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to my family

"Sólo es posible avanzar cuando se mira lejos,
sólo cabe progresar cuando se piensa en grande."
José Ortega y Gaset

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Abstract

If there is valuable information for predicting bond prices over time, how can we use this information to improve investor's risk-return trade-off and term structure modeling? This thesis aims at answering this question. This thesis star asking whether or not the liquidity premium of TIPS relative to Treasury bonds significantly predicts U.S bond returns. Next, given this empirical evidence, I assess the plausibility of liquidity as unspanned factor for the U.S. yield curve, and finally, I translate this empirical finding into an improved bond asset allocation and performance.

The first chapter discusses the predictive role of alternative measures of the liquidity premium of TIPS relative to Treasury bonds for government excess bond returns. The results show that the liquidity premium predicts positive (negative) TIPS (nominal Treasury) excess returns. The explanatory power of the TIPS liquidity premium is statistically significant and economically meaningful for short-term excess TIPS maturities and for long-term nominal Treasury bonds. I also find that the out-of-sample forecasting power of liquidity for nominal Treasury excess returns appears to have been addressed by the events during the recent financial crisis. By contrast, I have evidence of out-of-sample forecasting ability during both normal and bad times for TIPS' excess returns.

Motivated by this empirical findings, which suggest that bond excess returns can be predicted by liquidity risk, in the second chapter I explore whether or not the TIPS liquidity premium can be consider as an unspanned factor that forecast bond returns, but that is not necessarily spanned by the U.S. yield curve. In this chapter, I consider a joint Gaussian affine term structure model for zero-coupon U.S. Treasury and TIPS bonds, with an unspanned factor: liquidity risk. In the model, the liquidity factor is restricted to affect only the cross-section of yields but it is allowed to determine the bond risk premia. I present empirical evidence suggesting that the liquidity factor does not affect the dynamic of bonds under the pricing measure, but does affect them under the historical measure. Consequently, the information contained in the yield curve appears to be insufficient to completely characterize the variation in the price of curvature risk.

The question as to whether or not asset returns are predictable is of significant importance for portfolio choice. In their seminal papers, Merton (1969) and Samuelson (1969) show that if asset returns are independently and identically distributed (IID) over time, then the optimal asset allocation is constant over time. However, Kim and Omberg (1996), Brennan et al. (1997) and Viceira and Campbell

(1999) show that if asset returns are predictable, then the optimal asset allocation depends on the investment horizon and the predictive variables.

In the third chapter, I estimate the non-parametric optimal bond portfolio choice of a representative agent that acts optimally with respect to his/her expected utility one period forward, provided that he/she observes the ex-ante liquidity signal. Using daily observations of zero-coupon Treasury and TIPS bonds yields, I construct equally-weighted returns from 2004- 2012. Considering alternative measures of liquidity, I find that the liquidity differential between nominal and TIPS bonds appears to be a significant determinant of the portfolio allocation to U.S. government bonds. In fact, conditional allocations in risky assets decrease as market liquidity conditions worsen, and the effect of market liquidity decreases with the investment horizon. I also find that the bond return predictability translates into improved in-sample and out-of-sample asset allocation and performance.

Résumé

S'il existe de l'information intéressante pour prévoir les prix des titres du Trésor au fil du temps, comment peut-on utiliser cette information pour améliorer le rapport risque/rendement de l'investisseur et la modélisation de la structure par terme ? Cette thèse a pour objectif de répondre à cette question. Le premier chapitre analyse le rôle prédictif des mesures alternatives de la prime de liquidité des TIPS (Treasury Inflation-Protected Securities) par rapport aux titres du Trésor pour des rendements en excès des obligations gouvernementales. Les résultats montrent que la prime de liquidité prévoit des rendements en excès positifs (négatifs) pour les TIPS (Treasury nominales). Je trouve, également, que le pouvoir de prévision hors échantillon de la liquidité des rendements en excès des Treasury nominales paraît avoir été guidé par les évènements de la crise financière récente. Par contre, je trouve empiriquement qu'il y a également une capacité de prévision des rendements en excès des TIPS hors échantillon pendant les périodes normales aussi bien que pour les mauvaises périodes.

Dans le deuxième chapitre, j'examine si la prime de liquidité des TIPS peut être considérée comme un facteur dit unspanned (c'est-à-dire dont la valeur n'est pas une combinaison linéaire de la courbe des rendements) pour prévoir les rendements des obligations, mais qu'elle n'est pas nécessairement spanned par la courbe des taux des États Unis. Je considère un modèle affine et gaussien de la structure par terme d'obligations à coupon zéro du Trésor américain pour l'ensemble des Treasuries and TIPS, avec un facteur unspanned : le risque de liquidité. Dans ce modèle, le facteur de liquidité est contraint d'affecter seulement les rendements de coupe transversale, mais il permet de déterminer les primes de risque des obligations. L'évidence empirique suggère que le facteur de liquidité n'affecte pas la dynamique des obligations en vertu de la probabilité risque-neutre, mais qu'il affecte cette dynamique sous la mesure historique. Par conséquent, l'information contenue dans la courbe de rendement s'avère insuffisante pour caractériser complètement la variation du prix du risque de courbure.

Dans le troisième chapitre, j'estime, par des méthodes non paramétriques, le choix de portefeuille optimal d'obligations pour un agent représentatif qui agit d'une façon optimale par rapport à son utilité espérée sur la période suivante, à partir du signal de liquidité observé ex ante. Considérant les différentes mesures de liquidité, je trouve que le différentiel de liquidité entre les obligations nominales et les TIPS paraît être un facteur significatif de choix du portefeuille en obligations du gouvernement des États Unis. En effet, l'allocation conditionnelle en actifs risqués décroit avec la détérioration des conditions de liquidité du marché, et l'effet de liquidité du marché diminue avec l'horizon d'investissement. Je trouve également que la prévisibilité de rendement des obligations se traduit par une meilleure allocation et performance aussi bien intra que hors échantillon.

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General Introduction

The predictability of the return on financial assets is a question that has been debated among academics and practitioners since the end of the last century (1980's). Predictability is related to the possibility of generating an excess returns in using past information. In the literature, two well documented but competing explanations have been developed as a source of such predictability. The first one argues that predictability is the result of variation in the expected returns driven by economic fundamentals, while the second considers that predictability is attributed to market inefficiencies.

For the first explanation, Fama and French (1989) and Ferson and Harvey (1991) claim that the apparent predictability in long-horizon stock return indexes is due to business cycle movements and changes in aggregate risk premia. The economic explanation is that the level of risk aversion changes along with the economic cycle, being higher in situations of recession (thus higher the expected risk premium), and lower in situations of expansion (thus lower the expected risk premium). Assuming rationality, predictability should reflect the time-varying expected risk premium, which changes with economic conditions. Evidence that time variation in expected returns is related to business conditions is common to stocks and bonds, suggesting that predictability is real and rational (Fama (1990)), and consistent with inter-temporal asset pricing models.

In contrast, De Bond and Thaler (1984), Lehmann (1990), and Chopra et al. (1992) claim that such predictability is symptomatic of inefficient markets, markets populated with overreacting and irrational investors. Consequently, for the defenders of irrational behavior the arguments related with changes in the economic conditions are not convincing. The fact that time variation is common to stocks and bonds may just mean that non rational anomalies are correlated across assets and markets, both at the domestic and international level. Also, its relation to business conditions may just be a signal that common anomalies in different markets are related to business conditions. However, only under the joint hypothesis of market efficiency and risk neutrality, returns should not be predictable (Pesaran (2003)). As a consequence, market predictability on its own would not imply market inefficiency and irrational behavior. Indeed, what we should do is also study the risk aversion of the investor, as Rey (2004) argues.

Following both explanations, several papers suggest different predictor variables. A number of authors present empirical evidence of ex-post (or *in-sample*) stock return predictability. Fama and Schwert (1977), Rozeff (1984), Keim and Stambaugh

(1986), Campbell (1987), Campbell and Shiller (1991) and Fama and French (1989) show that excess returns could be successfully predicted based on lagged values of variables such as dividend-price ratio and dividend yield, earnings-price ratio and dividend-earnings ratio, interest rates and spreads, inflation rates, book-to-market ratio, volatility, investment-capital ratio, consumption, wealth, and income ratio, and aggregate net or equity issuing activity. However, studies of ex-ante (or *out-of-sample*) return predictability have found either that previous successful results were restricted to particular sub-samples (Pesaran and Timmermann (1995)) or that return predictability was a statistical illusion (see Bossaerts and Hillion (1999)).

Also, there are numerous studies that identify various financial and macroeconomic variables as predictors for the U.S. bond risk premia (expected excess returns). For instance, the term structure slope, the forward spread, the lagged excess returns, the Cochrane and Piazzesi (2005) tent-shaped factor, and macroeconomic fundamentals are some of the variables that have been identified as predictors for Treasury bonds (see Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005), Ludvigson and Ng (2009) and Cooper and Priestley (2009a)).

At the same time, several authors express concern that the apparent predictability of stock and bond returns might be spurious. Many of the predictor variables in the literature are highly persistent: Nelson and Kim (1993) and Stambaugh (1999) point out that persistence leads to biased coefficients in predictive regressions if innovations in the predictor variable are correlated with returns. In other words, the apparent predictability of stock and bond returns might be spurious, as many of the predictor variables are highly persistent, leading to possibly biased coefficients and incorrect t-tests in predictive regressions (see, for example, Nelson and Kim (1993), Cavanagh et al. (1995), and Stambaugh (1999)). These problems are exacerbated when large numbers of variables are considered and only results that are apparently statistically significant are reported.

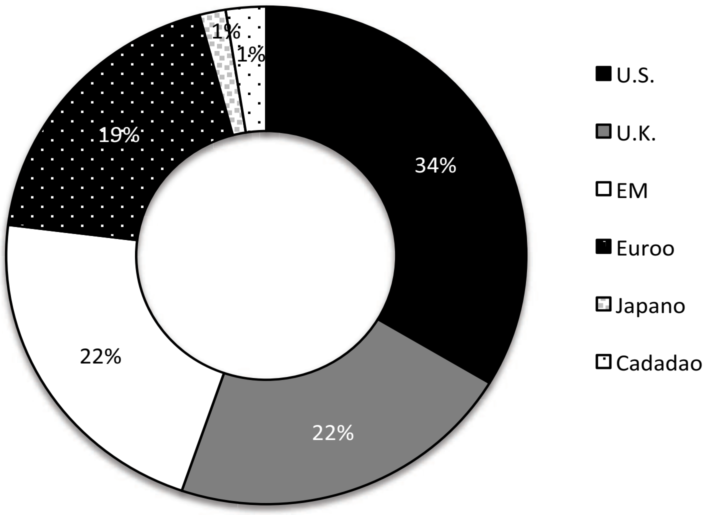
This thesis does not contribute directly to this debate. By contrast, it tries to give an answer to the following question: If there is valuable information for predicting bond prices over time, how can we use this information to improve investor's risk-return trade-off and term structure modeling? To do that, this thesis starts asking whether or not the liquidity premium of TIPS relative to Treasury bonds significantly predicts U.S bond returns. Next, given this empirical evidence, in Chapter 2, the plausibility of liquidity premium as unspanned factor for the U.S. yield curve is assessed. Finally, in Chapter 3, the empirical finding of predictability is translated into an improved bond asset allocation and performance.

The inception of inflation-linked bonds (ILBs), such as U.S. Treasury Inflation Protection Securities (TIPS) and UK Inflation-linked Gilts, was a fundamental innovation in the financial market. Inflation-linked bonds are financial instruments whose principal is adjusted by changes in the inflation rate. Therefore, ILBs potentially provide protection from inflation's effects, preventing investors from looses in their purchasing power.[1] The United Kingdom was the first to supplement its government bond issue program with inflation-linked bonds in 1981. This was followed by Australia in 1985, Canada in 1991, Sweden in 1994, the United States in 1997 (which created treasury inflation protected securities, or TIPS), France in 1998, Italy in 2003, Japan in 2004 (in spite of its deflationary environment), and Germany in 2006. In recent years, the linker market has also grown sharply in the emerging markets (particularly in Brazil, Mexico, Turkey, and South Africa). Figure 1 shows as of April 2013, the global market value of inflation-linked government bonds was approximately \$2.5 trillion. The United States is the largest issuer with \$845 billion, followed by the United Kingdom with \$549 billion, and the Euro zone with \$476 billion (France with \$244 billion, Italy with \$159 billion, and Germany with \$73 billion). Brazil is the largest among the emerging market issuers of inflation-linked bonds, with approximately \$262 billion outstanding (which is part of EM in Figure 1). Mexico is the second with \$55 billion, and in addition, other countries such as Turkey, South Africa, Chile, and Argentina have sizable and growing inflation-linked bond markets.

The largest and best established inflation-linked bond market is the TIPS bond market. TIPS has shown a consistent growth since its inception in 1997. In fact, the market capitalization has grown by more than thirty times, from \$33 billion dollars in 1997 to over \$1.200 billion in 2013. However, it has been characterized by be less liquid than nominal Treasury bond market. As a consequence, the lack of liquidity is thought to result in TIPS yields having a liquidity premium relative to nominal securities. Theoretically, a less liquid security carries higher liquidity risk, and thus must carry a higher yield (higher expected returns or risk premium as well) as an compensation for the incremental risks and higher costs of trading. This additional yield is the liquidity risk premium. The existence of this liquidity premium in TIPS yields, which is time varying, is well documented in the academic literature by Sack and Elsasser (2004), Shen (2006), Hordahl and Tristani (2010), Campbell et al. (2009), Dudley et al. (2009), Christensen and Gillan (2011), Gurkaynak et al. (2010), Pflueger and Viceira (2012), among others. For instance, D'Amico et al. (2010) estimate that the liquidity premium was about 1 percent in the early years of the TIPS program. Pflueger and Viceira (2012) find that the liquidity premium is around 40 to 70 basis points during normal times, but was more during the early

[1] Beyond to provide protection from inflation risk a number of potential benefits arise with the availability of inflation-linked bonds. Some benefits are investors' possibility to hedge inflation risk, better risk sharing for the economy, reduction of government borrowing cost, and one of the more important benefits is that the existence of the ILB markets gives the possibility to derive a market-determined measure of real rates and inflation expectations. However, inferring market expectations of inflation accurately, from the yield spread between nominal Treasuries and inflation-linked bonds, commonly known as break-even inflation rate (BEI), is a difficult task. This is due to that in terms of risk, the yield spread includes not only the nominal inflation risk they are exposed to the Treasury, also includes liquidity risk that is present in the market for ILBs.

Figure 1: World Government Inflation-linked bonds Markets Outstanding



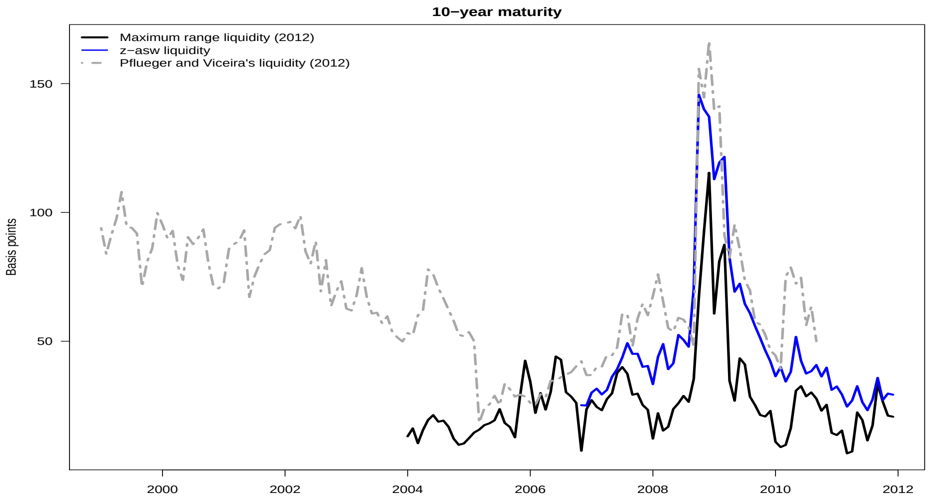
Source: Barclays Universal Government Inflation-linked all maturities Bond Index. USD Billions. Data as of April 2013. Euro includes France, Germany and Italy, and EM indicates Emerging Markets. Percentage values correspond to the proportion of U.S. TIPS on the total Index.

years of TIPS and during the 2008-2009 financial crisis.

In this thesis, I consider three alternative liquidity premium measures in TIPS. The first measure is the estimated liquidity premium by Pflueger and Viceira (2012). They estimate the TIPS liquidity by regressing the yield differential between nominal and inflation-linked treasury securities (commonly referred as cash Break-even inflation (BEI)), on several liquidity proxies in bond markets, and by controlling for inflation expectations. As they claim, their measure likely represents a combination of current ease of trading TIPS versus nominal U.S. Treasuries and the risk that the liquidity of TIPS might deteriorate. The second measure is the Christensen and Gillan (2011) TIPS liquidity measure, which is computed as the spread between the observed break-even inflation rates (BEI), which are defined as the difference between nominal and inflation-indexed bond yields, and the inflation swap rates which are considered as a synthetic BEI. In other words, they identify the liquidity component in TIPS yields using information from the bond market and also from the inflation swap market. By construction, this measure represents the combined liquidity premiums in TIPS and inflation swaps.

The third measure for the market liquidity premium in TIPS, is computed by looking at how inflation-linked asset swaps on nominal bonds correspond to inflation-linked ones. This bond asset swap spread captures the relative financing cost, the specialness and the balance sheet cost of TIPS over nominal Treasuries. This characteristic influences the ease of liquidating some securities and the attractiveness by which to hold them with respect to others. This spread should be a good market-based measure of the market perception (current and expected) of relative liquidity in bond market. Although this measure was first consider by Pflueger and Viceira (2012) as one of the liquidity proxies, in this thesis it is directly used as a measure of the TIPS liquidity premium. Figure 2 compares the three liquidity premium measures in a monthly frequency.

Figure 2: TIPS liquidity premium measures



U.S. monthly data from November 2006 to December 2012. The *z-asw* liquidity corresponds to the residual spread between TIPS and nominal 10-year bonds asset swaps calculated using end of the month data for nominal and TIPS z-spread asset swaps. The maximum range liquidity corresponds to the measure proposed by Christensen and Gillan (2011) for 10-year maturity calculated from January 2004 until December 2012, and Pflueger and Viceira (2012) estimation of the 10-year TIPS liquidity premium from January 1999 to December 2010.

The measures described above allow us to identify the relative liquidity premium between two comparable assets, in this case the cost derived from TIPS liquidity disadvantage relative to nominal bonds.[2] As a result, the liquidity measures described above meet the same definition of liquidity premium. In particular, the lack of liquidity is thought to result in TIPS yields having a liquidity premium relative to nominal securities. Consequently, liquidity refers to the total cost of all frictions (wider bid-ask spreads, lower trading volume, etc.) to trade off the less liquid asset beyond that of the more liquid asset against which it is being compared.[3]

[2] Absolute liquidity premium is defined as the price difference between the observed and the unobservable frictionless market outcome of a given asset. However, we work with the relative concept since it is extremely difficult to identify the unobservable frictionless price of an asset directly.

[3] In general, market liquidity is an elusive concept. While most observers would agree whether a given market is liquid or not, it is difficult to draw up a precise definition for it. This is because market liquidity is multi-faceted in the sense that the definition necessarily changes depending on what dimension one wishes to emphasize. Theoretically, market liquidity has four dimensions: *i*) *Tightness* which refers to the difference between buy and sell prices, for example the bid-

Additionally, the second and the third measure are characterized by being market-based measures of liquidity, in the sense that they are model-free and can be readily calculated using daily data. By contrast, the Pflueger and Viceira (2012) liquidity premium measure is model-dependent by construction, and it is only available on a monthly frequency.[4] Finally, it is important to highlight that there exists a close relationship between bond break-evens and inflation swap rates, because theoretically, both rates measure the markets' expectations of future inflation. However, the most recent crisis showed that U.S. cash and swap markets can diverge significantly, with each market driven by its specific dynamics. Asset swapping activity should theoretically hold the two markets together, but the empirical evidence suggests that such activity was not sufficient to offset diverging forces in stressed market conditions (see Gomez (2013) for a further discussion). Consequently, even though the second an the third measures of liquidity are highly correlated (which suggests that all of them are capturing similar information about the liquidity differential between nominal and TIPS yields), they are computed using information from different markets. Thus, it would make them capture different aspects of liquidity premium, especially in times of financial distress where each market tends to be driven by its specific dynamics, such as funding costs.

Recent studies have documented significant *in-sample* predictability of U.S. bond excess returns by means of liquidity. Fontaine and Garcia (2011) study the role of funding liquidity as a predictor variable for the U.S. bond risk premia. They find that funding liquidity, measured as the price difference between on-the-run versus off-the-run bonds, predicts a substantial share of the risk premium of Treasury bonds. In a more recent paper Pflueger and Viceira (2012) provide empirical evidence for liquidity as a source of predictability in U.S. inflation-indexed bonds *in-sample*. The first contribution of this thesis is to the recent literature on excess return

ask spread in a quote-driven market. *ii*) *Depth* relates to the size of transactions that can be absorbed without affecting prices. *iii*) *Immediacy* denotes the speed with which orders can be executed, and *iv*) *resiliency* the easiness with which prices return to normal after temporary order imbalances (For additional details see (BIS 1999)). However, a definition which seems to garner relatively wide support, is that a market is liquid if transactions can take place rapidly, and with little impact on price (BIS 1999). In other words, market liquidity is defined as the difference between the transaction price and the fundamental value of an asset.

[4] Literature on liquidity of the bond market has been characterize by using a variety of measures. The bid-ask spread, trading volume, trading frequency and average quote size, among others, have been widely used. However, Fleming (2001) point out that these proxies only measure the cost of executing a single trade of limited size, which is the case of bid-ask spread, or are indirect measures of liquidity or, more importantly, they are also associated with market volatility. Other widely used liquidity measure in the Treasury market is the difference between the yield of on-the-run and off-the-run security with similar cash flow characteristics (the on-the-run U.S. Treasury bond is the bond issued at the most recent auction, while the off-the-run bonds are the bonds issued at all the other auctions that are still outstanding). However, a drawback of this measure is that this spread can be difficult to interpret, and factors besides liquidity can cause on-the-run securities to trade at a premium, confounding the interpretation of the spread (See Fleming (2001) and Pasquariello and Vega (2009) and references therein).

predictability of inflation-linked bonds, by assessing not only the in-sample but also the out-of-sample predictive power of liquidity risk premium.

Controlling for typical excess return predictors such as the term structure slope and the tent-shaped factor of Cochrane and Piazzesi (2005), I document the predictive power that liquidity premium of different maturities have for nominal and inflation-linked excess returns in the U.S. bond market. This result is confirmed using different liquidity premium measures available in literature. That is, all measures seem to contain useful information used to predict excess returns in both nominal and inflation-linked bond markets. This means that predictability results are robust as to how the liquidity premium is measured.

While predictability has been well studied and documented in the literature in U.S. nominal Treasuries, in general, less has been done to provide empirical evidence for the predictability of returns in inflation-linked bonds, and no effort has been made to assess the predictability in an out-of-sample context. In-sample results indicate that the TIPS liquidity premium for different maturities, are a significant and economically relevant source of predictability for real excess returns across bonds of all maturities. I find adjusted R^2 values ranging over 6% to 36% across different maturities when I consider the *z-asw* liquidity premium as a predictive variable. Additionally, out-of-sample I find positive values for the R^2_{OS} proposed by Campbell and Thompson (2008). Furthermore, using the MSE-F test, the named ENC-NEW test, and the Giacomini and White (2006) test, I find evidence that the model with the liquidity premium and traditional factors outperform the constant expected return model (or the historical average excess return which is a popular benchmark). More so, liquidity does contain information about future real excess returns not captured by traditional predictor variables. However, rejecting the constant model null in favor of the linear model alternative provides evidence of predictability from a general set of predictor variables. However, it does not test whether or not the information content of liquidity does helps to predict excess returns. Therefore, as an additional exercise, I test whether or not liquidity variables have a forecasting ability for U.S. bond excess returns. To do so, I consider for the null hypothesis the linear model including traditional predictor variables, and for the alternative model the linear model but also including liquidity. I find that the unrestricted model forecasts are superior to the restricted ones, then, significantly, I conclude that the liquidity model contains information beyond the model with traditional factors.

These findings have important implications for term structure modeling and portfolio choice. First, in line with a recent topic of interest introduced by Duffee (2011) and Joslin et al. (2011), I conclude that the liquidity capture unspanned predictability in U.S. bond excess returns, in the sense that it contains predictive power beyond information contained in the yield curve. This result has relevant implications for the estimation of the U.S. yield curve using affine term structure models, as the traditional version of this class of models ignore information about expected excess returns contained in factors beyond the yield curve (Ang and Piazzesi (2003)). Second, the predictability of liquidity risk premium for U.S. excess return bonds is of importance for portfolio choice. The optimal portfolio allocation with independently and identically distributed (i.i.d.) returns can be materially altered when asset returns are predictable. In fact, if asset returns are i.i.d. over time, then the optimal asset allocation is constant over time, but if asset returns are predictable, then the optimal asset allocation depends on the investment horizon and the predictive variables. Therefore, to consider the effect of liquidity risk on optimal portfolio allocation brings the problem closer to that which investors are actually solving, and helps to better understand their optimal behavior.

The second contribution of this thesis is to assess the plausibility of liquidity as unspanned factor for the U.S. yield curve. In the second chapter, I consider a joint Gaussian affine term structure model for zero-coupon U.S. Treasury and TIPS bonds, with an unspanned factor: liquidity risk. The liquidity factor is restricted to affect the cross-section of yields but it is allowed to determine the bond risk premia. In other words, I am considering liquidity as an additional factor that does not span the yield curve but improves the estimation of bond risk premia. While macroeconomic variables (such as real output and inflation), have usually been proposed as unspanned factors, little attention has been paid to financial market variables as possible additional unspanned factors. As far as I know, the only paper considering financial factors in addition to spanned macro factors is Dewachter and Iania (2011). Considering the standard macro-finance model, they assess the relative importance of macro and financial shocks for the U.S. yield curve, by introducing additional liquidity-related and return forecasting factors. They find that the model considering liquidity and risk premium shocks significantly outperforms the standard macro factor models in fitting the yield curve. However, my work differs in a fundamental way from this paper, since I consider liquidity as an unspanned factor, and I use a different empirical approach.

I introduce the ordinary Gaussian ATSM framework, proposed in discrete time by Abrahams et al. (2013), for pricing inflation-linked bonds jointly with nominal bonds, so that both yield curves are affine in the state variables. However, in the spirit of Joslin et al. (2011), in addition to the yield curve risk (principal component factors), in this model I consider liquidity as a different source of risk, which is unspanned by the joint yield curve. Using this empirical model, I attempt to answer the following questions: (*i*) given that bond excess returns can be predicted by liquidity, can the liquidity premium be considered as a factor that forecasts bond returns but which is not spanned by the yield curve?; (*ii*) if so, does the variation in liquidity premium influence the shape of the yield curve? and finally, (*iii*) how does the market price liquidity risk in the U.S. government bond market?.

To answer the first question, I start by empirically testing the plausibility of the TIPS liquidity premium as an unspanned factor. I find that the TIPS liquidity premium fulfill the three empirical facts identified by Joslin et al. (2011), in the case of macroeconomic variables, as indicators of the presence of unspanned factor. First, the TIPS relative liquidity premium is not linearly spanned by the information in the joint yield curve. Second, the unspanned liquidity factor has a predictive power for excess returns in bond markets; and third, bond yields follow a low-dimensional factor model. Then, as is traditional in this empirical literature, I explore the inter-temporal associations between the TIPS liquidity premium and the traditional set of fundamentals that capture macro information affecting bond prices and the dynamic of the yield curve (such as the Federal Funds Rate (FFR_t), the Term spread ($TERM_t$), the default credit spread (CDS_t), the Market Volatility Index (VIX_t), among other variables). Additionally, I examine the empirical relationship between movements in the level, slope and curvature of the term structure of U.S. nominal and real interest rates, and TIPS liquidity premium shocks.

Results show that the TIPS liquidity premium increases in response to aggregate economic uncertainty shocks (represented by the VIX index), as well as to expected monetary tightening conditions (associated with a positive shock to the four-quarter-ahead eurodollar futures rate ($ED4_t$)). This means that greater economic uncertainty or doubtfulness about the near-term path of monetary policy would result in a higher liquidity risk, increasing liquidity premiums and deteriorating market liquidity. However, the TIPS liquidity premium decreases in response to increments in bond returns. This result indicates that investors will demand higher returns when liquidity conditions in TIPS bond markets worsen.Additionally, the TIPS liquidity premium influences the shape of the joint nominal and real yield curve. More so, shocks to nominal and real bond yield factors appear to have an effect on the liquidity premium. Additionally, this effect is meaningful given that (as previous empirical evidence has shown) yield curve factors are highly correlated with measures of inflation expectations and monetary policy instruments, which provides an explanation for this dynamic connection. Finally, from the estimation of a five factor model (including four principal components of zero coupon yields, plus the liquidity premium as pricing factors), I test for the presence of unspanned factors. I present empirical evidence suggesting that the the liquidity factor does not affect the dynamic of bonds under the pricing measure, but does affect them under the historical measure. Consequently, the information contained in the yield curve appears to be insufficient to completely characterize the variation in the price of curvature risk.

Overall, using the empirical model and additional empirical evidence, I conclude, first, that the TIPS liquidity premium is indeed an unspanned factor that helps to forecast U.S. bond risk premia, and that it is not linearly spanned by the information in the joint yield curve. Second, I show that the variation in the TIPS liquidity premium influences the shape of the yield curve. In fact, an increase in the TIPS liquidity premium lowers the nominal interest rates of all maturities. Similarly, the effect of a one-standard deviation shock to TIPS liquidity is positive for the slope factor, meaning that it makes the yield curve steeper. Thus, when liquidity conditions worsen in the TIPS market relative to the nominal market, nominal long-term interest rates change by much larger amounts than short-term rates. The curvature factor also increases in response to a liquidity shock, which indicates that the yield curve becomes more curved at the short end. Third, I find that only the liquidity factor significantly affects the market price of the curvature risk. This result is somewhat consistent with the results in Abrahams et al. (2013). They find that the liquidity factor significantly affects the market price of the curvature risk as well as that of the liquidity risk, however they consider liquidity as an additional spanned factor.

The final contribution of this thesis is to conditional optimal portfolio choice literature, by examining how changes in liquidity risk premium influences optimal portfolio allocations in U.S. government nominal and index-linked bonds. To do that, I assume that the investor makes decisions in real terms where the investment horizon is one-month, one-quarter and one-year. I only consider a short-term investor in the empirical analysis. The reason for this is related to the fact that for a buy-and-hold long-term investor, whose investment horizon perfectly matches the maturity of the bond, TIPS offer full protection against inflation if held until maturity. Similarly, an investor who adopts a buy-and-hold strategy for TIPS mitigates risk arising from illiquidity, given that he/she does not face higher costs of buying or selling the bond before it reaches maturity. However, TIPS are currently issued with only a few specific maturities: 5-year, 10-year and 30-year, therefore the investment horizon over which I consider investors who hold assets does not match the maturity of any outstanding TIPS. Hence, I study a short-term investor who maximizes real wealth but is not able to invest in a risk-less asset in real terms (given that TIPS are a risky asset both in nominal and in real terms), and also faces liquidity risk. Notice, however, that a short-term investor benefits from the availability of TIPS in terms of a wider investment opportunity set that allows an increase in the returns per unit of risk, investing even a small fraction of his wealth in TIPS (Cartea et al. (2012)).

The investor's problem is to choose optimal allocations to the risky asset as a function of predictor variable: the TIPS liquidity premium. As risky assets, I consider equally weighted bond portfolios (short-term and long-term), assuming an investor who is able to invest in only one risky asset. As result I differentiate various portfolio allocation problems: first, where the investor chooses between the portfolio of short-term or long-term Treasury bonds and a risk-free asset; and second, where the investor chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. Finally, I also study an investor with mean-variance (MV) and constant relative risk aversion (CRRA), with different degrees of risk aversion, in order to test the sensitivity of the optimal portfolio choice to the higher moments.

I make use of an econometric framework based on a portfolio choice problem of a single period investor, where the investor's problem is set up as a statistical decision problem, with asset allocations as parameters and the expected utility as the objective. The allocations are estimated by direct maximization of expected utility proposed by Brandt (1999). Results show that the liquidity premium seems to be a significant determinant of the portfolio allocation of U.S. government bonds. In fact, conditional allocations in risky assets decrease as liquidity conditions worsen. In particular, an increase in the liquidity differential between nominal and TIPS bonds leads to lower optimal portfolio allocations for nominal Treasury bonds, and also to lower optimal portfolio allocations in TIPS, but at different levels of liquidity. Additionally, the effect of liquidity is a decreasing function of investment horizons, in the sense that for the same degree of risk aversion the investor reacts less abruptly to an increase in the liquidity premium when he/she has a longer investment horizon. Furthermore, as the investment horizon becomes longer, the smaller the optimal portfolio weight, and so, the less is invested in the risky asset.

The above conclusions are not determined by the level of risk aversion or the investors preferences. The relation between optimal portfolio weights and the liquidity premium remains the same for different values of risk aversion, and also across investor preferences. These characteristics mainly change the level of the portfolio function, having a small impact on the shape of the function. In addition, results do not depend on a particular choice of the maturity of the liquidity premium (similar results are found when considering 10-year or 20-year liquidity premium), nor on a specific way to proxy liquidity (I have similar results with both liquidity premium measures).

From the standpoint of practical advice to portfolio investors, a final natural question to ask is whether or not the bond return predictability translates into improved out-of-sample asset allocation and performance. To answer this question, I compare the performance of the optimal portfolio choices of two investors: one investor who makes portfolio allocations based on the belief that bond returns are predictable by liquidity (conditional strategy); and the other who believes that bond returns are independent and identically distributed (i.i.d.), and ignores any evidence of bond return predictability in making his/her portfolios allocation choices (unconditional strategy). I conclude that the conditional strategy outperforms the unconditional strategy, improving not only the in-sample, but the also out-of-sample asset allocation and performance.

Overall, results show that optimal portfolios vary substantially with regards to predictor value. In particular, the effect of liquidity is a decreasing function of the investment horizon. Additionally, conditional allocations in risky assets decrease as liquidity conditions worsen. However, once the liquidity differential between U.S. nominal Treasury and TIPS bonds is sufficiently large, it leads to: (*i*) lower optimal portfolio allocations in TIPS; and (*ii*) higher optimal portfolio allocations on nominal bonds with respect to the risk-free bond. To summarize, this chapter suggests that market liquidity signals could provide valuable guidance to investors, and adds to the evidence found for stock portfolios by Ghysels and Pereira (2008), which suggests the existence of a dependence of the optimal portfolio choices on changes in liquidity.

Chapter 1

Liquidity Premium and Return Predictability in U.S. Inflation-linked Bonds Market

Abstract: This chapter discusses the predictive role of alternative measures of the liquidity premium of TIPS relative to Treasury bonds for government excess bond returns. The results show that the liquidity premium predicts positive (negative) TIPS (nominal Treasury) excess returns. The explanatory power of the TIPS liquidity premium is statistically significant and economically meaningful for short-term excess TIPS maturities and for long-term nominal Treasury bonds. I also find that the out-of-sample forecasting power of liquidity for nominal Treasury excess returns appears to have been addressed by the events during the recent financial crisis. By contrast, I have evidence of out-of-sample forecasting ability during both normal and bad times for TIPS' excess returns.
Key Words: Liquidity risk, inflation-linked government bonds, predictability, bond risk premia.
JEL classification: C13, C52, G11, G32.

1.1 Introduction

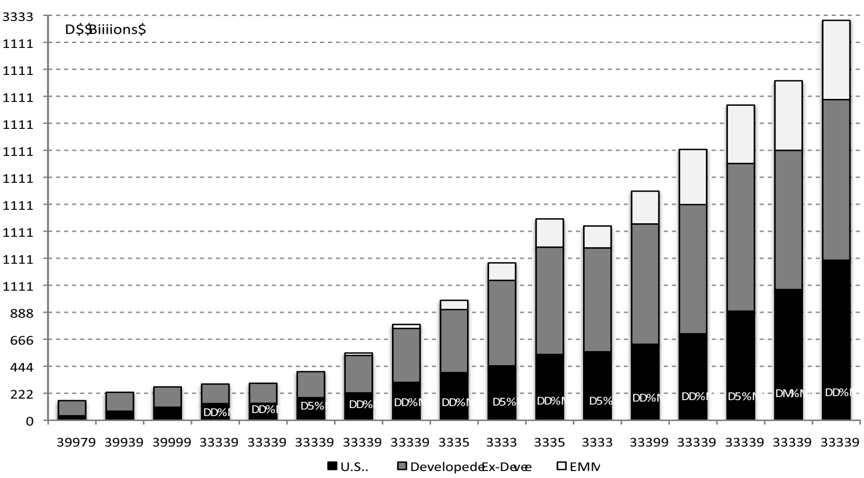
The inception of Inflation-linked bonds (ILBs), such as the U.S. Treasury Inflation Protection Securities (TIPS) and U.K. Inflation-linked Gilts, was a fundamental innovation in the financial market.[1] ILBs are financial instruments whose principal is adjusted by changes in the inflation rate. That means that the interest rate remains

[1] Although inflation-linked bonds cannot be considered as a new financial instrument (they were first issued by the State of Massachusetts in 1790), it appears clear that the issuance of this kind of bond and related inflation-linked derivatives offered important potential novel benefits to both sovereign issuers and private investors, and this was considered as a step conducive to the further broadening and deepening of financial markets.

fixed, but the actual money paid out increases or decreases depending on how the inflation rate has affected the adjusted principal. Therefore, ILBs potentially provide protection from the effects of inflation. However, they also offer additional benefits in a broader portfolio context; in particular, investors can take advantage of the diversification effects that ILBs provide, and of the improvement in the investment opportunity set in real terms, as well as serving as the safe asset for a long-term investor (see Cartea et al. (2012)).

The largest and best established inflation-linked bond market is the TIPS market. TIPS has shown a consistent growth since its inception in 1997. As Figure 1.1 shows, market capitalization has grown by more than thirty times, from \$33 billion dollars in 1997 to over \$1.200 billion in 2013. However, it has been characterized as being less liquid than the nominal Treasury bond market. Figure 1.2 plots monthly data on average trading volumes in U.S. Treasury securities from 2001 to 2014. It shows that trading activity in the TIPS is much lower than activity in nominal securities. In fact, it represents, on average, 2% of the daily total trading volume. However, as the bottom plot shows, it increases from \$1.9 billion on average during 2001 to \$11.7 billion in 2014, which represents a 500% increase in volume.

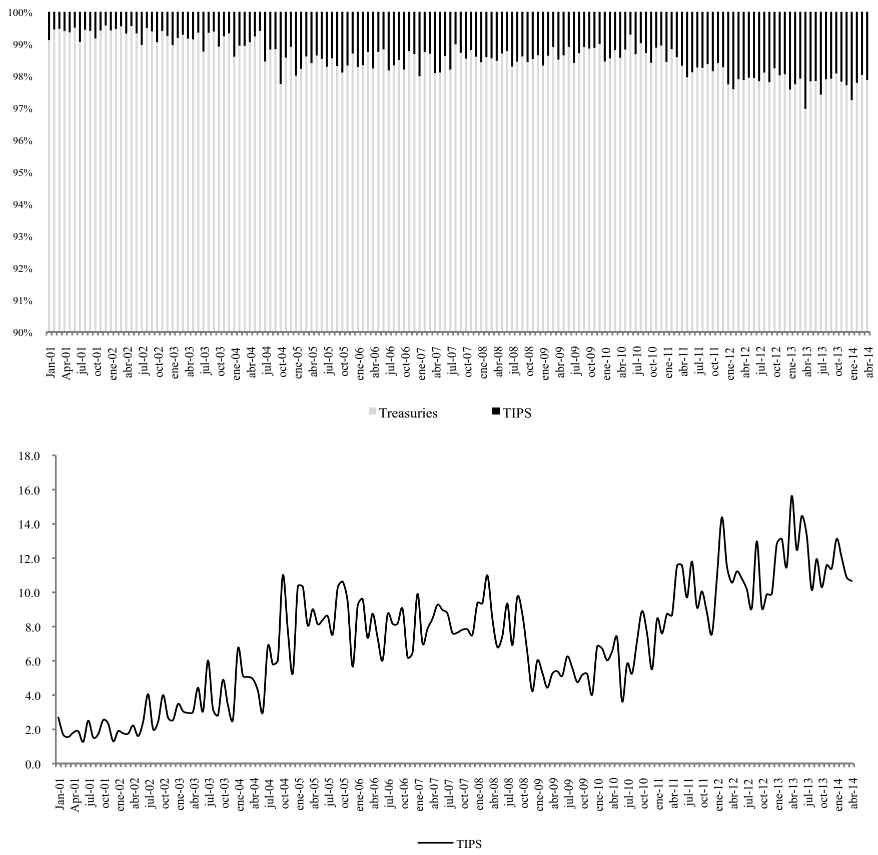
Figure 1.1: Growth of global inflation-linked bond market



Source: Barclays Universal Government Inflation-linked all maturities Bond Index. USD Billions. Annual data from 1997 to 2013. Developed Ex-U.S. indicates developed countries except U.S., and EM indicates Emerging Markets. Percentage values correspond to the proportion of U.S. TIPS on the total Index.

Theoretically, a less liquid security carries higher liquidity risk, and thus must carry a higher yield (higher expected returns or risk premium) as a compensation for the incremental risks and higher costs of trading. This additional yield is the liquidity risk premium. TIPS' lack of liquidity compares with nominal Treasuries, resulting

Figure 1.2: Average daily trading volume in U.S. Bond Market



Source: Federal Reserve Bank of New York. USD Billions. Monthly data from January, 2001 to April, 2014.

in TIPS yields including a liquidity premium relative to Treasuries.[2] The existence of this liquidity premium in TIPS yields, which is time varying, is well documented in the academic literature by Sack and Elsasser (2004), Shen (2006), Hordahl and Tristani (2010), Campbell et al. (2009), Dudley et al. (2009), Christensen and Gillan (2011), Gurkaynak et al. (2010), Pflueger and Viceira (2012), among others.

On the other hand, one stylized fact extensively documented empirically in the literature is that excess returns on Nominal Treasury bonds are predictable, however, much less has been done to explore the predictability of inflation-linked excess bond returns. The term structure slope, the forward spread, the lagged excess returns and the Cochrane and Piazzesi (2005) tent-shaped factor are some variables that have

[2] Liquidity risk premium is defined here as the total cost of all frictions to trade a relative less liquid asset beyond those of the more liquid asset against which it is being compared (Christensen and Gillan (2011)).

been identified as predictors for Treasury bonds (Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005)). Recent literature has studied the ability of macroeconomic fundamentals to predict nominal excess bond returns (see Ludvigson and Ng (2009) and Cooper and Priestley. (2009b)). The role of liquidity as a predictor variable for nominal bonds has also been studied by Fontaine and Garcia (2011). They find that funding liquidity, measured as the price difference between on-the-run versus off-the-run bonds, predicts a substantial share of the risk premium of Treasury bonds.

Nevertheless, literature for real excess return predictability is scarce and out-of-sample predictability has been not assessed. In a very recent paper Pflueger and Viceira (2012) provide empirical evidence for liquidity as a source of predictability in U.S. inflation-indexed bonds *in-sample*. I contribute to the recent literature on excess return predictability of inflation-linked bonds by assessing not only the in-sample but also the out-of-sample predictive power of liquidity risk premium.

I join Pflueger and Viceira (2012) in contributing empirical evidence for the rejection of the expectation hypothesis in the U.S. government inflation-linked bonds. In fact, I find that TIPS market liquidity contains information about future nominal and real (TIPS) excess bond returns. However, I go a step further in several areas. First, Pflueger and Viceira (2012) focus their analysis on a 10-year maturity. In contrast, I study the dynamics of liquidity across 5, 10 and 20 years to maturity, and test the hypothesis of predictability considering one-year excess returns from holding bonds with maturities of 3-, 4-, 5-, 6-, 10- and 20-year, and also consider an equally weighted bond portfolio return. Second, I undertake an extensive analysis of both in-sample and out-of-sample tests of bond returns predictability, and provide new evidence of a significant out-of-sample prediction in forecasting one-year TIPS excess returns using alternative measures of TIPS liquidity premium. Third, for robustness I investigate bond return predictability considering three different measures of TIPS liquidity premium, using a greater number of observations by sampling more frequently (daily observations) than in Pflueger and Viceira (2012).

In this paper, I use three alternative liquidity premium measures in TIPS. The first measure is the estimated liquidity premium by Pflueger and Viceira (2012). They estimate the TIPS liquidity by regressing the yield differential between nominal and inflation-linked treasury securities (commonly referred as cash Break-even inflation (BEI)), on several liquidity proxies in bond markets, and by controlling for inflation expectations. As they claim, their measure likely represents a combination of current ease of trading TIPS versus nominal U.S. Treasuries and the risk that the liquidity of TIPS might deteriorate.

Additionally, I use the Christensen and Gillan (2011) TIPS liquidity measure, which is computed as the spread between the synthetic and cash Break-even inflation rate, and represents the combined liquidity premiums in TIPS and inflation swaps. Finally, I measure the market liquidity premium in TIPS by looking at how inflation-linked asset swaps on nominal bonds correspond to inflation-linked ones. This bond asset swap spread captures the relative financing cost, the specialness and the balance sheet cost of TIPS over nominal Treasuries. This characteristic influences the ease of liquidating some securities and the attractiveness by which to hold them with respect to others. This spread should be a good market-based measure of the market perception (current and expected) of relative liquidity in bond market.

I calculate liquidity premium for 5-, 10-, 20- and 30-year maturity as the residual spread between TIPS and nominal z-spread asset swaps using daily data from November 2006 to December 2011. I call this measure the *z-asw* liquidity premium, denoted as $L_{n,t}^{z\text{-}asw}$. I find that this variable is highly correlated and shares the same dynamic pattern with other measures of relative bond liquidity premium proposed in literature. Additionally, it is strictly positive for all maturities and shows a peak in late 2008 during the financial crisis.

Controlling for typical excess return predictors such as the term structure slope and the tent-shaped factor of Cochrane and Piazzesi (2005), results indicate that the TIPS liquidity premium for different maturities, are a significant and economically relevant source of predictability for real excess returns across bonds of all maturities. I find adjusted R^2 values ranging over 6% to 36% across different maturities when I consider the *z-asw* liquidity premium as a predictive variable. Additionally, the TIPS liquidity premium also has strong out-of-sample forecasting power. In fact, I find positive values for the R_{OS}^2 proposed by Campbell and Thompson (2008). Furthermore, using the MSE-F test, the named ENC-NEW test, and the Giacomini and White (2006) test, I find evidence that the model with the liquidity premium and traditional factors outperform the constant expected return model. More so, liquidity does contain information about future real excess returns not captured by traditional predictor variables.

Similarly, I find that nominal Treasury excess returns are also predictable from the liquidity differential between U.S. inflation-linked and Treasury bonds. Remarkably, I find that the out-of-sample predictability is statistically significant for long-term maturity nominal bonds. I consider this result as very good, since the term structure slope and the tent-shaped factor of Cochrane and Piazzesi (2005) are very strong and encompass a large variety of information, and thus is quite difficult to beat in-sample, and especially out-of-sample. However, interestingly, the tent-shaped factor seems to have lost its predictive power during the sample period under analysis.

To verify the robustness of my findings, I conduct a sub-period analysis and conclude that out-of-sample forecasting power of liquidity for nominal Treasury excess returns seems to be addressed by the events during the crisis. By contrast, I have statistically significant evidence for both in-sample and out-of-sample forecasting ability during normal and bad times for TIPS excess returns.

My findings have important implications in terms of asset pricing and portfolio choice. First, in line with a recent topic of interest introduced by Duffee (2011) and Joslin et al. (2011), I conclude that the liquidity capture unspanned predictability in U.S. bond excess returns, in the sense that it contains predictive power beyond information contained in the yield curve. This result has relevant implications for the estimation of the U.S. yield curve using affine term structure models, as the traditional version of this class of models ignore information about expected excess returns contained in factors beyond the yield curve (Ang and Piazzesi (2003)). Second, the predictability of liquidity risk premium for U.S. excess return bonds is of importance for portfolio choice. The optimal portfolio allocation with independently and identically distributed (i.i.d.) returns can be materially altered when asset returns are predictable. In fact, if asset returns are i.i.d. over time, then the optimal asset allocation is constant over time, but if asset returns are predictable, then the optimal asset allocation depends on the investment horizon and the predictive variables. Therefore, to consider the effect of liquidity risk on optimal portfolio allocation brings the problem closer to that which investors are actually solving, and helps to better understand their optimal behavior.[3]

The rest of the paper is organized as follows. Section **??** provides a description of the liquidity premium measures available in literature, as well as an explanation of the alternative market-based measure for TIPS liquidity premium proposed. I describe the data and provide some basic statistics in Section **??**. Section 1.4 presents the empirical results for in-sample and out-of-sample predictability, plus some robustness results that include a sub-period analysis (during crisis and post-crisis results) with results considering monthly instead of daily frequency. Finally, Section 1.5 concludes.

1.2 Liquidity premium meausures for the U.S inflation-indexed bond market

1.2.1 Existing measures

Different practical approaches have been used to measure the liquidity differential between nominal Treasuries and TIPS yields. In general, two approaches have been implemented: market-based measures used by Christensen and Gillan (2011); and a regression procedure used by Pflueger and Viceira (2012).

To identify the liquidity component in TIPS yields, Christensen and Gillan (2011) use additional information from an inflation swap market. An inflation swap is a

[3] In a recent papers, Gomez (2014a) examines how changes in liquidity risk premium influences optimal portfolio allocations in U.S. government nominal and index-linked bonds. Additionally, Gomez (2014b) also examines the role of TIPS liquidity as an unspanned factor for the U.S. term structure.

bilateral contractual agreement. It requires, on one side (the inflation payer), to make periodic floating-rate payments linked to inflation, in exchange for predetermined fixed-rate payments from the other side (the inflation receiver). The most common contract is the zero-coupon inflation swap, which has the most basic structure with payments exchanged only on maturity.

The observed rates represent the fixed rate paid by the inflation receiver, which is the fixed rate agents are willing to pay in order to receive the cumulative rate of inflation during the swap life. Hence, the quoted rate can be viewed as a break-even inflation rate, which depends on the expected inflation over the life of the swap. Therefore, the quoted rate has been used to derive market-based measures of expectations for inflation. In theory, the inflation compensation implicit in the prices of nominal bonds relative to index-linked bonds should be the same as that embodied in inflation swap rates. The two should be consistent due to arbitrage.[4] Thus in a frictionless world the following equality must hold

$$IS_{n,t} = \pi^e_{n,t} = BEI_{n,t},$$

where $IS_{n,t}$ is the inflation swap rate, and $BEI_{n,t} = y^N_{n,t} - y^{TIPS}_{n,t}$ is the break-even inflation (commonly referred to as *cash break-even inflation rate*), which is defined as the yield difference between nominal Treasury bonds and TIPS of a corresponding maturity n.

However, the observed BEI and inflation swap rates are not equal. First, the $BEI_{n,t}$ contains information about market expected future inflation rates plus an inflation risk premium, and minus liquidity risk premium. Second, as occurs in the TIPS market, the market for inflation swaps is less liquid than the market for nominal Treasury bonds, such that the observed price of each asset should contain a non-negative time-varying liquidity premium that biases its yields upward (Christensen and Gillan (2011)). This means that inflation swap rates should be adjusted by liquidity risk. Then, the observed inflation swap rates (commonly referred as *Synthetic break-even rate*) is given by:

$$\widehat{IS}_{n,t} = IS_{n,t} + L^{IS}_{n,t},$$

where $L^{IS}_{n,t}$ is the liquidity premium included in the inflation swap rates.

Christensen and Gillan (2011) argue that the liquidity component in BEI can be identified from the difference between inflation swap rate and the cash BEI

$$\Delta_{n,t} = \widehat{IS}_{n,t} - \widehat{BEI}_{n,t} = L^{IS}_{n,t} + L^{ILB}_{n,t}. \tag{1.1}$$

[4] This is because the pay-offs of index-linked bonds can be replicated using inflation swap contracts. Two portfolios with identical future pay-offs should have the same price via arbitrage. Hence, with perfect markets we would expect perfect substitution between break-even rates available in the inflation swap and bond markets.

They call this measure ”maximum range TIPS liquidity premium”, and show that this result holds under two assumptions: *i)* the market for TIPS and inflation swaps are less liquid than the market for nominal Treasury bonds, and *ii)* the nominal Treasury yields we observe are very close to the unobservable nominal yields that would prevail in a frictionless world, that means $\hat{y}^N_{n,t} = y^N_{n,t}$. Under these assumptions, the difference between the two rates is the sum of the liquidity premiums in inflation swaps and TIPS.

Pflueger and Viceira (2012) also use an empirical approach by estimating the TIPS liquidity premium explicitly using a model. They regress the break-even inflation rate on a set of three measures of liquidity in bond markets: the nominal off-the-run spread, the relative TIPS transaction volume, and the difference between TIPS asset swap spreads and nominal U.S. Treasury asset swap spreads. They also control for inflation expectation using the survey long-term inflation expectations (π^{SPF}) and the Chicago Fed National Activity Index (CFNAI)

$$\widehat{BEI}_{n,t} - \pi^{SPF} = a_1 + a_2 X_t + a_3 CFNAI_t + \varepsilon_t,$$

where X_t is a vector containing the three liquidity proxies.

They obtain the TIPS liquidity premium as the negative of the variation in $\widehat{BEI}_{n,t} - \pi^{SPF}$ explained by the liquidity variables, while controlling for the CFNAI as a proxy of short-term inflation expectations. Hence, the estimated relative liquidity premium in TIPS yields is given by

$$\hat{L}^{PV}_{n,t} = -\hat{a}_2 X_t. \tag{1.2}$$

An increase in $\hat{L}^{PV}_{n,t}$ reflects a reduction in the liquidity of TIPS relative to nominal Treasury bonds. Given that their estimated liquidity reflects liquidity fluctuations in both nominal bonds and TIPS, they assume that the liquidity premium $\hat{L}^{PV}_{n,t}$ is entirely attributable to time-varying liquidity in TIPS rather than in nominal bonds.

1.2.2 An alternative measure for the liquidity differential between TIPS and Treasury yields

I am interested in identifying the relative liquidity premium between two comparable assets.[5] In particular, I use information provided by inflation-linked derivative markets to construct a market-based measure of TIPS liquidity premiums by comparing each TIPS asset swap spread rate versus its reference nominal bond.

[5] Absolute liquidity premium is defined as the price difference between the observed and the unobservable frictionless market outcome of a given asset. However, I work with the relative concept since it is extremely difficult to identify the unobservable frictionless price of an asset directly.

An asset swap is a derivative transaction used to convert the fixed cash flows of an asset into floating cash flows. Asset swaps are used to transform the cash flow characteristics of reference assets, so that investors can hedge the risks to create synthetic investments with more suitable cash flow characteristics. In the bond market, asset swaps typically take fixed cash flows on a bond and exchange them for Libor (i.e. floating rate payments), plus asset swap spread (ASW), which is paid over or under Libor. In particular, Treasury bond asset swaps involve transactions in which the investor acquires a bond position and then enters into an interest rate swap with maturity matching bonds, transforming the fixed coupon of the bond into a Libor based floating coupon. For an inflation-linked bond asset swap, the investor acquires a TIPS position, but now the investor enters into a series of zero-coupon inflation swaps (one for each bond payment), in order to strip out the inflation component of the fixed cash flows, and then enters into an interest rate swap.

One special feature of a bond asset swap is that it allows an investor to swap fixed rate payments on a bond to floating rate, maintaining the original credit exposure to the fixed rate bond. In an asset swap, the asset swap buyer takes on the credit risk of the bond. The purpose of the asset swap spread is to compensate the asset swap buyer for taking this risk. Thus, the spread above or below Libor reflects the credit spread difference between the bond and the swap rate. In other words, the swap spread is a measure of interbank risk; a negative (positive) asset swap level means that the credit rating of the bond issuer is better (worse) than the interbank market one.[6]

Credit risk is essential in order to be able to understand and compare the asset swap spread between nominal and inflation-linked bond asset swaps. A number of papers have shown that credit spreads may actually consist of both default and liquidity components.[7] The default component reflects differences between default risk from one issuer to another. In practice, the generic swap rate applies to top quality rated clients. Dealers use this rate as a reference and adjust it for default risk and other characteristics of the client.[8] However, Duffie and Huang (1996) show that such differences have little impact on swap rates, and empirical literature has shown that swaps spreads that arise from counterparty default risk are small, that is, about 1 basis point (see Grinblatt (2001) for a discussion). Thus, default risk can be assumed as symmetric between counterparties. Even more, investors in Treasury bonds and TIPS are both taking credit exposure to the U.S. government, and there is essentially zero default risk to U.S. government debt.

[6] For governments and for assets of better credit quality than AA-rated, the spread is normally negative, whereas for non-government credits with lower credit quality the spread is typically positive.

[7] See Liu et al. (2006) and all included references.

[8] Non-generic swaps usually differ from the generic swaps in terms of their rates, principal, or effective dates. Approximately 80% of all interest rate swaps are generic, however the underlying structure of the generic swap has been modified in a number of ways to accommodate different uses.

In addition, swap contracts could be also affected by liquidity risk if, for instance, the relevant Treasury bond trades *special* in the repo market (Liu et al. (2006)). Duffie (1996) shows that a bond that trades special in the repo market should trade at a price premium in the cash market.[9] This implies that more liquid Treasury bonds should trade more special in the repo market.[10] Hence relative specialness is considered to be a signal of the market perception of relative liquidity. A feature of the inflation-linked bonds is that the repo market does not trade them at the same level as the nominal bonds. More so, some counterparties refuse to take the inflation-linked bonds as collateral.[11] In fact, Barclays estimate that before the crisis and during standard market conditions, the repo was worth around -5 basis points on TIPS but -10 basis points on its reference nominal bond. As a result, this reflects the ease with which investors can obtain funding (funding liquidity) which reinforces the ease with which a bond is traded (asset liquidity), (see Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009), Lee (2013)).[12] In summary, the asset swap spread can be viewed as a bond specific measure, not only of credit risk implied in bond prices and yields, but also of the potential impact of transaction costs, differential taxation, institutional and foreign ownership of Treasury bonds and TIPS, as well as collateralization, market liquidity, supply and demand and other factors.

To identify the differential liquidity premium on TIPS yields, I use the difference between inflation-linked and nominal bond asset swap spreads. Through the asset swap spreads, I compare the relative value of bonds which have the same credit quality, but different characteristics. Nonetheless, the asset swap spread cannot readily be used for comparing the market's view of credit quality across different bonds, even when they have the same issuer. Therefore, I use the zero spread (z-spread) asset swap which is a useful measure of asset swap relative value, widely used by market practitioners.

The z-spread asset swap is a theoretical tool that allows us to compare asset swap levels across inflation-linked bonds, and also to compare them with asset swap levels of nominal bonds. This methodology avoids any issues with historical accretion (such as inflation accrual in the case of inflation-linked bonds) and smooths out

[9] Specialness means that an owner of a bond can lend this bond in the market and earn an additional return equal to the difference between overnight interest rates and the repo rate.

[10] The repo rate is a market driven rate so that if the bond is in very high demand in the repo market, this interest rate falls below market interest rates, so that a low repo rate is viewed as specialness of the bond.

[11] Two bonds have different collateral value if the cost of borrowing in a collateralized loan contract differs depending on the type of bond offered as collateral. The bond specific financing cost is referred to as the special repo rate. Thus, investors could improve the financing of bond positions if they trade special in the repurchase market.

[12] Funding also poses a serious risk to the asset swap buyer who will have an exposure to the cost of funding the asset on balance sheet. If there is a credit crisis, the cost of holding the asset may increase and this will reduce the profitability of the trade.

micro distortions providing a consistent measure for the asset swap.[13] The z-spread asset swap of a bond is defined as the basis point spread that would need to be added to the implied spot yield curve, such that the discounted cash flows of the a bond are equal to its present value. In other words, it is the spread that must be added to the curve being use for discounting, in order to generate a price that matches the market price. In fact, the present value of all future cash flows for a bond using prevailing spot rates could be greater than the price observed in the market. This difference arises because the market price incorporates additional factors such as liquidity risk. Hence, the z-spread quantifies the impact of additional factors that might be affecting the asset price. Additionally, the z-spread is calculated iteratively, which improves the accuracy of the value calculation as it uses the entire yield curve to value the cash flows.

Under the assumption that Fischer's equation holds, and given that the z-spread asset swap (z-asw) guarantees that discounted cash flows of the a bond are equal to its present value, I define relative liquidity as the difference between the z-asw spread of inflation-linked asset swaps with respect to its reference nominal bond

$$L_{n,t}^{z\text{-}asw} = z\text{-}asw_{n,t}^{TIPS} - z\text{-}asw_{n,t}^{N} \simeq L_{n,t}^{TIPS}. \tag{1.3}$$

This difference would capture the differential liquidity premium on TIPS yields, which is function of the relative financing cost, the specialness and the balance sheet cost of TIPS over nominal Treasury bonds. These characteristics influence the ease of liquidating some securities and the attractiveness by which to hold them with respect to others. Therefore, this spread should be a good market-based measure of the market perception (current and expected) of relative liquidity in the bond market. Finally, this spread should be non-negative, $L_{n,t}^{z\text{-}asw} \geq 0$ to be consistent with a liquidity premium measure.

This measure was first used, as one of the three liquidity proxies, by Pflueger and Viceira (2012) in their estimation of the TIPS liquidity premium. However, directly using this measure has the following advantages. First, it can be readily calculated using daily data. Second, as shown in the next section, this liquidity measure is the most strongly correlated with the other liquidity measures described in Section 2.1.

[13]Inflation-linked asset swap pricing is best thought of as akin to that in the nominal market, with the additional step of transforming all real cash flows into the nominal form (James (2010)). Hence, the inflation linked bond cash flows should be projected into nominal space. There are several ways this calculation can be done, three of which are particularly relevant to inflation-linked bonds: *i)* the par-par asset swap; *ii)* proceeds asset swap calculation; and *iii)* zero spread. The first two methods create inconsistencies comparing different bonds, and in particular comparing inflation-indexed bonds with nominal bonds. In particular, par-par asset swap net out the deviation from par of the bonds dirty cash price, however they tend to deviate from nominal par over their lifetime as they gain inflation accretion. Proceeds asset swaps avoid any issues with historical accretion, and hence are more appropriate for markets where bonds have been accreting inflation for years. However, it tends to exaggerate the value of the asset swap. For additional details about the different methods, see James (2010)

1.3 The data and basic statistics

1.3.1 Liquidity variables

I obtain daily nominal and TIPS z-spread asset swaps data from Barclays Live. The z-spreads are available starting in November 2006 until December 2011 for $n = 5, 10, 20, 30$ years to maturity. I have asset swaps on TIPS and nominal bonds for a set of outstanding bonds described on Table A1.1 in Appendix 1.5.[14] The residual spread between different TIPS and nominal z-spread asset swaps with the same maturity was calculated. Then the average spread across different assets for each maturity was computed, and corresponds to the TIPS liquidity premium measure that I use, $L^{z\text{-}asw}_{n,t}$ for n=5-,10-,20-,30-year maturities.

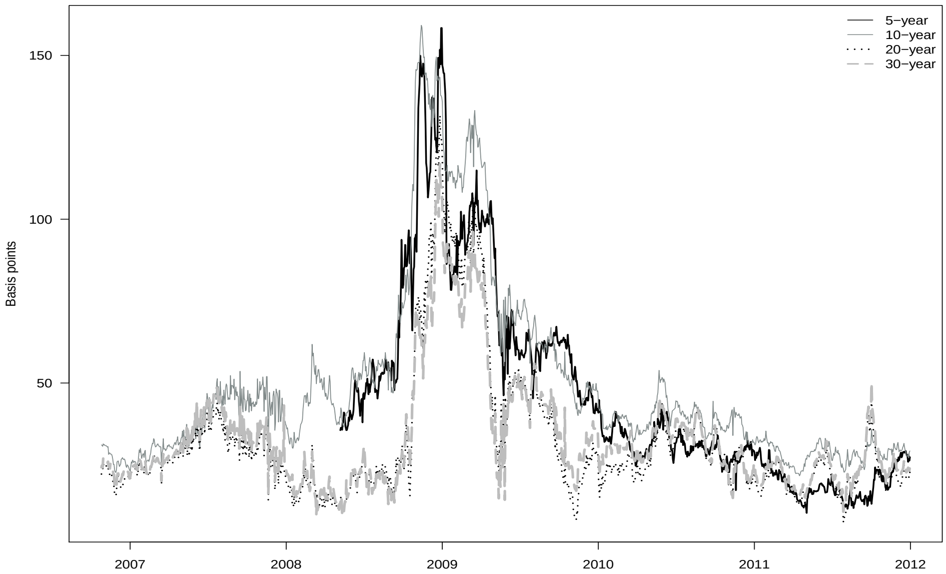
For comparison, I also consider the liquidity measures proposed by Christensen and Gillan (2011), and Pflueger and Viceira (2012) as described before, and defined by $\Delta_{n,t}$ in equation (1.1) and $\hat{L}^{PV}_{n,t}$ in equation (1.2), respectively. To construct the liquidity premium proposed by Christensen and Gillan (2011), I use daily estimates of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007), and Gurkaynak et al. (2010) for observed bond yields. This data set contains constant maturity yields for maturities from 2 to 20 years. For zero-coupon inflation swap rates, I use U.S. daily quotes from Barclays Live, which are converted into continuously compounded rates to make them comparable to the other interest rates. I compute this measure for 5, 10 and 20 years to maturity from January 2004 to December 2011. I also consider the liquidity premium proposed by Pflueger and Viceira (2012). They estimated the monthly liquidity premium from January 1999 to September 2010 for 10-year TIPS. [15]

Figure 1.3 presents the graphs for *z-asw* liquidity measure for all maturities and Table 1.1 (panel 1) shows a summary of statistics. We can see that $L^{z\text{-}asw}_{n,t}$ are strictly positive for all four maturities. Furthermore, the term structure of the combined liquidity premiums tends to be downward sloping with maturity which is consistent with results found by Christensen and Gillan (2011), however after 2010 it seems not to be the case, with results tending to be more flat. Over the whole sample the mean liquidity has been about 49 to 32 basis points. Additionally, the liquidity premium grew substantially during the financial crisis of 2008 and 2009. In fact, Figure 1.3 shows a peak in late 2008 during the financial crisis, right after the Lehman Brothers bankruptcy announcement. This is also consistent with previous results found in the literature.

[14] Asset swaps on bonds with less than 12 months to maturity are dropped from the estimation of the liquidity, because the effect of the indexation lag makes the prices of these securities erratic, as was noted by Gurkaynak et al. (2010). All other asset swaps are included in the calculation.

[15] I am very thankful to Luis Viceira and Carolin Pflueger for sharing their estimation of liquidity premium.

Figure 1.3: *z-asw* liquidity premium



The *z-asw* liquidity premium corresponds to the residual spread between TIPS and nominal bonds asset swaps calculated by using daily nominal and TIPS *z*-spread asset swaps data from November 1, 2006 to December 30, 2011.

Table 1.1: Summary Statistics of liquidity measures

Maturity	Corr.	Mean	Median	Std	Min	Max
z-asw Liquidity premium: $L^{z\text{-}asw}_{n,t}$						
5		47.09	36.12	31.53	10.31	158.38
10		49.59	40.88	28.63	21.69	159.18
20		32.38	25.92	21.28	7.53	131.69
30		33.16	28.01	17.90	9.68	116.88
Cristersen et al. (2011): $\Delta_{n,t}$						
5	0.8962	46.29	32.34	28.10	6.26	215.11
10	0.9338	29.43	25.35	16.98	4,50	118.75
20	0.9422	26.98	21.00	17.54	0.00	120.10
Pfluger and Vicerira (2012): $\hat{L}^{PV}_{n,t}$						
10	0.9357	55.98	50.05	26.97	25.00	152.60

U.S. daily data from November 1, 2006 to December 30, 2011 in basis points. The *z-asw* liquidity premium corresponds to the residual spread between TIPS and nominal bonds asset swaps calculated by using nominal and TIPS *z*-spread asset swaps rates. The other liquidity measures correspond to the TIPS liquidity proposed by Christensen and Gillan (2011), and the estimated 10-year TIPS liquidity premium estimated by Pflueger and Viceira (2012) which is available only in a monthly frequency. The correlation coefficients (Corr.) correspond to the linear association between *z-asw* liquidity premium and the corresponding measure proposed in literature for each maturity.

Panel 2, on Table 1.1, provides descriptive statistics for the maximum range proposed by Christensen and Gillan (2011), and panel 3 for Pflueger and Viceira (2012) provides the estimated liquidity available only on a monthly frequency. The first column shows the correlation coefficients between $L^{z\text{-}asw}_{n,t}$ liquidity measures and the other two measures by maturity. The sample correlation coefficients are more than 0.90 for the maximum range for all maturities and 0.94 for the 10-year liquidity of Pflueger and Viceira (2012). This suggests that all measures are capturing similar information about the yield difference between TIPS and U.S. Treasury bonds. Other columns shows the mean, median, standard deviation and the maximum and minimum value for each liquidity measure. Although there are differences in the magnitude of statistics calculated from the three liquidity measures, again $L_{n,t}^{z\text{-}asw}$ is close to the other two measures by maturity.

Figure 1.4 compares the two market-based measures for TIPS liquidity, that is the *z-asw* liquidity premium from liquidity measure proposed by Christensen and Gillan (2011). In general, both liquidity measures share the same dynamic pattern for all maturities. However, for the 5-year maturity the maximum range TIPS liquidity premium is slightly above $L_{n,t}^{z\text{-}asw}$; especially at the height of the financial crisis (about September 2008) with the Lehman Brothers bankruptcy announcement (this is indicated on the graph by the vertical dotted line). On the contrary, it is well below $L_{n,t}^{z\text{-}asw}$ measure for the 10-year maturity during most of the period sample, starting in August 2007 with the onset of the financial crisis and continuing until November 2011. For the 20-year maturity, the $L_{n,t}^{z\text{-}asw}$ measure is slightly below, particularly between late 2008 and early 2009. Additionally, despite general differences, both measures show sharp spikes in the liquidity premium, for different maturities, just after the height of the the financial crisis.

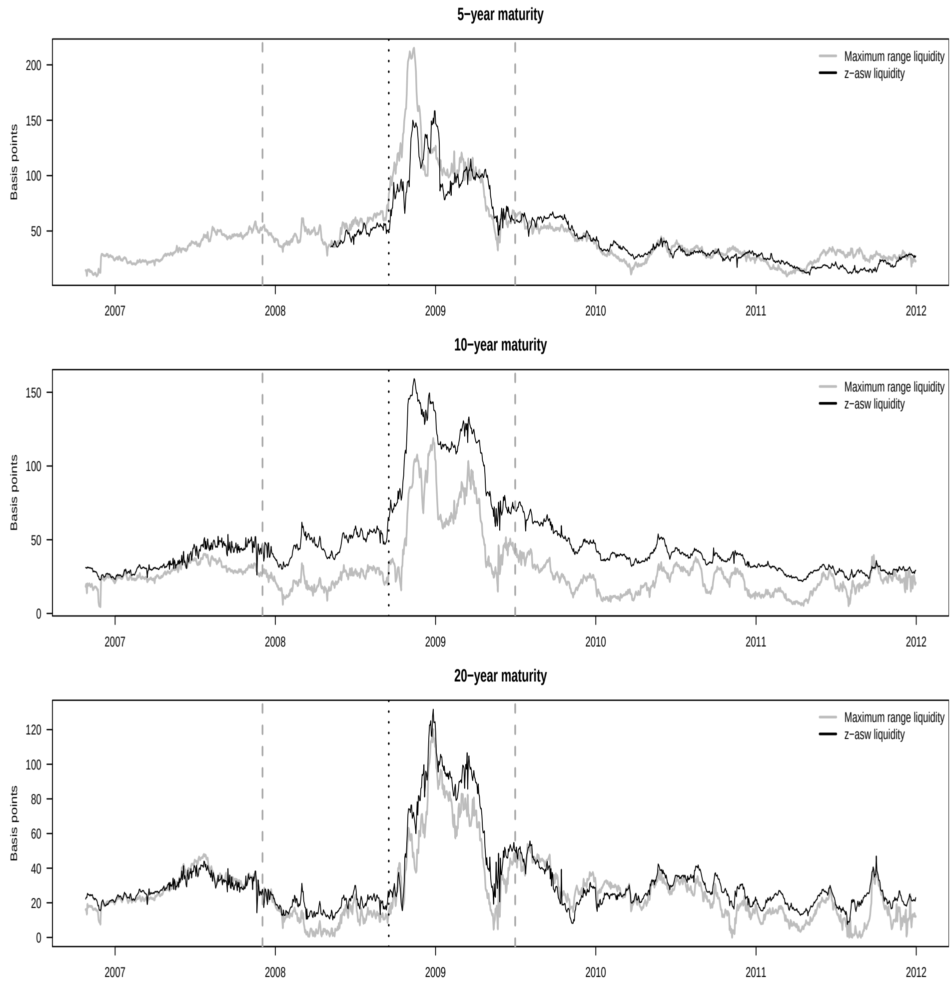
Figure 1.5 compares these two liquidity measures with respect to Pflueger and Viceira's (2012) estimated liquidity. I plot the three measures on a monthly frequency and just for the 10-year maturity.[16] Both the maximum range TIPS by Christensen and Gillan (2011) and the *z-asw* liquidity premium are below the premium estimated by Pflueger and Viceira (2012). The average difference is about 68 and 51 basis points with respect to the Christensen and Gillan (2011) and the *z-asw* liquidity premium, respectively.

Although, in general, I find that correlation is high and the dynamic pattern is the same overall for the three liquidity measures, there are some important deviations, especially at the 10-year maturity, which deserve more analysis. Theoretically, there exists a close relationship between bond break-evens and inflation swaps rates; in essence, both measure the markets' expectations of future inflation. However, the most recent crisis showed that the U.S. cash and swap markets can diverge significantly, with each market driven by its specific dynamics such as funding costs. Asset swapping activity should theoretically hold the two markets together (James (2012)), but the empirical evidence presented here shows that such activity was not sufficient to offset diverging forces in stressed market conditions.

In fact, Campbell et al. (2009) argue that investors supplying inflation protection,

[16] Table A1.2, Appendix 1.5 provides descriptive statistics for the three liquidity premium measures, but on a monthly frequency.

Figure 1.4: Comparison between U.S. daily liquidity premium measures

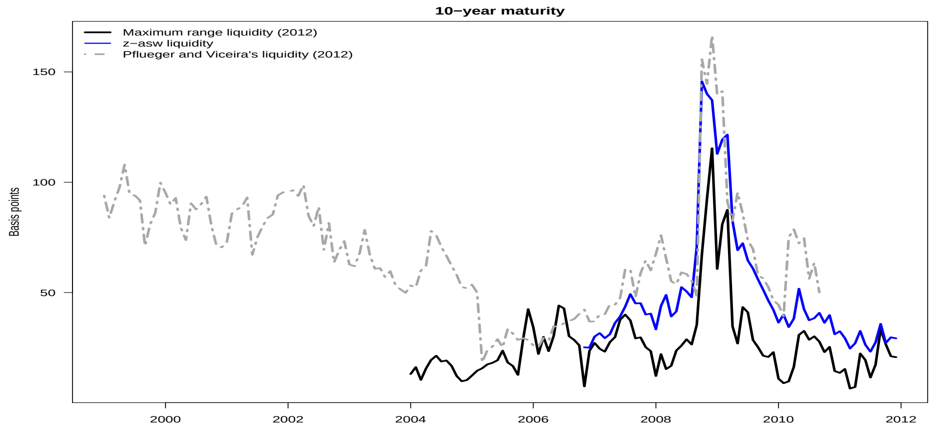


U.S. daily data from November 1, 2006 to December 30, 2011. The *z-asw* liquidity corresponds to the residual spread between TIPS and nominal bonds asset swaps calculated using nominal and TIPS z-spread asset swaps rates. The maximum range liquidity corresponds to the measure proposed by Christensen and Gillan (2011). Vertical dashed lines highlight the last NBER recession and the dotted line indicates the Lehman Brothers bankruptcy announcement.

through the inflation swap market, typically hedge their inflation swap positions by simultaneously taking long positions in TIPS and short positions in nominal Treasuries in the asset swap market. That means that the short part in the inflation swap (inflation payer) to hedge the assumed risk takes a long asset swap position on TIPS (floating rate payer) and a short asset swap position on nominal bonds (fixed rate payer). Thus, leveraged investors selling inflation protection in the inflation swap market face a positive financing cost derived from this strategy.

Therefore, the difference between synthetic and cash break-even rates reflects,

Figure 1.5: Comparison between U.S. monthly liquidity premium measures



U.S. monthly data from November 2006 to December 2012. The *z-asw* liquidity corresponds to the residual spread between TIPS and nominal 10-year bonds asset swaps calculated using end of the month data for nominal and TIPS z-spread asset swaps. The maximum range liquidity corresponds to the measure proposed by Christensen and Gillan (2011) for 10-year maturity calculated from January 2004 until December 2012, and Pflueger and Viceira (2012) estimation of the 10-year TIPS liquidity premium from January 1999 to December 2010.

among other things, the cost of manufacturing an inflation risk protection strategy, which is a financing cost (Campbell et al. (2009)). Additionally, Christensen and Gillan (2011) show that under competitive circumstances, and using this kind of hedging activity, the maximum range for the TIPS liquidity premium should be equal to the difference of TIPS and nominal Treasury asset swap spreads. However, this seems not to be the case for 5- and 10-year maturities over the sample period, particularly at the height of the financial crisis. Hence, asset swapping activity was not enough to hold the two markets together.

At the beginning of Lehman's collapse, the price of inflation-linked bonds started to cheapen relative to the equivalent nominal bonds. This came about as a result of a number of factors: *i*) investors switched their TIPS holdings in favor of nominal government bonds, that were perceived to have greater liquidity; *ii*) banks, in order to reduce their balance sheets, began to sell inflation-linked bonds that had been used to hedge inflation swap exposures; and *iii*) TIPS also cheapened because of the deterioration in credit markets, which affected TIPS more than nominal bonds because of their longer duration. Additionally, TIPS also cheapened relative to inflation swaps, reflecting the preference to hold swaps rather than balance-sheet-intensive bonds. As a result, there was a divergence in inflation expectations implied by synthetic and cash break-even rates.

The huge dislocation between inflation-linked and nominal government bonds created an opportunity for investors to benefit through asset swaps. In fact, investors would buy an inflation-indexed bond and swap out the inflation component, creating virtually the same risk exposure as an investment in a nominal Treasury bond but with much higher returns. Therefore, with a few sources of inflation supply, the asset swap market was a valuable source of inflation supply in the inflation derivatives market during the financial crisis.

As a result, asset swaps on inflation-linked bonds rapidly cheapened relative to historical levels. However, this cheapening was over and above the cheapening seen in the nominal asset swap market in the same period. Figure 1.6 shows the evolution of nominal asset swap z-spread on U.S nominal and TIPS bonds from November 2006 to December 2011, and shows that TIPS that were trading at Libor -25 basis points pre-Lehman announcement,were trading between Libor +100 and +150 basis points during the crisis. In consequence, the spread between nominal and inflation-linked bonds on asset swaps widened from around 20 basis points to well over 100 basis points. In essence, this spread represents an excessive relative liquidity premium, as the credit risk on the two bonds was more or less the same, since investors in Treasury bonds and TIPS are both taking credit exposure to the U.S. government. In summary, this behavior was the result of a "flight to quality" and drop in liquidity which favored nominal bonds, as they were less draining on bank balance sheets and a more stable source of funding for investors at that time (Pond and Mirani (2010)). In conclusion, during illiquid periods, the relative z-spread asset swap allows a more precise inference about how inflation-linked bonds perform relative to nominal bonds, therefore the spread between nominal and inflation-linked bonds on asset swaps seems to be a better proxy of liquidity in times of financial distress.[17] Additionally, during normal times this spread should be equal, or at least very close, to the spread between synthetic and cash break-even inflation rates.

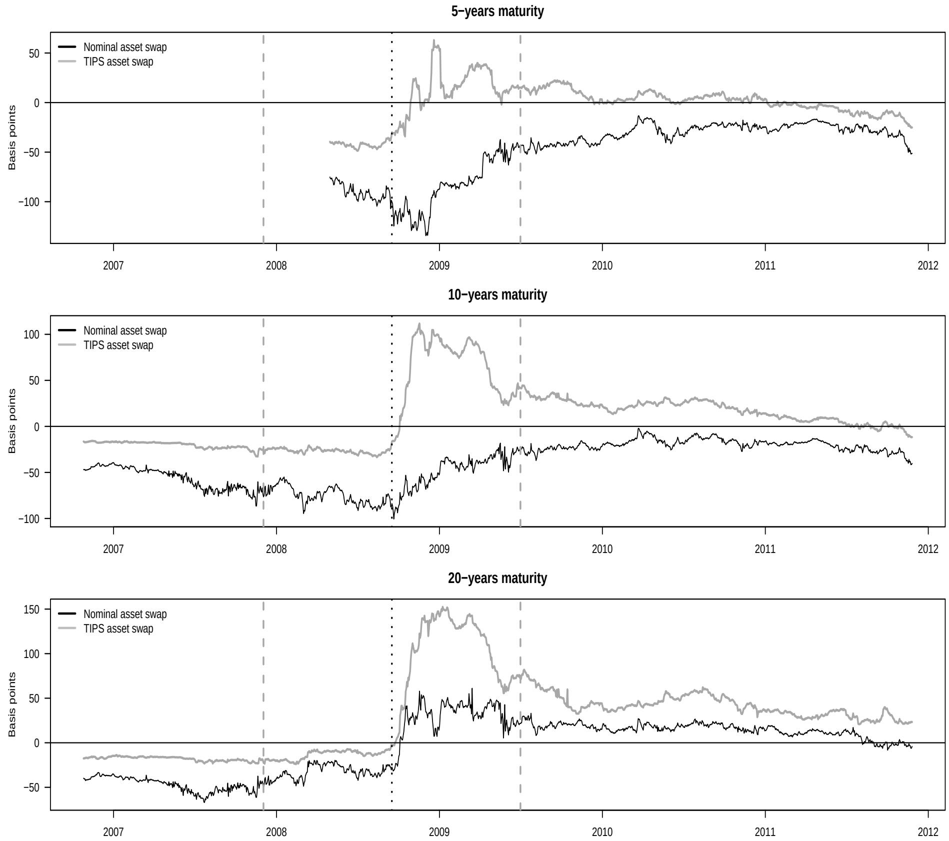
Figure 1.6 provides the historical level of asset swap spreads across the TIPS bonds, along with the asset swap spreads on similar maturity nominal Treasury bonds. Figure 1.6 shows that TIPS have traded significantly more cheaply during the whole sample period. In fact, after the crisis investors have traded the 20-year TIPS asset swap curve on average at Libor +40 basis points, while before crisis, they traded flat at Libor -15 basis points, as an example. For the 20-year Treasury asset swap were swapped on average at Libor +24, while before crisis at Libor -42. However, Figure 1.6 also shows that after the crisis the spread between TIPS asset swaps and those on nominal bonds has narrowed sharply since the peaks in October 2008.

1.3.2 Bond market variables

I am interested in determining whether or not the TIPS liquidity premium contains additional information over the existing traditional factors used to explain the bond

[17] It is well known that asset swaps are closely associated with credit derivatives such as credit default swaps (CDS), however Mayordomo et al. (2011) show that asset swap spread is a more accurate measure of credit risk than CDS during illiquid periods.

Figure 1.6: Nominal and TIPS z-spread asset swap rates by maturity



U.S. daily data from November 1, 2006 to December 30, 2011 for Nominal and TIPS z-spread asset swap rates. Vertical dashed lines highlight the last NBER recession and the dotted line indicates the Lehman Brothers bankruptcy announcement.

risk premium. To that end, I calculate annual excess returns from daily observations of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007), and Gurkaynak et al. (2010), respectively, available on the Federal Reserve website. The sample period is from November 2006 to December 2012 for yields, and from November 2007 to December 2012 for annual excess returns, with a daily frequency.

As predictor variables for excess bond returns I consider frequently reported variables, such as the term yield spread (TERM) and the Cochrane and Piazzesi (2005) tent-shaped factor (CP). I construct the term spread as the difference between market yield on U.S. Treasury securities at the 10-year constant maturity and the 1-year Treasury bill using data from the Federal Reserve Board statistical releases. Additionally, I construct the CP factor using 1 to 5-year zero coupon nominal bonds yields from Gurkaynak et al. (2007), and using the optimal weights found in Cochrane and Piazzesi (2005). Both variables are calculated using daily data for the period November 2006 to December 2011 (see Figure A1.2 in Appendix 1.5). Table 1.2 shows the summary statistic for bond excess returns as well as for its traditional predictor variables.

Table 1.2: Summary statistics of bond excess returns and bond factor variables

	TIPS excess returns				*Nominal excess returns*			
	Mean	Std	Max	Min	Mean	Std	Max	Min
$rx_{t+1}^{(3)}$	1.02	1.62	5.88	-3.45	1.15	0.59	2.50	-0.10
$rx_{t+1}^{(4)}$	1.50	1.80	6.96	-4.08	1.75	0.80	3.60	-0.40
$rx_{t+1}^{(5)}$	1.93	1.95	7.96	-4.56	2.29	1.01	4.60	-0.90
$rx_{t+1}^{(6)}$	2.31	2.09	8.89	-4.99	2.76	1.25	5.50	-1.40
$rx_{t+1}^{(10)}$	3.31	2.67	11.71	-7.17	3.92	2.50	9.91	-3.95
$rx_{t+1}^{(20)}$	4.25	4.46	15.26	-10.42	4.64	5.80	18.74	-14.09
$\overline{rx}_{t+1}^{(ST)}$	1.69	1.84	7.42	-4.27	1.99	0.87	4.10	-0.67
$\overline{rx}_{t+1}^{(LT)}$	3.78	3.45	12.16	-8.79	4.28	4.09	13.86	-9.02

	Bond Variables			
	Mean	Std	Max	Min
$TERM_t$	1.96	1.19	3.53	-0.48
CP_t	1.41	1.31	0.88	-0.52

$rx_{t+1}^{(n)}$ denotes the one-year bond excess log-returns on $n = 3-, 4-, 5-, 6-, 10-, 20-$ year maturity measured daily in percentage units. $\overline{rx}_{t+1}^{(ST)}$ is the average excess returns on three to six years to maturity bonds (equally weighted short-term portfolio) and $\overline{rx}_{t+1}^{(LT)}$ is the average excess returns on ten and twenty years to maturity bonds (equally weighted long-term portfolio). Term yield spread ($TERM_t$) is the difference between market yield on U.S. Treasury securities at 10-year constant maturity and 3 month T-bill. CP_t is the tent-shaped forward factor by Cochrane and Piazzesi (2005). The sample spans the period from 01/11/2007 to 30/12/2012 for excess bond returns and from 01/11/2006 to 30/12/2011 for factor variables.

Table 1.2 exhibits the annual excess log returns over the one-year yield bond for both nominal and TIPS bonds in percentage. It shows that longer term bonds experienced higher average returns than the shorter term maturities, suggesting the presence of a term premium in bond returns. Interestingly, TIPS have not outperformed comparable nominal bonds during the sample period considered here, meaning that the presence of a bond premium in TIPS during this period seems not to be confirmed. Cartea et al. (2012) have found that during last two U.S. recessions (March 2001 to November 2001, and December 2007 to June 2009) nominal bonds outperformed TIPS' returns, thus nominal bonds would be preferable during recessions than TIPS with similar a maturity. This is in line with Krishnamurthy and Vissing-Jorgensen (2012a) who argue that Treasury bonds have some of the same features as money, and that drives down the yield on Treasuries relative to assets that do not share these features. Additionally, Pflueger and Viceira (2012) suggest that their estimated relative liquidity premium might partly reflect a convenience yield on nominal bonds, rather than a liquidity discount specific to TIPS. In this case, TIPS are not undervalued securities, but instead investors may be willing to pay a liquidity premium on nominal Treasury bonds. In this case, the *z-asw* liquidity measure can also reflect the liquidity-based convenience yield differential between Treasury and TIPS bonds with identical maturities.

Finally, the long-term equally weighted TIPS portfolio has a mean annual excess return of 3.78% per year, while short-term they experience a mean return of 1.69%. The same occurs with equally weighted nominal bond portfolios: long-term nominal bond portfolios show higher annual realized excess log returns than short-term portfolios.

1.4 The predictive power of the liquidity premium

This section explores the potential predictability power that the TIPS liquidity premium could have for inflation-linked and Treasury excess bond returns. For robustness, I consider three measures of liquidity premium for TIPS: the *z-asw* liquidity premium $L_{n,t}^{z\text{-}asw}$; the maximum range liquidity premium $\Delta_{n,t}$ proposed by Christensen and Gillan (2011); and the 10-year estimated liquidity premium $L_{n,t}^{PV}$, as proposed by Pflueger and Viceira (2012).

1.4.1 Linear regression analysis

First, I regress individual bond excess returns considering the following standard predictive regression framework

$$rx_{t+1}^{(n)} = \alpha + \delta_n^{\top}\mathbf{M}_t + \beta_n^{\top}\mathbf{L}_t + \epsilon_{t+1}^{(n)}, \tag{1.4}$$

where $rx_{t+1}^{(n)}$ denotes annual excess log returns on n=3-, 4-, 5-, 6-, 10-, 20-year maturity, calculated as $rx_{t+1}^{(n)} = r_{t+1}^{(n)} - y_t^1$ with $r_{t+1}^{(n)}$ the holding one-year log return on a zero-coupon n-period bond, and where y_t^1 is the one year log yield. $\mathbf{M}_t$ denotes a two-dimensional vector of traditional predictor variables, which includes CP_t and $TERM_t$. $\mathbf{L}_t$ is a vector containing the liquidity premium measures. In particular, in this section, $\mathbf{L}_t$ contains either the maximum range $\Delta_{n,t}$ or the $L_{n,t}^{z\text{-}asw}$ liquidity premium for n= 5, 10, 20 years to maturity. The sample period is from November 1, 2006 to December 30, 2011 with a daily frequency. Finally, α is a constant term, and $\epsilon_{t+1}^{(n)}$ is a white noise error term.

To find the incremental contribution of $\mathbf{L}_t$ to bond excess return predictability, I estimate the equation (1.4) including only $\mathbf{M}_t$ which is the benchmark model, and then compare the adjusted R^2's to those from the forecasting competing model which also includes liquidity variables.

Additionally, as a robustness check as to how excess returns are computed, I estimate the coefficients by regressing the equally weighted bond excess return portfolio, computed as the average (across maturities) excess returns, on liquidity variables and/or controls variables. In this case, I consider the regression

$$\bar{rx}^{j}_{t+1} = \alpha + \delta^{\top}\mathbf{M}_t + \beta^{\top}\mathbf{L}_t + \bar{\epsilon}^{j}_{t+1} \tag{1.5}$$

where $\bar{rx}^{j}_{t+1}$ denotes the average excess returns for short-term (ST) or long-term (LT) maturities $j = ST, LT$. The equally weighted short-term bond portfolio is calculated as the average excess returns on three to six years to maturity bonds $\bar{rx}^{(ST)}_{t+1} = (1/4)\Sigma_n rx^{(n)}_{t+1}$ with $n = 3, 4, 5, 6$. Similarly, the equally weighted long-term (LT) bond portfolio is calculated as the average excess returns on 10 and 20 years to maturity bonds $\bar{rx}^{(LT)}_{t+1} = (1/2)\Sigma_n rx^{(n)}_{t+1}$ for $n = 10, 20$.

In order to test the in-sample forecasting ability of variables in equations (1.4) and (1.5), I employ the whole sample and conduct a Wald test for the null hypothesis that $H_0 : \beta_n = 0$ vs $H_1 : \beta_n \neq 0$. If the null hypothesis cannot be rejected at the desirable significance level, the predictive variable employed does not have any forecasting ability. I run standardized regressions, i.e. all variables are standardized in order to make the comparison across different predictors easier. Coefficients are estimated by ordinary least squares.

Literature highlights the importance of addressing the bias in estimates and t-statistics in predictive regressions with persistent variables, especially because strong autocorrelation might be induced from the overlapping scheme. The essential problem is to get the right standard errors, so I calculate standard errors and t-statistics using Newey and West (1987) correction based on the automatic lag selection. Additionally, given that Newey-West standard errors are based on asymptotic approximations that might be inadequate in finite samples, I follow Bouwman et al. (2012) using a bootstrap analysis to check the robustness of my inference in finite samples. In particular, I test for the significance of the variables of interest by constructing block bootstrap samples for $\mathbf{M}_t$, $\mathbf{L}_t$ and $rx^{(n)}_{t+1}$, and I report the p-values based on the bootstrap analysis. For the selection of optimal block lengths, I use the data-based procedure propose by Hall et al. (1995), which is one of the existing general block selection methods in the literature. The bootstrap procedure is described in Appendix 1.5.

TIPS excess returns

Table 1.3 presents analysis of the predictable variation in TIPS excess returns. Panel A presents a replication of the constrained regressions in Cochrane and Piazzesi (2005) but for inflation linked U.S. bond excess returns. This exercise does not confirm the stylized predictability results found for U.S. Treasury bonds that is, that the tent-shaped factor describes time variation in expected returns for all maturity bonds, nor does it predicts the TIPS' excess returns. This result provides support for the hypothesis that nominal bond term spread partly reflects the time-varying inflation risk premium, which affects the prices of nominal government bonds but not inflation-linked bonds. The coefficients of determination adjusted by degrees of freedom, $AdjR^2$, range between 0.08% and 7% for different maturities in individual regressions. For the short-term average excess returns regression, the $AdjR^2$ is equal to 0,3% while for the long-term average regression is 12.6%.

Table 1.3, panel B shows the results considering the *z-asw* liquidity premium for 10-year maturity. In this case, I run regressions including three predictor variables: CP_t, $TERM_t$ and $L^{z\text{-}asw}_{(10),t}$. In general, the estimated coefficients on liquidity variables have the expected positive sign and are statistically significant at 10% using the p-values based on the bootstrap procedure, except for excess log-returns on 10 and 20 years to maturity. The positive sign indicates that as liquidity conditions worsen (higher liquidity premium), returns are predicted to rise (higher excess returns) in order to compensate for the higher risk during bad times. The economic impact of liquidity is also important. For instance, one standard deviation move in $L^{z\text{-}asw}_{(10),t}$ leads to an increase of 90 basis points (0,49×1,84) in the short-term average TIPS excess returns. This magnitude is substantial relative to short-term average TIPS of 169 basis points. In the same way, one standard deviation move in the 5-, 20- and 30-year liquidity premiums results in an increase of 104, 94 and 98 basis points, respectively (see Table C1.2 in Appendix 1.5). I find similar results running the regressions using the TIPS liquidity premium measure proposed by Christensen and Gillan (2011). (Table C1.1, panel A, in Appendix 1.5, shows the results).

Additionally, $AdjR^2$ coefficients show an important increment with respect to the results in panel A. I find that the $AdjR^2$ is now ranging between 5% and 35% for different maturities. The increment for a short-term equally weighted portfolio is about 21%. This result shows that TIPS' excess returns, for the period under study, are mainly varying with liquidity risk. However, the effect is more pronounced for shorter maturity bonds, given that the change of the adjusted R^2 is larger compared to a reduced regression (which only includes the CP-factor and Term spread).

As a robustness test for the last results, I run regressions that ignores the traditional controls variables, i.e. the CP-factor and Term spread. In general, I find that results are robust, in the sense that signs and significance remain almost unchanged. The consistency of the sign, its size and the statistical significance provides evidence that liquidity variables have an important predictive power for TIPS' excess returns. In fact, the $AdjR^2$ decrease from 26% at the maturity of 3-year to 1,3% for 20-year bonds but are always higher than for the regressions including traditional controls (except for maturities beyond 10 years, i.e. 10- and 20-year maturity). The importance of traditional factors for longer-term maturities, as well as the importance of liquidity for shorter-term bonds, is also confirm by the value of the $AdjR^2$ reported by the equally weighted portfolio regressions, which are equal to 0,1% and 18,2% respectively. These results are not presented in this paper but are available upon request.

Table 1.3: TIPS excess returns predictability

Panel A: Traditional bond factors

n	CP	pv	$TERM$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,02	0,88 (0,89)	-0,16	0,44 (0,49)	2,54	-1,01	-357,87	-126,08	-2,03	0,98
4	0,00	0,98 (0,94)	-0,09	0,69 (0,73)	0,75	-1,70	-452,39	-161,06	-3,13	1,00
5	0,02	0,90 (0,98)	-0,02	0,93 (0,97)	0,08	-2,89	-512,47	-157,65	-3,98	1,00
6	0,02	0,87 (0,91)	0,04	0,85 (0,93)	0,22	-2,32	-371,61	-49,05	-2,15	0,98
10	0,01	0,92 (0,95)	0,23	0,29 (0,37)	5,12	0,51	241,13	173,29	2,18	0,01
20	0,05	0,68 (0,95)	0,42	0,06 (0,10)	7,14	0,49	136,94	85,01	1,14	0,13
$\bar{rx}^{ST}_{t+1}$	0,01	0,96 (0,99)	-0,05	0,82 (0,87)	0,26	-2,53	-521,27	-171,26	-4,46	1,00
$\bar{rx}^{LT}_{t+1}$	0,04	0,75 (0,97)	0,36	0,20 (0,26)	12,63	0,50	166,15	105,17	1,81	0,04

Panel B: Traditional bond factors and z-asw liquidity measure

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,08	0,53 (0,62)	-0,30	0,11 (0,37)	0,61	0,00 (0,02)	35,88	0,38	64,07	48,45	2,96	0,00
4	0,10	0,49 (0,62)	-0,21	0,29 (0,55)	0,54	0,00 (0,01)	26,71	0,09	7,75	10,75	1,50	0,07
5	0,10	0,50 (0,71)	-0,12	0,55 (0,55)	0,45	0,00 (0,05)	18,35	-0,49	-20,22	-6,49	-1,23	0,89
6	0,09	0,55 (0,67)	-0,04	0,85 (0,93)	0,36	0,00 (0,10)	11,94	-0,68	-23,54	-6,52	-1,80	0,96
10	0,03	0,83 (0,78)	0,21	0,33 (0,43)	0,07	0,65 (0,70)	5,61	0,57	363,16	251,05	2,27	0,01
20	0,01	0,93 (0,91)	0,47	0,04 (0,19)	-0,23	0,20 (0,67)	21,95	0,60	380,01	244,67	2,20	0,01
$\bar{rx}^{ST}_{t+1}$	0,09	0,51 (0,57)	-0,16	0,42 (0,22)	0,49	0,00 (0,10)	21,69	-0,23	-11,51	-1,42	-0,42	0,66
$\bar{rx}^{LT}_{t+1}$	0,02	0,87 (0,79)	0,39	0,10 (0,22)	-0,12	0,51 (0,59)	13,94	0,60	450,40	293,37	2,24	0,01

Continuation: TIPS excess returns predictability

Panel C: Traditional bond factors and common liquidity factor

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(pc)}$	pv			Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,06	0,66 (0,78)	-0,22	0,26 (0,57)	0,62	0,00 (0,02)			40,41	0,23	39,86	34,70	1,75	0,04
4	0,08	0,58 (0,72)	-0,14	0,49 (0,66)	0,57	0,00 (0,04)			32,55	-0,06	-6,02	4,19	0,61	0,27
5	0,09	0,57 (0,56)	-0,06	0,76 (0,86)	0,50	0,00 (0,04)			24,50	-0,65	-30,24	-10,49	-1,85	0,97
6	0,08	0,60 (0,71)	0,00	0,98 (0,98)	0,42	0,00 (0,10)			17,54	-0,76	-30,72	-8,76	-2,05	0,98
10	0,03	0,81 (0,83)	0,21	0,32 (0,35)	0,15	0,37 (0,43)			7,20	0,54	347,56	234,97	2,35	0,01
20	0,03	0,81 (0,89)	0,43	0,05 (0,22)	-0,16	0,35 (0,51)			19,63	0,56	315,18	198,70	2,05	0,02
$\bar{rx}^{ST}_{t+1}$	0,08	0,60 (0,70)	-0,10	0,64 (0,83)	0,53	0,00 (0,03)			27,65	-0,41	-22,62	-6,11	-1,08	0,86
$\bar{rx}^{LT}_{t+1}$	0,03	0,79 (0,79)	0,36	0,10 (0,18)	-0,05	0,79 (0,84)			12,84	0,57	373,09	237,62	2,13	0,02

Panel D: Traditional bond factors and both liquidity measures

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	$\Delta_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,08	0,55 (0,71)	-0,27	0,14 (0,33)	0,46	0,22 (0,36)	0,15	0,72 (0,79)	36,16	0,43	143,72	119,38	3,83	0,00
4	0,08	0,55 (0,71)	-0,14	0,49 (0,78)	0,24	0,55 (0,62)	0,30	0,51 (0,55)	27,84	0,03	3,75	23,22	1,10	0,13
5	0,08	0,59 (0,74)	-0,02	0,94 (0,99)	0,00	0,99 (0,99)	0,46	0,35 (0,55)	20,95	-0,89	-54,18	-12,16	-0,82	0,79
6	0,06	0,66 (0,73)	0,09	0,73 (0,78)	-0,22	0,64 (0,73)	0,59	0,25 (0,31)	16,26	-1,38	-63,03	-11,43	-1,80	0,96
10	-0,01	0,93 (0,96)	0,40	0,19 (0,36)	-0,74	0,14 (0,31)	0,84	0,09 (0,19)	14,23	0,57	352,64	293,67	2,37	0,01
20	-0,03	0,83 (0,83)	0,65	0,01 (0,13)	-1,01	0,04 (0,10)	0,80	0,06 (0,17)	29,75	0,67	431,39	309,39	2,68	0,00
$\bar{rx}^{ST}_{t+1}$	0,07	0,60 (0,72)	-0,07	0,75 (0,87)	0,10	0,82 (0,84)	0,40	0,40 (0,52)	23,64	-0,47	-34,97	-2,36	-0,25	0,60
$\bar{rx}^{LT}_{t+1}$	-0,02	0,85 (0,89)	0,58	0,04 (0,21)	-0,94	0,07 (0,25)	0,84	0,07 (0,19)	22,61	0,66	489,33	360,33	2,60	0,00

Results for the following regressions $rx^{(n)}_{t+1} = \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{n,t+1}$, where $rx_{n,t+1}$ denotes the annual TIPS excess log returns on $n = 3, 4, 5, 6, 10, 20$ years to maturity. I use $L^{z\text{-}asw}_{(5),t}$, $L^{z\text{-}asw}_{(10),t}$, and $L^{z\text{-}asw}_{(20),t}$ liquidity premium measure. M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. pv is the p-value calculated using the Newey-West correction for heteroscedasticity and autocorrelation with $T^{1/2}$ lags. The p-values based on the bootstrap analysis are presented in round brackets. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for the out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test, while GW is the Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and its corresponding p-value is given by $pval$. The sample spans the period from 01/11/2006 to 30/12/2011.

Next, I explore whether or not there exists a common and significant factor driving the variations of the liquidity premiums across maturities. To do that, I use a principal component analysis. The first three principal components of the *z-asw* liquidity premiums explain 92.45%, 6.59% and 0.96% of total variations, respectively. For the maximum range liquidity the three first principal components explain 85.31%, 11.12% and 3.56%, respectively. In both cases factor loading confirms that the first principal component is the most economically meaningful. I run all the regressions replacing the 10-year liquidity premium by the first principal component, denoted as $L^{z\text{-}asw}_{(pc)}$, which represents the liquidity premium factor common to all maturities.

Results presented in Table 1.3, panel C, are conclusive. Estimates for the common liquidity factor are positive and significant again for short-term excess return maturities. Using the first principal component, the $AdjR^2$ range from between 7% and 40% considering control variables. When I consider the first principal component from the Christensen and Gillan (2011) liquidity measures, I find similar values for $AdjR^2$ (much smaller values). In conclusion, it seems that there exists a common and significant factor driving the variations of the liquidity premium across maturities, which also predicts excess returns.

As a final exercise, I explore whether or not the maximum range for the TIPS liquidity by Christensen and Gillan (2011) and the *z-asw* liquidity premium measure contain the same information to forecast TIPS excess returns. Table 1.3, panel D, addresses this point by including both liquidity variables in the regression. Estimates for both liquidity variables are significant only for longer-term excess bond maturities (that is true only for the Newey and West p-value), and the $AdjR^2$ is equal to 14% and 29%, for 10- and 20-year maturity. Hence, higher $AdjR^2$ and the significance of the variables underline the hypothesis that both liquidity measures incorporate new information which helps to forecast inflation-linked excess returns but only for long-term bonds. For short-term bonds, liquidity predictors are redundant in the sense that they contain similar information to predict TIPS excess returns.

Overall, I conclude that there is encouraging evidence that the liquidity premium predicts TIPS excess returns, given that estimated coefficients are statistically and economically significant. Moreover, for all maturities the inclusion of liquidity increases the $AdjR^2$ whereby the strongest effect exists for shorter maturities and vanishes for long-term bonds. Additionally, by running a regression of the average (across maturities) excess returns, results are consistent with what I find for individual regressions. Therefore, I also conclude that the predictability power of the liquidity factor is robust as to how excess returns are computed.

Nominal Treasury excess returns

On the other hand, I explore whether or not the liquidity differential between TIPS and Treasury yields is an additional channel of predictability for U.S. Treasury excess returns. Again, I first present a replication of the constrained regressions in Cochrane and Piazzesi (2005), which is the benchmark model. Unexpectedly, the results are not consistent with Cochrane and Piazzesi (2005) results. The tent-shaped factor does not describe time variation in expected returns of any maturity bonds, for the period I am considering and using high frequency (daily) data. The term spread is only statistically significant for very short-term bond maturities and for long-term maturities. The coefficients of determination adjusted by degrees of freedom range from between 2% and 20% for different maturities as panel A of Table 1.4 shows. When I consider as dependent variable the average of excess returns across maturities, the $AdjR^2$ is equal to 3% and 8% for short-term and long-term maturities, respectively.

Results presented on Table 1.4, panel B, shows that nominal excess returns are also predictable from the liquidity differential between TIPS and Treasury bonds. First, the $AdjR^2$ are now ranging between 21% and 52%, showing an important increment when liquidity is included as an additional predictor. Second, the coefficient for liquidity variables are significant and negative, implying that a higher liquidity premium leads to lower excess returns in the Treasury bond market. The interpretation of the coefficient is straightforward. Investing under higher liquidity risk has to be compensated for by higher (excess) returns. For instance, an increase in one standard deviation in the 10-year liquidity premium results in a decrease of 0,46 standard deviations in the short-term average nominal excess returns, and a decrease of 0,69 standard deviations in the long-term average nominal excess returns. This means that a one standard deviation move in the 10-year liquidity premium of 30 basis points tends to go along with an decrease in 40 basis points in the short-term average nominal excess returns (0,46×0,87),-and of 282 basis points in the long-term (0,69×4,09). These magnitudes are also substantial relative to short- and long-term average nominal of 199 and 428 basis points. These empirical findings indicate that during bad times investors rush into nominal bonds, perhaps mainly on-the-run U.S. Treasuries, and they do not buy TIPS to the same degree. This is related to the "fight-to-quality" and "fight-to-liquidity" phenomena which coincide with higher market uncertainty and portfolio rebalances toward the saver, plus more liquid assets such as on-the-run bonds.

Liquidity premium seems to contain relevant information for forecasting the excess returns at an annual horizon, being the importance of the liquidity factor greater for longer-term bond excess maturities. In fact, it is statistically significant at 5%, and the $AdjR^2$ increases from nearly 24% at the shortest maturity to more than 50% at the longest considered maturity. However, the inclusion of the liquidity variable increases the $AdjR^2$ for all maturities (by 3% at the 2-year maturity and by

Table 1.4: Nominal Treasury excess returns predictability

Panel A: Traditional bond factors

n	CP	pv	$TERM$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,02	0,92 (0,97)	-0,45	0,00 (0,08)	20,65	-0,02	-4,50	15,47	-0,15	0,56
4	0,01	0,94 (0,95)	-0,30	0,05 (0,10)	9,21	-0,11	-24,49	-3,42	-0,70	0,76
5	0,00	0,99 (0,99)	-0,14	0,40 (0,58)	1,86	-0,09	-19,39	-6,29	-0,78	0,78
6	-0,02	0,91 (0,93)	0,02	0,93 (0,93)	2,07	0,02	4,67	3,63	0,21	0,42
10	-0,06	0,72 (0,81)	0,33	0,05 (0,10)	11,64	0,27	89,08	49,97	1,00	0,20
20	-0,01	0,95 (0,95)	0,26	0,10 (0,18)	6,87	0,12	33,55	23,42	1,11	0,13
$\bar{rx}^{ST}_{t+1}$	0,00	0,99 (0,99)	-0,18	0,27 (0,35)	3,23	-0,12	-25,01	-8,01	-0,90	0,82
$\bar{rx}^{LT}_{t+1}$	-0,03	0,89 (0,91)	0,29	0,09 (0,19)	8,42	0,18	52,25	32,25	1,10	0,12

Panel B: Traditional bond factors and z-asw liquidity measure

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,02	0,93 (0,93)	-0,41	0,01 (0,09)	-0,19	0,35 (0,50)	23,80	-0,37	-64,19	-25,14	-2,66	1,00
4	-0,05	0,79 (0,82)	-0,22	0,14 (0,41)	-0,35	0,07 (0,19)	20,56	-0,59	-88,74	-37,97	-4,02	1,00
5	-0,09	0,62 (0,72)	-0,02	0,88 (0,86)	-0,49	0,01 (0,10)	23,33	-0,50	-79,34	-27,41	-1,33	0,91
6	-0,12	0,43 (0,60)	0,15	0,31 (0,49)	-0,57	0,00 (0,04)	29,57	-0,20	-39,95	2,34	-0,01	0,50
10	-0,17	0,08 (0,23)	0,48	0,00 (0,10)	-0,63	0,00 (0,03)	47,13	0,48	216,60	168,97	1,05	0,15
20	-0,13	0,29 (0,59)	0,43	0,01 (0,10)	-0,70	0,00 (0,06)	51,73	0,54	283,41	190,18	1,60	0,05
$\bar{rx}^{ST}_{t+1}$	-0,08	0,64 (0,68)	-0,07	0,62 (0,73)	-0,46	0,01 (0,10)	22,15	-0,56	-85,37	-32,21	-1,72	0,96
$\bar{rx}^{LT}_{t+1}$	-0,14	0,19 (0,43)	0,45	0,01 (0,10)	-0,69	0,00 (0,05)	51,73	0,54	282,72	195,69	1,26	0,10

Continuation: Treasury excess returns predictability

Panel C: Traditional bond factors and common factor liquidity

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(pc)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,01	0,94 (0,96)	-0,43	0,00 (0,08)	-0,21	0,24 (0,41)	25,13	-0,37	-63,93	-24,52	-2,71	1,00
4	-0,04	0,84 (0,89)	-0,27	0,07 (0,35)	-0,37	0,03 (0,09)	22,19	-0,55	-84,85	-36,48	-5,01	1,00
5	-0,07	0,69 (0,79)	-0,09	0,53 (0,64)	-0,48	0,00 (0,06)	24,70	-0,45	-73,79	-26,85	-1,44	0,93
6	-0,09	0,52 (0,67)	0,07	0,66 (0,73)	-0,56	0,00 (0,08)	30,42	-0,16	-33,81	0,99	0,04	0,48
10	-0,14	0,18 (0,43)	0,38	0,01 (0,10)	-0,60	0,00 (0,03)	47,18	0,46	201,51	148,33	1,05	0,15
20	-0,10	0,39 (0,63)	0,32	0,02 (0,30)	-0,70	0,00 (0,06)	54,25	0,51	244,20	158,86	1,53	0,06
$\bar{rx}^{ST}_{t+1}$	-0,06	0,70 (0,75)	-0,14	0,36 (0,50)	-0,46	0,00 (0,09)	23,76	-0,51	-80,43	-31,43	-1,94	0,97
$\bar{rx}^{LT}_{t+1}$	-0,12	0,30 (0,41)	0,35	0,01 (0,25)	-0,68	0,00 (0,02)	53,49	0,51	248,28	165,49	1,26	0,10

Panel D: Traditional bond factors and both liquidity measures

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	$\Delta_{(10y)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,02	0,88 (0,94)	-0,61	0,00 (0,09)	0,68	0,18 (0,32)	-0,89	0,03 (0,12)	33,62	0,15	43,34	32,64	1,54	0,06
4	-0,01	0,94 (0,93)	-0,41	0,04 (0,20)	0,45	0,41 (0,52)	-0,82	0,05 (0,13)	28,84	-0,10	-22,40	-7,60	-1,23	0,89
5	-0,06	0,71 (0,75)	-0,18	0,35 (0,49)	0,18	0,74 (0,82)	-0,68	0,10 (0,21)	29,05	-0,15	-30,40	-6,60	-0,43	0,67
6	-0,10	0,50 (0,63)	0,03	0,87 (0,85)	-0,07	0,89 (0,93)	-0,52	0,18 (0,39)	32,87	0,00	0,26	20,00	0,50	0,31
10	-0,16	0,10 (0,19)	0,46	0,01 (0,04)	-0,55	0,08 (0,16)	-0,08	0,75 (0,80)	47,21	0,48	223,59	175,56	1,09	0,14
20	-0,13	0,28 (0,51)	0,43	0,02 (0,15)	-0,73	0,01 (0,03)	0,03	0,90 (0,87)	51,74	0,54	276,24	193,72	1,16	0,12
$\bar{rx}^{ST}_{t+1}$	-0,05	0,75 (0,85)	-0,24	0,22 (0,35)	0,25	0,65 (0,72)	-0,72	0,09 (0,20)	28,55	-0,16	-33,09	-9,98	-0,69	0,76
$\bar{rx}^{LT}_{t+1}$	-0,14	0,19 (0,41)	0,45	0,01 (0,10)	-0,69	0,02 (0,10)	0,00	0,99 (0,99)	51,73	0,53	274,43	197,03	1,05	0,15

Results for the following regressions $rx^{(n)}_{t+1} = \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{n,t+1}$, where $rx_{n,t+1}$ denotes the annual Treasury excess log returns on $n = 3, 4, 5, 6, 10, 20$ years to maturity. I use $L^{z\text{-}asw}_{(5),t}$, $L^{z\text{-}asw}_{(10),t}$, and $L^{z\text{-}asw}_{(20),t}$ liquidity premium measure. M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. pv is the p-value calculated using the Newey-West correction for heteroscedasticity and autocorrelation with $T^{1/2}$ lags. The p-values based on the bootstrap analysis are presented in round brackets. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for the out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test, while GW is the Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and its corresponding p-value is given by $pval$. The sample spans the period from 01/11/2006 to 30/12/2011.

more than 40% at the 20-year maturity). Likewise, the importance of liquidity for longer-term maturities is also supported by results obtained from regressions with the liquidity factor as they only predictor variable. $AdjR^2$ increases from 8% at 3-year maturity to 32% at 20-year maturity, being these values greater than reported in Panel A.[18] I find similar results with the Christensen and Gillan (2011) measure of TIPS liquidity (see Table C1.1 Appendix 1.5). This result is also confirmed considering other liquidity premium maturities, as is shown in Appendix 1.5, Table C1.3.

I find also negative and significant coefficients on the first principal component of the *z-asw* liquidity premiums, as panel C shows. It indicates that when the TIPS liquidity premium increase relative to nominal Treasury, then the Treasury excess returns fall. I find an economically significant decrease in the average nominal excess returns of 40 basis points (0,46×0,87) and 278 basis points (0,68×4,07) for short- and long-term maturities. I find approximately the same effect regardless of the maturity for the liquidity premium, which is also the case for TIPS excess returns.

To sum up, I find that variables commonly used in predicting nominal bond excess returns are not useful predictors in the context of the inflation-linked bond market. Even more, the tent-shape factor seems to no longer be a predictor variable for nominal Treasury bond returns, at least during the period under study. However, the liquidity premium for different maturities appears as a significant and economically relevant source of predictability for government bond excess returns. In fact, I find a higher $AdjR^2$ when I consider $L^{z\text{-}asw}_{n,t}$ liquidity premium measure, or the maximum range liquidity premium $\Delta^{(n)}_t$ proposed by Christensen and Gillan (2011) in addition to traditional predictors.

1.4.2 Out-of-sample analysis

In the previous section, judging by the Adjusted R^2 values, I show that the models do well in explaining bond excess returns in-sample. However, high in-sample R^2 does not imply good out-of-sample performance of predictor variables as shown by Goyal and Welch (2003). Recent return predictability literature highlights the importance of conduct out-of-sample tests for analyzing return predictability.

In this section, the out-of-sample tests allow me to confront the question of whether or not the forecast of excess returns using a different set of predictors are better than those based on using the historical average. Any evidence of out-of-sample forecasting ability goes toward nullifying the suggestion that the in-sample predictability is driven by a small sample bias. To assess the out-of-sample predictability, I consider for the null hypothesis the constant expected return model

[18] $AdjR^2$ for equally weighted portfolio regressions are equal to 21% for short-term maturities and 30% for long-term maturities. These results are available upon request.

(restricted model), which is the popular benchmark model used in literature (see Campbell and Thompson (2008) and Welch and Goyal (2008) among many others). The alternative or unrestricted model corresponds to the model including predictor variables

$$\begin{aligned} H_0 &: rx_{t+1}^{(n)} = \alpha + e_{t+1}^{(n)} \\ H_1 &: rx_{t+1}^{(n)} = \alpha + \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{t+1}^{(n)}. \end{aligned}$$

The total sample T is divided into the first p in-sample observations and the last q out-of-sample observations. To create the first out-of-sample forecast, I make use of the in-sample portion of the sample and estimate the OLS parameters of the unrestricted model. Then from the estimated equation I create the first out-of-sample forecast for the unrestricted form of the model:

$$\widehat{rx}_{p+(h-1)+1}^{(n)} = \hat{a} + \hat{\alpha}_n^\top \mathbf{M}_p + \hat{\beta}_n^\top \mathbf{L}_p,$$

with h as the forecast horizon, and I define the unrestricted forecast error as

$$u_{p+(h-1)+1}^{U(n)} = rx_{p+(h-1)+1}^{(n)} - \widehat{rx}_{p+(h-1)+1}^{(n)},$$

where $rx_{p+(h-1)+1}^{(n)}$ is the realized corresponding excess return value. In order to create the next forecasts, I recursively expand the in-sample portion of the sample and repeat the whole procedure through to the end of the available sample. I end up with $T-p-h+1$ out-of-sample unrestricted forecast errors $\left\{u_{t+(h-1)}^{U(n)}\right\}_{t=p+1}^{T-(h-1)}$.

To produce the out-of-sample forecast for the restricted form of the model, I compute the historical average excess return estimated using data up to p

$$\widetilde{rx}_{p+(h-1)+1}^{(n)} = \frac{1}{t}\sum_{s=1}^{t} rx_s^{(n)},$$

where the out-of-sample forecast error

$$u_{p+(h-1)+1}^{R(n)} = rx_{p+(h-1)+1}^{(n)} - \widetilde{rx}_{p+(h-1)+1}^{(n)},$$

and again I end up with $\left\{u_{n,t+(h-1)}^{R(n)}\right\}_{t=p+1}^{T-(h-1)}$ out-of-sample restricted forecast errors.

To assess the out-of-sample predictability of predictor variables, $\mathbf{M}_t$ and $\mathbf{L}_t$, I use different metrics. The first metric is the out-of-sample R^2 suggested by Campbell and Thompson (2008), which is denoted by R_{OS}^2, and is defined as

$$R_{OS}^{2(n)} = 1 - \frac{\Sigma_{t=p+1}^{T-(h-1)}\left(rx_{t+(h-1)}^{(n)} - \widehat{rx}_{t+(h-1)}^{(n)}\right)}{\Sigma_{t=p+1}^{T-(h-1)}\left(rx_{t+(h-1)}^{(n)} - \widetilde{rx}_{n,t+(h-1)}^{(n)}\right)}, \tag{1.6}$$

which measures the reduction in the mean squared forecast error for the unrestricted regression model. This means that the $\widehat{rx}_{n,p+(h-1)+1}$ unrestricted forecast model outperforms the benchmark restricted model when $R^2_{OS} > 0$, while an $R^2_{OS} < 0$ suggests that the restricted model predicts returns better than the other models.[19]

The other metrics I report statistically test the ability of a factor to improve the predictability of the benchmark model. In particular, I report the F-test developed by McCracken (2007):

$$MSE - F^{(n)} = (T - p - h + 1)\frac{MSE_R^{(n)} - MSE_U^{(n)}}{MSE_U^{(n)}},$$

where MSE_R denotes the mean squared error from the constant expected return model and MSE_U is the unrestricted model. For this test, the null hypothesis is that MSE associated with the restricted model is less or equal to the corresponding value for the unrestricted model. The alternative hypothesis is that the MSE associated with the unrestricted model is lower in comparison to the restricted model.

Additionally, I report the encompassing test extended by Clark and McCracken (2001) named ENC-NEW test[20]

$$ENC - NEW^{(n)} = \frac{\Sigma_{t=p+1}^{T-(h-1)}((e_t^{(n)})^2 - e_t^{(n)}\epsilon_t^{(n)})}{MSE_U^{(n)}},$$

where e_t is the vector of rolling out-of-sample errors from the constant returns model and $\epsilon_t^{(n)}$ is the vector of rolling out-of-sample errors from the model including predictor variables. The null hypothesis is that the unrestricted model cannot improve the forecast associated with the restricted model. The alternative hypothesis is that the unrestricted model has additional information that can improve the forecast obtained from the restricted model.

Finally, I use the Giacomini and White (2006) (GW) test. It is constructed under the assumption that the forecasts are generated using a moving data window and test the null hypothesis of equal forecasting accuracy between the unrestricted and restricted model. The GW test has the ability to take into account parameter uncertainty when evaluating the performance of different forecasting models. Therefore, the rejection of the null hypothesis is indeed due to poor restricted model performance, and is not the result of parameter uncertainty.

The out-of-sample tests are performed on the last two years of the sample from January 2010 to December 2011, which means that 830 daily observations (sixty

[19] Campbell and Thompson (2008) suggest that any value of R^2_{OS} that is above 0.5% for monthly data and 1% for quarterly data can signal an economically meaningful degree of returns predictability.

[20] Harvey et al. (1998) first propose this test which asks if the forecast from one model encompasses the forecast from another. Later Clark and McCracken (2001) extend the test by deriving the nonstandard asymptotic distribution.

percent of the sample) are used to provide the first estimate. This leaves me with 518 observations to use in computing the out-of-sample test statistics for $h = 21$.[21] Notice that the out-of-sample test does not include the crisis period.

The out-of-sample predictive ability metrics are shown in the last columns of Table 1.3 for TIPS and Table 1.4 for Treasury bonds, considering both individual and average (across maturity) excess return regressions. Results show that the models including the forecasting variables perform worse than the constant model for short-term TIPS maturities, as indicated by the negative estimates for R^2_{OS}. However, I find that regressions including the *z-asw* TIPS liquidity measure produce positive R^2_{OS} for very short TIPS maturities.

The estimates of $MSE-F$ and $ENC-NEW$ tend to reject the null hypothesis at 1% level in the same cases where R^2_{OS} is positive, meaning that one accepts that predictor variables add forecasting power to the null model. The last two columns of Table 1.3 report the Giacomini and White (2006) test statistic and corresponding p-values. There is no disagreement between the R^2_{OS}, $MSE-F$, $ENC-NEW$ and the GW test results, for both Treasury and TIPS excess returns. Each time I have a negative value for R^2_{OS}, it is also matched by not rejecting the null hypothesis for the GW test and also for the other two tests.

The out-of-sample results associated with U.S. Treasury nominal excess returns show that predictor variables have positive estimates for R^2_{OS} but only for long-term maturities, considering both individual and equal-weighted excess returns. This indicates that the forecast using liquidity in addition to traditional bond factors statistically outperform those using a constant. The corresponding estimates associated with $MSE-F$ and $ENC-NEW$ are statistically significant. The GW test tends to reject the null hypothesis less often than the other tests.

Rejecting the constant model null in favor of the linear model alternative provides evidence of predictability from a different set of predictor variables. However, it does not test whether or not the information content of liquidity does helps to predict excess returns. Therefore, as an additional exercise, I test whether or not liquidity variables have a forecasting ability for U.S. bond excess returns. To do so, I consider for the null hypothesis the linear model including traditional predictor variables, and

[21] When I decide on the sample-split parameter, I face the usual trade-off between reliability of the out-of-sample results and estimation. If I limit the out-of-sample forecasts to very recent periods, I have very few out-of-sample observations to use in calculating the out-of-sample test statistics. This makes the inference regarding out-of-sample predictability less reliable. If I begin the out-of-sample forecasts very early in the sample, I do not have many in-sample observations available to estimate the predictive regression models used to generate the initial out-of-sample forecasts. As a consequence, I follow the common decision in the literature to use 40% to produce the out-of-sample statistics.

for the alternative model the linear model but also including liquidity

$$H_0 : rx_{t+1}^{(n)} = \alpha + \delta_n^\top \mathbf{M}_t + e_{t+1}^{(n)}$$
$$H_1 : rx_{t+1}^{(n)} = \alpha + \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{t+1}^{(n)}.$$

If the unrestricted model forecasts are superior to the restricted ones, then, significantly, I conclude that the liquidity model contains information beyond the model with traditional factors. R^2_{OS}, $MSE-F$, $ENC-NEW$ and the GW test are reported in Table C1.4 in Appendix 1.5. I find little discrepancy between results considering different benchmark models. Panel A shows that the 10-year liquidity premium significantly predicts TIPS excess returns in out-of-sample regressions, however it does not predict nominal excess returns.

In summary, the results presented in this subsection show that predictor variables including liquidity premium have a significantly out-of-sample predictability for government excess returns. In particular, the out-of-sample predictability is statistically significant only for long-term bond maturities in the case of the U.S. Treasury, however liquidity seems to be a weak predictor of nominal excess returns. Conversely, I have strong evidence of out-of-sample predictability for TIPS excess returns. In this case, results suggest that liquidity is a significant predictor for TIPS: even more liquidity contains information about future real excess returns not captured by traditional predictor variables.

1.4.3 Robustness

Subperiod analysis

In the previous section, I investigated predictability of TIPS liquidity during the 2006-2012 period, which includes the major financial crisis that affected the economy over the years 2007-2009. In this section, the goal is to assess whether or not the value of TIPS liquidity varies during both normal and stress times, which would mean that its predictive ability is pervasive over time.

To do so, I split the sample into two sub-periods, crisis period (November 2006 to December 2009) and after crisis period (January 2010 to December 2012), and I repeat the analysis presented previously. I show that the TIPS liquidity factor would have provided a good measure of the extreme illiquidity tensions that arose during the crisis. In fact most results are magnified when I only consider data from 2006 up to the end of 2009. In terms of out-of-sample returns predictability, I confirm that the difference in out-of-sample forecasting power between the model with the liquidity variables and the benchmark constant expected return model is statistically significant for TIPS excess returns of shorter-term maturities in both sub-periods, while liquidity seems to be important mainly during the crises time for Treasury excess returns.

Table 1.5: TIPS excess bond returns predictability: sub-periods results

Panel A: Crisis (2007- 2009)

n	CP	pv	$TERM$	pv			AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,30	0,26 (0,51)	-0,34	0,03 (0,21)			6,83	0,63	97,86	96,64	1,35	0,09
4	-0,29	0,28 (0,54)	-0,36	0,02 (0,19)			7,41	0,61	98,17	97,88	1,24	0,11
5	-0,29	0,30 (0,50)	-0,37	0,02 (0,16)			7,93	0,59	105,29	106,41	1,16	0,12
6	-0,29	0,31 (0,46)	-0,38	0,02 (0,22)			7,97	0,57	119,00	122,95	1,07	0,14
10	-0,30	0,32 (0,44)	-0,32	0,06 (0,19)			6,29	0,51	179,37	209,56	0,77	0,22
20	-0,21	0,45 (0,67)	-0,20	0,41 (0,54)			2,83	0,36	92,97	141,86	0,39	0,35
$\bar{rx}^{ST}_{t+1}$	-0,30	0,29 (0,46)	-0,36	0,02 (0,51)			7,58	0,60	105,56	106,40	1,19	0,12
$\bar{rx}^{LT}_{t+1}$	-0,26	0,38 (0,62)	-0,26	0,22 (0,42)			4,39	0,44	120,22	159,54	0,56	0,29
n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,16	0,39 (0,64)	-0,69	0,00 (0,14)	0,98	0,00 (0,07)	46,10	0,80	94,01	110,03	1,69	0,05
4	0,17	0,36 (0,57)	-0,71	0,00 (0,08)	0,98	0,00 (0,06)	46,64	0,79	94,97	112,07	1,56	0,06
5	0,17	0,36 (0,54)	-0,72	0,00 (0,06)	0,97	0,00 (0,07)	46,85	0,77	99,95	118,72	1,49	0,07
6	0,17	0,38 (0,56)	-0,72	0,00 (0,07)	0,96	0,00 (0,10)	46,16	0,77	108,04	129,24	1,47	0,07
10	0,13	0,56 (0,65)	-0,64	0,00 (0,13)	0,89	0,00 (0,10)	38,73	0,72	138,83	170,60	1,46	0,07
20	0,11	0,69 (0,74)	-0,44	0,06 (0,19)	0,68	0,01 (0,10)	21,91	0,58	74,75	92,70	0,92	0,18
$\bar{rx}^{ST}_{t+1}$	0,17	0,37 (0,55)	-0,71	0,00 (0,05)	0,97	0,00 (0,07)	46,57	0,78	99,70	118,08	1,53	0,06
$\bar{rx}^{LT}_{t+1}$	0,12	0,63 (0,72)	-0,54	0,01 (0,17)	0,79	0,00 (0,05)	30,08	0,65	90,45	112,02	1,13	0,13

Panel A: After Crisis (2010- 2012)

n	CP	pv	$TERM$	pv			AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,09	0,62 (0,73)	0,14	0,53 (0,63)			1,59	-0,24	-39,31	-17,91	-4,44	1,00
4	0,10	0,62 (0,72)	0,23	0,29 (0,35)			3,24	-0,05	-11,00	-3,71	-0,58	0,72
5	0,09	0,65 (0,66)	0,29	0,16 (0,32)			4,78	0,13	33,02	26,44	1,19	0,12
6	0,06	0,71 (0,76)	0,33	0,10 (0,19)			5,72	0,24	69,44	67,39	1,68	0,05
10	-0,05	0,76 (0,83)	0,21	0,32 (0,38)			2,18	-0,16	-36,79	16,23	-1,00	0,84
20	-0,06	0,76 (0,77)	0,13	0,55 (0,64)			0,92	-0,22	-48,31	-2,69	-1,08	0,86
$\bar{rx}^{ST}_{t+1}$	0,09	0,64 (0,76)	0,25	0,23 (0,34)			3,79	0,04	8,87	9,37	0,45	0,33
$\bar{rx}^{LT}_{t+1}$	-0,06	0,76 (0,79)	0,15	0,48 (0,58)			1,25	-0,20	-44,93	2,42	-1,08	0,86
n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,01	0,89 (0,82)	0,34	0,00 (0,11)	0,86	0,00 (0,08)	36,68	0,39	58,68	71,11	1,67	0,05
4	0,02	0,88 (0,85)	0,42	0,00 (0,00)	0,83	0,00 (0,07)	35,38	0,49	80,04	92,74	2,03	0,02
5	0,01	0,90 (0,84)	0,46	0,00 (0,03)	0,75	0,00 (0,06)	31,02	0,51	83,26	98,13	1,89	0,03
6	0,01	0,97 (0,87)	0,46	0,00 (0,02)	0,59	0,00 (0,08)	22,11	0,37	59,14	80,21	1,58	0,06
10	-0,03	0,85 (0,77)	0,16	0,40 (0,46)	-0,19	0,16 (0,29)	3,90	-0,11	-24,85	7,85	-0,69	0,75
20	-0,02	0,92 (0,88)	0,02	0,90 (0,67)	-0,44	0,00 (0,10)	10,19	-0,12	-25,68	19,02	-0,61	0,73
$\bar{rx}^{ST}_{t+1}$	0,01	0,91 (0,86)	0,43	0,00 (0,72)	0,79	0,00 (0,08)	33,36	0,51	78,10	91,47	1,94	0,03
$\bar{rx}^{LT}_{t+1}$	-0,02	0,89 (0,89)	0,06	0,75 (0,02)	-0,38	0,00 (0,12)	8,20	-0,11	-24,59	15,01	-0,63	0,74

Results for the following regressions $rx^{(n)}_{t+1} = \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{n,t+1}$, where $rx_{n,t+1}$ denotes the annual TIPS excess log returns on $n = 3, 4, 5, 6, 10, 20$ years to maturity. I use $L^{z\text{-}asw}_{(10),t}$ liquidity premium measure. M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. pv is the p-value calculated using the Newey-West correction for heteroscedasticity and autocorrelation with $T^{1/2}$ lags. The p-values based on the bootstrap analysis are presented in round brackets. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for the out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test, while GW is the Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and its corresponding p-value is given by $pval$. The sample spans the period from 01/11/2006 to 30/12/2011.

Table 1.6: Treasury excess bond returns predictability: sub-periods results

Panel A: Crisis (2007- 2009)

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,22	0,12 (0,28)	-0,32	0,12 (0,19)			24,23	0,71	582,16	514,42	1,46	0,07
4	0,20	0,13 (0,35)	-0,31	0,10 (0,22)			22,15	0,75	735,58	634,23	1,55	0,06
5	0,17	0,18 (0,31)	-0,28	0,10 (0,22)			17,01	0,73	658,94	552,23	2,19	0,01
6	0,13	0,30 (0,41)	-0,23	0,15 (0,28)			10,68	0,64	418,16	340,59	2,41	0,01
10	0,01	0,96 (0,96)	0,03	0,81 (0,90)			0,07	0,12	32,47	32,33	-0,30	0,62
20	0,12	0,41 (0,59)	0,09	0,50 (0,68)			0,80	0,53	265,95	179,86	1,09	0,14
$\bar{rx}^{ST}_{t+1}$	0,18	0,16 (0,32)	-0,28	0,11 (0,24)			17,71	0,74	676,81	567,28	2,02	0,02
$\bar{rx}^{LT}_{t+1}$	0,09	0,50 (0,63)	0,08	0,56 (0,67)			0,46	0,42	174,46	115,94	0,78	0,22
n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,11	0,28 (0,41)	-0,07	0,71 (0,78)	-0,69	0,00 (0,00)	43,70	0,76	738,79	757,76	1,53	0,06
4	-0,14	0,09 (0,28)	-0,05	0,71 (0,71)	-0,72	0,00 (0,00)	43,51	0,80	942,69	905,79	1,54	0,06
5	-0,18	0,01 (0,06)	-0,01	0,89 (0,91)	-0,73	0,00 (0,00)	39,07	0,81	1003,65	910,68	1,67	0,05
6	-0,21	0,00 (0,01)	0,03	0,72 (0,83)	-0,72	0,00 (0,00)	31,81	0,77	807,51	702,24	1,93	0,03
10	-0,26	0,00 (0,00)	0,23	0,00 (0,00)	-0,56	0,00 (0,00)	13,08	0,49	232,88	220,04	1,03	0,15
20	-0,37	0,00 (0,02)	0,46	0,00 (0,03)	-1,03	0,00 (0,05)	44,42	0,80	954,66	834,77	2,80	0,00
$\bar{rx}^{ST}_{t+1}$	-0,17	0,01 (0,06)	-0,02	0,87 (0,92)	-0,73	0,00 (0,00)	39,73	0,81	1028,17	942,78	1,59	0,06
$\bar{rx}^{LT}_{t+1}$	-0,35	0,00 (0,00)	0,41	0,00 (0,00)	-0,92	0,00 (0,00)	35,22	0,73	654,25	564,22	3,47	0,00

Panel A: After Crisis (2010- 2012)

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,06	0,51 (0,74)	0,90	0,00 (0,00)			40,63	0,75	729,42	629,53	1,82	0,03
4	-0,10	0,33 (0,65)	0,85	0,00 (0,01)			35,67	0,74	691,58	671,77	1,91	0,03
5	-0,14	0,28 (0,58)	0,77	0,00 (0,02)			29,62	0,63	406,63	451,49	1,47	0,07
6	-0,16	0,26 (0,48)	0,69	0,00 (0,03)			24,46	0,48	222,38	287,44	1,10	0,14
10	-0,19	0,29 (0,56)	0,49	0,01 (0,13)			13,02	0,07	19,20	74,79	0,73	0,23
20	-0,12	0,60 (0,66)	0,25	0,21 (0,41)			3,69	-0,15	-30,57	9,32	0,20	0,42
$\bar{rx}^{ST}_{t+1}$	-0,13	0,28 (0,52)	0,79	0,00 (0,05)			31,01	0,66	454,43	488,83	1,55	0,06
$\bar{rx}^{LT}_{t+1}$	-0,14	0,50 (0,68)	0,33	0,10 (0,25)			6,06	-0,09	-20,36	23,74	0,38	0,35
n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,06	0,55 (0,74)	0,89	0,00 (0,00)	-0,05	0,40 (0,74)	40,75	0,78	836,14	783,06	1,82	0,03
4	-0,08	0,42 (0,70)	0,79	0,00 (0,03)	-0,24	0,00 (0,12)	38,46	0,70	570,13	533,01	1,76	0,04
5	-0,10	0,34 (0,58)	0,68	0,00 (0,03)	-0,37	0,00 (0,03)	36,10	0,59	341,27	323,54	1,81	0,04
6	-0,11	0,29 (0,53)	0,59	0,00 (0,06)	-0,45	0,00 (0,02)	34,20	0,46	207,33	204,07	1,70	0,04
10	-0,13	0,27 (0,55)	0,35	0,03 (0,30)	-0,59	0,00 (0,03)	29,35	0,14	38,73	68,25	1,05	0,15
20	-0,06	0,73 (0,80)	0,10	0,56 (0,50)	-0,66	0,00 (0,09)	24,14	-0,01	-2,74	56,20	-0,07	0,53
$\bar{rx}^{ST}_{t+1}$	-0,10	0,34 (0,63)	0,71	0,00 (0,04)	-0,35	0,00 (0,05)	36,72	0,61	369,34	348,08	1,78	0,04
$\bar{rx}^{LT}_{t+1}$	-0,08	0,59 (0,64)	0,18	0,31 (0,40)	-0,65	0,00 (0,06)	25,77	0,02	5,02	54,09	0,15	0,44

Results for the following regressions $rx^{(n)}_{t+1} = \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{n,t+1}$, where $rx_{n,t+1}$ denotes the annual Treasury excess log returns on $n = 3, 4, 5, 6, 10, 20$ years to maturity. I use $L^{z\text{-}asw}_{(10),t}$ liquidity premium measure. M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. pv is the p-value calculated using the Newey-West correction for heteroscedasticity and autocorrelation with $T^{1/2}$ lags. The p-values based on the bootstrap analysis are presented in round brackets. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for the out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test, while GW is the Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and its corresponding p-value is given by $pval$. The sample spans the period from 01/11/2006 to 30/12/2011.

It is important to bear in mind that during this period the Federal Reserve (FED) undertook important initiatives to combat the financial crisis. One of them was to implement the Large-scale Asset Purchase (LSAP) program, which is an unconventional monetary policy developed during the period from 2008 to 2013, with three quantitative easing (QE) rounds.[22] The decision to purchase large volumes of assets came first from November 2008 through to March 2010, when the FED bought of \$1.35 trillion of mortgage-backed securities (MBS) plus \$300 billion of Treasuries. Then, from November 2010 to the end of June 2011, the FED undertook a second LSAP program (QE2) involving the purchase of 600 billion in longer-term Treasuries (in particular, securities that mature between two and ten years). Finally, on September 13, 2012, the Federal Open Market Committee (FOMC) announced a third round of quantitative easing, (QE3). This time the Fed purchased \$40 billions in mortgage-backed securities per month, and on December 12, 2012, FOMC authorized up to \$45 billion in U.S. Treasuries in addition to the MBS purchases.

Krishnamurthy and Vissing-Jorgensen (2012b) suggest that the impact of FED purchases of long-term assets on expected inflation was large and positive (inflation expectations increased by between 36 and 95 basis points). Expected inflation increased substantially due to QE1 and modestly due to QE2, implying that reductions in real rates were larger than reductions in nominal rates. Christensen and Gillan (2012) also conclude that the second round of the LSAP program helped improve the TIPS market functioning on purchase dates and throughout the program by reducing the liquidity premiums that investors would have demanded if the purchases had not been conducted. Besides, Gagnon et al. (2011) argue that the LSAP program appeared to improve market liquidity in general. In fact, purchases of agency debt and MBS began at a time when liquidity in these markets was poor, and spreads to Treasury yields were unusually wide. As result, spreads of agency debt and MBS yields narrowed relative to Treasury yields, and spreads between on-the-run and off-the-run Treasury securities also narrowed.

In particular, predictability results for TIPS excess returns are reported in Table 1.5 for the crisis period (Panel A) and after crisis (Panel B). Results for the regression (including traditional bond factors) leads to the same conclusions during the crisis period as found when considering the whole period. In this period, neither the tent-shape factor, nor the term spread describe time variations in the TIPS expected returns of all maturity bonds. The same results also hold after the crisis.

Results for the liquidity variable are magnified during the crisis (in the sense that coefficient values are bigger), however in the after-crisis period results are similar to

[22] LSAP changed market expectations of future asset purchases by the FED and, consistent with the efficient market hypothesis, immediately affected asset prices. These announcements potentially affect asset prices through different channel: signaling and portfolio balance channel, but also by affecting liquidity and credit risk. Traditionally, for Treasury securities (a market with extraordinarily high liquidity and nearly no credit risk), the two main channels are signaling and portfolio balance.

what I find for the whole sample. In fact, a one standard deviation move in the 10-year liquidity premium of 30 basis points tends to accompany an increase in 178 basis points (0,97×1,84) in the average TIPS excess returns, while after the crisis the equally weighted excess returns increases in 145 basis points (0,79×1,84). Interestingly, for this period the liquidity premium are also statistically significant and economically meaningful for long-term maturity bonds, both individually or for the equally weighted portfolio. This seems to indicate that during the financial crisis the liquidity premium helped to predict excess returns for all maturities.

With respect to out-of-sample results, Table 1.5 shows that models including the forecasting variables perform better than the constant model, as indicated by the positive estimates for R^2_{OS} for all maturities. This implies that the unrestricted predictive regression model has a lower average mean-squared prediction error than the historical average return model. Similarly, $MSE-F$ and $ENC-NEW$ provide evidence regarding the ability of the three predictor variables to forecast out-of-sample. However, during the crisis period the GW test statistic tends not to reject the null hypothesis of equal forecast ability between both models when liquidity is not included. But the null hypothesis is rejected for all maturities when liquidity is included as a predictor variable. This result is confirmed by results in Table C1.4 in Appendix 1.5, where the linear model including liquidity (unrestricted model) outperforms, not only the constant model, but also the model with traditional predictor variables (benchmark model). This suggests that the linear model including liquidity as a predictor variable does a better job at forecasting excess returns than the historical average return model or than the linear model with traditional bond factors. In conclusion, liquidity has a statistically significant forecasting power for TIPS excess returns both during the financial crisis and after the crisis.

Table 1.6 reports the same set of results but for the U.S. Treasury nominal bonds. I find similar results as for TIPS, in the sense that the liquidity premium is statistically significant and economically meaningful considering each maturity individually or for the equally weighted portfolio. The effect of liquidity is also magnified during the crisis: the average Nominal excess returns decrease during crisis period in 64 basis points (-0,73×0,87) for short-term maturity portfolio and 376 basis points (-0,92×4,09) for long-term portfolio bond. With the exception of long-term maturity bonds, the term spread is statistically significant after the crisis period. This could be result of the higher inflation expectation of investors derived from the LSAP program.

The main result for Treasuries is that the liquidity premium seems to be important mainly during the crisis. In fact, between 1% and 24% of the variation in excess returns is explained by traditional factors during the crisis. Adding liquidity as a predictor variable produces an important increase in the R^2 coefficient, which rises up to a range of between 13% and 45%. However, R^2 values after crisis are almost the same in the two regressions: R^2 range between 4% and 40% for traditional factor versus 24% and 41% adding liquidity, having only an important increment for longer-term maturities. This results seems to be confirmed by the out-of-sample results during the crisis, in the sense that traditional bond factors have predictive out-of-sample power for shorter-term maturities, while liquidity seems to also help predict the longer-term maturities. On the contrary, results after the crisis period show that liquidity does not play a very important role as a predictor for nominal excess returns. These results are supported by findings in Table C1.4 in Appendix 1.5.

In the absence of the FED's quantitative easing operations (i.e. for the crisis period), the liquidity coefficient is negative and significant for all maturities (see Table 1.6, panel A). However, the second round of the FED's market operations artificially led to an excess demand for Treasury and also for TIPS bonds. Moreover, these market operations were viewed as a signal of the willingness to calm outcomes of the financial crisis, which led to a lower risk premium.[23] Therefore, the forecasting properties of the liquidity differential between Treasuries and TIPS yields for nominal excess bond returns seem to have been more pronounced during the low liquidity conditions observed with the extreme credit market disturbances in 2008.

Overall, the in-sample and out-of-sample forecasting power of liquidity for nominal bond excess returns seems to be addressed by the events during the crisis. By contrast, I have evidence of out-of-sample forecasting ability during both normal and bad times for TIPS excess returns. Additionally, I find that the effect of liquidity is magnified during the crisis.

Monthly returns

I investigate the validity and robustness of my results using monthly returns. I repeat the analysis presented in Section 4.1 and 4.2, using monthly estimates of yearly bond excess returns. Tables 1.7 and 1.8 report results from the in-sample and out-sample forecasting regression for 3-, 4-, 5-, 6-, 10- and 20-year log excess bond returns. In this case in addition to the *z-asw* liquidity premium measure and the maximum range for the TIPS liquidity by Christensen and Gillan (2011), I also use the 10-year estimated liquidity premium of Pflueger and Viceira (2012).

Interestingly, with monthly observations I confirm the stylized predictability results found for U.S. Treasury bonds, which is that the tent-shaped factor describes time variation in expected returns (using the bootstrap p-values the tent-shaped factor is statistically significant at 10% level). By contrast for TIPS, I obtain the same results as before, that is, neither nominal term spread nor the CP factor predict real excess returns.

[23] In fact, Gagnon et al. (2011) show that LSAP announcements lowered the risk premium of long-term interest rates.

Table 1.7: Nominal Treasury excess returns predictability: monthly data

Panel A: Excess returns and traditional bond factors

n	CP	pv	$TERM$	pv			AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,52	0,00 (0,11)	-0,68	0,00 (0,01)			51,39	0,61	19,17	19,98	2,78	0,00
4	0,50	0,00 (0,12)	-0,52	0,00 (0,01)			36,24	-0,01	-0,13	5,72	0,58	0,28
5	0,43	0,01 (0,09)	-0,35	0,01 (0,05)			21,63	-0,85	-5,51	0,80	-0,61	0,73
6	0,35	0,05 (0,17)	-0,18	0,20 (0,35)			11,84	-1,24	-6,64	-0,73	-1,51	0,93
10	0,13	0,48 (0,62)	0,19	0,19 (0,24)			6,85	-0,98	-5,93	-1,45	-2,38	0,99
20	0,08	0,75 (0,84)	0,09	0,61 (0,70)			1,87	-0,57	-4,34	-1,57	-2,20	0,99
$\bar{rx}^{ST}_{t+1}$	0,45	0,00 (0,05)	-0,39	0,01 (0,03)			24,82	-0,64	-4,68	1,51	-0,43	0,67
$\bar{rx}^{LT}_{t+1}$	0,35	0,80 (0,78)	0,43	0,80 (0,53)			1,68	-1,36	-6,92	-2,10	-3,29	1,00

Panel B: TIPS liquidity measures

z-asw liquidity premium measure

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,41	0,04 (0,11)	-0,54	0,00 (0,02)	-0,28	0,11 (0,31)	57,29	0,69	17,17	12,88	2,52	0,01
4	0,31	0,14 (0,26)	-0,30	0,08 (0,14)	-0,47	0,01 (0,11)	52,33	0,18	2,68	3,54	-0,10	0,54
5	0,19	0,38 (0,54)	-0,06	0,71 (0,72)	-0,60	0,00 (0,06)	47,72	-0,35	-3,14	0,69	-2,25	0,99
6	0,08	0,72 (0,77)	0,14	0,39 (0,37)	-0,67	0,00 (0,04)	44,76	-0,57	-4,36	0,73	-3,40	1,00
10	-0,15	0,46 (0,57)	0,53	0,00 (0,00)	-0,71	0,00 (0,02)	43,37	-0,13	-1,41	3,12	-1,51	0,93
20	-0,25	0,19 (0,31)	0,48	0,01 (0,02)	-0,82	0,00 (0,03)	50,99	0,09	1,21	2,19	-1,29	0,90
$\bar{rx}^{ST}_{t+1}$	0,22	0,31 (0,42)	-0,12	0,46 (0,50)	-0,57	0,01 (0,08)	48,14	-0,23	-2,26	0,88	-1,82	0,97
$\bar{rx}^{LT}_{t+1}$	-0,80	0,55 (0,57)	1,80	0,25 (0,01)	-2,86	0,02 (0,02)	26,27	-0,31	-2,85	-0,63	-1,78	0,96

Pflueger and Viceira liquidity premium measure

n	CP	pv	$TERM$	pv	$L^{PV}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,37	0,08 (0,19)	-0,44	0,03 (0,11)	-0,36	0,05 (0,24)	58,79	0,66	23,79	18,76	2,44	0,01
4	0,28	0,20 (0,34)	-0,18	0,33 (0,39)	-0,52	0,02 (0,10)	51,75	0,33	5,87	7,65	0,46	0,32
5	0,16	0,47 (0,56)	0,06	0,73 (0,75)	-0,63	0,01 (0,10)	44,34	-0,23	-2,22	2,67	-0,72	0,77
6	0,06	0,81 (0,84)	0,26	0,14 (0,16)	-0,69	0,01 (0,07)	39,20	-0,54	-4,23	1,41	-1,62	0,95
10	-0,18	0,44 (0,51)	0,66	0,00 (0,01)	-0,73	0,00 (0,07)	36,93	-0,25	-2,40	1,96	-2,67	1,00
20	-0,30	0,16 (0,28)	0,67	0,00 (0,01)	-0,90	0,00 (0,03)	48,12	-0,06	-0,69	1,04	-2,02	0,98
$\bar{rx}^{ST}_{t+1}$	0,19	0,40 (0,51)	0,00	0,99 (0,99)	-0,61	0,01 (0,07)	45,80	-0,09	-1,00	3,21	-0,50	0,69
$\bar{rx}^{LT}_{t+1}$	-0,96	0,57 (0,35)	2,43	0,13 (0,01)	-3,08	0,04 (0,04)	24,00	-0,60	-4,51	-1,11	-2,25	0,99

Results for the following regression $rx^{(n)}_{t+1} = \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{n,t+1}$ where $rx_{n,t+1}$ denotes the annual Treasury excess log returns on $n = 3, 4, 5, 6, 10, 20$ years to maturity. I use $L^{z\text{-}asw}_{(10),t}$ and $L^{PV}_{(10),t}$ liquidity premium measures. M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. pv is the p-value calculated using the Newey-West correction for heteroscedasticity and autocorrelation with $T^{1/2}$ lags. The p-values based on the bootstrap analysis are presented in round brackets. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ GW is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test and Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and it corresponding p-value. The sample spans the period from 2006.11 to 2010.12

Table 1.8: TIPS excess returns predictability: monthly data

Panel A: Excess returns and Traditional bond factors

n	CP	pv	$TERM$	pv			AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,07	0,86 (0,90)	-0,04	0,90 (0,91)			0,77	-2,17	-23,20	-6,00	-2,45	0,99
4	-0,05	0,88 (0,93)	0,01	0,98 (0,98)			0,28	-2,24	-12,08	-2,00	-2,40	0,99
5	-0,06	0,87 (0,91)	0,05	0,89 (0,90)			0,36	-2,12	-6,95	0,01	-2,09	0,98
6	-0,06	0,85 (0,88)	0,08	0,81 (0,85)			0,73	-1,65	-4,19	1,21	-1,57	0,94
10	-0,12	0,69 (0,79)	0,20	0,52 (0,62)			4,08	0,09	0,27	2,79	0,21	0,42
20	-0,16	0,46 (0,56)	0,40	0,14 (0,23)			14,36	0,00	0,02	1,15	-0,25	0,60
mean.st	-0,06	0,87 (0,91)	0,03	0,93 (0,95)			0,34	-2,31	-9,03	-0,79	-2,27	0,99
mean.lt	-0,50	0,45 (0,62)	1,13	0,64 (0,35)			5,52	-0,31	-0,61	0,01	-1,12	0,87

Panel B: TIPS liquidity measures

z-asw liquidity premium measure

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,20	0,42 (0,53)	-0,36	0,19 (0,28)	0,67	0,00 (0,06)	33,64	-2,97	-44,98	-9,80	-3,61	1,00
4	0,19	0,45 (0,59)	-0,28	0,34 (0,47)	0,61	0,00 (0,06)	27,08	-4,72	-29,05	-3,75	-2,77	1,00
5	0,16	0,51 (0,58)	-0,21	0,49 (0,61)	0,54	0,01 (0,07)	21,43	-3,39	-13,58	1,81	-1,61	0,95
6	0,13	0,60 (0,70)	-0,15	0,64 (0,73)	0,47	0,02 (0,09)	17,11	-1,18	-5,65	4,78	-0,68	0,75
10	0,00	0,99 (0,99)	0,07	0,83 (0,86)	0,28	0,15 (0,39)	9,97	0,50	5,28	9,53	1,33	0,10
20	-0,16	0,56 (0,66)	0,40	0,17 (0,32)	0,00	0,99 (0,99)	14,36	0,51	7,49	7,50	1,29	0,10
mean.st	0,17	0,50 (0,61)	-0,24	0,43 (0,57)	0,57	0,00 (0,09)	23,77	-4,72	-18,38	-0,21	-2,17	0,98
mean.lt	-0,33	0,77 (0,76)	0,93	0,74 (0,51)	0,42	0,81 (0,70)	6,11	-0,16	-1,38	0,67	-0,68	0,75

Pflueger and Viceira liquidity premium measure

n	CP	pv	$TERM$	pv	$L^{PV}_{(10y)}$	pv	AdjR^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,22	0,44 (0,57)	-0,48	0,10 (0,21)	0,68	0,00 (0,06)	26,71	-4,96	-49,20	-9,89	-4,15	1,00
4	0,21	0,44 (0,58)	-0,40	0,20 (0,31)	0,63	0,00 (0,07)	22,53	-7,07	-23,56	-2,53	-3,64	1,00
5	0,18	0,49 (0,62)	-0,32	0,33 (0,46)	0,56	0,01 (0,07)	18,47	-4,73	-11,40	1,43	-2,33	0,99
6	0,15	0,56 (0,66)	-0,25	0,46 (0,58)	0,50	0,01 (0,05)	15,10	-1,61	-5,21	3,75	-1,03	0,85
10	0,01	0,96 (0,97)	0,01	0,98 (0,98)	0,30	0,12 (0,36)	9,34	0,53	4,12	7,45	1,23	0,11
20	-0,17	0,55 (0,66)	0,41	0,19 (0,35)	-0,02	0,93 (0,95)	14,38	0,51	5,26	5,04	1,30	0,10
$\bar{rx}^{ST}_{t+1}$	0,19	0,48 (0,60)	-0,35	0,27 (0,40)	0,59	0,00 (0,10)	19,99	-6,78	-15,76	0,00	-3,00	1,00
$\bar{rx}^{LT}_{t+1}$	-0,33	0,77 (0,78)	0,86	0,76 (0,60)	0,41	0,82 (0,71)	5,96	-0,15	-0,92	0,60	-0,76	0,78

Results for the following regression $rx^{(n)}_{t+1} = \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{n,t+1}$ where $rx_{n,t+1}$ denotes the annual TIPS excess log returns on $n = 3, 4, 5, 6, 10, 20$ years to maturity. I use $L^{z\text{-}asw}_{(10),t}$ and $L^{PV}_{(10),t}$ for liquidity premium measures. M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. pv is the p-value calculated using the Newey-West correction for heteroscedasticity and autocorrelation with $T^{1/2}$ lags. The p-values based on the bootstrap analysis are presented in round brackets. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ GW is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test and Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and it corresponding p-value. The sample spans the period from 2006.11 to 2010.12

Additionally, I find that all TIPS liquidity premium measures have a positive impact on TIPS excess returns, which means that increasing illiquidity in the TIPS market leads to higher TIPS bond excess returns one year ahead, and a negative impact on nominal excess returns, which is consistent with previous results. Additionally, the TIPS liquidity variables add to the explanatory power of the traditional bond factors. For instance, when I add 10-year *z-asw* liquidity premium to the term spread and CP factor, the explanatory power increases on average by 12% for the equally weighted Treasury bond portfolio, and by 11% for equally weighted TIPS excess portfolio returns, being highly statistically and economically significant in both bases.

I find similar in-sample results considering the *z-asw* liquidity premium measure, the maximum range for the TIPS liquidity by Christensen and Gillan (2011) and the 10-year estimated liquidity premium of Pflueger and Viceira (2012). This is also the case for the out-of-sample results. Models for TIPS excess returns with the *z-asw* liquidity premium measure or the estimated TIPS liquidity by Pflueger and Viceira (2012) produce positive values for R^2_{OS} at the 10- and 20-year maturity. Additionally, the difference in out-of-sample forecasting power between the models with the liquidity variables and the benchmark model is statistically significant for longer-term maturities also. In contrast, for Treasury excess returns, all market liquidity variables appear to add out-of-sample forecasting power, being this result also confirmed by the GW test.

Overall, these results reflect the robustness of TIPS market liquidity as a predictive variable for bond excess returns. They also show that liquidity premium measures calculated from information coming from different markets, or by using an estimation strategy, contain all useful information necessary in order to predict excess returns in both nominal and inflation-linked bond markets, at least in-sample.

1.5 Conclusions

The goal of this paper is to determine whether or not liquidity in inflation-linked bonds contain additional information over the existing traditional factors in explaining U.S. bond risk premium. To that end, I use a market-based measure of liquidity premium for inflation-linked bonds computed from the pricing information in the nominal and inflation-linked asset swap markets, which I call *z-asw* liquidity premium.

The relative *z*-spread asset swap between a TIPS and its nominal bond can be viewed as a measure of the richness/cheapness of a cash break-even rate versus the inflation swaps curve (Pond and Mirani (2010)). In fact, the popularity of the asset swap during the crisis came about as a result of an important cheapening of TIPS following the collapse of Lehman Brothers in September 2008. TIPS also cheapened relative to inflation swaps, reflecting the preference to hold swaps rather than balance-sheet-intensive bonds. This behavior was the result of a "flight-to-quality" and drop in liquidity which favored nominal bonds, as they were less draining on bank balance sheets and a more stable source of funding for investors at that time. Therefore, the spread between nominal and inflation-linked bonds on asset swaps offers a more precise inference about how inflation-linked bonds perform relative to nominal bonds, mainly during the financial distress, and therefore seems to be a good proxy of the TIPS liquidity premium.

Using this measure, I document the predictive power that liquidity premium of different maturities have for nominal and inflation-linked excess returns in the U.S. bond market. This result is confirmed using other liquidity premium measures available in literature. While predictability has been well studied and documented in the literature in U.S. nominal Treasuries, in general, less has been done to provide empirical evidence for the predictability of returns in inflation-linked bonds, and no effort has been made to assess the predictability in an out-of-sample context. Filling this gap is the main contribution of this paper.

A number of key results emerge from this analysis. First, the *z-asw* liquidity measure is strictly positive for all four maturities and shows a peak in late 2008 during the financial crisis, which is consistent with results found in previous literature, and in particular with the maximum range for the TIPS liquidity proposed by Christensen and Gillan (2011), and the 10-year estimated liquidity premium of Pflueger and Viceira (2012). More so, I find that predictability results are robust as to how the liquidity premium is measured. That is, all measures seem to contain useful information used to predict excess returns in both nominal and inflation-linked bond markets, at least in-sample.

Second, focusing on return predictability from the *z-asw* liquidity premium, results show that controlling for typical excess returns predictors such as the term structure slope and the recently proposed tent-shaped factor of Cochrane and Piazzesi (2005), liquidity premium for different maturities is a significant and economically relevant source of predictability for government excess returns. I find that one standard deviation moves the 10-year liquidity premium results in an increase of 94 basis points in short-term equally weighted TIPS portfolio returns. Adjusted R^2 values range from 6% to 36% for different maturities.

Nominal excess returns are also predictable from the liquidity differential between Treasury and inflation-linked bond yields. I find that the $AdjR^2$ ranges from between 21% and 52% for different maturities when the 10-year *z-asw* liquidity premium is considered as the predictor. In this case, I find that one standard deviation move in the 10-year liquidity premium of 30 basis points tends to go along with an increase in 40 basis points in a short-term equally weighted Treasury bond portfolio returns.

As a check for the in-sample results, I investigate whether or not inferences on the statistical significance of the parameters estimates are inadequate in finite samples.

To that end, I conduct a bootstrap analysis, and report the bootstrapped p-values. I find that there is a reduction in the extent of statistical significance but in most of the cases the bootstrapped p-values do not lead to changes in the above conclusions.

Third, by performing an out-of-sample analysis, I compare the predictive ability of the model with the liquidity variable (in addition to the traditional factors), to the constant expected return model, which is a popular benchmark forecasting model in literature. Results suggest that the linear model including liquidity as the predictor variable does a better of job forecasting excess returns than the historical average return model (mainly for shorter-term maturities). However, this is not the case for Treasury excess returns where liquidity does not seem to help out-of-sample. These results are confirmed when I compare the linear model including traditional predictor variables with the alternative linear model including, in addition, liquidity. I regard this result as expected, since the tent-shaped factor and term spread are very strong predictors for the U.S. Treasury bonds, and encompass a large range of information, thus are hard to beat out-of-sample.

Finally, the TIPS liquidity factor reflects the extreme illiquidity tensions that arose during the crisis, being most of the predictability results magnified during the financial crisis, i.e. when I only consider data from 2006 up to the end of 2009. In fact, one standard deviation move in the 10-year liquidity premium tends to go along with a increase (decrease) in 178 basis points (64 basis points) in the short-term equally weighted Treasury (nominal Treasury) bond portfolio returns. In terms of out-of-sample returns predictability, I confirm that the difference in out-of-sample forecasting power between the model with the liquidity variable and the two benchmark models (the constant expected return model and the traditional factor linear model) is statistically significant for TIPS excess returns in both sub-periods, while liquidity seems to be important mainly during the crisis for Treasury excess returns. This might be a result of the FED's quantitative easing operations, which improved the whole market liquidity conditions mitigating the extreme tensions on the market.

Appendix A1

Additional data descriptions

Table A1.1: Nominal and TIPS asset swaps outstanding

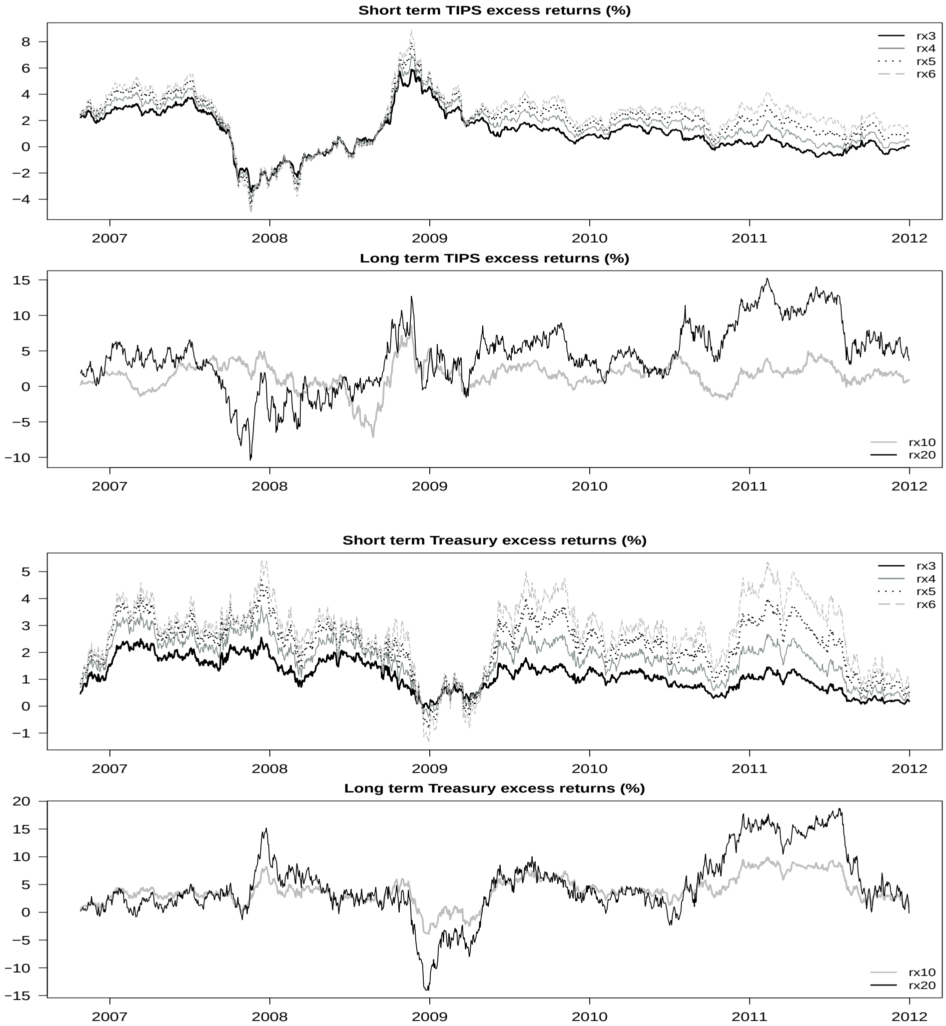
Coupon	Issued Date	Maturity Date
5-year bonds maturity		
3,375	04/30/07	04/15/12
0,625	04/30/08	04/15/13
1,250	04/30/09	04/15/14
0,125	04/29/11	04/15/16
10-year bonds maturity		
3,375	01/15/02	01/15/12
3,000	07/15/02	07/15/12
1,875	07/15/03	07/15/13
2,000	01/15/04	01/15/14
2,000	07/15/04	07/15/14
1,625	01/18/05	01/15/15
1,875	07/15/05	07/15/15
0,500	04/30/10	04/15/15
2,000	01/17/06	01/15/16
2,500	07/17/06	07/15/16
20-year bonds maturity		
2,375	07/30/04	01/15/25
2,000	01/31/06	01/15/26
2,375	01/31/07	01/15/27
1,750	01/31/08	01/15/28
30-year bonds maturity		
3,625	04/15/98	04/15/28
2,500	01/30/09	01/15/29
3,875	04/15/99	04/15/29
3,375	10/15/01	04/15/32
2,125	02/26/10	02/15/40
2,125	02/28/11	02/15/41

Table A1.2: Summary Statistics of monthly liquidity measures

Maturity	Corr.	Mean	Median	Std	Min	Max
z-asw Liquidity premium						
5		46.21	34.34	32.13	11.86	158.38
10		49.54	40.02	29.41	23.22	145.70
20		32.88	25.55	22.54	8.89	124.48
30		33.96	28.61	19.16	12.10	109.72
Cristersen and Gillan (2011)						
5	0.7204	40.18	30.24	29.26	10.43	193.20
10	0.9181	26.80	23.52	17.82	6.50	115.39
20	0.9511	24.67	20.82	18.40	0.94	114.76
Pfluger and Vicerira (2011)						
10	0.5434	105.84	104.14	32.70	64.00	255.60

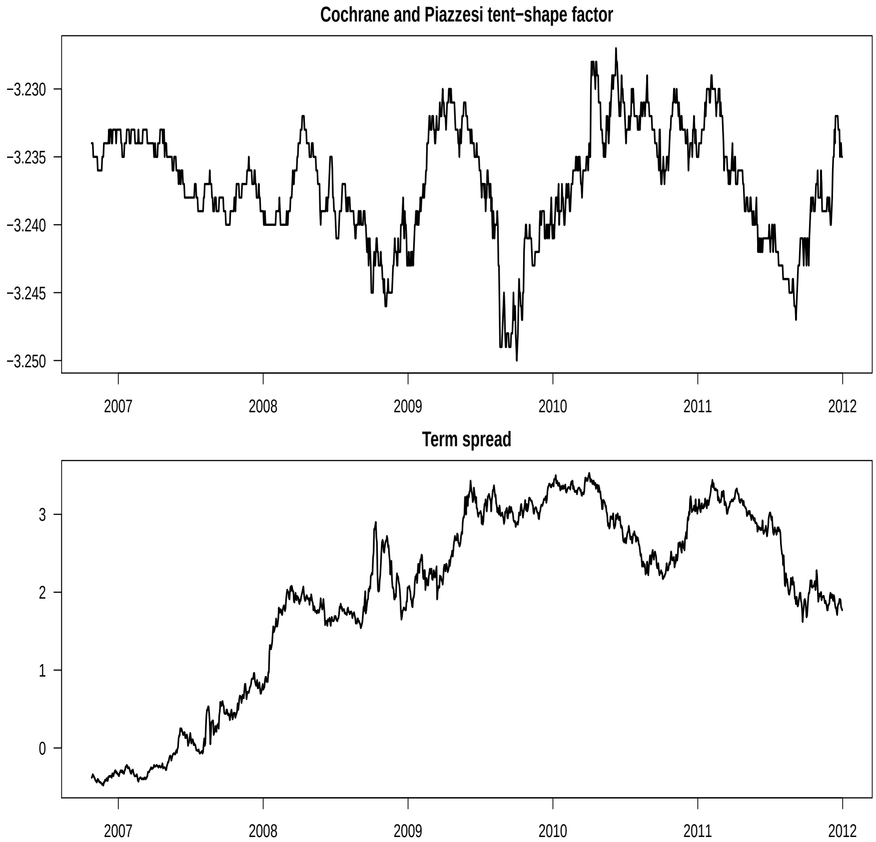
All liquidity measures correspond to end of the month values. The *z-asw* liquidity premium is the residual spread between TIPS and nominal bonds asset swaps calculated using daily nominal and TIPS z-spread asset swaps data from November 2006 to November 2011. The other liquidity measures correspond to the model-independent maximum range for the TIPS Liquidity by Christensen and Gillan (2011) calculated from January 2004 to November 2011 and the estimated 10-year TIPS liquidity premium estimated by Pflueger and Viceira (2012) from January 1999 to September 2010. The correlation coefficients correspond to the linear association between *z-asw* liquidity measure and the corresponding measure proposed in literature by maturity.

Figure A1.1: TIPS and nominal Treasury excess log-returns



Annual excess log returns on $n = 3, 4, 5, 6, 10, 20$ years to maturity, calculated as $rx_{t+1}^{(n)} = r_{t+1}^{(n)} - y_t^1$ with $r_{t+1}^{(n)}$ the log holding one year return on a zero-coupon n-period bond and y_t^1 is the one year log yield. The sample spans the period from 01/11/2006 to 30/12/2011.

Figure A1.2: Traditional bond factors



The $z\text{-}asw$ is the tent-shape Cochrane and Piazzesi (2005) factor and the Term spread correspond to the difference between long-term government yield bond (10-year bond) and short-term yield (1-year bond). The sample spans the period from 01/11/2006 to 30/12/2012.

Appendix B1

Block boostrap algorithm

Algorithm 1.1 Block boostrap (l denotes block size)

Require: Considering the bond return regression: $rx_{t+1}^{(n)} = \alpha + \beta_n^\top \mathbf{X}_t + \epsilon_{t+1}^{(n)}$.
Ensure: Bootstrap samples for $\mathbf{X}_t$ and $rx_{t+1}^{(n)}$
1: Estimate a first-order VAR model for $\mathbf{X}_t$, given by: $\mathbf{X}_{t+1} = \theta + \Phi_t \mathbf{X}_t + \nu_{t+1}$ where $\mathrm{var}(\nu_{t+1}) = \Sigma_\nu$
2: Obtain the standardized residuals define as: $\eta_t = \Sigma^{-1/2}\nu_t$ where $\Sigma^{-1/2}$ is the inverse of the Choleski factorization of Σ_ν.
3: Construct bootstrap samples for $\mathbf{X}_t$ by resampling from the standardized residuals η_t.
4: Construct bootstrap samples of $rx_{t+1}^{(n)}$ by using the bootstrap samples of $\mathbf{X}_t$ and resampling blocks of l subsequent residuals $\varepsilon_{t+1}^{(n)}$.
5: Repeat the bootstrap procedure 10.000 times.

Appendix C1

Additional results regression

Table C1.1: Excess returns predictability and Christersen and Gillan (2011) liquidity measure

Panel A: TIPS excess returns

n	CP	pv	$TERM$	pv	$\Delta_{(10),t}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,05	0,74 (0,80)	-0,16	0,46 (0,68)	0,56	0,00 (0,06)	33,79	-0,17	-35,05	6,79	-0,02	0,51
4	0,07	0,65 (0,74)	-0,09	0,70 (0,70)	0,52	0,00 (0,07)	27,19	-0,63	-77,61	-22,29	-1,65	0,95
5	0,08	0,63 (0,71)	-0,02	0,93 (0,96)	0,46	0,00 (0,07)	20,95	-1,48	-101,45	-35,18	-3,48	1,00
6	0,07	0,64 (0,73)	0,04	0,85 (0,89)	0,40	0,00 (0,19)	15,72	-1,42	-93,22	-24,72	-3,23	1,00
10	0,04	0,80 (0,84)	0,23	0,30 (0,39)	0,17	0,25 (0,54)	7,99	0,54	400,36	272,57	2,36	0,01
20	0,04	0,76 (0,81)	0,42	0,06 (0,22)	-0,11	0,49 (0,71)	18,27	0,52	246,04	153,74	1,69	0,05
$\bar{rx}^{ST}_{t+1}$	0,07	0,66 (0,62)	-0,05	0,83 (0,88)	0,49	0,00 (0,08)	23,53	-1,16	-91,50	-30,19	-2,77	1,00
$\bar{rx}^{LT}_{t+1}$	0,04	0,76 (0,79)	0,36	0,10 (0,31)	0,00	0,98 (0,99)	12,63	0,54	304,27	191,71	1,93	0,03

Panel B: Nominal excess returns

n	CP	pv	$TERM$	pv	$\Delta_{(10),t}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,02	0,90 (0,96)	-0,45	0,00 (0,09)	-0,28	0,00 (0,07)	28,32	-0,22	-43,48	-15,32	-1,44	0,93
4	-0,04	0,80 (0,91)	-0,30	0,03 (0,26)	-0,42	0,00 (0,04)	26,59	-0,31	-56,46	-24,91	-3,66	1,00
5	-0,07	0,63 (0,64)	-0,14	0,34 (0,51)	-0,52	0,00 (0,01)	28,69	-0,17	-35,40	-11,59	-0,52	0,70
6	-0,09	0,45 (0,48)	0,02	0,91 (0,96)	-0,58	0,00 (0,02)	32,82	0,06	16,58	21,50	0,67	0,25
10	-0,13	0,19 (0,37)	0,33	0,02 (0,15)	-0,57	0,00 (0,05)	43,77	0,49	231,59	153,81	1,48	0,07
20	-0,09	0,51 (0,63)	0,26	0,08 (0,40)	-0,63	0,00 (0,04)	45,65	0,44	190,05	121,26	1,82	0,03
$\bar{rx}^{ST}_{t+1}$	-0,06	0,65 (0,72)	-0,18	0,22 (0,50)	-0,50	0,00 (0,01)	27,87	-0,23	-45,08	-17,55	-1,04	0,85
$\bar{rx}^{LT}_{t+1}$	-0,10	0,39 (0,59)	0,29	0,04 (0,26)	-0,62	0,00 (0,03)	46,34	0,48	216,62	138,41	1,89	0,03

Results for the following regression $rx^{(n)}_{t+1} = \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{n,t+1}$ where $rx_{n,t+1}$ denotes the annual TIPS excess log returns on $n = 3, 4, 5, 6, 10, 20$ years to maturity. I use $\Delta_{(10),t}$ the maximum range for the TIPS liquidity by Christensen and Gillan (2011) as liquidity measure. M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. pv is the p-value calculated using the Newey-West correction for heteroscedasticity and autocorrelation with $T^{1/2}$ lags. The p-values based on the bootstrap analysis are presented in round brackets. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ GW is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test and Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and it corresponding p-value. The sample spans the period from 01/11/2006 to 30/12/2011.

Table C1.2: TIPS excess returns predictability: z-asw liquidity measure other maturities

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(5y)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,01	0,85 (0,92)	0,12	0,34 (0,56)	0,71	0,00 (0,11)	37,42	0,72	91,46	65,58	13,35	0,00
4	0,02	0,80 (0,93)	0,23	0,02 (0,21)	0,63	0,00 (0,10)	34,60	0,59	35,25	25,86	5,41	0,00
5	0,01	0,90 (0,97)	0,33	0,00 (0,10)	0,52	0,00 (0,10)	29,66	0,29	7,65	6,82	1,66	0,05
6	-0,01	0,93 (0,99)	0,39	0,00 (0,13)	0,41	0,00 (0,10)	23,89	-0,04	-0,74	1,06	-0,01	0,50
10	-0,07	0,45 (0,46)	0,54	0,05 (0,29)	0,06	0,68 (0,69)	12,45	0,51	216,87	142,33	2,34	0,01
20	-0,08	0,36 (0,53)	0,63	0,12 (0,38)	-0,30	0,07 (0,40)	16,51	0,55	377,28	239,90	3,05	0,00
$\bar{rx}^{ST}_{t+1}$	0,01	0,91 (0,95)	0,28	0,01 (0,10)	0,57	0,00 (0,10)	31,80	0,45	14,63	11,64	2,63	0,00
$\bar{rx}^{LT}_{t+1}$	-0,08	0,39 (0,62)	0,62	0,09 (0,21)	-0,17	0,28 (0,49)	14,39	0,55	405,37	259,36	2,78	0,00

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(20y)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,01	0,96 (1,00)	-0,21	0,30 (0,56)	0,58	0,00 (0,02)	35,50	-0,25	-54,15	-5,95	-0,31	0,62
4	0,03	0,84 (0,99)	-0,14	0,52 (0,71)	0,54	0,00 (0,05)	29,79	-0,69	-89,56	-29,40	-2,10	0,98
5	0,04	0,79 (0,85)	-0,06	0,77 (0,89)	0,49	0,00 (0,10)	23,70	-1,47	-108,05	-38,00	-3,82	1,00
6	0,05	0,78 (0,82)	0,00	0,98 (0,96)	0,43	0,00 (0,10)	18,10	-1,32	-91,53	-21,79	-2,50	0,99
10	0,02	0,87 (0,95)	0,21	0,33 (0,98)	0,19	0,24 (0,31)	8,60	0,55	348,40	236,71	2,37	0,01
20	0,04	0,71 (0,93)	0,42	0,05 (0,35)	-0,09	0,61 (0,72)	17,87	0,51	216,02	134,22	1,63	0,05
$\bar{rx}^{ST}_{t+1}$	0,03	0,83 (0,83)	-0,10	0,66 (0,76)	0,51	0,00 (0,09)	26,05	-1,20	-101,55	-35,20	-3,88	1,00
$\bar{rx}^{LT}_{t+1}$	0,04	0,75 (0,83)	0,36	0,10 (0,25)	0,02	0,92 (0,95)	12,66	0,53	263,60	165,38	1,92	0,03

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(30y)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,02	0,88 (0,94)	-0,22	0,29 (0,52)	0,59	0,00 (0,04)	36,73	-0,21	-43,62	-1,05	-0,16	0,56
4	0,05	0,76 (0,85)	-0,14	0,51 (0,67)	0,56	0,00 (0,03)	31,55	-0,62	-77,92	-24,36	-1,89	0,97
5	0,06	0,72 (0,82)	-0,07	0,76 (0,82)	0,51	0,00 (0,09)	25,66	-1,38	-95,24	-33,47	-4,14	1,00
6	0,06	0,71 (0,77)	0,00	1,00 (1,00)	0,45	0,00 (0,06)	19,96	-1,26	-80,11	-20,06	-2,94	1,00
10	0,03	0,84 (0,84)	0,21	0,34 (1,00)	0,21	0,24 (0,28)	9,32	0,54	331,12	223,80	2,39	0,01
20	0,04	0,72 (0,86)	0,42	0,05 (0,39)	-0,08	0,66 (0,77)	17,81	0,51	226,66	141,18	1,66	0,05
$\bar{rx}^{ST}_{t+1}$	0,05	0,76 (0,84)	-0,10	0,64 (0,85)	0,53	0,00 (0,05)	27,87	-1,11	-89,12	-30,46	-3,90	1,00
$\bar{rx}^{LT}_{t+1}$	0,04	0,75 (0,85)	0,36	0,11 (0,72)	0,03	0,89 (0,89)	12,70	0,53	272,21	170,98	1,93	0,03

Results for the following regression $rx^{(n)}_{t+1} = \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{n,t+1}$ where $rx_{n,t+1}$ denotes the annual TIPS excess log returns on $n = 3, 4, 5, 6, 10, 20$ years to maturity. I use $L^{z\text{-}asw}_{(5),t}$, $L^{z\text{-}asw}_{(10),t}$, and $L^{z\text{-}asw}_{(20),t}$ liquidity premium measure. M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. pv is the p-value calculated using the Newey-West correction for heteroscedasticity and autocorrelation with $T^{1/2}$ lags. The p-values based on the bootstrap analysis are presented in round brackets. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ GW is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test and Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and it corresponding p-value. The sample spans the period from 01/11/2006 to 30/12/2011.

Table C1.3: Nominal Treasury excess returns predictability: z-asw liquidity measure other maturities

n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(5y)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,00	0,99 (0,97)	-0,13	0,30 (0,50)	-0,08	0,83 (0,93)	21,90	-0,47	-75,99	-32,99	-4,56	1,00
4	-0,04	0,86 (0,89)	0,08	0,48 (0,56)	-0,27	0,48 (0,64)	17,74	-0,75	-102,64	-39,04	-2,84	1,00
5	-0,09	0,71 (0,80)	0,27	0,08 (0,13)	-0,43	0,22 (0,56)	19,64	-0,67	-96,01	-22,25	-1,20	0,88
6	-0,13	0,53 (0,75)	0,41	0,06 (0,10)	-0,55	0,08 (0,42)	30,84	-0,34	-60,95	12,23	-0,23	0,59
10	-0,19	0,12 (0,44)	0,61	0,05 (0,31)	-0,68	0,00 (0,10)	49,92	0,45	193,10	191,56	1,20	0,12
20	-0,14	0,26 (0,50)	0,37	0,32 (0,57)	-0,81	0,00 (0,09)	54,58	0,56	298,26	229,41	1,10	0,14
$\bar{rx}^{ST}_{t+1}$	-0,08	0,73 (0,79)	0,22	0,11 (0,24)	-0,40	0,27 (0,52)	16,58	-0,73	-101,00	-28,42	-1,46	0,93
$\bar{rx}^{LT}_{t+1}$	-0,16	0,19 (0,54)	0,45	0,20 (0,50)	-0,79	0,00 (0,09)	53,85	0,54	280,92	228,93	1,07	0,14
n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(20y)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,00	1,00 (1,00)	-0,42	0,00 (0,05)	-0,31	0,00 (0,05)	30,05	-0,22	-43,86	-14,15	-1,50	0,93
4	-0,01	0,95 (0,96)	-0,26	0,07 (0,19)	-0,44	0,00 (0,04)	28,49	-0,32	-57,96	-25,13	-4,04	1,00
5	-0,03	0,84 (0,90)	-0,09	0,55 (0,58)	-0,54	0,00 (0,01)	30,58	-0,20	-40,36	-15,30	-1,08	0,86
6	-0,05	0,70 (0,77)	0,07	0,64 (0,77)	-0,59	0,00 (0,03)	34,64	0,02	5,27	12,17	0,58	0,28
10	-0,09	0,42 (0,55)	0,38	0,01 (0,07)	-0,59	0,00 (0,05)	45,48	0,45	193,94	124,78	1,65	0,05
20	-0,04	0,75 (0,84)	0,32	0,03 (0,10)	-0,65	0,00 (0,02)	48,69	0,40	157,89	98,30	1,89	0,03
$\bar{rx}^{ST}_{t+1}$	-0,03	0,84 (0,83)	-0,13	0,37 (0,66)	-0,52	0,00 (0,01)	29,99	-0,26	-48,99	-20,18	-1,77	0,96
$\bar{rx}^{LT}_{t+1}$	-0,06	0,65 (0,82)	0,34	0,02 (0,08)	-0,64	0,00 (0,05)	49,05	0,43	180,22	112,06	2,20	0,01
n	CP	pv	$TERM$	pv	$L^{z\text{-}asw}_{(30y)}$	pv	Adj R^2	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,01	0,97 (0,99)	-0,43	0,01 (0,12)	-0,27	0,03 (0,10)	28,09	-0,24	-46,45	-15,40	-1,73	0,96
4	-0,02	0,91 (0,94)	-0,26	0,09 (0,41)	-0,40	0,00 (0,06)	24,90	-0,34	-60,79	-26,04	-4,41	1,00
5	-0,04	0,80 (0,93)	-0,09	0,56 (0,64)	-0,49	0,00 (0,03)	25,77	-0,23	-44,00	-16,12	-1,14	0,87
6	-0,06	0,66 (0,73)	0,07	0,66 (0,75)	-0,55	0,00 (0,04)	29,39	0,00	0,70	11,40	0,56	0,29
10	-0,10	0,37 (0,48)	0,38	0,01 (0,13)	-0,56	0,00 (0,03)	42,26	0,45	194,55	128,35	1,51	0,07
20	-0,06	0,65 (0,83)	0,32	0,02 (0,33)	-0,65	0,00 (0,02)	48,70	0,42	171,55	108,31	1,79	0,04
$\bar{rx}^{ST}_{t+1}$	-0,04	0,81 (0,84)	-0,13	0,39 (0,61)	-0,48	0,00 (0,02)	25,47	-0,28	-52,46	-21,00	-1,83	0,97
$\bar{rx}^{LT}_{t+1}$	-0,07	0,55 (0,70)	0,35	0,01 (0,19)	-0,63	0,00 (0,04)	47,96	0,44	190,85	120,81	2,02	0,02

Results for the following regression $rx^{(n)}_{t+1} = \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon_{n,t+1}$ where $rx_{n,t+1}$ denotes the annual TIPS excess log returns on $n = 3,4,5,6,10,20$ years to maturity. I use $L^{z\text{-}asw}_{(5),t}$, $L^{z\text{-}asw}_{(10),t}$, and $L^{z\text{-}asw}_{(20),t}$ liquidity premium measure. M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. pv is the p-value calculated using the Newey-West correction for heteroscedasticity and autocorrelation with $T^{1/2}$ lags. The p-values based on the bootstrap analysis are presented in round brackets. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ GW is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test and Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and it corresponding p-value. The sample spans the period from 01/11/2006 to 30/12/2011.

Table C1.4: Alternative benchmark model for out-of-sample analysis

Panel A: Whole sample period

	Nominal Treasury bonds						*TIPS bonds*					
n	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv	n	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	-0,34	-60,84	-23,85	-1,38	0,92	3	0,69	234,20	158,00	5,52	0,00
4	-0,43	-71,59	-22,49	-1,01	0,84	4	0,66	153,89	105,01	3,81	0,00
5	-0,38	-65,25	-11,26	-0,54	0,70	5	0,62	99,02	69,92	2,94	0,00
6	-0,22	-43,76	7,78	-0,05	0,52	6	0,50	57,18	43,79	1,95	0,03
10	0,28	92,90	93,73	0,27	0,39	10	-0,11	-27,92	-13,00	-1,50	0,93
20	0,48	219,09	160,02	1,02	0,15	20	0,21	67,92	39,27	5,41	0,00
$\bar{rx}^{ST}_{t+1}$	-0,39	-67,41	-14,30	-0,63	0,74	$\bar{rx}^{LT}_{t+1}$	0,65	113,59	79,32	3,34	0,00
$\bar{rx}^{ST}_{t+1}$	0,44	189,12	146,69	0,64	0,26	$\bar{rx}^{LT}_{t+1}$	0,12	41,09	23,26	3,52	0,00

Panel B: Crisis period

n	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv	n	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,16	45,59	49,03	1,32	0,09	3	0,45	19,86	27,66	1,77	0,04
4	0,18	50,79	58,04	0,68	0,25	4	0,45	21,15	28,88	1,88	0,03
5	0,28	91,75	97,59	0,49	0,31	5	0,45	23,72	31,35	2,03	0,02
6	0,37	141,60	139,34	0,83	0,20	6	0,45	27,12	34,65	2,19	0,01
10	0,42	176,44	155,67	1,13	0,13	10	0,43	40,80	45,98	2,68	0,00
20	0,58	325,97	337,22	1,35	0,09	20	0,35	28,12	27,87	1,75	0,04
$\bar{rx}^{ST}_{t+1}$	0,28	91,69	93,57	0,50	0,31	$\bar{rx}^{ST}_{t+1}$	0,45	23,14	30,82	2,01	0,02
$\bar{rx}^{LT}_{t+1}$	0,54	277,34	270,81	1,22	0,11	$\bar{rx}^{LT}_{t+1}$	0,38	29,66	31,28	2,08	0,02

Panel C: After Crisis period

n	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv	n	R^2_{OS}	$MSE-F$	$ENC-NEW$	GW	pv
3	0,08	21,97	25,34	0,50	0,31	3	0,71	138,78	149,70	3,96	0,00
4	-0,19	-37,38	-9,16	-2,10	0,98	4	0,68	115,08	127,18	3,75	0,00
5	-0,07	-16,49	24,25	-0,54	0,70	5	0,52	69,15	82,61	1,71	0,04
6	0,07	16,68	65,44	0,33	0,37	6	0,15	16,58	29,29	0,87	0,19
10	0,26	83,37	154,51	0,61	0,27	10	0,25	77,43	78,06	3,06	0,00
20	0,33	117,99	221,31	0,55	0,29	20	0,31	103,68	146,58	1,15	0,13
$\bar{rx}^{ST}_{t+1}$	-0,11	-23,99	16,33	-0,82	0,79	$\bar{rx}^{ST}_{t+1}$	0,61	84,60	97,47	2,42	0,01
$\bar{rx}^{LT}_{t+1}$	0,31	108,82	203,31	0,53	0,30	$\bar{rx}^{LT}_{t+1}$	0,30	102,25	132,03	1,54	0,06

The R^2_{OS} is the Campbell and Thompson (2008) coefficient for out-of-sample exercise. $MSE-F$ is the McCracken (2007) test, $ENC-NEW$ GW is the Harvey et al. (1998) and Clark and McCracken (2001) forecasts encompassing test and Giacomini and White (2006) statistics used to test the null hypothesis of equally of forecast accuracy between the unrestricted and restricted model and it corresponding p-value. The benchmark model in this case is $H_0 : rx^{(n)}_{t+1} = \alpha + e^{(n)}_{t+1}\delta_n^\top \mathbf{M}_t$, while the competing model is $H_1 : rx^{(n)}_{t+1} = \alpha + \delta_n^\top \mathbf{M}_t + \beta_n^\top \mathbf{L}_t + \epsilon^{(n)}_{t+1}$. I use $L^{z\text{-}asw}_{(10),t}$ liquidity premium measure and M_t includes term yield spread ($TERM_t$) and Cochrane and Piazzesi (2005) factor (CP_t). All variables are standardized and coefficients are estimates by OLS using annual overlapping excess returns. The total sample spans the period from 01/11/2006 to 30/12/2011.

Chapter 2

Is the TIPS Liquidity Premium Unspanned by the U.S. Term Structure of Interest Rates?

Abstract: In this paper, I consider a joint Gaussian affine term structure model for zero-coupon U.S. Treasury and TIPS bonds, with an unspanned factor: liquidity risk. In the model, the liquidity factor is restricted to affect only the cross-section of yields but it is allowed to determine the bond risk premia. This is motivated by the fact that bond excess returns can be predicted by the TIPS liquidity premium, therefore liquidity can be considered as an unspanned factor that forecasts bond returns but does not span the yield curve. I present empirical evidence suggesting that the liquidity factor does not affect the dynamic of bonds under the pricing measure, but does affect them under the historical measure. Consequently, the information contained in the yield curve appears to be insufficient to completely characterize the variation in the price of curvature risk.
Key Words: Liquidity risk, inflation-indexed bond market, affine term structure, unspanned factors, predictability.
JEL classification: C13, C52, G11, G32.

2.1 Introduction

Traditionally, no arbitrage affine term structure models (ATSM) assume that the yield curve is jointly spanned by all state variables. Empirical evidence initially suggest that the yield curve is sufficiently described by three latent yield factors, which are often called ”level”, ”slope” and ”curvature” (see Litterman and Scheinkman (1991), Ang and Piazzesi (2003) and Diebold and Li (2006)). More recently, Cochrane and Piazzesi (2005), Cochrane and Piazzesi (2008), and Duffee (2011) highlight the importance of additional factors, and Adrian et al. (2013) show that the first five principal components of Treasury yields are needed in order to explain Treasury returns. However, the yield curve does not contain all available information to forecast future excess bond returns. In fact, Ludvigson and Ng (2009) argue that: *"real and inflation factors have important forecasting power for future excess returns on U.S. government bonds, however this behavior is ruled out by the affine term structure models where the forecastability of bond returns and bond yields is completely summarized by the cross-section of yields or forward rates".* Subsequently, Joslin et al. (2011) determine that the additional information in macroeconomic variables that predicts excess bond returns is not perfectly spanned by the yield curve.

While macroeconomic variables (such as real output and inflation), have usually been proposed as unspanned factors, little attention has been paid to financial market variables as possible additional unspanned factors.[1] This paper examines the role of the liquidity risk premium as an unspanned factor for the U.S. term structure. In particular, the aim is to determine whether or not liquidity risk has an impact on bond investment decisions, apart from the effect of the traditional bond yield factors. This is motivated by recent empirical findings suggesting that bond excess returns can be predicted by liquidity risk, therefore it could be considered as an unspanned factor that forecast bond returns, but that is not necessarily spanned by the yield curve.

A variable is unspanned if its value is not related to the contemporaneous cross section of interest rates, but if helps to forecast future excess returns on the bonds (i.e. term structure risk premia). There are numerous studies that identify various financial and macroeconomic variables as predictors for the U.S. bond risk premia (expected excess returns). For instance, the term structure slope, the forward spread, the lagged excess returns, the Cochrane and Piazzesi (2005) tent-shaped factor, and macroeconomic fundamentals are some of the variables that have been identified as predictors for Treasury bonds (see Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005), Ludvigson and Ng (2009) and Cooper and Priestley (2009a)). The role of liquidity as a predictor variable also has been studied by Fontaine and Garcia (2011), Pflueger and Viceira (2012) and Gomez (2013). They provide empirical evidence for liquidity as a source of predictability for U.S. Treasury bonds, Treasury Inflation-Protected bonds (TIPS), or for both.[2]

[1] As far as I know, the only paper considering financial factors in addition to spanned macro factors is Dewachter and Iania (2011). Considering the standard macro-finance model, they assess the relative importance of macro and financial shocks for the U.S. yield curve, by introducing additional liquidity-related and return forecasting factors. They find that the model considering liquidity and risk premium shocks significantly outperforms the standard macro factor models in fitting the yield curve. However, my work differs in a fundamental way from this paper, since I consider liquidity as an unspanned factor, and I use a different empirical approach.

[2] TIPS are bonds issued by the U.S. Treasury Department, where the principal is indexed by inflation. TIPS bonds pay a semi-annual coupon equal to the product of the fixed nominal coupon rate and the inflation adjusted principal. The principal is adjusted by the Consumer

Unspanned factors in macro-finance term structure models are a recent topic of interest. The identification of unspanned risk is important as the traditional spanned factors that are able to capture the cross section of interest rates are not able to completely explain the physical dynamics of the data. However, literature has concentrated on an extensive search to find spanned variables embedded in the U.S. term structure. As result, a set of candidates have been identified where, besides the traditional bond yield factors, macroeconomic fundamentals are the most popular (see Cochrane and Piazzesi (2005), Hordahl et al. (2006), Cochrane and Piazzesi (2008), Kim (2009), Cooper and Priestley (2009a), Ludvigson and Ng (2009), Orphanides and Wei (2010) and Chernov and Mueller (2012) among others). Based on this evidence, macro-finance models were proposed by Ang and Piazzesi (2003), Moench (2008), Diebold et al. (2006), Dewachter et al. (2006), Dewachter and Lyrio (2006), Rudebusch and Wu (2008), Bekaert et al. (2010) and Dewachter and Iania (2011). However, the assumption underlying these models is that macroeconomic fundamentals are fully spanned by the term structure; an assumption that is not supported by the empirical evidence.

In response to this, Duffee (2011), Joslin et al. (2011) and Boos (2011) introduce a new branch of affine term structure models, where state variables have an effect on bond risk premia but do not span the cross-sectional distibution of yields. In particular, Duffee (2011) introduces unspanned hidden factors and documents that are an economically important component of bond risk premia. Joslin et al. (2011) explicitly apply these unspanned factors to observed macroeconomic variables (the inflation rate and industrial production growth), and show that shocks to those variables have a significant effect on the U.S. term premia. Simultaneously, Boos (2011) extends a term structure model of the Ang and Piazzesi (2003) class with unspanned macro factors, and provides an example with survey data on expected inflation to filter an unspanned factor.[3]

In this paper, I consider a joint Gaussian affine term structure model for zero-coupon U.S. Treasury and TIPS bonds, with an unspanned factor: liquidity risk. The liquidity factor is restricted to affect only the cross-section of yields, but it is

Price Index Urban Non-Seasonally Adjusted (CPI-U NSA). Thus, these securities are designed to remove the inflation risk of an investment and, as a result, to offer investors a security that would enable them to hedge inflation.

[3] Based on this initial research, some related work has also emerged. Wright (2011) look at unspanned macro factors in several countries besides the U.S. Li et al. (2011) investigates the difference between unspanned and traditional macro-finance models, and Barrillas (2011) researched the optimal bond portfolio choice given unspanned macro models. Additionally, the existence of unspanned volatility factors is another recent area of interest that was first seen in the fixed income market. Collin-Dufresne and Goldstein (2002), Heidari and Wu (2003) and Collin-Dufresne et al. (2009) define unspanned stochastic volatility as being those factors driving Cap and Swaption implied volatilities that do not drive the term structure of interest rates. In other words, they show that trading in underlying bonds does not span the term structure of interest rates.

allowed to determine the bond risk premia. In other words, I consider liquidity as an additional factor that does not span the yield curve, but improves the estimation of bond risk premia. Using this empirical model, I attempt to answer the following questions: (*i*) given that bond excess returns can be predicted by liquidity, can the liquidity premium be considered as a factor that forecasts bond returns but which is not spanned by the yield curve?; (*ii*) if so, does the variation in liquidity premium influence the shape of the yield curve? and finally, (*iii*) how does the market price liquidity risk in the U.S. government bond market?.

Theoretically, less liquid securities carry higher liquidity risk, and thus must carry a higher yield (higher expected returns or risk premia as well) as a compensation for the incremental risk and the higher cost of trading. This additional yield is the liquidity risk premium. TIPS' lack of liquidity compares with nominal Treasuries results in TIPS yields having a liquidity premium relative to Treasuries. In fact, the liquidity differential of TIPS relative to Treasury bonds has been well documented in the literature (see Sack and Elsasser (2004), Shen (2006), Hordahl and Tristani (2010), Campbell et al. (2009), Dudley et al. (2009), Christensen and Gillan (2011), Gurkaynak et al. (2010), Pflueger and Viceira (2012)).

I identify the liquidity component in TIPS yields through the difference between the observed break-even inflation rates (BEI) and the inflation swap rates, which is considered synthetic BEI. This measure was first introduced by Christensen and Gillan (2011), and it combines information from the U.S. bond market with information from the inflation-indexed swaps market, which is recognized as the market that trades the most liquid inflation derivatives in the over-the-counter (OTC) market. The particular choice of this measure for the liquidity premium is motivated by the fact that: *i*) it is highly correlated with other measures of the TIPS liquidity premium available in the literature, which suggests that they are all capturing similar information about the liquidity differential between nominal and TIPS yields; *ii*) U.S. bond excess returns can be predicted by this liquidity measure;[4] and *iii*) it is a market-based measure of liquidity which is straightforward to compute.

I start by empirically testing the plausibility of the TIPS liquidity premium as an unspanned factor. I find that the TIPS liquidity premium fulfill the three empirical facts identified by Joslin et al. (2011) in the case of macroeconomic variables. First, the TIPS relative liquidity premium is not linearly spanned by the information in the joint yield curve. Second, the unspanned liquidity factor has a predictive power for excess returns in bond markets; and third, bond yields follow a low-dimensional factor model. Then, I explore the inter-temporal associations between the TIPS

[4] Using this measure of liquidity premium, in addition to other measures, Gomez (2013) finds that, controlling for typical excess returns predictors (such as the term structure slope and the tent-shaped factor of Cochrane and Piazzesi (2005)), the TIPS liquidity premium, for different maturities, is a significant and economically relevant source of predictability for real and nominal excess returns across maturities.

liquidity premium and the traditional set of fundamentals that capture macro information affecting bond prices and the dynamic of the yield curve (such as the Federal Funds Rate (FFR_t), the Term spread ($TERM_t$), the default credit spread (CDS_t), the Market Volatility Index (VIX_t), among other variables). Overall, I conclude that the dynamic of the TIPS liquidity premium appears to be driven by a relatively wide set of economic variables. In particular, the TIPS liquidity premium increases in response to aggregate economic uncertainty shocks (represented by the VIX index), as well as to expected monetary tightening conditions (associated with a positive shock to the four-quarter-ahead eurodollar futures rate ($ED4_t$)). This means that greater economic uncertainty or doubtfulness about the near-term path of monetary policy would result in a higher liquidity risk, increasing liquidity premiums and deteriorating market liquidity. However, the TIPS liquidity premium decreases in response to increments in bond returns. This result indicates that investors will demand higher returns when liquidity conditions in TIPS bond markets worsen.

Next, I examine the empirical relationship between movements in the level, slope and curvature of the term structure of U.S. nominal and real interest rates, and TIPS liquidity premium shocks. As is traditional in this empirical literature, I infer the relationships between yield movements and shocks in liquidity using impulse responses (IRFs) technique implied from a VAR model. Results show that the TIPS liquidity premium influences the shape of the joint nominal and real yield curve. More so, shocks to nominal and real bond yield factors appear to have an effect on the liquidity premium. Additionally, this effect is meaningful given that (as previous empirical evidence has shown) yield curve factors are highly correlated with measures of inflation expectations and monetary policy instruments, which provides an explanation for this dynamic connection.

Finally, I estimate the joint pricing model of TIPS and Treasury bonds by using the three-step linear regression procedure introduced by Adrian et al. (2013), and adapted by Abrahams et al. (2013) in the case of joint bond pricing. This procedure has the advantage of being easily implementable, computationally efficient, it allows a large number of pricing factors, and can accommodate unspanned factors. From the estimation of a five factor model (including four principal components of zero coupon yields, plus the liquidity premium as pricing factors), I test for the presence of unspanned factors. I present empirical evidence suggesting that the the liquidity factor does not affect the dynamic of bonds under the pricing measure, but does affect them under the historical measure. Consequently, the information contained in the yield curve appears to be insufficient to completely characterize the variation in the price of curvature risk.

The rest of the paper is organized as follows. Section 2 discusses the related literature. I further discuss what motivates the inclusion of the TIPS liquidity premium as an unspanned factor in a joint pricing model. In Section 3, I describe the joint term structure model for nominal Treasury and Inflation-Linked Bonds (ILBs), and the estimation procedure. I describe the data and the set of pricing factors in Section 4. Section 5 presents the main empirical findings. Section 6 concludes.

2.2 Multifactor affine Gaussian term structure models

2.2.1 Related Literature

Most of the current literature has used the joint term structure of nominal Treasury and Inflation-Linked Bonds (ILBs)[5] yields to infer inflation expectations and real interest rates.[6] The issuance of Inflation-Linked Bonds provides the possibility to derive a market-determined measure of real rates[7] and inflation expectations through the break-even inflation rates (BEI). The BEI are defined as the differential between yields on nominal Treasury securities and ILBs of comparable maturities, and are recognized as a source of information about the market's inflation expectation.[8]

Christensen et al. (2010) estimate an affine arbitrage-free Nelson-Siegel model for a joint representation of nominal and real yield curves, which allows them to decompose BEI rates of any maturity into inflation expectations and inflation risk premia. Similarly, Adrian and Hao (2010) estimate the term structure of inflation expectations from an eight-factor term structure model of real and nominal yield curves. Chen et al. (2010) study the term structure of inflation risk premia using TIPS and a two-factor CIR term structure model with correlated real and inflation rates. D'Amico et al. (2010) consider a three- and four-factor Gaussian term structure model of interest rates and inflation, and show the importance of using TIPS for accurate predictions of inflation. Chernov and Mueller (2012) combine nominal yields, surveys of inflation forecasts and data from inflation-indexed bonds to characterize the term structure of inflation expectations from a dynamic macro-finance model. Grishchenko and Huang (2013) using an arbitrage free and model free

[5] An inflation-linked bond (commonly referred as linker) is similar to a nominal bond such as a Treasury bond. The only difference is that both its principal (the final payment at maturity) and its coupon (the interest rate paid during the life of the bond) are linked to an inflation index. This means that the investor receives the real (i.e. adjusted for inflation) face value of the bond at maturity, and the real value of the interest rate in the meantime.

[6] The real interest rate is the sum of the risk-free real rate and the inflation risk premium. Policymakers looking only at interest rates on conventional nominal bonds lack information about each separate component. From the information provided by the ILBs market it is possible to infer whether or not movements in nominal interest rates reflect changes in market expectations about inflation, changes in real interest rates, or even changes in inflation risk premia.

[7] This is because the indexed bonds provide a direct measure of real returns.

[8] Break-even rates, however, do not in general reflect only inflation expectations. They also include inflation risk premia, notably to compensate investors for inflation risk, and also to compensate for differential liquidity risk in the nominal and index-linked bond markets.

approach (which does not impose restrictions on real and nominal term structures), estimate the term structure of real interest rates, expected inflation, and inflation risk premia.

Besides Campbell and Shiller (1996) and Evans (1998) (who were the first to provide evidence on inflation risk premium using U.K. data), all aforementioned papers are based on U.S. TIPS data, and highlight the role of liquidity premium embedded in TIPS yields. Using different proxies for the latent liquidity premium, most of these papers have considered liquidity adjusted TIPS yield curves to avoid distortions on yields. In contrast, studies by D'Amico et al. (2010), Christensen et al. (2010), and Abrahams et al. (2013) examine liquidity premium in an asset pricing framework. Christensen et al. (2010) estimate the model with a real level factor (that is specific to TIPS yields only), a nominal level factor for nominal yields, and a common slope and curvature factors. They find that the four-factor joint model properly captures the dynamics of both the nominal and real Treasury yield curves, and allows them to decompose BEI rates. D'Amico et al. (2010) provide a measure of liquidity premium by introducing a separate (fourth) factor into their term structure model of nominal and real yields. They show that ignoring the TIPS liquidity premia leads to counterintuitive implications for inflation expectations and inflation risk premia, and produces large pricing errors for TIPS. Alternatively, Abrahams et al. (2013) consider a specification using three yield factors (extracted from the cross-section of Treasury yields), two real pricing factors (extracted from BEI), and one liquidity factor. They find that the liquidity factor does not significantly add to time variation in the market price of risk, except during the financial crisis.

As a result of the strong development over recent years of markets for inflation-linked instruments, derivatives such as inflation-linked swaps have appeared as an alternative outstanding source of information about private sector inflation expectations.[9] This market has also been used for the identification of the parameters of a joint model of nominal and real term structures. In fact, Haubrich et al. (2012) estimate a term structure model of real and nominal yields using data on nominal Treasury yields, survey forecasts of inflation, and inflation swap rates. They claim that inflation swaps can be more reliable indicator of real yields because they are less prone to uncertain changes in market liquidity conditions.

Unlike most studies, this paper focuses on the role of the TIPS liquidity premium as an additional risk factor, which is considered to be unspanned by the cross-section of bond yields. This unspanned factor would affect the pricing of risk, but not the cross sectional fit of the yield curve. In other words, it would change the $\mathbb{P}$-dynamics but not the $\mathbb{Q}$-dynamics. Following Abrahams et al. (2013), I consider a

[9] A fixed zero-coupon inflation swap is a bilateral contract that enables an investor to secure an inflation-protected return with respect to an inflation index. The inflation buyer (also called the inflation receiver) pays a predetermined fixed rate, and in return receives inflation-linked payment(s) from the inflation seller (also called the inflation payer).

joint Gaussian affine model for zero-coupon nominal Treasury and TIPS yields, but incorporate an unspanned factor: liquidity risk. Following Haubrich et al. (2012), I use different sources of data (nominal Treasury yields, TIPS yields and inflation swap rates) to estimate the parameters of the model. However, I do not use inflation swaps together with nominal Treasuries as an alternative measure of real yields. Instead, I use information on zero-coupon inflation swaps to identify the physical liquidity risk premium, which arise from the liquidity differential between Treasuries and TIPS bonds.

2.2.2 TIPS liquidity as unspanned factor for U.S. bond yields

What literature has done for the joint pricing of the Treasury and TIPS yields is to include the TIPS liquidity as an additional spanned factor. D'Amico et al. (2010) and Abrahams et al. (2013) model the impact of liquidity on nominal and real yields including TIPS liquidity as a spanned pricing factor. As is commonly found in this literature, D'Amico et al. (2010) use principal components extracted from TIPS yields as pricing real factors. In contrast, Abrahams et al. (2013) assume that liquidity is observed through a composite factor which measures the relative TIPS liquidity premium. It is computed as the weighted average of two observable indicators: the average absolute TIPS yield curve fitting error from the Nelson-Siegel-Svensson model of Gurkaynak et al. (2010), and the 13-week moving average of the ratio of primary dealers' Treasury transaction volumes relative to TIPS transaction volumes.

In contrast, to model the impact of TIPS relative liquidity on nominal and real yields, I assume that liquidity is an unspanned observed factor. The plausibility of TIPS liquidity as an unspanned factor is motivated by three empirical facts (which were pointed out by Joslin et al. (2011) in the case of macroeconomic factors). First, the TIPS relative liquidity premium is not linearly spanned by the information in the joint yield curve. In fact, using a regression analysis D'Amico et al. (2010) find that there exists a factor that is important for explaining the variations in TIPS yields, but is not as crucial for modeling nominal interest rates. They argue that this factor is related to the illiquidity of the TIPS market. Their conclusion is confirmed by conducting a principal components analysis, which suggests that there is at least one state variable which drives innovations in TIPS yields, but does not affect innovations in Treasury yields.

Second, the unspanned liquidity factor has predictive power for excess returns in bond markets. As mentioned before, Gomez (2013) finds that the TIPS liquidity premium is a significant and economically relevant source of predictability for real and nominal excess returns across maturities. These results are obtained after controlling for typical excess return predictors, such as the term structure slope and the tent-shaped factor of Cochrane and Piazzesi (2005). Similar results are also found by Pflueger and Viceira (2012) using an alternative measure of liquidity premium. Third, the cross-section of bond yields is well described by a low-dimensional set of risk factors. Literature has determined that three, four or five factors are often enough to explain nearly all of the cross-sectional variations in U.S. Treasury yields (see Litterman and Scheinkman (1991), Ang and Piazzesi (2003), Diebold and Li (2006), Cochrane and Piazzesi (2005), Cochrane and Piazzesi (2008), and Duffee (2011)). I confirm these empirical facts considering a longer sample size (from January, 2004 to December 2013), a higher frequency (daily observations), and also a different source of data than in previous studies (zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007), and Gurkaynak et al. (2010)). Results are presented in Section 5.

I identify the liquidity component in TIPS yields through the difference between the observed break-even inflation rates and the inflation swap rates, which are considered as a synthetic BEI

$$\Delta_{n,t} = IS_{n,t} - BEI_{n,t} = L^{IS}_{n,t} + L^{TIPS}_{n,t}, \tag{2.1}$$

where $BEI_{n,t} = y^{N}_{n,t} - y^{TIPS}_{n,t}$ is the (cash BEI) break-even inflation rates, which are defined as the difference between nominal and inflation-indexed bond yields, and $IS_{n,t}$ is the inflation swap rates (synthetic BEI) for the corresponding maturity n.

This measure was first used by Christensen and Gillan (2011), and it combines information from the U.S. bond market with information from the inflation-indexed swaps market, which are the most liquid inflation derivative contracts traded in the over-the-counter market.[10] Christensen and Gillan (2011) argue that this difference measures the liquidity premium in inflation swaps as well as the liquidity premium in TIPS, so that it can be seen as a maximum range of liquidity premia for the TIPS market.

An important feature of zero-coupon inflation-indexed swaps (ZCIIS) is that the pricing model for nominal and inflation-linked (real) bonds would determine inflation swap rates. In fact, Mercurio (2005) (who was the first to study the pricing of ZCIIS) shows that the price of inflation-indexed swaps can be expressed as a function of

[10] Different practical approaches have been used to measure the liquidity differential between Nominal Treasuries and TIPS yields. In general, two approaches have been implemented: market-based measures used by Christensen and Gillan (2011) and Gomez (2013), and a regression procedure used by Pflueger and Viceira (2012). The three measures are highly correlated with a sample correlation coefficient greater than 0.90 over the period 2006-2012, which suggests that all measures are capturing similar information about the liquidity differential between nominal and TIPS yields. Additionally, Fleming and Krishnan (2012), using novel tick data from the inter-dealer market, calculate typical liquidity proxies such as bid-ask spread, trading volume, trading frequency and, quote size and incidence. However, they highlight the limitations of the bid-ask spread and quoted depth as liquidity measures in the TIPS market.

zero-coupon Treasury and inflation-linked bonds.[11]

A swap is an agreement between two counter parties in order to exchange cash flows. The agreement specifies the cash flows and the dates when they are to be paid. In particular, in a ZCIIS, one party pays a fixed interest rate (commonly referred to as inflation swap rate (IS)), and receives the inflation rate over the specified time period. The inflation rate is calculated as the percentage return of the consumer price index, therefore while the fixed payment is known at the start date of the swap, the floating payment is not. As the name indicates, a ZCIIS has only one time interval $[t_0, T]$, with payments at time T and no intermediary payments.

Consider a payer ZCIIS that starts at time t_0, has a payment date at time T, and a swap rate equal to IS. The fixed amount (fixed leg) payed at maturity is equal to

$$(1 + IS)^{T-t_0} - 1,$$

and a floating amount received (floating leg) at maturity is

$$\frac{I_T}{I_{t_0}} - 1,$$

where I_t represents a price index. Then, the payoff to the holder of the ZCIIS is given by

$$Z_0(T, IS) = \frac{I_n}{I_{t_0}} - (1 + IS)^{T-t_0}. \tag{2.2}$$

Let $Z_0(t, T, IS)$ denote the price of ZCIIS at time t, $t_0 < t < T$. Mercurio (2005) shows that under standard no arbitrage opportunities the inflation-linked floating leg is equal to

$$Z_0(t, T, IS) = \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{i=0}^{T-1} r_{t+i}^N \right) \left(\frac{I_{t+T}}{I_t} - 1 \right) \right], \tag{2.3}$$

where r_t^N is the nominal short interest rate. But given that the nominal price of a real zero-coupon bond at time t, denoted by $P_{t,T}^R$, equals the nominal price of the contract paying off one unit of the price index at the bond maturity

$$I_t P_{t,T}^R = I_t \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{i=0}^{T-1} r_{t+i}^R \right) \right] = \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{i=0}^{T-1} r_{t+i}^N \right) \right],$$

then, the price of ZCIIS is equal to

$$Z_0(t, T, IS) = \left(\frac{I_{t+T}}{I_t} \right) P_{t,T}^R - P_{t,T}^N, \tag{2.4}$$

[11] Hinnerich (2008) extends the work by Mercurio (2005) studying the pricing of zero-coupon inflation-indexed swaps and other derivatives (year-on-year inflation-indexed swaps, zero-coupon inflation-indexed swaptions and options on TIPS) using an extended HJM framework allowing for both jumps and stochastic volatility.

which at time t_0 is

$$Z_0(t_0, T, IS) = P^R_{t_0,T} - P^N_{t_0,T},$$

where $P^N_{t,T}$ the price in dollars at time t of a nominal zero-coupon bond.

This result allows us to strip (with no ambiguity) real zero-coupon bond prices from the quoted prices of zero-coupon inflation-indexed swaps. Additionally, as Haubrich et al. (2012) claim, real yields on inflation-linked bonds can be derived as the difference between equivalent maturity nominal yields and inflation swap rates, and these synthetic real yields are less prone to uncertain changes in liquidity than TIPS yields. For this reason, inflation swaps can be a more reliable indicator of real yields. Finally, Mercurio (2005) shows that the price of ZCIIS is model-independent, in the sense that no assumptions on the dynamics of the assets are needed to price them.

2.3 The model

In this section, I introduce the ordinary Gaussian ATSM framework, proposed in discrete time by Abrahams et al. (2013), for pricing inflation-linked bonds jointly with nominal bonds, so that both yield curves are affine in the state variables. However, in the spirit of Joslin et al. (2011), in addition to the yield curve risk (principal component factors), in this model I consider liquidity as a different source of risk, which is unspanned by the joint yield curve.

2.3.1 Setup

Consider a discrete time environment. Let $P^N_{t,n}$ denote the price in dollars at time t of a nominal zero-coupon bond that pays out one dollar at the maturity date, n. Let I_t be any stochastic process at time t. By $P^R_{t,n}$ I denote the price in dollars at time t of a contract that pays out I_t dollars at time n. If I_t denotes a Consumer price index (CPI) at time n, then it is the price at time t of a contract that at maturity will pay out the dollar value of one CPI-unit at time maturity. Hence, in this case, $P^R_{t,n}$ is the price of an inflation-linked zero-coupon bond, which I will refer to henceforth as a real bond.

Assume that a liquid riskless nominal zero-coupon bond price at time t with maturity n, is given by

$$P^N_{t,(n)} = \mathbb{E}^{\mathbb{Q}}_t \left[\exp\left(-\sum_{i=0}^{n-1} r^N_{t+i} \right) \right] = \exp(A^N_n + \boldsymbol{B}'^N_n \boldsymbol{X_t}), \tag{2.5}$$

where $\mathbb{E}^{\mathbb{Q}}_t$ denotes the expected value at time t under the risk-neutral measure $\mathbb{Q}$, and r^N_t is the nominal risk free interest rate. Similarly, the price at time t of a inflation-linked zero coupon bond that matures at time n is equal to

$$P^R_{t,(n)} = \mathbb{E}^{\mathbb{Q}}_t \left[\exp\left(-\sum_{i=0}^{n-1} r^R_{t+i} \right) \right] = \exp(A^R_n + \boldsymbol{B}'^R_n \boldsymbol{X_t}), \tag{2.6}$$

where r^R_t is the real interest rate. In this case inflation-linked bonds are priced by discounting future cash flows using a real short rate. Note also that the real short rate is equal to the difference between nominal and inflation rate, $r^R_t = r^N_t - \pi_{t+1}$.

Working in a general affine framework, I assume that the dynamics of the $K \times 1$ vector of state variables $\boldsymbol{X_t}$, under the historical measure $\mathbb{P}$, is given by

$$\boldsymbol{X_{t+1}} = \boldsymbol{\Theta_1} + \boldsymbol{\Theta_2} \boldsymbol{X_t} + \boldsymbol{\nu_{t+1}}, \tag{2.7}$$

where $\boldsymbol{\Theta_1}$ is a $K \times 1$ vector, $\boldsymbol{\Theta_2}$ is a $K \times K$ matrix, and $\boldsymbol{\nu_t}$ is $K \times 1$ a vector which is assumed *iid* Gaussian with mean $\mathbb{E}^{\mathbb{P}}_t[\boldsymbol{\nu_{t+1}}] = \boldsymbol{0}$ and variance $\mathbb{V}^{\mathbb{P}}_t[\boldsymbol{\nu_{t+1}}] = \boldsymbol{\Sigma}$.

Under the assumption of no arbitrage opportunities, there exists a nominal pricing kernel M^N_t such that

$$P^N_{t,(n)} = \mathbb{E}_{t+1}[M^N_{t+1} P^N_{t+1,(n-1)}], \tag{2.8}$$

where the nominal pricing kernel is assumed exponentially affine

$$M^N_{t+1} = \exp\left(-r^N_t - \frac{1}{2} \boldsymbol{\Lambda}'^N_t \boldsymbol{\Lambda}^N_t - \boldsymbol{\Lambda}'^N_t \boldsymbol{\Sigma}^{-1/2} \boldsymbol{\nu_{t+1}} \right), \tag{2.9}$$

and I assume the nominal risk free interest rate and the price of risk $\boldsymbol{\Lambda_t}$ are also functions of the state variables

$$r^N_t = \delta_0 + \boldsymbol{\delta'_1} \boldsymbol{X_t}, \tag{2.10}$$

$$\boldsymbol{\Lambda}^N_t = \boldsymbol{\Sigma}^{1/2}(\boldsymbol{\lambda_0} + \boldsymbol{\lambda'_1} \boldsymbol{X_t}), \tag{2.11}$$

where δ_0 is a constant, $\boldsymbol{\lambda_0}$ and $\boldsymbol{\delta}_1$ are a $K \times 1$ vectors, and $\boldsymbol{\lambda_1}$ is a $K \times K$ matrix.

Since I also need to price a real bond, I derive the real pricing kernel M^R_{t+1}. No arbitrage condition requires consistency between the nominal and inflation-linked bonds, such that the inflation-linked bond can be see as a nominal bond paying realized inflation upon maturity (in the spirit of D'Amico et al. (2010)). Therefore, the following relationship for nominal and real pricing kernels should hold

$$M^R_t = M^N_t \left(\frac{I_{t+n}}{I_t} \right),$$

where $\frac{I_{t+n}}{I_t} = \exp(\Sigma^n_{i=1} \pi_{t+i})$. π_t denotes the log inflation rate which is based on changes in the prices levels $\pi_t = \ln(\frac{I_t}{I_{t-1}})$. Assuming that one-period log inflation rate is also an affine function of state variables

$$\pi_t = \pi_0 + \boldsymbol{\pi}'_1 \boldsymbol{X}_t, \tag{2.12}$$

where π_0 is a constant, and $\boldsymbol{\pi}_1$ is a $K \times 1$ vector. Due to the previous assumptions, the real pricing kernel is related to the nominal pricing kernel via

$$M_{t+1}^R = \exp\left(-r_t^N + \pi_{t+1} - \frac{1}{2}\boldsymbol{\Lambda'}_t^N \boldsymbol{\Lambda}_t^N - \boldsymbol{\Lambda'}_t^N \boldsymbol{\Sigma}^{-1/2}\boldsymbol{\nu_{t+1}}\right), \tag{2.13}$$

and the price of a one-period real bond is equal to $P_{t,(1)}^R = \mathbb{E}_{t+1}(M_{t+1}^R)$, therefore the real spot rate is equal to

$$\begin{aligned} r_t^R &= -\log(P_{t,(1)}^R) = -\mathbb{E}_{t+1}(M_{t+1}^R) - \frac{1}{2}\mathbb{V}_{t+1}(M_{t+1}^R) \\ &= -r_t^N + \pi_0 + \boldsymbol{\pi}_1'(\boldsymbol{\Theta_1} + \boldsymbol{\Theta_2 X_t}) - \frac{1}{2}(\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1)'\boldsymbol{\Sigma}^{1/2}\boldsymbol{\Sigma}^{1/2}(\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1) \\ &\quad - (\boldsymbol{\lambda_0} + \boldsymbol{\lambda_1' X_t})'\boldsymbol{\Sigma}^{1/2}\boldsymbol{\Sigma}^{1/2}(\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1). \end{aligned}$$

As usual, this equation implies the generalized Fisher decomposition of the real rate into the nominal rate, expected inflation, the convexity term, and the inflation risk premium, respectively.

Notice that the real short rate and the vector of real prices of risk are also affine functions of the state variables. In fact, using the last expression for the short real rate and given the equations (2.8) and (2.9), it is possible to write (2.12) as

$$\begin{aligned} M_{t+1}^R &= \exp\left(-r_t^N + \pi_0 + \boldsymbol{\pi}_1'(\boldsymbol{\Theta_1} - \boldsymbol{\Theta_2 X_t}) - \boldsymbol{\pi'}_1\boldsymbol{\nu_{t+1}} - \frac{1}{2}\boldsymbol{\Lambda'}_t^N\boldsymbol{\Lambda}_t^N - \boldsymbol{\Lambda'}_t^N\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\nu_{t+1}}\right) \\ &= \exp\left(-r_t^R - \frac{1}{2}(\boldsymbol{\pi}_1' - \boldsymbol{\lambda_0} - \boldsymbol{\lambda_1' X_t})'\boldsymbol{\Sigma}^{1/2}\boldsymbol{\Sigma}^{1/2}(\boldsymbol{\pi'}_1 - \boldsymbol{\lambda_0} - \boldsymbol{\lambda_1' X_t})\right) \\ &\quad \exp\left(-(\boldsymbol{\pi'}_1 - \boldsymbol{\lambda_0} - \boldsymbol{\lambda_1' X_t})\boldsymbol{\Sigma}^{1/2}\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\nu_{t+1}}\right) \\ &= \exp\left(-r_t^R - \frac{1}{2}\boldsymbol{\Lambda'}_t^R\boldsymbol{\Lambda}_t^R - \boldsymbol{\Lambda'}_t^R\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\nu_{t+1}}\right), \end{aligned} \tag{2.14}$$

where the real short rate can be rewritten as

$$\begin{aligned} r_t^R &= -\delta_0 + \pi_0 - (\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1)'\boldsymbol{\Theta_1} + (\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1)'\boldsymbol{\Theta_2 X_t} \\ &\quad - \frac{1}{2}(\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1)'\boldsymbol{\Sigma}^{1/2}\boldsymbol{\Sigma}^{1/2}(\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1) - (\boldsymbol{\lambda_0} + \boldsymbol{\lambda_1' X_t})'\boldsymbol{\Sigma}^{1/2}\boldsymbol{\Sigma}^{1/2}(\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1) \\ &= -\delta_0 + \pi_0 - (\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1)'(\boldsymbol{\Theta_1} - \boldsymbol{\Sigma\lambda_0}) - \frac{1}{2}(\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1)'\boldsymbol{\Sigma}^{1/2}\boldsymbol{\Sigma}^{1/2}(\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1) \\ &\quad + (\boldsymbol{\delta}_1 - \boldsymbol{\pi}_1)'(\boldsymbol{\Theta_2} + \boldsymbol{\Sigma\lambda_1'})\boldsymbol{X_t} \\ &= \omega_0 + \boldsymbol{\omega_1' X_t}, \end{aligned} \tag{2.15}$$

and $\boldsymbol{\Lambda'}_t^R = (\boldsymbol{\pi}_1 - \boldsymbol{\lambda_0} - \boldsymbol{\lambda_1 X_t})'\boldsymbol{\Sigma}^{1/2}$ can also be written as

$$\boldsymbol{\Lambda_t^R} = \boldsymbol{\Sigma}^{1/2}(\boldsymbol{\gamma_0} + \boldsymbol{\gamma_1' X_t}), \tag{2.16}$$

where $\boldsymbol{\gamma_0} = \boldsymbol{\pi}_1 - \boldsymbol{\lambda_0}$ is a $K \times 1$ vector and $\boldsymbol{\gamma_1} = \boldsymbol{\lambda_1}$ is a $K \times K$ matrix. Thus there exist also a real pricing kernel M_t^R, which allow us to price real bonds, such that

$$P_{t,(n)}^R = \mathbb{E}_{t+1}[M_{t+1}^R P_{t+1,(n-1)}^R]. \tag{2.17}$$

Risk neutral dynamic

Under the absence of arbitrage opportunity, there exists a risk neutral probability measure $\mathbb{Q}$ under which the state variables follows

$$\boldsymbol{X_{t+1}} = \boldsymbol{\Theta_1^*} + \boldsymbol{\Theta_2^*} \boldsymbol{X_t} + \boldsymbol{\nu_{t+1}^*}, \tag{2.18}$$

with $\boldsymbol{\Theta_1^*} = \boldsymbol{\Theta_1} - \boldsymbol{\lambda_0}$, $\boldsymbol{\Theta_2^*} = \boldsymbol{\Theta_2} - \boldsymbol{\lambda_1}$, and $\boldsymbol{\nu_{t+1}^*} = \boldsymbol{\nu_{t+1}} + \boldsymbol{\Sigma}^{-1/2}\boldsymbol{\Lambda}_t$. I assume that under $\mathbb{Q}$ the innovations $\boldsymbol{\nu_{t+1}^*}$ are also *i.i.d* Gaussian with mean $\mathbb{E}_t^{\mathbb{Q}}[\boldsymbol{\nu_{t+1}^*}] = \boldsymbol{0}$ and variance $\mathbb{V}_t^{\mathbb{Q}}[\boldsymbol{\nu_{t+1}^*}] = \boldsymbol{\Sigma}$.

Pricing functions

Given the above general set up, the log nominal bond price can be expressed as a follows

$$\log P_{t,n}^N = A_n^N + \boldsymbol{B}_n'^N \boldsymbol{X_t}.$$

By replacing the pricing kernel in equation (2.4) I obtain that the coefficients are determined by the following difference equations

$$\begin{aligned} A_n^N &= A_{n-1}^N + \boldsymbol{B}_{n-1}'^N \boldsymbol{\Theta_1^*} + \frac{1}{2}\boldsymbol{B}_{n-1}'^N \boldsymbol{\Sigma} \boldsymbol{B}_{n-1}^N - \delta_0, \\ \boldsymbol{B}_n^N &= \boldsymbol{B}_{n-1}'^N \boldsymbol{\Theta_2^*} - \boldsymbol{\delta_1}, \\ A_0^N &= 0, \quad \boldsymbol{B}_0^N = \boldsymbol{0}. \end{aligned} \tag{2.19}$$

Similarly, the log price for an inflation-indexed bond is also an affine function of the state variables

$$\log P_{t,n}^R = A_n^R + \boldsymbol{B}_n'^R \boldsymbol{X_t},$$

where

$$\begin{aligned} A_n^R &= A_{n-1}^R + (\boldsymbol{B}_{n-1}'^R + \boldsymbol{\pi}_1)\boldsymbol{\Theta_1^*} + \frac{1}{2}(\boldsymbol{B}_{n-1}^R + \boldsymbol{\pi}_1)'\boldsymbol{\Sigma}(\boldsymbol{B}_{n-1}^R + \boldsymbol{\pi}_1) - \delta_0 - \pi_0, \\ \boldsymbol{B}_n^R &= (\boldsymbol{B}_{n-1}'^R + \boldsymbol{\pi}_1)'\boldsymbol{\Theta_2^*} - \boldsymbol{\delta_1'}, \\ A_0^R &= 0, \quad \boldsymbol{B}_0^R = \boldsymbol{0}. \end{aligned} \tag{2.20}$$

Unspanned liquidity factor

Duffee (2011), Joslin et al. (2011) and Boos (2011) introduce a term structure model featured by unspanned factors, which do not affect the dynamics of bonds under the risk neutral probability measure, $\mathbb{Q}$, but do affect them under the historical measure, $\mathbb{P}$. The assumption that a given factor does not affect bond yields under the $\mathbb{Q}$ measure can be implemented by imposing the restriction that the corresponding element of $\boldsymbol{B}_n^i$, for $i = N, R$ and maturities $n = 1, \cdots, h.$, be equal to zero (see Adrian et al. (2013)).

Following Adrian et al. (2013), this restriction is incorporated by the partition of the factor vector $\boldsymbol{X_t}$ into spanned factors $\boldsymbol{X_t^s}$ with nonzero risk exposures, and unspanned factor l_t which has zero risk exposure

$$\begin{bmatrix} \boldsymbol{X_{t+1}^s} \\ l_{t+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Theta}_1^s \\ \theta_1^l \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Theta}_2^{ss} & \boldsymbol{\Theta}_2^{sl} \\ \boldsymbol{\Theta}_2^{ls} & \theta_2^{ll} \end{bmatrix} \begin{bmatrix} \boldsymbol{X_t^s} \\ l_t \end{bmatrix} + \begin{bmatrix} \boldsymbol{\nu_{t+1}^s} \\ \nu_{t+1}^l \end{bmatrix}$$

where $\boldsymbol{X_t^s}$ is a $K_s \times 1$ vector such that $\boldsymbol{X_t}$ is of dimension $K \times 1$ with $K = K_s + 1$. $\boldsymbol{\Theta}_2^{ss}$ is the upper $K_s \times K_s$ matrix, and $\boldsymbol{\Theta}_2^{sl}$ and $\boldsymbol{\Theta'}_2^{ls}$ are $K_s \times 1$ vectors.

According to Joslin et al. (2011), unspanned factors should fulfill two conditions: not be linearly spanned by the information in the joint yield curve; and have predictive power for excess returns in bond markets. To be consistent with these properties the upper right vector $\boldsymbol{\Theta}_2^{sl}$ has to be equal to zero. Therefore, under the risk neutral probability measure

$$\begin{bmatrix} \boldsymbol{X_{t+1}^s} \\ l_{t+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Theta}_1^s - \boldsymbol{\lambda}_0^s \\ \theta_1^{*l} - \lambda_0^l \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Theta}_2^{ss} - \boldsymbol{\lambda}_1^{ss} & \boldsymbol{0} \\ \boldsymbol{\Theta}_2^{ls} - \boldsymbol{\lambda}_1^{ls} & \theta_2^{ll} - \lambda_0^{ll} \end{bmatrix} \begin{bmatrix} \boldsymbol{X_t^s} \\ l_t \end{bmatrix} + \begin{bmatrix} \boldsymbol{\nu_{t+1}^s} \\ \nu_{t+1}^l \end{bmatrix}$$

This restriction eliminates the possibility of any influence of liquidity factor on spanned factors, and also implies that $\boldsymbol{\delta_1'} = [\boldsymbol{\delta'}_1^s \quad 0]$, so that the short rate does not load on the unspanned factor.

2.3.2 Estimation

For the estimation, I use the three step linear regression approach introduced by Adrian et al. (2013), and adapted by Abrahams et al. (2013) to the joint pricing of TIPS and Treasuries bonds. This procedure has the advantage of being easily implementable, computationally efficient and can accommodate unspanned factors.

This approach uses excess holding period returns to estimate the model parameters. The log excess holding return of a bond with maturity n is equal to

$$rx_{t+1,(n-1)}^i = r_{t+1,(n-1)}^i - r_t,$$

where $r^i_{t+1,(n-1)} = \ln P^i_{t+1,(n-1)} - \ln P^i_{t,(n)}$ is the holding one-period log-return on a zero-coupon n-period bond for $i = N, R$. Using equations (2.15) and (2.16) with equation (2.6) in the last definition yields

$$\begin{aligned} rx^N_{t+1,(n-1)} &= \alpha^N_{n-1} + \boldsymbol{B}'^N_{n-1}\boldsymbol{\Theta}_2\boldsymbol{X}_t + \boldsymbol{B}'^N_{n-1}\boldsymbol{X}_{t+1} \\ rx^R_{t+1,(n-1)} &= \alpha^R_{n-1} + (\boldsymbol{B}^R_{n-1} + \boldsymbol{\pi}_1)'\boldsymbol{\Theta}_2\boldsymbol{X}_t + (\boldsymbol{B}^R_{n-1} + \boldsymbol{\pi}_1)'\boldsymbol{X}_{t+1} - \pi_{t+1}, \end{aligned} \tag{2.21}$$

where

$$\begin{aligned} \alpha^N_{n-1} &= -\left(\boldsymbol{B}'^N_{n-1}\boldsymbol{\Theta}_2 + \frac{1}{2}\boldsymbol{B}'^N_{n-1}\boldsymbol{\Sigma}\boldsymbol{B}^N_{n-1}\right), \\ \alpha^R_{n-1} &= -\left((\boldsymbol{B}^N_{n-1} + \boldsymbol{\pi}_1)'\boldsymbol{\Theta}_2 + \frac{1}{2}(\boldsymbol{B}^N_{n-1} + \boldsymbol{\pi}_1)'\boldsymbol{\Sigma}(\boldsymbol{B}^N_{n-1} + \boldsymbol{\pi}_1)\right), \end{aligned}$$

then, stacking these equation in a system I have

$$\begin{bmatrix} rx^N_{t+1,(n-1)} \\ rx^R_{t+1,(n-1)} \end{bmatrix} = \begin{bmatrix} \alpha^N_{n-1} \\ \alpha^R_{n-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}^N_{n-1} \\ \boldsymbol{B}'^R_{n-1} + \boldsymbol{\pi}'_1 \end{bmatrix} \boldsymbol{\Theta}_2\boldsymbol{X_t} + \begin{bmatrix} \boldsymbol{B}^N_{n-1} \\ \boldsymbol{B}'^R_{n-1} + \boldsymbol{\pi}'_1 \end{bmatrix} \boldsymbol{X_{t+1}}$$

To estimate the parameters, the system is stacked across maturities and time periods into the vector $\boldsymbol{R}_{t+1} = \boldsymbol{R}\boldsymbol{X}_{t+1} + \boldsymbol{e}_{t+1}$ of observed returns, assuming $\boldsymbol{e}_{t+1}$ is the vector of return pricing errors (returns measurement errors) which is mean zero, conditionally independent, and serially uncorrelated

$$\boldsymbol{R} = \boldsymbol{\alpha}\boldsymbol{i}'_T + \boldsymbol{B}\boldsymbol{\Theta}_2\boldsymbol{X}_- + \boldsymbol{B}\boldsymbol{X} + \boldsymbol{E}, \tag{2.22}$$

where $\boldsymbol{R}$ is a $N \times T$ matrix of observed returns, $\boldsymbol{\alpha}$ is a $N \times 1$ vector, $\boldsymbol{i}_T$ is a $T \times 1$ vector of ones, $\boldsymbol{B}$ and $\boldsymbol{\Theta}_2$ are $N \times K$ and $K \times K$ matrices, respectively. Finally, $\boldsymbol{X}_- = [\boldsymbol{X}_0, \boldsymbol{X}_1, \cdots, \boldsymbol{X}_{T-1}]$ is a $K \times T$ matrix of lagged pricing factors, and $\boldsymbol{X}$ is a $K \times T$ matrix of contemporaneous pricing factors. $\boldsymbol{E}$ is a $N \times T$ matrix.

Based on equation (2.21), Abrahams et al. (2013) propose the following procedure to estimate the joint pricing model for Treasuries and TIPS, which is based on a set of linear regressions and is a variant of the estimator introduced by Adrian et al. (2013):

First step: Estimate $[\hat{\boldsymbol{\Theta}}_1, \hat{\boldsymbol{\Theta}}_2]$ and the innovation $\hat{\boldsymbol{\nu}}$ from the regression of factors on their lagged values by OLS. Compute $\hat{\boldsymbol{\Sigma}} = T^{-1} \cdot \hat{\boldsymbol{\nu}}\hat{\boldsymbol{\nu}}'$. This provides the estimated dynamic under $\mathbb{P}$ of the pricing factors.

Second step: Following equation (2.21) regress excess returns on a constant, lagged and contemporaneous pricing factors by Seemingly Unrelated regression (SUR) regression. The estimated coefficients from this regression are $[\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{B}}\hat{\boldsymbol{\Theta}}_2, \hat{\boldsymbol{B}}]$ and the innovation $\hat{\boldsymbol{E}}$. Compute $\hat{\boldsymbol{\Sigma}}_E = T^{-1} \cdot \hat{\boldsymbol{E}}\hat{\boldsymbol{E}}'$.

Third step: Estimate $\hat{\boldsymbol{\Theta}}^*_2$, which is the factor parameter loading matrix under $\mathbb{Q}$, from the cross sectional regression of the return exposures to the lagged pricing factors $\hat{\boldsymbol{B}}\hat{\boldsymbol{\Theta}}_2$ onto return exposures to contemporaneous pricing factors $\hat{\boldsymbol{B}}$ by GLS regression

$$\hat{\boldsymbol{\Theta}}_2^* = (\hat{\boldsymbol{B}}'\hat{\boldsymbol{\Sigma}}_E^{-1}\hat{\boldsymbol{B}})^{-1}\hat{\boldsymbol{B}}'\hat{\boldsymbol{\Sigma}}_E^{-1}\hat{\boldsymbol{B}}\hat{\boldsymbol{\Theta}}_2.$$

Fourth step: Estimate $\hat{\boldsymbol{\Theta}}_1^*$ by first running an additional SUR regression to obtained estimates for $[\boldsymbol{\omega}, \boldsymbol{\Omega}]$

$$\boldsymbol{R} = \boldsymbol{\omega}\boldsymbol{i}_T' + \boldsymbol{\Omega}(\hat{\boldsymbol{\Theta}}_2^*\boldsymbol{X}_- + \boldsymbol{X}) + \boldsymbol{\varepsilon}.$$

and then computing the GLS estimator for $\hat{\boldsymbol{\Theta}}_1^*$

$$\hat{\boldsymbol{\Theta}}_1^* = (\hat{\boldsymbol{\Omega}}'\hat{\boldsymbol{\Sigma}}_E^{-1}\hat{\boldsymbol{\Omega}})^{-1}\hat{\boldsymbol{\Omega}}'\hat{\boldsymbol{\Sigma}}_E^{-1}\left(\boldsymbol{\omega} + \frac{1}{2}\boldsymbol{\Omega}'\hat{\boldsymbol{\Sigma}}_E^{-1}\boldsymbol{\Omega}\right).$$

Fifth step: Given that the factor parameters under $\mathbb{Q}$ are related to the market price of risk parameters via a linear relationships, it is possible to compute

$$\hat{\boldsymbol{\lambda}}_0 = \hat{\boldsymbol{\Theta}}_1 - \hat{\boldsymbol{\Theta}}_1^*,$$
$$\hat{\boldsymbol{\lambda}}_1 = \hat{\boldsymbol{\Theta}}_2 - \hat{\boldsymbol{\Theta}}_2^*.$$

On the other hand, the parameters governing the evolution of the short nominal interest rate and the inflation rate as a function of the state variables describe by equations (2.6) and (2.8), respectively, need to be estimated. δ_0 and $\boldsymbol{\delta}_1$ are estimated by the OLS regression of the short rate onto a constant and the vector of pricing factors. The value of π_0, which represents the long-run mean of inflation, is predetermined using different sources of information such as theory, empirical evidence and previous literature (π_0 is fixed at 2.5%).[12] $\boldsymbol{\pi}_1$ is estimated according to equation (2.12) by regressing the demeaned inflation rate ($\pi_t - \pi_0$) onto the vector of state variables.

2.4 Data and factor construction

I first describe the data and then the estimation technique used to compute the pricing factor variables. I also present the empirical evidence supporting the plausibility of TIPS liquidity premium as an unspanned factor for the yield curve.

Data

I obtained daily observations of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007), and Gurkaynak et al. (2010), respectively,

[12] This is because it cannot be precisely pinned down by the sample period considered in this study.

available at the Federal Reserve web site. The sample period is from January 2004 to December 2013, which covers most of history available for TIPS.[13] This dataset is well known, allowing a good comparison with previous work.

Given my purpose of relating the yield curve with an illiquidity variable (which is a related to greater price uncertainty and volatility), I am not interested in monthly data, as is usual in this literature, but rather in daily yield curve data. Following the literature, I am not interested in the very long-end of the yield curve (maturities above 10 years), while, in contrast, I am interested in a richer set of yield curve points for short- and medium-term residual maturities than those presented in Gurkaynak et al. (2007) and Gurkaynak et al. (2010). However, this dataset consists of a fitted function which smooths across maturities. In particular, what Gurkaynak et al. (2007) do is to estimate the Svennson (1994) six-parameter function for instantaneous forward rates

$$f^n = \beta_0 + \beta_1 e^{-n/\tau_1} + \beta_2 (n/\tau_1) e^{-n/\tau_1} + \beta_3 (n/\tau_2) e^{-n/\tau_2}.$$

The parameters $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$ are published along with the estimated zero coupon curve.

I use the appropriate formula and these parameters to compute the implied zero-coupon yields for a set of additional relevant intra-year maturities. I end up with a daily time-series of zero-coupon yields for the 14 maturities considered in Diebold et al. (2006): 12-, 15-, 18-, 21-, 24-, 30-, 36-, 48-, 60-, 72-, 84-, 96-, 108- and 120-months. I use these yield curve data to estimate the yield curve latent factors: level, slope and curvature. However, for the estimation I only use 12-, 24-, 36-,..., 108- and 120-month nominal yields ($N^N = 9$) and 24-, 36-, 48-,..., 108- and 120-month TIPS yields ($N^R = 8$). For this cross-section, I calculate one-month holding period returns.

Additionally, I use the one-month Treasury yield from Gurkaynak et al. (2007) as the nominal risk-free rate. Finally, to measure liquidity, I use the market-based measure proposed by Christensen and Gillan (2011), which is defined as the difference between synthetic and cash break-even inflation rates, as in equation (2.1). To compute the break-even inflation rates, I use the daily estimates of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007) and Gurkaynak et al. (2010), from January 2004 to December 2012. For zero-coupon inflation swap rates, I use U.S. daily quotes from Barclays Live, which I convert into continuously compounded rates to make them comparable to the other interest rates. I compute this measure from January 2004 to December 2013 for n=10-year maturity.

[13] Treasury inflation-protected securities were introduced by the the U.S. government in 1997, however the Gurkaynak et al. (2010) data base does only contains complete information for all maturities starting from January 2004. Consequently, the first eight years were not considered in order to avoid noise in the estimation.

Spanned pricing factors

As is common in the literature, I perform principal components analysis to extract the spanned pricing factors of the model from yields. Panel A of Table 2.1 reports the correlations between the first three principal component factors extracted from U.S. nominal Treasury yields and from TIPS yields, in isolation from each other. A total number equal to $K = K^N + K^R = 3 + 3 = 6$ of spanned model factors are computed. Table 2.1 shows that the pricing factors extracted from Treasuries and TIPS yields are highly correlated, exhibiting a linear correlation of 84%, 77% and 59%, respectively.

Table 2.1: Unconditional correlation between yield factors

		Real factors			
		PC_1	PC_2	PC_3	Liquidty
Nominal factors	PC_1	0,8393	0,1521	-0,3127	-0,0306
	PC_2	0,1825	0,7741	0,3851	0,0111
	PC_3	0,2407	-0,2557	0,5845	0,3788
	Liquidity	0,4080	-0,4094	0,2957	

		Orthogonal real factors		
		PC_1	PC_2	PC_3
Real factors	PC_1	-0,2621	0,0113	-0,0004
	PC_2	0,2967	-0,3256	0,0250
	PC_3	-0,0987	0,0905	-0,5490

Panel A reports the correlations between the first three principal components for U.S. daily Treasury yields and U.S. daily TIPS yields from January 1, 2004 to December 30, 2011. Panel B reports the correlations between the first principal component from the residuals of regressions of break-even inflation rates on nominal principal components and the liquidity factor, and the first three principal components for U.S. daily TIPS yields.

Consequently, I use the same two sets of principal components considered by Abrahams et al. (2013). They propose to extract $K^N = 3$ principal components from nominal Treasury yields. Then, to reduce the unconditional collinearity among the pricing factors, they obtain additional factors as the first $K^{OR} = 3$ principal components from the residuals of regressions of break-even inflation rates on the K^N nominal principal components, as well as the liquidity factor

$$BEI_{n,t} = c + b_1 PC^N_{1,t} + b_2 PC^N_{2,t} + b_3 PC^N_{3,t} + b_4 \Delta_{10,t} + e_t, \tag{2.23}$$

where n=24-, 30-, 36-, 48-, 60-, 72-, 84-, 96-, 108- and 120-months.[14] These factors are called orthogonal real factors.

Table 2.2 shows that more than 98% of the variations in daily changes of 1-, 2-, 3-, ..., and 10-year nominal yields can be explained by the first three

[14] I obtained indistinguishable results from the residuals of regressions of TIPS yields on the K^N nominal principal components as well as the liquidity factor.

principal components. A similar percentage of the variation in TIPS yields, as well as in the residuals of the regressions of break-even inflation rates on nominal principal components and liquidity, can also be explained by the first three principal components. This is line with the empirical observation found by Joslin et al. (2011) that bond yields follow a low-dimensional factor model, which is reflected in the fact that three factors appear to explain nearly all of the cross-sectional variation in yields.

Table 2.2: Variance explained by principal components

	Nominal factors	Real factors	Orthogonal real factors
PC_1	0,94608	0,96027	0,95550
$PC_1 + PC_2$	0,99854	0,99905	0,99567
$PC_1 + PC_2 + PC_3$	0,99993	0,99996	0,99932

Nominal factors correspond to the first three principal components for U.S. daily Treasury yields from January 1, 2004 to December 30, 2011. Real factors correspond to the first three principal components for U.S. daily TIPS yields. Finally, orthogonal real factors correspond to the first principal component from the residuals of regressions of break-even inflation rates on nominal principal components and the liquidity factor, for the same sample period.

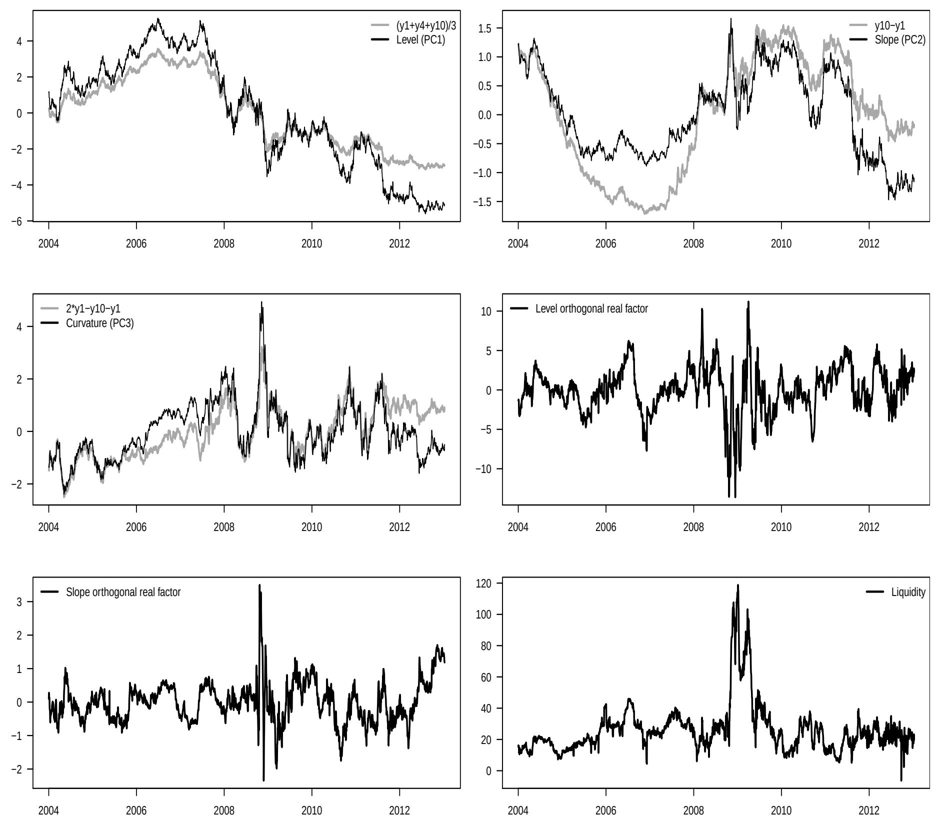
Panel B of Table 2.1 reports the correlations between the first three principal component factors extracted from nominal yields alone, and the first three principal component factors extracted from the residuals of regression of break-even inflation rates on nominal principal components and liquidity (i.e. orthogonal real factors). It is important to note that, the first, the second and the third factors largely retain their interpretations as the level, slope and curvature. This conclusion is based on the fact that they still keep an important correlation with the first, the second and the third real factors, respectively. This is confirmed by Figure A2.1, in Appendix 2.6, which plots the factor loadings of the first three principal components of yields for each set of bonds, and also for the residuals from the regression (**??**). As usual, each line in these graphs represents how yields of various maturities change when a factor moves. Graphs show that the level factor is almost flat, meaning that a level factor shock changes the interest rates of all maturities by almost identical amounts. The slope factor rises monotonically through all maturities, and the curvature factor is curved at the short-end of the yield curve.

Finally, Figure 2.1, plots the level (L_t^N), the slope (S_t^N) and the curvature (C_t^N) nominal factor, along with the orthogonal real factors (level (L_t^{OR}) and slope (S_t^{OR}) which correspond to the first two principal components of the residuals from equation (2.23)), and the liquidity premium factor (Δ_t). Factors are constructed using principal components analysis after the data series are demeaned, and divided by their respective standard deviation to make them comparable units[15] (summary

[15]Notice that the standard deviation of the principal components is not set to one.

statistics are available in Appendix 2.6, Table A2.1). Nominal factors are plotted together with their empirical proxies: the average of short-, medium- and long-term yields for the level factor, the difference between long- and short-term yields for the slope factor, and the difference between twice medium-term yields with respect to the sum of short- and long-term yields for the curvature factor. In all cases, the principal component factor and their standard empirical proxy are closely linked. Additionally, the level and slope factors display very high persistence, while the curvature is less persistent.

Figure 2.1: Nominal and orthogonal real yield factors and liquidity premium



Level, slope and curvature correspond to the three principal components from nominal Treasury yields of maturities for $n =$ 6-month, 1-, 2-, ..., 10- and 20-years. Orthogonal real factors correspond to the first two principal component from the residuals of regressions of break-even inflation rates on nominal principal components and the liquidity factor. Liquidity factor corresponds to the TIPS liquidity premium measure proposed by Christensen and Gillan (2011). Sample spans from January 1, 2004 to December 30, 2011.

2.5 Empirical results

2.5.1 Testing the empirical plausibility of the TIPS liquidity premium as an unspanned factor

As I already mentioned in Section 2.2.2, the plausibility of the TIPS liquidity premium as an unspanned factor is motivated by three empirical facts: *i*) the TIPS relative liquidity premium is not linearly spanned by the information in the joint yield curve. *ii*) the unspanned liquidity factor has a predictive power for excess returns in bond markets; and *iii*) bond yields follow a low-dimensional factor model.

To empirically test the first fact, I consider the projection of liquidity onto the principal components of yields on U.S. Treasury nominal and TIPS zero-coupon bonds, with maturities of 12- through 120-months. Results presented in Table 2.3, suggest that nominal and TIPS yields contain a factor that is not spanned by the traditional yield curve factors. In fact, the projection of liquidity onto the first three principal components gives an adjusted R^2 of 14%, thus approximately 86% of the variation in liquidity arises from risks distinct from the traditional nominal yield factors. Similarly, the adjusted R^2 in the case of the real yield factors is about 42%, which is much more higher than in the case of nominal factors. However, 58% of the variation in TIPS liquidity still arises from risks distinct from the real yield factors.

Table 2.3: TIPS liquidity unspanned factor

	Coefficient	t-stat	AdjR^2
A. Nominal factors			
PC_1^N	-0,161	-0,250	0,145
PC_2^N	0,246	0,101	
PC_3^N	57,869	2,169	
B. Real factors			
PC_1^R	2,237	3,746	0,421
PC_2^R	-11,856	-4,316	
PC_3^R	54,981	3,585	

Panel A regresses TIPS liquidity on the first three principal components for U.S. daily Treasury yields from January 1, 2004 to December 30, 2011. Panel B regresses TIPS liquidity on the first three principal components for U.S. daily TIPS yields using the same sample period. TIPS liquidity corresponds to the TIPS liquidity premium measure proposed by Christensen and Gillan (2011).

Nevertheless, results in Table 2.4 suggest that there exists a factor that is important for explaining the variations in TIPS yields, and also for modeling nominal interest rates. Following D'Amico et al. (2010), I regress the 10-year break-even inflation rate on the first principal components of yields

$$BEI_{10,t} = \alpha + \beta_1 PC^i_{1,t} + \beta_2 PC^i_{2,t} + \beta_3 PC^i_{3,t} + \beta_4 \Delta_{10,t} + e_t,$$

where $i=$ Nominal (N) or TIPS yields (R). Results show that 31% of the variation in the break-even inflation rate is explained by the first three principal components of nominal yields. Once I include liquidity in this regression, the adjusted R^2 is about 77%. Similarly, this occurs when I consider the first three principal components of TIPS yields. In this case, the adjusted R^2 is about 45%, and rises up to 73% when the liquidity factor is included. In the regression of the 10-year break-even inflation rate on the liquidity factor the adjusted R^2 is about 62%.

Table 2.4: Regression of the break-even inflation rates onto yield and liquidity factors

A. Individual factors

	Const	PC_1	PC_2	PC_3	Liquidity	AdjR^2
Nominal factors						
Coef	2,34	0,05	-0,02	-1,45		0,31
t-stat	33,88	3,06	-0,36	-2,31		
Real factors						
Coef	2,34	-0,02	0,32	-2,11		0,45
t-stat	35,42	-0,96	4,00	-4,18		
Liquidity factor						
Coef	2,84				-0,02	0,62
t-stat	49,56				-7,76	

B. Combined factors

	Const	PC_1	PC_2	PC_3	Liquidity	AdjR^2
Nominal factors + Liquidity						
Coef	2,81	0,05	-0,02	-0,42	-0,02	0,77
t-stat	58,66	5,37	-0,65	-1,71	-10,13	
Real factors + Liquidity						
Coef	2,79	0,02	0,12	-1,17	-0,02	0,73
t-stat	70,57	1,88	2,60	-3,51	-11,88	

Panel A regresses 10-year break-even inflation rate on TIPS liquidity and the first three principal components for U.S. daily Treasury yields from January 1, 2004 to December 30, 2011. Panel B regresses 10-year break-even inflation rate on TIPS liquidity on the first three principal components for U.S. daily TIPS yields using the same sample period. TIPS liquidity corresponds to the TIPS liquidity premium measure proposed by Christensen and Gillan (2011).

With respect to the second fact, the unspanned TIPS liquidity factor forecasts bond excess returns if liquidity significantly improves the yields-only forecast. To examine this, I explore whether or not the liquidity premium has considerable predictive power for excess returns, over and above PC_t^i where $i=$ Nominal (N) or TIPS yields (R). Table 2.5 shows the AdjR^2 values for individual bond excess returns considering the following standard predictive regression framework

$$rx_{t+1}^{(n)} = \alpha + \beta_1 PC_{1,t}^i + \beta_2 PC_{2,t}^i + \beta_3 PC_{3,t}^i + \beta_3 \Delta_{10,t} + \epsilon_{t+1}^{(n)},$$

where $rx_{t+1}^{(n)}$ denotes annual excess log returns on n=2-, 3-, 4-, 5-year maturity, calculated as $rx_{t+1}^{(n)} = r_{t+1}^{(n)} - y_t^1$, where $r_{t+1}^{(n)}$ is the holding one-year log-return on a zero-coupon n-period bond, and y_t^1 is the one year log-yield.

Table 2.5: AdjR^2 values

	Nominal excess returns			TIPS excess returns		
n	PC_t^N	$PC_t^N + \Delta_t$	Δ_t	PC_t^R	$PC_t^R + \Delta_t$	Δ_t
2	32.23	32.34	1.21	35.78	44.81	10.70
3	19.26	19.37	0.89	30.56	31.94	7.34
4	11.05	11.98	0.56	12.61	13.46	7.01
5	8.33	8.47	0.47	8.43	8.57	4.67

Panel A regresses 10-year break-even inflation rate on TIPS liquidity and the first three principal components for U.S. daily Treasury yields from January 1, 2004 to December 30, 2011. Panel B regresses 10-year break-even inflation rate on TIPS liquidity on the first three principal components for U.S. daily TIPS yields using the same sample period. TIPS liquidity corresponds to the TIPS liquidity premium measure proposed by Christensen and Gillan (2011).

Table 2.5 shows the adjusted R^2 of regression forecasts with a combined set of yields and liquidity factors. The first column represents the adjusted R^2 of regressions, which includes yield factors such as instruments, while the second column includes yields and liquidity. Comparing the adjusted R^2 of regressions with the yields-only factors, leads to the conclusion that the liquidity variable contains information that is unspanned by yields.

This evidence is in line with results found by Gomez (2013). Using a similar predictive regression framework, she documents the predictive power that the liquidity premium of different maturities has for nominal and inflation-linked excess returns in the U.S. bond market. Controlling for typical excess returns predictors such as the term structure slope and the recently proposed tent-shaped factor of Cochrane and Piazzesi (2005), she finds that the 10-year liquidity premium is significant and an economically relevant source of predictability for government excess returns. Statistically, the liquidity differential between Treasury and inflation-linked bond yields (measured using the Christensen and Gillan (2011) liquidity premium), increases the Adj-R^2 and also the R^2_{OS} for Nominal and TIPS one-year excess returns of different maturities, as shown in Table 2.6. For instance, Adj-R^2 for nominal excess returns ranges between 2% - 20% considering traditional factors, however once the liquidity factor is included the Adj-R^2 values rise, ranging between 27% - 33%.

In summary, results from the regressions presented earlier confirm that the TIPS liquidity premium (which represents the liquidity differential between U.S. Treasury and TIPS bonds) is to some extent unspanned by the nominal and real yield curves, and forecast bond excess returns along with yield curve information. As a result, I find empirical evidence to suggest that the TIPS liquidity is not spanned by the yield

Table 2.6: Bond excess returns predictability: Adj-R^2 and R^2_{OS} values

	Traditional factors		Including liquidity	
	In-sample	Out-of-sample	In-sample	Out-of-sample
Nominal	2% - 20%	-0.1% 0.02%	27% - 33%	-0.3% - 0.06%
TIPS	0.1% - 3%	-3% - 1.0%	16% - 36%	-1.4% - 0.2%

Results extracted from Table C1.1. AdjR^2 is the goodness-of-fit measure for the in-sample predictive regression model of one-year holding excess returns onto term structure slope, tent-shaped factor and liquidity premium. The R^2_{OS} is the Campbell and Thompson (2008) coefficient for out-of-sample exercise. The sample spans the period from 01/11/2006 to 30/12/2011.

curve, but it is important for enhancing bond return predictability. Consequently, liquidity premium could be included as an additional unspanned forecasting variable, not only in forecasting regressions, but also in term structure models.

2.5.2 TIPS liquidity premia, macro variables and U.S. bond returns

Macro-finance models have linked the term structure of yields to macroeconomic variables, in addition to common latent yield curve factors in a term structure model setting. Additionally, the existing empirical literature shows that some of the macroeconomic information that forecasts bond returns is embedded in the yield curve, and is thus also captured by yield curve factors. However, a significant part of the macroeconomic information shows that the forecasts bond risk premium is unspanned by the yield curve. This means that macroeconomic variables have further predictive power in addition to the yield curve factors.

In last section, I provide empirical evidence supporting the TIPS liquidity premium as an unspanned factor that helps to forecast bond risk premium, and that is not linearly spanned by the information in the joint yield curve. In this section, I am interested in analyzing the potential bi-directional feedback from the liquidity to the macro yield factors, and back again. Indeed, I explore the inter-temporal associations between the TIPS liquidity premium, macroeconomic variables (that have traditionally been considered as macro factors that affect bond prices, and the dynamic of the yield curve), and return bond variables. This analysis allows me to determine what relationships hold between the TIPS liquidity premium, macro variables, and bond returns. In particular, I address the following questions: (i) do macroeconomic variables and bond returns impact liquidity premium?, and (ii) does liquidity premium provide information about future macroeconomic variables?.

To answer these questions, I closely follow Goyenko et al. (2011). They study the connection between economic environment, prices and liquidity in the U.S. nominal bond market, finding that liquidity conditions are significantly affected by the economic variables. This relationship remains unstudied for the TIPS bond market. Thus, it would be useful to extend this analysis considering the nominal Treasury and TIPS bonds, in order to understand the dynamic relationship of the liquidity differential between nominal and TIPS bonds, and the economic environment.

Macro and bond variables

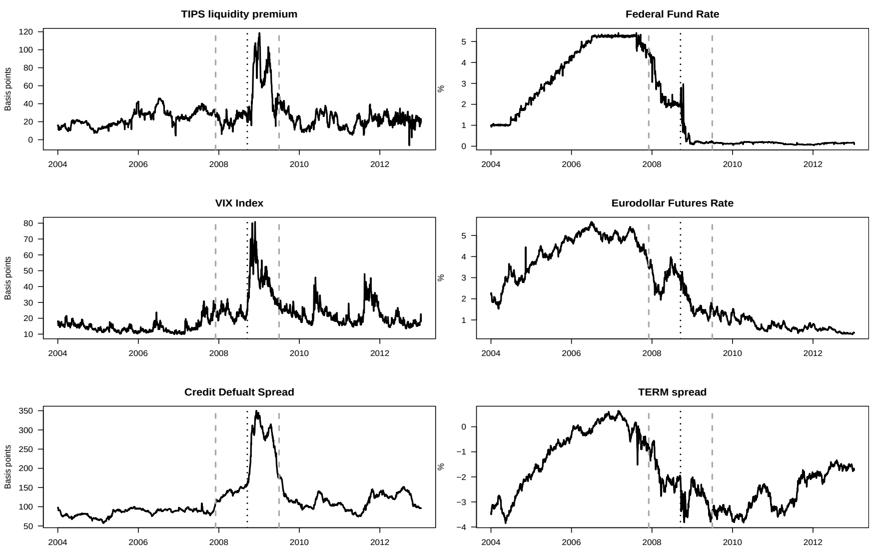
Empirical work on macro-finance models of term structures preferably include very similar choices of macroeconomic variables. In fact, inflation variables (such as consumer price index, personal consumption expenditures price index, blue chip inflation survey, spot market price index for all commodities, etc), and real activity variables (such as industrial production, the Federal Fund Rate, unemployment, output gap, CFNAI Index, among other variables) have been included in the traditional set of fundamentals that capture macro information affecting bond prices and the dynamic of the yield curve. However, given that I am not interested in monthly data (as is usual in this literature), but rather in daily yield curve data, I only consider variables available on a daily frequency.[16]

First, I capture the effect of the monetary policy by the Federal Funds Rate (FFR_t). Bernanke and Blinder (1992) and Laurent (1989) provide evidence that the Federal Funds Rate is a good indicator of monetary policy actions. The hypothesis is that if the Federal Reserve pursues an expansionary monetary policy (dropping the rate), the increase in funds could cause higher order inflows into government bonds, and potential changes in their liquidity conditions. In other words, decreases in the FFR (a looser monetary policy), could increase liquidity because of the decrement in the financing cost. I obtain daily data from the Federal Reserve Bank of St. Louis.

Rudebusch et al. (2006) use the four-quarter-ahead Eurodollar futures rate ($ED4_t$) as the macro factor. They argue that this variable captures financial market expectations for the future path of policy over the next 12 months. In fact, the rate on the Eurodollar futures with four quarters to expiration captures aspects of the stance of the monetary policy, that may not be adequately represented by the current and past Federal Funds Rate. Furthermore, this variable helps to capture changes in the expected future path of policy that may not be adequately accounted for by the lags of a VAR model. Central banks use forward guidance to affect the long-term interest rates and stimulate the economy. Forward guidance can reduce or increase the uncertainty around the expected policy rate with an explicit numerical instruction. An unexpectedly rise (fall) in the $ED4_t$ rate indicates an

[16]The selection of macroeconomic time series obviously comes with a selection bias that possibly undetermines the results of the analysis. The main selection criteria applied here obey practical reasons. The macroeconomic time series are chosen based on their availability. Data must be available on a daily frequency and cover the same period of time January 1, 2004 until December 31, 2013.

Figure 2.2: Macroeconomic variables



U.S. daily data from January 2, 2004 to December 31, 2012. Vertical dashed lines highlight the last NBER recession and the dotted line indicates the Lehman Brothers bankruptcy announcement.

increase (decrease) in the uncertainty about the future monetary policy (tighter monetary policy), and (worsen) financial market conditions. Thus, I would expect a positive relationship between liquidity premium and the uncertainty surrounding the future path of the policy. For instance, greater uncertainty about the near-term path of monetary policy would result in a higher liquidity risk, increasing liquidity premiums and deteriorating market liquidity. I obtain daily prices from Chicago Mercantile exchange (CME).

As a business cycle proxy, I use the nominal term yield spread, which has been also associated with the slope of the yield curve. The term spread between the yield on long-term bonds and short-term bonds has been widely used as a proxy for business conditions.[17] I construct the Term spread ($TERM_t$) as the difference between market yield on U.S. Treasury securities at 10-year constant maturity and three months T-bill, using daily data from the Federal Reserve Board statistical releases. Fama (1990) argues that term premiums tend to increase with maturity during good times, but during recessions long-term yields exceed those of short-term, producing a sharp decline in the slope of the yield curve and frequently by an inversion of the yield curve. Thus, I would expect a negative relationship between the term spread and liquidity premium. This is because volume injected into the

[17]The term spread is shown to decrease near peaks of economic activity and increase near economic troughs.

bond market may be reduced during a recession, when an investor will demand a greater return motivated by the illiquidity condition of the market.

I also consider the default credit spread (CDS_t) as a business cycle proxy. Chen (1991) argue that the default spread measured as the spread of lower- to higher-grade bonds is a proxy for business conditions. They argue that when business conditions are poor, spreads are likely to be high, and when business conditions are strong, spreads are likely to be low. Studies by Fama and French (1989), Fama (1990) and Jensen et al. (1996), find that the default spread also captures variations in expected returns in response to business conditions. Fama and French (1989), Fama (1990) and Schwert (1990) show that the required returns demanded by investors vary over the business cycle. Thus, I would expect a positive relationship between the default spread and TIPS liquidity premium, because of the same reason outlined before for the Term spread (however in this case it operates in the inverse sense). I measure default spread as the difference in yield between Baa and Aaa-rated corporate bonds with long-term maturity. I obtained daily data from the Federal Reserve Bank of St. Louis.[18]

In addition, I use the Market Volatility Index (VIX_t) from the Chicago Board of Options Exchange as a proxy of changes in aggregate economic uncertainty. This is a widely used measure of ex-ante volatility for aggregate stock markets, however Bloom (2009) shows that the VIX index is highly correlated with cross-sectional measures of dispersion based on firm-level profit growth and stock returns, industry-level total factor productivity (TFP) growth, and GDP forecasts, which suggests that the aggregate stock market volatility is a reasonable proxy for overall economic uncertainty. I would expect than more uncertainty increases the liquidity premium in the inflation-linked bond market.

To explore the connection between liquidity and bond variables, I consider bond returns as bond market variables. I calculate returns from quoted prices for nominal Treasury and TIPS bonds considering two maturities range: short- and long-term. In particular, I use 5-year bonds (RN_t^5 and RR_t^5) to capture price variation in short maturity and 10-years (RN_t^{10}, RR_t^{10}) for long-term maturity. Theoretically, investors would require lower returns on assets with relatively high liquidity because they face lower cost of capital. Thus, I would expect a negative relationship between TIPS bond returns and the liquidity premium.

Impulse response functions

I run a daily VAR specification considering the maximum range liquidity premium Δ_t as a measure for the TIPS liquidity premium. Additionally, I include nominal and

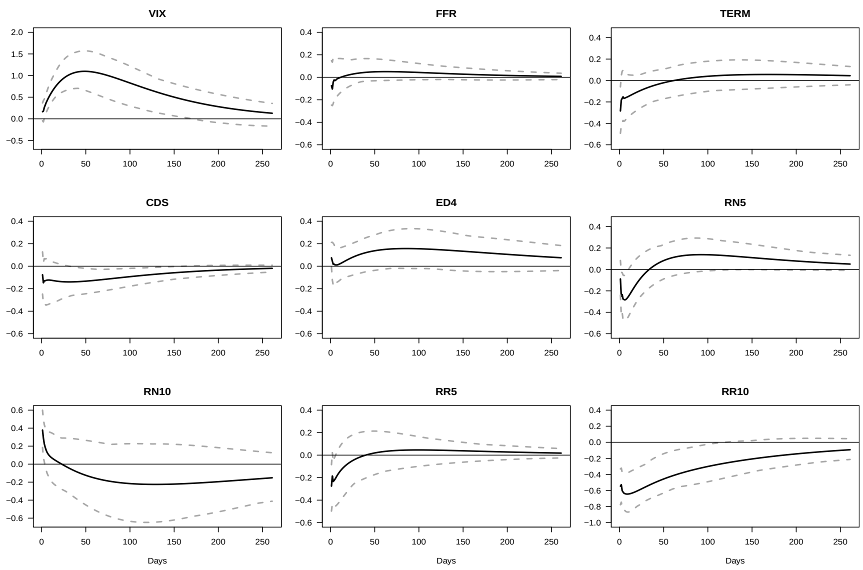
[18] This data correspond to Moody's seasoned Aaa corporate bond yield. Moody's tries to include bonds with remaining maturities as close as possible to 30 years. They drop bonds if the remaining life falls below 20 years, if the bond is susceptible to redemption, or if the rating changes.

TIPS bond market variables: for short (RN_t^5 and RR_t^5) and long (RN_t^{10} and RR_t^{10}) maturities. Finally, I include macroeconomic variables that comprise the Federal Fund Rate (FFR_t), the four-quarter-ahead Eurodollar futures rate ($ED4_t$), the Market Volatility Index (VIX_t) and the credit default spread (CDS_t). The number of lags in the VAR is chosen using a parsimony strategy based on the following informational criteria: Hannan-Quinn (HQ), Aikaike (AIC), Schwarz (SC) and Final Prediction Error (FPE). This means, I chose the minimum lag suggested by the four criteria, which is 1 lag. I order the variables accordingly to the conventional practice in macro-finance literature, and follow the order used in Goyenko et al. (2011). In the beginning, I place macro-variables: the VIX_t, FFR_t and $ED4_t$ followed by the business cycle proxy $TERM_t$ and CDS_t. Next, I place bond market variables as follows: RN_t^5, RR_t^5, RN_t^{10} and RR_t^{10}, and finally I include the TIPS bond liquidity variable Δ_t.

Figure 2.3 illustrates the response of the TIPS liquidity premium to a unit standard deviation change in a particular variable, traced forward over a period of 260 days (one year). The graphs depict the effect of a one-time shock in a particular variable on the current and future value of the liquidity premium. Dashed lines represent bootstrap 95% confidence bands derived via 1.000 bootstrap simulations. The first graph (Figure 2.3) presents the impulse-response function for the 10-year TIPS liquidity premium to a shock to the VIX index. The IRF indicates that the TIPS liquidity premium increases in response to a one standard deviation shock to the VIX index, being a statistically significant effect. A shock to the $TERM$ appears to have a significant impact on liquidity premium, having a negative initial effect on liquidity premium. This is consistent with the fact that a positive shock to the term spread (the slope of the yield curve upward), generates an increase in the volume injected into the bond market, which in turn improves the liquidity conditions on the market. As a result, investors demand a lower premium to hold TIPS. A positive shock to $ED4$ rate predicts an increase in the TIPS liquidity risk premium. This is also related to the fact that investors will demand higher risk premiums when uncertainty around future policy rates is higher. Positive shocks to the FFR and CDS appears not to have a statistically significant impact on the TIPS liquidity premium.

Innovations on 5- and 10-year TIPS bond returns seems to have a significant impact on liquidity premia. They have a negative initial effect, being a result consistent with the fact that an up-market move has positive effects on liquidity conditions (reducing the liquidity premium). This result was also found by Goyenko et al. (2011) in the case of nominal Treasuries, and by Chordia et al. (2003) for the stock market. A positive shock to 5-year nominal Treasury bond return has a significant negative impact on liquidity, however it has a positive shock to the 10-year nominal bond return predicts, and in contrast, a positive and a significant impact on the TIPS liquidity premium. This result indicates that an up-market move on long-term nominal bonds has adverse contemporaneous effects on liquidity

Figure 2.3: Response of the TIPS liquidity premium to macroeconomic and bond variables

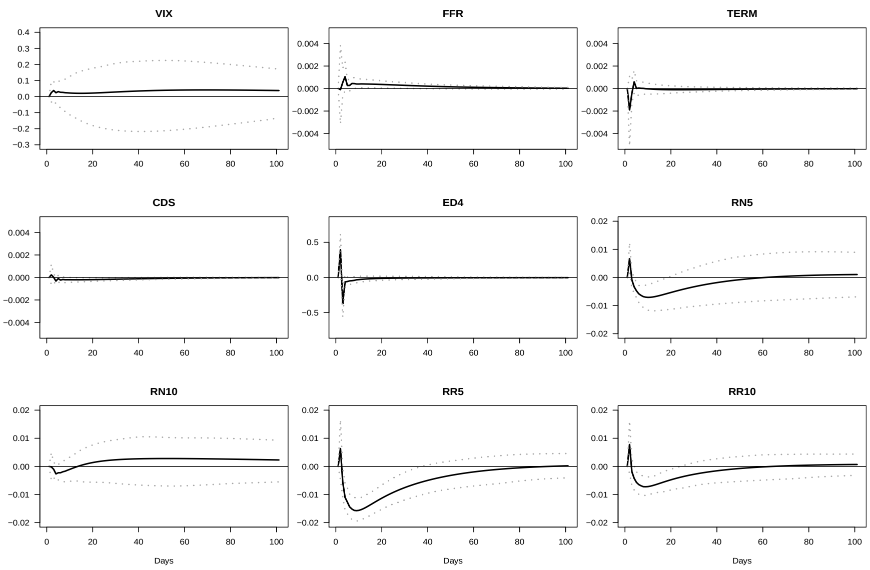


This figure plots impulse response functions for TIPS liquidity premium (the response variable) to a unit standard deviation change in a particular variable, traced forward over a period of one year. Response to Cholesky one standard deviation. Dashed lines represent bootstrap 95% confidence bands derived via 1.000 bootstrap simulations. U.S. daily data from January 2, 2004 to December 31, 2012.

conditions in the TIPS bond markets. Nevertheless, lagged effects are negative.

On the other hand, Figure 2.4 presents IRFs of the macro and bond returns variables in the VAR system. In this case, the impulse-response functions represent the answer of particular variable to a unit standard deviation change in the liquidity premium. I find that the current effect of one standard deviation shock to the TIPS liquidity premium is positive across bond returns maturities, for both nominal and TIPS bonds (except for 10-year nominal bond returns). This result indicates that investors could demand higher returns when the liquidity conditions in the TIPS bond markets worsen. The FFR, the term spread and the default spread appears not to have an statistically significant answer to a positive shock to the TIPS liquidity. The lagged effects of the liquidity shock vanish after a few days of the initial shock. However, they are economically meaningful. Indeed, one standard deviation change in the TIPS liquidity premium is contemporaneously associated with an expansionary monetary policy (a decrease in FFR), downturn in economic conditions (a decrease in $TERM$, and increase in CDS), and higher aggregate economic uncertainty (an increase in VIX index). In contrast, a shock to the TIPS liquidity only is statistically and economically significant for Eurodollar rates. Thus, a four-quarter Eurodollar rate seems to increase in response to a positive shock in

Figure 2.4: Response of the macroeconomic and bond variables to TIPS liquidity premium



This figure plots impulse response functions for Macro and Bond variables (the response variables) to a unit standard deviation change in TIPS liquidity premium, traced forward over a period of 100 days. Response to Cholesky one standard deviation. Dashed lines represent bootstrap 95% confidence bands derived via 1.000 bootstrap simulations. U.S. daily data from January 2, 2004 to December 31, 2012.

the TIPS liquidity risk premium.

Overall, I conclude that the dynamic of the TIPS liquidity premium appears to be driven by a relatively wide set of economic variables. In particular, the TIPS liquidity premium increases in response to aggregate economic uncertainty shocks (represented by VIX index), as well as to expected future monetary tightening conditions (associated with a positive shock to $ED4$). Additionally, the TIPS liquidity premium decreases in response to increments in bond returns. Nevertheless, the liquidity premium is a statistically significant determinant only for bond returns.

2.5.3 Does the variation in liquidity influence the shape of the yield curve?

Empirical studies have traditionally tried to directly model the relationships between bond yields and macro variables by using vector autoregressive (VAR) models. Even though the VAR approach has some limitations, it is very flexible, and the implied impulse response functions and variance decompositions give insights into the empirical relationships between factor-shocks and movements in the yield curve (Ang and Piazzesi (2003)). In this section, I examine the empirical relationship between movements in the level, slope and curvature of the term structure of U.S. nominal and real interest rates, and the TIPS liquidity premium shocks. I infer the relationship between yield movements and shocks in liquidity premium using impulse responses (IRFs) implied from a VAR model. I consider two groups of impulse responses in turn: yield curve responses to liquidity shocks; and liquidity responses to yield curve shocks.

The VAR model is estimated with the principal components formed from the set of nominal and TIPS yields described in Section 4. I order the term structure factors prior to the TIPS liquidity premium variable, as follows: L_t^N, S_t^N, C_t^N, L_t^{OR}, S_t^{OR} and Δ_t.[19] The number of lags in each VAR is chosen using the same set of informational criteria used before, being the minimum lag suggested by the four criteria equal to 2.

Figure 2.5 illustrates the response of a particular variable to a unit standard deviation change in the TIPS liquidity premium, traced forward over a period of 200 days. In other words, the graphs depict the effect of one-time shock in liquidity on the current and future value of the particular yield factor. Dashed lines represent bootstrap 95% confidence bands derived via 1.000 bootstrap simulations.

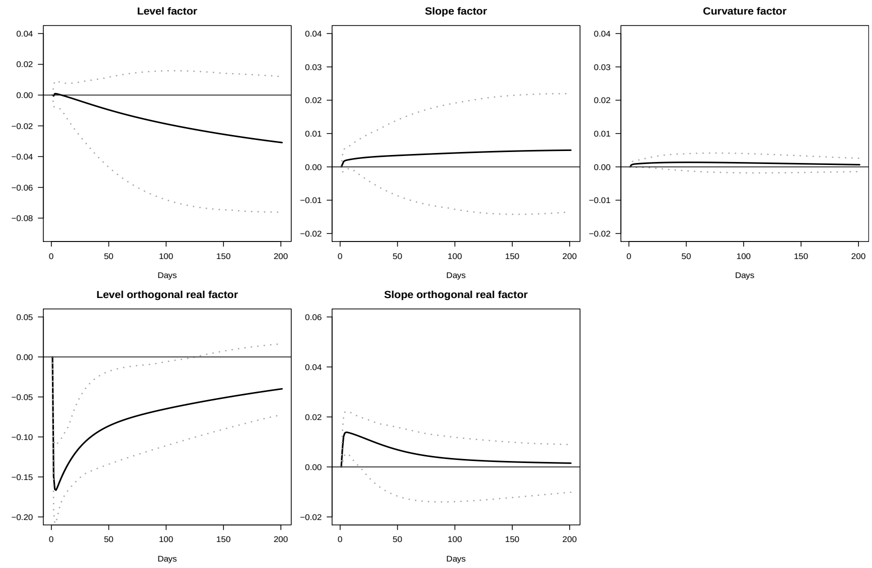
The first graph (Figure 2.5), presents the impulse-response function for level nominal factor. The result indicates that the level factor first increases in response to a one-standard deviation shock to liquidity, but then starts to decrease a few days after the initial shock, becoming negative after that. This result indicates that an increase in the TIPS liquidity premium lowers the nominal interest rates of all maturities. The second graph shows that the effect of a one-standard deviation shock to the TIPS liquidity is positive for the slope factor, meaning that it makes the yield curve steeper. Thus, when liquidity conditions worsen in the TIPS market relative to the nominal market, nominal long-term interest rates change by much larger amounts than short-term rates. The effect persists for at least one year, being the cumulative slope impact approximately equal to 1.06% in the first year. The curvature factor also increases in response to a liquidity shock, as Graph 3 shows, which indicates that the yield curve becomes more curved at the short end. The effect is persistent, however a shock to liquidity appears not to have a significant impact on any of the nominal factors.

With respect to real factors, Graph 4 revels that a shock to the TIPS liquidity premium predicts an important negative current impact for the orthogonal real level factor, with this impact gradually increasing toward zero after the initial shock. The contemporaneous effect is about -0.15%. This means that a one-standard deviation shock to liquidity decreases the orthogonal real factor by 0.15 percentage. In other words, if the liquidity premium rises by 16.55 basis points, the general level of real interest rates would lower by 0.15%. Finally, Graph 5, illustrates the response of

[19] I also examine the robustness of the results to alternative identification strategies. For instance, I obtain similar results ordering the variables as Δ_t, L_t^N, S_t^N, C_t^N and L_t^{OR}, S_t^{OR}.

the orthogonal real slope factor to a unit standard deviation change in the liquidity premium. The slope real factor rises in response to a liquidity shock, with response decaying slowly.

Figure 2.5: Impulse response function of nominal and orthogonal real factors to liquidity shock



Level, slope and curvature correspond to the three principal components from nominal Treasury yields of maturities for $n =$ 6-month, 1-, 2-, ..., 10- and 20-years. Orthogonal real factors correspond to the first two principal component from the residuals of regressions of break-even inflation rates on nominal principal components and the liquidity factor. Liquidity factor corresponds to the TIPS liquidity premium measure proposed by Christensen and Gillan (2011). Sample spans from January 1, 2004 to December 30, 2011.

Figure 2.6 provides the response of liquidity to perceived changes in the nominal and real yield traditional factors. In this figure, the responses give the basis points change in the liquidity premium to a one-standard deviation shock to yield factors.[20] The first graph displays the impulse response to a level shock. The level shock has a initial negative impact on the relative liquidity of TIPS with respect to nominal bonds, being the immediate impact about -20 basis points (-0.2) decrease. The liquidity response turns positive after about four months.

Nonetheless, while the estimated impulse responses of liquidity to a level shock are mostly insignificant, they are economically meaningful. In fact, the level factor (or general level of interest rates), has been associated with the bond market's perception of the long-run inflation rate by several papers that have explored

[20]The standard deviation are equal to 3.07 for L_t^N, 0.736 for S_t^N, 0.105 for C_t^N, 2.92 for L_t^{OR}, 0.61 for S_t^{OR}, and 16.15 basis points for Δ_t. All variables have a zero mean by construction, except liquidity which has a 22.26 basis points mean.

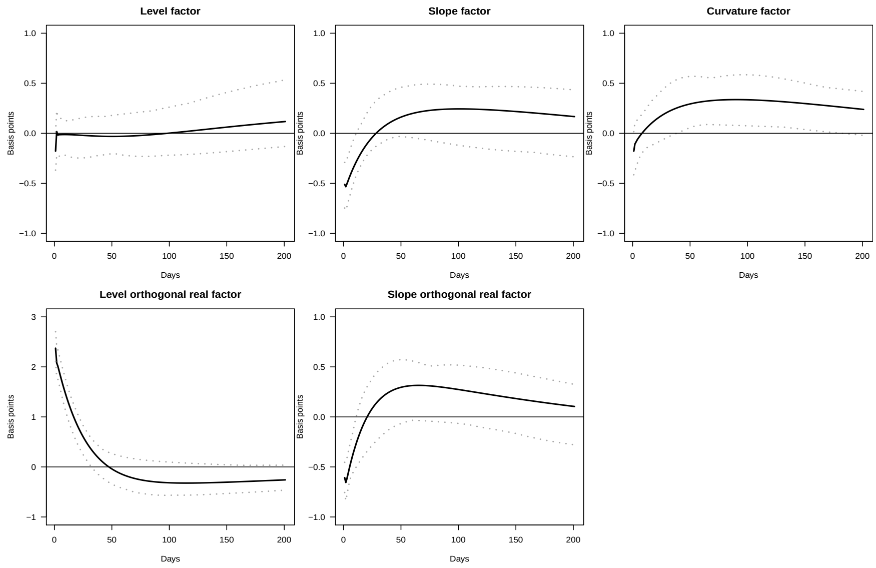
macroeconomic influences on the yield curve (see Dewachter and Lyrio (2006), Diebold et al. (2006), Rudebusch and Wu (2007), among others). Under this interpretation, an increase in the level factor (i.e. an increase in future perceived inflation), generates an expectation of higher inflation risk, which lowers the (ex ante) real interest rate. This may increase the demand of TIPS, which increases the price of those bonds while simultaneously causing yields to decrease. Thus the yield gap between TIPS and nominal Treasury bonds becomes wider, reflecting the persistent inflation concerns of the market, and also the potential changes in the liquidity conditions. Additionally, an increase in the level factor raises the FFR, which is related to a tightening in the monetary policy. However, during the sample period considered in this paper, the Federal Reserve has accommodated only a small portion of the expected rise in inflation. In contrast, since 2008 the federal funds target has been as low as it can be, fixed by the Fed at zero lower bound.

Furthermore, even before the financial downturn began in 2007, real interest rates had fallen sharply, especially over the past six years. One direct consequence of low real interest rates is that bond returns (and in general, asset returns) are expected to be highly volatile. In fact, when the real interest rate is unusually low, then the asset prices will become sensitive to information about dividends or risk premia in the distant future (Kocherlakota (2014)). This would produce an increment in the TIPS liquidity risk premium.

A shock to slope factor has a negative initial impact on liquidity premium, starting to increase, and becoming positive, approximately 30 days after the initial shock. In fact, a one-standard deviation shock to the slope factor results in an initial decrease in liquidity of about -0.5166% (which is equal to 51.66 basis points). Similarly, the TIPS liquidity premium responds negatively to an increase in the curvature factor. In this case, the TIPS liquidity premium decreases initially by approximately 18.71 basis points. After that, the liquidity premium rapidly increases, becoming positive after a few days, and reaching its maximum level two months after the initial shock.

The slope factor (or tilt of the yield curve), has been related to monetary policy actions, and in particular to future interest rate movements. Diebold et al. (2006) show that there is a close connection between the slope factor and the instrument of monetary policy (which is the FFR). The hypothesis is that if the Federal Reserve pursues an expansionary monetary policy (dropping the rate), the increase in funds could cause higher order inflows into nominal government bonds, and potential changes in their liquidity conditions. In other words, decreases in the FFR (a looser monetary policy) would increase liquidity because of the reduction in the financing cost. It is natural to think that the liquidity risk for TIPS is correlated with the small liquidity risk that exists for nominal Treasury notes. It is also is widely accepted that if there is a small liquidity risk associated with holding nominal Treasury bonds, there is an even larger liquidity risk associated with holding TIPS. Consequently, decreases in the FFR would increase liquidity in both markets, which means a reduction of the liquidity premium demanded by investors to hold TIPS, given the decrease in liquidity risk.

Figure 2.6: Impulse response function of liquidity to yields factor shocks



Level, slope and curvature correspond to the three principal components from nominal Treasury yields of maturities for $n =$ 6-month, 1-, 2-, ..., 10- and 20-years. Orthogonal real factors correspond to the first two principal component from the residuals of regressions of break-even inflation rates on nominal principal components and the liquidity factor. Liquidity factor corresponds to the TIPS liquidity premium measure proposed by Christensen and Gillan (2011). Sample spans from January 1, 2004 to December 30, 2011.

The sample period under analysis includes the last financial crisis, and the Federal Reserve's unprecedented response to the crisis. In the first half of 2004, the Federal Open Market Committee (FOMC) was particularly attentive to the possibility that economic growth would accelerate unexpectedly, leading to inflationary pressures. Despite judging that inflationary pressures would be temporary, a tightening in the monetary policy seemed appropriate, so they increased the target. Then, beginning in September 2007, in a series of 10 moves, the federal funds target was reduced from 5.25% to a range of 0% to 0.25% on December 16, 2008, as a response to the unusually severe crisis. Before 2008, short-term interest rates had never reached the zero lower bound. However, rates remained there for several years after that. With the federal funds target at the zero lower bound, the Federal Reserve attempted to provide stimulus through unconventional policies, such as quantitative easing (QE). This is a program where the government buys a large quantity of illiquid assets in order to affect their prices and yields. Even so, the large-scale asset purchase (LSAP) program appeared to improve market liquidity in general. Christensen and Gillan (2012) show that the second round of the LSAP program helped to improve the TIPS market functioning on purchase dates, and throughout the program, by reducing the liquidity premiums that investors would have demanded if the purchases had not been conducted. The observed events over the sample period suggest that under an overall uncertainty in the market, the TIPS liquidity premium has not responded to conventional monetary policy actions, such as lowered the federal funds rate, but instead it has decreased in response to unconventional policies.

Finally, the effect of a one-standard deviation shock to the orthogonal real level factor forecast has a big positive initial impact on the liquidity premium, starting to decrease and becoming negative (essentially in a permanent way), after 50 days of the initial shock. In particular, following an increase of one standard deviation in the real factor, the liquidity differential between Treasuries and TIPS yields initially increases by approximately 238 basis points. After that, the effect starts to decrease, being mostly significant within the first two months. Finally, the TIPS liquidity premium responds in a similar way to a one-standard deviation shock to real slope factor. The initial effect is negative, starting to rapidly increase, and becoming positive approximately 20 days after the initial shock.

Results in this section show that the TIPS liquidity premium influences the shape of the joint nominal and real yield curve. It has an economically significant impact on nominal yield factors, and also a statistically significant effect on real factors. On the other hand, shocks to nominal and real bond yield factors appear to have an effect on the liquidity premium. Additionally, this effect is meaningful given that (as previous empirical evidence has shown) the yield curve factors are highly correlated with measures of inflation expectations and monetary policy instruments, which provides an explanation for this dynamic connection.

2.5.4 Estimation of the five factor model: testing for the presence of unspanned factors

From the estimation of a five-factor Gaussian term structure model presented in Section 3 (including four principal components of zero coupon yields, plus the liquidity premium as pricing factors), I am interested in testing for the presence of unspanned factors. I do so by checking whether or not particular columns of $\boldsymbol{B}'$ are equal to zero. Let $\boldsymbol{b}_i$ a particular column of $\boldsymbol{B}'$, then based on the asymptotic distribution of the factor risk exposures $\boldsymbol{B}'$ derived by Adrian et al. (2013), and under the null hypothesis $Ho : \boldsymbol{b}_i = \boldsymbol{0}_{N\times 1}$, the Wald statistic is given by

$$W_{\boldsymbol{b}_i} = \hat{\boldsymbol{b}}_i' \hat{\boldsymbol{\Upsilon}}_{\boldsymbol{b}_i}^{-1} \hat{\boldsymbol{b}}_i \quad \sim^a \chi^2_{(N)},$$

with $\boldsymbol{\Upsilon}_{\boldsymbol{B}} = \sigma^2(\boldsymbol{I} \otimes \boldsymbol{\Sigma}^{-1})$. The Wald statistic has an asymptotic chi-squared distribution with N degrees of freedom. I start by assessing the relative importance of each of the model factors in explaining the cross-sectional variation of nominal Treasury returns, TIPS returns, and their joint cross-section.

Table 2.7: Significance of the $\boldsymbol{B}$: Wald statistics

Factor	Nominal ($\boldsymbol{B}^N$)	TIPS ($\boldsymbol{B}^R$)	Both ($\boldsymbol{B}^N$,$\boldsymbol{B}^R$)
PC_1^N	43.130	21.725	26.038
PC_2^N	42.416	19.749	23.990
PC_3^N	14.098	20.419	34.517
PC_1^{OR}	20.242	40.015	60.257
Δ_{10}	11.139	11.237	12.351
Critical value			
$\chi^2_{(N,\alpha=0.05)}$	$\chi^2_9 = 16.91$	$\chi^2_8 = 15.50$	$\chi^2_{17} = 27.58$

This table provides Wald statistics for tests of whether the risk factor exposures of a subset of returns associated with individual pricing factors are jointly different from zero. $\boldsymbol{B}^N$ denotes Wald statistics for the risk exposures of all nominal Treasury returns to a given factor, $\boldsymbol{B}^R$ denotes Wald statistics for the risk exposures of all TIPS returns to a given factor, and ($\boldsymbol{B}^N$,$\boldsymbol{B}^R$) are the corresponding Wald statistics for the joint cross-section of returns.

Table 2.7 provides the Wald statistics and their associated p-values, for tests of whether or not the risk factor exposures associated with individual pricing factors are jointly different from zero. As indicated by the associated Wald statistics in the first column of Table 2.7, nominal Treasury returns are significantly exposed to all three principal components extracted from nominal Treasury yields, as well as to the first principal component extracted from orthogonalized breakeven. However, I do not reject the null hypothesis that the liquidity factor has zero $\boldsymbol{B}$. Similarly, TIPS returns co-move strongly with innovations to all traditional spanned pricing factors of the model. However, this is not the case for the liquidity premium factor. Moreover, considering the joint cross-section of nominal Treasury and TIPS returns, I find that the liquidity factor is not associated with significant risk exposure. These findings, are in line with the empirical evidence presented before (in Section 5.1), and justify the assumption of treating the liquidity premium factor as unspanned in the specification.

Next, given the evidence presented before, I conclude that the liquidity factor does not affect the dynamics of bonds under the pricing measure, but does affect them under the historical measure. Thus, I estimate the model by imposing the restriction that the corresponding elements of $\boldsymbol{B}_n$ are exactly equal to zero. Thereafter, I assess whether or not a given risk factor is priced in the cross-section of Treasury and TIPS returns. Table 2.8 provides the estimated market price of risk parameters for the five-factor model (four spanned factors and one unspanned factor), as well as the associated standard errors (SE). I find that the level is an important driver of the market price of slope risk. I also find that the slope and curvature are important drivers of the market price of slope risk. Similarly, the price of curvature risk is driven by the level, slope and curvature nominal factors. The price of the level real risk (which corresponds to the first principal component from orthogonalized breakevens) is driven only by the slope factor.

Table 2.8: Market prices of risk: unspanned specification

Factor	λ_0	$\lambda_{1.1}$	$\lambda_{1.2}$	$\lambda_{1.3}$	$\lambda_{1.4}$	$\lambda_{1.5}$	W_Λ
PC_1^N	0.2313	**0.0607**	-0.1221	-0.1863	0.0055	-0.0010	37.75
SE	(0.3175)	(0.0231)	(0.1167)	(0.4701)	(0.2283)	(0.1112)	
PC_2^N	**0.0621**	0.0210	**-0.0427**	**-0.0827**	0.0013	-0.0003	13.50
SE	(0.0367)	(0.0271)	(0.0135)	(0.0543)	(0.0264)	(0.0129)	
PC_3^N	-0.0087	**-0.0022**	**0.0070**	**0.0144**	-0.0003	**0.0003**	10.01
SE	(0.0090)	(0.0010)	(0.0003)	(0.0013)	(0.0006)	(0.0001)	
PC_1^{OR}	-0.1728	-0.0185	**-0.1359**	-0.3650	0.0156	-0.0041	11.97
SE	(0.3304)	(0.0241)	(0.1125)	(0.4891)	(0.2375)	(0.1157)	

This table contains the estimates of the market price of risk parameters λ_0 and λ_1. Nominal factors PC_t^N correspond to the first three principal components for U.S. daily Treasury yields from January 1, 2004 to December 30, 2011. Orthogonal real factors PC_t^{OR} correspond to the first principal component from the residuals of regressions of break-even inflation rates on nominal principal components and the liquidity factor, for the same sample period. Liquidity premium unspanned factor $\Delta_{10,t}$ corresponds to the measure proposed by Chirstersen and Gillan (2011) for the liquidity differential between TIPS and nominal Treasury yields. Standard errors (SE) are reported in parenthesis.

Finally, only the liquidity factor significantly affects the market price of the curvature risk. This result can be interpreted as indicating that the information contained in the yield curve is insufficient to completely characterized the variation in the price of curvature risk. This result is somewhat consistent with the results in Abrahams et al. (2013). They find that the liquidity factor significantly affects the market price of the curvature risk as well as that of the liquidity risk, however they consider liquidity as an additional spanned factor.

To summarize the pricing implications of the model, I test the null hypothesis that the different rows of Λ (which includes $\lambda_{0.i}$ and $\lambda_{1.i}$, and is denoted by $\boldsymbol{\lambda}_i'$) are equal to zero. Then the corresponding Wald test for the $Ho : \boldsymbol{\lambda}_i' = \mathbf{0}_{1\times(K+1)}$ is given by

$$W_{\boldsymbol{\Lambda}_i} = \hat{\boldsymbol{\lambda}}_i' \hat{\boldsymbol{\Upsilon}}_{\boldsymbol{\lambda}_i}^{-1} \hat{\boldsymbol{\lambda}}_i \quad \sim^a \chi^2_{(K+1)},$$

has an asymptotic chi-squared distribution with $K+1$ degrees of freedom. The last column in Table 2.8 provides the Wald statistic values (the critical value is equal to $\chi^2_6 = 12.59$ for a significance level of $\alpha = 5\%$). I find that the level and slope risks are priced in the five-factor model. This is not a surprising result given that the level and slope risks capture the first and second largest share of the cross-sectional variation of yields. However, the curvature risk appears not to be priced at $\alpha = 5\%$, although most of the individual elements of λ_1 (for the second row of $\boldsymbol{\lambda}$) are significantly different from zero. The orthogonal level factor (as measured by the exposure to the first principal component from the residuals of regressions of break-even inflation rates on nominal principal components and the liquidity factor), is priced in the model.

2.6 Conclusion

In this paper, I consider a joint Gaussian affine term structure model for zero-coupon U.S. Treasury and TIPS bonds, with an unspanned factor: liquidity risk. The liquidity factor is restricted to affect the cross-section of yields but it is allowed to determine the bond risk premia. In other words, I am considering liquidity as an additional factor that does not span the yield curve but improves the estimation of bond risk premia. I use different sources of data (nominal Treasury yields, TIPS yields and inflation swap rates) to estimate the parameters of the model. In particular, I use information on zero-coupon inflation swaps to identify the physical liquidity risk premium, which arises from the liquidity differential between Treasuries and TIPS bonds.

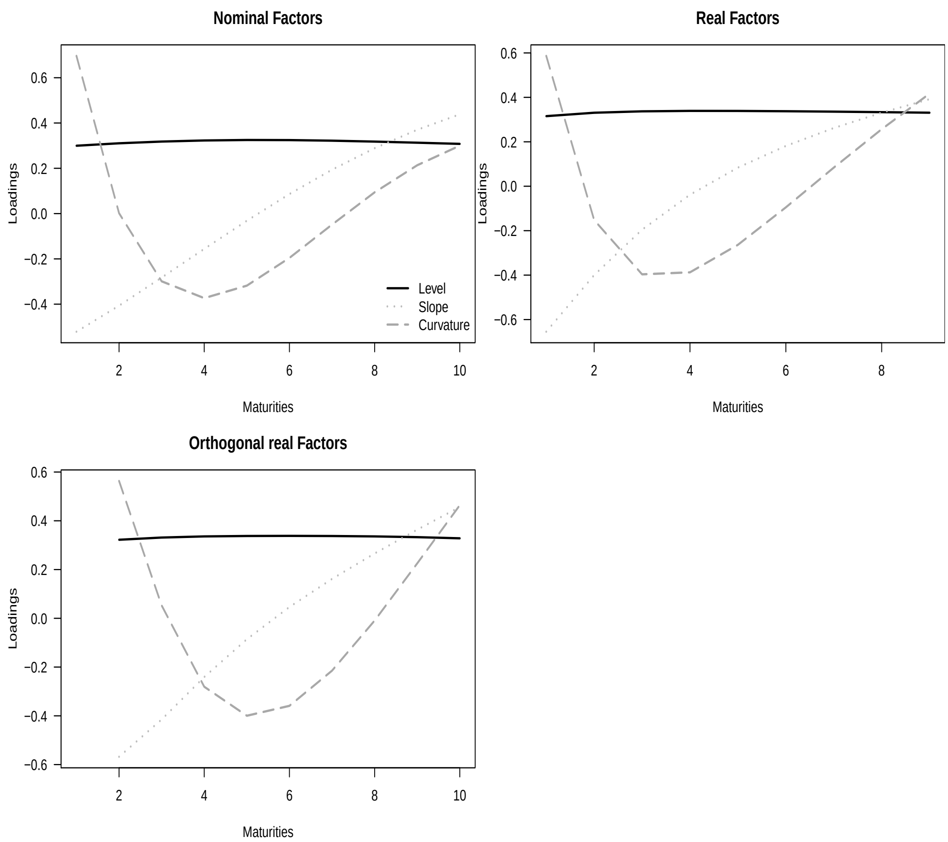
Using this empirical model and additional empirical evidence, I conclude, first, that the TIPS liquidity premium is indeed an unspanned factor that helps to forecast U.S. bond risk premia, and that it is not linearly spanned by the information in the joint yield curve. Second, I show that the variation in the TIPS liquidity premium influences the shape of the yield curve. In fact, an increase in the TIPS liquidity premium lowers the nominal interest rates of all maturities. Similarly, the effect of a one-standard deviation shock to TIPS liquidity is positive for the slope factor, meaning that it makes the yield curve steeper. Thus, when liquidity conditions worsen in the TIPS market relative to the nominal market, nominal long-term interest rates change by much larger amounts than short-term rates. The curvature factor also increases in response to a liquidity shock, which indicates that the yield curve becomes more curved at the short end.

Third, I conclude that the liquidity factor only significantly affects the market price of curvature risk. This result can be interpreted as indicating that the information contained in the yield curve is insufficient to completely characterize the variation in the price of curvature risk. This result is somewhat consistent with the results in Abrahams et al. (2013). They find that the liquidity factor significantly affects the market price of curvature risk as well as that of liquidity risk, when they consider liquidity as an additional spanned factor. I leave it to future work to consider additional unspanned factors (such as real output and inflation), and to perform out-of-samples exercises in order to compare different factor model specifications.

Appendix A2

Factor loadings

Figure A2.1: Factor loading



Level, slope and curvature correspond to the three principal components from nominal Treasury yields of maturities for $n =$ 6-month, 1-, 2-, ..., 10- and 20-years. Orthogonal real factor correspond to the first principal component from the residuals of regressions of break-even inflation rates on nominal principal components and the liquidity factor. Liquidity factor corresponds to the TIPS liquidity premium measure proposed by Christensen and Gillan (2011). Sample spans from January 1, 2004 to December 30, 2011.

Table A2.1: Summary statistics of factor variables

	L_t^N	S_t^N	C_t^N	L_t^{OR}	S_t^{OR}	Δ_t
Min	-5,60	-1,47	-0,24	-13,60	-2,34	-6,28
Median	0,16	-0,07	-0,01	0,07	-0,03	23,28
Mean	0,00	0,00	0,00	0,00	0,00	26,22
Max	5,26	1,67	0,49	11,20	3,50	118,80
Stdev	3,07	0,73	0,11	2,93	0,61	16,15

Panel A regresses TIPS liquidity on the first three principal components for U.S. daily Treasury yields from January 1, 2004 to December 30, 2011. Panel B regresses TIPS liquidity on the first three principal components for U.S. daily TIPS yields using the same sample period. TIPS liquidity corresponds to the TIPS liquidity premium measure proposed by Christensen and Gillan (2011).

Appendix B2

VAR regression

Table B2.1: Descriptive statistics for the macro variables and bond returns

	Macro variables					Returns				Liquidity
	VIX_t	$FFR_{10,t}$	$TERM_t$	CDS_t	$ED4_t$	R_5^N	$R_{10,t}^N$	$R_{5,t}^{TIPS}$	$R_{10,t}^{TIPS}$	Δ_t
Mean	20,79	1,90	1,88	1,18	253,94	1,67	2,27	0,67	1,00	26,24
Stdev	10,19	1,98	1,26	0,56	175,06	0,46	0,42	0,46	0,34	16,17
Skew.	2,17	0,62	-0,36	2,40	0,26	-1,00	-0,30	-1,20	-0,39	2,72
Kurt.	9,10	1,76	1,93	8,59	1,51	3,00	3,69	3,71	4,38	12,37
Percentile										
5%	11,20	0,08	-0,31	0,71	45,00	0,68	1,46	-0,39	0,33	10,63
50%	17,83	1,00	2,05	0,98	227,75	1,82	2,30	0,81	1,05	23,32
95%	42,83	5,25	3,56	2,85	521,00	2,20	2,89	1,16	1,48	60,08
Cross correlations										
vix	1,00									
FFR	-0,43	1,00								
TERM	0,41	-0,87	1,00							
CDS	0,82	-0,36	0,27	1,00						
ED4	-0,40	0,94	-0,75	-0,32	1,00					
RN5	-0,16	0,54	-0,11	-0,23	0,67	1,00				
RN10	0,11	0,08	0,28	-0,02	0,20	0,79	1,00			
RR5	-0,01	0,50	-0,11	-0,02	0,64	0,94	0,73	1,00		
RR10	0,05	0,19	0,15	-0,02	0,30	0,78	0,88	0,81	1,00	
liq	0,63	-0,02	-0,04	0,78	0,03	-0,01	0,04	0,21	0,08	1,00
Auto correlations										
1-day	0,98	1,00	1,00	1,00	1,00	0,93	0,62	0,94	0,60	0,98
2-day	0,97	1,00	1,00	1,00	1,00	0,92	0,59	0,93	0,56	0,97
5-day	0,95	1,00	0,99	0,99	0,99	0,91	0,59	0,92	0,58	0,94
22-day	0,83	0,99	0,96	0,95	0,98	0,85	0,53	0,86	0,51	0,75
Unit root test										
DF p-value	0,09	0,37	0,16	0,52	0,38	0,07	0,01	0,00	0,00	0,05

Chapter 3

Conditional Portfolio Choice in the U.S. Bond Market: The Role of Liquidity

Abstract: In this chapter, I estimate the non-parametric optimal bond portfolio choice of a representative agent that acts optimally with respect to his/her expected utility one period forward, provided that he/she observes the ex ante liquidity signal. Using daily observations of zero-coupon Treasury and TIPS bonds yields, I construct equally-weighted returns from 2004-2012. Considering alternative measures of liquidity, I find that the liquidity differential between nominal and TIPS bonds appears to be a significant determinant of the portfolio allocation to U.S. government bonds. In fact, conditional allocations in risky assets decrease as market liquidity conditions worsen, and the effect of market liquidity decreases with the investment horizon. I also find that the bond return predictability translates into improved in-sample and out-of-sample asset allocation and performance.
Key Words: Liquidity risk, optimal portfolio allocation, predictability, bond risk premia, non-parametric estimation.
JEL classification: C13, C52, G11, G32.

3.1 Introduction

Numerous empirical studies conclude that excess bond returns are predictable in the sense that they depend on the current value of some predictor variables. In addition, the term structure slope, the forward spread, the lagged excess returns, the Cochrane and Piazzesi (2005) tent-shaped factor, and macroeconomic fundamentals are some of the variables that have been identified as predictors for Treasury bonds (Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005), Ludvigson and Ng (2009) and Cooper and Priestley. (2009b)). The role of market liquidity as a predictor variable for government bonds has been studied more recently by Fontaine and Garcia (2011), Pflueger and Viceira (2012) and Gomez (2013). They provide empirical evidence for liquidity as a source of predictability for U.S. Treasury bonds, U.S. Treasury Inflation-protected bonds (TIPS), or for both.

The question as to whether or not asset returns are predictable is of significant importance for portfolio choice. In their seminal papers, Merton (1969) and Samuelson (1969) show that if asset returns are independently and identically distributed (IID) over time, then the optimal asset allocation is constant over time. However, Kim and Omberg (1996), Brennan and Lagnado (1997) and Viceira and Campbell (1999) show that if asset returns are predictable, then the optimal asset allocation depends on the investment horizon and the predictive variables.

While some studies provide insight into the role of liquidity as a predictor variable, few studies examine the effect of liquidity risk on optimal portfolio allocation. Ghysels and Pereira (2008) provide empirical evidence that the relevance of liquidity for stock portfolio choice depends on both the asset and the investment horizon. Garleanu (2009) studies portfolio choice and pricing in markets in which trading may take place with considerable delay, and shows that the liquidity level has a strong impact on portfolio choice. This paper focuses on examining how changes in liquidity risk premium influences optimal portfolio allocations in U.S. government nominal and index-linked bonds.

Throughout this paper, I assume that the investor makes decisions in real terms where the investment horizon is one-month, one-quarter and one-year. I only consider a short-term investor in the empirical analysis. The reason for this is related to the fact that for a buy-and-hold long-term investor, whose investment horizon perfectly matches the maturity of the bond, TIPS offer full protection against inflation if held until maturity.[1] Similarly, an investor who adopts a buy-and-hold strategy for TIPS mitigates risk arising from illiquidity, given that he/she does not face higher costs of buying or selling the bond before it reaches maturity. However, TIPS are currently issued with only a few specific maturities: 5-year, 10-year and 30-year, therefore the investment horizon over which I consider investors who hold assets does not match the maturity of any outstanding TIPS.[2] Hence, I study a

[1] TIPS are a useful hedge against inflation, but they do not guarantee a real rate of return. This is because the mechanics of adjusting for inflation for TIPS limit the exactness of the inflation adjustment and allow only approximate inflation hedges especially at high inflation levels. In fact, for TIPS, the reference price index is the non-seasonally adjusted CPI-U, and the indexation lag is three months. Therefore, TIPS operate with an indexation lag of three months. In other words, it takes three months from the incidence of price inflation (the month when a reference index reading is recorded) until it is incorporated into the coupon payment of the inflation-linked bond. Consequently, the indexation lag affects how well TIPS compensate for contemporaneous inflation, and prevents TIPS from guaranteeing a specified real return.

[2] U.S. Treasury inflation-protected securities were introduced in January 1997. TIPS bonds have been offered in 5-, 10-, 20-, and 30-year denominations. However, TIPS that have less than one year remaining to maturity are not easy to find in the secondary market, given that they have

short-term investor who maximizes real wealth but is not able to invest in a risk-less asset in real terms (given that TIPS are a risky asset both in nominal and in real terms), and also faces liquidity risk. Notice, however, that a short-term investor benefits from the availability of TIPS in terms of a wider investment opportunity set that allows an increase in the returns per unit of risk, investing even a small fraction of his wealth in TIPS (Cartea et al. (2012)).

The investor's problem is to choose optimal allocations to the risky asset as a function of predictor variable: the TIPS liquidity premium. As risky assets, I consider equally weighted bond portfolios on short-term bonds (1 to 10 years maturity); and on long-term bonds (11 to 20 years maturity), each of them are computed for Treasury bonds and for TIPS. The existence of a TIPS liquidity premium is well established. In fact, TIPS bonds have been characterized by being less liquid than nominal Treasury bonds.[3] TIPS' lack of liquidity compares with nominal Treasuries results in TIPS yields having a liquidity premium relative to Treasuries.[4] Since this liquidity premium is unobservable, different alternative ways of proxing liquidity have been proposed in literature. In particular, I test two market-based measures for the liquidity differential between inflation-indexed bonds and nominal bonds proposed by Christensen and Gillan (2011) and Gomez (2013). The first one is computed as the spread between synthetic and cash break-even inflation rates, while the second one corresponds to the asset swap spread on similar maturity inflation-linked and Treasury bonds. Both measures allow us to identify the relative liquidity premium between two comparable assets, which in this case arise from the cost derived from TIPS liquidity disadvantage relative to Nominal bonds.

The particular choice of these two measures for liquidity is motivated by the fact that: *i*) even though they are highly correlated (which suggests that all of them are capturing similar information about the liquidity differential between nominal and TIPS yields), they are measured using information from different markets, which would allow them to capture different aspects of the liquidity premium, especially in times of financial distress where each market tends to be driven by its specific dynamics, such as funding costs.[5] Next, I am interested in testing if the optimal

extremely high transaction costs.

[3] The existence of this liquidity premium in TIPS yields has been well documented in the academic literature by Sack and Elsasser (2004), Shen (2006), Hordahl and Tristani (2010), Campbell et al. (2009), Dudley et al. (2009), Christensen and Gillan (2011), Gurkaynak et al. (2010), Pflueger and Viceira (2012), among others.

[4] Liquidity risk premium is defined here as the total cost of all frictions to trade a relative less liquid asset beyond those of the more liquid asset against which it is being compared (Christensen and Gillan (2011)).

[5] Theoretically, there exists a close relationship between bond break-evens and inflation swaps rates. In essence, both measure the markets' expectations of future inflation. However, the most recent crisis showed that U.S. cash and swap markets can diverge significantly, with each market driven by its specific dynamics such as funding costs. Asset swapping activity should theoretically hold the two markets together, but the empirical evidence, discussed by Gomez (2013), shows that such activity is not sufficient to offset diverging forces in stressed market conditions.

portfolio choice depends on a particular choice to proxy liquidity premium, *ii*) they are market-based measures of liquidity which is straightforward to compute, and by construction they are also model-free.

Finally, I consider the portfolio policy of an investor who is able to invest in only one risky asset, and I differentiate various portfolio allocation problems: first, where the investor chooses between the portfolio of short-term or long-term Treasury bonds and a risk-free asset; and second, where the investor chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. I also study an investor with mean-variance (MV) and constant relative risk aversion (CRRA), with different degrees of risk aversion, in order to test the sensitivity of the optimal portfolio choice to the higher moments.

There are a series of ways in which this study contributes to the literature. First, it incorporates financial information (liquidity premium) in an asset allocation context, and shows how this can be of significance for both a mean-variance and a CRRA investor. Second, it focuses on a bond portfolio choice that is relatively unexplored in the literature, since the majority of the studies on asset allocation examine stock-only portfolios. Brennan and Lagnado (1997) (who were the first to analyze portfolio choice in the presence of time-varying expected returns), point out that the degree of the asset return predictability has a significant effect on the composition of the optimal portfolio. Therefore, the evidence in favor of bond return predictability (by means of variables such as liquidity) imply that a bond portfolio setting provides a robust framework to examine. Additionally, bonds-only portfolios are extremely important for the fund management industry and for central banks, as well as for liquidity, and inflation risk are highly relevant for insurance and the risk management of pension funds. Third, I examine portfolio choice among multiple government bonds with different maturities. More so, I consider both the U.S. Treasury bonds and inflation-linked bonds in the investor's asset menu.

I make use of an econometric framework based on a portfolio choice problem of a single period investor, where the investor's problem is set up as a statistical decision problem, with asset allocations as parameters and the expected utility as the objective. The allocations are estimated by direct maximization of expected utility proposed by Brandt (1999). A number of key results emerge from this analysis. First, the liquidity premium seems to be a significant determinant of the portfolio allocation of U.S. government bonds. In fact, conditional allocations in risky assets decrease as liquidity conditions worsen. In particular, an increase in the liquidity differential between nominal and TIPS bonds leads to lower optimal portfolio allocations for nominal Treasury bonds, and also to lower optimal portfolio allocations in TIPS, but at different levels of liquidity. Additionally, the effect of liquidity is a decreasing function of investment horizons, in the sense that for the same degree of risk aversion the investor reacts less abruptly to an increase in the liquidity premium when he/she has a longer investment horizon. Furthermore, as the investment horizon becomes longer, the smaller the optimal portfolio weight, and so, the less is invested in the risky asset.

The above conclusions are not determined by the level of risk aversion or the investors preferences. The relation between optimal portfolio weights and the liquidity premium remains the same for different values of risk aversion, and also across investor preferences. These characteristics mainly change the level of the portfolio function, having a small impact on the shape of the function. In addition, results do not depend on a particular choice of the maturity of the liquidity premium (similar results are found when considering 10-year or 20-year liquidity premium), nor on a specific way to proxy liquidity (I have similar results with both liquidity premium measures).

From the standpoint of practical advice to portfolio investors, a final natural question to ask is whether or not the bond return predictability translates into improved out-of-sample asset allocation and performance. To answer this question, I compare the performance of the optimal portfolio choices of two investors: one investor who makes portfolio allocations based on the belief that bond returns are predictable by liquidity (conditional strategy); and the other who believes that bond returns are independent and identically distributed (i.i.d.), and ignores any evidence of bond return predictability in making his/her portfolios allocation choices (unconditional strategy). I conclude that the conditional strategy outperforms the unconditional strategy, improving not only the in-sample, but the also out-of-sample asset allocation and performance.

The rest of the paper is organized as follows. Section 2 defines the conditional portfolio choice problem, provides a description of the liquidity premium measures available in the literature and presents the non-parametric estimation technique used. I describe the data and provide some basic statistics in Section 3. Section 4 presents the empirical results for different bond portfolios, different types of investors and different investment horizons. Section 5 concludes.

3.2 The conditional optimal portfolio problem

The traditional problem of optimal portfolio choice considers an investor which maximizes the conditional expected utility of next period's wealth under a budget constraint. Merton (1969) provides the solution, where the investor can trade continuously in a finite set of stocks and bank account. However, given that the stocks and bonds differ in many ways, the theory of portfolio management does not apply as it stands to bond portfolios (see Ekeland and Taflin (2005) for a discussion of this point). For the bond market, Schroder and Skiadas (1999), Ekeland and Taflin (2005), Ringer and Tehranchi (2006) and Liu (2007) have studied this problem using a theoretical approach. In particular, Ekeland and Taflin (2005) and Ringer and

Tehranchi (2006) set up, and solve the problem of managing a bond portfolio by optimizing (over all self-financing trading strategies for a given initial capital), the expected utility of the final wealth. Thus, optimal portfolio at time t is a linear combination of self-financing instruments, each one with a fixed time to maturity. Under this set up the value of the portfolio changes only because the bond prices change. Price bonds behave like price stocks, that is, it depends only on the risk it carries and not on time to maturity.

The impact of return predictability on optimal portfolio choice have also been considered in literature. Initially, it was studied under the assumption of no parameter or model uncertainty by Viceira and Campbell (1999), Balduzzi and Lynch (1999), Wachter (2002), Munk et al. (2004). Subsequently, Barberis (2000) incorporates parameter uncertainty, but does not allow for dynamic learning. More recently, Brandt et al. (2005) consider learning about other parameters of the return processes in addition to the predictive relation.

Various other papers investigate the effects of an aversion against ambiguity about the return process on portfolio choice (Maenhoud (2006), Liu (2010), Liu (2011), Chen et al. (2011) and Branger et al. (2013)). There is also a growing literature on portfolio selection that incorporates return predictability with transaction costs, started by Lynch and Balduzzi (2000), Brandt et al. (2004) and recently by Lynch and Tan (2011), and Garleanu and Pedersen (2009). Empirical studies also have been undertaken by Brandt (1999), Ait-Sahalia and Brandt (2001), and Brandt and SantaClara (2006) consider different predictive variables, while Ghysels and Pereira (2008) have the only paper that includes liquidity as a predictor variable (except for stock portfolio allocation problem).

On the other hand, the impact of inflation on portfolio choice also has also been considered in the literature. An initial extension of the Markowitz problem was introduced in the 1970s by Biger (1975), Friend et al. (1976), Lintner (1975) and Solnik (1978), among others. Intertemporal portfolio choice problem under inflation risk was studied by Campbell and Viceira (2001) in discrete time, and by and Brennan and Xia (2002) in continuous time. Both works tell us that a long-term, risk-averse investor prefers the indexed bond or a perfect substitution of indexed bond in order to hedge against the inflation risk. However, in these papers all relevant state variables are assumed observable and the probability distributions of all processes are assumed known. Bensoussan et al. (2009) and Chou et al. (2010) relax that restriction by assuming that the expected inflation rate is unobservable to the investor.

More recently, motivated by the fact that all these papers disregard model uncertainty (inflation model misspecification), Munk and Rubtsov (2012) solve a stock-bond-cash portfolio choice problem for a risk- and ambiguity-averse investor in a setting where the inflation rate and interest rates are stochastic and the expected inflation rate is unobservable. Also, De Jong and Zhou (2013) investigate the optimal portfolio and consumption policies for a finite horizon investor in a life-cycle model with habit formation and inflation risk.

Most of the existing studies on portfolio choice (with or without inflation risk), focus on stock-only portfolios (Viceira and Campbell (1999), Barberis (2000), Wachter (2002)), or examine the stock-bond mix portfolio choice (Munk et al. (2004)). Given the extensive literature for equity markets, it is surprising to note that no effort has been undertaken to examine the influence of liquidity in government bond portfolio choice. Filling this gap is one contribution of this paper. To follow, I define the investor's maximization problem, describe the conditioning information, and finally, introduce the estimation technique.

3.2.1 Investor utility maximization

Portfolio choice without inflation

Ekeland and Taflin (2005) and Ringer and Tehranchi (2006) express the solution of optimal portfolio choice as portfolios of self-financing trading strategies which naturally include stocks and bonds. In particular, they fix a utility function u and a planning horizon $T > 0$, and consider the functional $J(\varphi) = \mathbb{E}^{\mathbb{P}}[u(W_T^{\varphi})]$ where W_T^{φ} is the accumulated wealth at time T generated by the self-financing trading strategy φ. The goal is to characterize the strategy that maximizes J.

Following on from this literature, I consider the problem of optimal portfolio choice when the traded instruments are a set of risky bonds and a risk-less bond. In particular, and without loss of generality, I consider a bond market where only zero-coupon bonds are available. Fixing a utility function $u(W_{t+1})$ and a planning horizon $T > 0$, I consider an investor who maximizes the conditional expected utility of next period's wealth, subject to the budget constraint:

$$\begin{aligned} &\max_{\alpha_t \in \mathcal{A}(\varphi)} \quad \mathbb{E}[u(W_{t+1}) \mid Z_t] \\ &\text{subject to:} \quad W_{t+1} = W_t[R_{f,t+1} + \alpha_t(R_{b,t+1} - R_{f,t+1})] \end{aligned} \tag{3.1}$$

where W_{t+1} is the accumulated wealth at time $t+1$ generated by the self-financing trading strategy φ (which belongs to the set of admissible self-financing strategies denoted by $\mathcal{A}$), α_t represents the proportion of wealth invested in a risky bond with return $R_{b,t+1}$ and the remaining proportion $1 - \alpha_t$ is invested in risk-free bond with return $R_{f,t+1}$. The expectation is conditional on a state variable Z_t. The investor can have three different horizons: one-month, one-quarter or one-year (this represents the difference between t and $t+1$).

The weight that maximizes the expected utility function is the solution to the following Euler optimality condition

$$\mathbb{E}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha} \mid Z_t\right] = 0. \tag{3.2}$$

In particular, the solution of the investor's problem is the mapping from the state variable Z_t to the portfolio weights

$$\alpha_t = \alpha(Z_t), \tag{3.3}$$

and it denotes the portfolio choice of observing a signal $Z_t = z$.

The relation between the portfolio policy and the predictability of individual moments of the returns given the predictor Z_t depends on the specification of the utility function. I consider two types of investor preferences: mean-variance (MV) and power-utility (CRRA) preferences. An investor with mean-variance preferences maximizes

$$\max_{\alpha_t \in \mathcal{A}(\varphi)} \quad \mathbb{E}[W_{t+1} \mid Z_t] - \frac{\gamma}{2}\mathbb{V}[W_{t+1}^2 \mid Z_t], \tag{3.4}$$

where $\gamma > 0$ represents the coefficient of absolute risk aversion. The investor portfolio policy when the choice includes a risk-free rate is proportional to the conditional mean-variance ratio of the tangency portfolio

$$\alpha_t^{tg} = \frac{1}{\gamma W_t}\frac{\mathbb{E}[R_{t+1}^{tg} \mid Z_t]}{\mathbb{V}[R_{t+1}^{tg}]},$$

where R_{t+1}^{tg} is the return of the tangency portfolio. The reason I consider MV preferences is because it can be stated as a primitive, or can be derived as a special case of expected utility theory. Also, under MV preferences, portfolio weights depend exclusively and analytically on the two first moments of returns, which serve as benchmark case in this study.[6]

I also consider the most popular objective function in the portfolio choice literature, which is an investor with CRRA or power utility. In this case, the investor solves the following problem

$$\max_{\alpha_t \in \mathcal{A}(\omega)} \begin{cases} \mathbb{E}\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma}\right] & \text{if } \gamma > 1 \\ \mathbb{E}\left[\log(W_{t+1})\right] & \text{if } \gamma = 1 \end{cases} \tag{3.5}$$

subject to the budget constraint in (3.1), and where $\gamma > 0$ now measures the coefficient of relative risk aversion. As is well known, unlike mean-variance preferences, CRRA does not permit a closed form solution to the investor's portfolio problem. However, I consider CRRA preferences to be able to test whether or not an investor cares about higher order moments of the return distribution.

[6] Although the limitations of mean-variance analysis are well established in portfolio theory, its relative simplicity and easy intuition contributes to its continued use among investment professionals, in theoretical and empirical studies.

Portfolio choice with inflation

In this section, I follow Cartea et al. (2012) who solve the optimal portfolio choice problems for investors concerned with maximizing real wealth. Here, I assume that investors make allocation decisions in real terms, and are worried about the purchasing power of their terminal wealth, and do not suffer from money illusion. As before, I consider the optimal investment allocation of investors who are not worried about what may happen beyond the immediate next period but rather, care about the purchasing power of their wealth.

To avoid exposure to inflation risk, investors can: (*i*) invest in a risk-less asset in real terms; and/or (*ii*) invest in assets that covary with inflation. However, in this empirical analysis I only consider investors who have a maximum investment horizon of 1-year; they cannot find TIPS with this maturity and thus they are not able to invest in a risk-less real asset. Additionally, given that real interest rate changes affect TIPS returns, investors consider TIPS as a risky asset in both nominal and real terms.

An investor with MV or CRRA preferences maximizes the same problem in (3.4) and (3.5), respectively, but are now subject to the budget constraint

$$W_{t+1}^{R} = W_t^{R}[R_{f,t+1} + \alpha_t(R_{b,t+1} - R_{f,t+1})],$$

where W_{t+1}^{R} is now the terminal real wealth, and $R_{b,t+1}$ and $R_{f,t+1}$ are real risky and risk-free bond returns, respectively, as already seen.[7] In the absence of a real risk-free asset investors face inflation risk and deal with this through the covariances between the returns of risky assets and inflation. Securities which are correlated with inflation help to hedge against inflation, reducing the portfolio variance in real terms.

3.2.2 Liquidity measures

It is generally acknowledged that liquidity is important for asset pricing. At a theoretical level, two main views (not mutually exclusive), have been advanced to explain why liquidity should be priced by financial markets: illiquidity (*i*) creates trading costs; and (*ii*) can itself create additional risk. The first view holds that illiquid securities must provide investors with a higher than expected return to compensate for their larger transaction costs, controlling for fundamental risk. This view was first proposed and tested by Amihud and Mendelson (1986) for stock-market data, and by Amihud and Mendelson (1991) for fixed-income security markets. The second view suggests that liquidity is priced not only because it creates trading costs, but also because it is itself a source of risk, since it changes

[7] In this case, the real risk-free bond returns is calculated as $R_{f,t+1} - \pi_{t+1}$, where π_{t+1} is the log inflation rate.

unpredictably over time, as developed by Pastor and Stambaugh (2003). These two views have resulted in considerable literature on the relation between returns and liquidity.

On the other hand, the existence of differences in market liquidity conditions between nominal and inflation-indexed Treasury securities is well known. Different practical approaches have been used to measure this liquidity differential. In general, two approaches have been implemented: market-based measures used by Christensen and Gillan (2011) and Gomez (2013); and a regression procedure used by Pflueger and Viceira (2012).

Christensen and Gillan (2011) identify the liquidity component in TIPS yields using information from the bond market and also from the inflation swap market. An inflation swap is a bilateral contractual agreement. It requires one party (the inflation payer), to make periodic floating-rate payments linked to inflation, in exchange for predetermined fixed-rate payments from a second party (the inflation receiver). The most common contract is the zero-coupon inflation swap, which has the most basic structure with payments exchanged only on maturity.

The rates observed, $IS_{n,t}$, represent the fixed rate paid by the inflation receiver, that is, the rate that fixed rate agents are willing to pay (receive) in order to receive (pay) the cumulative rate of inflation during the life of the swap. Hence the quoted rate can be also viewed as a break-even inflation rate (BEI), which depends on expected inflation over the life of the swap. Thus, it is possible to use the quoted rate to derive market-based measures of expectations for inflation.

In theory, the inflation compensation implicit in the prices of nominal bonds relative to index-linked bonds should be the same as that found in inflation swap rates. The two should be consistent due to arbitrage.[8] Thus, in a frictionless world this equality must hold

$$IS_{n,t} = \pi^e_{n,t} = BEI_{n,t},$$

where $BEI_{n,t}$ denotes the cash break-even inflation rate and $\pi^e_{n,t}$ is the expected average inflation rate for the next n years.

However, in reality the cash BEI and inflation swap rates are not equal. As occurs in the ILB market, the market for inflation swaps are less liquid than the market for nominal Treasury bonds, such that the observed price of each asset should contain a non-negative time-varying liquidity premium that biases its yields upwards (Christensen and Gillan (2011)). That means that inflation swap rates should be adjusted by liquidity risk. The observed inflation swap rate (commonly referred as

[8] That is because the pay-offs of index-linked bonds can be replicated using inflation swap contracts. Two portfolios with identical future pay-offs should have the same price via arbitrage. Hence, with perfect markets we would expect perfect substitution between break-even rates available in the inflation swap and bond markets

synthetic break-even rate) are given by

$$\widehat{IS}_{n,t} = IS_{n,t} + L^{IS}_{n,t},$$

where $L^{IS}_{n,t}$ is the liquidity premium included in the inflation swap rates.

Christensen and Gillan (2011) argue that the liquidity component in BEI identify from the difference between observed BEI and inflation swap rate

$$\Delta_{n,t} = \widehat{IS}_{n,t} - \widehat{BEI}_{n,t} = L^{IS}_{n,t} + L^{ILB}_{n,t}, \tag{3.6}$$

They showed that this result hold under two assumptions: *i)* the market for ILBs and inflation swaps are less liquid than the market for nominal Treasury bonds; and *ii)* the nominal Treasury yields we observe are very close to the unobservable nominal yields that would prevail in a frictionless world, that means $\hat{y}^N_{n,t} = y^N_{n,t}$. Under these assumptions, the difference between the two rates is the sum of the liquidity premiums in TIPS and inflation swaps.

In a recent paper, Gomez (2013) measures the market liquidity premium in TIPS by looking at how inflation-linked asset swaps on nominal bonds corresponds to inflation-linked ones. The idea is that this asset swap spread captures the relative financing cost, the special nature and the balance sheet cost of TIPS over nominal Treasuries. These characteristics make some securities easier to liquidate and more attractive to hold than others, so this spread should be a good market-based measure of the market perception of relative liquidity in a bond market.

An asset swap is a derivative transaction that results in a change in the form of future cash flows generated by an asset. In the bond markets, asset swaps typically take fixed cash flows on a bond and exchanges them for Libor (i.e. floating rate payments) plus asset swap spread (ASW), which can be positive or negative. Thus, an asset swap is equivalent to buying a bond and entering into an interest rate swap with maturity matching the bond.[9]

The *z-asw* spread between a nominal and inflation-linked asset swaps, is given by:

$$L^{z\text{-}asw}_{n,t} = z\text{-}asw^{ILB}_{n,t} - z\text{-}asw^{N}_{n,t} = L^{ILB}_{n,t}. \tag{3.7}$$

This spread should be non-negative, $L^{z\text{-}asw}_{n,t} \geq 0$, and equal to the liquidity premium in the inflation linked bond.

Pflueger and Viceira (2012) estimate the TIPS liquidity premium explicitly using a model. They regress the break-even inflation rate on a set of three measures of liquidity in bond markets: the nominal off-the-run spread, relative TIPS transaction

[9] As for a nominal asset swap, the proceeds of a bond are exchanged against a floating rate interest payment, however the proceeds are not fixed but inflation-linked. Thus, a dealer might buy an indexed bond via a repo, provide an inflation-indexed cash flow to the market via an inflation swap and hedge its position with a standard interest rate swap.

volumes and the difference between TIPS asset-swap-spreads and nominal U.S. Treasury asset-swap spreads. They also control for inflation expectation using the survey of long-term inflation expectations (π^{SPF}) and the Chicago Fed National Activity Index (CFNAI). They estimate

$$\widehat{BEI}_{n,t} - \pi^{SPF} = a_1 + a_2 X_t + a_3 CFNAI_t + \varepsilon_t,$$

where X_t is a vector containing our three liquidity proxies. They obtain the TIPS liquidity premium as the negative of the variation in $\widehat{BEI}_{n,t} - \pi^{SPF}$ explained by the liquidity variables, while controlling for the CFNAI as a proxy of short-term inflation expectations. Hence, the estimated relative liquidity premium in TIPS yields equals

$$\hat{L}^{PV}_{n,t} = -\hat{a}_2 X_t. \tag{3.8}$$

An increase in $\hat{L}^{PV}_{n,t}$ reflects a reduction in the liquidity of TIPS relative to nominal Treasury bonds. Given that their liquidity estimate most likely reflects liquidity fluctuations in both nominal bonds and in TIPS, they assume that the liquidity premium $\hat{L}^{PV}_{n,t}$ is entirely attributable to time-varying liquidity in TIPS rather than in nominal bonds.

The measures described above allow us to identify the relative liquidity premium between two comparable assets, in this case the cost derived from TIPS liquidity disadvantage relative to nominal bonds.[10] As a result, the liquidity measures described above meet the same definition of liquidity premium. Specifically, liquidity refers to the total cost of all frictions (wider bid-ask spreads, lower trading volume, etc.) to trade off the less liquid asset beyond that of the more liquid asset against which it is being compared. However, I will use the two model-independent measures of liquidity premium to examine whether or not the liquidity differential between inflation-indexed bonds and nominal bonds (liquidity premium), represented by Z_t, constitute relevant conditioning information in the portfolio choice problem.

The reason for that particular choice is twofold. I use the market-based measures of liquidity because they are model-free and can be readily calculated using daily data, while Pflueger and Viceira (2012) liquidity premium is model-dependent by construction and it is only available on a monthly frequency.[11] Second, there exists a close relationship between bond break-evens and inflation swap rates, because theoretically, both rates measure the markets' expectations of future inflation. However, the most recent crisis showed that U.S. cash and swap markets can

[10] Absolute liquidity premium is defined as the price difference between the observed and the unobservable frictionless market outcome of a given asset. However, we work with the relative concept since it is extremely difficult to identify the unobservable frictionless price of an asset directly.

[11] Pflueger and Viceira (2012) estimated liquidity premium from January 1999 to September 2010, only for 10-year TIPS and in a monthly frequency. Consequently, it does not have enough sample points to be considered in this study (74 observations).

diverge significantly, with each market driven by its specific dynamics. Asset swapping activity should theoretically hold the two markets together, but the empirical evidence suggests that such activity was not sufficient to offset diverging forces in stressed market conditions (see Gomez (2013) for a further discussion). Consequently, even though the Christensen and Gillan (2011) and Gomez (2013) measures are highly correlated (which suggests that all of them are capturing similar information about the liquidity differential between nominal and TIPS yields), they are measured using information from different markets. Thus, it would make them capture different aspects of liquidity premium, especially in times of financial distress where each market tends to be driven by its specific dynamics, such as funding costs.

3.2.3 Non-parametric estimation

I use the methodology proposed by Brandt (1999) and Ait-Sahalia and Brandt (2001). They apply a standard generalized method of moments (GMM) technique to the conditional Euler equation that characterizes the investor's portfolio choice problem. In particular, it consists of replacing the conditional expectation with sample analogues, computed only with returns realized in a given state of nature where the forecasting variable level is $Z_t = \bar{z}$. Brandt (1999) suggests estimating the conditional expectation with a standard non-parametric regression. Ait-Sahalia and Brandt (2001) suggest a semiparametric approach to address the issue of which predictors are important for the portfolio choice when a large number of them are available.

Let a neighborhood of Z be $Z \pm h$ for some bandwidth $h > 0$. When the investor is characterized by the power utility, a simple non-parametric estimator of the conditional Euler equation is given by the Nadaraya-Watson estimator, where the moment condition is given by:

$$\hat{\mathbb{E}}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha} \mid Z_t = \bar{z}\right] = \frac{1}{Th}\frac{\Sigma_{t=1}^{T}\left(\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right)k(Z_t, \bar{z}, h)}{\Sigma_{t+1}^{T}k(Z_t, \bar{z}, h)} = 0, \quad (3.9)$$

where $k(Z_t, \bar{z}, h)$ is the kernel function which is assumed to be Gaussian. I apply exactly identified GMM to equation (3.2) to obtain $\hat{\alpha}(Z)$ which is a consistent estimate for the unknown optimal portfolio choice $\alpha(Z)$ (See Ait-Sahalia and Brandt (2001) for asymptotic properties of this estimators). The conventional solution to optimize the classical trade-off between variance and bias is to choose a bandwidth of the form: $h = \lambda\sigma_z T^{-1/K+4}$, where λ is a constant, K is the number of predictor variables and σ_z is the standard deviation of the predictor Z (see Hardle and Marron (1985)).

Finally, the optimal unconditional portfolio weight is compute by applying a standard GMM procedure to the unconditional Euler equation. In this case the moment condition is:

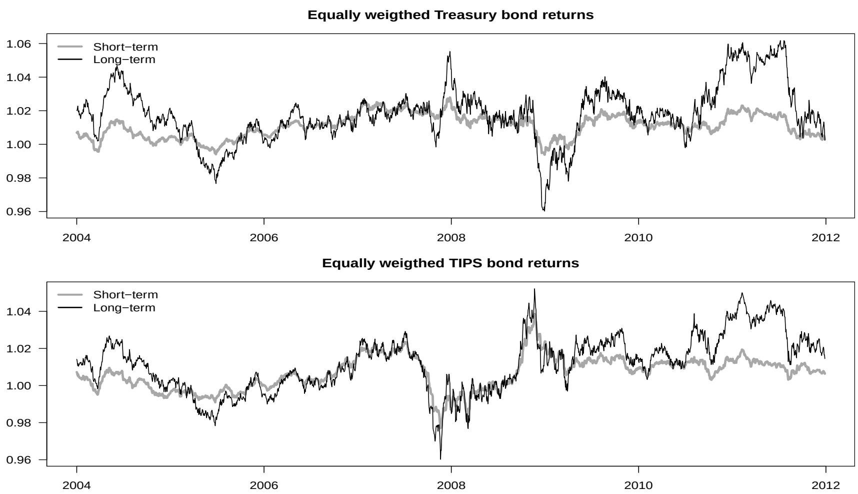
$$\hat{\mathbb{E}}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right] = \frac{1}{T}\Sigma_{t=1}^{T}\left(\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right) = 0, \qquad (3.10)$$

which yields the same results that directly compute weights from equation (3.3).

3.3 The Data and basic statistics

I am interested in the analysis of the empirical time-series relationship between optimal bond portfolio allocations and alternative measures of liquidity. To that end, I calculate monthly, quarterly and annual holding period returns from daily observations of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007) and Gurkaynak et al. (2010) for observed bond yields, respectively, available through the Federal Reserve web site. This data set contains constant maturity yields for maturities of 2 to 20 years. I construct equally weighted bond portfolios on short-term bonds (1 to 10 years maturity) and on long-term bonds (11 to 20 years maturity), each of them computed for Treasury bonds and for TIPS, ending up with four risky assets presented in Figure 3.1. The sample period is from January 2, 2004 to December 31, 2012.

Figure 3.1: Yearly return portfolios



Equally-weighted U.S. Government bond return portfolios calculated using daily data from January 2, 2004 to December 30, 2011.

For the same period, I also collect information on one-year Treasury bills from the Federal Reserve Board statistical releases. Following Ait-Sahalia and Brandt

(2001) and Ghysels and Pereira (2008) I assume Treasury bill is risky-free, and I fix the risk-free rate at its historical average. They argue that the constant risk-free rate assumption guarantees that any difference in the optimal portfolio functions across frequencies is solely due to the relation between returns and liquidity. In summary, the asset universe consists of the short-term Treasury bonds (weight α_{NS}), the long-term Treasury bonds (weight α_{NL}), the short-term TIPS (weight α_{RS}), the long-term Treasury bonds (weight α_{RL}) and the risk-free assets (weight α_{rf}).

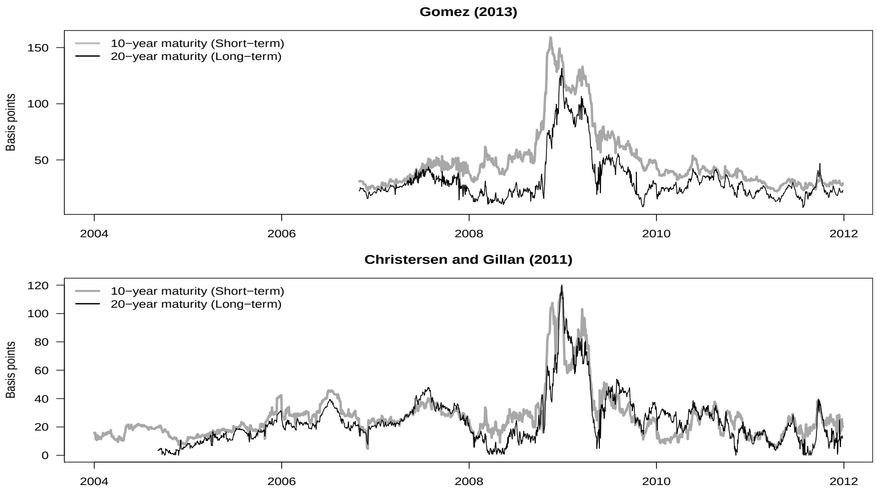
For liquidity, I use two-market based measures available on a daily frequency. The first measure is the liquidity measure proposed by Christensen and Gillan (2011). The data used to construct the liquidity premium proposed by Christensen and Gillan (2011) corresponds to daily estimates of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007) and Gurkaynak et al. (2010) for observed bond yields. For zero-coupon inflation swap rates, I use U.S. daily quotes from Barclays Live, which I have converted into continuously compounded rates to make them comparable to the other interest rates. I compute their liquidity measure, denoted by $\Delta_{n,t}$, for 10- and 20- years to maturity from January 2004 to December 2011.

The second measure is the asset swap liquidity premium used by Gomez (2013). I obtain daily nominal and TIPS z-spread asset swaps data from Barclays Live, starting in November 2006 until December 2011 for 10-years maturity (short-term portfolios liquidity) and 20-years maturity (long-term portfolios liquidity).[12] The residual spread between different TIPS and nominal z-spread asset swaps with the same maturity was calculated. Next, the average spread across different assets for each maturity was computed, and this corresponds to my liquidity premium measure, $L^{z\text{-}asw}_{n,t}$ for $n = 10, 20$ years maturity.

In Figure 3.2, I plot the evolution of liquidity premium measures $L^{z\text{-}asw}_{n,t}$ and $\Delta_{n,t}$ for short-term portfolios and long-term portfolios at a daily frequency. One can see that the values for both measures are strictly positive. Furthermore, $L^{z\text{-}asw}_{n,t}$ liquidity premiums tend to be downward sloping with maturity, indicating that the shorter-term liquidity premium is greater than the longer-term, especially during the crisis time. However, it seems not to be the case when liquidity is measured using the Christensen and Gillan (2011) measure. Additionally, the magnitude of the liquidity premium varies across measures. In fact, over the whole sample the mean short-term liquidity has been about 49 basis points for $L^{z\text{-}asw}_{n,t}$ compared with 29 basic points for $\Delta_{n,t}$. What is clear in both measures is that the liquidity premium grew substantially during the financial crises of 2008 and 2009. In fact, liquidity shows a peak in late 2008 during the financial crisis. In summary, although they are very similar and seem to be consistent (in the sense that they are able to capture the

[12] Asset swaps on bonds with less than 12 months to maturity are dropped from the estimation of the liquidity, because the effect of the indexation lag makes the prices of these securities erratic as was noted by Gurkaynak et al. (2010). All other asset swaps are included in the calculation.

Figure 3.2: Liquidity measures



The *z-asw* liquidity corresponds to the residual spread between TIPS and nominal bonds asset swaps calculated using daily data from November 1, 2006 to December 30, 2011. The maximum range liquidity corresponds to the difference between cash and synthetic break-even inflation rates proposed by Christensen and Gillan (2011) calculated using daily data from January 1, 2004 to December 30, 2012.

same events observed in the considered sample period), they show differences in the magnitude of statistics calculated from the two liquidity measures. For this reason, I am interested in testing whether or not the optimal portfolio choice depends on a particular way to measure liquidity premium.

Table 3.1 shows descriptive statistics of the liquidity predictors and holding period government bond portfolio returns, for the three investment horizons: one-month, one-quarter and one-year. The first lines in each panel show the mean, standard deviation, skewness and kurtosis for each liquidity measure and returns. By construction, and to facilitate the interpretation of the results, liquidity measures have a mean zero and standard-deviation equal to one (i.e. they have been standardized). The correlation coefficients between liquidity measures are more than 0.90. This suggests that all measures are capturing similar variations in the market bond yields. Also, there is evidence of fat tails in returns, especially at the shorter investment horizon. This tail risk suggests that the distribution is not normal, but skewed, and has fatter tails. The fatter tails increase the probability that an investment will move beyond three standard deviations. Nominal returns are negatively correlated with liquidity while TIPS returns are positively correlated. This means that as liquidity conditions worsen (higher liquidity premium), TIPS returns rise in order to compensate for the higher risk in bad times.

The following lines show the autocorrelation coefficients for different lags, which do not suggest persistence in most of the variables, especially at any frequency. The last line shows the p-value for the Dickey and Fuller test. The p-value for the Dickey and Fuller tests suggest the rejection of the null of a unit root for both short-term and long-term returns, and Christensen and Gillan (2011) 10-years liquidity. However, $L_{n,t}^{z\text{-}asw}$ seems not to be stationary. Given that the non-parametric approach requires stationary data, I would need transform each of those variables in order to make them a stationary series. However, it is not clear in Figure 3.2 that liquidity is not stationary. First, they are not moving along a decreasing or increasing time trend, and second, there are upward peaks related to the financial crisis, but before and after that they seems stationary. Consequently, I decided to work with the original series.

3.4 Empirical results

3.4.1 Unconditional portfolio weights

The goal in this section is to characterize the unconditional portfolio choice which serves as a benchmark for the conditional problem. Table 3.2 presents estimates of unconditional portfolio choices of investors with MV and CRRA preferences with different risk aversion degrees of $\gamma = 2$, 5, 10 and 20, and for three investment horizons. The entries in each column correspond to a portfolio choice between Treasury bills (assumed as risk-free) and one of the four different equally-weighted portfolio bonds: short-term nominal bonds (NS), long-term nominal bonds (NL), short-term TIPS (RS) or long-term TIPS (RL). That they do not impose short-sell constraints suggests a less realistic environment, mainly because the Markowitz portfolio tends to have very large quantities of individual assets (sometimes unreasonably so), I do not impose this restriction to make my results comparable with previous papers.

Several well-known features of optimal portfolio choice emerge. Consider the mean-variance portfolio choice weights. First, risk aversion affects how much wealth the investor allocates to risky securities instead of to the risk-free Treasury bill. The more risk-averse the investor, the less they will invest in the risky bond, so that long positions in risky bonds goes down with a higher degree of risk aversion. Second, given that this investor is forming his portfolio using only bonds and the risk-free Treasury bill, he/she will not want to short-sell the risky asset but rather will want to buy it on the margin (i.e. $\alpha > 1$). That means investors borrow money at risk-free rates and go long in risky bonds. For instance, an investor with an annual investment horizon and $\gamma = 20$ borrows 39% of wealth at the risk-free rate to invest a total of 139% in short-term nominal bonds portfolios. Finally, we see less large quantities of short-sales $(1 - \alpha)$ or, in some cases, no short-sales for the risk-free Treasury bill, for the same degree of risk aversion as the investment horizon increases. For example, an investor with $\gamma = 20$ goes short in the risk-free bond at the monthly frequency

Table 3.1: Descriptive Statistics for the portfolio measures of liquidity and bond returns

	Short-term				Long-term			
	$\Delta_{10,t}$	$L^{z\text{-}asw}_{10,t}$	R^N_{t+1}	R^{TIPS}_{t+1}	$\Delta_{20,t}$	$L^{z\text{-}asw}_{20,t}$	R^N_{t+1}	R^{TIPS}_{t+1}
Panel A: Monthly frequency								
Mean	0.00	0.00	1.04	1.02	0.00	0.00	1.06	1.03
Stdev	1.00	1.00	0.02	0.02	1.00	1.00	0.04	0.03
Skewness	2.58	1.98	0.03	-0.34	1.91	2.27	0.49	0.10
Kurtosis	11.15	6.33	3.70	6.30	8.36	7.97	5.60	5.91
Percentiles								
5%	-0.95	-0.85	1.01	0.99	-1.17	-0.86	0.99	0.98
50%	-0.19	-0.31	1.05	1.02	-0.18	-0.32	1.06	1.03
95%	2.15	2.54	1.07	1.05	2.06	2.75	1.11	1.07
Cross correlations								
$\Delta_{n,t}$	1.00				1.00			
$L^{z\text{-}asw}_{n,t}$	0.91	1.00			0.93	1.00		
R^N_{t+1}	0.05	0.07	1.00		-0.13	-0.11	1.00	
R^{TIPS}_{t+1}	0.33	0.28	0.46	1.00	0.18	0.15	0.59	1.00
Auto correlations								
1-day	0.99	1.00	0.95	0.96	0.99	0.99	0.95	0.94
2-day	0.98	0.99	0.91	0.92	0.98	0.98	0.90	0.89
5-day	0.95	0.98	0.80	0.78	0.95	0.96	0.77	0.72
22-day	0.76	0.89	0.07	0.06	0.77	0.81	-0.06	-0.11
Unit root test								
DF p-value	0.02	0.53	0.01	0.01	0.14	0.36	0.01	0.01
Panel B: Quarterly frequency								
Mean	0.00	0.00	1.05	1.02	0.00	0.00	1.06	1.03
Stdev	1.00	1.00	0.03	0.03	1.00	1.00	0.07	0.05
Skewness	2.58	1.98	0.04	-0.55	1.91	2.27	0.28	-0.26
Kurtosis	11.15	6.33	2.80	6.78	8.36	7.97	3.32	4.27
Percentiles								
5%	-0.95	-0.85	1.00	0.98	-1.17	-0.86	0.95	0.95
50%	-0.19	-0.31	1.04	1.02	-0.18	-0.32	1.06	1.04
95%	2.15	2.54	1.09	1.07	2.06	2.75	1.17	1.11
Cross correlations								
$\Delta_{n,t}$	1.00				1.00			
$L^{z\text{-}asw}_{n,t}$	0.91	1.00			0.93	1.00		
R^N_{t+1}	-0.14	-0.12	1.00		-0.23	-0.30	1.00	
R^{TIPS}_{t+1}	0.37	0.31	0.28	1.00	0.23	0.16	0.59	1.00
Auto correlations								
1-day	0.99	1.00	0.98	0.99	0.99	0.99	0.98	0.98
2-day	0.98	0.99	0.96	0.97	0.98	0.98	0.96	0.95
5-day	0.95	0.98	0.92	0.93	0.95	0.96	0.91	0.89
22-day	0.90	0.96	0.86	0.86	0.90	0.93	0.85	0.81
Unit root test								
DF p-value	0.02	0.53	0.01	0.01	0.14	0.36	0.01	0.01

but goes long in both long-term nominal bonds and the risk-free bond at longer investment horizons. The same situation occurs with long-term bonds with respect to short-term ones in the sense that we see less large quantities for a portfolio of long-term vs short-term bonds. This indicates that a smaller portion of the portfolio is devoted to risky assets as investment horizons increase or when long-run assets are available.

Results for CRRA preferences are very similar to those for MV. In theory, what differentiates a Mean-variance investor from a CRRA investor is that the latter has

Continuation: Descriptive Statistics

	Short-term				Long-term			
	$\Delta_{10,t}$	$L^{z\text{-}asw}_{10,t}$	R^{N}_{t+1}	R^{TIPS}_{t+1}	$\Delta_{20,t}$	$L^{z\text{-}asw}_{20,t}$	R^{N}_{t+1}	R^{TIPS}_{t+1}
Panel A: Annual frequency								
Mean	0.00	0.00	1.06	1.04	0.00	0.00	1.10	1.06
Stdev	1.00	1.00	0.04	0.05	1.00	1.00	0.09	0.08
Skewness	2.58	1.98	-0.14	0.02	1.91	2.27	0.16	0.06
Kurtosis	11.15	6.33	2.33	3.06	8.36	7.97	3.70	2.76
Percentiles								
5%	-0.95	-0.85	0.99	0.96	-1.17	-0.86	0.94	0.93
50%	-0.19	-0.31	1.06	1.04	-0.18	-0.32	1.09	1.07
95%	2.15	2.54	1.12	1.11	2.06	2.75	1.29	1.21
Cross correlations								
$\Delta_{n,t}$	1.00				1.00			
$L^{z\text{-}asw}_{n,t}$	0.91	1.00			0.93	1.00		
R^{N}_{t+1}	-0.50	-0.53	1.00		-0.60	-0.62	1.00	
R^{TIPS}_{t+1}	0.36	0.30	-0.04	1.00	0.00	0.03	0.46	1.00
Auto correlations								
1-day	0.99	1.00	0.99	1.00	0.99	0.99	0.99	0.99
2-day	0.98	0.99	0.98	0.99	0.98	0.98	0.98	0.98
5-day	0.95	0.98	0.96	0.97	0.95	0.96	0.95	0.95
22-day	0.76	0.89	0.83	0.86	0.77	0.81	0.78	0.82
Unit root test								
DF p-value	0.02	0.53	0.23	0.09	0.14	0.36	0.05	0.02

The *z-asw* liquidity premium corresponds to the residual spread between TIPS and nominal bonds asset swaps calculated using nominal and TIPS z-spread asset swaps rates. The other liquidity measure corresponds to the TIPS Liquidity proposed by Christensen and Gillan (2011). U.S. daily data from January 1, 2004 to December 30 2012 in basis points.

a preference for higher order moments and not only for the expected return and its variance, thus their risky position depends on relative risk aversion. However, empirical results in Table 3.2, show that investors seem not to be primarily affected in their decisions by the first two return moments. So, the effect of higher order moments of CRRA investors seem not to be strong enough, especially for TIPS. The biggest holding difference is for short-term nominal bonds at the monthly frequency, where CRRA investors with different levels of risk aversion tend to hold larger quantities.

There are important differences in the optimal portfolio weights between short-term and long-term nominal bonds with both types of preferences. In fact, equally risk-averse investors tend to hold bigger positions on short-term bonds relative to long-term ones, i.e. the short-term bond weight typically exceeds the long-term weight for the same kind of bond. However, these differences become smaller when the investment horizon become longer. Bonds with a longer maturity will usually pay a higher interest rate than shorter-term bonds. However, long-term bonds have greater duration than short-term bonds, so interest rate changes will have a greater effect on long-term bonds than on short-term bonds. As a result, investors are more conservative holding smaller positions in long-term bonds relative to short-term bonds, given that they would offer greater stability and lower risk.

Investors also hold bigger positions in nominal bonds relative to TIPS bonds. These differences could be attributed, at least in the case of CRRA investor, to the negative skewness in short-term TIPS bond returns for monthly and quarterly frequency, as Table 3.1 shows. Investors prefer positive skewness, because it implies a low probability of obtaining a large negative return. Then, investors tend to the extreme portfolios (Sharpe ratio driven, skewness driven or kurtosis driven) and avoid being stuck in the middle.

3.4.2 Conditional portfolio weights

Non-parametric optimal portfolio function

In this section I present the optimal portfolio weights as function of the liquidity differential between inflation-indexed bonds and nominal bonds (liquidity premium), represented by Z_t. I apply the utility maximization framework presented above with respect to Z_t. For each kernel grid point,[13] I optimize the portfolio weight by maximizing the representative agent's marginal utility in that state using a GMM inference technique. The portfolio weights that follow from the optimization of the expected utility under MV and CRRA preferences are presented in this section.

Table 3.3 shows estimates of the optimal conditional portfolio choice of investors (Weight) and their corresponding standard errors (Std) obtained by applying the Politis and Romano (1994) bootstrap procedure which is described in Appendix 3.5. I use this stationary bootstrap procedure to preserve autocorrelation properties of the data in the bootstrap samples.[14] The standard errors are presented only in order to assess the precision of the non-parametric method used. Each panel shows a different investment horizon (monthly, quarterly and annual), and they present the portfolio allocation problems considered before: two, where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset, and another two where the investor chooses between a portfolio of short-term or long-term TIPS and a risk-free asset, with each of them considering a MV and a CRRA investor.

[13] I define fifteen not evenly spaced realizations of the liquidity ranging from its mean minus one standard deviation to its means plus three standard deviations, which correspond to the interior 95% of the empirical distribution of the liquidity premium. Alternatively, I also define fifteen not evenly spaced realizations of the liquidity ranging between its minimum and maximum value, however results are broadly the same with both grids.

[14] This method is a variation of the standard block bootstrap that manages to create bootstrap series that are strictly stationary which accounts for the autocorrelation in the data.

Table 3.2: Unconditional Portfolio Weights

	Mean-Variance investor				Power Utility investor			
	Treasury		TIPS		Treasury		TIPS	
γ	Short-term	Long-term	Short-term	Long-term	Short-term	Long-term	Short-term	Long-term
				Monthly frequency				
2	79.75	15.58	33.53	12.32	139.32	10.15	7.26	6.57
	[29.70]	[3.48]	[5.73]	[1.80]	[5.91]	[0.77]	[1.16]	[0.45]
5	31.90	6.23	13.41	4.93	64.58	5.11	7.26	3.63
	[11.88]	[1.39]	[2.29]	[0.72]	[10.65]	[0.65]	[1.16]	[0.59]
10	15.95	3.12	6.71	2.46	30.15	2.79	3.97	1.95
	[5.94]	[0.70]	[1.15]	[0.36]	[6.50]	[0.41]	[0.72]	[0.35]
20	7.98	1.56	3.35	1.23	14.51	1.43	2.06	1.00
	[2.97]	[0.35]	[0.57]	[0.18]	[3.60]	[0.22]	[0.39]	[0.19]
				Quarterly frequency				
2	29.45	6.54	10.11	5.31	29.35	6.94	6.00	4.00
	[6.30]	[0.81]	[1.93]	[0.80]	[1.57]	[0.64]	[0.92]	[0.58]
5	11.78	2.61	4.05	2.12	14.04	3.01	2.88	1.82
	[2.52]	[0.33]	[0.77]	[0.32]	[1.45]	[0.36]	[0.60]	[0.35]
10	5.89	1.31	2.02	1.06	6.98	1.50	1.51	0.94
	[1.26]	[0.16]	[0.39]	[0.16]	[0.77]	[0.18]	[0.33]	[0.19]
20	2.95	0.65	1.01	0.53	3.44	0.75	0.77	0.47
	[0.63]	[0.08]	[0.19]	[0.08]	[0.39]	[0.09]	[0.17]	[0.09]
				Annual frequency				
2	13.91	4.71	3.45	3.23	14.78	3.62	3.45	3.20
	[1.60]	[0.64]	[0.73]	[0.37]	[1.34]	[0.39]	[0.84]	[0.49]
5	5.57	1.88	1.38	1.29	6.30	1.77	1.41	1.35
	[0.64]	[0.26]	[0.29]	[0.15]	[0.73]	[0.28]	[0.36]	[0.23]
10	2.78	0.94	0.69	0.65	3.11	0.92	0.71	0.68
	[0.32]	[0.13]	[0.15]	[0.07]	[0.37]	[0.15]	[0.18]	[0.12]
20	1.39	0.47	0.35	0.32	1.54	0.46	0.35	0.34
	[0.16]	[0.06]	[0.07]	[0.04]	[0.18]	[0.08]	[0.09]	[0.06]

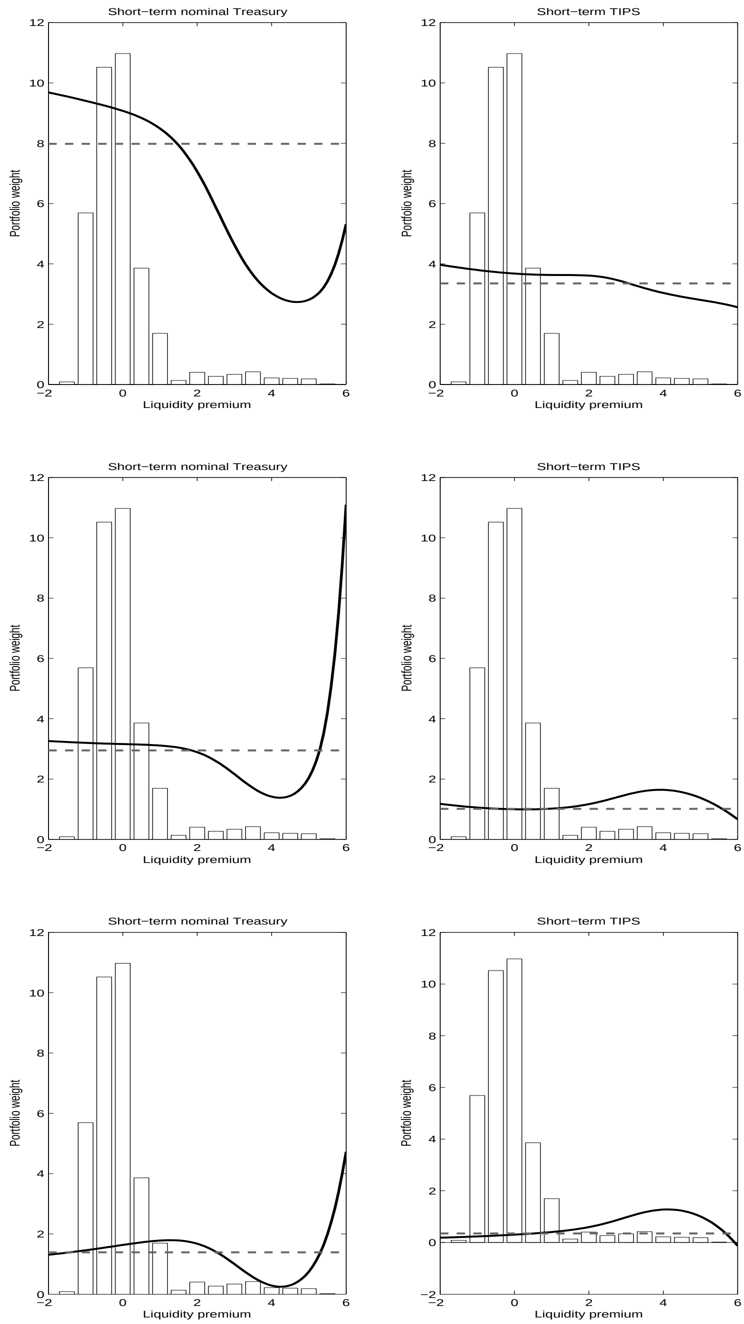
This table shows estimates of the optimal unconditional portfolio choice of investors. This is computed by applying a standard GMM procedure to the unconditional euler equation (3.10). Each panel shows a different investment horizon: monthly, quarterly and annual. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. Weights in the table correspond to the risky asset. In brackets are the Newey-West (12 lags) standard errors. I used U.S. data from January 1, 2004 to December 30 2011.

Table 3.3: Conditional Portfolio Weights ($\gamma = 20$)

	Mean-Variance investor								Power Utility investor							
	Treasury				TIPS				Treasury				TIPS			
	Short-term		Long-term		Short-term		Long-term		Short-term		Long-term		Short-term		Long-term	
Z	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev
								Monthly frequency								
-1	9.41	0.72	2.05	0.17	3.80	0.50	1.31	0.19	7.00	0.00	2.21	0.26	2.13	0.52	0.99	0.18
0	9.07	0.72	2.02	0.17	3.68	0.55	1.33	0.19	7.00	0.00	2.10	0.24	2.01	0.52	0.98	0.18
2	7.06	0.93	1.42	0.20	3.61	0.66	1.38	0.21	7.00	0.00	1.51	0.23	1.90	0.57	1.03	0.18
4	2.99	0.95	0.41	0.17	3.15	0.50	0.82	0.20	7.00	0.00	0.55	0.32	2.56	0.77	1.19	0.26
5	2.31	NaN	0.27	NaN	3.16	NaN	0.65	NaN	7.00	0.00	0.33	0.42	3.33	1.06	1.12	0.43
								Quarterly frequency								
-1	1.46	0.16	0.75	0.07	0.24	0.07	0.51	0.08	3.48	0.36	0.88	0.10	0.74	0.15	0.46	0.08
0	1.63	0.16	0.73	0.07	0.30	0.08	0.49	0.09	3.48	0.36	0.85	0.09	0.71	0.16	0.43	0.08
2	1.68	0.21	0.61	0.08	0.60	0.10	0.58	0.11	3.51	0.39	0.68	0.08	0.79	0.16	0.49	0.09
4	0.24	0.21	0.07	0.11	1.41	0.19	0.76	0.21	2.43	1.05	0.08	0.14	1.38	0.28	0.84	0.17
5	-0.21	NaN	-0.03	NaN	1.65	NaN	0.77	NaN	2.15	1.44	-0.03	0.19	1.88	0.47	1.04	0.27
								Annual frequency								
-1	0.24	0.08	0.59	0.06	0.24	0.08	0.29	0.04	1.63	0.17	0.71	0.07	0.23	0.08	0.31	0.05
0	0.30	0.08	0.62	0.06	0.30	0.09	0.30	0.05	1.78	0.17	0.72	0.07	0.30	0.08	0.32	0.05
2	0.60	0.09	0.50	0.08	0.60	0.11	0.39	0.06	1.72	0.17	0.42	0.08	0.52	0.09	0.38	0.06
4	1.41	0.21	-0.18	0.08	1.41	0.22	0.68	0.12	0.26	0.31	-0.18	0.07	1.20	0.16	0.76	0.12
5	1.65	NaN	-0.32	NaN	1.65	NaN	0.67	NaN	-0.23	0.42	-0.35	0.10	1.71	0.25	1.08	0.20

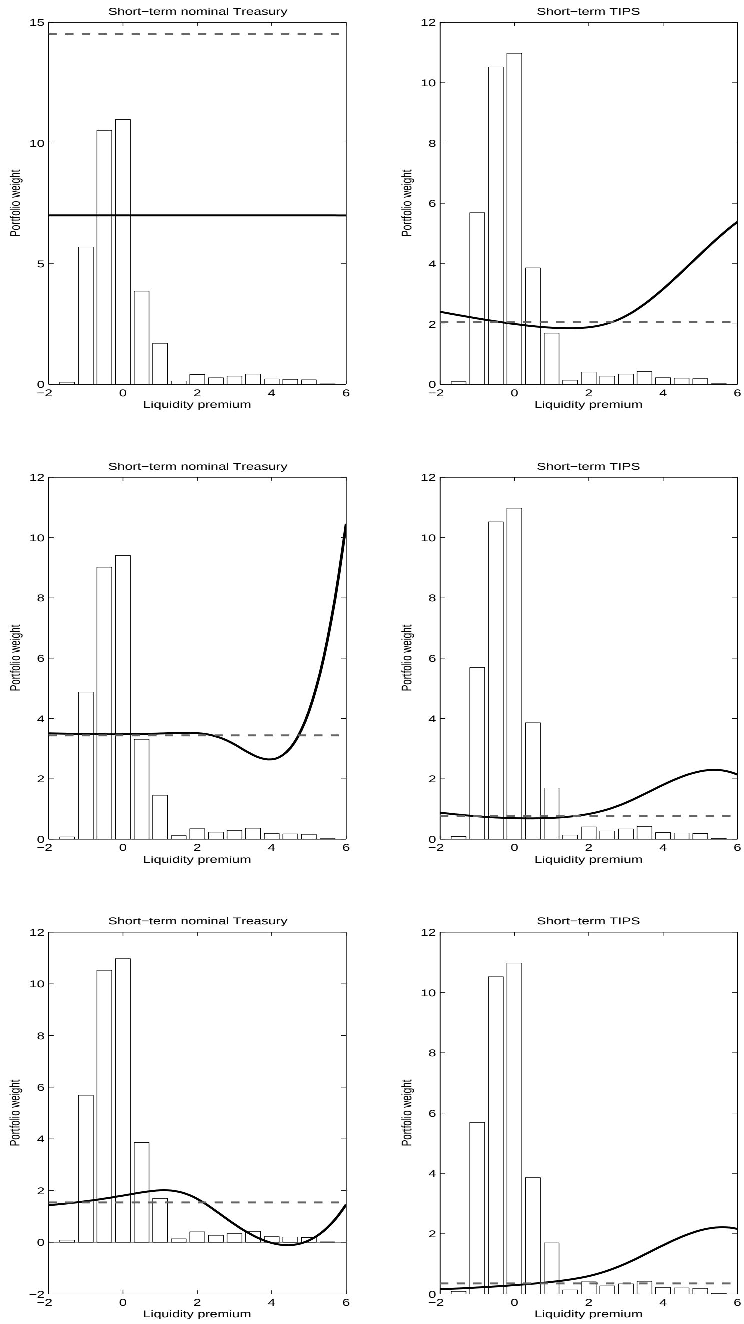
This table shows estimates of the optimal conditional portfolio choice of investors. This is computed by applying a standard GMM procedure to the conditional euler equation (3.9). Each panel shows a different investment horizon: monthly, quarterly and annual. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. Weights correspond to the risky asset. Standard errors (Std) are obtained applying the Politis and Romano (1994) bootstrap procedure. I used U.S. data from January 1, 2004 to December 30 2011.

Figure 3.3: Optimal portfolio weights as a function of 10-year liquidity premium (Mean-Variance investor)



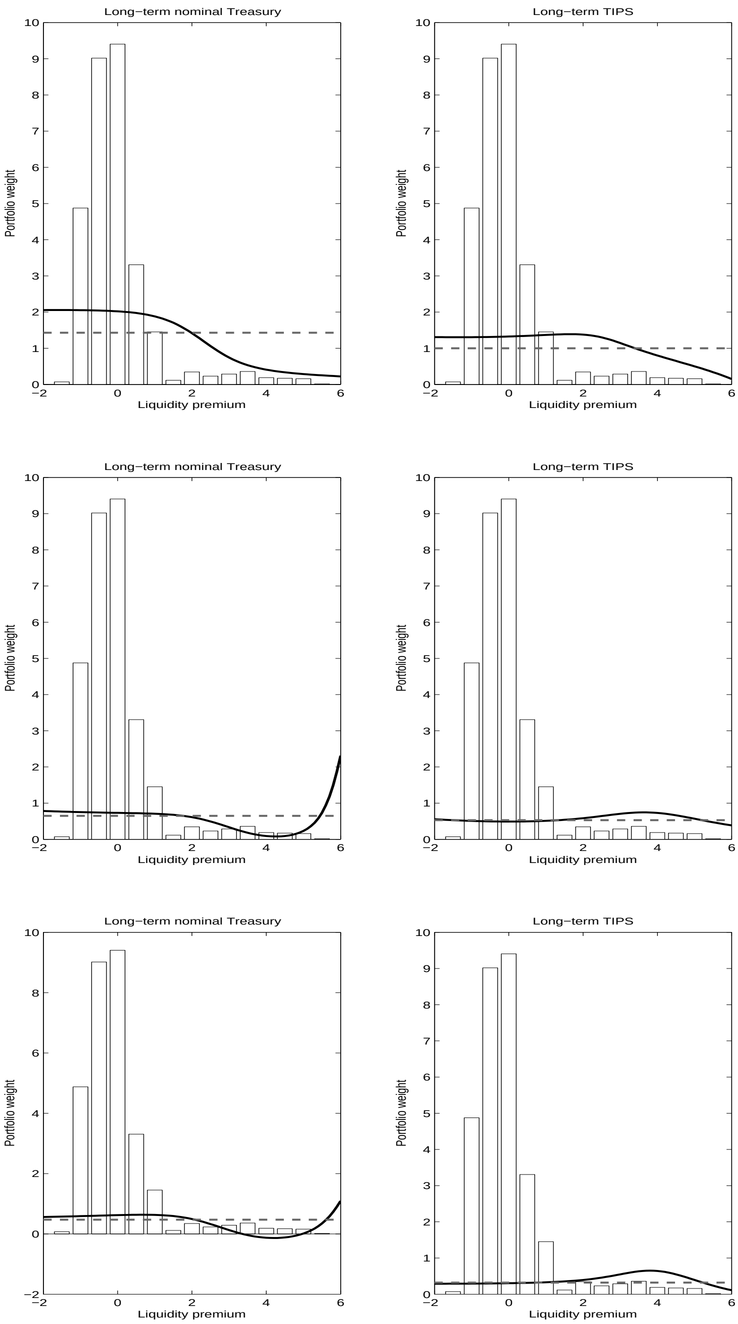
In each panel the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The bold line represents the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. In the first row, both the investment horizon and the rebalancing frequency are one-month; in the second row, one-quarter; and in the third, one-year.

Figure 3.4: Optimal portfolio weights as a function of 10-year liquidity premium (CRRA investor)



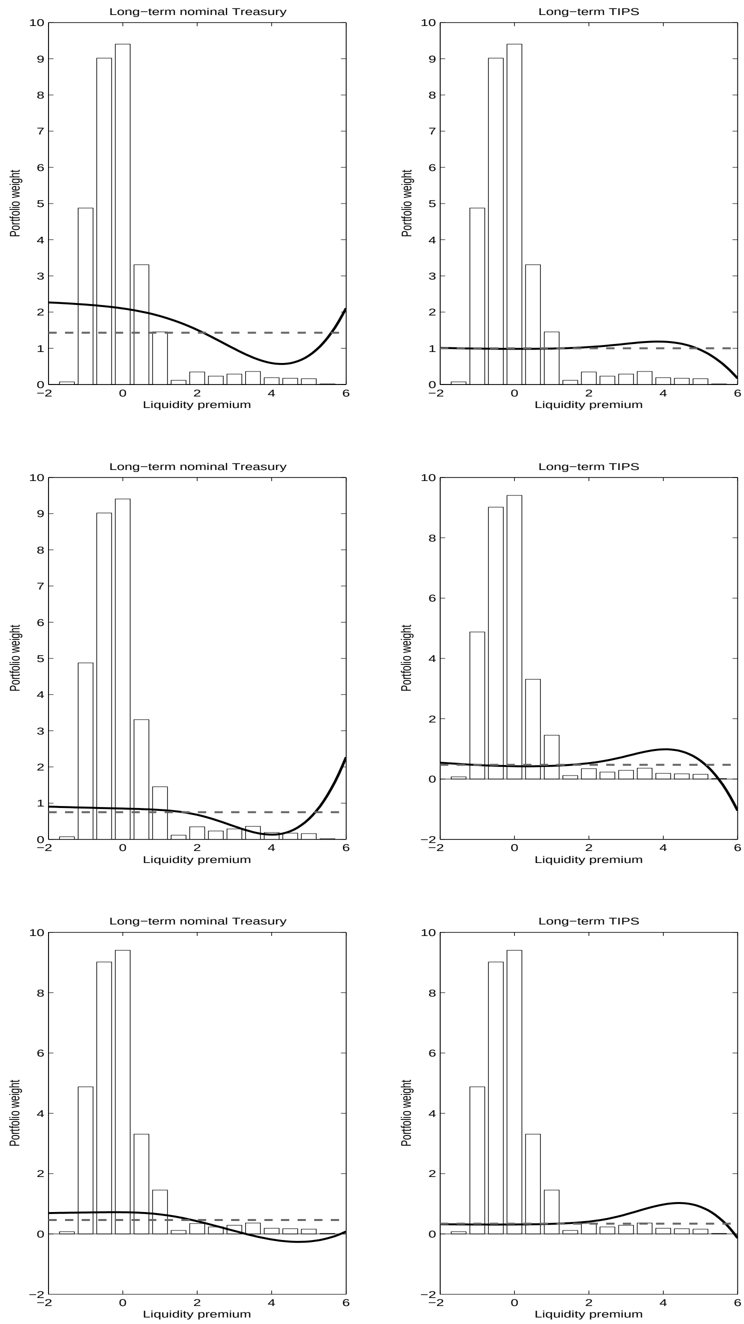
In each panel the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The bold line represents the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. In the first row, both the investment horizon and the rebalancing frequency are one-month; in the second row, one-quarter; and in the third, one-year.

Figure 3.5: Optimal portfolio weights as a function of 10-year liquidity premium (Mean-variance investor)



In each panel the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The bold line represents the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. In the first row, both the investment horizon and the rebalancing frequency are one-month; in the second row, one-quarter; and in the third, one-year.

Figure 3.6: Optimal portfolio weights as a function of 10-year liquidity premium (CRRA investor)



In each panel the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The bold line represents the optimal fraction of wealth allocated to the respective equally-weigthed U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. In the first row, both the investment horizon and the rebalancing frequency are one-month; in the second row, one-quarter; and in the third, one-year.

Figures 3.3 to 3.6 are the companion graphs to Table 3.3. Each figure shows the optimal portfolio weight as a function of liquidity $\alpha(Z_t)$ represented by the bold line. Additionally, in each figure the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram of liquidity premium (scaled to add up to 30). The left column contains the optimal fraction of wealth allocated to the respective equally-weighted U.S. nominal bond portfolio, and the right column to the equally-weighted TIPS bond portfolio. Finally, in the first row, both the investment horizon and the rebalancing frequency are one-month, in the second row, one-quarter, and in the third row, one-year.

Results presented in Table 3.3 and in Figures 3.3 to 3.6 correspond to the case when the coefficient of relative risk aversion is equal to $\gamma = 20$. The results for the other degrees of risk aversion considered in the unconditional case are not presented here in order to save space. They are available upon request.

A number of results emerge from this analysis. First, the liquidity premium seems to be a significant determinant of the portfolio allocation to U.S. government bonds. For instance, for a MV investor and at the monthly horizon, liquidity is a strong determinant of the allocation to short-term and long-term nominal bonds, with the optimal weight ranging from 9.41 at Liquidity= (-1) to 2.31 at Liquidity=5, as Table 3.3 shows. This indicates that an increase in the liquidity premium (i.e., liquidity conditions worsen) is accompanied by a strong decrease in the optimal allocation in short-term nominal bonds.

I have a similar result for the long-term nominal bonds with weights ranging from 2.05 to 0.27. Furthermore, liquidity also seems to be an important determinant of the allocation to TIPS. In this case, an increase in liquidity premium produces a decrease in the optimal allocation to both short-term, and long-term TIPS bonds. However, the effect is less strong with weights ranging from 3.80 to 3.16 for short-term, and from 1.31 to 0.65 for long-term for liquidity ranging between -1 and 5, respectively.

At quarterly and annual frequencies, optimal allocation still responds to changes in liquidity but mainly at high levels of liquidity premium. What we see is that the conditional weight is very close to the unconditional weight for low levels of liquidity (i.e. liquidity= -1 to 2), however optimal allocation starts to respond to changes in the liquidity when market liquidity conditions worsen (i.e. liquidity > 2). Interestingly, the investor tends to substitute cash for nominal bonds, and TIPS bonds for cash when the liquidity rises above its mean plus about 4 standard deviations, as Figures 3.3 and 3.5 show.

Second, conditional allocations in risky assets decrease as liquidity conditions worsen. In particular, an increase in the liquidity differential between nominal and TIPS bonds lead to: lower optimal portfolio allocations on nominal Treasury bonds, and also lower optimal portfolio allocations in TIPS, but at different levels of liquidity. When the liquidity premium is low (i.e. the liquidity differential between nominal and TIPS bonds is small), we see that the optimal allocation to either nominal or TIPS bonds is mostly unresponsive to liquidity premium, and it is very close to unconditional allocation. This occurs in the negative range of liquidity and also in the center of the distribution.

When the liquidity premium is high (i.e. in presence of big liquidity differentials between nominal and TIPS bonds), portfolio allocation on both nominal bonds and TIPS bonds decreases. However, this occurs at different levels of liquidity. In particular, the investor starts to decrease their position in nominal bonds at liquidity=2, but when there is insufficient liquidity, the investor holds a larger position in nominal bonds. On the other hand, portfolio allocation on TIPS bonds behaves in the reverse direction. That is, the investor only decreases asset allocation to TIPS in the upper positive part of liquidity (i.e. when the liquidity premium is very high), while between liquidity=2 and liquidity=4 TIPS bonds allocations increases, being above the unconditional value. Thus, in general, portfolio allocation for each type of bonds (nominal and TIPS) moves in cycles and each of them has its own cycle. Typically, when one type of bond is performing well, the other may not be performing as well in terms of liquidity, and the allocation rule reflects this situation.

Third, I find in general that the shape of the optimal portfolio policy functions of mean-variance and CRRA investors, with the same degree of risk aversion, are similar even though they have different levels (see Figure B3.1 in the Appendix 3.5). This suggest that investors seems to be primarily affected in their decisions by the first two return moments. Thus, the effect of higher order moments of CRRA investors exist but it seems not to be strong enough. However, this is not true at the monthly frequency. In this case, portfolio policies differ substantially which can be attributed to time variation in the higher order moments of the return distribution. This result is not induced by the choice of the kernel bandwidth, given that I explicitly control for it by constraining the kernel to be the same for the mean-variance and the CRRA preferences.[15]

Fourth, the effect of liquidity is a decreasing function of the investment horizon. For a given degree of risk aversion, the size of the optimal portfolio weight differs considerably across investment horizons. I find that as investment horizons became

[15]Non-parametric methods are typically indexed by a bandwidth or tuning parameter which controls the degree of complexity. The choice of bandwidth is often critical to implementation. In this application, the bandwidth is given by: $h = \lambda \sigma_z T^{-1/K+4}$, where $K = 1$ which is the dimension of Z (I am considering only one predictor variable which is liquidity), $\sigma(Z)$ is the standard deviation of the predictor variable, $T = 2086$ is the sample size and λ is a constant. For a big enough value of λ, I obtain a flat portfolio weight and small λ produce a very noise portfolio weight function. I consider values ranging from 9 to 3 for λ. These values guarantee bigger weight to an observation located at the mean of liquidity variable (which is zero), smaller weights to observations located one standard deviation away from the mean ($Z_t = \pm 1$), and even smaller weights to observations located two standard deviation away from the mean, etc. The results presented in this section correspond to $\lambda = 6$.

longer, the smaller the optimal portfolio weight, and the less that is invested in the risky asset. In particular, for the same degree of risk aversion investors react less abruptly to an increase in the liquidity premium when the investment horizon is one-year, than when the investment horizon is one-month.

For instance, we can see from Table 3.3 that when liquidity is equal to its mean ($Z_t = 0$) a MV investor with $\gamma = 20$ reduces the cash holdings from 2.02 to 0.62 when the investment horizon increases from one-month to one-year. This means that the investor borrows 102% of wealth at the risk-free rate to invest a total of 202% in short-term nominal bonds when the investment horizon is one-month. However, when the investment horizon becomes larger, the investor takes a long position in both assets holding 62% of their wealth in short-term nominal bonds and 38% in cash. The same occurs when I consider a CRRA investor. For example, considering the same case, but for long-term TIPS bonds, a CRRA investor reduces their bonds positions from 98% to 32%, as Table 3.3 shows.

Fifth, different degrees of risk aversion mainly change the level of the portfolio function but have little impact on the shape of this function, as is shown in Figure 3.7. In this figure, I only plot the portfolio policies for the long-term nominal (left column) and TIPS bonds (right column) for a one-year investment horizon. The first row in the figure corresponds to a mean-variance investor, and the second row to a CRRA investor. Finally, in each panel bold black lines represent an investor with $\gamma = 5$, the bold grey line with $\gamma = 10$ and the dotted line with $\gamma = 20$. Looking at Figure 3.7, we see that the more risk-averse the investor becomes, the smaller the optimal portfolio weight, so the less that is invested in the risky asset. Furthermore, more risk-averse investors react less abruptly to an increase in the liquidity premium.

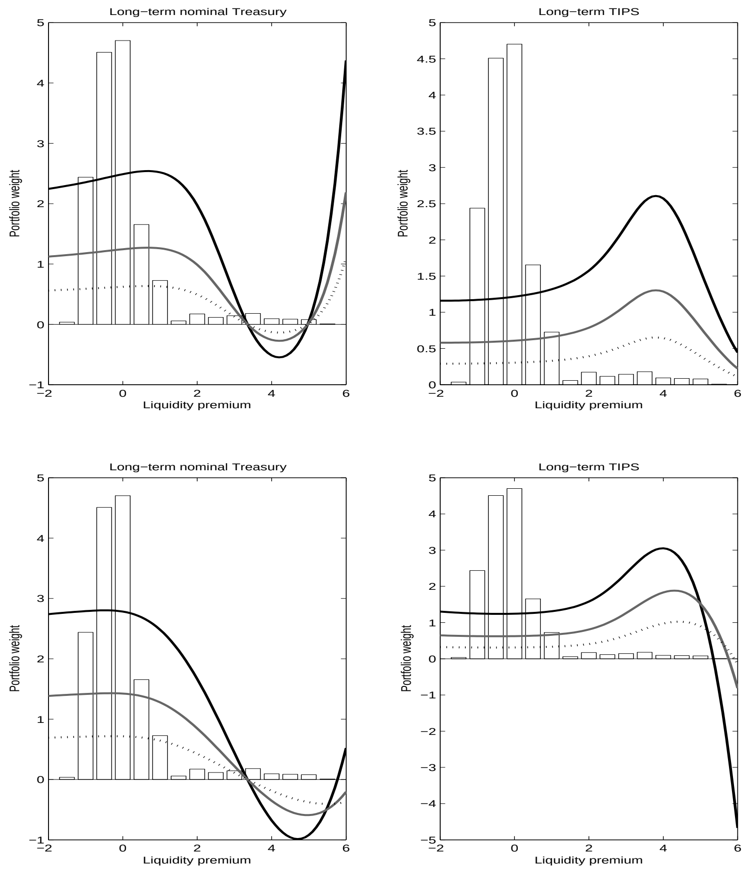
To summarize, and in general, results consistently show that the optimal allocation to short-term or long-term bonds is mostly unresponsive to changes in liquidity conditions at low levels (i.e. at liquidity= -1 to 4). However once liquidity reaches certain levels (liquidity > 4), which indicates that market liquidity conditions have worsened, then the investor starts to respond by decreasing the positions in TIPS and increasing the position in nominal bonds.

Additionally, the above conclusion is not determined by the level of risk aversion, the investment horizon or the investor preferences. The relation between optimal portfolio weights and liquidity premium remains the same for different values of risk-aversion, different investment horizons and also across investors' preferences. The characteristics mainly change the level of the portfolio function that have a small impact on the function shape, except for the monthly frequency.

Do weights really respond to changes in liquidity?

The main question of this paper is whether or not the weights respond to changes in liquidity. To test whether or not a portfolio weight is statistically different from

Figure 3.7: Optimal portfolio weights as a function of 10-year liquidity premium (Mean-variance and CRRA investor with different values for γ)



The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The lines represent the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. Bold black line represent an investor with $\gamma = 5$, the bold grey line for $\gamma = 10$ and dotted line for $\gamma = 20$. In the first row correspond to the case of mean-variance investor and the second row to the CRRA investor. The investment horizon and the rebalancing frequency in this figure correspond to one-year.

zero is pointless in this context, simply because it does not provide an answer for the question asked above. What I do next, following Ghysels and Pereira (2008), is to statistically test this question by using the following approximation:

$$H_0 : \frac{\partial \alpha(Z)}{\partial Z} |_{Z=\bar{Z}} \quad \cong \frac{\alpha(\bar{Z}+0.1) - \alpha(\bar{Z}-0.1)}{0.2} = 0 \tag{3.11}$$

where the first derivative of $\alpha(Z)$ is approximated by a finite difference which allows me to compute the slope of the optimal portfolio weight function at each value of the predictor variable.

Table 3.4 shows the point estimate slopes and t-stat computed using the standard errors obtained also from the Politis and Romano (1994) stationary bootstrap procedure. I draw one main conclusion from this table which is consistent with the results presented above. It is clear that optimal portfolio policy is not linear or constant in liquidity. For the two investor preferences the short-term nominal and the TIPS bonds portfolio policy responds to changes in liquidity. This conclusion is derived from the fact that the null hypothesis is rejected indicating that all slopes are statistically significant at the 10% level or less. The only case where slopes are not statistically significant is for short-term TIPS bonds with MV preferences. The other case where we can not reject the null hypothesis is for short-term nominal bonds with CRRA preferences. In this case, the optimal portfolio function is constant but smaller than the unconditional weight.

For long-term TIPS, $\alpha(Z_t)$ is almost constant and statistically not different from zero over the negative range of liquidity until $Z_t = 2$. After that the slopes are positive and over the last range of liquidity they are negative and statistically significant. I find the same results for both investor preferences. The optimal portfolio function for long-term nominal bonds goes in the opposite way. It starts by being flat and statistically not different from zero, then slopes become negative, and over the the end range of liquidity, slopes are positive and statistically significant.

Overall, I can conclude that optimal portfolio choice is unresponsive over the negative and first positive range of liquidity, however portfolio allocations start to react as liquidity conditions worsen. This conclusion regarding the general shape of the portfolio weight functions is reliable in the sense that non-parametric techniques used here produce a consistent estimator of the portfolio functions.

Robustness analysis: Parametric portfolio functions

Parametric portfolio functions

To analyze whether or not the shape of the optimal portfolio functions presented in this section are robust to a particular choice for the constant λ, I also estimate a parametric portfolio function. This is to confirm the results obtained above. In accordance with the shapes of the portfolio functions obtained before, I use a third degree polynomial in liquidity (Z_t) to approximate the estimated non-parametric portfolio policy function

$$\alpha^p(Z_t) = a_0 + a_1 Z_t + a_2 Z_t^2 + a_3 Z_t^3. \tag{3.12}$$

The parametric optimal portfolio weight is computed by applying a standard GMM procedure to the conditional Euler equation. In this case the moment

Table 3.4: Point estimates for the slope of the conditional portfolio weight function

	Mean-Variance investor								Power Utility investor							
	Treasury				TIPS				Treasury				TIPS			
	Short-term		Long-term		Short-term		Long-term		Short-term		Long-term		Short-term		Long-term	
Z	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat
Monthly frequency																
-1	-0.30	-1.62	-0.01	-0.31	-0.15	-1.14	0.01	0.18	0.00	-0.03	-0.08	-1.49	-0.14	-2.02	-0.01	-0.53
0	-0.40	-2.19	-0.06	-1.10	-0.08	-0.58	0.03	0.76	0.00	-0.26	-0.15	-2.39	-0.11	-1.63	0.00	-0.05
2	-2.08	-3.33	-0.67	-4.76	-0.07	-0.35	-0.07	-0.62	0.00	0.07	-0.47	-3.96	0.05	0.58	0.06	1.42
4	-1.01	-2.01	-0.19	-1.64	-0.14	-0.52	-0.24	-2.75	0.00	0.61	-0.31	-2.06	0.66	1.68	-0.01	-0.03
4.5	-0.71	-1.67	-0.14	-1.27	-0.03	-0.05	-0.18	-2.51	0.00	-1.76	-0.23	-1.62	0.76	1.77	-0.07	-0.31
6.0	2.43	4.31	-0.10	-1.30	-0.01	-0.05	-0.13	-2.35	0.00	0.65	0.42	0.93	0.91	2.24	-0.06	-0.28
Quarterly frequency																
-1	0.17	4.26	-0.03	NaN	0.06	3.08	-0.03	-1.60	-0.02	-0.24	-0.03	-1.08	-0.05	-2.43	-0.03	-1.06
0	0.18	2.81	-0.02	-1.00	0.08	3.95	0.00	-0.05	0.01	0.10	-0.03	-1.04	-0.02	-1.10	-0.01	-0.72
2	-0.35	-2.85	-0.18	-4.56	0.28	6.51	0.10	2.42	-0.08	-0.33	-0.23	-3.31	0.13	3.78	0.08	3.26
4	-0.62	-3.18	-0.17	-1.98	0.31	2.52	0.01	0.04	-0.55	-1.05	-0.18	-2.29	0.46	2.21	0.24	1.84
4.5	-0.53	-2.82	-0.13	-1.61	0.27	2.18	0.00	0.03	-0.44	-0.84	-0.14	-1.79	0.48	2.18	0.23	1.71
6.0	0.93	2.76	0.65	1.65	-0.31	1.96	-0.05	-0.91	0.57	0.54	0.45	1.68	0.53	2.07	-0.43	-1.08
Annual frequency																
-1	0.06	3.23	0.03	3.00	0.06	2.78	0.01	0.66	0.14	5.12	0.02	1.12	0.05	3.06	0.00	-0.15
0	0.08	4.26	0.03	1.48	0.08	3.39	0.02	1.46	0.15	3.41	-0.02	-0.61	0.07	3.74	0.01	0.64
2	0.28	6.16	-0.26	-4.02	0.28	5.94	0.10	4.41	-0.45	-3.13	-0.29	-5.80	0.19	5.90	0.08	3.34
4	0.31	1.83	-0.21	-3.64	0.31	2.04	0.04	0.54	-0.65	-3.75	-0.24	-4.48	0.48	3.96	0.30	2.78
4.5	0.24	1.26	-0.15	-2.61	0.24	1.54	-0.01	-0.21	-0.52	-3.04	-0.18	-3.45	0.50	3.66	0.31	2.55
6.0	0.87	1.45	0.54	2.34	-0.33	1.98	-0.11	-0.76	0.37	1.92	0.06	1.57	0.12	3.25	-0.34	-2.48

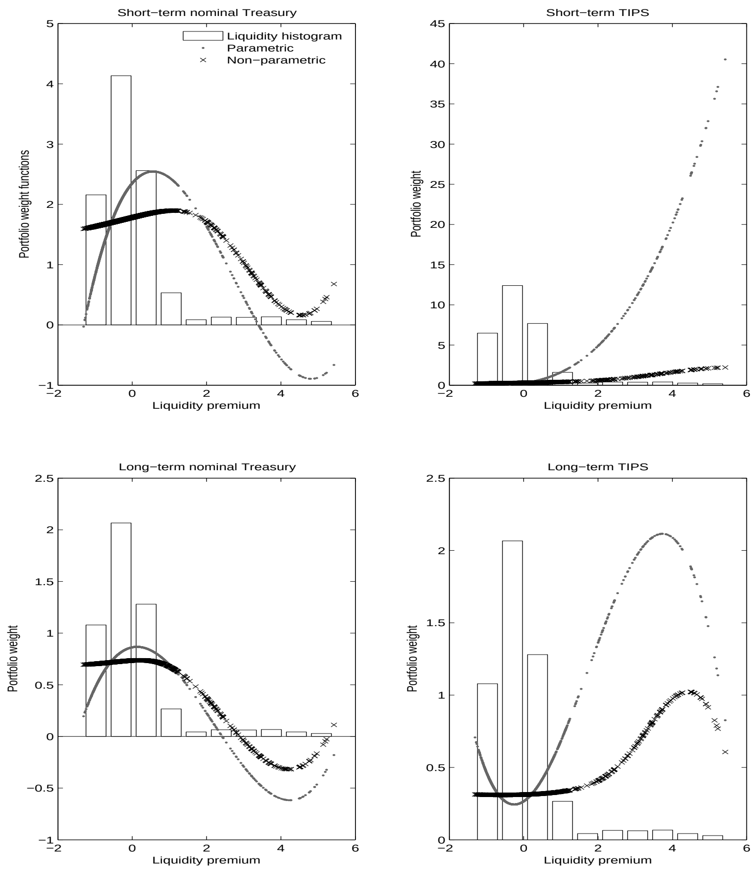
This table shows the point estimates slopes and their standard errors obtained from Politis and Romano (1994) stationary bootstrap procedure. This is computed by approximating the first derivative of $\alpha(Z)$ by equation (3.11). Each panel shows a different investment horizon: monthly, quarterly and annual. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. I used U.S. data from January 1, 2004 to December 30 2011.

condition is

$$\hat{\mathbb{E}}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha} \mid Z_t = \bar{z}\right] = \frac{1}{T}\Sigma_{t=1}^{T}\left(\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right) \otimes g(Z_t) = 0, \quad (3.13)$$

where $g(Z_t) = [1, Z, Z^2, Z^3]$. The constants a_0, a_1, a_2, a_3 are reported in Table 3.5, and a comparison between the parametric and non-parametric optimal portfolio functions is plotted in Figure 3.8.

Figure 3.8: Optimal portfolio weights as a function of 10-year liquidity premium (Parametric vs Non-parametric functions)



The bars in the background represent the histogram (scaled to add up to 10) of liquidity premium. The lines represent the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. The investment horizon and the rebalancing frequency in this figure correspond to one-year and I assume CRRA preferences.

Figure 3.8 compares the parametric optimal portfolio function, obtained from the polynomial model, with the non-parametric function in the case of CRRA preferences. What we see is that both functions have approximately the same shape (except for short-term TIPS), which is an indication that results are broadly consistent. I confirm that portfolio policies are nonlinear, and the comparison of policies at different points of the liquidity distribution shows a large variation in the optimal allocation, being the effect of liquidity strong when it is greater than three standard deviations above its mean. However, results must be interpreted with caution since the data density at the margins of the empirical distribution of the predictor variable is small. It is well known that an empirical distribution is a noisy model of the true distribution in the tail area.

Table 3.5 gives estimates of the parameter a_i for the optimal portfolio function defined in equation (3.12) for a CRRA investor. I will focus on the annual frequency, which contains the companion results for Figure 3.8, and we see that most of the coefficients are statistically significant at 5% o more. a_1, which is the slope of the portfolio function in the center of the distribution ($Z_t = 0$), is positive and statistically significant for short-term nominal and TIPS bonds, but it is not statistically significant for long-term bonds. This means that portfolio weights do not respond to changes in liquidity at the central range of liquidity. The result is consistent with the non-parametric results.

Alternative market-based measure of liquidity

As an additional robustness check, I also consider an alternative market-based measure of liquidity. This examines whether or not the results presented here depend on a particular way to proxy the liquidity differential between inflation-indexed bonds and nominal bonds (liquidity premium), represented by Z_t. Figure C3.1, in Appendix 3.5, shows the optimal fraction of wealth allocated to equally-weighted U.S. bond portfolios as a function of liquidity premium measures using Gomez (2013). Looking at Figure C3.1, first I confirm the conclusion that liquidity constitutes relevant conditioning information in the portfolio choice problem. Second, I conclude that results are robust to the liquidity premium measure used, in the sense that the shape of the optimal allocation policy is approximately the same with both measures of liquidity. Finally, liquidity measures are available for different maturities (10-years and 20-years), however results do not depend on a particular choice of the maturity of the liquidity premium (results are available upon request). This implies that both market-based measures of liquidity are capturing time variations in investment opportunities.

3.4.3 Does bond return predictability imply improved asset allocation and performance?

From the standpoint of practical advice to portfolio investors, an additional natural question to ask is whether or not the bond return predictability translates into

Table 3.5: Parametric conditional portfolio weight function (CRRA investor)

Monthly frequency

	Short-term Nominal				Short-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	10.81	-6.35	25.75	-4.61	1.65	-1.19	2.17	0.06
t-stat	4.37	-2.17	2.00	-1.92	3.53	-1.76	2.29	0.11
p-value	0.00	0.03	0.05	0.06	0.00	0.08	0.02	0.91
	Long-term Nominal				Long-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	2.06	-0.51	0.08	-0.01	0.91	0.00	0.30	-0.06
t-stat	9.05	-1.42	0.40	-0.42	4.21	0.00	1.41	-1.63
p-value	0.00	0.16	0.69	0.67	0.00	1.00	0.16	0.10

Quarterly frequency

	Short-term Nominal				Short-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	3.48	-0.01	0.05	-0.03	0.49	0.02	2.06	-0.38
t-stat	6.64	-0.02	0.09	-0.30	2.62	0.05	4.05	-4.14
p-value	0.00	0.99	0.93	0.77	0.01	0.96	0.00	0.00
	Long-term Nominal				Long-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	0.83	-0.20	-0.01	0.00	0.35	-0.06	0.52	-0.10
t-stat	6.67	-1.37	-0.07	0.04	3.41	-0.33	3.00	-3.07
p-value	0.00	0.17	0.94	0.97	0.00	0.74	0.00	0.00

Annual frequency

	Short-term Nominal				Short-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	2.35	0.72	-0.72	0.09	0.25	0.62	0.60	0.12
t-stat	8.19	3.35	-4.16	2.46	1.99	2.05	2.38	0.54
p-value	0.00	0.00	0.00	0.01	0.05	0.04	0.02	0.59
	Long-term Nominal				Long-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	0.86	0.07	-0.28	0.04	0.27	0.17	0.31	-0.06
t-stat	7.27	0.93	-4.44	3.65	3.49	1.39	2.93	-2.59
p-value	0.00	0.35	0.00	0.00	0.00	0.16	0.00	0.01

Each panel gives estimates of the parameter a_i for the optimal portfolio function defined in equation (3.12). These estimates are computed through GMM on the moment condition (3.13), using a Newey and West estimator of the spectral density matrix. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. Each panel shows a different investment horizon: monthly, quarterly and annual. I used U.S. data from January 1, 2004 to December 30 2011.

improved out-of-sample asset allocation and performance. The idea is that at the start of each period (one-month, one-quarter or one-year), one investor makes portfolio allocations based on the belief that bond returns are predictable by liquidity. I compare his/her performance to that of another investor who believes that bond returns are independent and identically distributed (i.i.d.), and ignores any evidence of bond return predictability in making his/her portfolios allocation choices.

I used rolling estimation approach, which consists of estimating a series of out-of-sample portfolio returns by using a rolling estimation window over the entire data set. Specifically, I choose an estimation window of length M=260 days (1 year). In each day, starting from $t = M + 1$, I use the data in the previous M days to estimate the optimal portfolio weights. In other words, each investor has an investment horizon of one-year and uses all data available until period $T-M$ to choose his/her first portfolio weights. Next, I use those weights to compute the portfolio returns. Repeating this procedure, involve adding the information for the next period in the data set and dropping the earliest period (keeping the window length fixed), until the end of the data set is reached. In this way, I obtain a time series of portfolio returns for each (unconditional and conditional) strategy.

To compute out-of-sample performance of this two different strategies, I compute the out-of-sample Sharpe ratio of strategy j, defined as the sample mean of out-of-sample excess returns (over the risk-free asset), μ_j, divided by their sample standard deviation, σ_j, for strategy $j = U, C$

$$SR_j = \frac{\mu_j}{\sigma_j}. \tag{3.14}$$

In addition, I calculate the certainty equivalent rates of return (CER) for each strategy to judge its relative performance. The CER represents the risk-free rate of return that investor is willing to accept instead of undertaking the risky portfolio strategy. Formally, I compute the CER of strategy j

$$CER_j = \mu_j - \frac{\gamma}{2}\sigma_j^2, \tag{3.15}$$

where μ_j and σ_j^2 are the mean and variance of out-of-sample excess returns for strategy $j = U, C$. To test whether or not the Sharpe ratios, and the certainty equivalent returns of two strategies are statistically distinguishable, I test the following null hypothesis $Ho : SR_U - SR_C$ and $Ho : CER_U - CER_C$. This difference represents the gain (or loss) in returns from investing in unconditional strategy versus conditional strategy. I compute the p-value of the differences by using the Politis and Romano (1994) stationary bootstrap procedure ($pv - boot$).[16] Finally, an useful benchmark are the in-sample Sharpe ratios and the certainty equivalent returns (to assess the effect of estimation error), calculated for the different portfolio strategies by using the entire time series of excess returns.

Table 3.6 shows results assuming both investors are mean-variance optimizer with a one-year investment horizon, and $\gamma = 10$. Panel A shows the CER and the SR calculated with the entire data set (in-sample analysis). The in-sample Sharpe ratios are all positive (except for short-term nominal bonds), being the performing of the conditional strategy better than the unconditional strategy for all portfolios.

[16] I replicate the process described in Appendix 3.5 1000 times. For each such replication, I compute the optimal allocations for each investor through one year (260 days). At every point in time, the investors are allowed to utilize just the information available up to that point in time. I calculate the difference in certainty equivalent between the two strategies and the adjusted Sharpe ratio for each replication. Finally, I count the proportion of times in 1000 replications that these differences exceed the certainty equivalent and adjusted Sharpe ratio based on the original data for a given set of results.

Table 3.6: Sharpe ratios and certainty equivalent returns (Mean-Variance investor with $\gamma = 10$)

Panel A: In- sample results

		Unconditional		Conditional		Differential		Ho:differential=0	
		SR	CER	SR	CER	$SR_U - SR_C$	$CER_U - CER_C$	$pv - boot$	$pv - boot$
Treasury	Short-term	0.2891	1.5005	-0.1912	0.3447	0.4812	1.1558	0.0460	0.0484
	Long-term	0.1286	0.5358	0.3632	0.6010	-0.2346	-0.0652	0.0479	0.0014
TIPS	Short-term	0.2188	0.3602	0.4114	0.9541	-0.1926	-0.5939	0.0480	0.0004
	Long-term	0.1399	0.3502	0.8102	0.8110	-0.6703	-0.4608	0.0404	0.0004

Panel B: Out-of-sample results

		Unconditional		Conditional		Differential		$p - value$	
		SR	CER	SR	CER	$SR_U - SR_C$	$CER_U - CER_C$	$pv - boot$	$pv - boot$
Treasury	Short-term	0.2823	1.5886	-0.2261	0.3204	0.5084	1.2682	0.0553	0.0548
	Long-term	0.1220	0.5540	0.3413	0.6456	-0.2193	-0.0916	0.0504	0.0040
TIPS	Short-term	0.2085	0.3351	0.3520	1.1332	-0.1435	-0.7981	0.0541	0.0005
	Long-term	0.1295	0.2894	0.7521	0.8846	-0.7551	-0.5952	0.0464	0.0006

This table reports the out-of-sample CER returns for two different investor strategies: unconditional (bond returns are i.i.d) and conditional (bond returns are predictable) strategy. The $p - values$ of the difference between SR, and CER from each strategy are obtained applying the Politis and Romano (1994) bootstrap procedure. The complete data set correspond to U.S. data from January 1, 2004 to December 30 2011.

For instance, for a nominal long-term portfolio the Sharpe ratio of unconditional strategy is equal to 0.12 versus 0.36 of the conditional strategy, indicating that with the conditional strategy the investor takes on less risk to achieve the same return. For the same portfolio, the CER_U is equal to 0.53 vs 0.60 of the CER_C. This means that an investor requires a higher risk-free return to give up the opportunity to invest in the portfolio following a conditional strategy.

Similarly, the difference between the in-sample SR for the unconditional and conditional strategy shows the loss (given that I obtain negative values) from investing, based on the belief that bond returns are i.i.d. This means that the bond return predictability translates into improved in-sample asset allocation and performance. The comparison of in-sample certainty equivalent returns and their differences, confirms the conclusions from the analysis of Sharpe ratios. Finally, the difference between the Sharpe ratios and certainty equivalent returns of each strategy are statistically significant in all cases, as $pv - boot$ values indicate.

Next, I assess the magnitude of the potential gains that can actually be realized by an investor, using the out-of-sample performance of the strategies. From panel B of Table 3.6, we see that in all cases the SR for the portfolios from the conditional strategy is much higher than for the unconditional strategy. I find the same results for CER. This means that a conditional strategy outperforms the unconditional strategy. This suggests also that conditional strategy might improve, not only in-sample but also out-of-sample performance. The significance of the CER differential and the SR differential, which is measure using the stationary bootstrap technique proposed by Politis and Romano (1994), implies that this result is statistically significant.

Finally, the difference between the in-sample and out-of-sample strategies allows me to gauge the severity of the estimation error. From the out-of-sample Sharpe ratio, reported in Panel B of Table 3.6, the unconditional strategy does not have a substantially lower Sharpe ratio and certainty equivalent returns out-of-sample than in-sample. This means that the effect of estimation error seems not to be so large. Consequently, it does not erodes the gains from optimal diversification given that differences turn out not to be economically important.

3.5 Conclusions

Although many studies on the liquidity premium have been conducted, the implications for investors are rarely addressed in any detail. In order to draw conclusions from the effect of the liquidity risk premium from an investor's point of view, it is necessary to specifically analyze optimal portfolio compositions in realistic settings. This is the focus of this paper.

I consider the portfolio problem of a mean-variance and a power utility investor whose portfolio choices are between the asset of interest and a risk-free asset. The investor's problem is to choose optimal allocations to the risky asset as a function of predictor value: liquidity premium. In this paper, I use two alternative measures recently proposed in the literature for the liquidity differential between inflation-indexed bonds and nominal bonds. These assess whether or not liquidity changes influence optimal portfolio allocations in the U.S. government bond market. While these issues have been well studied for stock-only portfolios, in general, less has been done to provide empirical evidence for the optimal portfolio choice of a utility-maximizing risk-averse investor, conditional upon observing a particular liquidity signal.

Overall, results show that optimal portfolios vary substantially with regards to predictor value. In particular, the effect of liquidity is a decreasing function of the investment horizon. Additionally, conditional allocations in risky assets decrease as liquidity conditions worsen. However, once the liquidity differential between U.S. nominal Treasury and TIPS bonds is sufficiently large, it leads to: (*i*) lower optimal portfolio allocations in TIPS; and (*ii*) higher optimal portfolio allocations on nominal bonds with respect to the risk-free bond. To summarize, this paper suggests that market liquidity signals could provide valuable guidance to investors, and adds to the evidence found for stock portfolios by Ghysels and Pereira (2008), which suggests the existence of a dependence of the optimal portfolio choices on changes in liquidity.

Appendix A3

Bootstrap procedure

Algorithm 3.1 Politis and Romano (1994) stationary bootstrap procedure

Require: Considering the equally weighted bond returns portfolio R_t

Ensure: that the data are re-sampled in blocks where the block length has a geometric distribution with a mean of $1/q$.

1: Randomly select an observation, say, R_t^N, from the original time series

2: With a fixed probability q, select the next observation randomly from the original time series, and with probability $(1-q)$ select it as the next observation to R_t (i.e., select R_{t+1}^N) from the original time series.

3: Repeat this process to generate a pseudo time series of desired length.

4: Construct bootstrap samples of $rx_{t+1}^{(n)}$ by using the bootstrap samples of $\mathbf{X}_t$ and resampling blocks of w subsequent residuals $\varepsilon_{t+1}^{(n)}$.

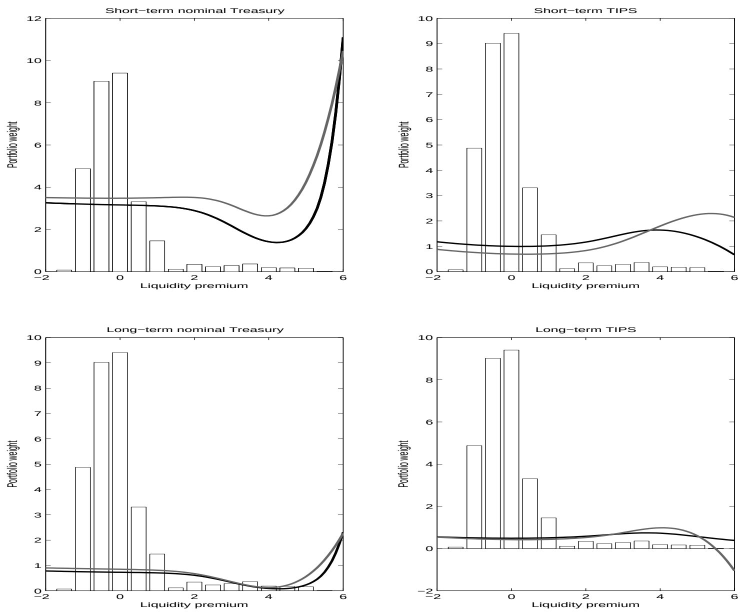
5: Repeat the bootstrap procedure 1000 times.

Appendix B3

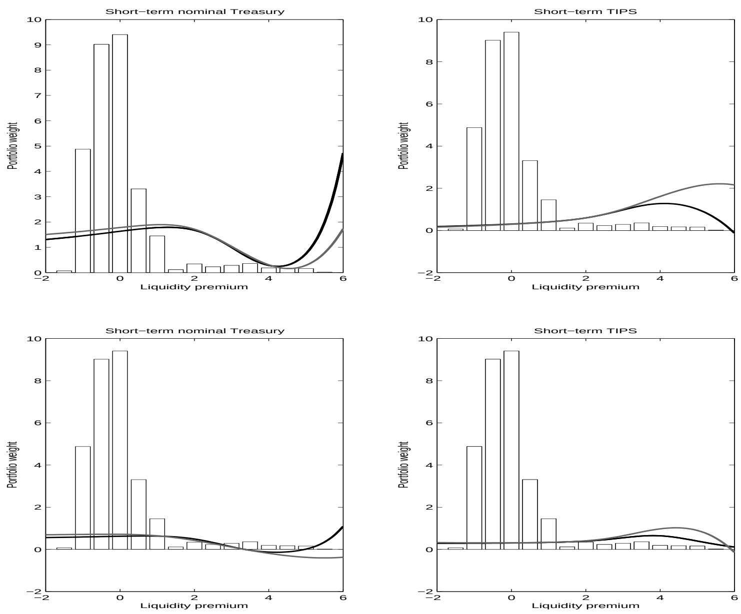
MV vs CRRA optimal portfolio functions

Figure B3.1: MV vs CRRA optimal portfolio weights

(a) Quaterly frequency



(b) Annual frequency



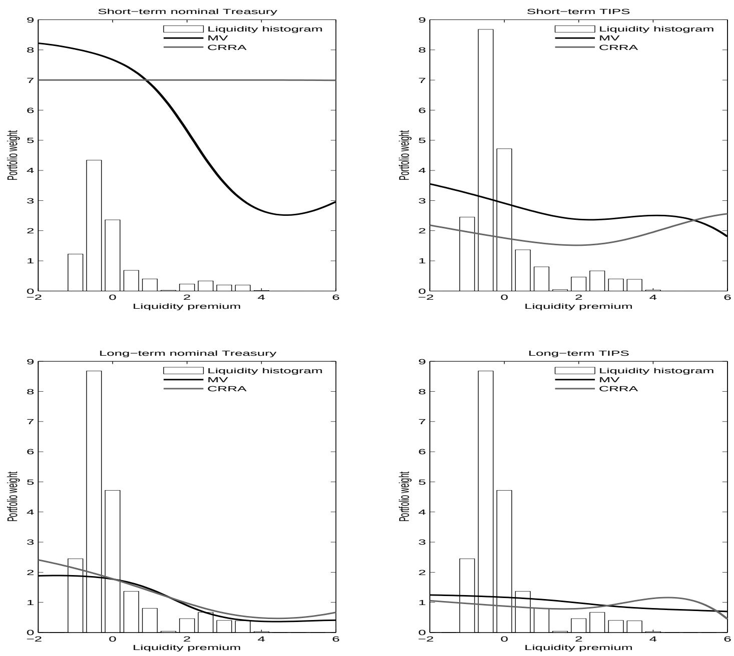
The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The black line represent line represent the optimal fraction of wealth allocated to equally-weighted U.S. bond portfolio as a function of liquidity premium for a mean-variance investor. The grey line represents the same for CRRA investor.

Appendix C3

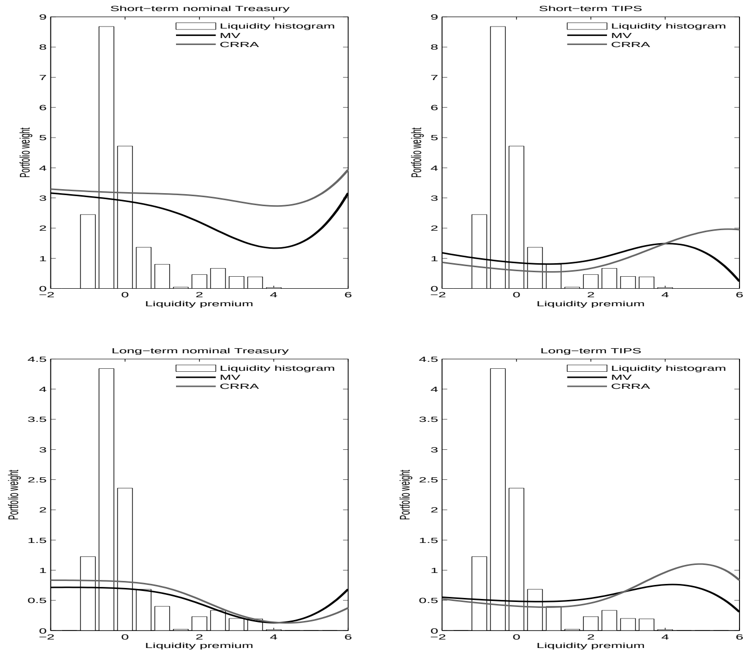
Optimal portfolio functions considering an alternative liquidity premium measure

Figure C3.1: Optimal portfolio weights as a function of 10-year liquidity premium: $L_{10,t}^{z-asw}$

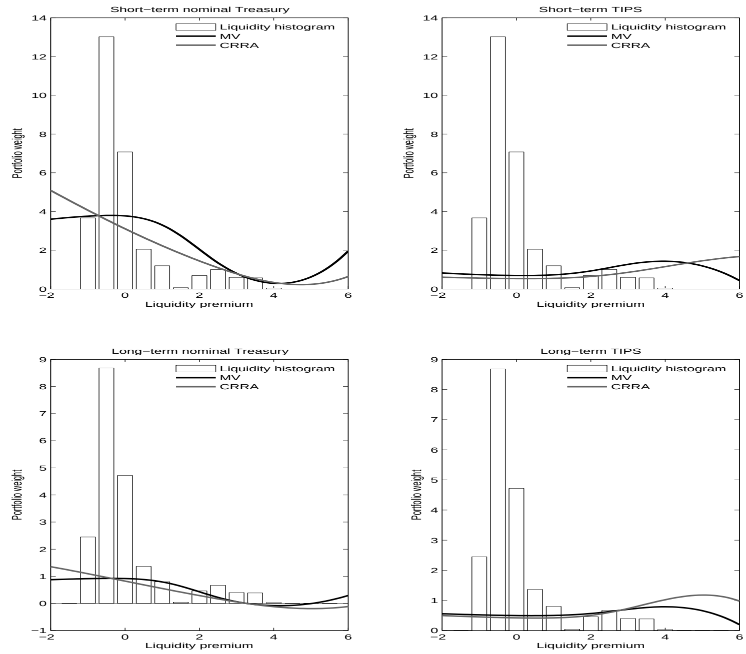
(a) Monthly frequency



(a) Quarterly frequency



(b) Annual frequency



The bars in the background represent the histogram (scaled to add up to 20) of liquidity premium. The lines represent the optimal fraction of wealth allocated to equally-weighted U.S. bond portfolio as a function of liquidity premium.

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