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How to Design Infrastructure Contracts in a Warming World?

A Critical Appraisal of Public-Private Partnerships

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Abstract. We analyze how uncertainty regarding future climate conditions affects the design of concession contracts, organizational forms and technological choices in a principal-agent context with dynamic moral hazard, limited liability and irreversibility constraints. The prospect of future, uncertain productivity shocks on the returns on the firm?s effort creates an option value of delaying efforts which exacerbates agency costs. Contracts and organizational forms are drafted to control this cost of delegated flexibility. Our analysis is relevant for infrastructure sectors that are sensitive to changing weather conditions and sheds a pessimistic light on the relevance of Public-Private Partnerships in this context.

Keywords: Public-private partnerships, concession contracts, climate change, irreversibility, agency costs.

JEL Codes: D82, L32, Q54.

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1 Introduction

Climate change raises specific and unprecedented challenges for large investments in infrastructure. Transport, water and sewage, and energy networks are sectors typically characterized by significant sunk investments and technological choices that are locked in over several decades. Over such periods, changing weather conditions are expected, albeit unpredictable. Certainly, the deep current scientific uncertainty regarding future climate conditions, potentially coupled with an increasing likelihood of very large catastrophic events, makes long-term decisions in such sectors quite a challenge for practitioners.¹ Climate change might have significant implications for future investments and maintenance of existing assets in key sectors, which in turn might affect upfront investments both in size and quality.

Since they involve long term contracts, often covering twenty or thirty years, the socalled Public-Private Partnerships (PPPs) are particularly sensitive to climate change hazards. Over recent years, a number of countries, including the US as well as several European and emerging countries, have increasingly relied on PPPs to respond to their investments needs in sectors involving Long-Lived Capital Stock (LLKS).² Because it entails delegation to the private sector of key decisions over both the structure of initial investments and the subsequent management of assets over a long period, this contracting mode has also been viewed as an attractive response to important shortages in public funds. Beyond this public finance motivation, the efficiency gains of PPPs have also been repeatedly emphasized in contexts where relationships between public bodies and the private sector are plagued with agency costs, contract incompletenesses and transaction costs as pointed out by a burgeoning literature.³

Equipped with a model of long-term contracting tailored to the specificities that climate change brings to the agency relationship between public bodies and firms, this paper aims at analyzing the suitability of the standard PPP model in coping efficiently with climate change-related uncertainty. In a nutshell, we argue that long-term contracting is plagued with new agency costs of delegated flexibility that may be better controlled when parties wait until uncertainty on climate conditions is resolved to draft new arrangements. This offers thus a rather pessimistic view on the benefit of PPPs in that context.

¹Weitzman (2009) and Hallegate (2009).

²See for instance, the exhaustive evidence and discussion in Engel, Fischer and Galetovic (2008) and Estache and Iimi (2011), and the statistics reported by Engel, Fischer and Galetovic (2011) on the growing importance of PPPs in Europe and in the U.S., with a fivefold increase between 1998-2007 and 2008-2010.

³Benz, Grout and Hallonen (2001), Bennet and Iossa (2006), Hart (2003), Iossa and Martimort (2008), Martimort and Pouyet (2008), Engel, Fisher and Galetovic (2010), among many others.

Climate change and infrastructure investment. The current process of anthropogenic climate change will dramatically affect the environment in which long-term economic decisions are made. What makes this process peculiar is the large and growing uncertainty on future values of environmental parameters. Indeed, global scenarios about climate change include relatively large confidence intervals (IPCC, 2007). This makes it difficult to pinpoint more than broad probability distributions for future outcomes, and to rule out disastrous collapses.

Some evidence exists of a link between anthropogenic greenhouse gases (GHG) concentration and local extreme events, such as heat waves, floodings and precipitations.⁴ However, uncovering the exact channels and providing precise future projections appear to be beyond current scientific possibilities.⁵ As a result, climate risk assessments for specific businesses such as utilities are severely limited by the coarse spatial resolution of climate models and the ensuing lack of clear understanding of how global climate change translates at the local level (IFC, 2010).

Climate change related hazards are especially relevant for infrastructure on several fronts. First, the accelerating rate of climate change implies that long-lived investments will have to cope, during their lifetime, with a broader range of climatic conditions. Power plants typically last for at least 30 to 40 years; energy distribution networks and water and transportation infrastructures are built to last for periods of time in between 30 and 100 years.⁶ Such assets are thus likely to experience large variations in average temperature conditions, precipitations, etc., over their life cycle. This is particularly true for developing countries, where there are both large scale needs for infrastructure investment and it is widely expected that the impact of climate change will be stronger (World Development Report, 2010).

Second, the very nature of infrastructure investments implies a crucial sensitivity to climate hazards. Water collection and distribution networks, as well as hydroelectric power plants,⁷ are dependent on precipitations, rivers and glacial runoffs, drought and floods. Significant changes along these dimensions would imply major impacts on the availability of water for human consumption and irrigation and significant needs for adaptation of water management networks to deal with risks of scarcity and contamination among others.⁸ Similarly, physical infrastructures, such as roads and

⁴See Stott, Stone and Allen (2004), Kunreuther and Michel-Kerjan (2009), Pall et al. (2011), and Min, Zhang, Zwiers and Hegerl (2011).

⁵Stone and Allen (2005). Piao et al. (2010).

⁶Shalizi and Lecocq (2009), Hallegate (2009).

⁷See the case of the Brazilian Belo Monte dam, world's third largest projected dam. http://www.globalpost.com/dispatch/brazil/110203/drought-belo-monte-dam, last consulted Feb., 23rd, 2011.

⁸Piao et al. (2010); The Center for Health and the Global Environment (2005).

bridges, are critically sensitive to extreme temperatures: More intense heat waves will increase deterioration of traditional asphalt roads, while those in low-lying areas will require additional investments to be protected against floods. Energy plants and distribution networks efficiency is also affected by extreme temperatures, precisely when these conditions also generate demand peaks. For example, increasing reliance on nuclear power would imply greater needs and more difficulties to rely on water for cooling.⁹ Additionally, the process of rapid urbanization, especially in the developing world, leads to an ever greater concentration of investments and services, making equipments and networks even more sensitive to stress on environmental resources and to localized extreme events such as storms or floods.

Finally, the mounting pressures to introduce innovative technologies that mitigate the impact of infrastructure on climate change¹⁰ are also likely to significantly increase costs. For example, for the first time in mankind's history, the current change in energy technological mix away from fossil fuels implies a shift towards less efficient energy sources.¹¹ Indeed, leading renewables are characterized among others by lower energy density and greater intermittency, translating into higher costs. Similarly, water management systems face the challenge of shifting from purely mitigating technologies to ones that address the underlying causes of incident threats.¹² This complex technological evolution is clearly affected by the specific relationship between infrastructure investors, who are often private, and public decision-makers.

Overview of the model. We consider a two-period relationship between a public authority (the principal) and a firm (the agent) for the provision of a public service. Contracts are plagued with dynamic moral hazard and uncertainty on climate change is resolved over time. The firm exerts non-verifiable efforts (or investments) in each period of the relationship. Efforts are privately costly but yield, with some probability, some extra social value beyond a base level. The firm is protected by limited liability and must receive rents in each period to exert efforts.¹³

In contrast with standard dynamic agency models,¹⁴ which assume that effort only pays off in the current period, the first important specificity of our environment is the existence of an irreversibility constraint that links efforts at different points in time. More precisely, the second-period effort cannot be lower than the first-period one. Such linkage is indeed quite natural for infrastructure projects, whose development can of-

⁹World Development Report (2010).

¹⁰Shalizi and Lecocq (2009), Davis et al. (2010).

¹¹Kerr (2010).

¹²Vörösmarty et al. (2010).

¹³Laffont and Martimort (2002, Chapter 4).

¹⁴Rogerson (1985) and Olhendorf and Schmitz (2008) among many others.

ten be decomposed into different stages (the decomposition between "build and operate" in the vocable coined by PPPs practitioners). For instance, if a bigger infrastructure is chosen earlier on, more follow-up investments and maintenance are certainly needed later on. In between these two stages, uncertainty on climate shocks is resolved and affects the second-period return on the agent's effort.

Because of the initial uncertainty over future productivity shocks, there is an option value of waiting until such uncertainty gets resolved before undertaking any investment with long-term irreversible effects. This important lesson is well-known from the seminal works of Arrow and Fisher (1974), Henry (1974), Dixit and Pindyck (1994), and Kolstad (1996). In the context of our model, a high (resp. low) first-period effort makes it more (resp. less) likely that the irreversibility constraint binds. Incentives to keep flexibility call for reducing the first-period investment. Contrary to this earlier literature, which focused on the consequences of such irreversibility in a non-strategic context, we embedd these flexibility motives in an agency relationship.

The consequences are twofold. First, uncertainty on climate changes makes it impossible to write long-term contracts conditional on future climate contingencies. Delegation to the private sector takes place in a highly incomplete contracting environment. Second, the principal and the agent may disagree on how much flexibility to keep and, even if they agree under some circumstances, it may be at the cost for the principal of giving up more rent to the agent. The combination of irreversibility and ex ante uncertainty on future productivity shocks creates an option value of delaying first-period investment to keep flexibility in second-period effort. In an agency context, this exacerbates the difficulty of providing incentives in earlier phases of the project.

Considering different scenarios corresponding to various degrees of contractual incompleteness, our analysis unveils how the agency problem between the public sector and the private sector is actually exacerbated by the firm's incentives to underinvest in the earlier period so as to maintain flexibility for the future.

Overview of our findings. The very logic of the irreversibility literature immediately explains why the firm's incentives to keep flexibility for the second period dampens its first-period effort. In an agency context, those distorted incentives directly impact on the design of incentive schemes over both periods of the relationship since the principal wants to control of how much flexibility is delegated to the agent. This impact comes through two different channels. First, a *Commitment Effect* captures how a lower first-period effort affects agency costs not only in the first but also in the second period whenever the firm is actually constrained by its earlier commitment. Second, a *Flexibility Effect* measures the sensitivity of a second-period effort to incentives under all favorable circumstances where this earlier commitment is not binding. It turns out

that those two effects go in opposite directions.

To illustrate, consider the case of an agent who is less responsive to incentive payments at lower levels of effort. Technically speaking, this means that his effort supply is less elastic for lower rewards. To incentivize first-period effort despite the countervailing impact of the agent's incentives for keeping flexibility, the principal must thus raise significantly the first-period payment and shift more rent towards the agent in that period. The *Commitment Effect* then pushes the first-period reward ups. *A contrario*, since the firm's effort supply is more elastic for higher rewards, it becomes less attractive to raise second-period payments to incentivize the second-period effort under all favorable realizations of the productivity shock. The optimal contract entails decreasing incentives over time. By a reverse argument, increasing incentives arise when the effort supply is non-increasing.

Our analysis also unveils to what extent other organizational and technological choices have attractive properties in view of reducing the agency costs of delegated flexibility.

Echoing the theoretical literature on the costs and benefits of PPPs in agency contexts,¹⁵ we first show that a commitment to unbundle the different stages of the project between different firms prevents the perverse incentives for flexibility. Along the same lines, keeping contracts somewhat incomplete and short-term reaches an even better outcome even if a single firm is in charge at different points in time. With such shortterm contracting, the principal can delay future rounds of contracting until productivity shocks are known. This shifts the flexibility motives from the agent to the principal himself, reduces agency costs and definitively improves contractual performances.

Finally, we demonstrate that it might be optimal to adopt a more costly technology if it improves flexibility against environmental shocks. This highlights why the current process of increased environmental uncertainty may entail a shift towards more flexible, although possibly less efficient, technologies and suggests a possible substitutability between contractual and technological choices.¹⁶

Organization of the paper. Section 2 presents the model. Section 3 offers two polar benchmarks. In the first one, the principal can perfectly control efforts. We recast there standard results of the irreversibility literature in the framework of our model. The

¹⁵See, among others, Benz, Grout and Hallonen (2001), Hart (2003), Bennet and Iossa (2008), Martimort and Pouyet (2008), Iossa and Martimort (2008) for static models, Schmitz (2005) and Iossa and Martimort (2011) for dynamic ones.

¹⁶A fairly large literature has dealt with the issue of technological change in the presence of climate change, mostly in the context of growth models, as exemplified by the early work of Nordhaus (1994) and more recently by Acemoglu et al. (2012) work on directed technical change. However, to our knowledge none of these contributions have analyzed incentives for technological change in the presence of significant uncertainty on climate change.

second scenario addresses what happens with myopic players who do not care about the impact of their current decisions on the future. Section 4 analyzes the agency relationship between the government and the firm to whom it delegates the two-stage project. We attach particular attention to the agent's intertemporal incentive constraint and the new agency costs of delegated flexibility. Section 5 characterizes optimal contracts. This analysis stresses that optimal contracts result from a trade-off between a *Commitment Effect* and a *Flexibility Effect*. Section 6 highlights the role of various organizational choices that reduces those agency costs. Section 7 addresses technological choices. Section 8 offers a critical view of the PPP model in light of our findings. Finally, Section 9 concludes highlighting some avenues for future research. Proofs are relegated to an Appendix.

2 The Model

Technology. Consider a public-private partnership contract between a government (hereafter often referred to as the "principal") and a private firm (the "agent") for the provision of a public service (for instance energy, water, sanitation or transportation). Typically, PPP contracts may grant a concession for twenty to thirty years to the private sector. For the purpose of the model, we only consider two stages: an initial investment period at date 1 and a follow-up investment or maintenance period at date 2.

We denote by δ the common discount factor of these players.¹⁷ This parameter can also be viewed as an index of the length of the accounting period.

In the first stage, the agent invests in an infrastructure which basic social value is $S_0 > 0$. If the design turns out to be successful, this social value increases to $S_0 + S$, where S > 0. The project is successful with probability e_1 , where e_1 is the firm's effort in the first period. This effort may be viewed as a (size-related) investment that affects the project's social value (for instance the quality of a water or sanitation network, or the design of a transport system).

Exerting such effort has a cost $\psi(e_1)$ for the agent.¹⁸ Following the agency literature,¹⁹ the quantity $R(e) = e\psi'(e) - \psi(e)$ denotes the agent's *liability rent* when exerting effort *e*. Anticipating on what follows, this quantity is the amount that must be con-

¹⁷The *Stern report* (2007) suggests to take a social discount rate of 1.3 percent to reflect the compounding effects of future growth rate and marginal utility of consumption. Dasgupta (2007) discusses the ethical arguments behind this choice.

¹⁸For technical reasons, and unless stipulated otherwise, we assume that $\psi(0) = \psi'(0) = 0$, $\psi'(e) \ge 0$, $\psi''(e) > 0$, and $\psi'''(e) \ge 0$ and that $\psi(\cdot)$ is convex enough to ensure that optimal efforts are interior to the interval (0, 1) under all circumstances below. Note that the Inada condition $\psi'(1) = +\infty$ would be sufficient in this respect.

¹⁹See Laffont and Matimort (2002, Chapter 4) for instance.

ceded to the agent when agency costs undermine delegated management.²⁰ For further references, we also denote by $\varphi \equiv \psi'^{-1}$ the inverse function of ψ' . As we will see below, $\varphi(t)$ is the agent's effort supply when he receives a reward t in case of successful investment.²¹ Let then $\varepsilon(t) = \frac{t\varphi'(t)}{\varphi(t)}$ denote the supply elasticity. Much of our results below will depend on the monotonicity properties of this elasticity. One can verify that $\varepsilon(\cdot)$ is non-decreasing if and only if $\frac{e\psi''(e)}{\psi'(e)}$ is non-increasing.²² In other words, a greater elasticity of effort supply means that the marginal disutility of effort has itself a lower elasticity, which amounts to saying that the firm is less responsive to incentives as its effort increases.

In the second period of the relationship, the agent must perform some complementary investment to keep the infrastructure viable or exert some maintenance of existing assets. This investment is again successful and yields an extra return S, with some probability that will be specified below as depending on the agent's second-period effort e_2 , as well as on a productivity shock. Exerting this second-stage effort costs $\psi(e_2)$.

Irreversibility. We follow the standard approach of the irreversibility literature (Arrow and Fisher 1974, Henry 1974, and Dixit and Pindyck 1994, among others) and assume that the first-period effort affects the second-period production function: Once an infrastructure of a given size has been set up, the firm must ensure a minimum follow-up level of investment/maintenance. This irreversibility is thus captured through the following simple *intertemporal irreversibility constraint*:

$$e_2 \ge e_1.^{23}$$
 (1)

Uncertainty on Climate Shocks. Uncertain climate shocks affect the second-period probability of the project generating social value, which we write as θe_2 if second period effort is e_2 . The parameter θ is a productivity shock linked to climate change. This shock is distributed over a finite support $[0, \overline{\theta}]$ according to a cumulative distribution F with an everywhere positive and atomless density f = F'. Observe that, under the worst scenario (i.e., $\theta = 0$), the project no longer generates any value in the second

²⁰Our technical assumptions imply that R(0) = 0, $R'(e) = e\psi''(e) \ge 0$ and $R''(e) = e\psi'''(e) + \psi''(e) > 0$. ²¹It is immediate to check that $\varphi(\cdot)$ is increasing $(\varphi'(t) = \frac{1}{\psi''(\varphi(t))} > 0)$ and concave $(\varphi''(t) = \psi''(e) + \psi''(e) + \psi''(e) = 0$.

 $^{-\}frac{\psi^{\prime\prime\prime}(\varphi(t))}{\psi^{\prime\prime3}(\varphi(t))} \le 0).$

^{ψ} ^{($\psi(c)$)} ²²For instance, $\psi(e) = \lambda \frac{e^{1+\alpha}}{1+\alpha}$ (with $\alpha \ge 0$ and $\lambda > 0$) is such that $\dot{\varepsilon}(t) \equiv 0$, while $\psi(e) = \lambda \frac{exp(re)-1-re}{r}$ (with r > 0 and $\lambda > 0$) is such that $\dot{\varepsilon}(t) \le 0$, and $\psi(e) = \lambda \log(1+e^2)$ (with $\lambda > 0$) is such that $\dot{\varepsilon}(t) \ge 0$.

²³To interpret this constraint, one may think of a water or a road project, which long-term social value depends on realizing additional investments that are positively related to the initial sunk investment.

period. This captures the possibility that climate shocks may have a really detrimental impact on welfare.²⁴

Let $E_{\theta}(\cdot)$ denote the expectation operator with respect to θ . For simplicity, we assume that $E_{\theta}(\theta) = 1$. On "average", the probabilities of success in the first and second periods are identical if efforts at those dates are the same. In other words, agents believe that shocks follow no intertemporal trend. This simplifying assumption allows us to focus on the pure role of uncertainty in affecting first-period investment.

Indeed, date 2 realizations of the productivity shock θ are uncertain at date 1. However, this shock becomes common knowledge at the time of choosing second-period effort. As we will see below, the irreversibility constraint (1) is only binding following an adverse evolution of the environment, since it reduces the marginal return on second-period effort. This justifies reducing investment at an earlier stage even in the absence of any agency problem. Our analysis will unveil how incentives for flexibility are modified in an agency context.

Contracts. The firm's efforts in both periods are non-verifiable. The firm is protected by limited liability and cannot make losses in any period.²⁵ Indeed, profits in any period are redistributed as dividends to the firm's owners in the same period.²⁶ In lines with standard moral hazard problems under limited liability, incentives can thus only be provided by rewarding the firm in case the incremental social value *S* is realized.²⁷ Together, nonverifiability and limited liability create agency costs. A convenient design of dynamic contracts will limit the *liability rent* that accrues to the firm.

For most of the paper, the relationship between the government and the firm is run by a long-term contract that covers both periods. Although, the productivity shock θ is common knowledge at date 2 and ex post verifiable, the contingencies underlying the resulting θ cannot be foreseen ex ante. Thus, long-term contracts contingent on those shocks cannot be written and the only feasible contracts entail a pooling second-period payment independent of θ . This assumption captures the extreme incompleteness surrounding contracting when climate may evolve in unpredictable ways.

²⁴This simple formalization of climate shocks is in lines with the practical outcomes of even the most sophisticated modeling and simulation exercises currently available. Indeed, these exercises offer rough assessments of the impact of extreme events, such as probabilities of exceeding given risk thresholds (e.g., Pall et al., 2011).

²⁵Alternatively, the firm is not sufficiently diversified and can be viewed as being infinitely risk-averse below zero wealth; an assumption that may be relevant for ventures involved in large projects, which may have a constrained access to the financial market.

²⁶This simplifying assumption implies that the firm cannot build a buffer of wealth to relax its liability constraint in the future.

²⁷This assumption echoes real-world practices. PPP contracts often specify revenues sharing rules. We refer to Iossa and Martimort (2008, 2011) for an in-depth description.

Let denote by (t_1, t_2) such long-term contract, i.e., a profile of per-period rewards to the firm following good performances in periods 1 and 2 respectively.

Remark 1 Although contracts may depend on calendar time, history-dependent contracts are ruled out. In full generality, the second-period reward could depend on whether a success or a failure took place earlier on. With obvious notations, such history-dependent contract would write as $\{t_1, t_2(S), t_2(F)\}$. The role of history-dependent contracts in dynamic agency models is by now well known, especially since Rogerson (1985) in settings where agents are risk averse and, more recently, Olhendorf and Schmitz (2008) when risk-neutral agents are protected by limited liability as in our context. These papers show that using memory (and especially, setting up larger second-period rewards in case of earlier success, i.e., $t_2(S) > t_2(F)$) provides cheaper incentives. Ruling out history dependence allows us to focus on new issues that, in a full-fledged model, would superimpose to those well-known from this earlier literature.²⁸

Preferences. Up to some constant terms related to the basic social value of the infrastructure S_0 , the principal's intertemporal payoff can be written as:

$$V(t_1, t_2, e_1, e_2(\cdot)) = e_1(S - t_1) + \delta E_\theta(\theta e_2(\theta t_2, e_1))(S - t_2),$$
(2)

where the second-period effort $e_2(\theta t_2, e_1)$ depends a priori on the realized productivity shock, the second-period incentive reward t_2 and the earlier effort e_1 .²⁹

Taking into account that the firm controls investments in both periods, its intertemporal profit can be expressed as:

$$U(t_1, t_2) = \max_{e_1} \left\{ e_1 t_1 - \psi(e_1) + \delta E_\theta \left(\max_{e_2 \ge e_1} \theta e_2 t_2 - \psi(e_2) \right) \right\}.$$
 (3)

²⁹We simplify the analysis by giving zero weight to the firm's profit in the government's objectives. This is in particular relevant when foreign firms are involved in managing and building key infrastructures. Following Baron and Myerson (1982), our results would be robust if the firm's profit receives a non-negative weight $\alpha < 1$ in the government's objective function.

²⁸Several justifications can be given to rule out history-dependent contracts. First, governments in charge of implementing new infrastructures may have to rely on regulatory agencies and dedicated bureaucracies to obtain information on performances, a process that might not be immune to capture and manipulations. Stationary contracts are less sensitive to such manipulations and may be attractive in this respect. Second, history-dependent contracts require that the government be able to commit to delay rewards. In contexts with weak contractual enforcement, the government may renege on his commitment, and the firm may have little choice except to accept new contractual terms. (In a related context, Laffont and Tirole (1993, Chapter 8, p. 349) discuss the limited feasibility of delayed payments in dynamic contractual relationships.) Third, our model can be reinterpreted *mutatis mutandis* as if the efforts were observable but contracts were incomplete in the following sense: Suppose that e_i is no longer a probability but an input, whose social return is e_1S in the first period and θe_2S in the second. Those returns are no longer random. With that alternative interpretation in mind, the payments t_i/S can be viewed as a per unit price paid to the firm. Because returns are no longer random, there is only "one" past history.

This yields the following expression of the second-period effort $e_2(\theta t_2, e_1)$ as:

$$e_2(\theta t_2, e_1) = \arg \max_{e_2 \ge e_1} \theta e_2 t_2 - \psi(e_2)$$
 (4)

3 Useful Benchmarks

3.1 Optimal Flexibility without Agency Costs

As a benchmark, consider the hypothetical setting where the government invests in both periods by himself (or, more precisely controls the effort of a public enterprise).³⁰ Alternatively, this setting also corresponds to the case where efforts are verifiable and can be contracted upon ex ante. This section will thus recap basic results from the irreversibility literature to facilitate future comparisons.

Suppose that the principal knows θ before choosing his second-period effort. An effort plan $(e_1^i, e_2^i(\cdot))$ solves the following intertemporal problem:³¹

$$\max_{e_1} \left\{ e_1 S - \psi(e_1) + \delta E_\theta \left(\max_{e_2 \ge e_1} \theta e_2 S - \psi(e_2) \right) \right\}.$$

From there, we immediately obtain the expression of the second-period effort:

$$e_2(\theta S, e_1) = \max\{\varphi(\theta S), e_1\}$$

The second-period effort is constrained by the earlier commitment only if the productivity shock is sufficiently adverse, i.e., $\theta \leq \frac{\psi'(e_1)}{S}$. Then, the marginal return on second-period effort is too small to ensure that the irreversibility constraint is slack.

Inserting the expression of the second-period effort into the principal's intertemporal payoff, we obtain:

$$\max_{e_1} \left\{ e_1 S - \psi(e_1) + \delta\left(\int_0^{\frac{\psi'(e_1)}{S}} (\theta e_1 S - \psi(e_1)) f(\theta) d\theta + \int_{\frac{\psi'(e_1)}{S}}^{\overline{\theta}} R(\varphi(\theta S)) f(\theta) d\theta\right) \right\}.$$

Optimizing yields an optimal level of effort in the first-period e_1^i worth:

$$e_1^i = \varphi(\zeta(\delta, 1)S) < \varphi(S), \tag{5}$$

³⁰We follow Sappington and Stiglitz (1987) in assuming that such public firm has access to the same technology as the private sector.

³¹Where the superscript i is meant for "informed".

where $\zeta(\delta, 1)$ is the unique solution in (0, 1) to the equation

$$\zeta(\delta, 1) = 1 - \delta \int_0^{\zeta(\delta, 1)} F(\theta) d\theta.$$
(6)

Everything happens as if the value of the first-period investment was reduced to take into account the flexibility motives. Note that a myopic principal, i.e., one with $\delta = 0$, would choose an effort level $\varphi(S)$. To gain flexibility over a wider region of possible realizations of θ , a less myopic principal reduces his first-period investment below that amount. This delays effort provision till the productivity shock θ is known.

Observe that the threshold value of the productivity shock $\zeta(\delta, 1)$ below which an earlier commitment is binding (i.e., $e_2(\theta S, e_1) = e_1$ when $\theta \leq \zeta(\delta, 1)$) is below the mean:

$$\zeta(\delta, 1) < 1 = E_{\theta}(\theta).$$

This illustrates the principal's bias towards keeping flexibility even in the absence of any agency problem. Later on, we will see how this threshold changes when agency costs are explicitly taken into account.

Finally, observe that the second-period effort $e_2^i(\theta)$ is easily derived as:

$$e_2^i(\theta) = \varphi(\max\{\zeta(\delta, 1), \theta\}S)$$

Remark 2 As δ increases, $\zeta(\delta, 1)$ decreases and the less myopic principal underinvests more.³²

3.2 Myopic Players and Agency Costs

Consider the case of myopic players who care only about the first period, i.e., $\delta = 0$. In that context, flexibility motives are *de facto* irrelevant; neither the principal nor the agent anticipate an impact of their first-period choices on the future. The first-period relationship boils down to a static model where the firm receives a reward *t* in case of success. We first derive the optimal static contract in this environment.

Incentive compatibility leads to the following expression of effort at that date:

$$e_1 = \arg \max_{\tilde{e}_1} \tilde{e}_1 t - \psi(\tilde{e}_1) \Leftrightarrow e_1 = \varphi(t).$$

³²As δ decreases towards zero and the second contracting period matters less, $\zeta(\delta, 1)$ obviously converges to one. When the principal is myopic, his first-period effort is a constraint on his future choice only when productivity shocks are below the mean.

This allows us to express the principal's expected payoff as:

$$(S-t)\varphi(t).$$

The optimal stationary reward t(1) maximizes this expression, solving the familiar first-order condition:³³

$$S = t(1) + \frac{\varphi(t(1))}{\varphi'(t(1))}.$$
(7)

The principal faces a rent-efficiency trade-off. Increasing the reward boosts incentives and raises the probability of success but it also decreases the principal's net surplus.

Note that the optimal reward t(1) is closer to S as the elasticity of effort supply $\varepsilon(\cdot)$ is itself greater, i.e., when the marginal disutility of effort has a greater elasticity. In other words, as $\varepsilon(\cdot)$ increases, the rent-efficiency trade-off is further tilted towards leaving more rent to the agent.

Consider now the second period and suppose, as a benchmark, that there is no irreversibility constraint. The probability of success is now affected by a common knowledge shock θ so that it becomes θe . We are here interested in how the agent's optimal reward varies with the productivity shock.

Incentive compatibility now becomes:

$$e_2 = \arg \max_{\tilde{e}_2} \theta \tilde{e}_2 t - \psi(\tilde{e}_2) \Leftrightarrow e_2 = \varphi(\theta t).$$

Equation (7) is modified accordingly as:

$$S = t(\theta) + \frac{\varphi(\theta t(\theta))}{\theta \varphi'(\theta t(\theta))}.$$
(8)

Consider now two productivity shocks, $\theta \ge \theta'$. Observe that $t(\theta) \ge t(\theta')$ (resp. \le) if, for any t, $\frac{\theta t \varphi'(\theta t)}{\varphi(\theta t)} \ge \frac{\theta' t \varphi'(\theta' t)}{\varphi(\theta' t)}$ (resp. \le), which holds if and only if the elasticity of effort supply $\varepsilon(\cdot)$ is non-decreasing (resp. non-increasing).

In other words, when this elasticity is non-decreasing, the agent's effort supply is less responsive to incentives if a more favorable shock hits in the second period. Optimal rewards must be increased to provide incentives. The trade-off between efficiency and extraction of the firm's liability rent is tilted in favor of the firm, and more rent must be given up by the principal. When the probability of success is affected by productivity shocks, the monotonicity of $\varepsilon(\cdot)$ thus gives some insights about how stringent agency costs are. Those insights will be useful to build further intuition for the shape

³³Under the assumptions made on ψ (and thus φ), the problem is quasi-concave. Indeed, the function $t + \frac{\varphi(t)}{\varphi'(t)}$ is increasing in t when φ is concave (i.e., ψ' is convex).

of the firm's intertemporal incentive problem in Section 4 below.

4 Delegated Flexibility: Incentive Compatibility

Suppose now that the government delegates the tasks of investing to the firm. This section analyzes incentive compatibility constraints in that scenario.

Thanks to the concavity of the firm's objective function in (3), we get the following expression for second-period incentive compatibility, where again we make the dependence of second-period effort on e_1 and t_2 explicit:

$$e_{2}(\theta t_{2}, e_{1}) = \arg\max_{e_{2} \ge e_{1}} \theta e_{2} t_{2} - \psi(e_{2}) \Longrightarrow e_{2}(\theta t_{2}, e_{1}) = \max\{\varphi(\theta t_{2}), e_{1}\}.$$
 (9)

The irreversibility constraint is again binding for adverse shocks, i.e., when θ is low enough, as it was already the case without any agency problem. For such adverse realizations of the shock, the firm would like to exert less second-period effort than what it has already committed to through its first-period investment. Because disinvesting is not possible, the firm underinvests in the first period to keep some flexibility.

This is of course very similar to the incentive problem faced by the principal when he invests by himself. The important issue investigated in this section is to what extent the agent and his principal evaluate differently those incentives for flexibility and whether it can be a source of extra rent for the agent.

To understand the firm's incentives to underinvest, let us insert the expression of $e_2(\theta t_2, e_1)$, given in (9), into (3). The firm's intertemporal payoff becomes:

$$U(t_1, t_2) = \max_{e_1} e_1 t_1 - \psi(e_1) + \delta \left(\int_0^{\frac{\psi'(e_1)}{t_2}} (\theta e_1 t_2 - \psi(e_1)) f(\theta) d\theta + \int_{\frac{\psi'(e_1)}{t_2}}^{\bar{\theta}} R(\varphi(\theta t_2)) f(\theta) d\theta \right).$$

Optimizing with respect to e_1 yields the following expression of the first-period incentive compatibility constraint:

$$\psi'(e_1) = t_1 - \delta t_2 \int_0^{\frac{\psi'(e_1)}{t_2}} F(\theta) d\theta.$$
 (10)

To better understand the design of an optimal contract, let us introduce the scale parameter γ such that $t_2 = \gamma t_1$. For future reference, and much in the spirit of what we did in Section 3.1, let us also denote by $\zeta(\delta, \gamma)$ the unique solution in the interval $(0, \gamma^{-1})$ to the following equation:

$$\zeta(\delta,\gamma) = \frac{1}{\gamma} - \delta \int_0^{\zeta(\delta,\gamma)} F(\theta) d\theta.$$
(11)

The agent constrained by his initial investment, i.e., $e_2 = e_1$, when $\theta \leq \zeta(\delta, \gamma)$.

That $\zeta(\delta, \gamma)$ is decreasing in γ simply means that the first-period effort decreases as the first-period reward decreases relatively to the second-period one. This boosts flexibility and makes it more attractive to invest in the second period.

With those notations at hands, the firm's first-period incentive constraint becomes:

$$e_1(t_1, t_2) = \varphi\left(\zeta(\delta, \gamma)t_2\right) \equiv \varphi\left(\zeta\left(\delta, \frac{t_2}{t_1}\right)t_2\right)$$
(12)

where the dependence on the whole profile of rewards (t_1, t_2) is made explicit.

Remark 3 Let us briefly come back on our assumption that there is "no intertemporal trend" in productivity shocks (i.e., $E_{\theta}(\theta) = 1$). The possibility of such trends (i.e., $E_{\theta}(\theta) < 1$, resp. >, for a decreasing, resp. decreasing, trend) can be equivalently modeled by assuming that first- and second-period surplus differ, i.e., $S_2 < S_1$ (resp. >). This is just a matter of renormalization. When the principal controls efforts, the first-period decision will be biased towards a lower (resp. higher) degree of flexibility. Formally, the irreversibility constraint is binding for $\theta \leq \zeta\left(\delta, \frac{S_2}{S_1}\right) = \frac{S_1}{S_2} - \delta \int_0^{\zeta\left(\delta, \frac{S_2}{S_1}\right)} F(\theta) d\theta$.

The incentive constraint (12) looks pretty similar to (5), which describes the principal's optimal choices. Had the principal used a stationary contract $t_1 = t_2$, the irreversibility constraint would again be binding if and only if $\theta \le \zeta(\delta, 1)$. With stationary contracts, the principal and the agent agree on what should be the optimal level of flexibility. They are constrained by their first-period effort choices over the same range of realizations of the productivity shock even though these choices may differ.

The next property is useful to build intuition on our future results:

Lemma 1 An increase in t_2 boosts first-period incentives

$$\frac{\partial \psi'(e_1(t_1, t_2))}{\partial t_2} > 0$$

Raising t_2 of course boosts the second-period effort when (1) is slack but at the same time it also makes it more attractive to invest in the first period since the irreversibility constraint is less likely to be binding.

5 Delegated Flexibility: Optimal Contracts

5.1 Stationary Contracts

Given the large uncertainty surrounding climate change, it might sometimes be difficult to distinguish between several pre-defined contracting periods with different climatic conditions. In other words, the productivity shocks might just arise at a date that is left unspecified in the contract. Such extra degree of contract incompleteness can be modeled by assuming that parties can only use stationary contracts. In this section, we investigate the possible distortions in that case. This analysis is useful in view of building intuition for the more complex case of non-stationary contracts.

Let denote by $t = t_1 = t_2$ such stationary reward. As already mentioned, with stationary contracts, the principal and the agent agree on what should be the optimal level of flexibility. The irreversibility constraint is again binding if and only if $\theta \leq \zeta(\delta, 1)$. From (12), the first-period effort becomes

$$e_1(t) = \varphi(\zeta(\delta, 1)t) \tag{13}$$

(where we make explicit the dependence on *t*) while the second-period effort is

$$e_2(\theta, t) = \varphi(\max\{\theta, \zeta(\delta, 1)\}t).$$
(14)

With those expressions of efforts, the principal's intertemporal payoff becomes:

$$(1+\delta)(S-t)\Phi(t,\delta) \tag{15}$$

where

$$\Phi(t,\delta) = \frac{1}{1+\delta} \left(\varphi(\zeta(\delta,1)t) + \delta \int_0^{\bar{\theta}} \theta \varphi(\max\{\theta,\zeta(\delta,1)\}t) f(\theta) d\theta \right).$$
(16)

This expression of the principal's welfare highlights that, with stationary contracts, everything happens as if the environment was itself stationary with an "average" effort supply for each period (or probability of success) being now given by $\Phi(t, \delta)$.

Optimal stationary contracts. Optimizing the principal's intertemporal payoff yields a familiar expression for the optimal per-period reward $t^s(\delta)$:

$$S = t^{s}(\delta) + \frac{\Phi(t^{s}(\delta), \delta)}{\frac{\partial \Phi}{\partial t}(t^{s}(\delta), \delta)}.$$
(17)

Comparative statics. To highlight the different effects at play in determining $t^s(\delta)$, it is useful to view $\Phi(t, \delta)$ as the product of two factors:

$$\Phi(t,\delta) = \varphi(\zeta(\delta,1)t) \left(1 + \frac{\delta}{1+\delta} \left(\int_{\zeta(\delta,1)}^{\bar{\theta}} \theta\left(\frac{\varphi(\theta t)}{\varphi(\zeta(\delta,1)t)} - 1 \right) f(\theta) d\theta \right) \right).$$
(18)

Taking log-derivatives allows us to decompose the elasticity of $\Phi(t, \delta)$ as the sum of a *Commitment* and a *Flexibility Effects*:

$$\frac{t\frac{\partial\Phi}{\partial t}(t,\delta)}{\Phi(t,\delta)} = \underbrace{\frac{\zeta(\delta,1)t\varphi'(\zeta(\delta,1)t)}{\varphi(\zeta(\delta,1)t)}}_{\text{Commitment Effect}} + \underbrace{\frac{\frac{\delta}{1+\delta}t\left(\int_{\zeta(\delta,1)}^{\bar{\theta}}\theta\frac{\partial}{\partial t}\left(\frac{\varphi(\theta t)}{\varphi(\zeta(\delta,1)t)}\right)f(\theta)d\theta\right)}{1+\frac{\delta}{1+\delta}\left(\int_{\zeta(\delta,1)}^{\bar{\theta}}\theta\left(\frac{\varphi(\theta t)}{\varphi(\zeta(\delta,1)t)}-1\right)f(\theta)d\theta\right)}.$$
(19)

To understand the respective role of those two effects, consider first *a fictitious static model* where the probability of success would be $\varphi(\zeta(\delta, 1)t)$ and the principal's payoff $(S-t)\varphi(\zeta(\delta, 1)t)$. In such fictitious model, only the *Commitment Effect* would thus matter and the optimal reward $\hat{t}(\delta)$ would solve:

$$S = \hat{t}(\delta) + \frac{\varphi(\zeta(\delta, 1)\hat{t}(\delta))}{\zeta(\delta, 1)\varphi'(\zeta(\delta, 1)\hat{t}(\delta))}.$$
(20)

The *Commitment Effect* captures the idea that the (first-period) reward *t* has an impact not only on the first-period effort but also on the second-period one as long as the irreversibility constraint is binding. This effect is partially dissipated by the agent's incentives for flexibility. Indeed, everything happens as if only a fraction $\zeta(\delta, 1)$ of the first-period reward affects the first-period effort, or equivalently as if a "virtual" productivity shock $\zeta(\delta, 1) < 1$ had hit in that first period. Taking into account that $\zeta(\delta, 1) < 1$, Proposition 1 is an immediate implication of our findings in Section 3.2.

Proposition 1

$$\varepsilon(\cdot)$$
 non-decreasing (resp. non-increasing) $\Rightarrow \hat{t}(\delta) \le t(1)$ (resp. \ge). (21)

The *Commitment Effect* reduces (resp. increases) the optimal reward compared with the myopic scenario when $\varepsilon(\cdot)$ is non-decreasing (resp. non-increasing). The intuition is similar to that we developed in Section 3.2. In this fictitious static model, the "virtual" productivity shock $\zeta(\delta, 1) < 1$ makes the firm more (resp. less) responsive to incentives at that date when $\varepsilon(\cdot)$ is non-decreasing (resp. non-increasing). The firstperiod rent-efficiency trade-off is thus tilted towards the principal (resp. the agent).

As can be seen from (19), the *Flexibility Effect* measures instead the relative impact of the (second-period) reward *t* on the second-period probability of success over all favorable events $\theta \ge \zeta(\delta, 1)$ where the irreversibility constraint is slack. The sign of this effect follows from next Lemma.

Lemma 2

$$\varepsilon(\cdot)$$
 non-decreasing (resp. non-increasing) $\Rightarrow \frac{\partial}{\partial t} \left(\frac{\varphi(\theta t)}{\varphi(\theta' t)} \right) \ge 0$ (resp. $\le) \quad \forall t, \quad \forall \theta \ge \theta'.$

Applying Lemma 2 to the case where $\theta \ge \theta' = \zeta(\delta, 1)$, we observe that the *Flexibility Effect* boosts (resp. reduces) the elasticity of $\Phi(t, \delta)$ when $\varepsilon(\cdot)$ is non-decreasing (resp. non-increasing). This effect matters when the irreversibility constraint is slack over the second period, i.e., precisely when the *Commitment Effect* does not matter. Proposition 2 compares now $\hat{t}(\delta)$ with $t^s(\delta)$. It confirms that the *Flexibility Effect* always attenuates the *Commitment Effect* and brings the optimal reward closer to t(1).

Proposition 2

$$\varepsilon(\cdot)$$
 non-decreasing (resp. non-increasing) $\Rightarrow t^{s}(\delta) \ge \hat{t}(\delta)$ (resp. \le). (22)

This proposition points the trade-off between commitment and flexibility.³⁴ Whenever the *Commitment Effect* reduces (resp. increases) the stationary reward, the *Flexibility Effect* makes it attractive to increase (resp. decrease) this reward to benefit from a greater flexibility when the irreversibility constraint does not bind.

Intuitively, the delegated incentives for keeping flexibility make the firm play on the intertemporal distribution of rents it can get over both periods; pushing forward (resp. backward) rents when $\varepsilon(\cdot)$ is non-decreasing (resp. non-increasing). Those incentives decrease the first-period effort pushing it towards regions where the firm is more responsive to incentives when $\varepsilon(\cdot)$ is non-decreasing (resp. non-increasing). This first-period force leads the principal to reduce (resp. increase) the optimal reward accordingly. At the same time, the principal would like to increase (resp. decrease) the second-period reward to enjoy more of the firm's flexibility following good shocks.

³⁴Boyer and Robert (2006) also analyzed the trade-off between commitment and flexibility in a framework where private information is the source of the intertemporal linkage across periods.

5.2 Non-Stationary Contracts

Non-stationary policies might a priori be attractive because the principal may no longer be as much torn between the conflicting forces of the *Commitment* and *Flexibility Effects* as shown above.

With non-stationary policies, the principal might want to boost first-period incentives by offering greater rewards earlier on, i.e., $t_1 > t_2$. Although it improves firstperiod investment, this policy makes it also more likely that the irreversibility constraint (1) binds. In other words, the project may start "big" and generate unnecessary constraints under adverse circumstances later on. On the contrary, the principal might want to favor adaptation and flexibility in the second period. This is obtained by boosting second-period incentives with increasing rewards, i.e., $t_1 < t_2$. In that case, the project may start "small".

Definition 1 A profile has decreasing (resp. increasing, stationary) incentives when $\gamma < 1$ (resp. >, =).

In terms of actual PPP practices, several contractual dimensions that shift the power of incentives, including asset ownership, duration and compensation terms, might be relevant to think of applications of the different scenarios analyzed here. For example, the shift from having high-powered incentives earlier on to low-powered incentives later in the relationship that arises when $\gamma < 1$, might be viewed as a feature of PPP projects such as 'Build-Operate-Transfer' (BOT), in which ownership is relinquished to the public sector at the end of the contracting period. Alternatively, concession contracts contemplating ultimate divestiture of assets such as 'Build-Operate-Own' (BOO), i.e., in which operators end up being owners at the end of the franchise, may be good proxies for the case $\gamma > 1$.³⁵

Each of these non-stationary policies may turn out to be optimal, depending again on the properties of the elasticity of effort supply.

Proposition 3 Assume quasi-concavity of this principal's objective function in (t_2, γ) where $\gamma = \frac{t_2}{t_1}$. The optimal long-term contract $(t_1^*(\delta), t_2^*(\delta) = \gamma^*(\delta)t_1^*(\delta))$ is such that:

$$\varepsilon(\cdot)$$
 non-decreasing (resp. non-increasing, stationary) $\Rightarrow \gamma^*(\delta) \ge 1$ (resp. \le , =). (23)

In the context of non-stationary contracts, we can think of the *Flexibility Effect* as a tendency to increase the relative reward $\frac{t_2}{t_1}$, i.e., to raise γ , while the *Commitment Effect* corresponds instead to the tendency to decrease this relative reward, i.e., to reduce γ .

³⁵We develop this discussion further in Section 8.

When $\varepsilon(\cdot)$ is non-decreasing, we know from Section 5.1 that raising the secondperiod reward is attractive because the firm is then more responsive to incentives following favorable productivity shocks. With non-stationary contracts, the *Flexibility Effect* then dominates and leads to a non-decreasing profile of rewards ($\gamma > 1$). When $\varepsilon(\cdot)$ is instead non-increasing, the firm is less responsive to incentives following favorable shocks. The *Commitment Effect* dominates and leads to a non-increasing profile of rewards ($\gamma < 1$).

With stationary contracts, the principal and the agent agree on how much flexibility should be kept. Since optimal contracts might be non-stationary, a conflict might exist between the principal and the agent on the optimal degree of flexibility. This conflicts reflects the pattern of intertemporal rewards.

Corollary 1

 $\varepsilon(\cdot)$ non-decreasing (resp. non-increasing, constant) $\Rightarrow \zeta(\delta, \gamma^*(\delta)) \le \zeta(\delta, 1)$ (resp. $\ge, =$). (24)

6 Organizational Choices

Much of the debate on the costs and benefits of PPPs over more traditional forms of procurement hinges on the comparison of agency and transaction costs involved under alternative organizational scenarios.

Under bundling, viewed as a metaphor for the case of PPPs, different tasks corresponding to different stages of a project are performed by the same firm or consortium. Instead, under unbundling, different firms are in charge with different stages of the project. The thrust of the existing literature is that bundling may be beneficial when it internalizes some contractual externalities, making it cheaper to provide incentives when different tasks are jointly controlled by the same firm.

Our dynamic agency model, where efforts are performed sequentially, raises similar issues. As we shall see below, the comparison of agency costs under alternative scenarios nevertheless depends on the degree of contractual incompleteness.

6.1 The Benefits of Unbundling

To get at the costs and benefits of bundling tasks, consider first a scenario in which the principal commits ex ante to deal with two different firms, which act at different dates and receive the corresponding payments t_1 and t_2 . Under unbundling, the firm in charge in the first period does not internalize the impact of its first-period effort on future effort, so that:

$$e_1 = \varphi(t_1).$$

This implies that the second-period effort is constrained whenever $\varphi(\theta t_2) \leq \varphi(t_1)$ or

$$\theta \le \frac{1}{\gamma} = \frac{t_1}{t_2}$$

Clearly, the irreversibility constraint is now binding more often than if bundling was chosen. It turns out that this decreased flexibility benefits the principal. Indeed, the principal prefers that the first-period agent does not anticipate the consequences of his own effort on future choices. Instead, under bundling, the principal cannot prevent the first-period agent from keeping some flexibility for his own future choices. Taking a broader perspective, this result is another instance of a basic principle of multi-tasking models à la Holmström and Milgrom (1991), applied to the PPP literature: Bundling tasks may increase first-period agency costs. In our context, this principle strikes again. Part of the first-period reward gets dissipated by the agent's incentives for flexibility.

Proposition 4 *With long-term contracts and unbundling, the first-period firm's effort responds only to first-period incentives. First-period incentives are cheaper than under bundling. Unbundling is the principal's preferred organizational form.*

Importantly, Proposition 4 suggests that it is worth limiting contract length in uncertain environments. This points at a novel cost of PPPs.

6.2 Short-Term Contracts

Let us come back to our initial bundling scenario, with a single firm in charge over both periods, but assume now that no long-term contract can be signed. In such a highly incomplete contracting environment, parties leave open the possibility of drafting new contracts when the productivity shock becomes common knowledge and verifiable. A priori, the cost of such scenario could be the principal's limited ability to control the agent's first-period effort through a commitment to second-period rewards. The benefit is that, by waiting for the relevant information, second-period incentives can be better tailored to the realization of the productivity shock. In other words, short-term contracts allow the principal to better control the degree of flexibility kept by the firm. We will see below that this control can indeed be perfect.

Under short-term contracting, the principal forms a conjecture on the first-period (non-observable) effort, say e_1^e , at the time of drafting second-period contracts. In the

second-period, the spot contract implements the optimal reward $t_2(\theta) = t(\theta)$ exactly as in Section 3.2 as long as the second-period effort it induces is unconstrained by the first-period choice, i.e., when

$$\varphi(\theta t(\theta)) \ge e_1^e. \tag{25}$$

Instead, when this inequality does not hold, the principal offers a second-period reward $t_2(\theta)$ that just implements a second-period effort worth e_1^e .

$$\varphi(\theta t_2(\theta)) = e_1^e \Leftrightarrow \theta t_2(\theta) = \psi'(e_1^e).$$
(26)

Observe that $\theta t(\theta)$ is increasing in θ ; in other words, the second-period effort increases with the productivity shock.³⁶ Hence, there exists a cut-off $\theta^*(e_1^e)$ such that the irreversibility constraint is only binding (resp. slack) for all $\theta \leq \theta^*(e_1^e)$ (resp. \geq).

Using the expression of $t_2(\theta)$ coming from (26) when the irreversibility constraint is binding, we can rewrite the firm's intertemporal payoff as:

$$\max_{e_1} e_1 t_1 - \psi(e_1) + \delta \left(\int_0^{\theta^*(e_1^e)} (\psi'(e_1^e) e_1 - \psi(e_1)) f(\theta) d\theta + \int_{\theta^*(e_1^e)}^{\bar{\theta}} R(\varphi(\theta t(\theta))) f(\theta) d\theta \right).$$

Optimizing and taking into account that conjectures are correct at equilibrium leads to the surprisingly simple expression of the first-period incentive constraint:

$$e_1 = \varphi(t_1). \tag{27}$$

This is the same incentive constraint as with a myopic firm that does not take the impact of its first-period effort on future opportunities into account. Intuitively, the fact that second-period rewards depends only on the principal's conjecture on the first-period effort and not on its exact value makes it now irrelevant for the agent to try to manipulate the irreversibility constraint by choosing the first-period effort. In other words, the principal's ability to delay second-period contracting untill the productivity shock becomes common knowledge removes all the agent's incentives for keeping flexibility. Even with a single firm in charge at both dates, the principal can now replicate the unbundling scenario. The extra benefit compared with Section 6.1 is that date 2 contract can now depend explicitly on the realization of θ .

Proposition 5 With short-term contracts,

³⁶Indeed, since φ is concave, we have: $\frac{d}{d\theta}(\theta t(\theta)) = \frac{S}{2 - \frac{\varphi(\theta t(\theta))\varphi''(\theta t(\theta))}{(\varphi'(\theta t(\theta)))^2}} > 0.$

• The optimal first-period reward t_1^{st} is such that:

$$\zeta(\delta, 1)S = t_1^{st} + \frac{\varphi(t_1^{st})}{\varphi'(t_1^{st})};$$
(28)

• The second period reward $t_2(\theta)$ is such that:

$$t_{2}(\theta) = \begin{cases} \frac{t_{1}^{st}}{\theta} & \text{if } \theta \leq \zeta(\delta, 1) \\ t(\theta) & \text{if } \theta \geq \zeta(\delta, 1). \end{cases}$$
(29)

The first-period agency cost on the right-hand side of (28) has the same form as that obtained with myopic players in Section 3.2. The left-hand side is nevertheless different. Everything happens as if the the first-period social value of the project was discounted by the familiar term $\zeta(\delta, 1)$, i.e., by the same amount as with no delegation (see Section 3.1). In other words, the principal is not myopic himself.

As a result, rewards and efforts are reduced in that scenario:

$$t_1^{st} < t(1).$$

Written in terms of effort, (28) becomes:

$$\zeta(\delta, 1)S = \psi'(e_1^{st}) + R'(e_1^{st}).$$
(30)

From this, it becomes straightforward to observe that:

$$e_1^{st} < e_1^i.$$

Compared with the scenario in Section 3.1, effort is downward distorted to reduce the firm's rent.

With short-term contracts, the principal is now able to better control the agent. By designing contracts once shocks are common knowledge, the principal keeps full control on the degree of flexibility needed in the first period.

Remark 4 Proposition 5 also describes the outcome that is achieved under an unbundling scenario when the principal can commit upfront to offer a second-period reward that depends on the productivity shocks. Of course, such possibility would require that climate contingencies are perfectly foreseen, an extreme assumption.

7 Technological Choices

An important question for long-lived infrastructures in the context of climate change is to determine how flexible technologies should be. When designing a long-term project, contracting parties may indeed opt for technologies with low exposure to climatic hazards even if such choice is ex ante costly.³⁷ For example, a water company may chose to invest in safer extraction technologies to limit subsequent risk of contamination, or a road concessionaire may include in the project design a number of features that reduce the exposure of the road itself to floods and heavy precipitations.

To model such issues, we consider the choice within a continuum of potential technologies indexed by some parameter α which characterizes the level of exposure to the productivity shock θ . Adopting an α -technology costs $C(\alpha)$ (with C(0) = C'(0) = 0, $C'(\alpha) \ge 0$ and $C''(\alpha) > 0$). Assuming that this technological choice is verifiable, it is just an accounting convention to consider that the principal fully bears that cost. Importantly, the choice of a less costly α -technology exacerbates the impact of climate shocks on productivity. More precisely, we assume that the distribution $F(\cdot|\alpha_1)$ is a mean-preserving spread transformation of $F(\cdot|\alpha_2)$ whenever $\alpha_1 < \alpha_2$. Assuming differentiability of $f(\theta|\alpha)$ in α , this simply means that:

$$\int_{0}^{\theta} \frac{\partial F}{\partial \alpha}(x|\alpha) dx < 0 \quad \forall \theta \in (0,\bar{\theta}) \text{ and } \int_{0}^{\bar{\theta}} \frac{\partial F}{\partial \alpha}(x|\alpha) dx = 0.$$
(31)

A lower α -technology, which is less costly, implies more uncertainty on θ around the "mean scenario" $E_{\theta}(\theta) = 1$. Instead, by investing more ex ante, parties ensure that random productivity shocks will be closer to that mean.

For simplicity, we restrict our analysis to the case of stationary contracts $t_1 = t_2 = t$ although similar insights would apply in less incomplete environments. Making the dependence of $\zeta(\delta, 1, \alpha)$ on α explicit, the first-period incentive compatibility constraint becomes:

$$e_1(t) = \varphi(\zeta(\delta, 1, \alpha)t) \tag{32}$$

with

$$\zeta(\delta, 1, \alpha) = 1 - \delta \int_0^{\zeta(\delta, 1, \alpha)} F(\theta | \alpha) d\theta < 1.$$
(33)

Lemma 3 The firm chooses less flexibility as α increases:

$$\frac{\partial \zeta}{\partial \alpha}(\delta, 1, \alpha) > 0. \tag{34}$$

³⁷For earlier references on technological flexibility, we refer to Stigler (1939), Jones and Ostroy (1984), Vives (1989), and Boyer and Moreaux (1989).

Investing into a technology with a greater α reduces uncertainty on the productivity shock and makes it less crucial to keep flexibility. The incentive compatibility constraint (32) comes "closer" to that found when the firm is myopic.

Before characterizing the optimal technology, observe that myopic parties would never choose to make any such investment. Beyond that benchmark, we have:

Proposition 6 Assume that $\delta > 0$ and $\frac{d}{dt}(\varphi(t) + t\varphi'(t)) \leq 0$, for all t. Investing in a α -technology which reduces uncertainty is optimal.

This result points a possible substitutability between contractual and technological choices. When technologies are better able to cope with climate uncertainty, there is certainly less need to distort intertemporal incentives with long-term contracts. Pushing further this idea, PPPs might then be viewed as more attractive than in our baseline scenario.

8 Discussion

The three key variables that affect contracting patterns in our analysis are the distribution of the productivity shock θ , the discount factor δ , and the firm's elasticity of effort supply $\varepsilon(t)$. One may wonder first how these variables relate to standard contracting features and, second, how they might also depend on various institutional constraints, such as financial restrictions, regulatory or political uncertainty, etc.

We may assimilate an increase in δ to a longer project duration, while tightened financial constraints or increased uncertainty in the political environment may instead reduce this variable. The distribution of productivity shocks may be considered as more uncertain (in the sense of Section 7) when projects have a higher intrinsic exposure to climate shocks and possibly more stringent ex ante obligations to cater to regulatory or environmental issues. Finally, as far as the elasticity of effort supply is concerned, we observed earlier that $\varepsilon(t)$ is non-increasing if $e\psi''(e)/\psi'(e)$ is non-decreasing in *e*. This is more likely when $\psi''(e)$ is quickly increasing in *e*. Any regulatory policy or financial constraint that makes it more costly to provide effort in this strong sense might also correspond to such non-increasing supply elasticity.

Longer project duration or more exposure to adverse climate shocks both reduce $\zeta(\delta, \gamma)$. This exacerbates the value of keeping flexibility. On the contrary, stronger financial constraints or greater political and regulatory uncertainty might push towards more myopic behavior not only by reducing the discount factor but also because the

elasticity of effort supply might become non-increasing which also favors decreasing incentives.

It may be interesting to reflect upon the relevance of our conclusions along two dimensions. First, we may now give hints on the value of contracting in sectors in which projects are commonly managed as PPPs, such as energy production or distribution, water and sanitation networks, transport project, and local public goods. Second, we may assess the performances of different forms of PPP contracts typically found in practice, such as management contracts, concessions, Build-Operate-Transfer (BOT).

As for projects across sectors, local public goods are probably those for which the concerns for commitment dominate those for flexibility. Instead, water and sanitation networks, which typically are governed by longer term concessions and cumulate high exposure to climate hazard, are projects with high demand for flexibility. Transport projects and energy endeavors might fall in between. Power production projects, with their important exposure to climatic shocks and strong mitigation requirements, are probably closer to water networks. We expect the trade-off between commitment and flexibility to be more favorable to flexibility in water and energy production PPPs than for energy distribution, transport and other local public goods.

Looking now at different forms of PPP contracts, some of the contractual dimensions that shift the power of incentives, including asset ownership and duration, can be linked to the shape of the rewards schedule. As mentioned earlier, decreasing incentives, i.e., a shift from high-powered incentives earlier on to low-powered incentives later on, may correspond to PPP contracts contemplating the return of assets ownership to the public sector at the end of the contracting period, such as Build-Operate-Transfer (BOT). On the contrary, concession or Build-Operate-Own (BOO) contracts, which include the divestiture of assets to private operators at the end of the period, would have increasing incentives over time.

Although other factors certainly play a role, the recent trend in water management away from PPPs may indicate that local governments tend to shy away from contractual arrangements which induce growing rents to private operators. We thus conjecture that the shift towards decreasing incentives as a result of growing climate-related uncertainties is more likely for water and energy production PPPs than for energy distribution, transport and other local public goods. It may also translate into shorter contracts with lower-powered incentives schemes, and as well as a lower propensity to divest assets in the long run.

As a result, our work casts doubts on the benefits of bundling, i.e., of PPPs of any form, in a context in which unbundling may be welfare improving, unless sufficient scope for technological improvements exists, which is again less likely in the water sector than in energy. In environments in which climate-related uncertainty has strong bites, and for projects sharing the characteristics pointed out, the use of relatively short-term contracts, such as service or management contracts, not to exceed 3 to 5 years, may be preferred.

This paper's conclusions thus superimpose to the costs and benefits of PPPs already identified in the existing literature (e.g., Engel et al., 2008), shifting the standard trade-offs discussed there. Moreover, as discussed above, the impact is likely to differ across types of projects and sectors. While this bears some resemblance to Iossa and Martimort (2008) conclusion that in "fast-moving" sectors bundling might come at the cost of added rigidity, the potential substitutability between organizational and technological choices means that climate change may be less of a strain for PPPs in sectors with rapidly evolving characteristics.

9 Conclusion

Considering that climate change related hazards are especially relevant for infrastructure sectors, we have questioned how climate uncertainty might affect long-term contractual relationships such as PPPs. Our main results were to show how the classical underinvestment effect found with irreversible investments under uncertainty is modified by agency costs. The contractual response to those new agency costs of delegated flexibility depends on properties of the firm's effort supply. When the elasticity of effort supply is non-increasing (resp. non-decreasing), optimal profiles of rewards are nonincreasing (resp. no-decreasing) over time. This suggests that 'BOT' or 'BOO' forms of PPPs may be good proxies depends on fine details of cost functions. We conjectured that such decreasing incentives are likely for water and energy production PPPs, and to a lesser extent for energy distribution, transport and other local public goods, putting some stress on the long-run viability of PPPs in these sectors.

Beyond, our analysis has shown that coping with climate uncertainty certainly requires a careful analysis of the joint design of institutional, contractual and technological constraints. Short-term contracts, unbundling of tasks, flexible technologies are all tools that help reducing the new agency costs of delegated flexibility.

Our analysis leaves a number of important issues unsettled. Whether institutional, contractual and technological choices are complements or substitutes in coping with climate hazards is an important question that was only briefly touched upon above.³⁸ In this respect, an open question is whether flexible technologies become more attrac-

³⁸This theme echoes the important insights due to Holmström and Milgrom (1991) and Milgrom and Roberts (1990), and Athey and Schmutlzer (1995) in more general frameworks.

tive when governments cannot easily commit to long-term contracts because of political or fiscal pressures to renege on the agreements signed with the private sector.

Another important question that would deserve further thoughts is how financial and regulatory constraints may impact investments and adoption of new technologies in infrastructure sectors. Our paper indicates that a first effect would come from how those constraints shape the cost of effort but the exact channels remain to be unveiled. Making progresses on that front is clearly a relevant issue from a policy point of view, especially in a context in which much hope is placed on the development of green technologies to face mounting environmental challenges.

Our approach to the problem of delegated flexibility has been by and large normative. Of course, political agendas may push public decision-makers to adopt objective which differ from an ideal scenario. Environmental policies and concerns for flexibility may be high on the agenda of politicians at the time of the inceptions of long-term contracts but may fade later on. This suggests that it could be interesting to extent our framework to account for evolving preferences of the government and possibly with renegotiation of its objectives.

Finally, investments on long-lived assets in infrastructure sectors plays a key role in fostering growth. Climate hazards might thus impact on growth rates not only directly by putting long-lived assets at risk but also by affecting contracts and investments in those sectors. Again, the full consequences of this intriguing channel between climate and growth remain to be studied. We intend to explore some of those questions in future research.

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10 Appendix

Proof of Lemma 1. Observe that:

$$\psi'(e_1(t_1, t_2)) = \zeta\left(\delta, \frac{t_2}{t_1}\right) t_2.$$

Differentiating (11) with respect to t_2 yields:

$$\frac{\partial \psi'(e_1(t_1, t_2))}{\partial t_2} = \zeta \left(\delta, \frac{t_2}{t_1}\right) + \frac{t_2}{t_1} \frac{\partial \zeta}{\partial \gamma} \left(\delta, \frac{t_2}{t_1}\right).$$

Taking into account the definition of $\zeta(\delta, \gamma)$ in (11), we first obtain:

$$\frac{\partial \zeta}{\partial \gamma}(\delta,\gamma) = -\frac{1}{\gamma^2 (1 + \delta F(\zeta(\delta,\gamma)))} < 0.$$
(A1)

Inserting into the above expression, we finally obtain:

$$\frac{\partial \psi'(e_1(t_1, t_2))}{\partial t_2} = \zeta\left(\delta, \frac{t_2}{t_1}\right) - \frac{\frac{t_1}{t_2}}{1 + \delta F\left(\zeta\left(\delta, \frac{t_2}{t_1}\right)\right)}.$$
(A2)

Using again (11), observe that:

$$\zeta(\delta,\gamma)\left(1+\delta F\left(\zeta\left(\delta,\gamma\right)\right)\right)-\frac{1}{\gamma}=\delta\left(\zeta\left(\delta,\gamma\right)F\left(\zeta\left(\delta,\gamma\right)\right)-\int_{0}^{\zeta\left(\delta,\gamma\right)}F(\theta)d\theta\right)$$

Integrating by parts, we find:

$$\int_0^{\zeta(\delta,\gamma)} \theta f(\theta) d\theta = \zeta(\delta,\gamma) F(\zeta(\delta,\gamma)) - \int_0^{\zeta(\delta,\gamma)} F(\theta) d\theta.$$

Hence, we get:

$$\zeta(\delta,\gamma)\left(1+\delta F\left(\zeta(\delta,\gamma)\right)\right) - \frac{1}{\gamma} = \delta\left(\int_{0}^{\zeta(\delta,\gamma)} \theta f(\theta)d\theta\right) > 0 \tag{A3}$$

where the last inequality follows, since $\zeta(\delta, \gamma) > 0$. It follows that:

$$\frac{\partial \psi'(e_1(t_1, t_2))}{\partial t_2} = \frac{\delta \int_0^{\zeta\left(\delta, \frac{t_2}{t_1}\right)} \theta f(\theta) d\theta}{1 + \delta F\left(\zeta\left(\delta, \frac{t_2}{t_1}\right)\right)} > 0$$

which ends the proof.

Proof of Lemma 2. For $\theta \ge \theta'$, observe that:

$$\frac{\partial}{\partial t} \left(\frac{\varphi(\theta t)}{\varphi(\theta' t)} \right) = \frac{\theta \varphi'(\theta t)}{\varphi(\theta' t)} - \frac{\theta' \varphi(\theta t) \varphi'(\theta' t)}{\varphi^2(\theta' t)} = \frac{\varphi(\theta t)}{t \varphi(\theta' t)} \left(\varepsilon(\theta t) - \varepsilon(\theta' t) \right).$$

Taking $\theta' = \zeta(\delta, 1)$, we get that $\frac{\partial}{\partial t} \left(\frac{\varphi(\theta t)}{\varphi(\zeta(\delta, 1)t)} \right) \ge 0$ (resp. \le) if $\varepsilon(\cdot)$ is non-decreasing (resp. non-increasing).

Proof of Proposition 2. Observe that $\varphi' > 0$ implies that $\frac{\varphi(\theta t)}{\varphi(\zeta(\delta,1)t)} - 1 > 0$ for $\theta > \zeta(\delta,1)$ so that the denominator of the second term on the right-hand side of (19) is positive. Therefore, we have:

$$\frac{\frac{\partial \Phi}{\partial t}(t,\delta)}{\Phi(t,\delta)} \ge \frac{\zeta(\delta,1)\varphi'(\zeta(\delta,1)t)}{\varphi(\zeta(\delta,1)t)} \text{ (resp. \le)} \quad \forall t$$
(A4)

$$\Leftrightarrow \int_{\zeta(\delta,1)}^{\bar{\theta}} \theta \frac{\partial}{\partial t} \left(\frac{\varphi(\theta t)}{\varphi(\zeta(\delta,1)t)} \right) f(\theta) d\theta \ge 0 \text{ (resp. \le)} \quad \forall t.$$
(A5)

Applying Lemma 2 for $\theta' = \zeta(\delta, 1)$ yields:

$$\varepsilon(\cdot)$$
 non-decreasing (resp. non-increasing) \Rightarrow (A4) holds. (A6)

The comparison bewteen $t^s(\delta)$ and $\hat{t}(\delta)$ immediately follows.

Proof of Proposition 3. Equipped with the definition of the first-period effort given in (12), we can rewrite the principal's intertemporal payoff as a function of (t_2, γ) (instead of (t_1, t_2)):

$$V(t_2, \gamma) = \left(S - \frac{t_2}{\gamma}\right)\varphi\left(\zeta(\delta, \gamma)t_2\right)$$
$$+\delta\left(S - t_2\right)\left(\varphi\left(\zeta(\delta, \gamma)t_2\right)\int_0^{\zeta(\delta, \gamma)}\theta f(\theta)d\theta + \int_{\zeta(\delta, \gamma)}^{\bar{\theta}}\theta\varphi\left(\theta t_2\right)f(\theta)d\theta\right).$$
 (A7)

We first compute the derivatives of $V(t_2, \gamma)$ with respect to γ and t_2 respectively:

$$\frac{\partial V}{\partial \gamma}(t_2,\gamma) = t_2 \left(\frac{\varphi(\zeta(\delta,\gamma)t_2)}{\gamma^2} + \frac{\partial \zeta}{\partial \gamma}(\delta,\gamma)\varphi'(\zeta(\delta,\gamma)t_2) \left(\left(S - \frac{t_2}{\gamma}\right) + \delta(S - t_2) \int_0^{\zeta(\delta,\gamma)} \theta f(\theta)d\theta \right) \right) \right) \tag{A8}$$

$$\frac{\partial V}{\partial t_2}(t_2,\gamma) = -\frac{1}{\gamma}\varphi\left(\zeta(\delta,\gamma)t_2\right) - \delta\left(\varphi\left(\zeta(\delta,\gamma)t_2\right) \int_0^{\zeta(\delta,\gamma)} \theta f(\theta)d\theta + \int_{\zeta(\delta,\gamma)}^{\bar{\theta}} \theta\varphi\left(\theta t_2\right) f(\theta)d\theta \right) + \zeta(\delta,\gamma)\varphi'\left(\zeta(\delta,\gamma)t_2\right) \left(S - \frac{t_2}{\gamma} + \delta(S - t_2)\zeta(\delta,\gamma)\varphi'\left(\zeta(\delta,\gamma)t_2\right) \int_0^{\zeta(\delta,\gamma)} \theta f(\theta)d\theta \right) + \delta(S - t_2) \int_{\zeta(\delta,\gamma)}^{\bar{\theta}} \theta^2 \varphi'(\theta t_2) f(\theta)d\theta. \tag{A9}$$

Assuming quasi-concavity of $V(t_2, \gamma)$, the first-order condition $\frac{\partial V}{\partial t_2}(t_2, \gamma) = 0$ defines implicitly an optimal reward in terms of γ , say $t_2^*(\gamma)$. The optimality condition with respect to γ can then be written as:

$$0 = \frac{\partial V}{\partial \gamma}(t_2^*(\gamma), \gamma).$$

By definition of the optimal stationary contract $t^s(\delta)$, we have $t^s(\delta) = t_2^*(1)$ since indeed, fixing $\gamma = 1$, the optimality condition with respect to t_2 amounts to:

$$\frac{\partial V}{\partial t_2}(t^s(\delta), 1) = 0,$$

which rewrites as (17).

Assuming that the principal's objective is quasi-concave, the optimal value $\gamma^*(\delta)$ satisfies the following condition:

$$\gamma^*(\delta) \ge 1 \Leftrightarrow \frac{\partial V}{\partial \gamma}(t^s(\delta), 1) \ge 0.$$

Evaluating the expression for $\frac{\partial V}{\partial \gamma}(t_2, \gamma)$ from (A8) at $(t_2, \gamma) = (t^s(\delta), 1)$, we get:

$$\frac{\partial V}{\partial \gamma}(t^s(\delta), 1) = t^s(\delta) \left(\varphi(\zeta(\delta, 1)t^s(\delta)) + \frac{\partial \zeta}{\partial \gamma}(\delta, 1)\varphi'(\zeta(\delta, 1)t^s(\delta))(S - t^s(\delta)) \left(1 + \delta \int_0^{\zeta(\delta, 1)} \theta f(\theta) d\theta \right) \right)$$
(A10)

Inserting the expression of $S - t^s(\delta)$ taken from (17) into (A10), we obtain:

$$\frac{\partial V}{\partial \gamma}(t^{s}(\delta),1) = t^{s}(\delta)\varphi'(\zeta(\delta,1)t^{s}(\delta)) \left(\frac{\varphi(\zeta(\delta,1)t^{s}(\delta))}{\varphi'(\zeta(\delta,1)t^{s}(\delta))} - \frac{1+\delta\int_{0}^{\zeta(\delta,1)}\theta f(\theta)d\theta}{1+\delta F(\zeta(\delta,1))}\frac{\Phi(t^{s}(\delta),\delta)}{\frac{\partial\Phi}{\partial t}(t^{s}(\delta),\delta)}\right).$$
(A11)

Using (A3), we find:

$$\zeta(\delta, 1) = \frac{1 + \delta \int_0^{\zeta(\delta, 1)} \theta f(\theta) d\theta}{1 + \delta F(\zeta(\delta, 1))}.$$
(A12)

Inserting (A12) into (A11), yields:

$$\frac{\partial V}{\partial \gamma}(t^s(\delta), 1) = t^s(\delta)\varphi'(\zeta(\delta, 1)t^s(\delta)) \left(\frac{\varphi(\zeta(\delta, 1)t^s(\delta))}{\varphi'(\zeta(\delta, 1)t^s(\delta))} - \zeta(\delta, 1)\frac{\Phi(t^s(\delta), \delta)}{\frac{\partial \Phi}{\partial t}(t^s(\delta), \delta)}\right).$$
(A13)

In particular, we get:

$$\frac{\partial V}{\partial \gamma}(t^s(\delta), 1) \ge 0 \Leftrightarrow \frac{\frac{\partial \Phi}{\partial t}(t^s(\delta), \delta)}{\Phi(t^s(\delta), \delta)} \ge \frac{\zeta(\delta, 1)\varphi'(\zeta(\delta, 1)t^s(\delta))}{\varphi(\zeta(\delta, 1)t^s(\delta))}.$$
(A14)

From (A6), we finally get:

 $\varepsilon(\cdot)$ non-decreasing (resp. non-increasing) $\Rightarrow \frac{\partial V}{\partial \gamma}(t^s(\delta), 1) \ge 0$ (resp. ≤ 0).

Finally, observe that, when $\varepsilon'(t) \equiv 0$, (19) implies:

$$\frac{\frac{\partial \Phi}{\partial t}(t,\delta)}{\Phi(t,\delta)} = \frac{\zeta(\delta,1)t\varphi'(\zeta(\delta,1)t)}{\varphi(\zeta(\delta,1)t)}.$$

(A13) can be simplified as:

$$\frac{\partial V}{\partial \gamma}(t^s(\delta), 1) = 0.$$

This ends the proof.

Proof of Corollary 1. Observe that $\zeta(\delta, \gamma)$ is decreasing in γ . Therefore, $\gamma^*(\delta) \ge 1$ implies

$$\gamma^*(\delta) \ge 1 \Leftrightarrow \zeta(\delta, \gamma^*(\delta)) \le \zeta(\delta, 1).$$

Taken with Proposition 3, this ends the proof.

Proof of Proposition 4. Under unbundling, we can express the principal's intertemporal payoff as:

$$V^{s}(t_{2},\gamma) = \left(S - \frac{t_{2}}{\gamma}\right)\varphi\left(\frac{t_{2}}{\gamma}\right) + \delta\left(S - t_{2}\right)\left(\varphi\left(\frac{t_{2}}{\gamma}\right)\int_{0}^{\frac{1}{\gamma}}\theta f(\theta)d\theta + \int_{\frac{1}{\gamma}}^{\bar{\theta}}\theta\varphi\left(\theta t_{2}\right)f(\theta)d\theta\right).$$
(A15)

Define now:

$$\tilde{V}(t_2,\gamma,\zeta) = \left(S - \frac{t_2}{\gamma}\right)\varphi\left(\zeta t_2\right) + \delta\left(S - t_2\right)\left(\varphi\left(\zeta t_2\right)\int_0^\zeta \theta f(\theta)d\theta + \int_\zeta^{\bar{\theta}} \theta\varphi\left(\theta t_2\right)f(\theta)d\theta\right).$$

Observe that the principal's intertemporal payoffs under bundling and unbundling are respectively such that:

$$V(t_2,\gamma) = \tilde{V}(t_2,\gamma,\zeta(\delta,\gamma)) \text{ and } V^s(t_2,\gamma) = \tilde{V}\left(t_2,\gamma,\frac{1}{\gamma}\right)$$

Observe also that

$$\frac{\partial \tilde{V}}{\partial \zeta}(t_2, \gamma, \zeta) = t_2 \varphi'(\zeta t_2) \left(S - \frac{t_2}{\gamma} + \delta \left(S - t_2 \right) \int_0^{\zeta} \theta f(\theta) d\theta \right).$$

Take any pair $(t_1, t_2 = \gamma t_1)$ such that $S \ge t_i$ for i = 1, 2. (The pair $t_2^* = t_2^*(\gamma^*)$ and $t_1^* = t_1^*(\gamma^*)/\gamma^*$ where t_2^* is the optimal second-period reward and γ^* the optimal ratio between first- and second-period reward under bundling satisfy those properties). Observe then that $V(t_2, \gamma, \zeta)$ is everywhere non-decreasing in ζ . Therefore, we get:

$$V(t_2,\gamma) \le V^s(t_2,\gamma).$$

Hence,

$$V(t_2^*, \gamma^*) \le V^s(t_2^*, \gamma^*)$$

and unbundling dominates.

Proof of Proposition 5. Observe that (27) implies that $\theta^*(e_1)$ is such that $\theta^*(e_1)t(\theta^*(e_1)) = t_1$ and that $\theta t_2(\theta) = \psi'(e_1) = t_1$ when the technological constraint is binding. With those observations, we can now write the principal's intertemporal payoff under short-term

contracting in terms of the first-period reward t_1 only as:

$$V^{st}(t_1) = (S-t_1)\varphi(t_1) + \delta\left(\int_0^{\theta^*(\varphi(t_1))} (\theta S - t_1)\varphi(t_1)f(\theta)d\theta + \int_{\theta^*(\varphi(t_1))}^{\bar{\theta}} (S - t(\theta))\theta\varphi(\theta t(\theta))f(\theta)d\theta\right)$$
(A16)

Optimizing $V^{st}(t_1)$ with respect to t_1 gives the following first-order condition for t_1^{st} :

$$\left(S\left(1+\delta\int_{0}^{\theta^{*}(\varphi(t_{1}^{st}))}\theta f(\theta)d\theta\right)-t_{1}^{st}\left(1+\delta F(\theta^{*}(\varphi(t_{1}^{st}))\right)\right)\varphi'\left(t_{1}^{st}\right)-\varphi\left(t_{1}^{st}\right)\left(1+\delta F(\theta^{*}(\varphi(t_{1}^{st})))\right)=0.$$

or

$$\left(\frac{1+\delta\int_{0}^{\theta^{*}(\varphi(t_{1}^{st}))}\theta f(\theta)d\theta}{1+\delta F(\theta^{*}(\varphi(t_{1}^{st})))}\right)S = t_{1}^{st} + \frac{\varphi(t_{1}^{st})}{\varphi'(t_{1}^{st})}.$$
(A17)

By definition of $t^*(\theta^*(\varphi(t_1^{st})))$, we have:

$$S = t(\theta^*(\varphi(t_1^{st})) + \frac{\varphi(\theta^*(\varphi(t_1^{st}))t_1^{st}(\theta^*(\varphi(t_1^{st}))))}{\theta^*(\varphi(t_1^{st}))\varphi'(t(\theta^*(\varphi(t_1^{st}))))}$$

Because $\theta^*(\varphi(t_1^{st}))t^*(\theta^*(\varphi(t_1^{st})))=t_1^{st}$, it follows that:

$$\theta^*(\varphi(t_1^{st}))S = t_1^{st} + \frac{\varphi(t_1^{st})}{\varphi'(t_1^{st})}$$

Putting together this condition with (A17), it follows that:

$$\theta^*(\varphi(t_1^{st})) = \frac{1 + \delta \int_0^{\theta^*(\varphi(t_1^{st}))} \theta f(\theta) d\theta}{1 + \delta F(\theta^*(\varphi(t_1^{st})))}.$$
(A18)

Comparing (A12) with (A18) gives us:

$$\zeta\left(\delta,1\right) = \theta^*(\varphi(t_1^{st})).$$

Inserting into (A17), we finally obtain (28).

Proof of Lemma 3. Differentiating (33) with respect to α , we get:

$$\frac{\partial \zeta}{\partial \alpha}(\delta, 1, \alpha) = -\frac{\delta \int_0^{\zeta(\delta, 1, \alpha))} \frac{\partial F}{\partial \alpha}(\theta | \alpha) d\theta}{1 + \delta F(\zeta(\delta, 1, \alpha) | \alpha)} < 0, \tag{A19}$$

where the inequality follows from (31). This immediately implies (34).

Proof of Proposition 6. Making explicit the dependence on α , we rewrite the principal's intertemporal payoff as

$$V(t,\alpha) = (1+\delta)(S-t)\Phi(t,\delta,\alpha) - C(\alpha),$$
(A20)

where the average discounted probability of success is now

$$\Phi(t,\delta,\alpha) = \frac{1}{1+\delta} \left(\varphi(\zeta(\delta,1,\alpha)t) + \delta \int_0^{\bar{\theta}} \theta \varphi(\max\{\theta,\zeta(\delta,1,\alpha)\}t) f(\theta|\alpha) d\theta \right).$$
(A21)

The first-order optimality condition immediately yields the by-now standard expression of the stationary transfer $t^s(\delta)$:

$$S = t^{s}(\delta) + \frac{\Phi(t^{s}(\delta), \delta, \alpha)}{\frac{\partial \Phi}{\partial t}(t^{s}(\delta), \delta, \alpha)}.$$
(A22)

The first-order optimality condition for the optimal investment level α^* writes as:

$$C'(\alpha^*) = (1+\delta)(S - t^s(\delta))\frac{\partial\Phi}{\partial\alpha}(t^s(\delta), \delta, \alpha^*),$$
(A23)

where

$$(1+\delta)\frac{\partial\Phi}{\partial\alpha}(t^{s}(\delta),\delta,\alpha^{*}) = \frac{\partial\zeta}{\partial\alpha}(\delta,1,\alpha)t^{s}(\delta)\varphi'(\zeta(\delta,1,\alpha^{*})t^{s}(\delta))\left(1+\delta\int_{0}^{\zeta(\delta,1,\alpha^{*})}\theta f(\theta|\alpha^{*})d\theta\right)$$
$$+\delta\int_{0}^{\bar{\theta}}\theta\varphi(\max\{\theta,\zeta(\delta,1,\alpha^{*})\}t^{s}(\delta))\frac{\partial f}{\partial\alpha}(\theta|\alpha^{*})d\theta.$$
(A24)

Integrating by parts, we get:

$$\int_0^{\zeta(\delta,1,\alpha^*)} \theta f(\theta|\alpha^*) d\theta = \zeta(\delta,1,\alpha^*) F(\zeta(\delta,1,\alpha^*)|\alpha^*) - \int_0^{\zeta(\delta,1,\alpha^*)} F(\theta|\alpha^*) d\theta.$$

Using (6) yields:

$$1 + \delta \int_0^{\zeta(\delta, 1, \alpha^*)} \theta f(\theta | \alpha^*) d\theta = \zeta(\delta, 1, \alpha^*) (1 + \delta F(\zeta(\delta, 1, \alpha^*) | \alpha^*)).$$

Using (A19), we now find:

$$\frac{\partial \zeta}{\partial \alpha}(\delta, 1, \alpha^*) \left(1 + \delta \int_0^{\zeta(\delta, 1, \alpha^*)} \theta f(\theta | \alpha^*) d\theta \right) = -\delta \zeta(\delta, 1, \alpha^*) \int_0^{\zeta(\delta, 1, \alpha^*)} \frac{\partial F}{\partial \alpha}(\theta | \alpha^*) d\theta.$$

Inserting into (A24) yields:

$$\frac{(1+\delta)}{\delta} \frac{\partial \Phi}{\partial \alpha} (t^{s}(\delta), \delta, \alpha^{*}) = \int_{\zeta(\delta, 1, \alpha^{*})}^{\bar{\theta}} \theta \varphi(\theta t^{s}(\delta)) \frac{\partial f}{\partial \alpha}(\theta | \alpha^{*}) d\theta$$
$$+ \int_{0}^{\zeta(\delta, 1, \alpha^{*})} \theta \varphi(\zeta(\delta, 1, \alpha^{*}) t^{s}(\delta)) \frac{\partial f}{\partial \alpha}(\theta | \alpha^{*}) d\theta$$
$$- \zeta(\delta, 1, \alpha^{*}) t^{s}(\delta) \varphi'(\zeta(\delta, 1, \alpha^{*}) t^{s}(\delta)) \int_{0}^{\zeta(\delta, 1, \alpha^{*})} \frac{\partial F}{\partial \alpha}(\theta | \alpha^{*}) d\theta.$$
(A25)

From (31), and especially $\int_{0}^{\bar{\theta}} \frac{\partial F}{\partial \alpha}(\theta | \alpha^*) d\theta = 0$, integrating by parts and using $\frac{\partial F}{\partial \alpha}(\bar{\theta} | \alpha^*) = \frac{\partial F}{\partial \alpha}(0 | \alpha^*) = 0$ yields

$$\int_0^{\bar{\theta}} \theta \frac{\partial f}{\partial \alpha}(\theta | \alpha^*) d\theta = 0,$$

which can be rewritten as

$$\int_{0}^{\zeta(\delta,1,\alpha^{*})} \theta \frac{\partial f}{\partial \alpha}(\theta | \alpha^{*}) d\theta = -\int_{\zeta(\delta,1,\alpha^{*})}^{\bar{\theta}} \theta \frac{\partial f}{\partial \alpha}(\theta | \alpha^{*}) d\theta.$$
(A26)

Integrating by parts and taking into account that $\int_0^{\overline{\theta}} (\theta - \zeta(\delta, 1, \alpha^*)) \frac{\partial f}{\partial \alpha}(\theta | \alpha^*) d\theta = 0$, (31) also implies:

$$\int_{0}^{\zeta(\delta,1,\alpha^{*})} \frac{\partial F}{\partial \alpha}(\theta|\alpha^{*})d\theta = -\int_{0}^{\zeta(\delta,1,\alpha^{*})} (\theta - \zeta(\delta,1,\alpha^{*})) \frac{\partial f}{\partial \alpha}(\theta|\alpha^{*})d\theta,$$

or

$$\int_{0}^{\zeta(\delta,1,\alpha^{*})} \frac{\partial F}{\partial \alpha}(\theta|\alpha^{*})d\theta = \int_{\zeta(\delta,1,\alpha^{*})}^{\bar{\theta}} (\theta - \zeta(\delta,1,\alpha^{*})) \frac{\partial f}{\partial \alpha}(\theta|\alpha^{*})d\theta.$$
(A27)

Using conditions (A26) and (A27) and inserting into (A24) finally gives:

$$(1+\delta)\frac{\partial\Phi}{\partial\alpha}(t^{s}(\delta),\delta,\alpha^{*}) = \delta \int_{\zeta(\delta,1,\alpha^{*})}^{\bar{\theta}} \frac{\partial f}{\partial\alpha}(\theta|\alpha^{*})v(\theta)d\theta,$$
(A28)

where

$$v(\theta) = \theta(\varphi(\theta t^s(\delta)) - \varphi(\zeta(\delta, 1, \alpha^*)t^s(\delta))) - (\theta - \zeta(\delta, 1, \alpha^*))t^s(\delta)\zeta(\delta, 1, \alpha^*)\varphi'(\zeta(\delta, 1, \alpha^*)t^s(\delta)).$$

Observe that

$$v(\zeta(\delta, 1, \alpha^*)) = v'(\zeta(\delta, 1, \alpha^*)) = 0$$

and

$$v''(\theta) = t^s(\delta)(2\varphi'(\theta t^s(\delta)) + \theta t^s(\delta)\varphi''(\theta t^s(\delta))) \le 0$$

under the assumption of Proposition 6.

Integrating by parts twice, we find:

$$\frac{(1+\delta)}{\delta}\frac{\partial\Phi}{\partial\alpha}(t^{s}(\delta),\delta,\alpha^{*}) = -\int_{\zeta(\delta,1,\alpha^{*})}^{\bar{\theta}}v''(\theta)\left(\int_{\theta}^{\bar{\theta}}\frac{\partial F}{\partial\alpha}(\theta'|\alpha^{*})d\theta'\right)d\theta > 0,$$

where the last inequality follows from the facts that $\int_{\theta}^{\overline{\theta}} \frac{\partial F}{\partial \alpha}(\theta'|\alpha^*) d\theta' > 0$ for all $\theta > 0$ under (31) and $v'' \leq 0$. Finally, inserting into (A23) yields $\alpha^* > 0$ when $\delta > 0$.