lambda_{Z0}\left(1-e^{\alpha \underline{t}_Z}\right)}{(\bar{q}-\bar{x}-\underline{y})} + \frac{\rho \lambda_{X0}}{\bar{Z}} \left[\underline{t}_Z - \left(\frac{\bar{q}-\underline{y}}{\bar{q}-\underline{y}-\bar{x}} \right) \frac{(1-e^{-\alpha \underline{t}_Z})}{\alpha} \right] \quad (71)$$

The first term of the right-hand-side of (71) is clearly negative, and the second term is also negative due to result (69). Hence, $\partial \lambda_{Z0}/\partial \underline{y} < 0$.

- Last, the partial derivative of (30) with respect to $\underline{y}$ is:

$$\frac{\partial \Phi}{\partial \underline{y}} = \frac{(c_y - c_x)}{\rho} \left[1 - e^{-\rho \underline{t}_Z} - \rho(\bar{q}-\underline{y}-\bar{x}) e^{-\rho \underline{t}_Z} \frac{\partial \underline{t}_Z}{\partial \underline{y}} - \rho \bar{x} e^{-\rho \bar{t}_Z} \frac{\partial \bar{t}_Z}{\partial \underline{y}} \right]$$

Using (66) and (67), it comes:

$$\frac{\partial \Phi}{\partial \underline{y}} = \frac{(c_y - c_x)}{\rho} \left\{ 1 - e^{-\rho \underline{t}_Z} - \rho e^{-\rho \underline{t}_Z} \frac{(1-e^{-\alpha \underline{t}_Z})}{\alpha} - \rho e^{-\rho \bar{t}_Z} \left[\underline{t}_Z - \frac{(1-e^{-\alpha \underline{t}_Z})}{\alpha} \right] \right\}$$

Since $\bar{t}_Z > \underline{t}_Z$, we have $-e^{-\rho \bar{t}_Z} > -e^{-\rho \underline{t}_Z}$ and then:

$$\begin{aligned} \frac{\partial \Phi}{\partial \underline{y}} &> \frac{(c_y - c_x)}{\rho} \left\{ 1 - e^{-\rho \underline{t}_Z} - \rho e^{-\rho \underline{t}_Z} \frac{(1-e^{-\alpha \underline{t}_Z})}{\alpha} - \rho e^{-\rho \underline{t}_Z} \left[\underline{t}_Z - \frac{(1-e^{-\alpha \underline{t}_Z})}{\alpha} \right] \right\} \\ \Leftrightarrow \frac{\partial \Phi}{\partial \underline{y}} &> \frac{(c_y - c_x)}{\rho} e^{-\rho \underline{t}_Z} \left(e^{\rho \underline{t}_Z} - 1 - \rho \underline{t}_Z \right) \end{aligned} \quad (72)$$

Since the function $h(z) = e^{\rho z} - \rho z - 1$ defined for $z > 0$ is such that $h(0) = 0$ and $h'(z) = \rho(e^{\rho z} - 1) > 0$, then $h(z) > 0$ for any $z > 0$ and $\partial \Phi/\partial \underline{y} > 0$.

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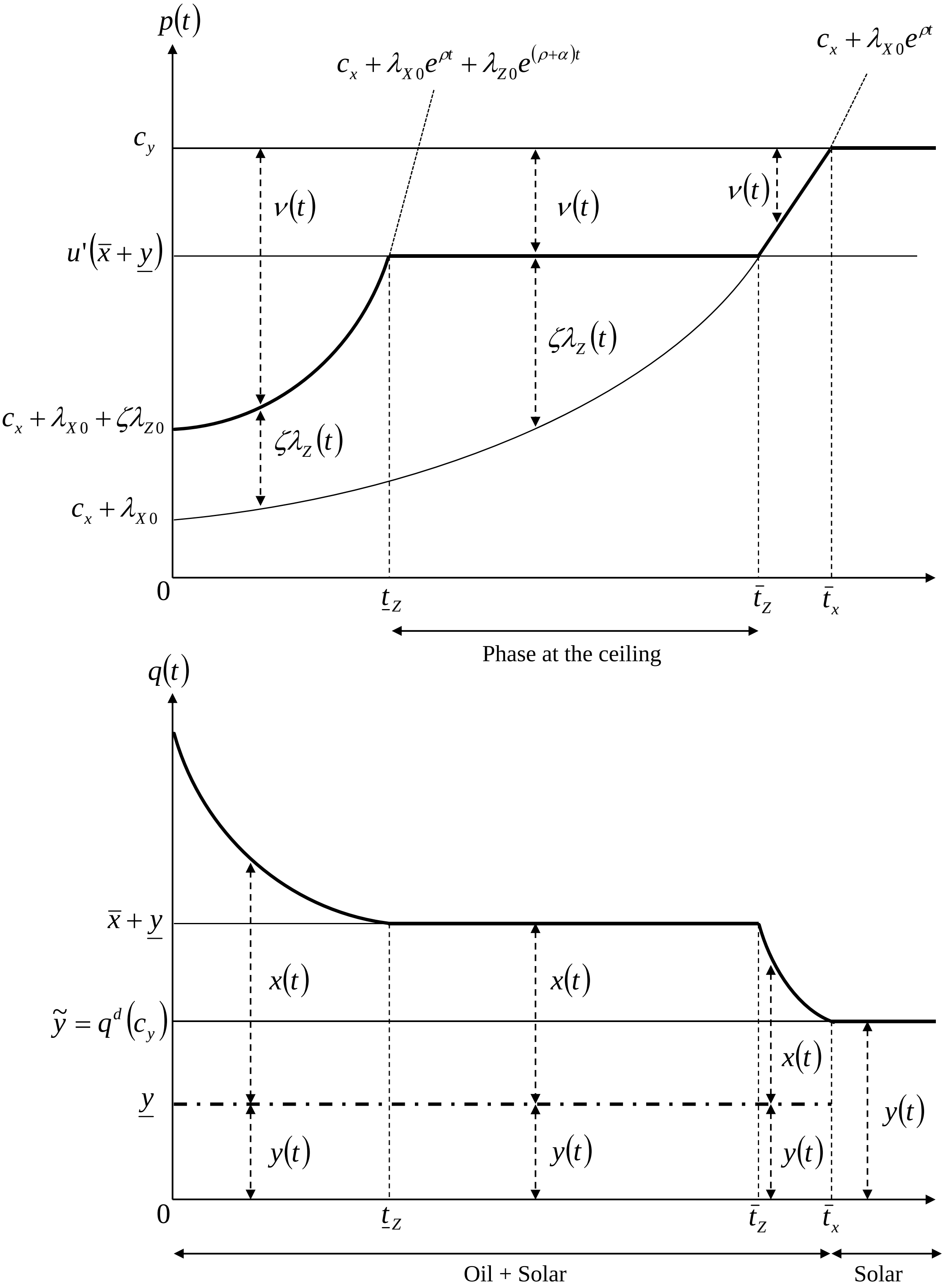

Figure 1: Time profiles of the energy price and consumption. The case of a quantity mandate

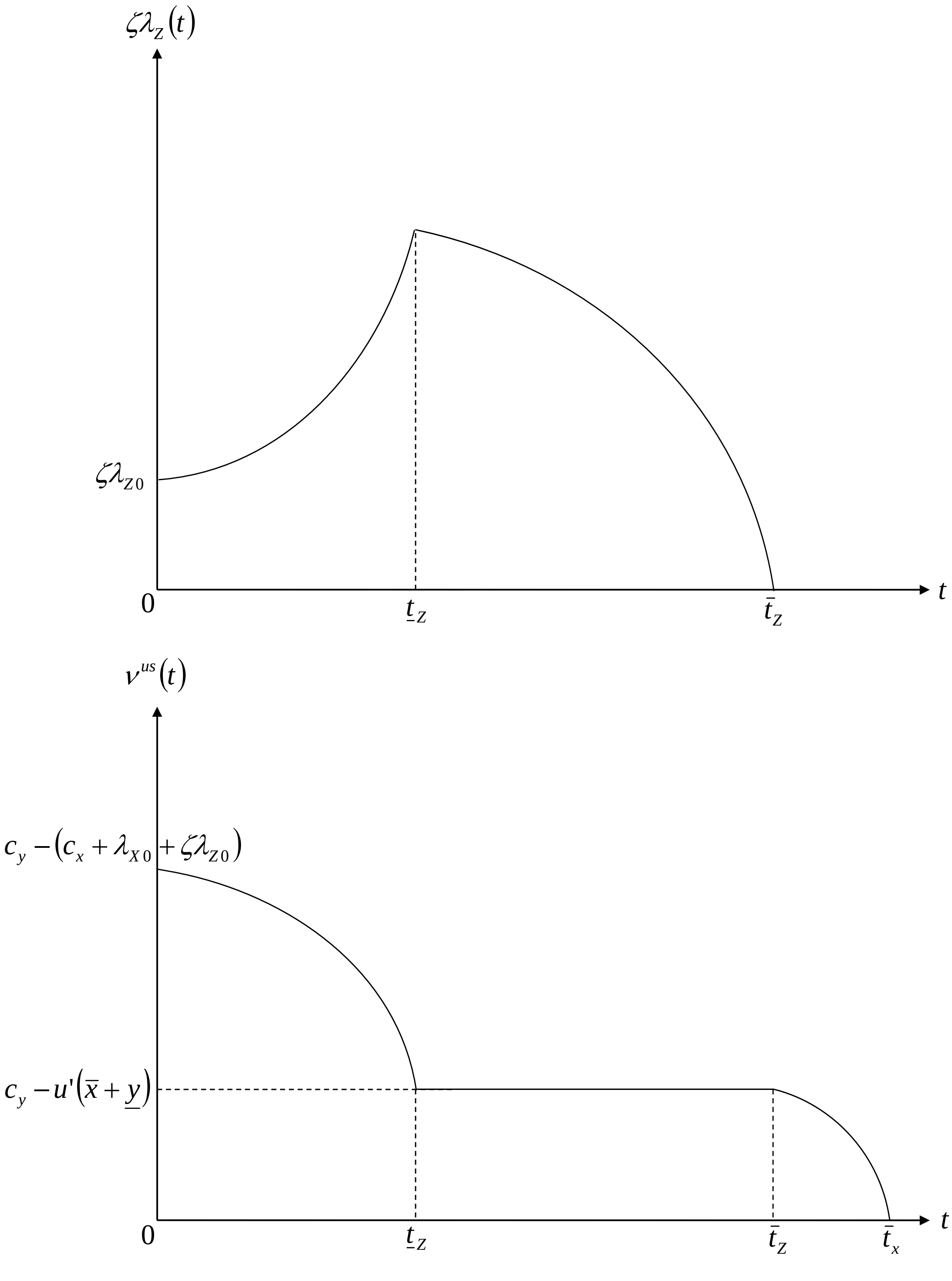

Figure 2: Time profiles of the carbon tax and the solar subsidy. The case of a quantity mandate

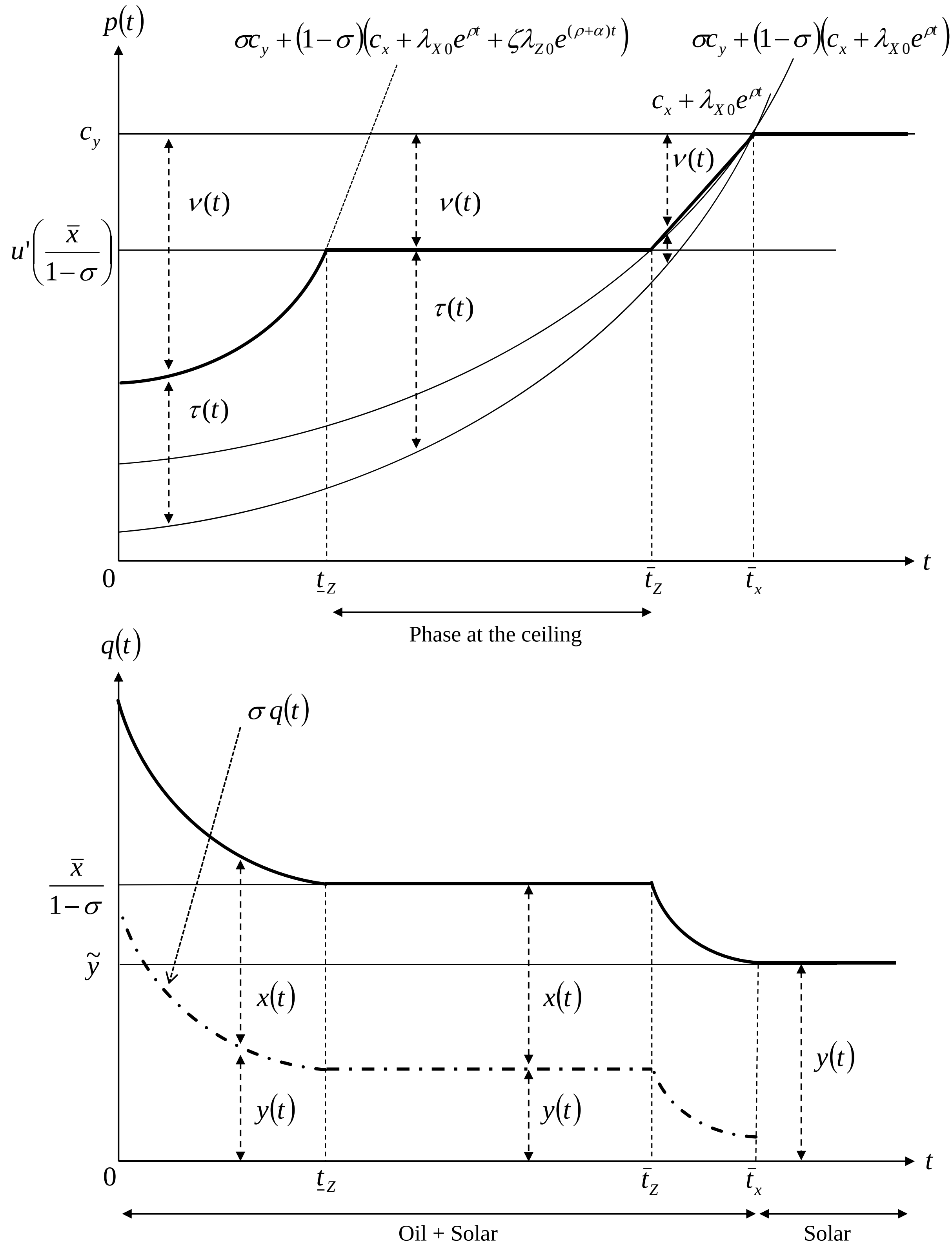

Figure 3: Time profiles of the energy price and consumption. The case of a proportional mandate

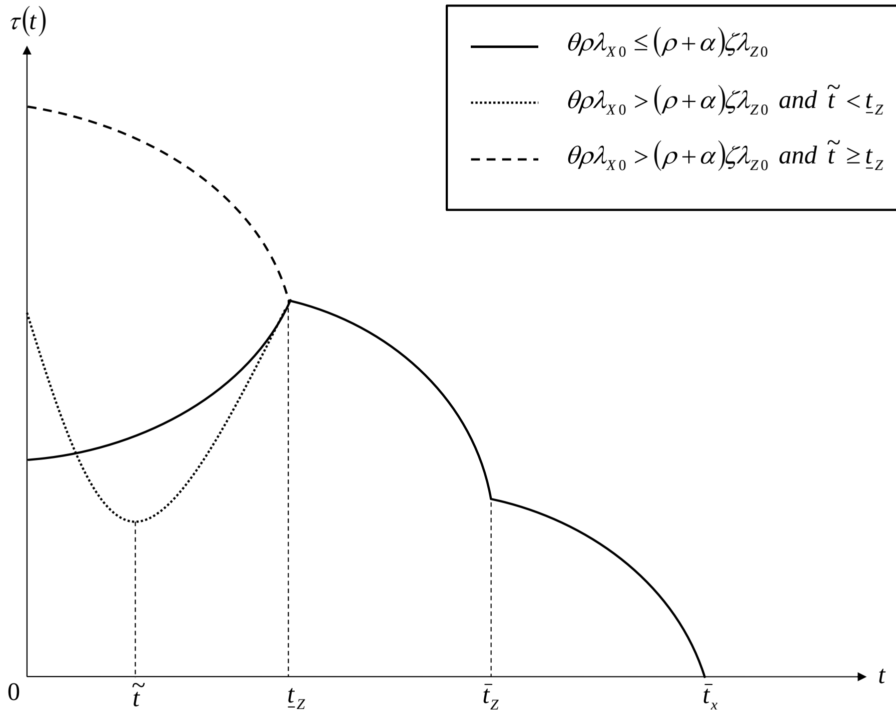

Figure 4: Time profile of the global tax on fossil fuel use with a proportional mandate

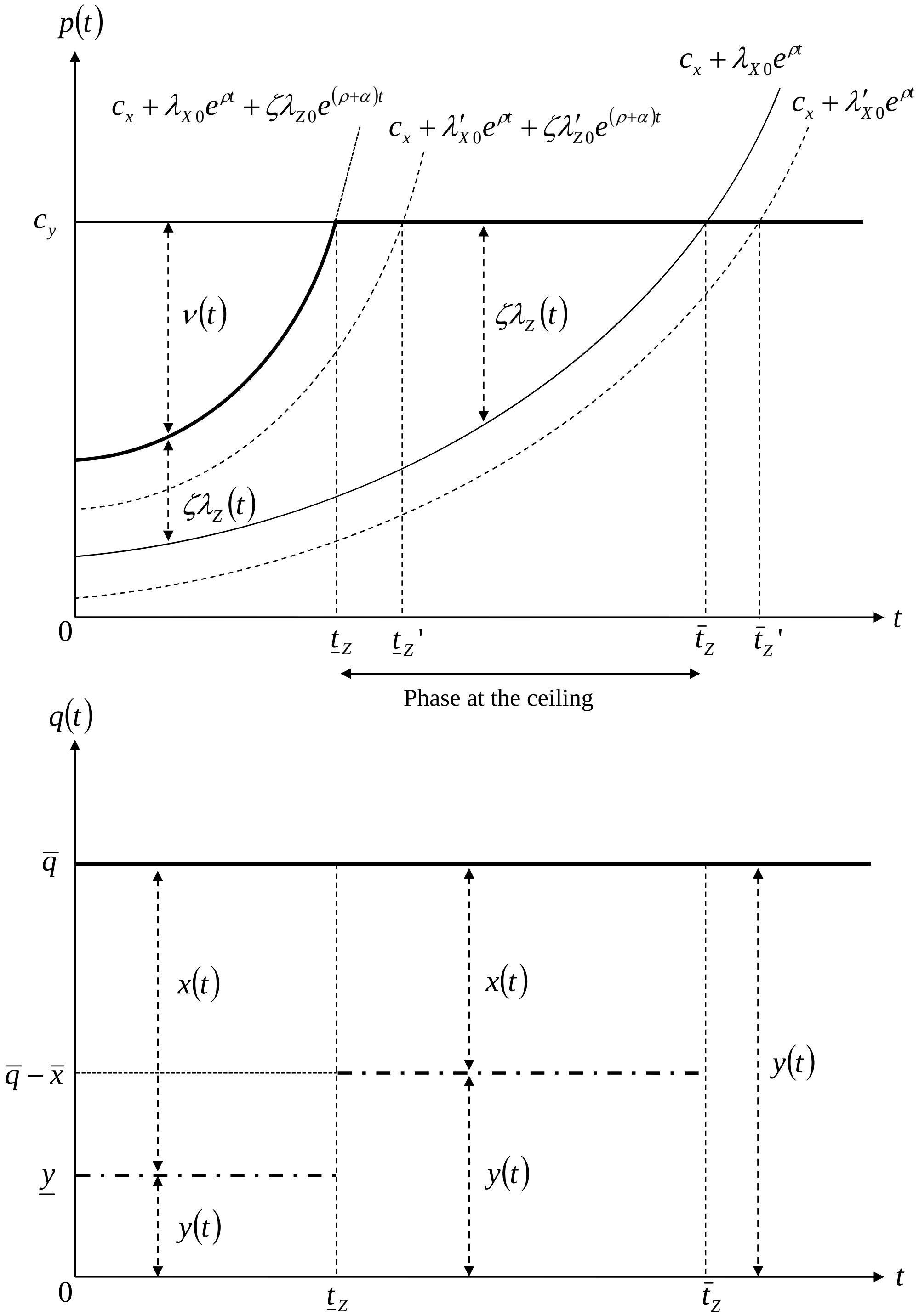

Figure 5: Time profiles of the energy price and consumption with a quantity mandate. The inelastic demand case

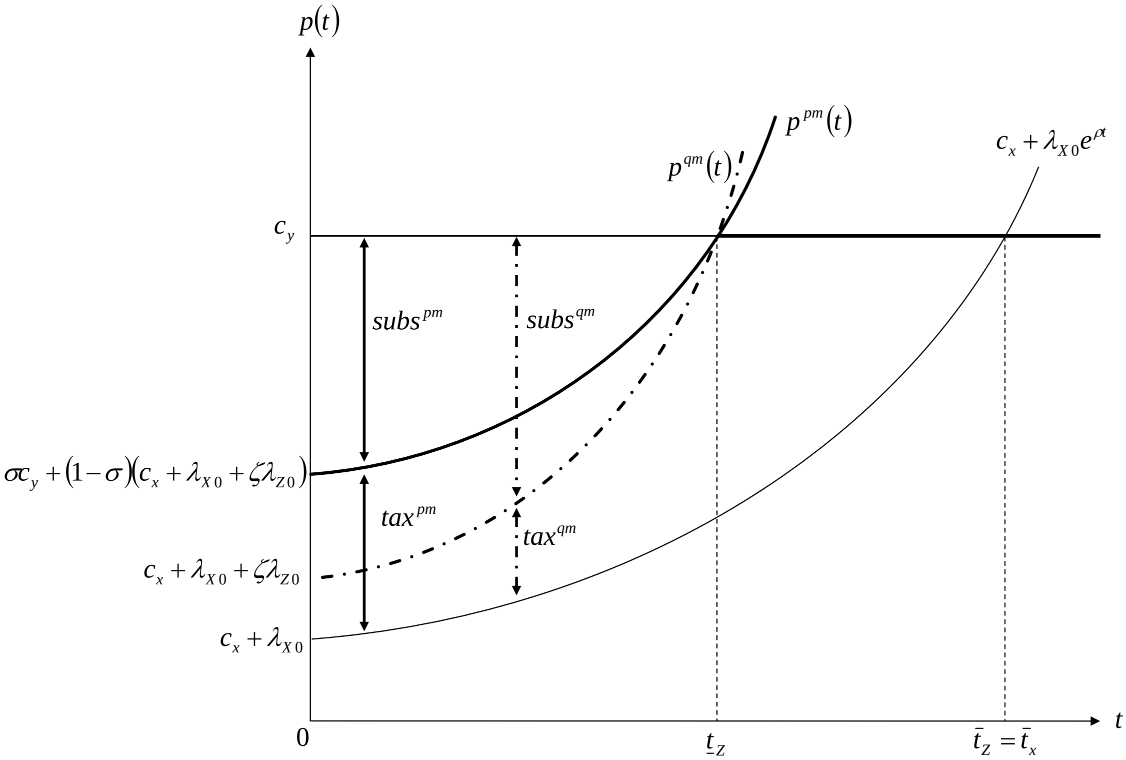

Figure 6: Comparison of the two systems. The inelastic demand case where $\underline{y} = \sigma\bar{q}$