On the Causes of the Great Inflation *

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Abstract

An influential paper by Clarida, Galí and Gertler (2000) –CGG– has attributed the great inflation of the 1970 to the violation of the Taylor principle in the conduct of US monetary policy (weak, indeterminacy inducing response to expected inflation). We evaluate this thesis in the context of a standard NK model against a version of the model that incorporates incomplete information about the true state of the economy and learning. The likelihood–based estimation of the model overwhelmingly favors the specification with indeterminacy over the alternative with determinacy, whether mis-perceptions are present or not.

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Introduction

The causes of the "great" inflation of the 1970s remain the subject of debate. One of the most popular explanations has been provided by Clarida et al. (2000) (henceforth CGG). CGG argue that the great inflation was caused by a well–meaning, non–opportunistic FED that committed an honest, technical mistake. In particular, the FED is alleged to have employed a policy rule that involved too weak of a reaction to expected inflation. This triggered —inadvertently— indeterminacies and allowed self–fulfilling inflation expectations to contribute to higher inflation. CGG claim that the empirical evidence favors their interpretation. When estimating a policy rule for the pre-Volcker period, they find that, unlike the rule for the post-Volcker era, it indeed violates the Taylor principle.

The CGG thesis has its critics. For instance, Orphanides (2004) has repeated the CGG exercise using *real time* data on inflation and output as well as a real time measure of potential output (partly constructed by the Commerce department). His main result is that the estimated interest rate rule does not differ significantly across the pre- and post Volcker periods. And that it satisfies determinacy in both periods. This result seems to owes much to the assumption that the FED in the 70s formulated monetary policy on the basis of a very large and persistent, perceived output gap. While some part of this gap came from measurement error in actual output, most of it arose from the estimate of potential output published by the Department of Commerce's in its monthly publication Business Conditions Digest[1].

An alternative test of the CGG thesis has been undertaken by Lubik and Schorfheide (2004) (L-S). They estimate a small scale, forward looking New Keynesian model without restricting the parameters to lie in the determinacy region. They specify a prior probability distribution over parameters that places equal weight on determinate and indeterminate regions of the parameter space and compute posterior odds ratios for these regions. Their main result regarding the policy rule supports the CGG claim of indeterminacy [2].

Does the L-S analysis provide an adequate framework for evaluating the case for indeterminacy? A possible problem with their approach -a problem acknowledged by them- regards its sensitivity to model mis–specification. In particular, the endogenous dynamics are richer in the indeterminacy region of the parameter space than in the determinacy region. Omitted propagation mechanisms under determinacy may thus bias the posteri-

[1]Of course, it is not known what measure of the output gap, if any, the FED actually utilized during that period.

[2]Collard and Dellas (2007) too evaluate the CGG thesis in the context of a calibrated DSGE model but find that the model can account for the 70s only in the face of an implausibly large negative productivity shock.

ors toward indeterminacy.[3] To deal with this problem they include additional sources of dynamics under determinacy (such as habit persistence and backward–price indexation) and find that they do not materially affect the results.

Our view is that such rigidities are unlikely to help the NK model generate large inflation volatility and persistence under determinacy[4]. Consequently, the case for determinacy may not have received a "fair hearing" in the L-S specification. In this paper we propose an alternative, more plausible source of inflation inertia that has been identified in the literature as a possible important contributor to the great inflation. Namely, misperceptions about the state of the economy and gradual learning, an element that has been emphasized by Orphanides. We examine whether the CGG thesis survives against this alternative specification.

Naturally, we are not the first ones to study the contribution of this type of misperceptions to the great inflation. There exists a large literature that has looked at this issue[5] and which claims that misperceptions may have indeed been the driving force behind the inflation of the 70s. Nevertheless, we find the message from this literature rather incomplete because of three reasons.

First, much of this literature has been conducted in the context of backward looking models (where all or some of the agents do not employ rational expectations). Second, with rare exceptions, the learning mechanism utilized is not rational. Typically, the monetary authorities are assumed to make use of plausible but arbitrary forecasting rules. Both of these features introduce extra degrees of freedom and are quite controversial. And third, and more importantly, these works restrict the estimation to the determinacy region of the parameter space, so they are not in a position to differentiate among competing explanations (for instance, mis-perceptions against indeterminacy) and select the one most consistent with the data.

Our analysis addresses all three problems. Relying on the empirical approach of L-S, we also estimate a version of the standard, rational expectations, New Keynesian model that includes mis-perceptions about the true state of the economy and learning. Learning is assumed to be rational (it is based on the Kalman filter). We find that the case of indeterminacy in the pre–Volcker period is overwhelming, independent of whether the alternative determinate model contains measurement error or not.

[3] Of course, such a potential problem is present in all system-based approaches to evaluation of indeterminacies.

[4] Collard and Dellas (2004) show that standard real rigidities -such as habit persistence, investment adjustment costs etc.- are not an important source of inflation inertia.

[5] See, for instance,Bullard and Eusepi (2005), Cukierman and Lippi (2005), Milani (2006), Nelson and Nicolov (2002), Primiceri (2006).

We also evaluate the case for indeterminacy using model specifications that closely follow the Orphanides reasoning. In particular, we estimate a version where the signals available to the agents/monetary authority are real time inflation and the real time output gap (the series used by Orphanides). And another version where the monetary policy rule involves a reaction to these signals only[6]. The case for indeterminacy remained overwhelming independent of the competing determinant model with mis-perceptions. We consider these results as providing strong support to the CGG thesis[7].

The rest of the paper is organized as follows. Section 1 describes the model. Section 2 presents the solution method and the estimation strategy. Section 3.3 discusses our results. A last section concludes.

1 The model

We consider the same standard prototype New–Keynesian model as that employed by L-S. *The* model consists of three equations: a forward–looking IS–curve (equation (45)), a New–Keynesian Phillips curve (equation (2)) and a monetary policy rule in the form of a simple Taylor rule (equation (3)). These equations are derived from a fully–fledged micro–founded dynamic general equilibrium model (see the Appendix for the details):

$$\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - \tau(\widehat{R}_t - \mathbb{E}_t\widehat{\pi}_{t+1}) + \widehat{g}_t \tag{1}$$

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1} + \kappa(\widehat{y}_t - \widehat{z}_t) \tag{2}$$

$$\widehat{R}_t = \rho_R\widehat{R}_{t-1} + (1-\rho_R)(\psi_\pi\widehat{\pi}_t + \psi_y\widehat{y}_t) + \varepsilon_t^R \tag{3}$$

where $\widehat{y}_t$, $\widehat{\pi}_t$ and $\widehat{R}_t$ denote output, inflation and the nominal interest rate, all measured as percentage deviations from the steady state[8]. $\mathbb{E}_t$ is the output gap expectation operator based on information up to period t. The economy is subjected to three shocks: a demand shock, g_t, a shock which shifts the marginal costs of production, z_t, and a monetary policy

[6] These variables cannot be treated as purely exogenous, this would render the nominal interest rate exogenous and the equilibrium indeterminate. We thus allowed them to respond to the endogenous variables of the model via either an error correction mechanism or a simple linear feedback rule.

[7] Of course, there may exist other models with mis-perceptions that could conceivably "beat" the model with indeterminacy. For instance, a model where policy-makers formed beliefs using a backward looking model while the agents used a forward looking model. While one can always claim that such an assumption would better capture the way of thinking of policy-makers at that time, we –as well as many others– find such an assumption both arbitrary and controversial.

[8] In this specification, potential output is deterministic and the misperception of the output gap arises exclusively from the measurement error in actual output. We have also used $\widehat{y}_t - \widehat{z}_t$ as the measure of output gap that is targeted by the policymakers. We will show later that the relative merits of the competing specifications do not vary with the measure of perceived output gap employed.

shock, ε_t^R. The two real shocks are assumed to follow AR(1) processes of the form:

$$\widehat{z}_t = \rho_z \widehat{z}_{t-1} + \varepsilon_t^z, \tag{4}$$

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \varepsilon_t^g \tag{5}$$

where $|\rho_z| < 1$ and $|\rho_g| < 1$. ε_t^z and ε_t^g are two gaussian white noise processes with mean zero and respective standard deviation σ_z and σ_g. Following Lubik and Schorfheide (2004), we allow for non–zero contemporaneous correlation ρ_{gz} between these two innovations. The monetary policy shock, ε_t^R, is assumed to be iid and normally distributed with mean zero and standard deviation σ_R.

As is well known in the literature, this model can be indeterminate over some region of the parameter space and sunspots may play a role. In this case we solve the model as explained in L-S. In the case of determinacy we consider two possibilities. In the first one, agents are assumed to have perfect information about the state of the economy. In the second one agents are assumed to have imperfect information in the sense that they do not observe the actual state of the economy, but only a signal. In this case the agents face a signal extraction problem which is solved using the Kalman filter. For more details, see the Appendix.

The case of perfect information is identical to that in L-S so we only discuss the case of imperfect information. Let the state of the economy be represented by two vectors $\widehat{X}_t^b$ and $\widehat{X}_t^f$. The first one includes the predetermined (backward looking) state variables, i.e. $\widehat{X}_t^b = (\widehat{R}_{t-1}, \widehat{z}_t, \widehat{g}_t, \widehat{\varepsilon_t^R})'$, whereas the second one consists of the forward looking state variables, i.e. $\widehat{X}_t^f = (\widehat{y}_t, \widehat{\pi}_t)'$. The model then admits the following representation

$$M_0 \begin{pmatrix} \widehat{X}_{t+1}^b \\ \mathbb{E}_t \widehat{X}_{t+1}^f \end{pmatrix} + M_1 \begin{pmatrix} \widehat{X}_t^b \\ \widehat{X}_t^f \end{pmatrix} = M_2 \varepsilon_{t+1} \tag{6}$$

where

$$M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\tau & 0 & 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 0 & 0 & \beta \end{pmatrix}$$

$$M_1 = \begin{pmatrix} -\rho_R & 0 & 0 & -1 & -(1-\rho_R)\psi_y & -(1-\rho_R)\psi_\pi \\ 0 & -\rho_z & 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & -\kappa & 0 & 0 & \kappa & -1 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon_{t+1} = \{\varepsilon^z_{t+1}, \varepsilon^g_{t+1}, \varepsilon^R_{t+1}\}$$

Thus the first row corresponds to the Taylor rule, the second, third and forth row to the demand, cost push shock and policy shock, the fifth row to the IS-curve and the sixth row to the Phillips curve. Let us denote the signal process by $\{S_t\}$. The measurement equation relates the state of the economy to the signal:

$$S_t = C \begin{pmatrix} \widehat{X}^b_t \\ \widehat{X}^f_t \end{pmatrix} + v_t. \tag{7}$$

Agents receive a four dimensional signal on the cost push, demand shock, monetary policy shock and nominal interest rate. Hence C equals:

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \rho_R & 0 & 0 & 1 & (1-\rho_R)\psi_y & (1-\rho_R)\psi_\pi \end{pmatrix}$$

The noise sequence $\{v_t\}$ is assumed to be a Gaussian white noise process which is uncorrelated and all leads and lags with $\{\varepsilon_t\}$. Moreover the signals are assumed to be uncorrelated so their contemporaneous covariance matrix is diagonal, $\Sigma_v = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2)$. We do not find it plausible to assume that nominal interest rates are observed with noise, so we will set $\sigma_4^2 = 0$.

The agents use the signals to infer the state of the economy. The solution concept is based on the Kalman filter and is explained in detail in the appendix to this paper. Let us denote $\widehat{X}_t = (\widehat{X}_t^{b\prime}, \widehat{X}_{t|t}^{b\prime})'$ where $\widehat{X}_{t|t}^b$ is the estimate of $\widehat{X}_t^b$ given the information up to and including period t. The solution to the model is a state equation of the form:

$$\widehat{X}_{t+1} = M_X \begin{pmatrix} \widehat{X}_t^b \\ \widehat{X}_{t|t}^b \end{pmatrix} + M_E \begin{pmatrix} \varepsilon_{t+1} \\ v_{t+1} \end{pmatrix} \tag{8}$$

The state vector is then related to the observations $\widehat{Y}_t$ by

$$\widehat{Y}_t = M_Y \widehat{X}_t \tag{9}$$

The observations consist of real GDP, y_t, (GDP deflator) inflation rate, π_t, and the federal fund rate, R_t, $\widehat{Y}_t = \{\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t\}$. The matrix M_Y selects the corresponding entries of $\widehat{X}_t$ and relates them to the observations.

2 Estimation procedure

The observation equation is:

$$Y_t = Y^\star + H\widehat{Y}_t \tag{10}$$

where the vectors Y_t and $Y^\star$ denote the time series and $Y^\star$ the steady state values. In our case the observations consist of HP detrended output, inflation and the real interest rate so that

$$Y_t = \begin{pmatrix} y_t \\ \pi_t \\ R_t \end{pmatrix} \qquad \text{and} \qquad Y^\star = \begin{pmatrix} 0 \\ \pi^\star \\ r^\star + \pi^\star \end{pmatrix}. \tag{11}$$

The matrix H is given by

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}.$$

The 4 reflects the fact that we use annualized inflation and interest rates.

2.1 The Prior Distribution

In order to make our results comparable to those of L-S we use their priors. The specification with the corresponding parameters is reported in Table 1. In the case of imperfect information Σ_v appears as an additional parameter (see equation(48)). When we assumed that the elements of Σ_v were distinct we were not able to identify $\sigma_1^2, \sigma_2^2, \sigma_3^2)$ separately. Thus we imposed $\sigma_1^2 = \sigma_2^2 = \sigma_3^2$. Note that this implies that the noise to signal ratio differs across shocks. We later discuss an alternative specification. As we have no particular information concerning this parameter, we take the same prior as for σ_z, the standard deviation of the marginal cost shock.

The parameters $\mu_{\zeta,R}$ $\mu_{\zeta,g}$ $\mu_{\zeta,z}$ are extra parameters that appear in the solution under indeterminacy and modify the transmission mechanism of the fundamental shocks (they correspond to $\mu_{\zeta,R}$ $\mu_{\zeta,g}$ $\mu_{\zeta,z}$ in L-S). If they are zero then the endogenous expectation formation does not abruptly change as the economy moves across the boundary between the determinacy and the indeterminacy region.

We collect all parameters into the vector θ and assume that the parameters are distributed a priori independently from each other. That is, the prior distribution of θ, $p(\theta)$, is just the product of the prior distributions for each parameter θ_i separately:

$$p(\theta) = \prod_i p_i(\theta_i).$$

Table 1: Summary of the prior distribution of parameters

Parameter	Support	Density	Mean	Standard deviation	95-percent HPD interval [a]
ψ_π	$\mathbb{R}^+$	Gamma	1.10	0.50	[0.35,2.27]
ψ_y	$\mathbb{R}^+$	Gamma	0.25	0.15	[0.05,0.62]
ρ_R	$[0,1)$	Beta	0.50	0.20	[0.13,0.87]
$\pi^\star$	$\mathbb{R}^+$	Gamma	4.00	2.00	[1.10,8.76]
$r^\star$	$\mathbb{R}^+$	Gamma	2.00	1.00	[0.55,4.37]
κ	$\mathbb{R}^+$	Gamma	0.50	0.20	[0.19,0.96]
τ^{-1}	$\mathbb{R}^+$	Gamma	2.00	0.50	[1.14,3.09]
ρ_g	$[0,1)$	Beta	0.70	0.10	[0.49,0.87]
ρ_z	$[0,1)$	Beta	0.70	0.10	[0.49,0.88]
σ_ζ	$\mathbb{R}^+$	Inv. Gamma	0.25	0.13	[0.12,0.57]
$\mu_{\zeta,R}$	$\mathbb{R}$	Normal	-0.01	1.00	[-1.96,1.95]
$\mu_{\zeta,g}$	$\mathbb{R}$	Normal	-0.01	1.00	[-1.97,1.96]
$\mu_{\zeta,z}$	$\mathbb{R}$	Normal	-0.00	1.00	[-1.97,1.94]
σ_R	$\mathbb{R}^+$	Inv. Gamma	0.31	0.17	[0.15,0.72]
σ_g	$\mathbb{R}^+$	Inv. Gamma	0.38	0.19	[0.18,0.86]
σ_z	$\mathbb{R}^+$	Inv. Gamma	1.00	0.51	[0.48,2.30]
$\rho_{g,z}$	$[-1,1]$	Normal	0.00	0.38	[-0.75,0.75]
σ_1	$\mathbb{R}^+$	Inv. Gamma	1.00	0.51	[0.48,2.30]

Note: The parameters are distributed independently from each other. [a] 95-percent highest probability density (HPD) credible intervals (see Geweke (2005), p.57)

2.2 Likelihood function

The computation of the likelihood function is an application of the Kalman filter to the state–space representation given by the state equation (8) and the measurement equations (9) and (11). Collecting all parameters in a vector θ and assuming normality, the observations Y_t given the past are normally distributed:

$$Y_t|\mathcal{Y}_{t-1} \sim \mathrm{N}\left(M_Y(\theta)X_{t|t-1}, M_Y(\theta)P_{t|t-1}M_Y(\theta)'\right)$$

where $\mathcal{Y}_{t-1} = \{Y_{t-1}, Y_{t-2}, \dots\}$ represents the entire history of the data. $X_{t|t-1}$ denotes the linear projection of X_t on $\mathcal{Y}_{t-1}$. Associated with this forecast is the mean squared error matrix $P_{t|t-1} = \mathbb{E}(X_t - X_{t|t-1})(X_t - X_{t|t-1})'$. Both $X_{t|t-1}$ and $P_{t|t-1}$ can be computed recursively starting with some initial values $X_{1|0}$ and $P_{1|0}$. As the model was obtained by linearization around the deterministic steady state, it seems natural to assume that the systems starts at the steady state which implies that $X_{1|0} = 0$. Likewise, we assume that $P_{1|0}$ corresponds to the steady state covariance matrix $P^\star$. This matrix can found by solving the matrix equation

$$P^\star = M_X(\theta)P^\star M_X(\theta) + M_E(\theta)\Sigma(\theta)M_E(\theta)$$

where $\Sigma(\theta) = \mathbb{E}(\varepsilon_t', v_t')'(\varepsilon_t', v_t')$.

Given these preliminaries, the likelihood function, $L(\theta|\mathcal{Y}_T)$, of the data $\mathcal{Y}_T$, T being the sample size, given the parameter vector θ is

$$L(\theta|\mathcal{Y}_T) = \prod_{t=1}^{T} f(Y_t|\mathcal{Y}_{t-1})$$

where f is the density function of the normal distribution given above. Clearly the log-likelihood function $\ell(\theta|\mathcal{Y}_T)$ is

$$\ell(\theta|\mathcal{Y}_T) = \log L(\theta|\mathcal{Y}_T) = \sum_{t=1}^{T} \log f(Y_t|\mathcal{Y}_{t-1}).$$

The density function of the posterior distribution of θ, $g(\theta|\mathcal{Y}_T)$, is then given by

$$g(\theta|\mathcal{Y}_T) = \frac{L(\theta|\mathcal{Y}_T) \times p(\theta)}{\int L(\theta|\mathcal{Y}_T)p(\theta)\mathrm{d}\theta}$$

where $p(\theta)$ is the prior distribution of θ.

3 Estimation Results

3.1 The Data

The data consists of quarterly observations of the inflation rate, the Federal Funds rate and the HP–filtered real GDP

—Figure 2 about here—

Figure 2 depicts the evolution of these three time series over the period 1960:I–1997:IV. This figure shows that the great inflation period started in the middle of the sixties, more precisely in 1967:1, and ended somewhere in the beginning of the eighties. There is actually no consensus on the exact ending date of the great inflation period. Some argue that it ended with the arrival of Paul Volcker as chairman of the Federal Reserve Board in 1979:II, while others date the end in 1982:IV. We have carried out the estimation with either break point. The results were the same, so we only report those corresponding to the 1979:II break point.

3.2 Discussion of the Empirical Results

For each of the three cases (indeterminacy, determinacy with perfect information and determinacy with imperfect information) we summarize the posterior distribution of the parameters by presenting some key statistics in a table and by plotting its marginal distributions. In addition we plot the impulse responses to the various shocks.

3.2.1 Case I: Indeterminacy

The key finding here regards ψ_π, the coefficient measuring the response of policy to inflation. As in L-S, it is distributed around a value well below one. The other coefficients take values that are similar to those reported in the literature.

The distributions of the indeterminacy parameters σ_ζ, $\mu_{\zeta,R}$, $\mu_{\zeta,g}$, and $\mu_{\zeta,z}$ are substantially shifted away from zero, which provides evidence against the continuity assumption (namely, the assumption that expectation formation does not abruptly change as the economy moves across the boundary between the determinacy and the indeterminacy region). Note also that the estimation provides considerable information with respect to the standard deviations of the shocks (see Figure 3).

The response of the economy to the shocks is summarized by the impulse response functions plotted in Figure 4. Monetary tightening, that is, an increase in the nominal interest rate, leads to a reduction in output and inflation. The adjustment of the economy after the shock is monotone, there is no hump. The demand shock has the expected effect: it increases output, inflation, and the nominal interest rate. The adjustment again is monotone with the exception of the interest rate. The marginal cost shock also exhibits the expected effects. A negative shock increases output and decreases inflation and the nominal interest rate.There is a hump–shaped adjustment of output and the interest rate. The effect of the sunspot shock is pretty much the same as the effect to the marginal cost shock.

Table 2: Summary of the posterior distribution in the case of indeterminacy

Parameter	Mode	Mean	Median	Standard deviation	95-percent HPD interval[a]
ψ_π	0.76	0.65	0.65	0.12	[0.43,0.89]
ψ_y	0.20	0.24	0.23	0.07	[0.12,0.38]
ρ_R	0.62	0.67	0.68	0.07	[0.54,0.81]
$\pi^\star$	4.75	4.43	4.49	1.10	[2.23,6.57]
$r^\star$	0.88	1.06	1.03	0.35	[0.39,1.74]
κ	0.46	0.37	0.35	0.13	[0.15,0.63]
τ^{-1}	2.05	1.78	1.73	0.45	[1.01,2.69]
ρ_g	0.82	0.76	0.77	0.06	[0.65,0.87]
ρ_z	0.72	0.74	0.75	0.06	[0.63,0.85]
σ_ζ	0.29	0.23	0.22	0.07	[0.11,0.37]
$\mu_{\zeta,R}$	-0.23	-0.56	-0.54	0.48	[-1.49,0.37]
$\mu_{\zeta,g}$	0.03	1.50	1.50	0.63	[0.41,2.81]
$\mu_{\zeta,z}$	-0.16	-0.67	-0.66	0.27	[-1.24,-0.20]
σ_R	0.20	0.20	0.20	0.02	[0.16,0.24]
σ_g	0.21	0.27	0.26	0.05	[0.18,0.37]
σ_z	1.20	1.36	1.32	0.21	[1.01,1.80]
$\rho_{g,z}$	0.98	0.52	0.49	0.19	[0.23,0.90]
marginal log-data density:					-267.804

[a] 95-percent highest probability density (HPD) credible intervals (see Geweke (2005), p.57)

—FIGURE 4 ABOUT HERE—

Note also that all impulse response function are qualitatively similar to those of L-S.

3.2.2 Case II: Determinacy with perfect information

The determinate model with perfect information has the least number of parameters and is therefore the most restrictive of all three cases. A summary of the corresponding a posteriori distribution is given in Table 3. Compared to the prior and to the previous case, the distribution of ψ_π is shifted to the right with a mean of 1.21 and 90–percent confidence interval of $[1.00, 1.43]$. Thus the monetary authority responds much stronger to an increase in inflation than under the previous specification. The response to the output gap is lower and in line with that reported in the literature. The distribution of the other parameters does not change much. The only noteworthy exception regards the equilibrium inflation rate, whose mean increases from 4.50 to 5.73 percent. The remaining parameters are also pretty much in line with the priors and similar to the indeterminacy case.

Table 3: Summary of the posterior distribution in the case of determinacy with perfect information

Parameter	Mode	Mean	Median	Standard deviation	95-percent HPD interval [a]
ψ_π	1.01	1.07	1.05	0.07	[1.00,1.21]
ψ_y	0.26	0.24	0.23	0.08	[0.10,0.40]
ρ_R	0.71	0.67	0.67	0.07	[0.54,0.80]
$\pi^\star$	5.25	5.54	5.54	0.67	[4.21,6.88]
$r^\star$	0.84	0.93	0.92	0.29	[0.37,1.50]
κ	0.37	0.42	0.40	0.16	[0.14,0.73]
τ^{-1}	1.95	2.06	2.05	0.44	[1.16,2.89]
ρ_g	0.87	0.86	0.87	0.03	[0.81,0.91]
ρ_z	0.73	0.75	0.76	0.04	[0.68,0.83]
σ_R	0.20	0.23	0.23	0.02	[0.18,0.27]
σ_g	0.18	0.21	0.21	0.03	[0.15,0.27]
σ_z	1.16	1.21	1.19	0.17	[0.92,1.54]
$\rho_{g,z}$	0.92	0.92	0.93	0.04	[0.83,0.98]
marginal log-data density:					-280.184

[a] 95-percent highest probability density (HPD) credible intervals (see Geweke (2005), p.57)

Figure 7 provides information on the dynamics of the system following a shock. As is well known, the model cannot generate any humps in the absence of price indexation and various real rigidities.

—FIGURE 7 ABOUT HERE—

3.2.3 Case III: Determinacy with imperfect information

Consider finally the case of determinacy with imperfect information on the part of the agents concerning the true state of the economy. The nominal interest rate is the only variable that is assumed to be perfectly observed. The shocks are observed with noise and so are output and inflation. In this specification we have one additional parameter -relative to the case with determinacy described in the previous section-, σ_1, the standard deviation of the signal on the imperfectly observed variables.

Table 4 gives a summary of the posterior distribution. Compared to the case with perfect information, there are some small differences regarding the parameters of the policy rule. In particular, the reaction to inflation is somewhat weaker while policy inertia and the response to the output gap greater. Moreover, the data provide information on the key parameter of interest, the variance of the signal. The posterior distribution of σ_1 is different from the prior one (see Figure **??**). Consequently, the addition of imperfect information makes a difference. Note also that the size of the standard deviation of σ_1 is quite plausible. For instance, the standard deviation of the measurement error in the inflation rate computed using real time data from the Philadelphia FED (the difference between the initial and final release) is about ... over the period ... Note also that it is lower than the value typically assumed in calibrated models in the literature (see Svensson and Woodford, 2003).

Despite the similarities in the central tendencies of the posterior distributions between the determinacy models with and without perfect information, there exists an important difference regarding the implied dynamics of inflation, in particular, following the demand shock. Comparison of Figures 7 and 10 shows that the impact effect is less strong and that there is hump–shaped response in the adjustment of inflation. Similarly, the effect of a marginal cost shock is less pronounced in the case of imperfect information compared to the case with perfect information. Both of these findings are consistent with the argument of Collard and Dellas (2006) who argue that mis–perceptions can be a major source of inertial behavior in the New Keynesian model. Note that the response of inflation to a monetary policy shock is short lived. This is a common finding in the literature (see Canova (2005)).

—FIGURE 10 ABOUT HERE—

Table 4: Summary of the posterior distribution in the case of determinacy with imperfect information

Parameter	Mode	Mean	Median	Standard deviation	95-percent HPD interval[a]
ψ_π	1.00	1.07	1.05	0.07	[0.99,1.21]
ψ_y	0.30	0.30	0.29	0.08	[0.16,0.48]
ρ_R	0.78	0.76	0.76	0.05	[0.66,0.85]
$\pi^\star$	4.91	5.70	5.65	0.71	[4.45,7.04]
$r^\star$	1.13	1.07	1.03	0.38	[0.37,1.77]
κ	0.32	0.33	0.31	0.11	[0.14,0.57]
τ^{-1}	2.28	2.03	1.99	0.54	[1.12,3.05]
ρ_g	0.90	0.87	0.87	0.03	[0.81,0.93]
ρ_z	0.79	0.75	0.75	0.04	[0.67,0.83]
σ_R	0.20	0.22	0.22	0.02	[0.18,0.27]
σ_g	0.18	0.27	0.26	0.05	[0.19,0.37]
σ_z	1.22	1.29	1.25	0.19	[0.96,1.67]
$\rho_{g,z}$	0.87	0.77	0.78	0.09	[0.60,0.92]
σ_1	0.50	0.72	0.69	0.19	[0.38,1.08]
marginal log-data density:					-280.927

[a] 95-percent highest probability density (HPD) credible intervals (see Geweke (2005), p.57)

3.2.4 Comparing the three cases

In addition to the plots and summary statistics of the posterior distribution of the parameters, it is instructive to compare the marginal log–data densities. These numbers are computed via the modified harmonic mean estimator proposed by Geweke (1999). They can be used to calculate the posterior probability of indeterminacy against determinacy (5). In the comparison of the determinacy model with perfect information to that with indeterminacy, the probability is 0.999 is favor of indeterminacy. Similar results obtain when we compare the specification with indeterminacy to that with imperfect information case. In this case too the probability in favor of indeterminacy is 0.999.

Table 5: Probabilities

Indet. vs Det. Perf. Info.	0.9999
Indet. vs Det. Imp. Info.	0.9999
Det. Imp. Info. vs Det. Perf. Info.	0.3223

3.3 Extensions

In models with measurement errors, the informational structure typically plays a crucial role for the results. Due to identification problems, we assumed in section that $\sigma_1 = \sigma_2 = \sigma_3$. Given the differences in the assumed priors of the standard deviation of the various shocks, this assumption implies that the signals about the monetary and demand shocks are more noisy than that about the technology shock (in relative terms). In order to investigate the role of this assumption we consider an alternative specification involving a similar signal to noise ratio across shocks. Namely, we set $\sigma_1 = k\sigma_R, \sigma_2 = k\sigma_g, \sigma_3 = k\sigma_z$ and impose a prior on the coefficient k. We re-estimated the models without finding any difference in the results relative to those reported in the previous section. In particular, as shown in table 6, the indeterminacy case still overwhelmingly dominates the two other alternative specifications.

Table 6: Probabilities

Indet. vs Det. Perf. Info.	0.9999
Indet. vs Det. Imp. Info.	0.9999
Det. Imp. Info. vs Det. Perf. Info.	0.5617

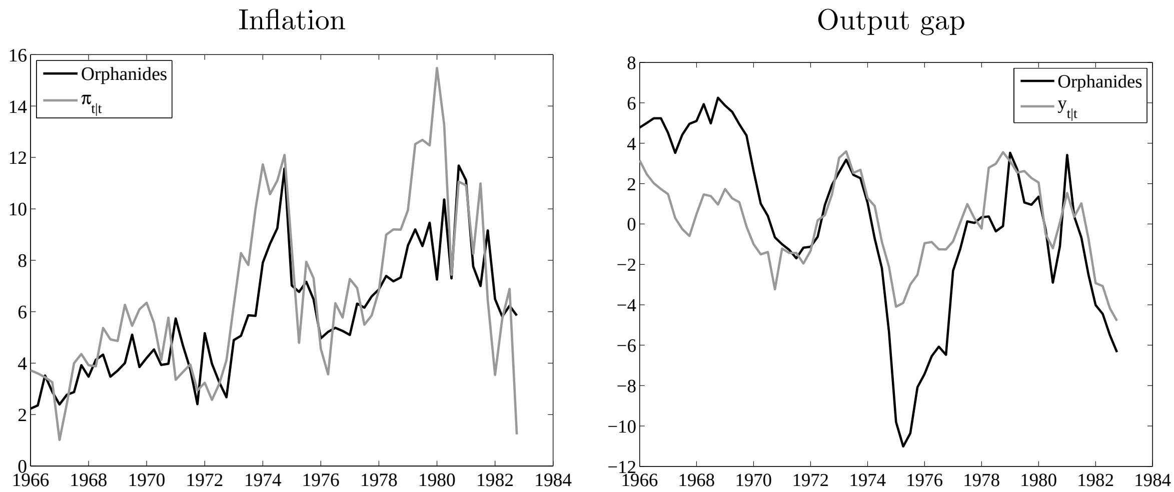
The specification of the signals so far has been standard. In order to bring our informational structure closer to that described by Orphanides, we re-estimated the model under the assumption that in addition to the perfect signal on R, the agents and the monetary authority receive as signals the real time values of inflation and output gap used by Orphanides in his assessment of the FED policy at the time. Table 7 represents the counterpart to table 5 under this specification. As can be seen, the case for indeterminacy remains overwhelming.

Table 7: Probabilities

Indet. vs Det. Perf. Info.	0.9999
Indet. vs Det. Imp. Info.	0.9999
Det. Imp. Info. vs Det. Perf. Info.	0.0000

Nonetheless, one may still argue that there is not enough misperception even under this specification. This is because the monetary authority does not respond exclusively to the signals. In particular, the monetary authority combines the signals with other information it has on the state of the economy in order to form perceptions about actual inflation and the output gap. And then it formulates policy on the basis of these perceptions. As can be seen in Figure 1, while the perception about inflation generated by the model is very similar to its real time –Orphanides– counterpart, that of the output gap is not. There

Figure 1: Real time inflation and Output Gap (Model vs Data)



exists an alternative formulation of the policy rule that incorporates the amount of misperceptions claimed by Orphanides. Namely, it involves reactions only to the real time inflation and output gap signals. That is, the policy rule now takes the form

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1-\rho_R)(\psi_\pi \widehat{\pi}_t^{\mathrm{O}} + \psi_y \widehat{y}_t^{\mathrm{O}}) + \varepsilon_t^R \tag{12}$$

where $\widehat{\pi}_t^{\mathrm{O}}$ and $\widehat{y}_t^{\mathrm{O}}$) are the Orphanides series. Of course, it is well known that an interest rule that contains only exogenous variables on the right hand side cannot support a determinate equilibrium. Consequently, we have augmented the model with equations relating Orphanides' perceived inflation and output gap to the model generated inflation and output gap. We did so in the context of either an error correction specification where the Orphanides series responds to its lagged value as well as to the model generated series. Or, within a specification where the Orphanides and model generated series differ by a random term. Again, as can be seen from table 8, utilizing the inflation and output gap data that may have been available to the FED as the *sole* drivers of policy does not decrease the odds in favor of indeterminacy[9].

We have also experimented with a large number of specifications involving different priors, signals, variables in the policy rule etc. In no case, was the indeterminate version threatened by any of its rivals.

[9]In this case, we estimate the model without learning as the path of output and inflation are exogenous as far as the monetary authority's policy is concerned.

Table 8: Probabilities

Indet. vs Det. Perf. Info.	0.9999
Indet. vs Det. Imp. Info.	0.9999
Det. Imp. Info. vs Det. Perf. Info.	0.0000

Where does this leave us? Based on our findings we think that one faces two alternatives. If one accepts that a forward looking, rational agents-expectations model (with sticky prices) provides the best description of the behavior of agents (private and policymakers alike) during that period, then one must necessarily also accept the CGG explanation of the great inflation. If one is willing to contemplate alternative specifications allowing for the presence also of backward looking agents, then alternative explanations may get a chance to rival the CGG thesis. As we stated in the introduction, we have not pursued the latter any further because we doubt that doing so represents -from a methodological point of view- as a compelling modelling exercise as the former one.

4 Conclusions

Inflation in the US reached very high levels during the 1970s, to a large extent due to what proved to be excessively loose monetary policy. A popular explanation of the great inflation attributes this looseness to inadvertent policy mistakes committed by a central bank that followed a rule. Such mistakes may arise even when the central bank is sufficiently averse to inflation if it does not fully understand the properties of the rule it uses (Clarida et al. (2000)). Lubik and Schorfheide (2004) have investigated this possibility and found that the data support a policy specification where monetary policy inadvertently destabilizes the economy, that is, an a policy associated with indeterminacy.

Our analysis has extended the Lubik and Schorfheide (2004) approach in order to address the possibility that it was imperfect information about the true *state* of the economy rather than lack of understanding of the properties of the policy rule that fueled the great inflation. This represents the view of Orphanides (2004). We found that, independent of the degree of mis-perceptions, the CGG thesis that the great inflation was caused by indeterminacy inducing policy was favored strongly by the data. We believe that if one is willing to accept that a forward looking NK model with rational agents and monetary authority is a reasonable representation of the US economy in the 70s, then one must also necessarily accept the CGG position.

References

Bullard, James and Stefano Eusepi, Did the Great Inflation Occur Despite Policymaker Commitment to a Taylor Rule?, *Review of Economic Dynamics*, 2005, *8* (2), 324–359.

Canova, Fabio, Monetary Policy and the evolution of US economy: 1948-2002, 2005. Mimeo.

Clarida, Richard, Jordi Galí, and Mark Gertler, Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory, *Quarterly Journal of Economics*, 2000, *115* (1), 147–180.

Collard, Fabrice and Harris Dellas, *The Great Inflation of the 1970s*, Working Paper 336, European Central Bank 2004.

—— and —— , *Misperceived Money and Inflation Dynamics*, mimeo 2006.

—— and —— , The Great Inflation of the 1970s, *Journal of Money, Credit and Banking*, 2007, *39* (3), 713–730.

Cukierman, Alex and Francesco Lippi, Endogenous Monetary Policy with Unobserved Potential Output, *Journal of Economics Dynamics and Control*, 2005, *29* (11), 1951–1983.

Geweke, John F., Using Simulation Methods for Bayesian Econometric Models: Inference, Development and Communication, *Econometric Reviews*, 1999, *18*, 1–126.

—— , *Contemporary Bayesian Econometrics and Statistics*, Hoboken, New Jersey: John Wiley & Sons, 2005.

Lorenzoni, G., *A Theory of Demand Shocks*, Unpublished manuscript, MIT 2006.

Lubik, Thomas A. and Frank Schorfheide, Testing for Indeterminacy: An Application to U.S. Monetary Policy, *American Economic Review*, 2004, *94* (1), 190–217.

Milani, Fabio, A Bayesian DSGE Model with Infinite-Horizon Learning: Do "Mechanical" Sources of Persistence Become Superfluous?, *International Journal of Central Banking*, 2006, *6*, 151–177.

Nelson, Edward and Kalin Nicolov, *Monetary Policy and Stagflation in the UK*, Discussion Paper 3458, CEPR 2002.

Orphanides, Athanasios, Monetary Policy Rules, Macroeconomic Stability and Inflation: A View from the Trenches, *Journal of Money Credit and Banking*, 2004, *36*, 151–177.

Primiceri, Giorgio E., Why Inflation Rose and Fell: Policy-Makers' Beliefs and U.S. Postwar Stabilization Policy, *Quarterly Journal of Economics*, 2006, *121*, 867–901.

Svensson, Lars and Michael Woodford, Indicator Variables for Optimal Policy, *Journal of Monetary Economics*, 2003, *50* (3), 691–720.

Figure 2: Data: Whole Sample Period

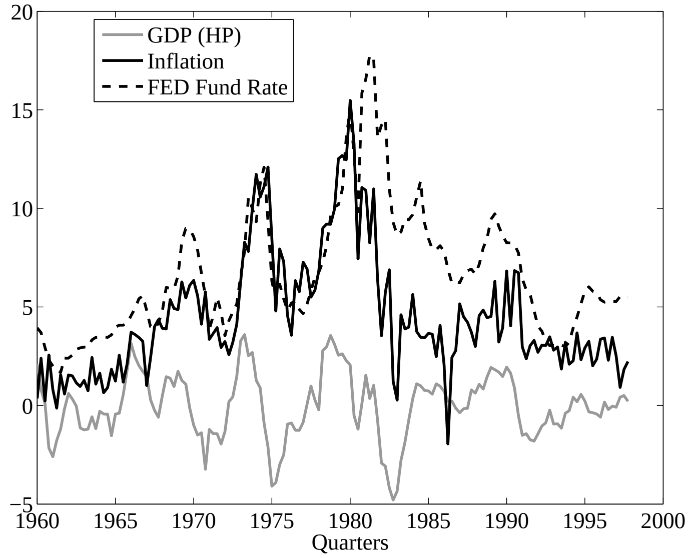
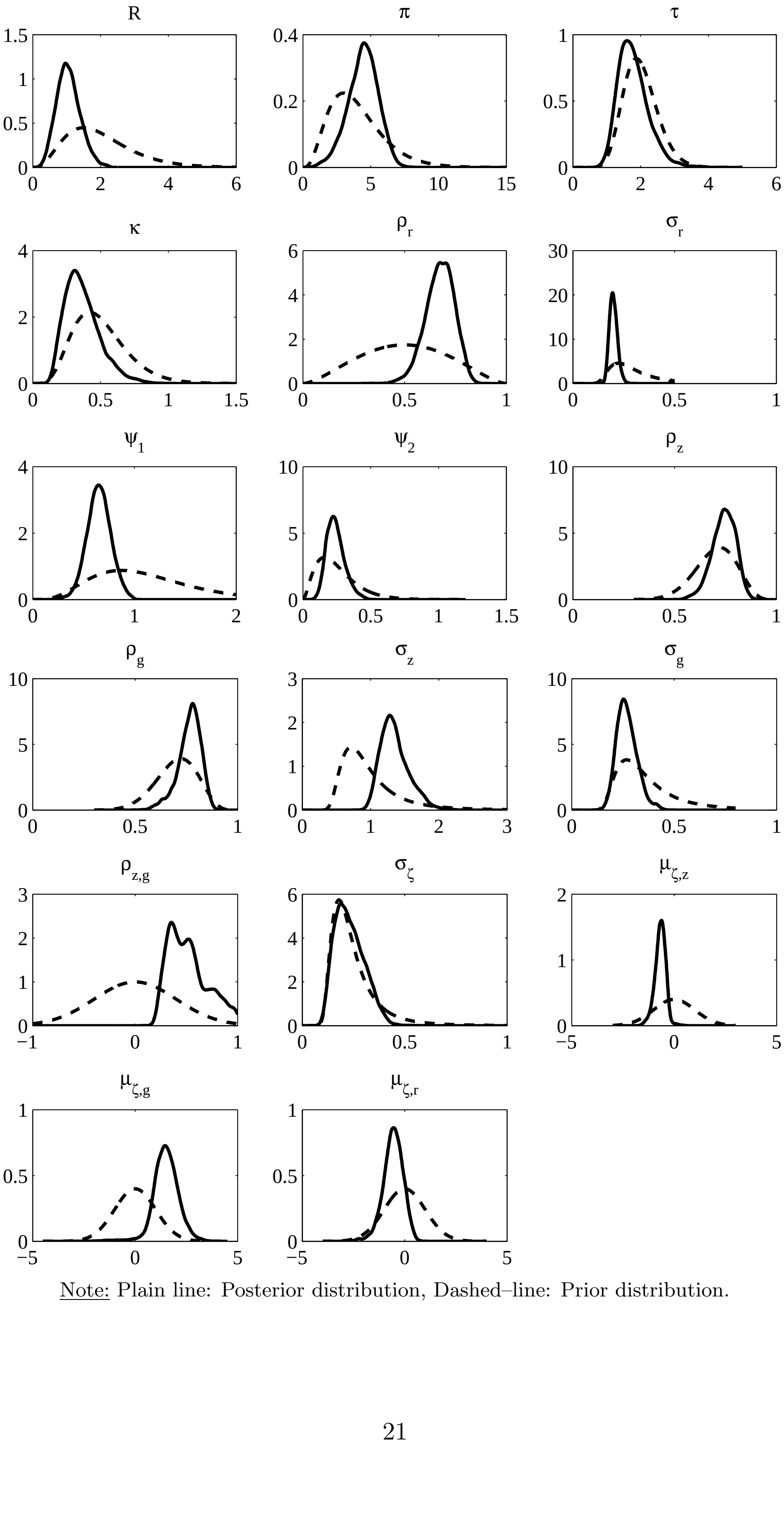
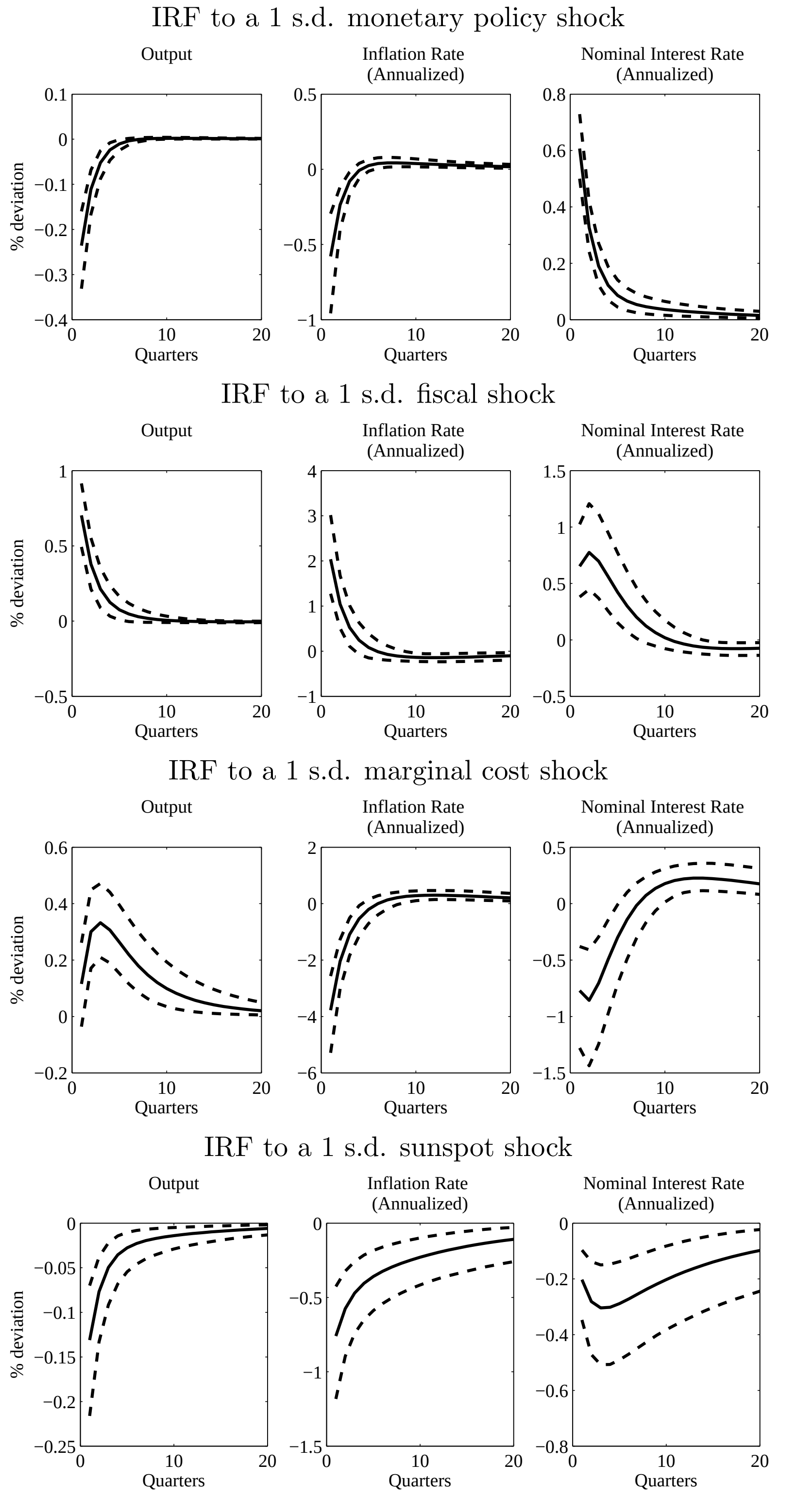


Figure 3: Prior vs Posterior Distributions (indeterminacy)



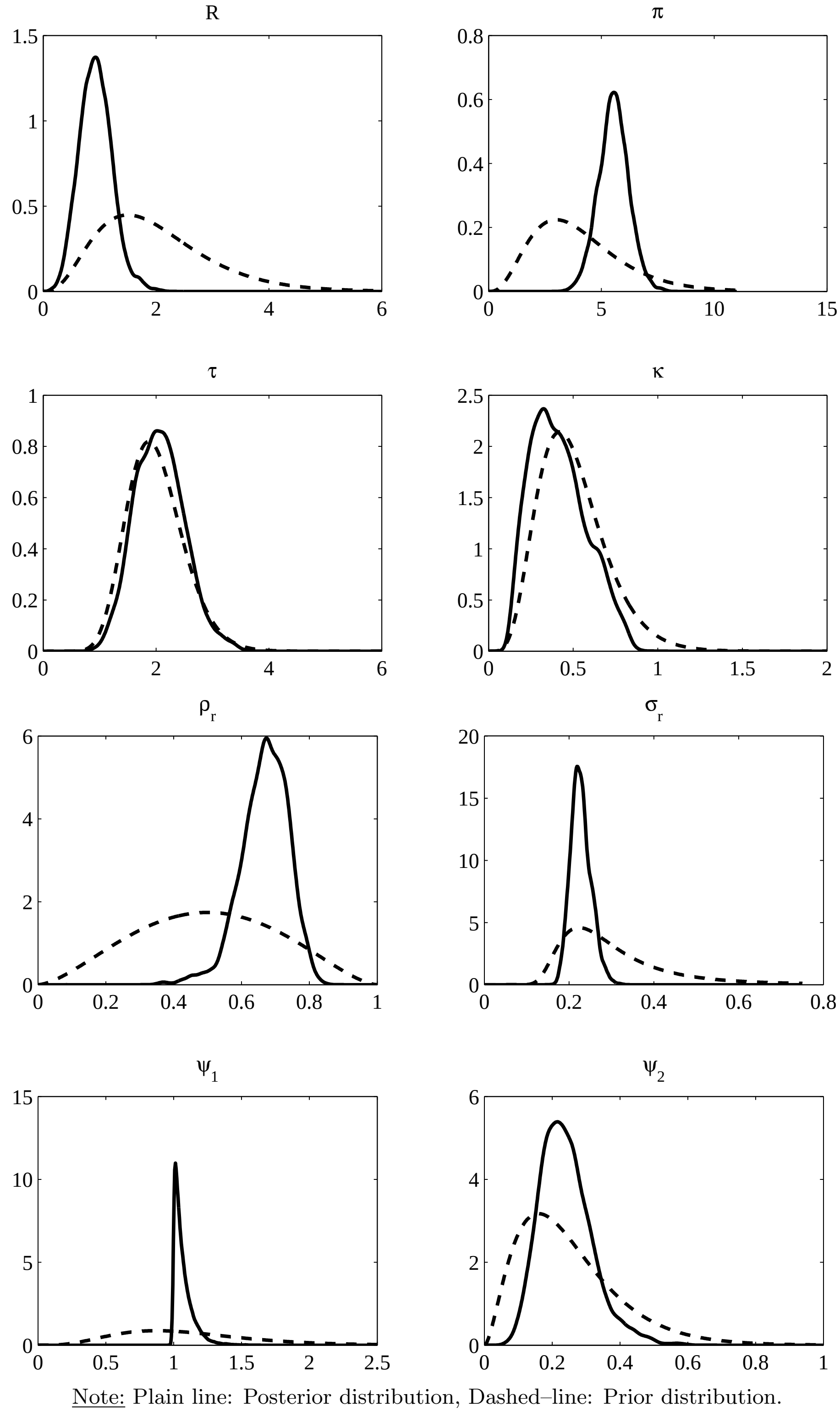
Note: Plain line: Posterior distribution, Dashed–line: Prior distribution.

Figure 4: IRFs under indeterminacy



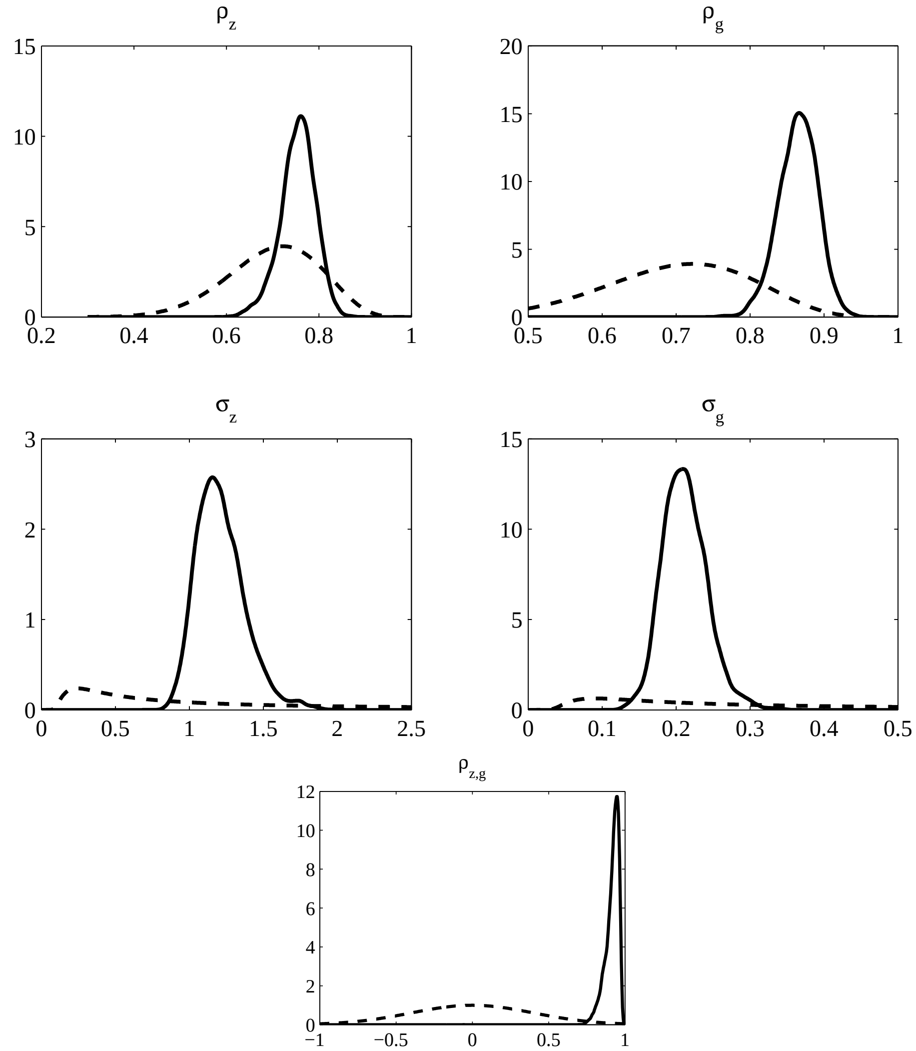
Note: Plain line: average across simulations, Dashed–line: 90% confidence interval.

Figure 5: Prior vs Posterior Distributions (Determinacy)



Note: Plain line: Posterior distribution, Dashed–line: Prior distribution.

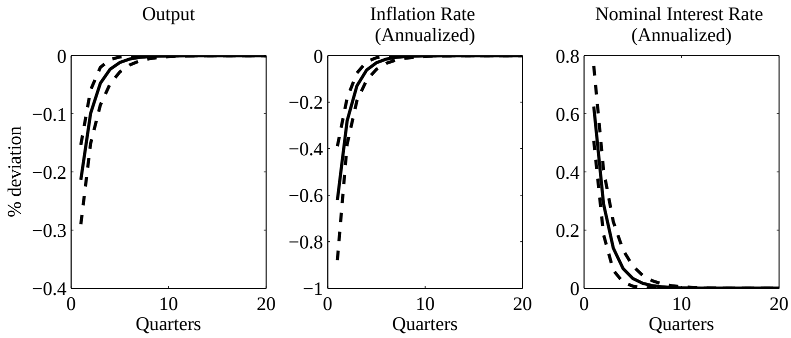
Figure 6: Prior vs Posterior Distributions (Determinacy)



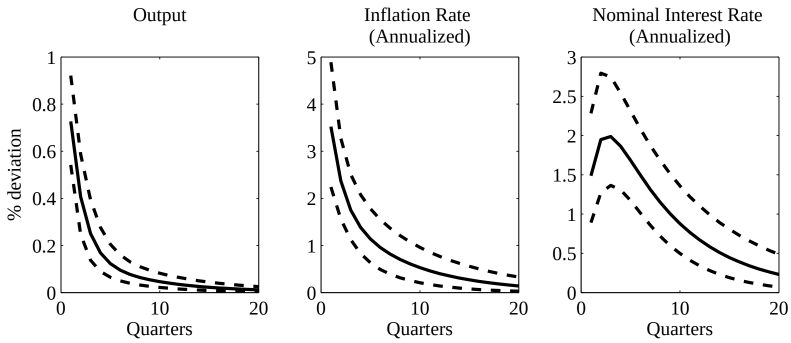
Note: Plain line: Posterior distribution, Dashed–line: Prior distribution.

Figure 7: IRFs under determinacy

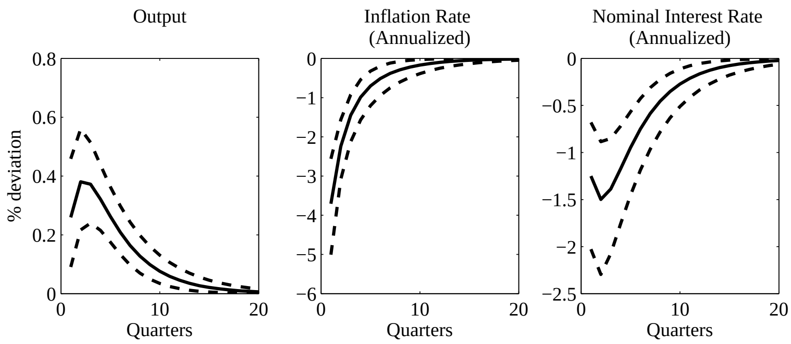
IRF to a 1 s.d. monetary policy shock



IRF to a 1 s.d. fiscal shock

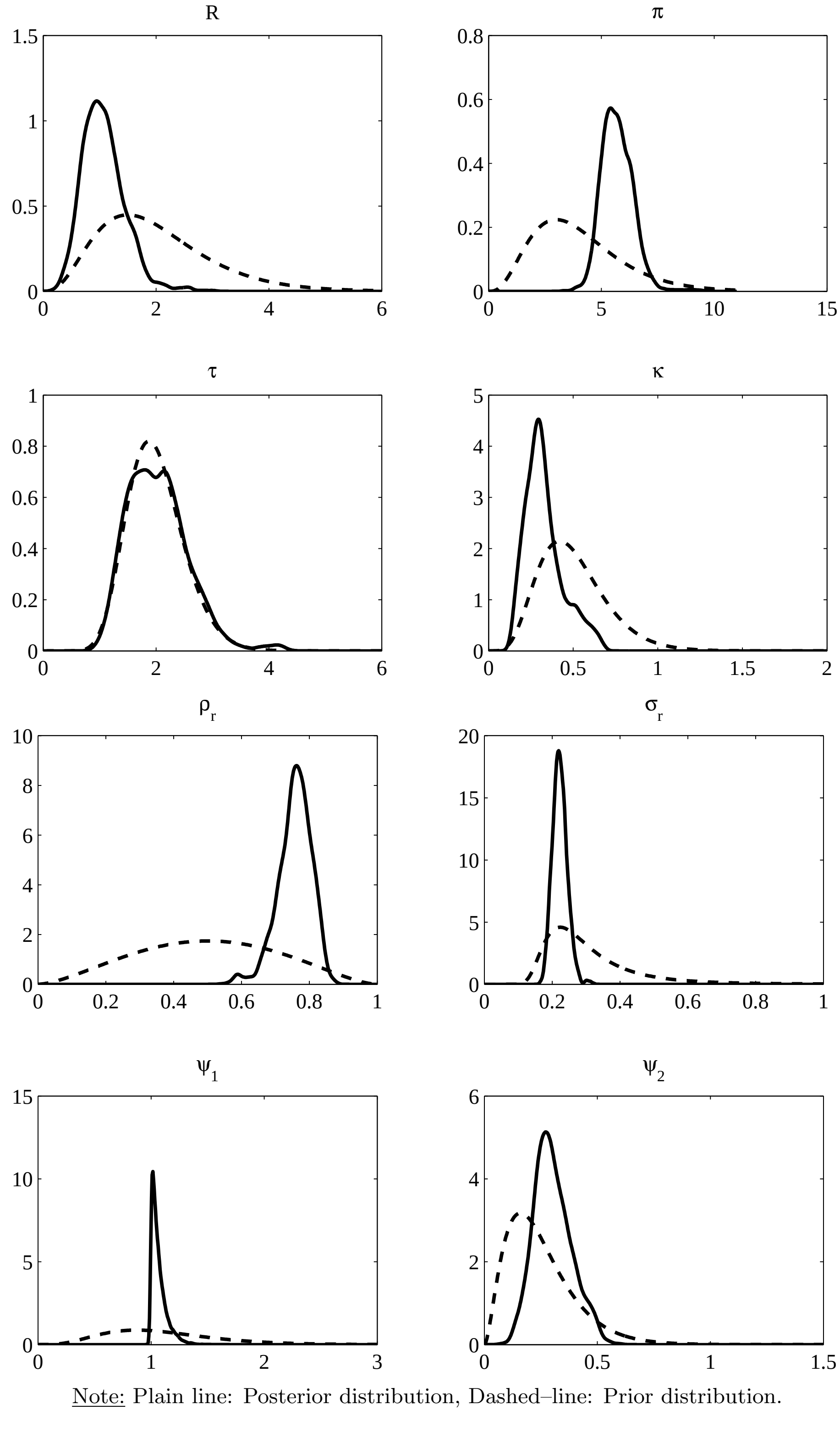


IRF to a 1 s.d. marginal cost shock



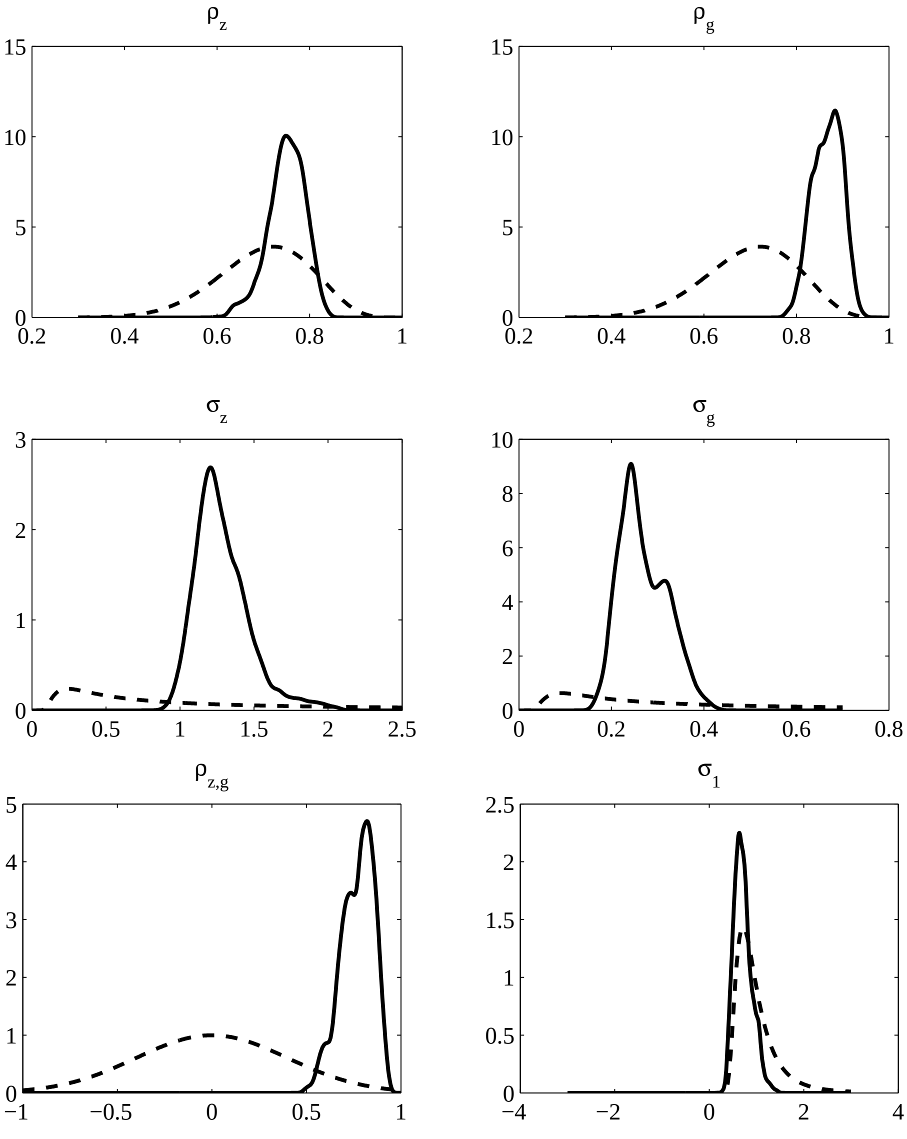
Note: Plain line: average across simulations, Dashed–line: 90% confidence interval.

Figure 8: Prior vs Posterior Distributions (imperfect information)



Note: Plain line: Posterior distribution, Dashed–line: Prior distribution.

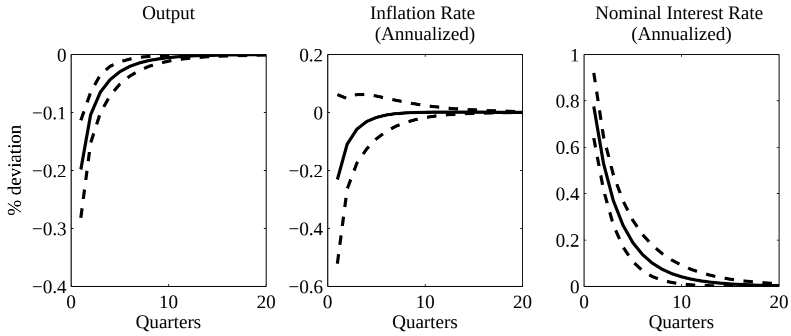
Figure 9: Prior vs Posterior Distributions (imperfect information)



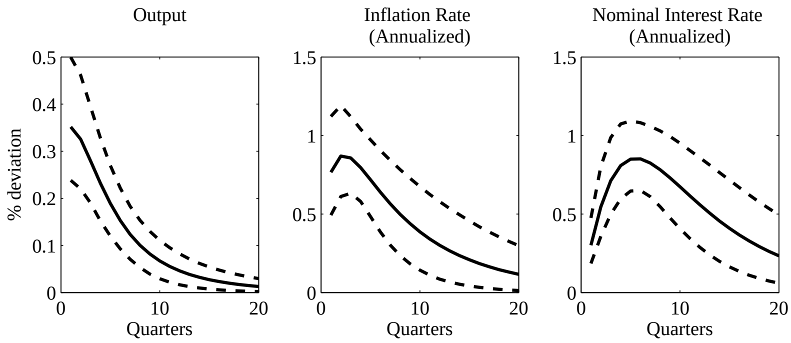
Note: Plain line: Posterior distribution, Dashed–line: Prior distribution.

Figure 10: IRFs under imperfect information

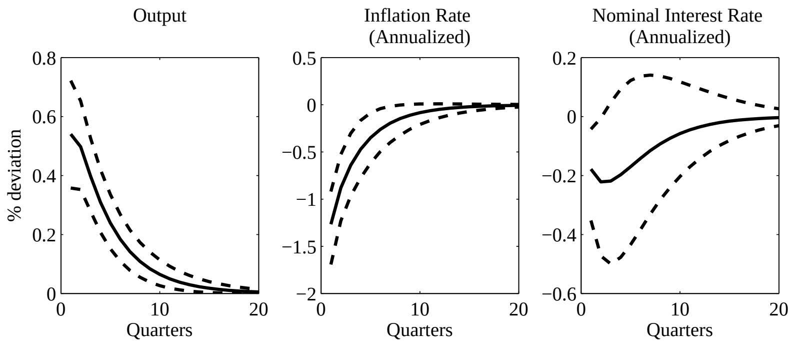
IRF to a 1 s.d. monetary policy shock



IRF to a 1 s.d. fiscal shock



IRF to a 1 s.d. marginal cost shock



Note: Plain line: average across simulations, Dashed–line: 90% confidence interval.

5 Appendix (Not intended for publication)

5.1 The Model

Household: The household solves the following maximization problem

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\frac{c_{t+s}^{1-\frac{1}{\tau}} - 1}{1 - \frac{1}{\tau}} - \alpha_h \frac{h_{t+s}^{1+\chi}}{1+\chi} \right)$$

subject to

$$\int \varrho(t+1|t) q_{t+1} + B_t + P_t c_t = q_t + P_t w_t h_t + R_{t-1} B_{t-1} + \Pi_t + \tau_t^g$$

τ_t^g denote government expenditures.

We have the following set of first order conditions

$$c_t^{-\frac{1}{\tau}} = \Lambda_t P_t \tag{13}$$

$$\alpha_h h_t^{\chi} = \Lambda_t P_t w_t \tag{14}$$

$$\Lambda_t = \beta R_t \mathbb{E}_t \Lambda_{t+1} \tag{15}$$

$$\varrho(t+1|t)\Lambda_t = \beta \Lambda_{t+1} f_{t+1|t} \tag{16}$$

Final good: There exists a final good, y_t, which is produced by perfectly competitive firms by combining intermediate goods, $y_t(j)$, according to the following technology

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} \mathrm{d}j \right)^{\frac{\theta}{\theta-1}}$$

This yields the following demand function

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} y_t$$

and free entry yields the aggregate price

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta} \mathrm{d}j \right)^{\frac{1}{1-\theta}}$$

Intermediate good: The intermediate good is produced by means of labor according to the following constant returns to scale technology

$$y_t(j) = a_t n_t(j)$$

a_t is a macroeconomic technological shock.

Each intermediate good firm has monopoly power and sets its price. Price changes are subject to quadratic adjustment costs. The firm solves:

$$\max \sum_{s=0}^{\infty} \int \varrho(t+s|t) \Bigg(P_{t+s}(j) a_t n_{t+s}(j) - P_{t+s} w_{t+s} n_{t+s}(j) \\ - P_{t+s} \frac{\varphi}{2} \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - \overline{\pi} \right)^2 y_{t+s} \Bigg)$$

subject to

$$a_t n_t(j) \leqslant \left(\frac{P_t(j)}{P_t} \right)^{-\theta} y_t \qquad (\widetilde{\nu}_t(j))$$

which yields the following set of first order conditions

$$\left(P_t(j) - \widetilde{\nu}_t(j)\right) a_t = P_t w_t \tag{17}$$

$$\varphi \frac{P_t}{P_{t-1}(j)} \left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\pi} \right) y_t = \left(\frac{P_t(j) - \theta \widetilde{\nu}_t(j)}{P_t(j)} \right) a_t n_t(j) \\ + \int \varrho_{t+1|t} \varphi \frac{P_{t+1} P_{t+1}(j)}{P_t^2(j)} \left(\frac{P_{t+1}(j)}{P_t(j)} - \overline{\pi} \right) y_{t+1} \tag{18}$$

Equilibrium The labor market equilibrium is given by

$$\int_0^1 n_t(j) \mathrm{d}j = h_t$$

The goods market equilibrium is

$$y_t = c_t + \tau_t^g + \int_0^1 \frac{\varphi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\pi} \right)^2 y_t \mathrm{d}j$$

Government expenditures are assumed to be proportional to output, $\tau_t^g = \zeta_t y_t$, where ζ_t is assumed to be an exogenous shock. Hence, the goods market equilibrium reduces to

$$(1 - \zeta_t) y_t = c_t + \int_0^1 \frac{\varphi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\pi} \right)^2 y_t \mathrm{d}j$$

Let us define $\lambda_t = P_t\Lambda_t$, $\nu_t(j) = \widetilde{\nu}_t(j)/P_t$, $\pi_t = P_t/P_{t-1}$, $\pi_t(j) = P_t(j)/P_{t-1}(j)$ and $p_t(j) = P_t(j)/P_t$. The equilibrium satisfies

$$
\begin{aligned}
(1-\zeta_t)y_t &= c_t + \int_0^1 \frac{\varphi}{2}(\pi_t(j)-\overline{\pi})^2 y_t \mathrm{d}j \\
h_t &= \int_0^1 n_t(j)\mathrm{d}j \\
y_t &= \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}}\mathrm{d}j\right)^{\frac{\theta}{\theta-1}} \\
y_t(j) &= a_t n_t(j) \\
1 &= \left(\int_0^1 p_t(j)^{1-\theta}\mathrm{d}j\right)^{\frac{1}{1-\theta}} \\
c_t^{-\frac{1}{\tau}} &= \lambda_t \\
\alpha_h h_t^{\chi} &= \lambda_t w_t \\
\lambda_t &= \beta R_t \mathbb{E}_t \frac{\lambda_{t+1}}{\pi_{t+1}} \\
\varrho(t+1|t)\lambda_t &= \beta \frac{\lambda_{t+1}}{\pi_{t+1}} f_{t+1|t} \\
\left(p_t(j)-\nu_t(j)\right) a_t &= w_t \\
\varphi\frac{\pi_t(j)}{p_t(j)}\left(\pi_t(j)-\overline{\pi}\right) y_t &= \left(1-\theta\nu_t(j)\right) a_t n_t(j) + \beta\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\varphi\frac{\pi_{t+1}(j)}{p_t(j)}\left(\pi_{t+1}(j)-\overline{\pi}\right) y_{t+1}
\end{aligned}
$$

The equilibrium can then be reduced to

$$
\begin{aligned}
(1-\zeta_t)y_t &= c_t + \int_0^1 \frac{\varphi}{2}(\pi_t(j)-\overline{\pi})^2 y_t \mathrm{d}j \\
h_t &= \int_0^1 n_t(j)\mathrm{d}j \\
y_t &= \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}}\mathrm{d}j\right)^{\frac{\theta}{\theta-1}} \\
y_t(j) &= a_t n_t(j) \\
\left(\int_0^1 p_t(j)^{1-\theta}\mathrm{d}j\right)^{\frac{1}{1-\theta}} &= 1 \\
\alpha_h h_t^{\chi} &= c_t^{-\frac{1}{\tau}} w_t \\
\left(p_t(j)-\nu_t(j)\right) a_t &= w_t \\
c_t^{-\frac{1}{\tau}} &= \beta R_t \mathbb{E}_t \frac{c_{t+1}^{-\frac{1}{\tau}}}{\pi_{t+1}} \\
\varphi \frac{\pi_t(j)}{p_t(j)}\left(\pi_t(j)-\overline{\pi}\right) y_t &= \left(1-\theta\nu_t(j)\right) a_t n_t(j) + \beta \mathbb{E}_t \frac{c_{t+1}^{-\frac{1}{\tau}}}{c_t^{-\frac{1}{\tau}}} \varphi \frac{\pi_{t+1}(j)}{p_t(j)}\left(\pi_{t+1}(j)-\overline{\pi}\right) y_{t+1}
\end{aligned}
$$

Note that we also have

$$
\pi_t \left(\int_0^1 (\pi_{t-1}(j)p_{t-1}(j))^{1-\theta}\mathrm{d}j\right)^{\frac{1}{1-\theta}} = 1
$$

Log–linear Representation

$$
\begin{aligned}
\widehat{y}_t &= \widehat{c}_t + \frac{\zeta}{1-\zeta}\widehat{\zeta}_t && (19) \\
\widehat{h}_t &= \int_0^1 \widehat{n}_t(j)\mathrm{d}j && (20) \\
\widehat{y}_t &= \int_0^1 \widehat{y}_t(j)\mathrm{d}j && (21) \\
\widehat{y}_t(j) &= \widehat{a}_t + \widehat{n}_t(j) && (22) \\
\int_0^1 \widehat{p}_t(j)\mathrm{d}j &= 0 && (23) \\
\chi\widehat{h}_t &= -\frac{\widehat{c}_t}{\tau} + \widehat{w}_t && (24) \\
\widehat{\nu}_t(j) &= \theta\widehat{p}_t(j) + (\theta-1)\left(\widehat{a}_t - \widehat{w}_t\right) && (25) \\
-\frac{\widehat{c}_t}{\tau} &= \widehat{R}_t + \mathbb{E}_t\left(-\frac{\widehat{c}_{t+1}}{\tau} - \widehat{\pi}_{t+1}\right) && (26) \\
\varphi\overline{\pi}^2\widehat{\pi}_t(j) &= -\widehat{\nu}_t(j) + \beta\varphi\overline{\pi}^2\mathbb{E}_t\widehat{\pi}_{t+1}(j) && (27)
\end{aligned}
$$

We also have

5.2 Perfect Information Case

Plugging (25) in (27), we have in a symmetric equilibrium

$$\varphi\overline{\pi}^2\widehat{\pi}_t = (\theta-1)\left(\widehat{w}_t - \widehat{a}_t\right) + \beta\varphi\overline{\pi}^2\mathbb{E}_t\widehat{\pi}_{t+1}$$

Making use of the fact that $\widehat{y}_t = \widehat{c}_t + \frac{\zeta}{1-\zeta}\widehat{\zeta}_t$, the equilibrium of the economy may be written as

$$\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - \tau(\widehat{R}_t - \mathbb{E}_t\widehat{\pi}_{t+1}) + \frac{\zeta}{1-\zeta}\left(\widehat{\zeta}_t - \mathbb{E}_t\widehat{\zeta}_{t+1}\right) \tag{28}$$

$$\widehat{w}_t = \frac{1+\chi\tau}{\tau}\widehat{y}_t - \chi\widehat{a}_t - \frac{\zeta}{\tau(1-\zeta)}\widehat{\zeta}_t \tag{29}$$

$$\widehat{\pi}_t = \frac{(\theta-1)(1+\chi\tau)}{\tau\varphi\overline{\pi}^2}\left(\widehat{y}_t - \frac{\tau(1+\chi)}{1+\chi\tau}\widehat{a}_t - \frac{\tau\zeta}{\tau(1-\zeta)(1+\chi\tau)}\widehat{\zeta}_t\right) + \beta\mathbb{E}_t\widehat{\pi}_{t+1} \tag{30}$$

Let us then define $\kappa \equiv \frac{(\theta-1)(1+\chi\tau)}{\tau\varphi\overline{\pi}^2}$, $\widehat{g}_t \equiv \frac{\zeta}{1-\zeta}\left(\widehat{\zeta}_t - \mathbb{E}_t\widehat{\zeta}_{t+1}\right)$ and $\widehat{z}_t \equiv \frac{\tau(1+\chi)}{1+\chi\tau}\widehat{a}_t + \frac{\tau\zeta}{\tau(1-\zeta)(1+\chi\tau)}\widehat{\zeta}_t$. This system can be rewritten as

$$\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - \tau(\widehat{R}_t - \mathbb{E}_t\widehat{\pi}_{t+1}) + \widehat{g}_t \tag{31}$$

$$\widehat{\pi}_t = \kappa\left(\widehat{y}_t - \widehat{z}_t\right) + \beta\mathbb{E}_t\widehat{\pi}_{t+1} \tag{32}$$

5.3 Imperfect Information Case

There are –at least– two alternative specifications of the imperfect information (signal extraction) problem. One specification involves the distinction between idiosyncratic and aggregate shocks. Suppose that the agents are subject to shocks that contain both idiosyncratic and common –aggregate– components and that the agents can only observe the combined shocks. If these two components have different stochastic processes then the agents need to solve a signal extraction problem (see Lorenzoni (2006)).

An alternative and simpler specification involves the assumption that all shocks are common but they are measured with error. Of course, this statement is technically equivalent to assuming that a suitable subset of the endogenous variables is measured with error. Otherwise, knowledge of the model would allow the agents to solve out for the true values of the shocks, eliminating the signal extraction problem. This is the standard practice in the literature (for instance, see Svensson and Woodford (2003)). Some may find the assumption that the individuals may lack perfect knowledge of some of their own variables questionable. But it can be defended on the basis that, for instance, even at the firm level the output and/or the inputs may not be measured contemporaneously without any error. This is precisely the assumption made in models of sticky information or inattentive agents (or in Svensson and Woodford (2003)). We have opted for this specification because of two reasons: First, its empirical implementation is straightforward as it only requires the specification of the signals and the noise in the measurement of the variables. And second, given the existence of real time data (for instance, at the Philadelphia FED) one can assess the plausibility of the estimated amount of noise in the model by comparing it to that present, say, in data revisions. A specification with idiosyncratic shocks, on the other hand, may require knowledge about (or assumptions on) the relative variance of idiosyncratic and aggregate shocks in the estimation of the model.

In what follows, we assume that productivity and fiscal expenditures are measured imperfectly. The agents are given noisy signals on some variables and make decisions based on this information set. We denote by $\mathscr{E}_t$ the expectation operator in this case. The log–linear representation of the equilibrium is given by

$$\widetilde{y}_t = \widetilde{c}_t + \frac{\zeta}{1-\zeta}\widetilde{\zeta}_t \tag{33}$$

$$\widetilde{h}_t = \int_0^1 \widetilde{n}_t(j)\mathrm{d}j \tag{34}$$

$$\widetilde{y}_t = \int_0^1 \widetilde{y}_t(j)\mathrm{d}j \tag{35}$$

$$\widetilde{y}_t(j) = \widetilde{a}_t + \widetilde{n}_t(j) \tag{36}$$

$$\int_0^1 \widetilde{p}_t(j)\mathrm{d}j = 0 \tag{37}$$

$$\chi\widetilde{h}_t = -\frac{\widetilde{c}_t}{\tau} + \widetilde{w}_t \tag{38}$$

$$\widetilde{\nu}_t(j) = \theta\widetilde{p}_t(j) + (\theta-1)\left(\widetilde{a}_t - \widetilde{w}_t\right) \tag{39}$$

$$-\frac{\widetilde{c}_t}{\tau} = \widetilde{R}_t + \mathscr{E}_t\left(-\frac{\widetilde{c}_{t+1}}{\tau} - \widetilde{\pi}_{t+1}\right) \tag{40}$$

$$\varphi\overline{\pi}^2\widetilde{\pi}_t(j) = -\widetilde{\nu}_t(j) + \beta\varphi\overline{\pi}^2\mathscr{E}_t\widetilde{\pi}_{t+1}(j) \tag{41}$$

$\widetilde{x}_t \equiv \widehat{x}_t + \xi_t^x$ denotes observed variables, where $\widehat{x}_t$ denotes the true value of x_t and ξ_t^x is the associated measurement error.

Making use of the fact that $\widetilde{y}_t = \widetilde{c}_t + \frac{\zeta}{1-\zeta}\widetilde{\zeta}_t$, the equilibrium of the economy may be written as

$$\widetilde{y}_t = \mathbb{E}_t\widetilde{y}_{t+1} - \tau(\widetilde{R}_t - \mathbb{E}_t\widetilde{\pi}_{t+1}) + \frac{\zeta}{1-\zeta}\left(\widetilde{\zeta}_t - \mathbb{E}_t\widetilde{\zeta}_{t+1}\right) \tag{42}$$

$$\widetilde{w}_t = \frac{1+\chi\tau}{\tau}\widetilde{y}_t - \chi\widetilde{a}_t - \frac{\zeta}{\tau(1-\zeta)}\widetilde{\zeta}_t \tag{43}$$

$$\widetilde{\pi}_t = \frac{(\theta-1)(1+\chi\tau)}{\tau\varphi\overline{\pi}^2}\left(\widetilde{y}_t - \frac{\tau(1+\chi)}{1+\chi\tau}\widetilde{a}_t - \frac{\tau\zeta}{\tau(1-\zeta)(1+\chi\tau)}\widetilde{\zeta}_t\right) + \beta\mathbb{E}_t\widetilde{\pi}_{t+1} \tag{44}$$

Let us define $\kappa \equiv \frac{(\theta-1)(1+\chi\tau)}{\tau\varphi\overline{\pi}^2}$, $\widetilde{g}_t \equiv \frac{\zeta}{1-\zeta}\left(\widetilde{\zeta}_t - \mathbb{E}_t\widetilde{\zeta}_{t+1}\right)$ and $\widetilde{z}_t \equiv \frac{\tau(1+\chi)}{1+\chi\tau}\widetilde{a}_t + \frac{\tau\zeta}{\tau(1-\zeta)(1+\chi\tau)}\widetilde{\zeta}_t$.

The system rewrites as

$$\widetilde{y}_t = \mathbb{E}_t\widetilde{y}_{t+1} - \tau(\widetilde{R}_t - \mathbb{E}_t\widetilde{\pi}_{t+1}) + \widetilde{g}_t \tag{45}$$

$$\widetilde{\pi}_t = \kappa\,(\widetilde{y}_t - \widetilde{z}_t) + \beta\mathbb{E}_t\widetilde{\pi}_{t+1} \tag{46}$$

These two equations together with the following interest rate policy rule comprise the entire model

$$\widetilde{R}_t = \rho_R\widetilde{R}_{t-1} + (1-\rho_R)(\psi_\pi\widetilde{\pi}_t + \psi_y\widetilde{y}_t) + \varepsilon_t^R$$

Let the state of the economy be represented by two vectors $\widetilde{X}_t^b$ and $\widetilde{X}_t^f$. The first one includes the predetermined (backward looking) state variables, i.e. $\widetilde{X}_t^b = (\widetilde{R}_{t-1}, \widetilde{z}_t, \widetilde{g}_t, \widetilde{\varepsilon}_t^R)'$, whereas the second one consists of the forward looking state variables, i.e. $\widetilde{X}_t^f = (\widetilde{y}_t, \widetilde{\pi}_t)'$. The model admits the following representation

$$M_0\begin{pmatrix}\widetilde{X}_{t+1}^b\\ \mathbb{E}_t\widetilde{X}_{t+1}^f\end{pmatrix} + M_1\begin{pmatrix}\widetilde{X}_t^b\\ \widetilde{X}_t^f\end{pmatrix} = M_2\varepsilon_{t+1} \tag{47}$$

where

$$M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ -\tau & 0 & 0 & 0 & 1 & \tau\\ 0 & 0 & 0 & 0 & 0 & \beta \end{pmatrix}$$

$$M_1 = \begin{pmatrix} -\rho_R & 0 & 0 & -1 & -(1-\rho_R)\psi_y & -(1-\rho_R)\psi_\pi\\ 0 & -\rho_z & 0 & 0 & 0 & 0\\ 0 & 0 & -\rho_g & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & -1 & 0\\ 0 & -\kappa & 0 & 0 & \kappa & -1 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon_{t+1} = \{\varepsilon_{t+1}^z, \varepsilon_{t+1}^g, \varepsilon_{t+1}^R\}$$

Thus the first row corresponds to the Taylor rule, the second, third and forth row to the demand, cost push shock and policy shock, the fifth row to the IS-curve and the sixth row to the Phillips curve. Let us denote the signal process by $\{S_t\}$. The measurement equation relates the state of the economy to the signal:

$$S_t = C\begin{pmatrix}\widetilde{X}_t^b\\ \widetilde{X}_t^f\end{pmatrix} + v_t. \tag{48}$$

Finally u and v are assumed to be normally distributed covariance matrices Σ_{uu} and Σ_{vv} respectively and $E(uv')=0$.

$X_{t+i|t}=E(X_{t+i}|\mathscr{I}_t)$ for $i \geqslant 0$ and where $\mathscr{I}_t$ denotes the information set available to the agents at the beginning of period t. The information set available to the agents consists of *i)* the structure of the model and *ii)* the history of the observable signals they are given in each period:

$$\mathscr{I}_t=\{S_{t-j}, j \geqslant 0, M_0, M_1, M_2, C, \Sigma_{uu}, \Sigma_{vv}\}$$

The information structure of the agents is described fully by the specification of the signals.

6 Solving the system

Step 1: We first solve for the expected system:

$$M_0\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t}\end{pmatrix}+M_1)\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t}\end{pmatrix}= \tag{49}$$

which rewrites as

$$\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t}\end{pmatrix}=W\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t}\end{pmatrix} \tag{50}$$

where

$$W=-M_0^{-1}M_1$$

After getting the Jordan form associated to (50) and applying standard methods for eliminating bubbles, we get

$$X^f_{t|t}=GX^b_{t|t}$$

From which we get

$$\begin{aligned} X^b_{t+1|t} &= (W_{bb}+W_{bf}G)X^b_{t|t}=W^bX^b_{t|t} & (51)\\ X^f_{t+1|t} &= (W_{fb}+W_{ff}G)X^b_{t|t}=W^fX^b_{t|t} & (52)\end{aligned}$$

Step 2: We go back to the initial system to get and write

Then, (**??**) rewrites

$$M_0\begin{pmatrix} X^b_{t+1} \\ X^f_{t+1|t}\end{pmatrix}+M_1\begin{pmatrix} X^b_{t} \\ X^f_{t}\end{pmatrix}=M_2u_{t+1}$$

Taking expectations, we have

$$M_0\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t}\end{pmatrix}+M_1\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t}\end{pmatrix}=0$$

Subtracting, we get

$$M_0 \begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} + M_1 \begin{pmatrix} X^b_t - X^b_{t|t} \\ X^f_t - X^f_{t|t} \end{pmatrix} = M_2 u_{t+1} \tag{53}$$

which rewrites

$$\begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} = W^c \begin{pmatrix} X^b_t - X^b_{t|t} \\ X^f_t - X^f_{t|t} \end{pmatrix} + M_0^{-1} M_2 u_{t+1} \tag{54}$$

where, $W^c = -M_0^{-1} M_1$. Hence, considering the second block of the above matrix equation, we get

$$W^c_{fb}(X^b_t - X^b_{t|t}) + W^c_{ff}(X^f_t - X^f_{t|t}) = 0$$

which gives

$$X^f_t = F^0 X^b_t + F^1 X^b_{t|t}$$

with $F^0 = -{W^c_{ff}}^{-1} W^c_{fb}$ and $F^1 = G - F^0$.

Now considering the first block we have

$$X^b_{t+1} = X^b_{t+1|t} + W^c_{bb}(X^b_t - X^b_{t|t}) + W^c_{bf}(X^f_t - X^f_{t|t}) + M^2 u_{t+1}$$

from which we get, using (51)

$$X^b_{t+1} = M^0 X^b_t + M^1 X^b_{t|t} + M^2 u_{t+1}$$

with $M^0 = W^c_{bb} + W^c_{bf} F^0$, $M^1 = W^b - M^0$ and $M^2 = M_0^{-1} M_2$.

We also have

$$S_t = C_b X^b_t + C_f X^f_t + v_t$$

from which we get

$$S_t = S^0 X^b_t + S^1 X^b_{t|t} + v_t$$

where $S^0 = C_b + C_f F^0$ and $S^1 = C_f F^1$

7 Filtering

Since our solution involves terms in $X^b_{t|t}$, we would like to compute this quantity. However, the only information we can exploit is a signal S_t that we described previously. We therefore use a Kalman filter approach to compute the optimal prediction of $X^b_{t|t}$.

In order to recover the Kalman filter, it is a good idea to think in terms of expectation errors. Therefore, let us define

$$\widetilde{X}_t^b = X_t^b - X_{t|t-1}^b$$

and

$$\widetilde{S}_t = S_t - S_{t|t-1}$$

Note that since S_t depends on $X_{t|t}^b$, only the signal relying on $\widetilde{S}_t = S_t - S^1 X_{t|t}^b$ can be used to infer anything on $X_{t|t}^b$. Therefore, the policy maker revises its expectations using a linear rule depending on $\widetilde{S}_t^e = S_t - S^1 X_{t|t}^b$. The filtering equation then writes

$$X_{t|t}^b = X_{t|t-1}^b + K(\widetilde{S}_t^e - \widetilde{S}_{t|t-1}^e) = X_{t|t-1}^b + K(S^0 \widetilde{X}_t^b + v_t)$$

where K is the filter gain matrix, that we would like to compute.

The first thing we have to do is to rewrite the system in terms of state–space representation. Since $S_{t|t-1} = (S^0 + S^1)X_{t|t-1}^b$, we have

$$\begin{aligned} \widetilde{S}_t &= S^0(X_t^b - X_{t|t}^b) + S^1(X_{t|t}^b - X_{t|t-1}^b) + v_t \\ &= S^0 \widetilde{X}_t^b + S^1 K(S^0 \widetilde{X}_t^b + v_t) + v_t \\ &= S^\star \widetilde{X}_t^b + \nu_t \end{aligned}$$

where $S^\star = (I + S^1 K)S^0$ and $\nu_t = (I + S^1 K)v_t$.

Now, consider the law of motion of backward state variables, we get

$$\begin{aligned} \widetilde{X}_{t+1}^b &= M^0(X_t^b - X_{t|t}^b) + M^2 u_{t+1} \\ &= M^0(X_t^b - X_{t|t-1}^b - X_{t|t}^b + X_{t|t-1}^b) + M^2 u_{t+1} \\ &= M^0 \widetilde{X}_t^b - M^0(X_{t|t}^b + X_{t|t-1}^b) + M^2 u_{t+1} \\ &= M^0 \widetilde{X}_t^b - M^0 K(S^0 \widetilde{X}_t^b + v_t) + M^2 u_{t+1} \\ &= M^\star \widetilde{X}_t^b + \omega_{t+1} \end{aligned}$$

where $M^\star = M^0(I - KS^0)$ and $\omega_{t+1} = M^2 u_{t+1} - M^0 K v_t$.

We therefore end–up with the following state–space representation

$$\widetilde{X}_{t+1}^b = M^\star \widetilde{X}_t^b + \omega_{t+1} \quad (55)$$

$$\widetilde{S}_t = S^\star \widetilde{X}_t^b + \nu_t \quad (56)$$

For which the Kalman filter is given by

$$\widetilde{X}_{t|t}^b = \widetilde{X}_{t|t-1}^b + PS^{\star\prime}(S^\star PS^{\star\prime} + \Sigma_{\nu\nu})^{-1}(S^\star \widetilde{X}_t^b + \nu_t)$$

But since $\widetilde{X}^b_{t|t}$ is an expectation error, it is not correlated with the information set in $t-1$, such that $\widetilde{X}^b_{t|t-1}=0$. The prediction formula for $\widetilde{X}^b_{t|t}$ therefore reduces to

$$\widetilde{X}^b_{t|t} = PS^{\star\prime}(S^\star PS^{\star\prime}+\Sigma_{\nu\nu})^{-1}(S^\star\widetilde{X}^b_t+\nu_t) \tag{57}$$

where P solves

$$P = M^\star PM^{\star\prime}+\Sigma_{\omega\omega}$$

and $\Sigma_{\nu\nu}=(I+S^1K)\Sigma_{vv}(I+S^1K)'$ and $\Sigma_{\omega\omega}=M^0K\Sigma_{vv}K'M^{0\prime}+M^2\Sigma_{uu}M^{2\prime}$

Note however that the above solution is obtained for a given K matrix that remains to be computed. We can do that by using the basic equation of the Kalman filter:

$$\begin{aligned}
X^b_{t|t} &= X^b_{t|t-1}+K(\widetilde{S}^e_t-\widetilde{S}^e_{t|t-1})\\
&= X^b_{t|t-1}+K(S_t-S^1X^b_{t|t}-(S_{t|t-1}-S^1X^b_{t|t-1}))\\
&= X^b_{t|t-1}+K(S_t-S^1X^b_{t|t}-S^0X^b_{t|t-1})
\end{aligned}$$

Solving for $X^b_{t|t}$, we get

$$\begin{aligned}
X^b_{t|t} &= (I+KS^1)^{-1}(X^b_{t|t-1}+K(S_t-S^0X^b_{t|t-1}))\\
&= (I+KS^1)^{-1}(X^b_{t|t-1}+KS^1X^b_{t|t-1}-KS^1X^b_{t|t-1}+K(S_t-S^0X^b_{t|t-1}))\\
&= (I+KS^1)^{-1}(I+KS^1)X^b_{t|t-1}+(I+KS^1)^{-1}K(S_t-(S^0+S^1)X^b_{t|t-1}))\\
&= X^b_{t|t-1}+(I+KS^1)^{-1}K\widetilde{S}_t\\
&= X^b_{t|t-1}+K(I+S^1K)^{-1}\widetilde{S}_t\\
&= X^b_{t|t-1}+K(I+S^1K)^{-1}(S^\star\widetilde{X}^b_t+\nu_t)
\end{aligned}$$

where we made use of the identity $(I+KS^1)^{-1}K\equiv K(I+S^1K)^{-1}$. Hence, identifying to (57), we have

$$K(I+S^1K)^{-1}=PS^{\star\prime}(S^\star PS^{\star\prime}+\Sigma_{\nu\nu})^{-1}$$

remembering that $S^\star=(I+S^1K)S^0$ and $\Sigma_{\nu\nu}=(I+S^1K)\Sigma_{vv}(I+S^1K)'$, we have

$$K(I+S^1K)^{-1}=PS^{0\prime}(I+S^1K)'((I+S^1K)S^0PS^{0\prime}(I+S^1K)'+(I+S^1K)\Sigma_{vv}(I+S^1K)')^{-1}(I+S^1K)S^0$$

which rewrites as

$$\begin{aligned}
K(I+S^1K)^{-1} &= PS^{0\prime}(I+S^1K)'\left[(I+S^1K)(S^0PS^{0\prime}+\Sigma_{vv})(I+S^1K)'\right]^{-1}\\
K(I+S^1K)^{-1} &= PS^{0\prime}(I+S^1K)'(I+S^1K)'^{-1}(S^0PS^{0\prime}+\Sigma_{vv})^{-1}(I+S^1K)^{-1}
\end{aligned}$$

Hence, we obtain

$$K=PS^{0\prime}(S^0PS^{0\prime}+\Sigma_{vv})^{-1} \tag{58}$$

Now, recall that

$$P = M^{\star} P M^{\star\prime} + \Sigma_{\omega\omega}$$

Remembering that $M^{\star} = M^0(I + KS^0)$ and $\Sigma_{\omega\omega} = M^0 K \Sigma_{vv} K' M^{0\prime} + M^2 \Sigma_{uu} M^{2\prime}$, we have

$$\begin{aligned}
P &= M^0(I - KS^0)P\left[M^0(I - KS^0)\right]' + M^0 K \Sigma_{vv} K' M^{0\prime} + M^2 \Sigma_{uu} M^{2\prime} \\
&= M^0\left[(I - KS^0)P(I - S^{0\prime}K') + K\Sigma_{vv}K'\right] M^{0\prime} + M^2 \Sigma_{uu} M^{2\prime}
\end{aligned}$$

Plugging the definition of K in the latter equation, we obtain

$$P = M^0\left[P - PS^{0\prime}(S^0 P S^{0\prime} + \Sigma_{vv})^{-1} S^0 P\right] M^{0\prime} + M^2 \Sigma_{uu} M^{2\prime} \tag{59}$$

8 Summary

We finally end–up with the system of equations:

$$\begin{aligned}
X_{t+1}^b &= M^0 X_t^b + M^1 X_{t|t}^b + M^2 u_{t+1} & (60) \\
S_t &= S_b^0 X_t^b + S_b^1 X_{t|t}^b + v_t & (61) \\
X_t^f &= F^0 X_t^b + F^1 X_{t|t}^b & (62) \\
X_{t|t}^b &= X_{t|t-1}^b + K(S^0(X_t^b - X_{t|t-1}^b) + v_t) & (63) \\
X_{t+1|t}^b &= (M^0 + M^1) X_{t|t}^b & (64)
\end{aligned}$$

to describe the dynamics of our economy.

This may be recast as a standard state–space problem

$$\begin{aligned}
X_{t+1|t+1}^b &= X_{t+1|t}^b + K(S^0(X_{t+1}^b - X_{t+1|t}^b) + v_{t+1}) \\
&= (M^0 + M^1)X_{t|t}^b + K(S^0(M^0 X_t^b + M^1 X_{t|t}^b + M^2 u_{t+1} - (M^0 + M^1)X_{t|t}^b) + v_{t+1}) \\
&= KS^0 M^0 X_t^b + ((I - KS^0)M^0 + M^1)X_{t|t}^b + KS^0 M^2 u_{t+1} + K v_{t+1}
\end{aligned}$$

Then

$$\begin{pmatrix} X_{t+1}^b \\ X_{t+1|t+1}^b \end{pmatrix} = M_{\mathrm{X}} \begin{pmatrix} X_t^b \\ X_{t|t}^b \end{pmatrix} + M_{\mathrm{E}} \begin{pmatrix} u_{t+1} \\ v_{t+1} \end{pmatrix}$$

where

$$M_{\mathrm{X}} = \begin{pmatrix} M^0 & M^1 \\ KS^0 M^0 & ((I - KS^0)M^0 + M^1) \end{pmatrix} \text{ and } M_{\mathrm{E}} = \begin{pmatrix} M^2 & 0 \\ KS^0 M^2 & K \end{pmatrix}$$

and

$$X_t^f = M_{\mathrm{F}} \begin{pmatrix} X_t^b \\ X_{t|t}^b \end{pmatrix}$$

where

$$M_{\mathrm{F}} = \begin{pmatrix} F^0 & F^1 \end{pmatrix}$$