European Economic Review 18 (1982) 209–219. North-Holland Publishing Company

A TEST OF THE EQUILIBRIUM HYPOTHESIS BASED ON INVENTORIES

A Communication

Georges DUCOS

University of Social Sciences, 31070 Toulouse, France

Jerry GREEN

Harvard University, Cambridge, MA 02138, USA

Jean-Jacques LAFFONT

University of Social Sciences, 31070 Toulouse, France
Ecole des Hautes Etudes en Sciences Sociales, 75006 Paris, France

1. Introduction

The theory of fix-price equilibria has offered a new challenging paradigm to economics. The empirical relevance of this paradigm remains to be proved, despite a number of efforts in this direction [see for example Fair and Jaffe (1972), Laffont and Garcia (1976)]. Most of the tests of the equilibrium hypothesis rely heavily on the specification of a price dynamics of a traditional type, i.e., price variations are proportional to excess demand.

Green and Laffont (1981) suggested to complete the static fix-price equilibria by a price dynamics, called anticipatory pricing, in which prices, at the beginning of the period, are set at the values which clear expected excess demands and remain fixed during the period. This procedure has the advantage of yielding a well defined and relatively simple dynamics of a number of key macroeconomic variables. The extremely simple model used in that paper gives in particular an inventory stock dynamic equation. The purpose of this note is to report our efforts to test the equilibrium hypothesis in that framework.

In section 2 the model is briefly summarized to obtain the inventories equation. In section 3, the likelihood functions for the equilibrium hypothesis and the disequilibrium hypothesis are obtained. Results for the test of the equilibrium hypothesis with French and American data are reported in section 4 where it is shown that the equilibrium hypothesis is rejected.

2. The model

2.1. Basic structure

We consider an economy with three commodities: money, a consumption good and labor, and two agents — a consumer and a producer — which are aggregates of large numbers of competitive agents.

Two stores of value are desired: money balances and inventories. Money balances are held only by the consumer; they are used to finance purchases of goods in excess of labor income. Inventories are owned exclusively by the producer; they result from an excess of output over sales and are used to fully execute desired transactions. The actual variation in these stocks is a composite of the intended and unintended changes.

At the beginning of each period, the level of inventories is known and prices are fixed at the values which would yield the Walrasian equilibrium if all random factors in the economy had their average levels. We will refer to this pricing as *anticipatory* pricing. Further the expected values of demands and supplies of good and labor which are functions of the nominal prices and the stocks of money and inventories, are known. These functions may differ from their expected values because of unforeseeable events. Thus, there is a tendency toward market clearing, but short-run disturbances continually keep it from being achieved.

2.2. The structural form

The structural form of the result is the following one [see Green and Laffont (1981) for more details]:

$$x_t^s = \underset{+}{\alpha_0}(s_t - \bar{s}) + \underset{+}{\alpha_1}(p_t - w_t) + \underset{+}{a}(l_t - l_t^d) + \varepsilon_t^1, \tag{1}$$

$$x_t^d = \underset{-}{\beta_1} p_t + \underset{+}{\beta_2} w_t + \underset{+}{b}(l_t - l_t^s) + \varepsilon_t^2, \tag{2}$$

$$l_t^d = \underset{-}{\gamma_0}(s_t - \bar{s}) + \underset{+}{\gamma_1}(p_t - w_t) + \underset{+}{c}(x_t - x_t^s) + \varepsilon_t^3, \tag{3}$$

$$l_t^s = \underset{-}{\delta_1} p_1 + \underset{+}{\delta_2} w_t + \underset{+}{d}(x_t - x_t^d) + \varepsilon_t^4, \tag{4}$$

$$x_t = \min(x_t^d, x_t^s), \tag{5}$$

$$l_t = \min(l_t^d, l_t^s), \tag{6}$$

where, all in period t,

x_t^s = desired level of supply (i.e., of actual sales) to the household sector;
x_t^d = desired level of good demand;
l_t^d, l_d^s = desired level of labor demand and labor supply, respectively;
x_t, l_t = actual, market-determined sales and employment;
p_t, w_t = logarithm of price level and wage rate, respectively;
s_t = inventory stocks at the start of the period;
$\bar{s}$ = desired level of inventory stocks in a steady-state;
$\varepsilon_t^1, \varepsilon_t^2, \varepsilon_t^3, \varepsilon_t^4$ = random errors.

The signs below the parameters indicate if the parameters are expected to be positive or negative.

The amount by which an agent is constrained below his desired level of purchase or sale in a market enters into the determination of his desired trade in the other market. The parameters a, b, c, d define the 'spillover effects.'

Since we treat the case of a constant money stock throughout, nominal prices and price expectations are sufficient to specify the level of real balances. Because we take the view that the unit of time is rather short compared with the planning horizon of the consumer, the principal determinant of the consumer's demand for real balances is its expectation of future prices and wages.

Moreover, quantities actually transacted are determined by the 'short side' of each market.

The basic assumption of the model is that p_t and w_t are set in advance at the level that would clear the market if there were zero errors in each of the behavioral equations. Defining these levels as p_t^* and w_t^*, we have

$$p_t^* = (s_t - \bar{s}) \frac{[\alpha_0(\delta_2 + \gamma_1) - \gamma_0(\beta_2 + \alpha_1)]}{(\beta_1 - \alpha_1)(\delta_2 + \gamma_1) - (\delta_1 - \gamma_1)(\beta_2 + \alpha_1)}, \tag{7}$$

$$w_t^* = (s_t - \bar{s}) \frac{[\gamma_0(\beta_1 - \alpha_1) - \alpha_0(\delta_1 - \gamma_1)]}{(\beta_1 - \alpha_1)(\delta_2 - \gamma_1) - (\delta_1 - \gamma_1)(\beta_2 + \alpha_1)}. \tag{8}$$

As the values $(\varepsilon_t^1, \varepsilon_t^2, \varepsilon_t^3, \varepsilon_t^4)$ are realized, (p_t^*, w_t^*) is not a Walrasian equilibrium price system in general. We get a fixed price equilibrium in which the quantities x_t and l_t serve as the equilibrating variables. That is, sales and employment are assumed varying until, at their equilibrium levels, the system of equations (1)–(6) is satisfied.

2.3. The reduced form

Let

$$X_t \equiv \alpha_0(s_t - \bar{s}) + \alpha_1(p_t^* - w_t^*) = \beta_1 p_t^* + \beta_2 w_t^*, \tag{9}$$

and

$$L_t \equiv \gamma_0(s_t - \bar{s}) + \gamma_1(p_t^* - w_t^*) = \delta_1 p_t^* + \delta_2 w_t^*. \tag{10}$$

Four cases must be distinguished to describe the reduced form.

Regime I: Keynesian unemployment

$$x_t^s > x_t^d = x_t, \qquad l_t^s > l_t^d = l_t. \tag{11}$$

We obtain the system

$$\left.\begin{aligned}
&x_t^s = X_t + \varepsilon_t^1,\\
&x_t^d = x_t = X_t + \{-bc\,\varepsilon_t^1 + \varepsilon_t^2 + b(\varepsilon_t^3 - \varepsilon_t^4)\}/\{1 - bc\},\\
&l_t^d = l_t = L_t + \{-c(\varepsilon_t^1 - \varepsilon_t^2) + \varepsilon_t^3 - bc\,\varepsilon_t^4\}/\{1 - bc\},\\
&l_t^s = L_t + \varepsilon_t^4.
\end{aligned}\right\} \tag{12}$$

Regime II: Under consumption

$$x_t^s > x_t^d = x_t, \qquad l_t^d > l_t^s = l_t. \tag{13}$$

We have

$$\left.\begin{aligned}
&x_t^s = X_t + \{\varepsilon_t^1 - ac\,\varepsilon_t^2 + a(-\varepsilon_t^3 + \varepsilon_t^4)\}/\{1 - ac\},\\
&x_t^d = x_t = X_t + \varepsilon_t^2,\\
&l_t^d = L_t + \{-c(\varepsilon_t^1 - \varepsilon_t^2) + \varepsilon_t^3 - ac\,\varepsilon_t^4\}/\{1 - ac\},\\
&l_t^s = l_t = L_t + \varepsilon_t^4.
\end{aligned}\right\} \tag{14}$$

Regime III: Repressed inflation

$$x_t^d > x_t^s = x_t, \qquad l_t^d > l_t^s = l_t. \tag{15}$$

We have

$$\left.\begin{aligned} x_t^s &= x_t = X_t + \{\varepsilon_t^1 - ad\varepsilon_t^2 + a(-\varepsilon_t^3 + \varepsilon_t^4\}/\{1-ad\}, \\ x_t^d &= X_t + \varepsilon_t^2, \\ l_t^d &= L_t + \varepsilon_t^3, \\ l_t^s &= l_t = L_t + \{d(\varepsilon_t^1 - \varepsilon_t^2) - ad\varepsilon_t^3 + \varepsilon_t^4\}/\{1-ad\}. \end{aligned}\right\} \tag{16}$$

Regime IV: Classical unemployment

$$x_t^d > x_t^s = x_t, \qquad l_t^s > l_t^d = l_t. \tag{17}$$

We have

$$\left.\begin{aligned} x_t^s &= x_t = X_t + \varepsilon_t^1, \\ x_t^d &= X_t + \{-bd\varepsilon_t^1 + \varepsilon_t^2 + b(\varepsilon_t^4 - \varepsilon_t^4)\}/\{1-bd\}, \\ l_t^d &= l_t = L_t + \varepsilon_t^3, \\ l_t^s &= L_t + \{d(\varepsilon_t^1 - \varepsilon_t^2) - bd\varepsilon_t^3 + \varepsilon_t^4\}/\{1-bd\}. \end{aligned}\right\} \tag{18}$$

The local stability conditions are

$$1-bc>0, \quad 1-ac>0, \quad 1-ad>0, \quad 1-bd>0. \tag{19}$$

These conditions imply the existence and uniqueness of the quantity constrained solution [Gourieroux et al. (1980)].

Moreover, each of the regimes is realised if the ε's lie in the following regions:

Regime I

$$V_t^1 - bV_t^2 > 0, \qquad cV_t^1 - V_t^2 > 0; \tag{20}$$

Regime II

$$V_t^1 - aV_t^2 > 0, \qquad cV_t^1 - V_t^2 < 0; \tag{21}$$

Regime III

$$V_t^1 - aV_t^2 < 0, \qquad dV_t^1 - V_t^2 < 0; \tag{22}$$

Regime IV

$$V_t^1 - bV_t^2 < 0, \qquad dV_t^1 - V_t^2 > 0; \tag{23}$$

where

$$V_t^1 = \varepsilon_t^1 - \varepsilon_t^2 \quad \text{and} \quad V_t^2 = \varepsilon_t^3 - \varepsilon_t^4.$$

In this model, inventories are entirely composed of unsold stocks of final goods. Because there is no depreciation, the change in stocks is simply the difference between production and sales,

$$s_{t+1} = s_t + gl_t - x_t, \tag{24}$$

where $g > 0$ is the marginal product of labor. Eq. (2.24) is a stochastic difference equation because l_t and x_t are random variables that depend on the underlying ε's:

Regime I

$$s_{t+1} = s_t + gL_t - X_t$$
$$+ \{c(b-g)\varepsilon_t^1 + (gc-1)\varepsilon_t^2 + (g-b)\varepsilon_t^3 + (1-cg)\varepsilon_t^4\}/\{1-bc\},$$

Regime II

$$= s_t + gL_t - X_t - \varepsilon_t^2 + g\varepsilon_t^4, \tag{25}$$

Regime III

$$= s_t + gL_t - X_t$$
$$+ \{(gd-1)\varepsilon_t^1 + d(a-g)\varepsilon_t^2 + a(1-gd)\varepsilon_t^3 + (g-a)\varepsilon_t^4\}/\{1-ad\},$$

Regime IV

$$= s_t + gL_t - X_t - \varepsilon_t^1 + g\varepsilon_t^3.$$

This relation is piecewise linear in the ε_t. Let $\phi^i(\varepsilon_t)$ denote the linear form in the ε_t associated with regime $i (i = 1, 2, 3, 4)$ in eq. (25).

Let

$$K_0 = \{(g\delta_1 - \beta_1)[\alpha_0(\delta_2 + \gamma_1) - \gamma_0(\beta_2 + \alpha_1)]$$
$$+ (g\delta_2 - \beta_2)[\gamma_0(\beta_1 - \alpha_1) - \alpha_0(\delta_1 - \gamma_1)]\}$$
$$/\{(\beta_1 - \alpha_1)(\delta_2 + \gamma_1) - (\delta_1 - \gamma_1)(\beta_2 + \alpha_1)\}. \tag{26}$$

We have

$$s_{t+1} = -K_0\bar{s} + (1+K_0)s_t + \phi(\varepsilon_t), \tag{27}$$

where $\phi(\varepsilon_t)$ is the function which has the value of $\phi^i(\varepsilon_t)$ whenever ε_t lies in regime i.

Let

$$A_0 = -K_0\bar{s} \quad \text{and} \quad A_1 = 1 + K_0, \tag{28}$$

$$s_{t+1} = A_0 + A_1 s_t + \phi(\varepsilon_t). \tag{29}$$

3. Test of the equilibrium hypothesis

The inventories equation may be used to test equilibrium versus disequilibrium. First let us write the inventories equation of the equilibrium model in which prices equate supply and demand after the ε's have taken their values.

Let

$$\Delta = (\beta_1 - \alpha_1)(\delta_2 + \gamma_1) - (\delta_1 - \gamma_1)(\beta_2 + \alpha_1) \cdot (1/\Delta), \tag{30}$$

$$M = [g(\delta_1 + \delta_2)\gamma_1 - \beta_1(\gamma_1 + \delta_2) + \beta_2(\delta_1 - \gamma_1)] \cdot (1/\Delta), \tag{31}$$

$$N = [g\delta_2(\beta_1 - \alpha_1) - g\delta_1(\beta_2 + \alpha_1) + \alpha_1(\beta_1 + \beta_2)] \cdot (1/\Delta). \tag{32}$$

If we make the equilibrium hypothesis, the inventories equation becomes

$$s_{t+1} = A_0 + A_1 s_t + \phi^{\text{eq}}(\varepsilon_t), \tag{33}$$

where

$$\phi^{\text{eq}}(\varepsilon_t) = M\varepsilon_t^1 - (M+1)\varepsilon_t^2 + N\varepsilon_t^3 - (N-g)\varepsilon_t^4. \tag{34}$$

Since $\phi^{\text{eq}}(\varepsilon_t)$ is a linear function of the ε_t^j, the mean of the error term is zero.

On the contrary, in the disequilibrium case, since the inventories equation is piecewise linear, the mean of the error term is different within each regime and the mean value of the error term averaged over the four regimes will not be zero. This implies a different estimate of the constant, in each regime, which will be biased away from A_0.

So, to test the equilibrium hypothesis, we can estimate the equation of the equilibrium model. Then the brute force procedure would be to estimate the disequilibrium system by maximum likelihood methods separately for each 4^T partions of the T data points, and to perform a likelihood ratio test.

In the empirical work we proceed as follows. We simplify the model by assuming that the main shocks occur on the demand function for good, i.e.,

$$\varepsilon_t^1 \equiv \varepsilon_t^3 \equiv \varepsilon_t^4 \equiv 0.$$

Then, two regimes only are possible, namely keynesian unemployment if $\varepsilon_t^2 < 0$ and repressed inflation if $\varepsilon_t^2 > 0$. The linear forms of the errors associated with each of these regimes are then

$$\phi^1(\varepsilon_t) = B_{21}\varepsilon_t^2 \quad \text{if} \quad \varepsilon_t^2 < 0 \qquad \text{where} \quad B_{21} = (gc-1)/(1-bc) < 0, \tag{35}$$

$$\phi^2(\varepsilon_t) = B_{22}\varepsilon_t^2 \quad \text{if} \quad \varepsilon_t^2 > 0 \qquad \text{where} \quad B_{22} = d(a-g)/(1-ad) < 0. \tag{36}$$

The inventories equations become

$$\left.\begin{array}{ll} s_{t+1} = A_0 + A_1 s_t + \underset{+}{B_{21}}\varepsilon_t^2 & \text{if keynesian unemployment,} \\ \text{and} & \\ s_{t+1} = A_0 + A_1 s_t + \underset{-}{B_{22}}\varepsilon_t^2 & \text{if repressed inflation.} \end{array}\right\} \tag{37}$$

So the equilibrium hypothesis amounts to $B_{21} = B_{22}$ that we test by the maximum likelihood ratio test. Since d is expected to be small, the variance in the keynesian regime is expected to be larger than in the repressed inflation regime.

From (37), ε_t^2 can be written

$$\varepsilon_t^2 = \mathbf{1}_{R^-}(s_{t+1} - A_0 - A_1 s_t)\{(s_{t+1} - A_0 - A_1 s_t)/B_{22}\} + \mathbf{1}_{R^+}(s_{t+1} - A_0 - A_1 s_t)\{(s_{t+1} - A_0 - A_1 s_t)/B_{21}\}. \tag{38}$$

Under the conditions $B_{22} \neq 0$ and $B_{21} \neq 0$, which are satisfied because of the assumptions of the model, this mapping is one to one. The conditional density function of $S_{t+1} = s_{t+1}$ knowing that $S_t = s_t$, $S_{t-1} = s_{t-1}, \ldots, S_1 = s_1$, for any given value of s_1, can be easily obtained by Jacobian technique,

$$f_t[s_{t+1}/s_t, s_{t-1}, \ldots, s_1] = \mathbf{1}_{R^-}(s_{t+1} - A_0 - A_1 s_t) f_{1t} + \mathbf{1}_{R^+}(s_{t+1} - A_0 - A_1 s_t) f_{2t}, \tag{39}$$

where

$$\left.\begin{aligned} f_{1t}(s_{t+1}) &= |1/B_{22}|g_t\{(s_{t+1}-A_0-A_1s_t)/B_{22}\}, \\ f_{2t}(s_{t+1}) &= |1/B_{21}|g_t\{(s_{t+1}-A_0-A_1s_t)/B_{21}\}, \end{aligned}\right\} \tag{40}$$

g_t being the density function of ε_t^2.

Then, we can derive the likelihood function

$$\begin{aligned} L = \prod_{t\in\theta_1} & |1/B_{22}|g_t\{(s_{t+1}-A_0-A_1s_t)/B_{22}\} \\ \times \prod_{t\in\theta_2} & |1/B_{21}|g_t\{(s_{t+1}-A_0-A_1s_t)/B_{21}\}, \end{aligned} \tag{41}$$

where

$$\theta_1 = \{t: s_{t+1}-A_0-A_1s_t<0\}, \qquad \theta_2 = \{t: s_{t+1}-A_0-A_1s_t>0\}.$$

Assuming ε_t^2 normally distributed with mean 0 and variance σ_2^2, we have

$$\begin{aligned} g_t(\varepsilon_t^2) = & \mathbf{1}_{R^-}(s_{t+1}-A_0-A_1s_t)(2\Pi\sigma_2^2)^{-\frac{1}{2}} \\ & \times \exp[-(1/2\sigma_2^2)\{(s_{t+1}-A_0-A_1s_t)/B_{22}\}^2] \\ & + \mathbf{1}_{R^+}(s_{t+1}-A_0-A_1s_t)(2\Pi\sigma_2^2)^{-\frac{1}{2}} \\ & \times \exp[-(1/2\sigma_2^2)\{(s_{t+1}-A_0-A_1s_t)/B_{21}\}^2]. \end{aligned} \tag{42}$$

4. Results

We carry out the test for France and the USA, using data on the manufacturing sector.[1] The equations used in both countries are of the form

$$s_t = a_0 s_{t-1} + a_1(\widehat{PI})_t + a_2(\widehat{w/p})_t + a_3,$$

where $(\widehat{PI})_t$ is a predictor of industrial production in the manufacturing sector and $(\widehat{w/p})_t$ is a predictor of the real wage.[2]

[1]The data for France were obtained from INSEE and OECD, and cover the period 1970–1978. The US data are from NBER and OECD, and cover the period 1971–1979. They are quarterly and seasonally adjusted.

[2]We used several specifications for predictors.

FRANCE — Equilibrium hypothesis

$$s_t = \underset{(0.034)}{0.816} s_{t-1} + \underset{(42)}{385.47} (\widehat{PI})_t + \underset{(7228)}{19197.45} (\widehat{w/p})_t - \underset{(6155)}{37370.95},$$

$$DW = 2.09, \qquad \bar{R}^2 = 0{,}998, \qquad \hat{\sigma}^2 = 1.02.10^6.$$

FRANCE — Disequilibrium hypothesis

$$s_t = 0.788 s_{t-1} + 371.18 (\widehat{PI})_t + 27342.12 (\widehat{w/p})_t - 40585.26,$$

$$\hat{\sigma}^2(\text{keynesian unemployment}) = 2.9.10^6,$$

$$\hat{\sigma}^2(\text{repressed inflation}) = 1.9.10^5.$$

The $\chi^2(1)$ statistics is 16, rejecting the equilibrium hypothesis at the 0.01 level.

USA — Equilibrium hypothesis

$$s_t = \underset{(0.040)}{0.925} s_{t-1} + \underset{(0.019)}{0.109} (\widehat{PI})_t + \underset{(2.848)}{5.491} (\widehat{w/p})_t - \underset{(5.992)}{7.812},$$

$$DW = 1.93, \qquad \bar{R}^2 = 0.996, \qquad \hat{\sigma}^2 = 0.284.$$

USA — Disequilibrium hypothesis

$$s_t = 0.912 s_{t-1} + 0.110 (\widehat{PI})_t + 3.567 (\widehat{w/p})_t - 4.464,$$

$$\hat{\sigma}(\text{keynesian unemployment}) = 0.617,$$

$$\hat{\sigma}(\text{repressed inflation}) = 0.004.$$

The $\chi^2(1)$ statistics is 43, rejecting the equilibrium hypothesis at the 0.01 level.

In both cases reported here as well as in all the various models we estimated, the equilibrium hypothesis is strongly rejected eventhough in most cases we certainly did not reach the global maximum for the likelihood function under the disequilibrium hypothesis (because this computation requires in principle the consideration of all partitions of the data in two subsets).

The disequilibrium theory predicts in addition that the variance under keynesian unemployment is larger than in the repressed inflation regime. This result is obtained in the regressions reported here and is robust to the cancellation of the data concerning the 1974–75 period (this asymmetry could be thought to be 'due to the sample'). However, this asymmetry has not been always obtained and is clearly sensitive to the specification of the model, in particular expectations, and the sample period. Further applied work will be required to ascertain how robust the specific nonlinear form implications of the disequilibrium theory are.

References

Fair, R.C. and D.M. Jaffee, 1972, Methods of estimation for markets in disequilibrium, Econometrica 40, 497–514.

Gourieroux, C., J.J. Laffont and A. Monfort, 1980, Disequilibrium econometrics in simultaneous equation systems, Econometrica 48, 75–96.

Green, J. and J.J. Laffont, 1981, Disequilibrium dynamics with inventories and anticipatory price-setting, European Economic Review 16, no. 1, 199–223.

Laffont, J.J. and R. Garcia, 1976, Disequilibrium econometrics for business loans, Econometrica 43, 1187–1204.