The Lindahl equilibrium in Schumpeterian growth models:

Knowledge diffusion, social value of innovations and optimal R&D incentives*

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Abstract

What is the social value of innovations in Schumpeterian growth models? This issue is tackled by introducing the concept of Lindahl equilibrium in a standard endogenous growth model with vertical innovations which is extended by explicitly considering knowledge diffusion. Assuming that knowledge diffuses on a Salop (1979) circle allows us to formalize the creation of the pools of knowledge in which research and development (R&D) activities draw from to produce innovations. Within this model, we compare two equilibria. The standard Schumpeterian equilibrium *à la* Aghion & Howitt (1992) is mainly characterized by incomplete markets since knowledge is not priced. It provides the usual private value of innovations. The Lindahl equilibrium is a benchmark enabling us to compute the system of prices that sustains the first-best social optimum, and thus to define and to determine analytically the social value of innovations. It provides a suitable methodology for revisiting issues involving the presence of knowledge, often studied in the industrial organization and endogenous growth literatures. This comparison sheds a new light on the consequences of non-rivalry of knowledge and of market incompleteness on innovators' behavior in the Schumpeterian equilibrium. We notably revisit the issues of Pareto sub-optimality and of R&D incentives in presence of cumulative innovations. Basically, the key externality triggered by market incompleteness implies that knowledge creation is indirectly funded by means of intellectual property rights on rival goods embodying knowledge. Therefore, because the private value of innovations differs from the social one, innovators are not given the optimal incentives.

Keywords: Schumpeterian growth theory - Lindahl equilibrium - Social value of innovations - Pareto sub-optimality - Cumulative innovations - Knowledge spillovers

JEL Classification: D52 - O31 - O33 - O40 - O41

1 Introduction

The first purpose of this paper is to precisely define and to analytically determine the social value of innovations in a standard endogenous growth model with vertical innovations. Since the starting point of our analysis of the decentralized economy is based on the equilibrium with creative destruction introduced by Aghion & Howitt (1992), we will refer to this type of growth model as Schumpeterian. Understanding what is the social value of innovations in Schumpeterian models is of several interests. First, this requires to provide a precise definition of an innovation. The main difficulty here lies in that, in standard growth theory, an innovation involves two types of goods: the new knowledge (a non rival good) inherent in this innovation and the intermediate good (a rival good) which embodies knowledge. Consequently, we define an innovation as a pair "new knowledge / intermediate good". Second, it also allows us to revisit key issues widely analysed in the economics of innovation. In particular, regarding welfare issues, we shed a new light on the Pareto sub-optimality of the standard Schumpeterian equilibrium *à la* Aghion & Howitt. In the same way, it enables us to come back to the issues of R&D incentives in presence of cumulative

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innovations. Third, and more generally, we clarify the nature of the equilibria usually considered in growth literature (as for instance the ones considered in Romer 1990, in Grossman & Helpman 1991, in Aghion & Howitt 1992, and in most of the subsequent literature); in particular, we analyse the consequences of the market incompleteness resulting from the fact that, in those equilibria, knowledge is not priced. Basically, this incompleteness is the keystone of our analysis.

In order to define and to compute the social value of innovations we use the concept of Lindahl equilibrium, which deeply departs from the Schumpeterian equilibrium generally studied in the standard literature. Indeed, since an innovation goes along with the creation of knowledge, one needs to construct a benchmark equilibrium which enables to price a non rival good. In our general equilibrium analysis, the Lindahl equilibrium provides naturally this benchmark in an Arrow-Debreu perspective. We know that this equilibrium exhibits, by construction, the system of prices that sustains the first-best social optimum in presence of non rival goods.[1] Accordingly, since the social value of an innovation is by definition its optimal value,[2] this value can be obtained using the Lindahl equilibrium.

Our methodology is twofold. First, we consider a model in line with the standard growth models with vertical innovations, as for instance the ones of Grossman & Helpman (1991) and Aghion & Howitt (1992). In this first step, the model is defined stricto sensu by technology and preferences, which is sufficient to characterize the first-best social optimum. The novelty of our formalization lies in the fact that we propose an extension of these models such that we explicitly introduce *knowledge diffusion* across sectors. Second, within this basic model, we study and compare two types of equilibria. The first one is the standard Schumpeterian equilibrium *à la* Aghion & Howitt (1992), in which markets are incomplete since knowledge is not priced. In this equilibrium, as usual in the literature, we define the *private value* of an innovation as the sum of the expected present values of the monopoly profits on intermediate goods received by the latest innovator who has been granted a patent. The second equilibrium we consider is the Lindahl one, in which markets are complete. In this equilibrium, we define the *social value* of an innovation as the sum of the expected present values of the income received by the innovator; at each date, this income is the sum of the Lindahl prices of all the users of each unit of knowledge inherent in her innovation. In the remaining of the introduction, we present the structure of the paper and we discuss the main results.

In Section 2, we present a scale-invariant fully endogenous growth model with vertical innovations. This model is standard except that we explicitly formalize knowledge diffusion (*i.e.* we detail the process underlying the "knowledge spillovers" often quoted in the standard literature), which allows us to apprehend better the non-rivalry property of knowledge. In this respect, we introduce the circle of Salop (1979). A continuum of intermediate sectors is located over the circle. Each of them has its own R&D activity producing knowledge, and produces an intermediate good that embodies this knowledge.[3] Furthermore, knowledge diffuses over the Salop circle, each intermediate sector simultaneously sending and receiving new knowledge. This formalization includes all possible cases ranging from intra-sectorial diffusion only (as for instance in the models of Grossman & Helpman 1991, Segerstrom 1998, or Peretto 1999) to global inter-sectorial diffusion (as for instance in Aghion & Howitt 1992, Young 1998, Howitt 1999, or Segerstrom 2000). In other words, it encompasses many models used in growth literature. Furthermore, it provides a mechanism through which the pools of knowledge in which R&D activities draw from to produce new knowledge are created. In particular, the broader the *scope of knowledge diffusion* is (*i.e.* the stronger the intensity of knowledge spillovers is), the wider these pools are. Regarding the innovation process in each sector, we use two basic assumptions to derive a fairly general law of knowledge accumulation: the creation of new knowledge depends on the level of R&D effort as well as on those pools of knowledge. Once we have presented the technology and the preferences of the model, we compute the first-best social optimum. We show notably that the optimal R&D effort, and thus the optimal growth rate of the economy, depend on the productivity of the labor devoted to R&D and on the productivity of the knowledge used by R&D activities. These two determinants of the innovation process are commonly found in the literature. The novelty in our formalization lies in the fact that this process also depends on the scope of knowledge diffusion, and thus on the size of the pools of knowledge. This novelty leads us to introduce the concept of *dimension of innovations*; this dimension is function both of the productivity of knowledge in R&D and of the scope of knowledge diffusion. This concept will turn out to particularly matter in our analysis, because the larger the dimension of innovations is, the stronger the externality

[1] For further details and discussion, one can for instance refer to Milleron (1972) or to Mas-Colell, Whinston & Green (1995 - Ch. 11 and Ch. 16).

[2] More precisely, we will see that the social value of an innovation is the optimal value of the new knowledge inherent in this innovation.

[3] As it will be underlined, a fundamental point is that, in each sector, the intermediate good embodies the whole stock of knowledge created so far in this sector: not only the knowledge newly produced by the latest innovator, but also the one produced by each of her predecessors.

triggered by the non-rivalry of knowledge is.

In Section 3, we consider the decentralized economy and we study two types of equilibria: the Schumpeterian equilibrium *à la* Aghion & Howitt and the Lindahl equilibrium. The former is standard; as mentioned above, it considers incomplete markets (knowledge is not priced). Accordingly, R&D is indirectly funded by monopoly profits on intermediate goods which embody knowledge: intellectual property rights (IPRs), like patents, on these rival goods are introduced as a means to provide incentives to firms to invest in R&D. Therefore, as usual in the literature, the value of innovations stems only from the stream of monopoly profits; that is why we refer to this value as the *private value of innovations*.

The Lindahl equilibrium has never been introduced in growth theory, at least to the best of our knowledge. In this equilibrium, knowledge is priced using Lindahl prices and thus markets are complete. Formally, when an innovation occurs in a given intermediate sector, each unit of knowledge inherent in this innovation has an impact on three types of activities in the economy: the production of final good, the production of the intermediate good of this sector, and the production of knowledge in all sectors up to which this knowledge diffuses. Then, each of these users pays each unit of knowledge at a personalized price (the Lindahl price) which is equal to the marginal profitability of that unit for this user. Finally, the innovator receives the sum of these Lindahl prices for each unit of knowledge inherent in her innovation. At this stage of the analysis, we know that we have determined the system of prices sustaining the first-best social optimum. Thus, we have obtained the instantaneous *optimal* value of each unit of knowledge. Since knowledge is infinitely-lived, we can determine the social value of each unit of knowledge as the discounted sum of these instantaneous social values. Finally, to obtain the *social value of an innovation*, we multiply the social value of each unit of knowledge by the number of units of knowledge inherent in this innovation.

The comparison between the two equilibria allows us to present the basic intuitions on the consequences of the non-rivalry of knowledge and of market incompleteness on the innovators' behavior in the Schumpeterian equilibrium. Note that, in order to focus on the distortions involved by market incompleteness, we correct the well known static monopoly distortion present in this equilibrium.

In Section 4, we use the concepts of Lindahl equilibrium and of social value of innovations to investigate and revisit issues related to R&D incentives and innovators' behavior.

In 4.1, we revisit the well known issue of the Pareto sub-optimality of the Schumpeterian equilibrium. We show that market incompleteness leads to a discrepancy between the private value of innovations and their social value because, in the Schumpeterian equilibrium, innovators do not fully internalize all the effects of knowledge creation in their behaviors. Accordingly, we demonstrate that R&D investment is under (*resp.* over) optimal in the Schumpeterian equilibrium if and only if the private value is below (*resp.* above) the social value. Furthermore, under optimal R&D investment is more likely to occur if the dimension of innovations is large (formally, if it is greater than a threshold determined in the paper). Indeed, in this case, because of the externality implied by market incompleteness, the monopoly profits are lower than the revenues that the innovators should get so that R&D would be optimally funded. Conversely, the smaller the dimension of innovations is, the weaker the externality is, and thus the more likely R&D investment will be over optimal.

In 4.2 and 4.3, we analyse more technically the incentives issues raised in 4.1 and we revisit R&D funding issues raised by the presence of cumulative innovations. These issues have been tackled in many papers both in industrial organization literature (e.g. Scotchmer 1991; Green & Scotchmer 1995; Bessen & Maskin 2009) and in growth literature (e.g. O'Donoghue & Zweimüller 2004; Acemoglu & Akcigit 2012; Chu, Cozzi & Galli 2012). Given the fact that an innovation may foster subsequent innovations in the same sector but also in other unrelated sectors, the typical focus is on issues regarding profit division between sequential innovators along a quality ladder, and on the appropriate design of intellectual property rights to provide sufficient incentives to innovators. Naturally, innovations are cumulative in the sense that the new knowledge builds on previously created knowledge. Providing the first-best prices system thanks to the Lindahl equilibrium enables us to determine exactly the payments that each innovator should receive in order to implement the social optimum. This establishes - within a dynamic general equilibrium framework - the suitable benchmark to tackle the issue of profit division in case of sequential innovations. Besides, the methodology we introduce points out the significance of distinguishing an intermediate good, which embodies knowledge, and this knowledge per se. Formally, we study the link between the social value of an intermediate good (which is given by its net surplus) and the value of the knowledge which is embodied in it. Then, we can better understand why the Schumpeterian decentralized economy may lead to too little or to too much R&D investment. Indeed, clearly distinguishing between knowledge and the intermediate good in which it is embodied (*i.e.* completing the markets as it is done in the Lindahl equilibrium) enables us to explain why innovators are not given the optimal incentives in the Schumpeterian equilibrium. The level of R&D investment in the Schumpeterian equilibrium *à la* Aghion

& Howitt deviates from the optimal one firstly owing to the well known appropriability issue: innovators do not extract the whole surplus. However, even if they were doing so, this equilibrium would still not provide the optimal R&D incentives for three reasons: two of them are static, and the third one is dynamic.

First, since an intermediate good is the result of several steps of invention, modification, and improvement in a given sector, it incorporates not solely the knowledge inherent in the latest innovation (*i.e.* the incremental knowledge) but also the whole knowledge produced by prior innovators in this sector. Thus, when the latest innovator sells an intermediate good, she implicitly sells the whole stock of knowledge created in the sector so far, that is the knowledge she created as well as all the knowledge created by each of her predecessors in this sector. Therefore, the quantity of knowledge sold is not the proper one; the latter should be the incremental flow of knowledge inherent in her innovation only (as shown in the Lindahl equilibrium). This first static distortion tends to provide too much R&D incentives to innovators, and thus to lead to an excessive level of resources devoted to R&D.

Second, this knowledge is not sold at the proper price. Indeed, each innovator does not take into account knowledge spillovers because the knowledge she creates is used *freely* to produce new knowledge in her sector as well as in other sectors (contrary to the Lindahl equilibrium in which each user of knowledge pays for it). This second static distortion tends to provide too little R&D incentives, and hence to lead to an insufficient level of R&D investment. Indeed, as argued by Green & Scotchmer (1995), "the social value of an early innovation includes the net social value of the applications it facilitates". Here, we determine exactly what part of the social value of an innovation is missing in the private value (resulting from the Schumpeterian equilibrium) when an innovation is assimilated only to the intermediate good in which the knowledge inherent in this innovation is embodied.

Third, in this Schumpeterian equilibrium, the period during which an innovation provides revenue to its producer is not the proper one. The underlying reason is as follows. Because of the creative destruction mechanism, an innovator has a monopoly whose lifespan is finite in average. However, since knowledge is infinitely-lived, the innovator should receive revenue forever (like in the Lindahl equilibrium). This dynamic distortion, which relates to the fact that intertemporal knowledge spillovers are underestimated, tends to provide too little R&D incentives.

Fundamentally, introducing the Lindahl equilibrium in a standard Schumpeterian growth model enables us to show that these distortions - which all arise from market incompleteness - have opposite effects on R&D incentives. This is precisely the reason why the Schumpeterian equilibrium *à la* Aghion & Howitt may lead to too little as well as too much R&D investment, and may thus exhibit an under-optimal or an over-optimal growth rate.

After the presentation of the model (Section 2), the analysis of the two equilibria (Section 3), and the study of some key issues tackled in the economics of innovation that we revisit thanks to the Lindahl equilibrium (Section 4), we conclude in Section 5, and we provide all computations in Section 6.

2 Model and welfare

In this section, we present a continuous-time scale-invariant fully endogenous Schumpeterian growth model, in which knowledge can diffuse, with more or less intensity, across the sectors' R&D activity. First, we present the technologies and the preferences, which are independent of the two concepts of equilibria that will be studied in Section 3 below. Then, we characterize the first-best social optimum.

2.1 Technologies and preferences

We begin this subsection by presenting a general law of knowledge accumulation. Then, we introduce explicitly inter-sectorial knowledge diffusion by exploiting the circular product differentiation model of Salop (1979). Finally, we plug this technology of knowledge creation in a standard endogenous growth model with vertical innovations in the line of Grossman & Helpman (1991) and Aghion & Howitt (1992).

There is a continuum Ω, of measure N, of intermediate sectors uniformly distributed on a clockwise oriented circle. Each sector ω, $\omega \in \Omega$, is characterized by a stock of knowledge χ_ω and by an intermediate good ω, produced in quantity x_ω, which embodies this stock of knowledge. As usual in endogenous growth theory, we assume that all sectors have an identical initial level of knowledge, *i.e.* $\chi_{\omega 0} = \chi_0, \forall \omega \in \Omega$.[4] Each sector has its own R&D activity which is dedicated to the creation of innovations. In the remaining of the paper, an innovation is defined as follows:

[4]The assumption of symmetry across sectors is standard in endogenous growth theory; see, for instance, Aghion & Howitt (1992 or 1998 - Ch. 3), or Peretto & Smulders (2002). For more details on this issue, the reader can refer to Peretto (1998, 1999) or to Cozzi, Giordani & Zamparelli (2007) in which the relevancy of the symmetric equilibrium is discussed.

Definition 1. *An innovation at date t in any sector ω, $\omega \in \Omega$, consist in i) an increase of $\Delta\chi_{\omega t}$ units of new knowledge in this sector, and ii) the embodiment of this new knowledge in the intermediate good ω.*

This definition - in which we carefully distinguish between the intermediate good embodying knowledge and knowledge itself - will turn to be crucial in Section 3 when we determine the values of innovations.

2.1.1 Knowledge accumulation

It is commonly agreed that new knowledge is produced using two types of inputs: rival goods (e.g. labor, physical capital, final good), and a non rival one (a stock of knowledge previously created). In the present model, like in the standard Schumpeterian growth theory, the mechanism at the source of the creation of knowledge relies on two core assumptions. Firstly, the innovation process is uncertain:

Assumption 1. *If $l_{\omega t}$ is the amount of labor devoted to R&D at date t in any intermediate sector ω, $\omega \in \Omega$, to move on to the next quality of intermediate good ω, innovations occur randomly with a Poisson arrival rate $\lambda l_{\omega t}$, $\lambda > 0$.*

Secondly, each R&D activity creates new knowledge making use of previously created knowledge. This idea is formalized by considering that, in order to produce new knowledge, in each sector ω, RD activity draws from a specific *pool of knowledge* $\mathcal{P}_{\omega t}$:

Assumption 2. *For any intermediate good ω, $\omega \in \Omega$, if an innovation occurs at date t, the increase in knowledge $\Delta\chi_{\omega t}$ (i.e. the quality improvement of the intermediate good) depends on the current size of the pool of knowledge in which this sector's R&D activity draws from: $\Delta\chi_{\omega t} = \sigma\mathcal{P}_{\omega t}, \forall\omega \in \Omega$, $\sigma > 0$.*

From Assumptions 1 and 2, one derives the law of motion of the average knowledge inherent in any sector ω:

Lemma 1. *Under Assumptions 1 and 2, the expected knowledge in any intermediate sector ω, $\omega \in \Omega$, is a differentiable function of time. The law of motion of the knowledge characterizing any intermediate sector ω is $\dot{\chi}_{\omega t} = \lambda\sigma l_{\omega t}\mathcal{P}_{\omega t}$, $\forall\omega \in \Omega$.*[5]

Proof. See Appendix 6.1.1.

As underlined for instance by Aghion & Howitt (1998), Howitt (1999), Jones (1999), Laincz & Peretto (2006), or Dinopoulos & Sener (2007), most growth models differ mainly in the specification of the knowledge production technology.[6] The law of knowledge accumulation derived in Lemma 1 encompasses several of the ones assumed in the standard fully endogenous growth theory. Indeed, depending on the specification of the pools of knowledge $\mathcal{P}_{\omega t}$, a large number of growth models can be obtained using the present formalization. In Appendix 6.1.2, we provide several illustrations: models without knowledge spillovers, models considering only intra-sectorial knowledge spillovers, models assuming knowledge spillovers which depend on average knowledge, models in which spillovers depend on the knowledge level of the frontier firms ("leading-edge technology") and models with global knowledge spillovers (*i.e.* knowledge diffuses to the whole economy).

2.1.2 Knowledge diffusion and pools of knowledge

In Lemma 1, it appears that the new knowledge created in each sector ω depends on the pool of knowledge $\mathcal{P}_{\omega t}$ used by this sector. Now, we propose a mechanism formalizing how these pools are shaped. As a matter of fact, the constitution of each of these pools relies on the influence that R&D activities have on each other. The significance of the interactions between sectors has universally been underlined. In particular, several empirical studies stress that R&D performed in one sector may produce positive spillovers effects in other sectors (see, for instance, Griliches 1992; Griliches 1995; or Hall, Mairesse & Mohnen 2010). As stated by Hall et al., "such spillovers are all the more likely and significant as the sender and the receiver are closely related". Moreover, as argued in Hall (2004), "it is safe to say that without

[5] The expectation operator is dropped to simplify notations: $\frac{\partial \mathbb{E}[\chi_{\omega t}]}{\partial t} \equiv \dot{\chi}_{\omega t}$.

[6] The overviews provided by these authors propose a classification of the various growth models according to their key result with respect to the presence of scale effects. Three classes of models emerge: endogenous growth models exhibiting this non desirable property (e.g. the models of Romer 1990, Grossman & Helpman 1991, or Aghion & Howitt 1992), semi-endogenous growth models introducing decreasing returns to scale to suppress scale effect (e.g. the models of Jones 1995, Kortum 1997, or Segerstrom 1998), and fully endogenous growth models, which eliminate scale effects by allowing for expansion in the number of sectors (e.g. the models of Aghion & Howitt 1998 - Ch. 12, Dinopoulos & Thompson 1998 Peretto 1998, Young 1998, Howitt 1999, Peretto 1999, or Aghion & Howitt 2009 - Ch. 4).

diffusion, innovation would have little social or economic impact. In the study of innovation, the word diffusion is commonly used to describe the process by which individuals and firms in a society/economy adopt a new technology, or replace an older technology with a newer". In direct line with these statements, we explicitly introduce a process of knowledge diffusion within and across sectors.[7]

Formally, each sector can be simultaneously a *sender* and a *receiver* of knowledge: in the following, the index h, $h \in \Omega$, is used to point out a sector from which knowledge χ_h diffuses (the sender); the index ω, $\omega \in \Omega$, is used to point out the sector that may potentially use this knowledge (the potential receiver). For any R&D activity ω, $\omega \in \Omega$, the disposable pool of knowledge, $\mathcal{P}_{\omega t}$, is composed of the knowledge produced in this sector so far, as well as of knowledge diffused from other sectors. For any sector h, let us define the *scope of diffusion* of knowledge χ_h, denoted by θ, where $1 \leq \theta \leq N$, as the measure of the subset of sectors of Ω which use this stock of knowledge. It is a measure of the intensity of inter-sectorial knowledge spillovers, which is comprised between one (only intra-sectorial spillovers, and thus no inter-sectorial spillovers) and N (global inter-sectorial spillovers). In the case of sector specific innovations ($\theta = 1$), there is no sector using knowledge χ_h besides sector h itself. In the case in which there is inter-sectorial knowledge diffusion ($\theta > 1$), we assume that knowledge diffuses symmetrically over the circle Ω. This formalization of knowledge diffusion implies the following:

Lemma 2. *The neighborhood of diffusion of knowledge χ_h inherent in sector h, $h \in \Omega$, is $\Omega_h \equiv [h - \theta/2\,;\, h + \theta/2]$, where $\Omega_h \subseteq \Omega$. Then, at each date t, in any intermediate sector ω, the pool of knowledge used by the R&D activity is $\mathcal{P}_{\omega t} = \int_{\Omega_\omega} \chi_{ht}\, dh$, $\forall \omega \in \Omega$.*

From Lemmas 1 and 2, it is straightforward to establish the following:

Proposition 1. *At each date t, in any intermediate sector ω, knowledge is produced along with*

$$\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \mathcal{P}_{\omega t}, \text{ where } \mathcal{P}_{\omega t} = \int_{\Omega_\omega} \chi_{ht}\, dh \text{ and } \Omega_\omega \equiv \left[\omega - \frac{\theta}{2}; \omega + \frac{\theta}{2}\right], \forall \omega \in \Omega \tag{1}$$

At each date t, the whole disposable knowledge in the economy is

$$\mathcal{K}_t = \int_\Omega \chi_{\omega t}\, d\omega, \tag{2}$$

and the initial stock $\mathcal{K}_0$ is normalized to one.

Proposition 1 underlines the fact that the R&D activity of a given sector always uses the knowledge accumulated so far in this sector and potentially captures part of the mass of the ideas created in all other ones. This subset of $\mathcal{K}_t$ is more or less large depending on θ, the scope of knowledge diffusion. This formalization generalizes the standard innovation-based endogenous growth theory. Indeed, depending on the choice of the parameter θ, one obtains a large collection of pools, $\mathcal{P}_{\omega t}$, and thus of models, ranging from models with only intra-sectorial knowledge diffusion and no inter-sectorial knowledge diffusion ($\theta = 1$), to models with global inter-sectorial diffusion ($\theta = N$), that is in which knowledge diffuses to the whole economy. In the corollary below, we recap these two polar cases; they exhibit laws of motion commonly used in growth models with vertical knowledge accumulation.

Corollary. *The two polar cases are summarized as follows.*

- *If $\theta = 1$, then knowledge spillovers are only intra-sectorial. One has:*

$$\Omega_\omega = \{\omega\}\,,\ \ \mathcal{P}_{\omega t} = \chi_{\omega t}, \text{ and thus } \dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \chi_{\omega t}, \forall \omega \in \Omega$$

- *If $\theta = N$, then knowledge spillovers are global. One has:*

$$\Omega_\omega = \Omega,\ \mathcal{P}_{\omega t} = \mathcal{K}_t, \text{ and thus } \dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \mathcal{K}_t, \forall \omega \in \Omega$$

Our formalization thus encompasses the one of growth models assuming that sectors do not share knowledge (as, for instance, in Grossman & Helpman 1991, in Segerstrom 1998, in Peretto 1999, in Acemoglu 2009 - Ch. 14, or in Aghion & Howitt 2009 - Ch. 4) and the one of growth models assuming

[7]The term "diffusion" has often been used to refer to the phenomenon involving that, as stated by Chari & Hopenhayn (1991), "there is a lag between the appearance of a technology and its peak usage". The fact that a lag can be involved by technology adoption remains to be explored within our model and is left for further research (one could for instance consider that the more distant two sectors are, the longer the lag in technology adoption). This temporal dimension of knowledge diffusion is undoubtedly important. However, in this paper, we abstract away from it by considering instantaneous diffusion, and we focus on "spacial diffusion".

that any sector benefits from the whole stock of disposable knowledge in the economy (as, for instance in Aghion & Howitt 1992, Young 1998, Howitt 1999, Segerstrom 2000).[8] In this second polar case, the expression of the law of knowledge accumulation, which is here endogenously derived from assumptions made in a stochastic quality ladders model, leads to a law of motion of the whole disposable knowledge which is formally identical to the knowledge production function initially introduced by Romer (1990).[9]

2.1.3 Other assumptions

The remaining assumptions are standard in Schumpeterian growth theory. Intertemporal preferences of the representative household are given by

$$\mathcal{U} = \int_0^\infty u(c_t) e^{-\rho t} dt, \tag{3}$$

where ρ is the subjective discount rate and $u(c_t)$ is the individual instantaneous utility at date t, which is given by $u(c_t) = \ln(c_t)$.[10] At each date t, each of the L identical households is endowed with one unit of labor that is supplied inelastically.[11] In order to remove the non-desirable scale effects property, we use the commonly shared assumption of proportionality between the size of the population and the number of sectors: $N = \gamma L$, where γ is a strictly positive parameter.[12] The total quantity of labor, L, is used to produce the final good and in R&D activities. Thus, the labor constraint is

$$L_t = L_t^Y + \int_\Omega l_{\omega t}\, d\omega \tag{4}$$

Besides labor, the production of the final good requires the use of all available intermediate goods, each of which is associated with its own level of knowledge. The final good production technology is

$$Y_t = (L_t^Y)^{1-\alpha} \int_\Omega \chi_{\omega t} (x_{\omega t})^\alpha d\omega \ , \ 0 < \alpha < 1 \tag{5}$$

The final good has two competing uses. Firstly, it is consumed by the representative household in quantity c_t. Secondly, it is used in the production of intermediate goods along with

$$x_{\omega t} = \frac{y_{\omega t}}{\chi_{\omega t}} \ , \ \omega \in \Omega \,, \tag{6}$$

where $y_{\omega t}$ is the quantity of final good used to produce $x_{\omega t}$ units of intermediate good ω. This usual technology illustrates the increasing complexity in the production of intermediate goods: the higher the level of knowledge, the more costly the production of the intermediate good embodying it. One gets the following constraint on the final good market:

$$Y_t = Lc_t + \int_\Omega y_{\omega t} d\omega \tag{7}$$

2.2 First-best social optimum

The first-best social optimum is the solution of the maximization of the representative household's discounted utility (3) subject to (1), (2), (4), (5), (6) and (7). Proposition 2 gives the complete characterization of the optimum. From now on, we denote by g_{z_t} the rate of growth, $\dot{z}_t / z_t$, of any variable z_t, and we use the superscript "^{o}" is for "social optimum".

[8] In these models, the increase in knowledge consecutive to the occurrence of an innovation in sector ω at date t depends on the level of knowledge reached in the most advanced sector (*i.e.* it is assumed that $\mathcal{P}_{\omega t} = \max\{\chi_{\omega t}, \omega \in \Omega_t\}$). Insofar as they consider spillovers depending on the knowledge level of the frontier firms ("leading-edge technology"), these models relate to a framework assuming global knowledge spillovers. A similar interpretation can be found, for instance, in Jones (1999), in Dinopoulos & Sener (2007), or in Ha & Howitt (2007).

[9] Differentiating (2) with respect to time yields $\dot{\mathcal{K}}_t = \int_\Omega \dot{\chi}_{\omega t}\, d\omega = \lambda\sigma \left(\int_\Omega l_{\omega t} d\omega\right) \mathcal{K}_t \Leftrightarrow \dot{\mathcal{K}}_t = \lambda\sigma L_t^R \mathcal{K}_t$, where $L_t^R = \int_\Omega l_{\omega t} d\omega$ is the total amount of labor used in R&D.

[10] The results are robust if one considers a more general C.E.S. instantaneous utility function of parameter ε, $u(c_t) = c_t^{1-\varepsilon}/(1-\varepsilon)$.

[11] The results are robust if one considers constant population growth.

[12] This assumption - which is a necessary condition to cancel scale effects in fully endogenous growth model - has been justified both theoretically and empirically (see, for instance, Jones 1999, Segerstrom 2000, Laincz & Peretto 2006, or Dinopoulos & Sener 2007). Besides, it is not needed for obtaining the main insight of this paper and is introduced only in order to consider a scale-invariant fully endogenous growth model.

Proposition 2. *In the first-best social optimum, the repartition of labor, the quantity of each intermediate good ω, and the growth rates are respectively*

$$L_t^{Yo} = L^{Yo} = \frac{\rho\gamma L}{\lambda\sigma\theta},\ l_{\omega t}^o = l^o = \frac{1}{\gamma} - \frac{\rho}{\lambda\sigma\theta},\ x_{\omega t}^o = x^o = \alpha^{\frac{1}{1-\alpha}}\frac{\rho\gamma L}{\lambda\sigma\theta},$$

$$\textit{and } g_{c_t}^o = g_{Y_t}^o = g_{\mathcal{K}_t}^o = g_{\chi_{\omega t}}^o = g^o = \frac{\lambda\sigma\theta}{\gamma} - \rho, \forall\omega\in\Omega, \forall t$$

Proof. See Appendix 6.2.

The three parameters λ, σ and θ all account for the productivity of R&D activities. The parameter λ stands for the productivity of the labor devoted to the creation of innovations (see Assumption 1). The parameter σ stands for the productivity of knowledge used by a given sector in the creation of new knowledge: it indicates to which extent the pools of knowledge contribute to the increases in knowledge when innovations occur (see Assumption 2). More precisely, σ is a measure of the contribution of the pool of knowledge to the height of the jump in knowledge $\Delta\chi_{\omega t} = \sigma\mathcal{P}_{\omega t}$. These two parameters are commonly used in standard growth theory. The present paper introduces a new parameter θ, $\theta \in [1; N]$, which is another measure of the efficiency of knowledge; more precisely, θ stands for the overall influence of an innovation in the constitution of these pools of knowledge. Hence, the product $\sigma\theta$ is an index of what we can call the "dimension" of innovations. It depends both on σ, the productivity of knowledge, and on θ, the scope of knowledge diffusion (*i.e.* the intensity of knowledge spillovers). Finally, these three parameters have the same impact on the first-best: an increase in any of them leads to a reallocation of labor from the production to R&D activity, and thus to a higher growth rate.

To conclude this section, let us underline that, in the technologies presented above in Subsection 2.1, each unit of knowledge χ_ω involved by any innovation produced in a given sector ω affects simultaneously the final good sector (see (5)), the intermediate good sector ω (see (6)), and a more or less significant range of R&D activities (see (1)): the presence of knowledge involves a public (*i.e.* non rival) good issue. In the standard literature, this issue is commonly referred to as "knowledge spillovers" and relates both to knowledge diffusion and to an implicit market incompleteness (in the decentralized economies generally studied, knowledge is not priced; see, for instance the equilibria studied in Romer 1990 and in Aghion & Howitt 1992). The issue of knowledge spillovers has extensively been tackled in the literature, but to the best of our knowledge, has never been linked to the Lindahl equilibrium. In Sections 3 and 4 below, we deal with this concept of spillover by analysing and comparing two equilibria, namely the standard Schumpeterian equilibrium *à la* Aghion & Howitt and the Lindahl one. In particular, we argue that knowledge diffusion refers to the fact that knowledge is a non rival good, which can be priced (as in the Lindahl equilibrium) or not (as in the Schumpeterian equilibrium).

3 Schumpeterian equilibrium *à la* Aghion & Howitt versus Lindahl equilibrium

In the previous section, we have seen that innovations involve both new units of knowledge (*i.e.* non rival goods) and intermediate goods (*i.e.* rival goods) in which this knowledge is embodied. In all what follows, it is fundamental to distinguish between the two.

In order to deal with the non-rivalry property of knowledge, the seminal papers by Romer (1990), Grossman & Helpman (1991), and Aghion & Howitt (1992), focused on a decentralized equilibrium with incomplete markets and imperfect competition. Indeed, in this type of equilibria, a market and a price are specified for intermediate goods that incorporate knowledge, but not for knowledge itself; positive monopoly profits on the sale of intermediate goods, which result from IPRs granted to innovators, are used as incentives to invest in the creation of knowledge. This framework, which has become the standard in endogenous growth literature, provides a realistic decentralized economy in which R&D activity is privately and indirectly funded: the value of innovations stems from the stream of monopoly profits on the sale of intermediate goods embodying knowledge. This is a key point that is discussed extensively below.

The main motivation of our study is to address the following question: what is the social value of innovations in a dynamic general equilibrium model in which knowledge accumulation plays a key part? To answer this question, we first define the *social value of an innovation in any sector* ω as the optimal value of the knowledge inherent in this innovation, $\Delta\chi_{\omega t}$. How shall it then be computed? For that purpose, contrary to what is done in the standard Schumpeterian equilibrium, we price knowledge, that is we complete the markets. Thus, we use the concept of Lindahl equilibrium which provides the system of prices that sustains the first-best optimum in an economy with non rival goods.

In Subsection 3.1, we present the concepts. In particular, as mentioned above, we define two types of equilibria: the standard Schumpeterian equilibrium *à la* Aghion & Howitt (1992), and the Lindahl equilibrium. Then, we define the value of innovations in each of them. In the Schumpeterian equilibrium, as in the standard literature, we consider the *private value* of innovations which results from the sale of intermediate goods.[13] In the Lindahl equilibrium we consider the *social value* of these innovations, which is obtained by introducing the concept of Lindahl prices in endogenous growth theory.[14] In Subsection 3.2, we compute explicitly these two equilibria and these two values of innovations.

In the remaining of the paper, we normalize the price of final good to one, and we denote respectively by w_t, r_t and $q_{\omega t}$ ($\omega \in \Omega$) the wage, the interest rate and the price of intermediate good ω at date t.

3.1 General definitions

In this subsection, we start by providing the formal definitions of the two equilibria and of the two values of innovations studied in this paper (see 3.1.1 and 3.1.2, respectively). Then, we underline the main differences between the Schumpeterian equilibrium and the Lindahl one (see 3.1.3).

3.1.1 Schumpeterian equilibrium and private value of innovations

First, we recall the main features of the standard Schumpeterian equilibrium *à la* Aghion & Howitt (1992) which involves a fundamental externality (knowledge is not priced). In order to indirectly fund knowledge creation, one generally considers the following assumptions inspired by Schumpeter's creative destruction mechanism. Once an innovation occurs, the resulting knowledge is embodied in an intermediate good; then, the innovator is granted an intellectual property right, like a patent, and monopolizes the production and sale of this private good until replaced by the next innovator. In such a decentralized economy, Pareto sub-optimality may arise (the equilibrium allocation of labor in R&D can either be sub-optimal or over-optimal). The static distortion resulting from the presence of monopolies on the production and sale of intermediate goods can be mitigated by an *ad valorem* subsidy ψ on each intermediate good demand. The dynamic distortion relates to the externality triggered by the fact that there is no market for knowledge; it can be corrected by a public tool φ which can consist in a subsidy or in a tax on the profits of R&D activities, depending on whether the R&D effort is sub-optimal or over-optimal.[15] To each public tools vector (ψ, φ) is associated a particular equilibrium. The formal definition of the Schumpeterian equilibrium is the following:

Definition 2. *At each vector of public policies (ψ, φ) is associated a particular Schumpeterian equilibrium* à la *Aghion & Howitt. It consists of time paths of set of prices*

$$\left\{ \left(w_t\left(\psi, \varphi\right), r_t\left(\psi, \varphi\right), \left\{ q_{\omega t}\left(\psi, \varphi\right)\right\}_{\omega \in \Omega} \right) \right\}_{t=0}^{\infty}$$

and of quantities

$$\left\{ \left(c_t\left(\psi, \varphi\right), Y_t\left(\psi, \varphi\right), \left\{ x_{\omega t}\left(\psi, \varphi\right)\right\}_{\omega \in \Omega}, L_t^Y\left(\psi, \varphi\right), \left\{ l_{\omega t}\left(\psi, \varphi\right)\right\}_{\omega \in \Omega}, \left\{ \chi_{\omega t}\left(\psi, \varphi\right)\right\}_{\omega \in \Omega} \right) \right\}_{t=0}^{\infty}$$

such that: the representative household maximizes her utility; firms maximize their profits; the final good market, the financial market and the labor market are perfectly competitive and clear; on each intermediate good market, the innovator is granted a patent and monopolizes the production and sale until replaced by the next innovator; and there is free entry on each R&D activity (i.e. the zero profit condition holds for each R&D activity).

This set of equilibria is characterized in Proposition 3 below. The agents' behaviors and the detailed computations are provided in Appendix 6.3.[16] In the Schumpeterian equilibrium, in any sector ω, $\omega \in \Omega$, the incumbent innovator having successfully innovated at date t receives, at any date $\tau > t$, the net profit $\pi_\tau^{x_\omega}(\psi, \varphi)$ with probability $e^{-\int_t^\tau \lambda l_{\omega u}(\psi,\varphi) du}$ (*i.e.* provided that there is no innovation upgrading

[13] Alternatively, we could refer to the private value of an innovation as its Schumpeterian value, its rival value, or its patent value.

[14] Alternatively, we could call the social value of an innovation, its Lindahl value, its non rival value, or its optimal value.

[15] We return in detail to the issue of Pareto sub-optimality in Subsection 4.1.

[16] The optimal public tools (ψ^o, φ^o) implementing the first-best optimum are provided below in the corollary to Proposition 3.

intermediate good ω between t and τ). Given the governmental intervention on behalf of R&D activities and the intermediate good production function (6), this profit writes:

$$\pi_{\tau}^{x_\omega}(\psi,\varphi) = (1+\varphi)\left[q_{\omega\tau}(\psi,\varphi)\,x_{\omega\tau}(\psi,\varphi) - y_{\omega\tau}(\psi,\varphi)\right] \\ = (1+\varphi)\left[q_{\omega\tau}(\psi,\varphi) - \chi_{\omega\tau}(\psi,\varphi)\right] x_{\omega\tau}(\psi,\varphi) \quad (8)$$

As usual in standard Schumpeterian growth theory, we define the private value of the latest innovation in sector ω as the sum of the present values of the incumbent's expected net profits on the sale of intermediate good ω. Using (8), one gets the following formal definition.

Definition 3. *Consider an innovation at date t in any sector $\omega, \omega \in \Omega$. Its private value is*

$$\Pi_{\omega t}^{x}(\psi,\varphi) = \int_t^{\infty} \pi_{\tau}^{x_\omega}(\psi,\varphi)\, e^{-\int_t^{\tau}[r_u(\psi,\varphi)+\lambda l_{\omega u}(\psi,\varphi)]du} d\tau, \\ \text{where } \pi_{\tau}^{x_\omega}(\psi,\varphi) \text{ is given by (8)} \quad (9)$$

To conclude, note that the free entry condition in any R&D activity ω is

$$w_t(\psi,\varphi) = \lambda \Pi_{\omega t}^{x}(\psi,\varphi) \quad (10)$$

The key point is that, because the knowledge inherent in innovations is not priced, the value of any innovation stems only from the stream of monopoly profits given by property rights. Besides, recall that in this type of equilibrium, because of the creative destruction mechanism, in each sector, at date t, only the latest innovator (*i.e.* the incumbent) receives positive profits.

3.1.2 Lindahl equilibrium and social value of innovations

Now, we focus on the main purpose of this paper, which is to answer the following questions: what is the social value of an innovation and how should it be computed? As explained above, the social value of an innovation is the optimal one. Moreover, since an innovation consists in the creation of new units of knowledge embodied in an intermediate good, one has to determine the social value of each unit of knowledge inherent in an innovation. Finally, since knowledge is a non rival good, we construct a *Lindahl equilibrium* (which differs significantly from the standard Schumpeterian equilibrium *à la* Aghion & Howitt 1992) as follows. First, we complete the markets, that is, contrary to what is done in standard Schumpeterian equilibria, we price knowledge.[17] Each user of knowledge pays each unit of knowledge at a personalized price (the Lindahl price) which is equal to the marginal profitability of this unit for this user (*i.e.* its willingnesses to pay), and each innovator receives the sum of these Lindahl prices for each unit of knowledge inherent in her innovation. Second, all rival goods are priced at their marginal cost.

Since each unit of knowledge is simultaneously used by three types of economic activities, we have to define a Lindahl price for each of them. Formally, we denote by $v_{\omega t}^{Y}$, $v_{\omega t}^{x}$, and $v_{\omega t}^{\chi_h}$, $h \in \Omega_\omega$, the Lindahl prices of one unit of knowledge $\chi_{\omega t}$, $\omega \in \Omega$, for the final sector, for the intermediate good sector ω, and for each R&D activity h using this knowledge χ_ω (*i.e.* for all $h \in \Omega_\omega$), respectively. These Lindahl prices are the instantaneous marginal profitabilities for the various users. Then, the instantaneous price received at date t by the producer for this unit of knowledge is $v_{\omega t} = v_{\omega t}^{Y} + v_{\omega t}^{x} + v_{\omega t}^{R\&D}$, where $v_{\omega t}^{R\&D} = \int_\Omega v_{\omega t}^{\chi_h} dh$ is the instantaneous value of this unit for the whole R&D activity of the economy.

From the first welfare theorem, we know that the Lindahl equilibrium is Pareto optimal (e.g. Mas-Colell, Whinston & Green 1995 - Ch. 16 - P. 570); this result will indeed be verified within this endogenous growth framework (see Propositions 2 and 5). Hence, as mentioned above, we are sure that the value of knowledge computed in this equilibrium is the optimal one, that is the social one. Consequently, we use the superscript "^{o}" (introduced above in Subsection 2.2) for all variables associated to this equilibrium. The Lindahl equilibrium is formally defined as follows.

Definition 4. *The Lindahl equilibrium consists of time paths of set of prices*

$$\left\{\left(w_t^o, r_t^o, \{q_{\omega t}^o\}_{\omega\in\Omega}, \{v_{\omega t}^{Yo}\}_{\omega\in\Omega}, \{v_{\omega t}^{xo}\}_{\omega\in\Omega}, \{v_{\omega t}^{\chi_h o}\}_{\omega\in\Omega, h\in\Omega}\right)\right\}_{t=0}^{\infty}$$

and of quantities

$$\left\{\left(c_t^o, Y_t^o, \{x_{\omega t}^o\}_{\omega\in\Omega}, L_t^{Yo}, \{l_{\omega t}^o\}_{\omega\in\Omega}, \{\chi_{\omega t}^o\}_{\omega\in\Omega}\right)\right\}_{t=0}^{\infty}$$

[17] It is undoubtedly a crucial question to understand why knowledge can be priced or not. Fundamentally, this refers to the non-rivalry property of knowledge, and thus to the issues of observability, information, or excludability. This remains out of the scope of the present paper.

such that: the representative household maximizes her utility; firms (final good producer, intermediate goods producers, R&D activities) maximize their profits; all rival goods markets (final good market, labor market, intermediate goods markets, and financial market) are perfectly competitive and clear; each user of knowledge pays each unit of knowledge at its Lindahl price; for each unit of knowledge inherent in her innovation, the innovator receives the sum of the Lindahl prices of this unit; and all knowledge expenditures are funded by the government.

We now define the *social value of an innovation*. First, since knowledge is infinitely-lived, the *social value of one unit of knowledge* $\chi_{\omega t}$ at date t is the discounted sum of these instantaneous social values:

$$V^o_{\omega t} = \int_t^\infty v^o_{\omega \tau} e^{-\int_t^\tau r^o_u du} d\tau, \text{ where } v^o_{\omega t} = v^{Yo}_{\omega t} + v^{xo}_{\omega t} + v^{R\&Do}_{\omega t} \tag{11}$$

Second, as explained above (see Definition 1 and Assumption 2), an innovation involves an increase in knowledge of $\Delta\chi_{\omega t} = \sigma \mathcal{P}_{\omega t}$ new units. Accordingly, the definition of the social value of an innovation in sector ω is the following:

Definition 5. *Consider an innovation at date t in any sector $\omega, \omega \in \Omega$. Its social value is*

$$\mathcal{V}^o_{\omega t} = \sigma \mathcal{P}^o_{\omega t} V^o_{\omega t}, \text{ where } V^o_{\omega t} \text{ is given by (11)} \tag{12}$$

Definition 5 provides, in a Schumpeterian growth model, a formal expression which corresponds precisely to the statement of Green & Scotchmer (1995) when they write that "the social value of an early innovation includes the net social value of the applications it facilitates". In the Lindahl equilibrium, at any date t, all innovators, whether the incumbent or any previous one, receive positive income; recall that, on the contrary, in the Schumpeterian equilibrium only the latest innovator receives positive income (monopoly profits).

Obviously, the realism of this type of decentralized economy is more than questionable. First, due to the non-convexities of the technologies in which knowledge is an input, R&D expenditures must be publicly funded.[18] This is strongly at odds with empirical evidences. Second, as stated in Mas-Colell, Whinston & Green (1995 - Ch. 11 and Ch. 16) in their analysis of the issues involved by the presence of public goods and by their funding, the concept of Lindahl equilibrium implies several caveats. They argue that "it is unlikely that the critical assumption of price taking will be satisfied". Moreover, the non-rivalry property of knowledge raises standard problems of public economics: they add that "the fact that the prices are personalized means that personal, private information is required". Finally, they insist on the fact that "this [type of equilibrium] requires that the public good be excludable". In the present paper, these issues of realism are irrelevant; indeed, what matters here is that the concept of Lindahl equilibrium allows us to compute the system of prices which sustains the first-best optimum, and thus to unveil the social value of innovations.

In order to clarify the concept of Lindahl equilibrium, we present the agents' behaviors. In particular, we determine the Lindahl prices of each unit of knowledge $\chi_{\omega t}$ produced in any sector ω. These personalized prices are given by the marginal profitabilities of this unit for its various users.

- In the final sector, the competitive firm maximizes its profit (purchases of knowledge not included) $\pi^Y_t = (L^Y_t)^{1-\alpha} \int_\Omega \chi_{\omega t}(x_{\omega t})^\alpha d\omega - w_t L^Y_t - \int_\Omega q_{\omega t} x_{\omega t} d\omega$. The first-order conditions yield:

$$w_t = (1-\alpha)\frac{Y_t}{L^Y_t} \text{ and } q_{\omega t} = \alpha (L^Y_t)^{(1-\alpha)} \chi_{\omega t}(x_{\omega t})^{\alpha - 1}, \forall \omega \in \Omega \tag{13}$$

At each date t, the marginal profitability of one unit of knowledge $\chi_{\omega t}$ produced in any sector ω for the final sector is:

$$v^Y_{\omega t} = \frac{\partial \pi^Y_t}{\partial \chi_{\omega t}} = (L^Y_t)^{(1-\alpha)}(x_{\omega t})^\alpha, \forall \omega \in \Omega \tag{14}$$

[18] Basically, the replication argument states that technologies display constant returns to scale with respect to private inputs and increasing returns to scale with respect to private and public factors taken jointly. In a competitive market, the payment of private factors fully exhausts revenue and firms are thus unable to pay for the public good they use; hence, such an equilibrium would exist only if the purchase of knowledge is entirely financed by public expenditures (see, for instance, Kaizuka 1965; Manning, Markusen & Mc Millan 1985; or Romer 1990).

• In any intermediate sector ω, $\omega \in \Omega$, as explained above, we assume in this framework that, contrary to the standard literature, intermediate goods producers behave competitively. The profit on the sale of intermediate good ω (purchases of knowledge not included) is $\pi_t^{x_\omega} = (q_{\omega t} - \chi_{\omega t})x_{\omega t}$. Perfect competition and (13) imply that:

$$q_{\omega t} = \chi_{\omega t} \text{ and } x_{\omega t} = x_t = \alpha^{\frac{1}{1-\alpha}} L_t^Y, \forall \omega \in \Omega \tag{15}$$

At each date t, the marginal profitability of one unit of knowledge $\chi_{\omega t}$ produced in any sector ω for the production of intermediate good ω is negative:

$$v_{\omega t}^x = \frac{\partial \pi_t^{x_\omega}}{\partial \chi_{\omega t}} = -x_{\omega t}, \forall \omega \in \Omega \tag{16}$$

• In any intermediate sector h, $h \in \Omega$, given the technology of production of innovations (1), the expected profit of R&D activities (purchases of knowledge not included) is $\pi_t^{\chi_h} = \lambda l_{ht} \mathcal{V}_{ht} - w_t l_{ht}$, where $\mathcal{V}_{ht}$ is the value of an innovation (*i.e.* the value of $\Delta\chi_{ht} = \sigma \mathcal{P}_{ht}$ new units of knowledge) in sector h. Perfect competition in any R&D activity h gives the following free-entry condition:

$$w_t = \lambda \mathcal{V}_{ht} = V_{ht} \lambda \sigma \mathcal{P}_{ht}, \forall h \in \Omega \tag{17}$$

In the present model, we consider explicitly inter-sectorial knowledge spillovers. Accordingly, the presence of knowledge produced by R&D activity ω in the pool used by R&D activity h is contingent on the scope of knowledge diffusion. This has a major implication in the determination of the marginal profitability of one unit of knowledge $\chi_{\omega t}, \omega \in \Omega$, for any R&D activity h, $h \in \Omega$. Indeed, the willingness to pay of R&D activity h for knowledge inherent in an innovation in sector ω depends on whether knowledge $\chi_{\omega t}$ belongs to the pool $\mathcal{P}_{ht}$ used by R&D activity h,[19] or not. In the former case, the marginal profitability is positive, in the latter, it is zero. Formally, one has

$$v_{\omega t}^{\chi_h} = \frac{\partial \pi_t^{\chi_h}}{\partial \chi_{\omega t}} = V_{ht} \lambda \sigma l_{ht} \frac{\partial \mathcal{P}_{ht}}{\partial \chi_{\omega t}} = \begin{cases} V_{ht} \lambda \sigma l_{ht}, \text{ if } h \in \Omega_\omega \\ 0, \text{ if } h \notin \Omega_\omega \end{cases}, \forall \omega \in \Omega \tag{18}$$

In Lemma 3, we recapitulate the results relative to the marginal profitabilities of one unit of knowledge $\chi_{\omega t}$, $\omega \in \Omega$, given by (14), (16) and (18), obtained by studying the individual behaviors of the various users of this unit.

Lemma 3. *At date t, the Lindahl prices (marginal profitabilities) of one unit of knowledge $\chi_{\omega t}$ produced in any sector ω, for the final good sector, the intermediate sector ω, $\omega \in \Omega$, and the R&D sector $h, h \in \Omega$, respectively are:*

$$v_{\omega t}^Y = (L_t^Y)^{(1-\alpha)} (x_{\omega t})^\alpha, \; v_{\omega t}^x = -x_{\omega t}, \; \textit{and } v_{\omega t}^{\chi_h} = \begin{cases} V_{ht} \lambda \sigma l_{ht}, \textit{ if } h \in \Omega_\omega \\ 0, \textit{ if } h \notin \Omega_\omega \end{cases}, \forall \omega \in \Omega$$

The instantaneous price received by the producer of one unit of knowledge $\chi_{\omega t}$ is $v_{\omega t} = v_{\omega t}^Y + v_{\omega t}^x + v_{\omega t}^{R\&D}$, where $v_{\omega t}^{R\&D} = \int_\Omega v_{\omega t}^{\chi_h} dh$.

It is not surprising that the marginal profitabilities are positive for the final good production and for R&D activities. Conversely, one may wonder why it is negative for the production of the associated intermediate good. This results from the fact that the intermediate good production technology accounts for the increasing complexity in the production of intermediate goods (see (6)).

3.1.3 R&D incentives: Schumpeterian equilibrium versus Lindahl equilibrium

We now summarize the main points developed in this section, and we relate them to the issue of R&D incentives. Let us first recall that an innovation in any given sector involves the creation of a flow of new knowledge which increases the stock of knowledge previously embodied in the intermediate good produced in this sector. This point is considered differently in the two equilibria presented above. Indeed, there is a fundamental difference between the Schumpeterian equilibrium and the Lindahl equilibrium which lies in the fact that markets are incomplete in the former (knowledge is not priced) while they are complete in the latter (knowledge is priced). Accordingly, R&D is not funded the same way in these two types of equilibria. In the Schumpeterian equilibrium, when an innovator (*i.e.* a producer of new knowledge)

[19] This is the case if $h \in \Omega_\omega \equiv [\omega - \theta/2 \,;\, \omega + \theta/2]$ (see Lemma 2), that is if sector h is located in the neighborhood of diffusion of knowledge χ_ω.

sells an intermediate good (a rival good), she *indirectly* sells knowledge (a non rival good).[20] Therefore R&D investments are indirectly funded by monopolies profits on a rival good embodying knowledge (see $\pi_{\tau}^{x_\omega}(\psi, \varphi)$ in (9) above). In the Lindahl equilibrium, R&D investments are *directly* funded by pricing knowledge (see the Lindahl prices, $v_{\omega t}^{Y}$, $v_{\omega t}^{x}$ and $v_{\omega t}^{\chi_h}, h \in \Omega_\omega$, in (12) above) and each intermediate good is priced at its marginal cost (see (15)).

This difference has several implications which turn out to particularly matter when it comes to study R&D incentives in presence of sequential and cumulative innovations, and in particular to revisit the issue of Pareto sub-optimality arising in the Schumpeterian equilibrium. We summarize these implications in three points.[21]

a) The quantities of knowledge sold - whether indirectly, as in the Schumpeterian equilibrium, or directly, as in the Lindahl equilibrium - differ in the two equilibria. In the Schumpeterian equilibrium, an entrant (*i.e.* the latest innovator) sells an intermediate good which embodies the whole stock of knowledge, $\chi_{\omega t}$, created in the sector so far (*i.e.* including the knowledge produced by its predecessors) and thus benefits from previously conducted R&D. In contrast, in the Lindahl equilibrium, the knowledge sold by an innovator includes only the incremental knowledge, $\Delta\chi_{\omega t} = \sigma\mathcal{P}_{\omega t}$, associated to her innovation. This difference can be clearly identified by comparing the expressions of the private value and of the social value of an innovation (see Definitions 3 and 5, respectively): the flow of knowledge $\sigma\mathcal{P}_{\omega t}$ does not appear in $\Pi_{\omega t}^{x}(\psi, \varphi)$ while it does in $\mathcal{V}_{\omega t}$. In terms of R&D incentives provided in the Schumpeterian equilibrium, this tends to lead to an excessive level of resources devoted to R&D.

b) This knowledge is not sold to the same economic agents in the two equilibria. In the Schumpeterian equilibrium, the innovator's behavior depends on the fact that the knowledge embodied in the intermediate good has an impact on two agents. First, the final sector; indeed, she is the only agent to whom the intermediate good - and thus the knowledge embodied in it - is sold. Second, the innovator herself, since she produces this intermediate good and since her production costs increase with the level of knowledge accumulated in the sector (see the expressions of the technology of intermediate good production (6) and of monopoly profits (8)). On the contrary, in the Lindahl equilibrium, each unit of knowledge inherent in an innovation is valued for all of its users: for the producer of the final good ($v_{\omega t}^{Y}$), for the producer of the intermediate good embodying this knowledge ($v_{\omega t}^{x}$), but also for all the R&D activities using this knowledge - the one of the intermediate sector in which it has been produced ($v_{\omega t}^{\chi_\omega}$), as well as the ones of a more or less wide range of intermediate sectors ($v_{\omega t}^{\chi_h}, \forall h \in \Omega_\omega$). Obviously, in the Schumpeterian equilibrium, each innovator neglects the fact that her innovation may be of some use for producing new knowledge in her sector as well as in other sectors. In terms of R&D incentives, the fact that the Schumpeterian equilibrium considers only the private value of innovations, and that this value disregards (both intra and inter-sectorial) knowledge spillovers, tends to yield to insufficient R&D effort. Thereupon, one can observe that, the more intense knowledge spillovers are (*i.e.* the larger θ is), the higher the value of one unit of knowledge for R&D activities is (indeed, from Lemma 3, one has $v_{\omega t}^{R\&D} = \int_{\Omega_\omega} v_{\omega t}^{\chi_h} dh$, where $v_{\omega t}^{\chi_h} > 0, \forall \omega \in \Omega_\omega \equiv [h - \theta/2\,;\, h + \theta/2]$), and thus the more this effect will be likely to imply a sub-optimal level of R&D investment.

c) Finally, the two equilibria differ regarding the period during which an innovation provides revenues to its producer. In the Schumpeterian equilibrium, since there is no distinction between an intermediate good and the knowledge embodied in it, one neglects that goods disappear while knowledge persists. Indeed, in each intermediate sector, the latest innovator (a producer of knowledge) monopolizes the production and sale of the intermediate good (which embodies knowledge) until replaced by the next innovator. This creative destruction mechanism implies that there is a "business stealing effect" in the sense that, in each sector, the patent owner has a monopoly whose lifespan is finite in average. Accordingly, the period during which an innovation yields some return is also finite in average. This appears in (9), in which the discount factor includes the term $\lambda l_{\omega u}(\psi, \varphi)$ (the rate of creative destruction). In the Lindahl equilibrium, in each intermediate sector, the instantaneous profit of the intermediate good producer is nil because of perfect competition and, as in the Schumpeterian case, this good disappears when a higher quality good can be produced (*i.e.* consecutively to each innovation). However, the innovator receives revenues forever because each unit of knowledge constituting her innovation is infinitely-lived and priced at each instant of time (at each date t, each innovator receives the sum of the Lindahl prices $v_{\omega t}$). This

[20] We return to this point below in Subsection 4.2 in which we compute the value of the stock of knowledge embodied in the intermediate good.

[21] Through those three points, we aim to clarify the notion of "intertemporal spillovers" already mentioned in the literature (see, for instance, in Romer 1990 or in Aghion & Howitt 1992).

clearly appears in (12), in which $\lambda l_{\omega u}(\psi,\varphi)$ does not appear in the discount factor. Regarding R&D incentives provided within the Schumpeterian equilibrium, this conduces to too little R&D investments.

This analysis reveals that the reason why Schumpeterian models typically predict that the allocation of resources in R&D - and thus the growth rate - can either be insufficient or excessive stems precisely from the fact that innovators do not consider the proper good, do sell it neither at the proper price, nor during the proper period of time. We return in detail to these issues in Section 4 below. In particular, in Subsection 4.1, we revisit the issue of Pareto sub-optimality of the Schumpeterian equilibrium by exhibiting how under (*resp.* over) optimal R&D investment relates to the fact that the private value of innovations can be below (*resp.* above) the social one, and by revealing the part played by the dimension of innovations $\sigma\theta$. In Subsection 4.2, we develop the ideas sketched in the paragraphs a), b) and c) by studying the link between the social value of each intermediate good (*i.e.* its social surplus) and the value of the knowledge which is embodied in it; then we come back to the issues of R&D incentives and welfare in the Schumpeterian equilibrium.

3.2 Characterization of the two equilibria and values of innovations

In the previous subsection, starting from the difference between knowledge and the intermediate good which embodies it, we have distinguished Schumpeterian and Lindahl equilibria. Then, this allowed us to define the private value and the social value of innovations. We now compute these two equilibria and the two corresponding values of innovations. The Schumpeterian equilibrium and the private value of innovations are computed in Propositions 3 and 4, respectively. The Lindahl equilibrium and the social value of innovations are computed in Propositions 5 and 6, respectively.

Proposition 3. *At each vector of public policies (ψ,φ) is associated a particular Schumpeterian equilibrium* à la *Aghion & Howitt. It is characterized, at each date t, as follows.*

- *The quantities and the growth rates are $L_t^Y(\psi,\varphi) = L^Y(\psi,\varphi) = \frac{\gamma L\left(\frac{\lambda}{\gamma}+\rho\right)}{\lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)}$; $l_{\omega t}(\psi,\varphi) = l(\psi,\varphi) = \frac{1}{\gamma} - \frac{L^Y(\psi,\varphi)}{\gamma L}, \forall\omega\in\Omega$;*

  $x_{\omega t}(\psi,\varphi) = x(\psi,\varphi) = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L^Y(\psi,\varphi), \forall\omega\in\Omega;$

  $g_{c_t} = g_{Y_t} = g_{\mathcal{K}_t} = g_{\chi_{\omega t}} = g(\psi,\varphi) = \lambda\sigma\theta\, l(\psi,\varphi), \forall\omega\in\Omega.$

- *The prices are $w_t(\psi,\varphi) = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t(\psi,\varphi)$; $r(\psi,\varphi) = g(\psi,\varphi)+\rho$ $q_{\omega t}(\psi,\varphi) = q_t(\psi,\varphi) = \frac{\mathcal{K}_t(\psi,\varphi)}{\alpha\gamma L}, \forall\omega\in\Omega$, in which $\mathcal{K}_t(\psi,\varphi) = e^{g(\psi,\varphi)t}$.*

Proof. See Appendix 6.3.

The first-best social optimum can be implemented within this Schumpeterian equilibrium *à la* Aghion & Howitt. The optimal set of public tools (ψ^o,φ^o) correcting the two distortions inherent in that equilibrium (namely the monopoly distortion and the market incompleteness) can be obtained by identifying the equilibrium growth rate and quantities of intermediate goods, $g(\psi,\varphi)$ and $x(\psi,\varphi)$, with the optimal ones, g^o and x^o.[22] One gets the following corollary to Proposition 3.

Corollary. *The optimal tools are $\psi^o = 1-\alpha$, and $\varphi^o = \frac{\sigma\theta}{\rho}\left(\frac{\lambda}{\gamma}+\rho\right)-2$.*

The optimal tool used to correct the static distortion entailed by monopolies is the usual subsidy ($\psi^o > 0$) on each intermediate good demand; the optimal tool correcting the externality entailed by market incompleteness can consist either in a subsidy ($\varphi^o > 0$) or in a tax ($\varphi^o < 0$) on monopoly profits, depending on whether the R&D effort is sub-optimal or over-optimal. It is here obvious that the optimal

[22] One gets:

$$\begin{cases} g(\psi^o,\varphi^o) = g^o \\ x(\psi^o,\varphi^o) = x^o \end{cases} \Leftrightarrow \begin{cases} L^Y(\psi^o,\varphi^o) = \frac{\rho\gamma L}{\lambda\sigma\theta} \\ \left(\frac{\alpha^2}{1-\psi^o}\right)^{\frac{1}{1-\alpha}} L^Y(\psi^o,\varphi^o) = \alpha^{\frac{1}{1-\alpha}}\frac{\rho\gamma L}{\lambda\sigma\theta} \end{cases} \Leftrightarrow \begin{cases} \varphi^o = \sigma\theta\left(\frac{\lambda}{\gamma\rho}+1\right)-2 \\ \psi^o = 1-\alpha \end{cases}$$

R&D policy should depend on $\sigma\theta$, the dimension of innovations.[23] Indeed, one has

$$\varphi^{o} \gtreqless 0 \Leftrightarrow \sigma\theta \gtreqless \frac{2\rho}{\frac{\lambda}{\gamma}+\rho} \equiv \tilde{\vartheta} \tag{19}$$

We return to these points in Subsection 4.1, in which we revisit the issue of Pareto sub-optimality of the Schumpeterian equilibrium. In Proposition 4, we provide the value of an innovation in the Schumpeterian equilibrium, that we have called *private value.*

Proposition 4. *Consider an innovation at date t in any sector $\omega, \omega \in \Omega$. Its private value is*

$$\Pi_{\omega t}^{x}(\psi,\varphi) = \Pi_{t}^{x}(\psi,\varphi) = \frac{1-\alpha}{\lambda}\left(\frac{\alpha^{2}}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_{t}(\psi,\varphi),$$

where $\mathcal{K}_t(\psi,\varphi) = e^{g(\psi,\varphi)t}$ and $g(\psi,\varphi) = \lambda\sigma\theta\left[\frac{1}{\gamma} - \frac{\frac{\lambda}{\gamma}+\rho}{\lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)}\right]$.

Proof. See Appendix 6.3 (see equation (43)).

Now, we depart from the standard Schumpeterian equilibrium *à la* Aghion & Howitt and focus on the Lindahl equilibrium. Since we know that a Lindahl equilibrium is Pareto optimal, the repartition of labor, the quantities of intermediate goods and the growth rates are the optimal ones (they are given in Proposition 2).[24] The system of prices that sustains the first-best is provided in Proposition 5 (as previously, the superscript "^{o}" is used for "social optimum").

Proposition 5. *In the Lindahl equilibrium, the system of prices is the following.*

- *The prices of rival goods are $w_t^o = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t^o$; $r_t^o = \frac{\lambda\sigma\theta}{\gamma}$; $q_{\omega t}^o = q_t^o = \chi_t^o = \frac{\mathcal{K}_t^o}{\gamma L}, \forall\omega \in \Omega$, where $\mathcal{K}_t^o = e^{g^o t}$ and $g^o = \frac{\lambda\sigma\theta}{\gamma} - \rho$.*

- *Each unit of knowledge is priced as follows:*

  - *The Lindahl prices of one unit of knowledge $\chi_{\omega t}$ for the final good sector, for the intermediate sector ω, and for the R&D activity of intermediate sector $h, h \in \Omega$, are respectively*

$$v_{\omega t}^{Yo} = \alpha^{\frac{\alpha}{1-\alpha}}\frac{\rho\gamma L}{\lambda\sigma\theta}; v_{\omega t}^{xo} = -\alpha^{\frac{1}{1-\alpha}}\frac{\rho\gamma L}{\lambda\sigma\theta}$$

$$\text{and } v_{\omega t}^{\chi_h o} = \begin{cases} \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\theta}\left(L - \frac{\rho\gamma L}{\lambda\sigma\theta}\right), \text{if } h \in \Omega_\omega \\ 0, \text{if } h \notin \Omega_\omega \end{cases}, \forall\omega \in \Omega.$$

  - *The instantaneous income received by the innovator for each unit of knowledge $\chi_{\omega t}$ she produced is*

$$v_{\omega t}^{o} = v_{\omega t}^{Yo} + v_{\omega t}^{xo} + v_{\omega t}^{R\&Do} = v^{o} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}L, \forall\omega \in \Omega,$$

  where $v_{\omega t}^{R\&Do}$ is the instantaneous value of one unit of knowledge $\chi_{\omega t}$ for the whole R&D activity of the economy, given by $v_{\omega t}^{R\&Do} = \int_\Omega v_{\omega t}^{\chi_h o} dh = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(L - \frac{\rho\gamma L}{\lambda\sigma\theta}\right), \forall\omega \in \Omega.$

Proof. See Appendix 6.4.

The Lindahl equilibrium allows us to determine the value of each unit of knowledge (the sum of the Lindahl prices). In the following corollary, we provide the Lindahl equilibrium value of each unit of knowledge produced in sector ω, $V_{\omega t}^{o}$. It is derived in Appendix 6.4 (see equation (56)). Alternatively, it can also be computed from its definition given in (12), using the expressions of the instantaneous social value, $v_{\omega t}^{o}$, and of the interest rate, r_t^o, provided in Proposition 5.

[23]From here, one could develop an analysis close to ones of Segerstrom (1998) or Li (2003) and study the relation between the optimal R&D subsidy (or tax) and the dimension of innovations. More details on this issue are provided in Section 4.

[24]This result is indeed verified in the Appendix 6.4 (see (59)).

Corollary. *The social value of one unit of knowledge* $\chi_{\omega t}$ *is*

$$V_{\omega t}^{o} = V^{o} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\gamma L}{\lambda\sigma\theta}, \forall \omega \in \Omega.$$

Finally, in Proposition 6, we provide the value of an innovation in the Lindahl equilibrium, that we have called *social value.*

Proposition 6. *Consider an innovation at date* t *in any sector* $\omega, \omega \in \Omega$*. Its social value is*

$$\mathcal{V}_{\omega t}^{o} = \mathcal{V}_{t}^{o} = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\lambda}\mathcal{K}_{t}^{o},$$

where $\mathcal{K}_t^o = e^{g^o t}$ *and* $g^o = \frac{\lambda\sigma\theta}{\gamma} - \rho$*.*

Proof. See Appendix 6.4 (see equation (60)).

To conclude this analysis, in the following corollary, we provide a somehow intuitive result which contributes to explain the link between the private value and the social value of an innovation.

Corollary. *The optimal private value of innovations - i.e. the private value when the two distortions present in the Schumpeterian equilibrium (namely the monopoly distortion and the market incompleteness) are corrected - is the social value:* $\Pi_t^x(\psi^o, \varphi^o) = \mathcal{V}_t^o$*.*

Proof. The proof is straightforward. Plugging the optimal tools (derived in the corollary to Proposition 3) into the expression of the private value of innovations given in Proposition 4, one gets the expression of the social value of innovations given in Proposition 6.

This result shows that one could have directly computed the social value of an innovation in the Schumpeterian equilibrium. In that respect, it would be sufficient to set the two public tools, ψ and φ, at their optimal level. In this case, the innovators are given the optimal incentives via the discounted sum of the expected monopoly profits monopoly, and thus the private value and the social value of innovations are the same. However, introducing the concept of Lindahl equilibrium allows us to understand what is fundamentally the social value of an innovation: since an innovation involves new knowledge, that is a non rival good, it is the sum of the Lindahl prices paid by all the users of this knowledge. We return to this point in the applications provided below in Section 4.

To conclude, let us note that all results established in Section 3.2 involve the parameter θ, which stands for the scope of knowledge diffusion. In particular, one can express these results in the two polar cases (intra-sectorial knowledge spillovers only and global knowledge spillovers) summarized in the corollary to Proposition 1. Fundamentally, introducing explicitly knowledge diffusion on the Salop circle enables us to precisely apprehend the non-rivalry property of knowledge.

4 Applications: Pareto sub-optimality of the Schumpeterian equilibrium and R&D incentives in presence of cumulative innovations

In Section 3, we have insisted on the necessity to distinguish the intermediate good from the knowledge which is embodied in it; thereby, we used the concept of Lindahl equilibrium to determine the social value of innovations. Now, we use this methodology to revisit key issues widely analysed in the economics of innovation. Subsection 4.1 focuses on the Pareto sub-optimality of the standard Schumpeterian equilibrium *à la* Aghion & Howitt (1992). We show that it basically results from market incompleteness, which implies that the private value of innovations might diverge from their social value. Furthermore, we underline the key part played by the dimension of innovations. In Subsection 4.2, we study the link between the social value of each intermediate good (*i.e.* its social surplus) and the value of the knowledge which is embodied in it. This enables us to come back to the reasons why, in the Schumpeterian equilibrium, the innovators are not given the optimal R&D incentives. In Subsection 4.3 we show how the Lindahl equilibrium enables us to shed a new light on the issue of R&D incentives in presence of cumulative innovations. In particular, we revisit, within a Schumpeterian growth model, some arguments found in Scotchmer (1991), in Green & Scotchmer (1995), or in Chu, Cozzi & Galli (2012), among others.

The issues studied in these three subsections are intrinsically related. They all stem from the fact that the Schumpeterian equilibrium exhibits incomplete markets, and that, in order to provide R&D incentives, it introduces intellectual property rights on rival goods in which knowledge is embodied.

4.1 Pareto sub-optimality revisited, market incompleteness and dimension of innovations

Many R&D-based endogenous growth models predict that the decentralized economy can lead to either an insufficient or an excessive allocation of resources in R&D activity, leading to an insufficient or excessive growth. This well-known result of Pareto sub-optimality, which can arise in vertical differentiation class of models (e.g. Grossman & Helpman 1991; Aghion & Howitt 1992) but also in expanding variety models *à la* Romer (e.g. Benassy 1998; Jones & Williams 2000; Alvarez-Pelaez & Groth 2005), has been extensively discussed in the growth literature. One can find several complementary approaches trying to understand why the equilibrium allocation can either be sub-optimal or over-optimal. The first one could be described as market oriented: Aghion & Howitt (1992, 1998) or Jones & Williams (2000), for instance, focus on the various market failures of the equilibrium considered to explain why Pareto sub-optimality may arise. Basically, it is generally argued that the surplus appropriability problem and knowledge spillovers both promote under-investment in R&D whereas creative destruction and duplication effects both foster over-investment in R&D. In a complementary approach, Grossman & Helpman (1991) or Segerstrom (1998), among others, relate the fact that there is too little or too much R&D to the "size of innovations" (*i.e.* to the height of the jumps on the quality ladder) but omit to consider inter-sectorial knowledge spillovers. Grossman & Helpman show that only intermediate-size innovations should be subsidized, while small and large-size innovations should be taxed, whereas Segerstrom finds that it is optimal to subsidize small-size innovations and to tax large-size innovations. Li (2003) generalizes Segerstrom's analysis by taking into account the effect of inter-sectorial spillovers. As already mentioned, and as usual in Schumpeterian growth theory, in the Schumpeterian equilibrium *à la* Aghion & Howitt (1992) studied in this paper, the *laisser faire* growth rate, $g(0,0)$, can either be sub-optimal, optimal, or over-optimal, depending on the values of the parameters of the model:

$$g(0,0) = \lambda\sigma\theta\left[\frac{1}{\gamma} - \frac{\frac{\lambda}{\gamma}+\rho}{\lambda(1+\alpha)}\right] \lesseqqgtr g^o = g(\psi^o,\varphi^o) = \frac{\lambda\sigma\theta}{\gamma} - \rho \tag{20}$$

The purpose of this subsection is to shed a new light on this Pareto sub-optimality issue; in what follows, we show that this issue is basically related to the market incompleteness of the Schumpeterian equilibrium. We know, from the corollary to Proposition 3, that the first-best optimum can be implemented in this equilibrium with two public tools: a subsidy ($\psi^o > 0$) on each intermediate good demand, and a subsidy ($\varphi^o > 0$) or a tax ($\varphi^o < 0$) on the innovator's monopoly profits. First, it can easily be verified that the static distortion entailed by monopolies leads to sub-optimal investment in R&D, and thus tends to lower down the growth rate. Indeed, one has $g(0,\varphi^o) < g^o$. In other words, if the dynamic distortion (more precisely, the market incompleteness) is perfectly corrected (*i.e.* if $\varphi = \varphi^o$) and if there is no subsidy to correct the static distortion (*i.e.* if $\psi = 0$), then the R&D investment in the Schumpeterian equilibrium is sub-optimal.[25] Second, if the static distortion is perfectly corrected (*i.e.* if $\psi = \psi^o$) and if the dynamic one is not (*i.e.* if $\varphi = 0$), then the R&D investment - and thus the growth rate - can be lower, equal, or greater the optimal ones. Formally, one has indeed $g(\psi^o,0) \lesseqqgtr g^o$. These results are summarized in Proposition 7.

Proposition 7. *The optimal set of public tools, (ψ^o,φ^o), that allows to implement the first-best social optimum within the Schumpeterian equilibrium* à la *Aghion & Howitt is such that one has the following inequalities between the decentralized and the optimal growth rates:*

$$g(0,0) \lesseqqgtr g^o,\ g(0,\varphi^o) < g^o \text{ and } g(\psi^o,0) \lesseqqgtr g^o$$

The inequalities exhibited in Proposition 7 show that the monopoly distortion necessarily leads to a sub-optimal investment in R&D, whereas the dynamic distortion can lead either to sub-optimal or to over-optimal level of R&D investment. Hence, it is by considering only the distortion resulting from market incompleteness that one shall understand accurately the issue of Pareto sub-optimality. That is why, in order to better understand this issue, in all what follows, the monopoly distortion is perfectly corrected, that is $\psi = \psi^o$.

[25] Note that an alternative argument would be the following: the growth rate in the case in which the monopoly distortion is corrected is always higher than in the *laisser faire* case: $g(\psi^o,0) > g(0,0)$.

Now, in order to examine thoroughly this Pareto sub-optimality issue and its link with market incompleteness, we present several equivalences.

i) The growth rate of the Schumpeterian equilibrium is lower (*resp.* greater) than the optimal one, that is the market incompleteness leads to a sub-optimal (*resp.* over-optimal) R&D investment, if and only if the optimal tool used to deal with this market incompleteness is a subsidy (*resp.* a tax). Formally, one has: $g(\psi^o, 0) \lesseqqgtr g^o \Leftrightarrow \varphi^o \gtreqqless 0$. This result is somehow intuitive and does not require any particular comment. For instance, it means that, if the R&D investment is insufficient, one has to subsidize innovators.

ii) The growth rate of the Schumpeterian equilibrium is lower (*resp.* greater) than the optimal one if and only if the private value of innovations in any sector ω is smaller (*resp.* higher) than the social one. Formally, one has the following equivalence: $g(\psi^o, 0) \lesseqqgtr g^o \Leftrightarrow \Pi^x_{\omega t}(\psi^o, 0) \lesseqqgtr \mathcal{V}^o_{\omega t}$, $\forall \omega, \forall t$. This equivalence has been obtained thanks to the comparaison between the Schumpeterian and the Lindahl equilibria, and more precisely between the private and the social values of innovations. It underlines that these two values are at the heart of the design of R&D incentives; we return to this point below. For instance, if the private value of innovations is lower than the social one, R&D investment is insufficient (and thus the growth rate is too low) because the are not enough R&D incentives (that is why, as explained in i) above, R&D has to be subsidized).

iii) The growth rate of the Schumpeterian equilibrium is lower (*resp.* greater) than the optimal one if and only if the dimension of innovations, $\sigma\theta$, is above (*resp.* below) the threshold $\tilde{\vartheta}$. Formally, one has the following equivalence: $g(\psi^o, 0) \lesseqqgtr g^o \Leftrightarrow \sigma\theta \gtreqqless \frac{2\rho}{\frac{\lambda}{\gamma}+\rho} \equiv \tilde{\vartheta}$. This last equivalence is fundamentally linked to market incompleteness; it emphasizes the key part played both by σ, the productivity of knowledge, and by θ, the scope of knowledge diffusion. We clarify this point below.

The three equivalences i), ii) and iii) are summarized in Proposition 8.

Proposition 8. *If the monopoly distortion only is corrected (i.e.* $\psi = \psi^o$ *and* $\varphi = 0$*), one has the following equivalences:*

$$g(\psi^o, 0) \lesseqqgtr g^o \Leftrightarrow \varphi^o \gtreqqless 0 \Leftrightarrow \Pi^x_{\omega t}(\psi^o, 0) \lesseqqgtr \mathcal{V}^o_{\omega t} \Leftrightarrow \sigma\theta \gtreqqless \frac{2\rho}{\frac{\lambda}{\gamma}+\rho} \equiv \tilde{\vartheta}, \forall \omega, \forall t$$

Proof. The proof of i), ii) and iii) are straightforward.

i) From Proposition 2 one has $g^o = \frac{\lambda\sigma\theta}{\gamma} - \rho$; and from Proposition 3 and its corollary, one gets $g(\psi^o, 0) = \lambda\sigma\theta\left[\frac{1}{\gamma} - \left(\frac{\lambda}{\gamma}+\rho\right)/2\lambda\right]$. Hence, one has $g(\psi^o, 0) \lesseqqgtr g^o \Leftrightarrow 0 \lesseqqgtr \frac{\sigma\theta}{\rho}\left(\frac{\lambda}{\gamma}+\rho\right) - 2 = \varphi^o$; this proves the first equivalence. ii) At each date t, one has, for all $\omega \in \Omega$, $\Pi^x_{\omega t}(\psi^o, 0) = \Pi^x_t(\psi^o, 0) = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\lambda}\mathcal{K}_t(\psi^o, 0)$, where $\mathcal{K}_t(\psi^o, 0) = e^{g(\psi^o,0)t}$ and $\mathcal{V}^o_{\omega t} = \mathcal{V}^o_t = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\lambda}\mathcal{K}^o_t$, where $\mathcal{K}^o_t = e^{g^o t}$. One immediately gets $g(\psi^o, 0) \lesseqqgtr g^o \Leftrightarrow \Pi^x_{\omega t}(\psi^o, 0) \lesseqqgtr \mathcal{V}^o_{\omega t}$, the second equivalence. iii) Then, the third equivalence is obtained directly from (19).

Proposition 8, which is illustrated by Figure 1, shows that, in the Schumpeterian equilibrium, innovators do not take into account the dimension of innovation, $\sigma\theta$, in their behavior. That is, they neither consider the productivity of knowledge σ, nor the scope of knowledge diffusion θ. This appears in some previous results. First, it can be seen if one returns to the expression of the quantities of labor devoted to R&D. In the first-best social optimum, one has $l^o = 1/\gamma - \rho/\lambda\sigma\theta$, which clearly depends on the dimension of innovations: the higher $\sigma\theta$, the more resources allocated to R&D in the first-best. To the contrary, the dimension of innovations has no effect on the Schumpeterian equilibrium allocation of labor: $l(\psi^o, 0) = 1/\gamma - (\lambda/\gamma + \rho)/2\lambda = 1/2\gamma - \rho/2\lambda$. Second, this can also be seen by returning to the expression of the optimal tool, $\varphi^o = \frac{\sigma\theta}{\rho}\left(\frac{\lambda}{\gamma}+\rho\right) - 2$, that allows to correct the market incompleteness. As mentioned in the comments of Proposition 3 above (see (19)), the higher $\sigma\theta$, the larger φ^o, and thus the more likely R&D should be subsidized. Conversely, the lower $\sigma\theta$, the smaller φ^o, and thus the more likely R&D should be taxed. Indeed, one has $\sigma\theta \gtreqqless \tilde{\vartheta} \Leftrightarrow \varphi^o \gtreqqless 0$.

The basic point is that, in the Schumpeterian equilibrium, because of market incompleteness, innovators' behavior is based on the monopoly profit on intermediate goods (rival goods) which embodies knowledge (a non rival good) and not on the value of this knowledge which depends on its Lindahl prices. This incompleteness has a straightforward consequence. If the dimension of innovations $\sigma\theta$ is large, the more likely R&D investment will be insufficient, that is why it should be subsidized. Conversely, if $\sigma\theta$ is small, the more likely R&D investment will be excessive and should be taxed.

To conclude, the question on whether there is too little or too much R&D investment in the Schumpeterian equilibrium arises from the R&D incentives introduced in this framework. In this equilibrium,

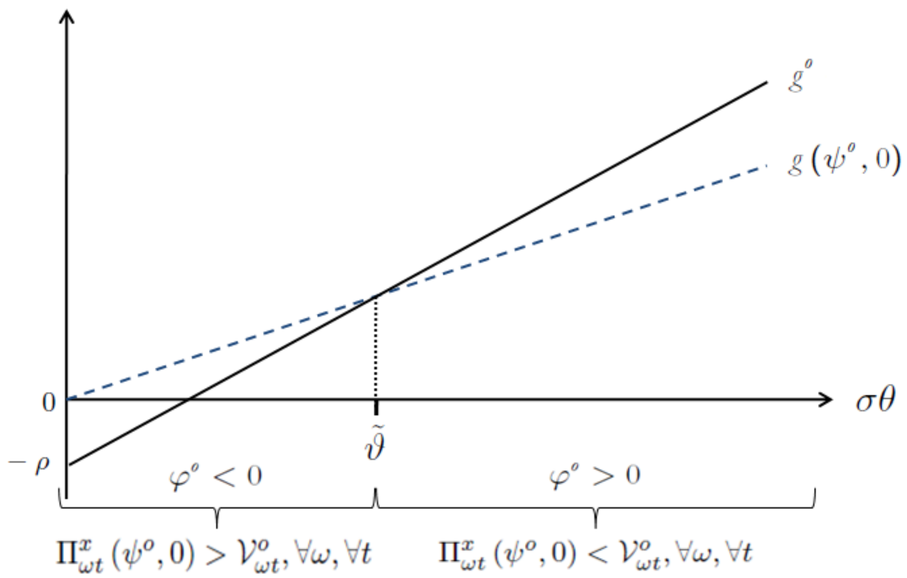

Figure 1: Pareto sub-optimality revisited

the innovators' behavior depends on the private value of innovations (see for instance the free entry condition (10)). In the Lindahl equilibrium, it depends on the social value of innovations (see for instance the free entry condition (17)). As exhibited in Proposition 8 and illustrated in Figure 1, the comparison of these two values allows us to apprehend better the Pareto sub-optimality issue. In Subsection 4.2, we investigate why these two values differ.

4.2 Social surplus of intermediate goods, social value of knowledge and R&D incentives

Our aim now is twofold. First, we study the link between the social value of each intermediate good and the value of the knowledge which is embodied in it. Second, we use this analysis to come back to the issues of R&D incentives and welfare in the Schumpeterian equilibrium.

The social value of an intermediate good is given by the social surplus, that we compute as follows. At the first-best, the inverse demand function of intermediate good ω at date t is obtained from (13). One has $q^o_{\omega t}(x_{\omega t}) = \alpha(L^{Yo}_t)^{(1-\alpha)}\chi^o_{\omega t}(x_{\omega t})^{\alpha-1}$, where L^{Yo}_t and $\chi^o_{\omega t}$ are given in Proposition 2. Then, the instantaneous net social surplus generated by the production and use of this intermediate good is $S^o_{\omega t} = \int_0^{x^o_{\omega t}} q^o_{\omega t}(x)dx - q^o_{\omega t}x^o_{\omega t}$, where $x^o_{\omega t}$ and $q^o_{\omega t}$ are given in Propositions 2 and 5, respectively. After computation, one gets

$$S^o_{\omega t} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\frac{\rho\gamma L}{\lambda\sigma\theta}\chi^o_{\omega t} \tag{21}$$

Expression (21) shows that the surplus depends on the stock knowledge $\chi^o_{\omega t}$ which is embodied in the intermediate good ω. Using the Lindahl prices allows us to exhibit the value of this knowledge in Proposition 9 below.

Proposition 9. *The instantaneous net social surplus generated by intermediate good ω at date t is the value of the stock of knowledge $\chi^o_{\omega t}$ embodied in it for the user and the producer of this good:*

$$S^o_{\omega t} = \left(v^{Yo}_{\omega t} + v^{xo}_{\omega t}\right)\chi^o_{\omega t}$$

Proof. The proof is straightforward. In Proposition 5, we have shown that the Lindahl prices of one unit of knowledge $\chi_{\omega t}$ for the final sector and for the intermediate sector ω are $v^{Yo}_{\omega t} = \alpha^{\frac{\alpha}{1-\alpha}}\frac{\rho\gamma L}{\lambda\sigma\theta}$ and $v^{xo}_{\omega t} = -\alpha^{\frac{1}{1-\alpha}}\frac{\rho\gamma L}{\lambda\sigma\theta}$, respectively. Summing $v^{Yo}_{\omega t}$ and $v^{xo}_{\omega t}$, one gets $(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\frac{\rho\gamma L}{\lambda\sigma\theta}$.

Proposition 9 shows that the surplus $S^o_{\omega t}$ is equal to the value of the stock of knowledge $\chi^o_{\omega t}$ for the two agents involved in the production and in the use of the intermediate good: the producer of this good (for whom, as seen in (16) and in Proposition 5, the Lindahl price $v^{xo}_{\omega t}$ is negative) and the final good producer (for whom, as seen in (14) and in Proposition 5, the Lindahl price $v^{Yo}_{\omega t}$ is positive).

Now, we use this result to study the issues of R&D incentives and welfare in the Schumpeterian equilibrium. More precisely, we explain why the incentives provided in that framework deviate from optimal ones which are given by the Lindahl equilibrium. In particular, we use the surplus to formalize the intuitions developed in 3.1.3.

Let us first recall that, in the Schumpeterian equilibrium, the incentives provided to innovators rest upon IPRs that ensure them monopoly power: as seen in Definition 3 (which gives the private value of innovations as the sum of the present values of the incumbent's expected net profits on the sale of intermediate goods), the key variable in the design of R&D incentives in each sector ω is $\pi_t^{x_\omega}(\psi, \varphi)$, the monopoly profit on intermediate good ω.

The first evident result is that the monopoly does not extract the whole surplus. This issue of appropriability has been extensively studied in the literature (see, for instance, Aghion & Howitt 1992, 1998 - Ch. 2, or Acemoglu 2009 - Ch. 14). At this stage of the analysis, one can easily observe that, even if the monopoly was able to extract the whole surplus at each date t, this would not provide the proper R&D incentives. The reason is twofold.

Firstly, as seen in (21) and in Proposition 9, the surplus directly depends on the whole quantity of knowledge embodied in intermediate good ω, $\chi_{\omega t}$. As explained in paragraph a) of 3.1.3, this *stock* $\chi_{\omega t}$ is different from the quantity of knowledge entailed by the latest innovation, that is from the *flow* $\Delta\chi_{\omega t}$. In any sector, the incumbent sells an intermediate good that embodies all the knowledge accumulated so far in this sector even though she has produced only part of this knowledge. This first effect results in too strong R&D inventives in the Schumpeterian equilibrium.

Secondly, the unit price of this knowledge $\chi_{\omega t}$ is not the one that would provide the proper incentives. Indeed, we have shown in Proposition 9 that the surplus $S_{\omega t}^o$ is equal to the value of $\chi_{\omega t}^o$ exclusively for two agents: the producer of the intermediate good and the final good producer. As mentioned in the paragraph b) of 3.1.3, the producer and the user of an intermediate good ω are not the only agents affected by the knowledge entailed by the innovation associated to this good. Indeed, so is any R&D activity whose pool of knowledge contains the knowledge embodied in intermediate good ω. Therefore, the unit price of knowledge appearing in the expression of the surplus given in Proposition 9, $v_{\omega t}^{Yo} + v_{\omega t}^{xo}$, does not provide the appropriate incentives to innovators since it does not include the Lindahl prices of the R&D activities. As argued in 3.1.2, the optimal price that the innovator should receive at each date t - that is the one that would give her the appropriate incentives - is $v_{\omega t}^o = v_{\omega t}^{Yo} + v_{\omega t}^{xo} + v_{\omega t}^{R\&Do}$. Accordingly, the surplus can be rewritten as follows:

$$S_{\omega t}^o = \left(v_{\omega t}^o - v_{\omega t}^{R\&Do}\right) \chi_{\omega t}^o \tag{22}$$

The expression (22) shows that, as argued throughout this paper and in particular in the paragraph b) of 3.1.3, knowledge is not priced in the Schumpeterian equilibrium. Therefore, this equilibrium omits the fact that knowledge produced in sector ω may have positive spillovers effects on other sectors' R&D activities: part of the social value generated by knowledge spillovers is neglected.[26] This second effect results in too weak R&D inventives.

The analysis conducted so far is static: it involves only the instantaneous profits, $\pi_t^{x_\omega}(\psi, \varphi)$, appearing in (9). Let us now consider the dynamic angle of the links between the surplus and the issues of R&D incentives and welfare. Recall that, as explained in the paragraph c) of 3.1.3, in the Schumpeterian equilibrium *à la* Aghion & Howitt each innovator receives the private value of her innovation; this value is computed on a period which is finite in average because the expected value of an intermediate good lifespan is finite (see Definition 3). On the contrary, in the Lindahl equilibrium, each innovator receives the social social value of this innovation; this value is computed on an infinite period (see Definition 5). Hence, the Schumpeterian equilibrium omits the fact that knowledge is infinitely-lived. This third effect results in too weak R&D inventives.

This leads us to come back to the issue of R&D incentives in presence of cumulative innovations.

4.3 Optimal R&D incentives in presence of cumulative innovations

In the previous subsections, we showed that, because it exhibits market incompleteness, the Schumpeterian equilibrium *à la* Aghion & Howitt (1992) does not provide the appropriate incentives for static and dynamic reasons. Regarding the static ones, there is first the appropriability issue; furthermore, even if the monopolies were able to extract the whole surplus, the incentives would not be the good ones since the surplus takes into account neither the proper quantity of knowledge ($\chi_{\omega t}$ instead of $\Delta\chi_{\omega t}$)

[26] The knowledge spillovers issue has extensively been tackled in the literature (e.g. Aghion & Howitt 1992, 1998, 2009; Li 2002; Peretto & Smulders 2002; Jones 2005; Acemoglu 2009). We think that introducing Lindahl prices - and thus the Lindahl equilibrium - in the analysis allows us to clarify this issue.

nor the proper prices ($v_{\omega t}^{R\&D} = \int_{\Omega_\omega} v_{\omega t}^{X_h} dh$ is missing). Regarding the dynamic one, the period during which an innovation yields some return is too short. The reason why there can be too little or too much R&D investment in the Schumpeterian equilibrium, which has been sketched above in 3.1.3, is now clearly identified: the issues of appropriability, of non proper prices, and of non proper duration lead to insufficient R&D investments; in contrast, the issue of non proper quantity of knowledge leads to excessive R&D investments. Basically, this underlines why it is crucial to distinguish between knowledge and the intermediate goods embodying it. More precisely, the comparison between the net social surplus generated by an intermediate good and the social value of the knowledge embodied in it enabled us to show how patenting intermediate goods embodying knowledge neglects several important features of the cumulativeness of innovation, and is therefore unlikely to provide optimal incentive to innovators.

In what should consist these optimal incentives? Let us briefly recall the main characteristics of the cumulative feature of innovations in our Schumpeterian growth model with explicit knowledge diffusion. There is a large number of agents, the innovators, who exchange knowledge, an infinitely-lived and non rival good. In each intermediate sector, innovators use knowledge produced in a subset of sectors whose measure depends of θ, the scope of knowledge diffusion (for example, only the knowledge produced in this sector in the polar case of only intra-sectorial knowledge spillovers, and the whole disposable knowledge in the economy in the polar case of global knowledge spillovers). Simultaneously, each innovator produces new knowledge which is used in a subset of sectors that is more or less large, depending on θ.

We have shown that the Lindahl equilibrium is the way to determine the income that each innovator should receive for an innovation created at date t in order to implement the first-best social optimum. This is line with ideas developed, for instance, in Tirole (1988), in Scotchmer (1991), or in Dasgupta, Mäler, Navaretti & Siniscalco (1996). Formally, in our Schumpeterian growth framework, this income is provided by the social value of the innovations, $\mathcal{V}_{\omega t}^{o}$. In the most general case, it is given by (12) in Definition 5; in the specified case, it is provided in Proposition 6.

5 Conclusion

This paper arises from the observation that, traditionally, the social value of innovations is not explicitly determined in standard growth theory. Our main purpose was to compute it. The Lindahl equilibrium enabled us to define and to determine analytically the social value of innovations in a Schumpeterian growth model.

Using the formalization of Salop (1979), we introduced knowledge diffusion in a standard endogenous growth model with vertical innovations following on from the seminal ones of Grossman & Helpman (1991) or Aghion & Howitt (1992). Intermediate sectors are uniformly distributed on the circle. In each intermediate sector, knowledge accumulates as innovations occur, and an intermediate good embodies this stock of knowledge. Accordingly, we defined an innovation as a pair "new knowledge / intermediate good" and, since knowledge is a non rival good, we explicitly formalized knowledge diffusion: the knowledge produced in each sector diffuses both within this sector and across sectors with more or less intensity. Since each sector is simultaneously sending and receiving new knowledge, in each of them, R&D activity draws knowledge from a pool which stems from knowledge diffusion. This formalization implies that the broader the scope of knowledge diffusion in the economy is (*i.e.* the stronger the intensity of knowledge spillovers is), the wider these pools are, and therefore the higher the optimal growth rate is. Basically, our framework encompasses all possible cases ranging from intra-sectorial diffusion only to global inter-sectorial diffusion, and allows us to revisit the standard (fully) endogenous growth literature.

Within this general model, we studied two equilibria. First, we recalled the standard Schumpeterian equilibrium *à la* Aghion & Howitt (1992). In this equilibrium, we computed the private value of innovations: as usual in the literature, it is the sum of the expected present values of the monopoly profits on intermediate goods received by the latest innovator who has been granted a patent. Second, we defined and we computed the Lindahl equilibrium. In this respect, we completed the markets by pricing knowledge; then, each innovator receives the sum of the Lindahl prices of all the users of each unit of knowledge inherent in her innovation. Because the Lindahl equilibrium provides the system of prices that sustains the first-best social optimum, this equilibrium enabled us to compute the social value of innovations. Since an innovation involves an increment of several units of knowledge, the social value of this innovation (*i.e.* the optimal value of the knowledge inherent in it) is thus obtained by multiplying this flow of knowledge by the sum of the expected present values of the Lindahl prices of each of these units.

Besides providing the system of first-best prices, and thus allowing to determine the social value of innovations, the Lindahl equilibrium is a benchmark which also makes possible to revisit key issues analysed in the economics of innovation. As a matter of fact, we think that this methodology is likely to be a suitable way to come back on issues involving the presence of knowledge, which are often studied in the industrial organization literature and in endogenous growth theory. Indeed, it enables to better understand the consequences of the non-rivalry of knowledge and of market incompleteness on the innovators' behavior. As a case in point, in this paper, we came back on the issue of Pareto sub-optimality of the standard Schumpeterian equilibrium and on the one of R&D incentives in presence of cumulative innovations within a dynamic general equilibrium framework. Basically, the key point here is that the fundamental externality resulting from market incompleteness implies that it is not possible to fund directly R&D activities. Indeed, in this type of decentralized economy, since knowledge is not priced, its creation is indirectly funded by means of intellectual property rights on intermediate goods embodying knowledge. Therefore, in the standard Schumpeterian equilibrium, because the private value of innovations differs from the social one, innovators are not given the optimal incentives.

Regarding the Pareto sub-optimality issue, we derived several results revealing the link between the level of R&D investment, the fact that the private value of innovations can be lower or greater than the social one, and the dimension of innovations (a variable which is a function of the productivity of knowledge in R&D and of the scope of knowledge diffusion). In particular, we showed that, when the monopoly distortion is corrected, the Schumpeterian equilibrium growth rate is sub-optimal (*resp.* over-optimal) if and only if the private value of innovations is below (*resp.* above) the social one. We also show that, if the dimension of innovations is large (*resp.* small), under (*resp.* over) optimal R&D investment is more likely to occur. Basically, our formalization enabled us to exhibit the following key fact: the impact of the externality triggered by market incompleteness - which implies a difference between the private and the social values of innovations - depends on the dimension of innovations.

In order to better apprehend this issue, we analysed the link between the surplus derived from the production and use of an intermediate good and the value of the knowledge embodied in it. Simultaneously, we shed a new light on the issue of optimal R&D incentives in presence of sequential and cumulative innovations within a Schumpeterian growth model which explicites knowledge diffusion. We showed that, even if the incumbent innovator was able to extract the whole surplus, the incentives provided in the Schumpeterian equilibrium would not be optimal for two static reasons and a dynamic one. First, when the latest innovator sells an intermediate good, she implicitly sells the whole stock of knowledge created in the sector so far; whereas she should sell only the incremental flow of knowledge inherent in her innovation (as it is shown in the Lindahl equilibrium). This first static distortion tends to provide too much R&D incentives to innovators and thus to lead to an excessive level of R&D investment. Second, the knowledge is valued exclusively for two agents: the intermediate good producer and the final good producer. However, it should also be valued for R&D activities using it, as it is the case in the Lindahl equilibrium. In other words, each innovator does not take into account the fact that the knowledge she creates is used by other R&D activities (*i.e.* knowledge spillovers are disregarded). This second static distortion tends to provide too little R&D incentives, and hence to lead to an insufficient level of R&D investment. Third, the period during which an innovation yields revenues is finite in average (the creative destruction implies that the monopoly power of the innovator has a lifespan which is finite in average). Yet, like in the Lindahl equilibrium, an innovator should receive revenue forever for the knowledge inherent in her innovation because it is infinitely-lived. This dynamic distortion tends to lead to to an insufficient level of R&D investment as well. The Lindahl equilibrium allowed us to determine what should be the optimal R&D incentives provided to each innovator in a dynamic general equilibrium model with cumulative innovations: each innovator should receive the social value of her innovation resulting from this equilibrium.

Certainly, as explained in Section 3 (see 3.1.2), the Lindahl equilibrium is clearly non realistic, contrary to the ones introduced initially by Romer (1990) or Aghion & Howitt (1992) for instance. However, our goal was not to provide an acceptable decentralized economy but to compute the social value of innovations and to understand better the consequences of the non-rivalry property of knowledge in an endogenous growth model. In particular, it is a benchmark to tackle the issue of the division of profit in case of sequential and cumulative innovations since it enables to determine the optimal sharing of profit among successive innovators.

To conclude, let us underline that the Lindahl equilibrium is a relevant concept in order to study the functioning of new technology sectors such as the software or biotechnology industries. Indeed, in these sectors, knowledge is embodied in intermediate goods that are akin to non rival goods (whereas they are rival in standard endogenous growth models). For instance, in the case of software, knowledge can be embodied in a CD-ROM, in a DVD, or even in other immaterial supports (e.g. via downloading) whose marginal costs of production are almost nonexistent. One faces here a weightless economy in which non rival goods - that are named "knowledge-products" by Quah (1997, 2001) or "information goods" by Scotchmer (2005) - do have a price. This price is positive (even though the marginal cost is zero) because these knowledge good can be directly protected by patents (several examples can be found in Chantrel, Grimaud & Tournemaine, 2012). Introducing these new sectors in a endogenous growth model with vertical innovations is left for future research; in any case, we are convinced that the Lindahl equilibrium is the proper benchmark since it provides the first-best prices.

6 Appendix

6.1 Law of Knowledge Accumulation

6.1.1 Proof of Lemma 1

Consider any given sector ω, $\omega \in \Omega$, and a time interval $(t, t+\Delta t)$. At date t, the knowledge in this sector is $\chi_{\omega t}$. Let k, $k \in \mathbb{N}$, be the number of innovations that occur during the interval $(t, t+\Delta t)$. Under Assumptions 1 and 2, the knowledge at date $t+\Delta t$, $\chi_{\omega\, t+\Delta t}$, is a random variable taking the values $\{\chi_{\omega t} + k\sigma\mathcal{P}_{\omega t}\}_{k\in\mathbb{N}}$ with associated probabilities $\left\{\frac{\left(\int_t^{t+\Delta t}\lambda l_{\omega u} du\right)^k}{k!} e^{-\int_t^{t+\Delta t}\lambda l_{\omega u} du}\right\}_{k\in\mathbb{N}}$. Accordingly, the expected level of knowledge at date $t+\Delta t$ is:

$$\mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] = \sum_{k=0}^{\infty} \frac{\left(\int_t^{t+\Delta t}\lambda l_{\omega u} du\right)^k}{k!} e^{-\int_t^{t+\Delta t}\lambda l_{\omega u} du}\left[\chi_{\omega t} + k\sigma\mathcal{P}_{\omega t}\right]$$

$$= \left[\chi_{\omega t}\sum_{k=0}^{\infty} \frac{\left(\int_t^{t+\Delta t}\lambda l_{\omega u} du\right)^k}{k!} + \sigma\mathcal{P}_{\omega t}\left(\int_t^{t+\Delta t}\lambda l_{\omega u} du\right)\sum_{k=1}^{\infty}\frac{\left(\int_t^{t+\Delta t}\lambda l_{\omega u} du\right)^{k-1}}{(k-1)!}\right] e^{-\int_t^{t+\Delta t}\lambda l_{\omega u} du}$$

The MacLaurin series $\sum_{k=0}^{K}\frac{\left(\int_t^{t+\Delta t}\lambda l_{\omega u} du\right)^k}{k!}$ converges to $e^{\int_t^{t+\Delta t}\lambda l_{\omega u} du}$ as $K \to \infty$. Thus, one gets:

$$\mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] = \left[\chi_{\omega t} e^{\int_t^{t+\Delta t}\lambda l_{\omega u} du} + \sigma\mathcal{P}_{\omega t}\left(\int_t^{t+\Delta t}\lambda l_{\omega u} du\right) e^{\int_t^{t+\Delta t}\lambda l_{\omega u} du}\right] e^{-\int_t^{t+\Delta t}\lambda l_{\omega u} du}$$

$$\Leftrightarrow \mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] = \chi_{\omega t} + \lambda\sigma\left(\int_t^{t+\Delta t} l_{\omega u} du\right)\mathcal{P}_{\omega t}$$

Let $\Lambda_{\omega u}$ denote a primitive of $l_{\omega u}$ with respect to the time variable u. Rewriting the previous expression, one exhibits the Newton's difference quotients of $\mathbb{E}\left[\chi_{\omega t}\right]$ and of $\Lambda_{\omega t}$:

$$\frac{\mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] - \chi_{\omega t}}{\Delta t} = \lambda\sigma\frac{\Lambda_{\omega t+\Delta t} - \Lambda_{\omega t}}{\Delta t}\mathcal{P}_{\omega t}$$

Finally, letting Δt tend to zero, one gets $\frac{\partial \mathbb{E}[\chi_{\omega t}]}{\partial t} \equiv \dot{\chi}_{\omega t} = \lambda\sigma l_{\omega t}\mathcal{P}_{\omega t}$. This proves that the expected knowledge in any sector ω, is a differentiable function of time. Its derivative gives the law of motion of the expected knowledge as given in Lemma 1, in which the expectation operator is dropped to simplify notations.

6.1.2 Particular Cases

The law of motion derived in Lemma 1 is quite general. Indeed, choosing particular specifications of the pools $\mathcal{P}_{\omega t}$, enables us to obtain several laws of knowledge accumulation commonly used in the fully endogenous growth Schumpeterian theory. We propose to classify the various models proposed in this literature in four main ranges according to the considered pools of knowledge (*i.e.* the considered types of knowledge spillovers).

No knowledge spillovers (neither inter nor intra-sectorial knowledge spillovers). In Barro & Sala-i-Martin (2003 - Ch. 6) or in Peretto (2007), for instance, the knowledge production technology uses final good only. In this extreme case, there are neither inter-sectorial nor intra-sectorial knowledge spillovers. A similar framework in which new knowledge is produced only with private inputs can also be considered using our formalization. Assume that $\mathcal{P}_{\omega t} = 1$; accordingly, one has $\dot{\chi}_{\omega t} = \lambda\sigma l_{\omega t}, \forall\omega \in \Omega$. In this case, the only input used in the production of knowledge is labor.

Only intra-sectorial knowledge spillovers (no inter-sectorial knowledge spillovers). In the models proposed by Grossman & Helpman (1991), Segerstrom (1998), Peretto (1999), Acemoglu (2009 - Ch. 14), or Aghion & Howitt (2009 - Ch. 4), among others, it is implicitly assumed that spillovers are only intra-sectorial (there are no spillovers across sectors): the pool of knowledge used in each sector comprises only the knowledge previously accumulated within this sector. This type of model can be obtained assuming that $\mathcal{P}_{\omega t} = \chi_{\omega t}, \forall \omega \in \Omega$. One gets the following knowledge production functions $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \chi_{\omega t}, \forall \omega \in \Omega$.

Knowledge spillovers depending on average knowledge. The models of Dinopoulos & Thompson (1998), Peretto (1998), Li (2003), among others, consider firm-specific knowledge production functions such that, as stated by Laincz & Peretto (2006), "spillovers depend on average knowledge". Surveying this literature, these authors formalize this assumption in equation (9) of their paper. One can equivalently refer to equations (7) and (9) in Jones (1999), to equations (13) and (14) in Dinopoulos & Sener (2007), to equation (5) in Ha & Howitt (2007), or to the framework used in Aghion & Howitt (2009 - Ch. 4). Using our notations, this normalization assumption gives the following knowledge production function $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \mathcal{P}_{\omega t}$, where $\mathcal{P}_{\omega t} = \int_{\Omega} \frac{\chi_{ht}}{N}\, dh, \forall \omega \in \Omega$. Here, the new knowledge produced in any given sector depends on a knowledge aggregator, which is the average knowledge within the whole economy.

This formalization has been introduced to remove the scale effect property while maintaining the endogenous ingredients of the seminal literature. Note however that it appears that the cases in which knowledge spillovers are only intra-sectorial and those in which they depend on average knowledge are closely related. Indeed, in both of these frameworks, there are no inter-sectorial knowledge spillovers: since one generally considers the symmetric case in which $\chi_{\omega t} = \chi_t, \forall \omega \in \Omega$ (e.g. Aghion & Howitt 1992, 1998 - Ch. 3; Peretto & Smulders 2002), one has $\mathcal{P}_{\omega t} = \frac{\chi_t}{N} \int_{\Omega} dh = \chi_t, \forall \omega \in \Omega$. Scale effects are canceled by removing inter-sectorial knowledge diffusion.[27]

Knowledge spillovers depending on the knowledge level of the frontier firms ("leading-edge technology"). In the models of Aghion & Howitt (1992), Young (1998), Howitt (1999), Segerstrom (2000), or Garner (2010), among others, the increase in knowledge consecutive to the occurrence of an innovation in sector ω at date t depends on the level of knowledge reached in the most advanced sector. This type of framework can be directly obtained from our formalization. Indeed, assuming $\mathcal{P}_{\omega t} = \chi_t^{max}$, where $\chi_t^{max} \equiv \max\{\chi_{\omega t}, \omega \in \Omega\}$, one gets $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \chi_t^{max}$, $\forall \omega \in \Omega$.

Global knowledge spillovers. A last range of models assumes that knowledge spillovers are global: each sector uses the whole disposable knowledge in the economy, that is $\mathcal{P}_{\omega t} = \int_{\Omega} \chi_{ht}\, dh = \mathcal{K}_t$. Accordingly, one gets the following knowledge production function:

$$\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \mathcal{K}_t, \forall \omega \in \Omega \tag{23}$$

Comments on the law of knowledge accumulation (23) are given in Section 2 (see 2.1.2). In particular, we show how it relates to the ones originally introduced in the seminal papers of Romer (1990) and Aghion & Howitt (1992) (see the comments of the corollary to Proposition 1).

6.2 First-Best Social Optimum - Proof of Proposition 2

The social planner maximizes the representative household's discounted utility (3) subject to (2), (1), (4), (5), (6) and (7). The maximisation program can be written as follows:

$$\begin{array}{l} \text{Max} \quad \mathrm{U}{=}\int_0^{\infty} \ln(c_t) e^{-\rho t} dt \text{ subject to} \\ \{\mathrm{c}_t\}_{t\in[0,\infty[} \\ \{\mathrm{L}_t^Y\}_{t\in[0,\infty[} \\ \{\mathrm{l}_{\omega t}\}_{t\in[0,\infty[,\, \omega\in\Omega} \\ \{x_{\omega t}\}_{t\in[0,\infty[, \omega\in\Omega} \end{array}
\begin{cases} \mathcal{K}_t = \int_{\Omega} \chi_{\omega t}\, d\omega \\ \dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \mathcal{P}_{\omega t} \ , \ \omega \in \Omega \\ \mathcal{P}_{\omega t} = \int_{\Omega_\omega} \chi_{ht}\, dh \ , \ \forall \omega \in \Omega \\ L = L_t^Y + \int_{\Omega} l_{\omega t}\, d\omega \\ Y_t = (L_t^Y)^{1-\alpha} \int_{\Omega} \chi_{\omega t} (x_{\omega t})^{\alpha} d\omega \\ x_{\omega t} = \frac{y_{\omega t}}{\chi_{\omega t}}, \ \omega \in \Omega \\ Y_t = Lc_t + \int_{\Omega} y_{\omega t} d\omega \end{cases}$$

where c_t, L_t^Y, $l_{\omega t}$ and $x_{\omega t}$, $\omega \in \Omega$, are the control variables, and $\chi_{\omega t}$, $\omega \in \Omega$, the continuum of state variables of the dynamic optimization problem.[28] We denote by respectively by $\iota_{\omega t}, \omega \in \Omega$, ν_t and μ_t,

[27] The link between the scale effect property and knowledge diffusion is analysed in detail in *Inter-sectorial Knowledge Diffusion and Scale Effects* by Gray & Grimaud (2013, working paper).

[28] Accordingly, note that the constraint relative to the law of motion of knowledge is in fact a continuum of constraints.

the co-state variables associated to the continuum of state variables, to the labor constraint, and to the final good resource constraint. After some rearrangement, one can write the Hamiltonian as:

$$\mathcal{H} = \ln(c_t)e^{-\rho t} + \mu_t \left[(L_t^Y)^{1-\alpha} \int_\Omega \chi_{\omega t}(x_{\omega t})^\alpha d\omega - Lc_t - \int_\Omega \chi_{\omega t} x_{\omega t} d\omega \right] + \nu_t \left[L - L_t^Y - \int_\Omega l_{\omega t} d\omega \right] + \int_\Omega \iota_{\omega t} \left[\lambda\sigma l_{\omega t} \int_{\Omega_\omega} \chi_{ht} dh \right] d\omega$$

The first-order conditions $\frac{\partial \mathcal{H}}{\partial c_t} = 0$, $\frac{\partial \mathcal{H}}{\partial L_t^Y} = 0$, $\frac{\partial \mathcal{H}}{\partial l_{it}} = 0$ $(i \in \Omega)$, $\frac{\partial \mathcal{H}}{\partial x_{it}} = 0$ $(i \in \Omega)$ and $\frac{\partial \mathcal{H}}{\partial \chi_{it}} = -\dot{\iota}_{it}$ $(i \in \Omega)$ respectively yield:[29]

$$c_t^{-1} e^{-\rho t} = \mu_t L \tag{24}$$

$$\mu_t(1-\alpha)\frac{Y_t}{L_t^Y} = \nu_t \tag{25}$$

$$\iota_{it}\lambda\sigma \int_{\Omega_\omega} \chi_{ht} dh = \nu_t, \forall i \in \Omega \tag{26}$$

$$\mu_t \left[\alpha (L_t^Y)^{1-\alpha} \chi_{it}(x_{it})^{\alpha-1} - \chi_{it} \right] = 0\ , \forall i \in \Omega \tag{27}$$

$$\mu_t \left[(L_t^Y)^{1-\alpha}(x_{it})^\alpha - x_{it} \right] + \lambda\sigma \int_{\Omega_i} \iota_{ht} l_{ht} dh = -\dot{\iota}_{it}, \forall i \in \Omega \tag{28}$$

From (27), one gets:

$$x_{it} = x_t = \alpha^{\frac{1}{1-\alpha}} L_t^Y\ ,\ \forall i \in \Omega \tag{29}$$

Plugging (29) in (5), and using the definition of the whole disposable knowledge in the economy (given by (2)), one gets:

$$Y_t = \alpha^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{K}_t, \text{ and thus } g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t} \tag{30}$$

Moreover, plugging (29) in the final good resource constraint, (7) becomes $Y_t = Lc_t + \alpha^{\frac{1}{1-\alpha}} L_t^Y \mathcal{K}_t$. Dividing both sides of this expression by Y_t and using the previous expressions of x_t and Y_t (respectively given in (29) and (30)), one obtains $Lc_t/Y_t = 1-\alpha$, yielding:

$$g_{c_t} = g_{Y_t} \tag{31}$$

Finally, the first-order conditions (25) and (28) become respectively

$$\mu_t(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t = \nu_t \quad \text{and} \quad \frac{\mu_t}{\iota_{it}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} L_t^Y + \lambda\sigma \int_{\Omega_i} \frac{\iota_{ht}}{\iota_{it}} l_{ht} dh = -g_{\iota_{it}}, \forall i \in \Omega \tag{32}$$

We now consider the usual symmetric case in which $l_{\omega t} = l_t$ and $\chi_{\omega t} = \chi_t$ $\forall \omega \in \Omega$. Accordingly, one has $\mathcal{P}_{\omega t} = \mathcal{P}_t = \theta\chi_t, \forall \omega \in \Omega$, and thus the following expression of the growth rate of the stocks of knowledge:

$$g_{\mathcal{K}_t} = g_{\chi_t} = \frac{\dot{\chi}_t}{\chi_t} = \lambda\sigma\theta l_t \tag{33}$$

Moreover, (26) becomes $\iota_{it}\lambda\sigma\theta\mathcal{K}_t/N = \nu_t, \forall i \in \Omega$. Hence, one has $\iota_{it} = \iota_t$, $\forall i \in \Omega$. Using (32) and the labor resource constraint, one gets $\frac{\mu_t}{\iota_t} = \frac{\lambda\sigma\theta}{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} N}$, and thus:

$$g_{\mu_t} = g_{\iota_t} = -\left(\frac{\lambda\sigma\theta L_t^Y}{N} + \lambda\sigma\theta l_t \right) = -\frac{\lambda\sigma\theta L}{N} = -\frac{\lambda\sigma\theta}{\gamma} \tag{34}$$

Furthermore, the labor constraint (4) is now:

$$L_t^Y + Nl_t = L \Leftrightarrow L_t^Y + \gamma L l_t = L \Leftrightarrow l_t = \frac{1}{\gamma}\left(1 - \frac{L_t^Y}{L}\right) \tag{35}$$

Finally, log-differentiating (24) gives $g_{c_t} + \rho = -g_{\mu_t}$; using (34) allows us to derive the optimal growth rate of per-capita consumption:

$$g_c^o = \frac{\lambda\sigma\theta}{\gamma} - \rho \tag{36}$$

[29] Plus the usual transversality conditions.

The social optimum is completely characterized by (29), (30), (31), (33), (35) and (36); and therefore by the following system of equations (the superscript "^{o}" is used for "social optimum"):

$$\begin{cases} g_c^o = \frac{\lambda\sigma\theta}{\gamma} - \rho & (l1) \\ g_{c_t}^o = g_{Y_t}^o = g_{L_t^Y}^o + g_{\mathcal{K}_t}^o = g_{L_t^Y}^o + \lambda\sigma\theta l_t^o & (l2) \\ l_t^o = \frac{1}{\gamma}\left(1 - L_t^{Yo}/L\right) & (l3) \\ x_t^o = \alpha^{\frac{1}{1-\alpha}} L_t^{Yo} & (l4) \end{cases} \tag{37}$$

From ($l1$), ($l2$) and ($l3$) one gets:

$$\frac{\lambda\sigma\theta}{\gamma} - \rho = g_{L_t^Y}^o + \lambda\sigma\theta\, l_t^o \Leftrightarrow g_{L_t^Y}^o = \frac{\lambda\sigma\theta L_t^{Yo}}{\gamma L} - \rho$$

In order to solve for L_t^{Yo}, we use a variable substitution. Let $X_t = 1/L_t^{Yo}$; one gets the following first-order linear differential equation: $\dot{X}_t - \rho X_t = -\lambda\sigma\theta/N$. Its solution is

$$X_t = \left(X_0 - \frac{\lambda\sigma\theta}{\rho N}\right)e^{\rho t} + \frac{\lambda\sigma\theta}{\rho N} \Leftrightarrow L_t^{Yo} = \frac{1}{\left(\frac{1}{L_0^{Yo}} - \frac{\lambda\sigma\theta}{\rho N}\right)e^{\rho t} + \frac{\lambda\sigma\theta}{\rho N}}$$

Using the transversality condition, it can be shown that L_t^{Yo} immediately jumps to its steady-state level $L^{Yo^{ss}} = \rho N/\lambda\sigma\theta$. The transversality condition is only satisfied when $L_t^{Yo} = L^{Yo^{ss}}, \forall t$. Thus, one has $g_{L_t^Y}^o = 0$.

Finally, replacing L_t^{Yo} in the system of equations (37) and using the assumption $N = \gamma L$, one obtains the characterization of the social optimum as exhibited in Proposition 2.

6.3 Schumpeterian equilibrium *à la* Aghion & Howitt (1992) and private value of innovations - Proof of Propositions 3 and 4

In this subsection, we provide the detailed analysis of a decentralised economy *à la* Aghion & Howitt (1992), we fully characterize the set of equilibria as functions of the public tools vector (ψ, φ), and we compute the private value of innovations. As stated in Definition 2, at each vector (ψ, φ) is associated a particular Schumpeterian equilibrium, which consists of time paths of set of prices

$$\left\{\left(w_t(\psi,\varphi), r_t(\psi,\varphi), \{q_{\omega t}(\psi,\varphi)\}_{\omega\in\Omega}\right)\right\}_{t=0}^{\infty}$$

and of quantities

$$\left\{\left(c_t(\psi,\varphi), Y_t(\psi,\varphi), \{x_{\omega t}(\psi,\varphi)\}_{\omega\in\Omega}, L_t^Y(\psi,\varphi), \{l_{\omega t}(\psi,\varphi)\}_{\omega\in\Omega}, \{\chi_{\omega t}(\psi,\varphi)\}_{\omega\in\Omega}\right)\right\}_{t=0}^{\infty}$$

such that: the representative household maximizes her utility; firms maximize their profits; the final good market, the financial market and the labor market are perfectly competitive and clear; on each intermediate good market, the innovator is granted a patent and monopolizes the production and sale until replaced by the next innovator; and there is free entry on each R&D activity (*i.e.* the zero profit condition holds for each R&D activity). For all the computations, we drop the (ψ, φ) part on the variables to simplify notations.

The representative household maximizes her intertemporal utility given by (3) subject to her budget constraint, $\dot{b}_t = w_t + r_t b_t - c_t - T_t/L$, where b_t denotes the per capita financial asset and T_t is a lump-sum tax charged by the government in order to finance public policies. This yields the usual Keynes-Ramsey condition:

$$r_t = g_{c_t} + \rho \tag{38}$$

In the final sector, the competitive firm maximizes its profit given by $\pi_t^Y = (L_t^Y)^{1-\alpha}\int_\Omega \chi_{\omega t}(x_{\omega t})^\alpha d\omega - w_t L_t^Y - \int_\Omega (1-\psi) q_{\omega t} x_{\omega t} d\omega$. The first-order conditions yield

$$w_t = (1-\alpha)\frac{Y_t}{L_t^Y} \quad \text{and} \quad q_{\omega t} = \frac{\alpha (L_t^Y)^{(1-\alpha)}\chi_{\omega t}(x_{\omega t})^{\alpha-1}}{1-\psi}\ , \ \forall\omega\in\Omega \tag{39}$$

Consider any sector ω, $\omega \in \Omega$. Given the governmental intervention on behalf of R&D activities, the incumbent innovator, having successfully innovated at date t, receives, at any date $\tau > t$, the net profit

$\pi_\tau^{x_\omega} = (1+\varphi)\left(q_{\omega t} x_{\omega t} - y_{\omega t}\right)$ with probability $e^{-\int_t^\tau \lambda l_{\omega u} du}$. Differentiating (9) with respect to time gives the standard arbitrage condition in each R&D activity ω:

$$r_t + \lambda l_{\omega t} = \frac{\dot{\Pi}^x_{\omega t}}{\Pi^x_{\omega t}} + \frac{\pi_t^{x_\omega}}{\Pi^x_{\omega t}} \ , \ \forall \omega \in \Omega \tag{40}$$

Besides, the incumbent monopoly in each intermediate good sector ω, $\omega \in \Omega$, maximizes the instantaneous profit $\pi_t^{x_\omega}$, where the demand for intermediate good ω, $x_{\omega t}$, is given by (39). After maximization, one obtains the usual symmetric use of intermediate goods in the final good production and mark-up on the price of intermediate goods:

$$x_{\omega t} = x_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y \quad \text{and} \quad q_{\omega t} = \frac{\chi_{\omega t}}{\alpha} \quad , \ \forall \omega \in \Omega \tag{41}$$

Together with the definition of the whole disposable knowledge in the economy (2), (41) allows us to rewrite the final good production function (5) and the wage expression given in (39) respectively as

$$Y_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{K}_t \text{ and } w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t \tag{42}$$

The free entry condition condition in any R&D activity ω is given by (10). One has $w_t = \lambda \Pi^x_{\omega t}$, where $\lambda \Pi^x_{\omega t}$ is the expected revenue when one unit of labor is invested in R&D (from Assumption 1), and w_t is the cost of one unit of labor (given in (42) above). This condition gives the private value of an innovation in sector ω at date t, as defined in (9):

$$\Pi^x_{\omega t} = \Pi^x_t = \frac{(1-\alpha)}{\lambda}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t, \forall \omega \in \Omega \tag{43}$$

Consequently, one has $\dot{\Pi}^x_{\omega t}/\Pi^x_{\omega t} = g_{\mathcal{K}_t}$ and $\pi_t^{x_\omega}/\Pi^x_{\omega t} = \frac{(1+\varphi)\lambda\alpha\chi_{\omega t} L_t^Y}{(1-\psi)\mathcal{K}_t}$, $\forall \omega \in \Omega$. Replacing in (40), one rewrite the arbitrage condition as follows:

$$r_t + \lambda l_{\omega t} = g_{\mathcal{K}_t} + \frac{(1+\varphi)\lambda\alpha L_t^Y \chi_{\omega t}}{(1-\psi)\mathcal{K}_t} \ , \ \forall \omega \in \Omega \tag{44}$$

Log-differentiating with respect to time the expression of the final good production function given in (42), gives:

$$g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t} \tag{45}$$

Furthermore, using (2), (6), and (41), the final good resource constraint (7) can be rewritten as $Y_t = Lc_t + \left[\alpha^2/(1-\psi)\right]^{\frac{1}{1-\alpha}} L_t^Y \mathcal{K}_t$. Dividing both sides by Y_t and using the expression of Y_t given in (42), one gets $Lc_t/Y_t = 1 - \alpha^2/(1-\psi)$. Log-differentiating this expression gives:

$$g_{Y_t} = g_{c_t} \tag{46}$$

As usually in the standard literature, we focus on the symmetric equilibrium, in which $l_{\omega t} = l_t$ and $\chi_{\omega t} = \chi_t$, $\forall \omega \in \Omega$.[30] Consequently, one has $\mathcal{K}_t = N\chi_t$. Hence, the growth rate of the whole disposable knowledge is $g_{\mathcal{K}_t} = g_{\chi_t}$. Moreover, the pools of knowledge and the laws of accumulation of knowledge in each sector ω are respectively given by $\mathcal{P}_{\omega t} = \mathcal{P}_t = \theta\chi_t$ and $\dot{\chi}_{\omega t} = \dot{\chi}_t = \lambda\sigma\theta\, l_t \chi_t$. Therefore, one has:

$$g_{\chi_{\omega t}} = g_{\chi_t} = g_{\mathcal{K}_t} = \lambda\sigma\theta\, l_t \ , \ \forall \omega \in \Omega \tag{47}$$

Finally, we can rewrite (44), the arbitrage condition in any R&D activity ω, $\omega \in \Omega$, as:

$$r_t + \lambda l_t = \lambda\sigma\theta l_t + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)N} \tag{48}$$

[30]See, for instance, Aghion & Howitt (1992, 1998 - Ch. 3) or Peretto & Smulders (2002). Besides, the relevancy of the symmetric equilibrium is discussed in details in Peretto (1998, 1999), or in Cozzi, Giordani & Zamparelli (2007).

The equilibrium quantities, growth rates and prices are characterized by equations (4), (38), (41), (42), (45), (46), (47) and (48):

$$\begin{cases} L_t = L_t^Y + Nl_t \\ r_t = g_{c_t} + \rho \\ x_{\omega t} = x_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y \quad \text{and} \quad q_{\omega t} = \frac{\chi_{\omega t}}{\alpha} \quad , \forall \omega \in \Omega_t \\ Y_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{K}_t \text{ and } w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t \\ g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t} \\ g_{Y_t} = g_{c_t} \\ g_{\chi_{\omega t}} = g_{\chi_t} = g_{\mathcal{K}_t} = \lambda\sigma\theta l_t \ , \ \forall \omega \in \Omega \\ r_t + \lambda l_t = \lambda\sigma\theta l_t + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)N} \end{cases} \tag{49}$$

From (38) and (48), one gets $g_{c_t} + \rho + \lambda l_t = \lambda\sigma\theta l_t + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)N}$; and from (45), (46) and (47), one gets $g_{c_t} = g_{Y_t} = g_{L_t^Y} + g_{\chi_t} = g_{L_t^Y} + \lambda\sigma\theta\, l_t$. Combining these two expressions, and using the labor constraint (4) and the assumption $N = \gamma L$ gives the following differential equation in L_t^Y.

$$g_{L_t^Y} - \frac{\lambda}{\gamma L}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right] L_t^Y = -\left(\frac{\lambda}{\gamma} + \rho\right) \tag{50}$$

Now, we use the following variable substitution: $X_t = 1/L_t^Y$. Log-differentiation with respect to time writes $g_{X_t} = -g_{L_t^Y}$. Substituting into (50) gives

$$-\, g_{X_t} - \frac{\lambda}{\gamma L}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\frac{1}{X_t} = -\left(\frac{\lambda}{\gamma} + \rho\right)$$

$$\Leftrightarrow \dot{X}_t - \left(\frac{\lambda}{\gamma} + \rho\right) X_t = -\frac{\lambda}{\gamma L}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]$$

The solution of this first-order linear differential equation is

$$X_t = e^{\left(\frac{\lambda}{\gamma}+\rho\right)t}\left(X_0 - \frac{1}{\frac{\lambda}{\gamma}+\rho}\frac{\lambda}{\gamma L}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\right) + \frac{1}{\frac{\lambda}{\gamma}+\rho}\frac{\lambda}{\gamma L}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]$$

Accordingly, one gets:

$$L_t^Y = \frac{1}{e^{\left(\frac{\lambda}{\gamma}+\rho\right)t}\left(\frac{1}{L_0^Y} - \frac{1}{\frac{\lambda}{\gamma}+\rho}\frac{\lambda}{\gamma L}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\right) + \frac{1}{\frac{\lambda}{\gamma}+\rho}\frac{\lambda}{\gamma L}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]}$$

Using the transversality condition in the program of the representative household, we can show that L_t^Y immediately jumps to its steady-state level $L^{Y^{ss}} = \left(\frac{\lambda}{\gamma} + \rho\right) / \left(\frac{\lambda}{\gamma L}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\right)$. The transversality condition is only satisfied when $L_t^Y = L_0^Y = \gamma L\left(\frac{\lambda}{\gamma} + \rho\right) / \left(\lambda\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\right), \forall t$. Thus, one has $g_{L_t^Y} = 0$.

Substituting into the system (49), one proves Propositions 3 and 4 in which we provide the complete characterization of the decentralized Schumpeterian equilibrium, and the private value of an innovation, respectively.

6.4 Lindahl equilibrium and social value of innovations - Proof of Propositions 5 and 6

The representative household maximizes her intertemporal utility given by (3) subject to her budget constraint, $\dot{b}_t = w_t + r_t b_t - c_t - T_t/L$, where b_t denotes the per capita financial asset and T_t is a lump-sum tax charged by the government in order to finance public policies and fund R&D expenditures. This yields the usual Keynes-Ramsey condition:

$$r_t = g_{c_t} + \rho \tag{51}$$

Differentiating (12) with respect to time gives an arbitrage condition stating that the rate of return is the same on the financial market and on any R&D investment:

$$r_t = \frac{v_{\omega t}}{V_{\omega t}} + \frac{\dot{V}_{\omega t}}{V_{\omega t}}, \forall \omega \in \Omega \tag{52}$$

As usual in the standard literature, we focus on a symmetric equilibrium in which $l_{\omega t} = l_t$ and $\chi_{\omega t} = \chi_t$, $\forall \omega \in \Omega$.[31] Consequently, since $N = \gamma L$, one has $\mathcal{K}_t = N\chi_t = \gamma L \chi_t$ and $L = L_t^Y + \gamma L l_t$. The pools of knowledge and the laws of accumulation of knowledge in each sector ω are now respectively given by $\mathcal{P}_{\omega t} = \mathcal{P}_t = \theta \chi_t$ and $\dot{\chi}_{\omega t} = \dot{\chi}_t = \lambda\sigma\theta l_t \chi_t$. Therefore, one has

$$g_{\chi_{\omega t}} = g_{\chi_t} = g_{\mathcal{K}_t} = \lambda\sigma\theta l_t, \forall \omega \in \Omega \tag{53}$$

Using (2), (6), and (15), the final good production function (5), the wage (13), and the final good resource constraint (7) can be rewritten respectively as

$$Y_t = \alpha^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{K}_t,\ w_t = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t, \text{and } Y_t = Lc_t + \alpha^{\frac{1}{1-\alpha}} L_t^Y \mathcal{K}_t \tag{54}$$

Dividing both sides of the expression of the final good resource constraint given in (54) by Y_t gives $Lc_t/Y_t = 1-\alpha$. Log-differentiating with respect to time this expression as well as the final good production function given in (54), one gets:

$$g_{c_t} = g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t} \tag{55}$$

From (17) and (54), one gets the following social value of one unit of knowledge χ_{ht}:

$$V_{ht} = V_t = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\gamma L}{\lambda\sigma\theta}, \forall h \in \Omega \tag{56}$$

Using (15), (56), the marginal profitabilities of knowledge given in Lemma 3 can be rewritten as:

$$v_{\omega t}^Y = \alpha^{\frac{\alpha}{1-\alpha}} L_t^Y,\ v_{\omega t}^x = -\alpha^{\frac{1}{1-\alpha}} L_t^Y \text{ and } v_{\omega t}^{\chi_h} = \begin{cases} \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(L - L_t^Y\right)}{\theta}, \text{if } h \in \Omega_\omega \\ 0, \text{if } h \notin \Omega_\omega \end{cases}, \forall \omega \in \Omega$$

Accordingly, the instantaneous social value of one unit of knowledge $\chi_{\omega t}$, at date t, is:

$$v_{\omega t} = v_{\omega t}^Y + v_{\omega t}^x + \int_\Omega v_{\omega t}^{\chi_h} dh = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} L, \forall \omega \in \Omega \tag{57}$$

From (56) and (57), one has $\dot{V}_{\omega t}/V_{\omega t} = 0$ and $v_{\omega t}/V_{\omega t} = \lambda\sigma\theta/\gamma, \forall \omega \in \Omega$. Thus, from the arbitrage condition (52), one obtains the equilibrium interest rate:

$$r_t = \frac{\lambda\sigma\theta}{\gamma} \tag{58}$$

The repartition of labor and the growth rates are characterized by (4), (51), (53), (55) and (58). From (51), (55) and (58), one gets $g_{c_t} = g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t} = \lambda\sigma\theta/\gamma - \rho$; and from (4) and (53), one gets $g_{\mathcal{K}_t} = g_{\chi_t} = \lambda\sigma\theta\left(1/\gamma - L_t^Y/\gamma L\right)$. From these two expressions, one obtains $g_{L_t^Y} - \frac{\lambda\sigma\theta}{\gamma L} L_t^Y = -\rho$. Using the variable substitution, $X_t = 1/L_t^Y$, one gets a first-order linear differential equation:

$$g_{X_t} + \frac{\lambda\sigma\theta}{\gamma L}\frac{1}{X_t} = \rho \Leftrightarrow \dot{X}_t - \rho X_t = -\frac{\lambda\sigma\theta}{\gamma L}$$

Its solution is $X_t = e^{\rho t}\left(X_0 - \frac{\lambda\sigma\theta}{\rho\gamma L}\right) + \frac{\lambda\sigma\theta}{\rho\gamma L}$. Hence, one obtains

$$L_t^Y = \frac{1}{e^{\rho t}\left(\frac{1}{L_0^Y} - \frac{\lambda\sigma\theta}{\rho\gamma L}\right) + \frac{\lambda\sigma\theta}{\rho\gamma L}}$$

Using the transversality condition of the program of the representative household, it can be shown that L_t^Y immediately jumps to its steady-state level $L^{Y^{ss}} = \rho\gamma L/\lambda\sigma\theta$. The transversality condition is only satisfied when $L_t^Y = L_0^Y, \forall t$. Hence, one has $L_t^Y = \rho\gamma L/\lambda\sigma\theta = L^{Y o}, \forall t$, and thus $g_{L_t^Y}^o = 0$.

[31] See footnote 30.

Thus, the repartition of labor, the quantities of intermediate good, the growth rates, and the quantities of knowledge are

$$L_t^Y = L^{Yo} = \frac{\rho\gamma L}{\lambda\sigma\theta}; l_{\omega t} = l^o = \frac{1}{\gamma} - \frac{\rho}{\lambda\sigma\theta}, \forall\omega \in \Omega;$$
$$x_{\omega t} = x^o = \alpha^{\frac{1}{1-\alpha}} \frac{\rho\gamma L}{\lambda\sigma\theta}, \forall\omega \in \Omega;$$
$$g_{c_t} = g_{Y_t} = g_{\mathcal{K}_t} = g_{\chi_{\omega t}} = g^o = \frac{\lambda\sigma\theta}{\gamma} - \rho, \forall\omega \in \Omega;$$
$$\chi_{\omega t} = \chi_t^o = \frac{\mathcal{K}_t^o}{\gamma L}, \forall\omega \in \Omega; \text{ and } \mathcal{K}_t^o = e^{g^o t} \tag{59}$$

This proves that the quantities and growth rates computed in the Lindahl equilibrium are indeed those of the first-best social optimum. The system of prices is as follows.

- The prices of rival goods are $w_t^o = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t^o$; $r_t^o = \frac{\lambda\sigma\theta}{\gamma}$; $q_{\omega t}^o = q_t^o = \chi_t^o = \frac{\mathcal{K}_t^o}{\gamma L}, \forall\omega \in \Omega$.

- Regarding the pricing of knowledge, one has the following results.

    - The personalized prices (Lindahl prices) of one unit of knowledge $\chi_{\omega t}$ for the final good sector, the intermediate sector ω, and R&D sector $h, h \in \Omega$, are $v_{\omega t}^{Yo} = \alpha^{\frac{\alpha}{1-\alpha}} \frac{\rho\gamma L}{\lambda\sigma\theta}, \forall\omega \in \Omega$; $v_{\omega t}^{xo} = -\alpha^{\frac{1}{1-\alpha}} \frac{\rho\gamma L}{\lambda\sigma\theta}, \forall\omega \in \Omega$; and $v_{\omega t}^{\chi_h o} = \begin{cases} \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\theta}\left(L - \frac{\rho\gamma L}{\lambda\sigma\theta}\right), \text{if } h \in \Omega_\omega \\ 0, \text{if } h \notin \Omega_\omega \end{cases}, \forall\omega \in \Omega$.

    - The instantaneous income received by the producer of one unit of knowledge $\chi_{\omega t}$ is $v_{\omega t}^o = v_{\omega t}^{Yo} + v_{\omega t}^{xo} + v_{\omega t}^{R\&Do} = v^o = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} L, \forall\omega \in \Omega$, where $v_{\omega t}^{R\&Do} = \int_\Omega v_{\omega t}^{\chi_h o} dh = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(L - \frac{\rho\gamma L}{\lambda\sigma\theta}\right), \forall\omega \in \Omega$.

This proves Proposition 5. Finally, as seen in (12), an innovation consists in an increase in knowledge of $\Delta\chi_{\omega t} = \sigma\mathcal{P}_{\omega t}$ new units; moreover, from (56), the social value of one unit of knowledge $\chi_{\omega t}$ at date t is $V_{\omega t}^o = \int_t^\infty v_{\omega t}^o e^{-\int_t^s r_u du} ds = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\gamma L/\lambda\sigma\theta$. Hence, the social value of an innovation in any sector ω is $\mathcal{V}_{\omega t}^o = \sigma\mathcal{P}_{\omega t}^o V_{\omega t}^o$, where $\mathcal{P}_{\omega t}^o = \theta\chi_{\omega t}^o = \theta\mathcal{K}_t^o/\gamma L$. Finally, one gets:

$$\mathcal{V}_{\omega t}^o = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}{\lambda}\mathcal{K}_t^o \tag{60}$$

This proves Proposition 6.

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