Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts

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Abstract

The paper shows that time-consistent, imperfectly targeted support to distressed institutions makes private leverage choices strategic complements. When everyone engages in maturity mismatch, authorities have little choice but intervening, creating both current and deferred (sowing the seeds of the next crisis) social costs. In turn, it is profitable to adopt a risky balance sheet. These insights have important consequences, from banks choosing to correlate their risk exposures to the need for macro-prudential supervision.

Keywords: monetary policy, funding liquidity risk, strategic complementarities, macro-prudential supervision

JEL numbers: E44, E52, G28.

One of the many striking features of the recent financial crisis is the extreme exposure of economically and politically sensitive actors to liquidity needs and market conditions:

- Subprime borrowers were heavily exposed to interest rate conditions, which affected their monthly repayment (for those with adjustable rate mortgages) and conditioned their ability to refinance (through their impact on housing prices).

- Commercial banks, which traditionally engage in transformation, had increased their sensitivity to market conditions. First, and arbitraging loopholes in capital adequacy regulation, they pledged substantial amounts of off-balance-sheet liquidity support to the conduits they designed.[1] These conduits had almost no equity on their own and rolled over commercial paper with an average maturity under one month. For many large banks, the ratio of asset backed commercial paper to the bank's equity was substantial (for example, in January 2007, 77.4% for Citibank and 201.1% for ABN Amro, the two largest conduit administrators).[2] Second, on the balance sheet, the share of retail deposits fell from 58% of bank liabilities in 2002 to 52% in 2007.[3] Third, going forward commercial banks counted on further securitization to provide new cash. They lost an important source of liquidity when the market dried up.

- Broker-dealers (investment banks) gained market share and became major players in the financing of the economy. Investment banks rely on repo and commercial paper funding much more than commercial banks do. An increase in investment banks' market share mechanically resulted in increased recourse to market financing.[4]

The overall picture is one of a wide-scale maturity mismatch. It is also one of substantial systematic-risk exposure, as senior CDO tranches, a good share of which was held by commercial banks, amounted to "economic catastrophe bonds".[5]

This paper argues that this wide-scale transformation is closely related to the unprecedented intervention by central banks and treasuries.[6] As described more in detail when we map out the interpretation of our model in terms of actual policies, roughly two categories of

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[1]See e.g. Figure 2.3 (page 95) in Acharya-Richardson (2009), documenting the widening gap between total assets and risk-weighted assets.

[2]See Table 2.1 (page 93) in Acharya-Richardson for the numbers for the 10 largest administrators.

[3]Source: Federal Reserve Board's Flow of Funds Accounts.

[4]Source: Federal Reserve Board's Flow of Funds accounts.

[5]To use Coval et al (2007)'s expression.

[6]Strikingly, by March 2009, the Fed alone had seen its balance sheet triple in size (to $ 2.7 trillion) relative to 2007.

interventions were pursued in order to facilitate financial institutions' access to refinancing. For lack of better words, we term them respectively interest rate and transfer policies. By interest rate policies, we have in mind various forms of government intervention which effectively lower borrowing costs for banks: lowering the Fed Funds rate to zero, extending debt guarantees to a wide range of financial institutions, accepting low-quality assets as collateral with low haircuts in loans or repurchasing agreements, purchasing commercial paper in the primary market etc. By transfer policies, we refer to interventions that primarily boost the net worth of financial institutions without lowering their borrowing cost: recapitalizations, the purchase of legacy assets at inflated prices etc. The distinction between these two categories is sometimes blurred in practice. From our perspective, the key distinguishing feature is whether the intervention under consideration reduces banks' borrowing costs or simply acts to boost their net worth.

In a nutshell, the central argument of the paper is that private leverage choices depend on the anticipated policy reaction to the overall maturity mismatch. Difficult economic conditions call for public policy to help financial institutions weather the shock. Policy instruments however are only imperfectly targeted to the institutions they try to rescue. For example, the archetypal non-targeted policy, lowering the Fed Funds rate, benefits financial institutions engaging in maturity mismatch, but its effects apply to the entire economy. An accommodating interest rate policy involves (a) an invisible subsidy from consumers to banks (the lower yield on savings transfers resources from consumers to borrowing institutions), (b) current costs, such as the (subsidized) financing of unworthy projects by unconstrained entities, and (c) differed costs (the sowing of seeds for the next crisis, both through incentives for maturity mismatch, going forward, and the authorities' loss of credibility).

While the first cost is proportional to the volume of refinancing, the other two are not and are instead akin to a fixed cost. This generates strategic complementarities in balance-sheet riskiness choices. It is ill-advised to be in a minority of institutions exposed to the shock, as policymakers are then reluctant to incur the "fixed cost" associated with active interest rate policy. By contrast, when everybody engages in maturity transformation, the central bank has little choice but intervening. Refusing to adopt a risky balance sheet then lowers banks' rate of return. It is unwise to play safely while everyone else gambles.[7]

The same insight applies when some players expose themselves to liquidity risk either

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[7]As Charles Prince, then CEO of Citigroup, famously stated in the summer of 2007: "*as long as the music is playing, you have to get up and dance*".

because they are unsophisticated[8] or because they engage in regulatory arbitrage.[9] Strategic complementarities then manifest themselves in the increased willingness of other actors to take on more liquidity risk due to the presence of unsophisticated players or regulatory arbitrageurs. A reinterpretation of our analysis is thus in terms of an amplification mechanism.

The paper's first objective is to develop a simple framework that is able to capture and build on these insights. Corporate entities (called "banks") choose their level of short-term debt (or, equivalently, whether to hoard liquid instruments in order to meet potential liquidity needs). In the basic model liquidity shocks are correlated, and so there is macroeconomic uncertainty. Maturity transformation is intense in the economy when numerous institutions take on substantial short-term debt. The issuance of short-term debt enables banks to increase their leverage and investment, but exposes them to a potential refinancing problem in case of a shock. When privileging leverage and scale, bankers thereby put at risk "banking stakeholders", a designation regrouping those agents who would be hurt in case banks have to delever: bankers themselves, industrial companies that depend on bank loans for their financing, and employees of those banks and industrial companies.[10]

Authorities maximize a weighted average of consumer surplus and banking stakeholders' welfare. Focusing in a first step on interest rate policy, they can, in case of an aggregate shock, facilitate troubled institutions' refinancing by lowering the effective interest rate at which banking entrepreneurs borrowing. However, loose interest rate policy, besides transferring resources from consumers to banks with refinancing needs, might for example facilitate the financing of unworthy projects (in the basic version) or entails future costs (future illiquidity of institutions or loss of credibility). This distortion is akin to a fixed cost, which is worth incurring only if the size of the troubled sector is large enough. We obtain the following insights:

- Excessive maturity transformation. The central bank supplies too much liquidity in the time-consistent outcome. Our theory therefore brings support to the view that authorities in the recent crisis had few options when confronted with the fait accompli, and that the crisis should have been contained ex ante through more careful prudential policies. While prudential supervision is traditionally concerned with the solvency of

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[8]Such players may for instance miscalibrate the risk involved in relying on funding liquidity or on securitization to cover their future needs, and thereby mistakenly engage in maturity mismatch.

[9]As was the case with largely underpriced liquidity support to conduits.

[10]Note that consumers may have multiple incarnations: As taxpayers/savers, they should oppose an intervention, while as employees of these corporate entities, they might welcome it. All these effects are taken into account in our welfare analysis.

individual institutions, our framework suggests the potential value of a new, macro-prudential approach, in which prudential regulators consider not only the individual institutions' transformation activities, but also the overall transformation of the strategic institutions.[11],[12]

- Optimal regulation. In our model, optimal regulation takes the form of a liquidity requirement or equivalently of a cap on short-term debt. Importantly, breaking down banks into smaller banks would achieve no benefit in our framework. The basic problem here is not too big too fail, but rather that the banks as a whole are doing too much maturity mismatch, and are taking on too much correlated risk.

- Regulatory pecking order. If regulation is costly, our model suggests that regulation should be confined to a subset of key institutions, the ones that authorities are the most tempted to bail-out ex post.[13]

- Endogenous macroeconomic uncertainty. We relax the correlated-shock assumption and let banks choose the correlation of their shock with that of other banks. We find that they actually choose to maximize the correlation of their shocks due to the nature of the policy response. This result runs counter to conventional wisdom. Financial theory (CAPM) predicts that, when faced with a choice among activities, a firm will want to take as much risk as possible in those states of nature in which the economy is doing well. That is, it will strive to be as negatively correlated as possible with the market portfolio.

- Sowing the seeds of the next crisis. Loose interest rate policy today increases the likelihood of future crises. First, they signal the central bank's willingness to accommodate maturity mismatches, and deprive it of future credibility. Second, they stimulate new

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[11]Although extremely imperfect, liquidity regulation does exist at the micro level (both through stress tests under Basel II, and through the definition of country-specific liquidity ratios).

[12]These questions are at the forefront of the regulatory reform agenda. The Financial Stability Forum (2009) calls for "a joint research program to measure funding and liquidity risk attached to maturity transformation, enabling the pricing of liquidity risk in the financial system" (Recommendation 3.2) and recommends that "the BIS and IMF could make available to authorities information on leverage and maturity mismatches on a system-wide basis" (Recommendation 3.3).

[13]These strategic institutions correspond to large retail banks (where size matters indirectly because of the disruption in the payment and credit systems, or because of the greater coverage in the media), or to other large financial institutions that are deeply interconnected with them through opaque transactions (as was the case recently with AIG or the large investment banks). They also include those with close connections with the central bank; in the latter respect, while starting with Barro-Gordon (1983) the literature on central bank independence as a response to time-inconsistency has emphasized political independence, our analysis stresses the need for independence with respect to the financial industry.

maturity mismatches through a price effect: They make short-term debt cheaper, encouraging maturity mismatches; and they provide a subsidy to capital, encouraging overall leverage.

Interest rate policies are rough instruments because they entail distortions. By contrast transfer policies do not entail similar distortions and instead involve only a subsidy. One might therefore conjecture that interest rate policy is a dominated instrument when such transfer policies are available. Relatedly, we need to check the robustness of the insights stated above to optimal policy interventions.

Accordingly, the second objective of the paper is to analyze the optimal bailout mix using a mechanism design approach. We allow authorities to operate direct transfers to institutions. However when implementing transfer policies, they face an asymmetry of information (they are unsure which banks are distressed or intact); consequently, direct transfers entail a different set of distortions, associated with wasted-support costs. We characterize the optimal policy intervention (interest rate and transfers) given informational constraints. We show that:

- Interest rate policy is actually always used in equilibrium; indeed transfers are not even used unless the crisis affects a large fraction of the banks, in which case interest rate policy and transfers are used in conjunction. The key insight is that interest rate policy is a market-driven solution, in that it benefits primarily those institutions with actual borrowing needs; put more technically, it helps screen out opportunistic institutions with limited refinancing needs. While transfers better focus on strategic actors, they entail a greater waste of resources by supporting entities that have no need for, or should not engage in refinancing.

- The insights gleaned for pure interest rate bailouts carry over to optimal bailouts: strategic complementarities in the size and quality of liquidity positions, excessive maturity transformation, pecking order of regulation and endogenous macroeconomic uncertainty.

The paper is organized as follows. Section I sets up the model. Section II analyses the commitment benchmark, where the central bank can announce and stick to an interest-rate policy. Section III performs the same exercise for the time-consistent outcome. Section IV draws the implications for regulation. Section V provides the two foundations for the hazards

of low interest rate policies as sowing the seeds for the next crisis. Section VI allows for the full range of policy instruments and derives the optimal bailout policy. Finally, Section VII concludes.

Relationship to the literature. Our paper is related to several disjoint bodies of literature. The importance of keeping interest low in recessions is classic in macroeconomic theory. Our paper contributes to this literature first by pointing out a new channel through which interest rate policy suffers from time inconsistency (of the kind emphasized by Finn E. Kydland and Edward Prescott 1977), and second by viewing interest rate policy in a broader bailout context in which support to institutions and asset prices are alternative instruments.

Potential macroeconomic shortages of liquidity exist if corporations are net lenders (Michael Woodford 1990) or if corporations are net borrowers and face macroeconomic shocks (Bengt Holmström and Jean Tirole 1998). The literature on aggregate liquidity has emphasized the role of governments in providing (possibly contingent) stores of value that cannot be created by the private sector. Like in Holmström and Tirole, liquidity support is viewed here as redistribution from consumers to firms in bad states of nature; it however is an ex post redistribution rather than a planned one, and it emphasizes the role of interest rates and more generally borrowing costs in enabling refinancing.

Time-inconsistency from rescuing banks and the resulting moral hazard problems in a single-bank context have been emphasized by numerous works, starting with Walter Bagehot (1873).

Through its emphasis on strategic complementarities, our paper is reminiscent of the wide body of literature on multiple equilibria in macroeconomics, starting with Peter A. Diamond (1982) and Russell W. Cooper and Andrew John (1988) (see, e.g., Cooper 1999 for a review). Our paper emphasizes the idea that strategic complementarities stem from the government's policy response. In that, it is particularly related to Stephen Morris and Hyun Song Shin (1998), Martin Schneider and Aaron Tornell (2004) and Romain Rancière, Aaron Tornell and Frank Westermann (2008). Morris and Shin, and Schneider and Tornell are concerned with exchange rates while Rancière, Tornell and Westermann focus on a risky technological choice. These papers posit that the government accommodates private agents once the latter have reached some exogenous threshold of private involvement (in speculation, currency mismatch or realized returns). This threshold gives rise to strategic complementarities. An important difference in our model is that the incentives to bail out and hence the policy reaction function are endogenized. This puts the time-inconsistency of

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policy at the center stage and has important positive implications for comparative statics, as well as normative consequences by allowing us to study the optimal design of regulation.

Viral Acharya and Tanju Yorulmazer (2007, 2008) study the incentives for banks to correlate the risks inherent in their investment choices. In Acharya-Yorulmazer (2007), the possibility for one bank to acquire the other pushes banks to minimize their correlation. However, they assume that when both banks fail, both banks are bailed out. If the bailout guarantee when both banks fail is worth more than the rent obtained by the surviving bank when only one bank fails and is sold to the other, then banks seek to maximize their correlation. Acharya-Yorulmazer (2008) introduces a richer model with fire-sales and makes the point that from an ex-post perspective, bailing out failed banks and subsidizing intact banks to take over failed banks have similar effects, but that the latter is preferable ex ante because it induces banks to differentiate their risks. There are important differences with our paper: first, in our setup, bank managers are indispensable to the project, so that intact banks are at no comparative advantage over outside investors when refinancing failed banks; second, these papers do not emphasize the role of untargeted policy instruments; third, they do not allow banks to vary the amount of risk that they take.[14]

Humberto N. Ennis and Todd Keister (2009, 2010) study a modification of the model of Douglas W. Diamond and Phillip H. Dybvig (1983) where a policy maker can only choose policies (such as deposit freezes) that are contingent on the fraction of agents who have already withdrawn their deposits, and that are efficient ex post. They point out that bank run equilibria can exist together with the efficient equilibrium. Our paper shares the idea that policy responses (under no commitment) can generate multiplicity. However, while they focus on the incentives of depositors to run, we analyze instead the ex ante choices of banks.

Diamond and Raghuram Rajan (2009) emphasize, as we do, that interest rate policy is time inconsistent and that low-interest rate policies may encourage excessive leverage. Interestingly, in their framework, because of an assumed form of market incompleteness (non-contingent deposits) absent in our model, optimal interest rate policy under commitment involves both low interest rates in bad times and high interest rates in good times.

Varadarajan V. Chari and Patrick J. Kehoe (2010) study a model in which inefficient ex-post liquidations are required for ex-ante efficiency, but the possibility of ex-post bailouts introduces a time-inconsistency problem. They show, as we do, that regulation in the form of specific ex-ante restrictions on private contracts can increase welfare. Interestingly, in

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[14]In our model, this occurs through the choice of short term debt, leverage, and maturity mismatch.

their setup, the ex-post cost of bailouts is that they trigger a bad continuation equilibrium of the policy game. In the best equilibrium, any deviation triggers the worst equilibrium, introducing a fixed cost from bailouts. This reputational mechanism is therefore similar to ours. We elaborate on this analogy in Section V.

Finally, the optimal regulation in our model bears some resemblance to Anil K. Kahsyap, Rajan and Jeremy Stein (2008). They propose replacing capital requirements by mandatory capital insurance policy, whereby banks are forced to hoard liquidity, in the form of T-bills.

I. The model

A. Banks

There are three periods, $t = 0, 1, 2$. Banking entrepreneurs have utility function $U = c_0 + c_1 + c_2$, where c_t is their date-t consumption. They are protected by limited liability and their only endowment is their wealth A at date 0. Their technology set exhibits constant returns to scale. At date 0 they choose their investment scale i and a level of short-term debt (see below). At date 1, a safe cash flow πi accrues, that can be used to pay back the short-term debt. Uncertainty bears on the investment project: It is intact with probability α, and distressed with probability $1 - \alpha$. Whether the project is intact or distressed depends on the realization of an aggregate shock—a "crisis". In other words, the shocks impacting the different banking entrepreneurs are perfectly correlated.[15]

If the project is intact, the investment delivers at date 1; it then yields, besides the safe cash flow πi, a payoff of $\rho_1 i$, of which $\rho_0 i$ is pledgeable to investors.[16] If the project is distressed, the project yields no payoff at date 1, except for the safe cash flow πi. It yields a payoff at date 2 if fresh resources j are reinvested. The project can be downsized to any level $j \leq i$. It then delivers at date 2 a payoff of $\rho_1 j$, of which $\rho_0 j$ is pledgeable to investors.[17] The

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[15]Later we will allow entrepreneurs to choose the correlation of their shock with those faced by other entrepreneurs.

[16]As usual, the "agency wedge" $\rho_1 - \rho_0$ can be motivated in multiple ways, including limited commitment, private benefits or incentives to counter moral hazard (see Section I.B; see also Holmström and Tirole 2010).

[17]Note that we are assuming that the manager is indispensable to the project. As a result, intact banks are at no advantage over consumers in buying or operating distressed banks. This assumption turns off a channel that could generate strategic substituabilities, whereby some institutions (banks, or other specialist buyers) overhoard liquidity to secure available resources when a lot of banks are distressed and attractive opportunities arise. On the other hand if banks are expected to be rescued (as is the case in the paper), specialist buyers have no incentive to hoard liquidity. See Acharya-Yorulmazer (2007, 2008), and chapter 7

following assumption will guarantee that the projects are attractive enough that banking entrepreneurs will always invest all their net worth.[18]

Assumption 1 *(high return)* $\rho_1 > 1 - \pi + 1 - \alpha$.

The interest rate is a key determinant of the collateral value of a project. It plays an important role in determining the initial investment scale i as well as the reinvestment scale j. We explain how interest rates are determined in Section I.B. In sum, the gross rate of interest is equal to 1 between dates 0 and 1. Between dates 1 and 2, the interest rate is equal to 1 in the absence of a crisis, and to $R \leq 1$ otherwise.[19] For the rest of the paper, we adopt the convention that R refers to the interest rate between dates 1 and 2 if there is a crisis.

At the core of the model is a maturity mismatch issue, where a long-term project requires occasional reinvestments. The bank has to compromise between initial investment scale i and reinvestment scale j in the event of a crisis. Maximizing initial scale i requires loading up on short-term debt and exhausting reserves of pledgeable income. This in turn forces the bank to downsize and delever in the event of a crisis. Conversely, limiting the amount of short-term debt to mitigate maturity mismatch requires sacrificing initial scale i.

The bank issues state-contingent short-term debt. It is always optimal to set short-term debt in event of no crisis equal to πi. We denote di (where $d \leq \pi$) the amount of short-term debt in the event of a crisis; we refer to it simply as short-term debt through-out the paper. The excess $xi \equiv (\pi - d)i$ of the safe cash flow πi over debt payments di represents cash available at date 1 in the event of a crisis (x is the analog of a liquidity ratio). We assume that any potential surplus of cash over liquidity needs for reinvestment—$\max\{(\pi - d)i - j(1 - \rho_0/R), 0\}$ —is consumed by banking entrepreneurs. The policy of pledging all cash that is unneeded for reinvestment is always weakly optimal. Pledging less is also optimal (and leads to the same allocation) if the entrepreneur has no alternative use of the unneeded cash to distributing to investors. However, if the entrepreneur can divert (even an arbitrarily small) fraction of the extra cash for her own benefit, then pledging the entire unneeded cash is *strictly* optimal.

At date 1, in the adverse state, the bank can issue new securities against the date-2

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in Holmström-Tirole (2010).

[18]This condition is intuitive: investing 1 at $t = 0$ and 1 at date 1 if a crisis occurs yields a return $\rho_1 + \pi$ and costs $1 + (1 - \alpha)$.

[19]In all the cases that we consider, it is always optimal for the central bank to set the interest rate to 1 at date 0 (see Section V), and also at date 1 if there is no crisis, but to some $R \leq 1$ at date 1 if there is a crisis.

pledgeable income $\rho_0 j$, and so its continuation $j \in [0, i]$ must satisfy:

$$j \leq (\pi - d)i + \frac{\rho_0 j}{R}$$

yielding continuation scale:

$$j = \min \left\{ \frac{x}{1 - \frac{\rho_0}{R}}, 1 \right\} i.$$

This formula captures the fact that lower interest rates facilitate refinancing. A banking entrepreneur would never choose to have excess liquidity and so we restrict our attention to $d \in [\pi - (1 - \rho_0/R), \pi]$ or equivalently $x \in [0, 1 - \rho_0/R]$.

The banks needs to raise $i - A$ from outside investors at date 0. Because the bank returns $di + (\pi - d)i + \rho_0 i$ to these investors in the good state and only di in the bad one, its borrowing capacity at date 0 is given by:

$$i - A = \alpha (\pi i + \rho_0 i) + (1 - \alpha)di$$

i.e.

$$i = \frac{A}{1 - \pi - \alpha \rho_0 + (1 - \alpha)x}.$$

The banking entrepreneur therefore maximizes over $d \in [\pi - (1 - \rho_0/R), \pi]$ or equivalently $x \in [0, 1 - \rho_0/R]$:

$$(\rho_1 - \rho_0)[\alpha i + (1 - \alpha)j] = (\rho_1 - \rho_0) \left[\frac{\alpha + (1 - \alpha)\frac{x}{1 - \rho_0/R}}{1 - \pi - \alpha \rho_0 + (1 - \alpha)x} \right] A$$

The banking entrepreneur loads up on short-term debt ($x = 0$, i.e., $d = \pi$) if and only if

$$\alpha + \pi > 1 + \alpha \rho_0 \left(\frac{1}{R} - 1 \right),$$

and takes on just enough short-term debt to be able to continue full scale ($x = 1 - \rho_0/R$, i.e., $d = \pi - 1 + \rho_0/R$) otherwise.

We assume that the banking entrepreneurs prefer to limit the amount of short-term debt to have enough liquidity to continue at full scale in the adverse state of nature in the relevant range of interest rates (which will be $[\rho_0, 1]$):[20]

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[20]Note that Assumption 2 implies that $1 - \pi - \alpha \rho_0 > 0$ which guarantees that investment is finite.

Assumption 2 *(demand for liquidity):* $\alpha + \pi < 1$.

B. Rest of the economy

Consumers born at date $t \in \{0, 1\}$ consume at date $t + 1$; so their utility is $u_t = c_{t+1}$. They are endowed with a large amount or resources (savings) s when born.

A short-term *storage technology* yields 1 in the next period for 1 invested today. In particular the natural rate of interest (the marginal rate of transformation) between dates 1 and 2 is $R = 1$. For the date-1 interest rate to be $R \neq 1$, the storage technology must be taxed at rate $1 - R$ (see later for interpretations). The proceeds are rebated lump sum to consumers at date 2. Throughout the paper, we assume that s is large enough to finance all the necessary investments in the projects of banking entrepreneurs at each date t. As a result, consumers always invest a fraction of their savings in the short-term storage technology.

This modelling device is a way to capture a range of policy interventions that reduce borrowing costs for banks. For instance, taxing the short-term storage technology and rebating the proceeds lump-sum to consumers is essentially equivalent to subsidizing investment in the banks and financing this subsidy by a lump-sum tax on consumers. We elaborate on this analogy below and propose several interpretations for which this way of modelling interest rate policy could be a convenient reduced form. For now, we do not introduce any other instrument. In Section VI, we allow for the full range of policy instruments and derive the optimal bailout policy.

Assumption 3 *(interest rate distortion): The set of feasible interest rates is $[\rho_0, 1]$. Furthermore, there exists a fixed distortion or deadweight loss $L(R) \geq 0$ when the interest rate R diverges from its natural rate: $L(1) = L'(1) = 0$, and L is decreasing on $[\rho_0, 1]$.*

The upper bound at 1 for the interest rate R is not crucial but simplifies the analysis. As we shall see below, it will be used to normalize the optimal interest rate under commitment to $R = 1$. One can justify this assumption by positing arbitrage (foreigners or some long-lived consumers would take advantage of $R > 1$) or by assuming that marginal distortions $L'(R)$ are very high beyond 1. But again, we want to emphasize that this particular assumption only simplifies the exposition and plays no economically substantive role in the analysis.

The lower bound at ρ_0 for the interest rate R is also without loss of generality. Indeed, as will become apparent, the central bank will never find it optimal to lower the interest rate in

times of crisis below ρ_0. At that interest rate, projects can continue at full scale even when no liquidity has been hoarded. Lowering the interest rate below ρ_0 would only increase the distortion associated with interest policy at no gain.

Assumption 4 *(consumers): Suppose that date-0 investment is equal to i, that banks hoard liquidity x and so can salvage $j = xi/\left(1 - \rho_0/R\right)$ in case of crisis. Then*

(i) if there is a crisis at date 1, date-1 consumer welfare is $V = -L(R) - (1 - R)\rho_0 j/R$;

(ii) if there is no crisis at date 1, date-1 consumer welfare is $V = -L\left(1\right) = 0$.

In (i), the second term in V stands for the implicit subsidy from savers to borrowing banks. Indeed date-1 consumers' return on their savings $\widetilde{s}$ is $R\widetilde{s} + (1 - R)\left(\widetilde{s} - \rho_0 j/R\right)$ (the last term representing the lump-sum rebate on the $\widetilde{s} - (\rho_0 j/R)$ invested in the storage technology), or $\widetilde{s} - (1 - R)\rho_0 j/R$.[21] Note that welfare V does not include the additional indirect benefits that firms' managers and workers might derive from banks functioning at high scale. More on this below. Finally, we ignore the welfare of date-0 consumers as they have constant utility $u_0 = s$.

Our modelling of interest rate policy deserves some comments. It is a stylized representation of some actual interest rate policies. Their common feature is to reduce borrowing costs for banks. We list a few of those below.

Interpretation 1. One case in point is unconventional monetary policy.[22] Extended debt guarantees by the government reduce the rate (R) paid both by constrained institutions that the government wants to help (the "banks") and by other borrowers. The subsidy is paid by taxpayers who end up bearing the risk of debt.[23] Similarly, accepting assets as collateral at low haircuts in loans or repurchasing agreements and directly purchasing commercial paper at favorable terms lower the effective interest rate faced by borrowers. Such interventions

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[21]Note that we use the notation $\tilde{s}$ instead of s for the savings of date-1 consumers. This is because under our Interpretation 1 below, some of the savings s of date-1 consumers are invested in alternative wasteful investment projects. As a result, only a part $\tilde{s}$ of their savings are split between reinvestment in banks and the short-term storage technology.

[22]See, e.g., Gertler-Karadi (2009) and Gertler-Kiyotaki (2009) for models with both conventional and unconventional monetary policies.

[23]One way to formalize this is as follows. Imagine that in case of a crisis, the pledgeable part of the return ρ_0 from reinvesment is the expectation of a random variable realized at $t = 2$ that takes the value $\rho_0^+ > \rho_0$ with probability λ and 0 with probability $1 - \lambda$. Banking entrepreneurs can issue (defaultable) debt with nominal value ρ_0^+. A guarantee from the government to deliver a fraction ϕ of the value of every debt contract in case of default then reduces the (gross) interest rate demanded by creditors by a proportion factor $\lambda/\left[\lambda + (1 - \lambda)\phi\right] < 1$.

involve (in expectation) a subsidy from savers to borrowers, reduce the marginal borrowing cost of banks, and are therefore captured by our model.

With this in mind, the deadweight loss may for instance result from the date-1 financing of projects that have negative net present value at the natural rate. Suppose that there is a distribution of financially unconstrained firms with projects that have unit cost and return κ with cumulative distribution function $H(\kappa)$. Then the deadweight loss if consumers and project owners are weighted equally is $L(R) = \int_R^1 (1 - \kappa)dH(\kappa)$. If the projects' owners receive welfare weight $\beta_u \leq 1$ relative to consumers instead, then $L(R) = (1 - R)[H(1) - H(R)] - \beta_u \int_R^1 (\kappa - R)dH(\kappa)$ still satisfies our assumptions.[24]

Moreover, with our assumed preferences for consumers, a tax on the storage technology between dates 1 and 2 combined with a lump sum rebate is exactly equivalent to a subsidy on reinvestment in banks between dates 1 and 2. The distortion behind $L(R)$ arises because these projects are subsidized at the same rate as reinvestment in banks (this implicitly assumes that the government cannot screen out these projects from genuine positive net present value bank projects).

Interpretation 2. Another interest rate policy captured in a stylized way by the model is conventional monetary policy. An interpretation closely related to Interpretation 1 relies on a view of the monetary transmission mechanism whereby higher reserves allow banks to lever more through access to cheap retail deposits, as deposit insurance tends to be underpriced (at least during hard times since it is not indexed on the banks' riskiness).[25] This involves an implicit subsidy to banks since this deposit insurance is backed by taxes on consumers. Increasing reserves (or reducing reserve requirements) therefore both reduces the borrowing cost of banks (i.e. lowers the effective interest rate face by banks) and involves a subsidy from taxpayers to borrowing banks.

Interpretation 3. The deadweight loss function L can also be interpreted as a reduced form of a more standard distortion associated with conventional monetary policy, as emphasized in the New-Keynesian literature. Here we have in mind not a short-term intervention, but a prolonged reduction of interest rates (a year to several years, think of Japan). Even though our model is entirely without money balances, sticky prices or imperfect competition, it captures a key feature of monetary policy in New-Keynesian models routinely used to discuss and model monetary policy. In New-Keynesian models, the nominal interest

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[24]Note that in this case, project owners are lumped with consumers, and their welfare is included in consumer welfare.

[25]See Stein (2010) for a detailed exposition of this view.

rate is controlled by the central bank. Prices adjust only gradually according to the New-Keynesian Phillips Curve, and the central bank can therefore control the real interest rate. The real interest rate regulates aggregate demand through a version of the consumer Euler equation—the dynamic IS curve. Without additional frictions, the central bank can achieve the allocation of the flexible price economy by setting nominal interest rates so that the real interest rate equals to the "natural" interest rate. Deviating from this rule introduces variations in the output gap together with distortions by generating dispersion in relative prices. To the extent that these effects enter welfare separately and additively from the effects of interest rates on banks' balance sheets—arguably a strong assumption—our loss function $L(R)$ can be interpreted as a reduced form for the loss function associated with a real interest rate below the natural interest rate in the New-Keynesian model.[26][27] Under this interpretation, monetary policy works both through the usual New-Keynesian channel and through its effects on banks via a version of the "credit channel".[28]

C. Welfare and policy-making

The authorities (the "central bank") control the date-1 real rate of interest.

Assumption 5 *(welfare function): At date 1, the central bank's objective function is a weighted average W of consumer welfare V and continuation scale j ($j = i$ if there is no crisis): $W = V + \beta j$. At date 0, the central bank's objective function is the expectation of its date-1 objective function.*

The second term βj in the social welfare function deserves some comments. One possible interpretation is as follows. Imagine that, say, three categories of banking stakeholders' benefit from the banks' ability to continue. First, and most obviously the banking entrepreneurs themselves: They receive rent $s_b j$, where s_b is the banks' stake in continuation. Second, the

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[26] Yet another cost, absent in cashless New Keynesian models, is the so called inflation tax and arises when money demand is elastic.

[27] Because they are not our focus, we imagine here that the traditional time-inconsistency problems associated with monetary policy in the New-Keynesian model have been resolved. As is well known, this is the case if a sales subsidy is available to eliminate the monopoly price distortion.

[28] There are two versions of the credit channel (see Bernanke-Gertler 1995 for a review): the "balance sheet channel" and the "bank lending channel". Our model is consistent with the former in its emphasis on the effect of interest rates on collateral value. It is consistent with the latter in that low interest rates boost the real economy by facilitating bank refinancing and thereby increasing the volume of loanable funds to the economy.

higher j, the better off their borrowers. Third, the workers working in banks and industrial companies; to the extent that they are better off employed (e.g., they receive an efficiency wage) and that preserved employment is related to j, then workers' welfare grows with j. Thus if s_f and s_w denote the stakes of the industrial firms and the workers, and if β_b , β_j and β_w denote the three categories of stakeholders' welfare weights or political influence, then $\beta j = \left(\beta_b s_b + \beta_f s_f + \beta_w s_w \right) j$. Assumptions 1 and 6 (see below) imply that $\beta_b \leq 1$: this guarantees that the central bank never seeks to lower interest rates more than what is necessary to guarantee reinvestment at full scale since the freed-up cash is appropriated by banking entrepreneurs and hence represents an unattractive transfer to them from consumers.

This can be formalized further along the lines of Holmström-Tirole (1997). We only sketch it here: At date 0, the bank makes an investment in knowledge/staff so as to be able to monitor a mass i of firms at date 1. These firms enter in a relationship with the bank at date 0; from then on, they share available resources in coalition with the banks. Independently of whether or not firms succeed, they each produce short-term profits π at date 1 (hence the mass i of firms produces short-term profits πi). Firms succeed (return r per firm) or fail (return 0). Success is guaranteed if the bank monitors, and the firm managers as well as the workers in the firm do not shirk. Otherwise success accrues with probability 0. Shirking on monitoring yields benefit b, shirking for a firm manager brings benefit f, and shirking for a firm worker brings benefit w. Therefore incentive payments b, f and w per firm are required to discipline the bank, the firm manager and the workers. For simplicity, we assume that workers are cashless. Firms are cashless at date 0; at date 1, each firm has resources π. If there is no crisis, then the return of firms (r or 0 per firm) occurs in period 1. A crisis means that firms to be monitored must invest 1 each at date 1 and the return on reinvested funds occurs in period 2. This model is summarized by the equations in Sections I.A and I.B with $\rho_0 = r - (b + f + w)$, $\rho_1 = r - (f + w)$, $s_b = b$, $s_f = f$ and $s_w = w$. The only difference is that the total return on a unit of successful investment is $r > \rho_1$. This difference, however, makes no difference to our analysis, since only a fraction ρ_0 of the return can be pledged to outside investors, and banking entrepreneurs get to keep the difference between ρ_1 and the fraction of the return pledged to outside investors.

A roadmap. We will analyze two situations: one where the central bank can commit at date 0 to a specific contingent policy at date 1, and the (probably more likely) alternative where the central bank lacks commitment and instead determines its policy at $t = 1$ with no regard for previous commitments. In both cases, banking entrepreneurs and consumers form expectations regarding the interest rate $R \in [\rho_0, 1]$ that will be set if a crisis occurs.

Note that we have not included the interest rate R_0 between dates 0 and 1 in the set of policy instruments. In Section V.A we relax this assumption and allow for the storage technology between dates 0 and 1 to be taxed, with the proceeds rebated lump sum to consumers. We show that both under commitment and under no commitment, $R_0 = 1$ is always chosen. This justifies proceeding under the assumption that $R_0 = 1$.

II. Commitment solution

This Section analyzes the equilibrium when the date-1 interest rate is chosen at date 0. Ex-ante welfare is

$$W^{\text{ex ante}}(R) \equiv \alpha[V(1) + \beta i(R)] + (1 - \alpha)[V(R) + \beta j(R)]$$

where

$$V(R) \equiv -[L(R) + (1 - R)\frac{\rho_0 j(R)}{R}]$$

and

$$j(R) = i(R) = \frac{A}{1 - \pi - \alpha\rho_0 + (1 - \alpha)(1 - \frac{\rho_0}{R})}.$$

Using $V(1) = 0$, we can write

$$W^{\text{ex ante}}(R) = \left[\beta - (1 - \alpha)\frac{1 - R}{R}\rho_0\right] i(R) - (1 - \alpha)L(R).$$

An increase in the interest rate $R \in [\rho_0, 1]$ in case of crisis reduces the distortion $L(R)$. It also reduces the banks' leverage and therefore investment $i(R)$, which involves a redistribution from banking stakeholders to the rest of the population. We assume that $\beta \leq 1 - \alpha + 1 - \pi - \rho_0$, which implies that $[\beta - (1 - \alpha)(1 - R)\rho_0/R] i(R)$ is non-decreasing in R and hence that the optimum under commitment is $R = 1$.

Note that this condition is equivalent to that making socially undesirable a date-0 unit lump-sum transfer from consumers to banks in the absence of interest rate policy ($R = 1$): such a transfer would have social welfare cost $1 - \beta[i(1)/A] = 1 - \beta/(1 - \alpha + 1 - \pi - \rho_0)$, where the term in brackets is the leverage multiplier.

Assumption 6 *(No ex-ante wealth transfer):* $\beta \leq 1 - \alpha + 1 - \pi - \rho_0$.

Proposition 1 *The optimal interest rate policy under commitment features $R^c = 1$.*

III. No-commitment solution

Let us now assume that the interest rate is set at date 1, without commitment. At date 0, investors and banking entrepreneurs form an expectation for the interest rate $R^* \in [\rho_0, 1]$ that the central bank will set if a crisis occurs. Based on this expectation, the representative bank invests at scale $i(R^*)$ and hoards just enough liquidity $x^* i(R^*)$ to be able to reinvest at full scale in the event of a crisis, where $x^* = 1 - (\rho_0/R^*)$.

The central bank's decision. At date 1, the central bank is not bound by any previous commitment and is free to set the interest rate to maximize welfare from date 1 on. We have to distinguish two cases depending on whether or not the economy is in a crisis.

If there is no crisis, it is optimal to set $R = 1$. There is no point in lowering the interest rate since all banks are intact. If there is a crisis, the central bank is confronted with the following trade-off. By setting a low interest rate, it can limit the amount of downsizing that banks have to undergo. But this comes at the cost of a large interest rate distortion. An additional cost comes together with the implicit subsidy to banks, in the form of a redistribution of resources from consumers to banking stakeholders.

Because $\beta_b \leq 1$, the central bank would never set $R < R^*$. Indeed, lowering R below R^* does not increase continuation scale but merely redistrbutes resources from consumers to banking entrepreneurs, and comes at the cost of a greater interest rate distortion. However, the central bank might be tempted to set $R > R^*$. In this case, banks are forced to downsize. The reinvestment scale is determined according to:

$$j = \frac{x^*}{1 - \frac{\rho_0}{R}} i(R^*) \iff j = \frac{1 - \frac{\rho_0}{R^*}}{1 - \frac{\rho_0}{R}} i(R^*).$$

Proceeding as in Section II, we can compute ex-post (date-1) welfare $W^{\text{ex post}}(R; R^*)$ in case of crisis when the central bank sets the interest rate to $R \geq R^*$ and agents contracted at date 0 anticipating an interest rate of R^*:

(1) $$W^{\text{ex post}}(R; R^*) = -L(R) + \left[\beta - (1 - R)\frac{\rho_0}{R}\right]\frac{1 - \frac{\rho_0}{R^*}}{1 - \frac{\rho_0}{R}} i(R^*).$$

At date 1, in the event of a crisis, the central bank sets $R \in [R^*, 1]$ so as to maximize $W^{\text{ex post}}(R; R^*)$. Denote by $\mathcal{R}(R^*)$ the set correspondence defined by

$$(2) \qquad \mathcal{R}(R^*) \equiv \arg\max_R W^{\text{ex post}}(R; R^*)$$

and let

$$w \equiv \beta - (1 - \rho_0).$$

The first term on the right-hand side of (1) is increasing in R with $L'(1) = 0$. The behavior of the second term on the right-hand side of (1) depends crucially on the sign of w. If $w \leq 0$, then it is increasing in R with a positive derivative at $R = 1$. In this case, $\mathcal{R}(R^*) = \{1\}$: There is no commitment problem. If $w > 0$ on the other hand, then this term is strictly decreasing in R with a negative derivative at $R = 1$ so that $\mathcal{R}(R^*) \subseteq [R^*, 1]$. We will focus on this latter case in the rest of the paper.

Assumption 7 *(Ex-post bailout temptation): $w = \beta - (1 - \rho_0) > 0$.*

Note that this assumption is consistent our previous assumptions, and in particular with Assumption 6: it is more tempting to transfer wealth to banks ex post than ex ante.

Equilibria. We are now in position to describe the set of equilibria of the no-commitment economy, parametrized by the interest rate R^{nc} set by the central bank in the event of a crisis. The equilibrium set $\{R^{nc}\}$ corresponds to the set of fixed points of:

$$(3) \qquad R^{nc} \in \mathcal{R}(R^{nc}).$$

Proposition 2 *To every solution R^{nc} of equation (3) corresponds an equilibrium where investors and banking entrepreneurs correctly anticipate that the central bank will set $R = R^{nc}$ if a crisis occurs, invest at scale $i(R^{nc})$, and issue short-term debt $(\pi - 1 + \rho_0/R^{nc}) i(R^{nc})$. Moreover, there exists $\chi > 0$ such that $[1 - \chi, 1] \subseteq \{R^{nc}\}$.*

The equilibrium of the commitment economy $R^{nc} = 1$ is always an equilibrium of the no-commitment economy. However, there are always other equilibria with $1 > R^{nc} \geq \rho_0$. The condition for R^{nc} to be an equilibrium, namely $R^{nc} \in \mathcal{R}(R^{nc})$, is equivalent to the following condition

$$(4) \qquad \frac{w\rho_0}{1 - \frac{\rho_0}{R}} \left(\frac{1}{R^{nc}} - \frac{1}{R} \right) i(R^{nc}) \geq L(R^{nc}) - L(R) \text{ for all } R \in [R^{nc}, 1].$$

The left-hand side of equation (4) represents the cost in terms of a lower reinvestment scale of setting a higher interest rate $R > R^{nc}$. The right-hand side of equation (4) represents the gain in terms of a lower interest rate distortion of setting such a higher interest rate. The interest rate R^{nc} is an equilibrium if and only if the cost exceeds the gain for all interest rates $R > R^{nc}$. The fact that a neighborhood $[1 - \chi, 1]$ of 1 is always part of the equilibrium set $\{R^{nc}\}$ follows direction from the fact that $L'(1) = 0$. Intuitively, the right-hand side of equation (4) is small compared to the left-hand side for R^{nc} close enough to 1.

It is illuminating to examine the necessary and sufficient condition for $R^{nc} = \rho_0$ to be an equilibrium. In this case, the banks hoard no liquidity and the optimal policy is either to let the banks fail (and set $R = 1$) or to make continuation self-financing ($R = \rho_0$). A bailout is chosen if $-L(\rho_0) - (1 - \rho_0)i(\rho_0) + \beta i(\rho_0) \geq 0$, or:

$$(5) \qquad \frac{wA}{1 - \pi - \alpha\rho_0} \geq L(\rho_0).$$

Corollary 1 *Suppose that condition (5) holds. Then $R^{nc} = 1$ and $R^{nc} = \rho_0$ are equilibria of the no-commitment economy.*

In words, if agents expect the central bank to adopt a tough stance by setting $R = 1$ in case of crisis, then banks choose a small scale $i(1)$ and hoard enough liquidity $(1 - \rho_0)i(1)$ to withstand the shock even if the central bank sets $R = 1$. In turn, the central bank has no incentive to lower the interest rate below 1. Conversely, if agents expect the central bank to adopt a soft stance by setting $R = \rho_0$ in case of a crisis, then banks choose a large scale $i(\rho_0)$ and hoard no liquidity. Then if a crisis occurs, banks can continue at a positive scale only if the central bank sets the interest rate at its lowest possible level $R = \rho_0$ and engineers an extreme bailout. In turn, this extreme bailout is the optimal course of action for the central bank.

Strategic complementarities. The possibility of multiple equilibria illustrates that banks' leverage decisions are strategic complements. These strategic complementarities result from the interaction of three ingredients: imperfect pledgeability on the banks' side, untargeted instruments and time inconsistency on the policy side. Each bank's leverage decision has an effect on the other banks through the policy reaction function in case of a crisis.

To see this more formally, let $x \in [0, 1 - \rho_0]$ be the liquidity choice of a particular bank and $\bar{x} \in [0, 1 - \rho_0]$ the choice of other banks. The central bank would never choose an

interest rate R lower than $\rho_0/(1-\bar{x})$. It sets the interest rate $R \in [\rho_0/(1-\bar{x}), 1]$ in order to maximize

$$(6) \qquad -L(R) + \frac{\beta + \rho_0 - \frac{\rho_0}{R}}{1 - \frac{\rho_0}{R}} \frac{A\bar{x}}{1 - \pi - \alpha\rho_0 + (1-\alpha)\bar{x}}.$$

Without further hypotheses, the objective function in this equation is not necessarily concave in R. For the sake of this discussion, assume that there is enough convexity in the loss function $L(R)$ so that the objective function (6) is indeed strictly concave in R for all values of $\bar{x}$. This guarantees the existence of a unique maximizer $R^*(\bar{x}) \in [\rho_0/(1-\bar{x}), 1]$. Because the objective function (6) has a negative cross partial derivative between R and $\bar{x}$, and that $\rho_0/(1-\bar{x})$ is increasing in $\bar{x}$, there exists $\tilde{x} \in [0, 1-\rho_0]$ such that: for $\bar{x} < \tilde{x}$ we have $R^*(\bar{x}) > \rho_0/(1-\bar{x})$ and $R^*(\bar{x})$ is decreasing in $\bar{x}$; for $\bar{x} \geq \tilde{x}$, we have $R^*(\bar{x}) = \rho_0/(1-\bar{x})$ and $R^*(\bar{x})$ is increasing in $\bar{x}$.

The particular bank under consideration chooses $x \in [0, 1-\rho_0/R^*(\bar{x})]$, where its objective function is

$$U(x, \bar{x}) = (\rho_1 - \rho_0) \frac{A\left[\alpha + (1-\alpha)\frac{x}{1-\frac{\rho_0}{R^*(\bar{x})}}\right]}{1 - \pi - \alpha\rho_0 + (1-\alpha)x}.$$

The best response of this particular bank is therefore given by $x(\bar{x}) \equiv 1 - \rho_0/R^*(\bar{x})$. It has the following properties: for $\bar{x} < \tilde{x}$, we have $x(\bar{x}) > \bar{x}$ and so $x(\bar{x})$ is decreasing in $\bar{x}$; for $\bar{x} \geq \tilde{x}$, we have $x(\bar{x}) = \bar{x}$ and so $x(\bar{x})$ is increasing in $\bar{x}$. The best response $x(\bar{x})$ is not increasing over the whole range of liquidity choices $[0, 1-\rho_0]$. However, because it is increasing over $[\tilde{x}, 1-\rho_0]$, there are strategic complementarities in liquidity choices over that range. Note also that $[\tilde{x}, 1-\rho_0]$ is the relevant range, since all equilibria are in that range.[29]

Comparative statics. There are two ways to perform comparative statics when there are multiple equilibria. One possibility is to use a selection criterion. For example, one could select the bank's preferred equilibrium, i.e., the one associated with the lowest interest rate $\min\{R^{nc}\}$. Another, more ambitious approach, which we pursue here, is to establish the set monotonicity (with respect to the inclusion order) of $\{R^{nc}\}$ with respect to parameters.

Below, we will establish a number of results of the following form: "the set of equilibrium interest rates $\{R^{nc}\}$ is expanding with respect to some parameter μ". By this we mean

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[29]In fact, with the extra assumptions that we have made for this discussion—that the the objective function in equation (6) is strictly concave—the set of possible equilibrium liquidity choices is simply $[\tilde{x}, 1-\rho_0]$, and the set of equilibrium interest rates is $[\rho_0/(1-\tilde{x}), 1]$. The set of equilibria need not be an interval when this extra assumption is not verified.

that for $\mu < \mu'$, the set of equilibrium interest rates associated with μ is included in the one associated with μ'. The minimum of the set of equilibrium interest rates $\min\{R^{nc}\}$ is weakly decreasing in μ. By contrast, the maximum of the set of equilibrium interest rates $1 = \max\{R^{nc}\}$ is invariant to μ.

Corollary 2 *The set $\{R^{nc}\}$ of equilibrium interest rates is expanding in the relative weight β of banking stakeholders in the central bank's objective function as well as with the size (as measured by A) of banks.*

Proof. Suppose that $R^{nc} \in \mathcal{R}(R^{nc})$ - i.e. that (4) holds- when β and A are set to some initial value. Then if β and A are increased to β' and A', (4) still holds. As a result it is still the case that $R^{nc} \in \mathcal{R}(R^{nc})$. ∎

Strategic complementarities associated with bigger, more powerful and more strategic banks are stronger. The fixed cost of a bailout is independent of the characteristics or choices of banks. By contrast, for any given interest rate anticipated by the banks, the benefits of a bailout increase with the size (as measured by A), the influence and the importance (as measured by β) of banks.

It is also interesting to perform comparative statics on the set of equilibria with respect to the severity of the crisis. To this end, we now consider an extension of the basic model where only a fraction γ of banks are distressed in the event of a crisis. The parameter γ indexes the severity of the crisis. We keep the probability $1 - \hat{\alpha}$ of a crisis constant and let the probability of being intact $\alpha \equiv \hat{\alpha} + (1 - \hat{\alpha})(1 - \gamma)$ adjust. The logic of the model is essentially unchanged. The only difference is that date-1 welfare $W^{\text{ex post}}(R; R^*)$ is now given by

$$(7) \quad W^{\text{ex post}}(R; R^*) = -L(R) + \left[\beta - (1 - R)\frac{\rho_0}{R}\right] \frac{1 - \frac{\rho_0}{R^*}}{1 - \frac{\rho_0}{R}} \frac{\gamma A}{1 - \pi - \alpha\rho_0 + (1 - \alpha)(1 - \frac{\rho_0}{R^*})}.$$

Corollary 3 *The set $\{R^{nc}\}$ of equilibrium interest rates is expanding in the severity of the crisis γ.*

One interpretation of Corollary 3 (whose proof is similar to that of Corollary 2) is that one could observe banks increasing their leverage i/A and decreasing their liquidity hoarding x as the severity of a crisis increases. This is particularly interesting since exactly the opposite would happen in a model with fixed or pre-committed interest rate. Indeed, the opposite

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conclusion is usually obtained in corporate finance. Strategic complementarities can therefore result in perverse comparative statics.

Endogenous correlation. We have so far assumed that the correlation of distress shocks across banks was exogenous. We now relax this assumption and allow banks to choose the correlation of their distress risk with other banks' distress risk. There is a continuum of states of the world indexed by $\theta \in [0, 1]$. The distribution of θ is uniform on $[0, 1]$. Each bank faces a given probability of distress $1 - \alpha$ but can decide how to spread this distress risk across the states of the world. That is, each bank can choose a probability of being intact α_θ in state θ subject to the constraint that $\alpha \geq \int \alpha_\theta d\theta$. Let $R_\theta \in [\rho_0, 1]$ be the interest rates that banking entrepreneurs expect the central bank to set in state θ.

The banking entrepreneur commits to repay $d_\theta i$ in state θ if $\alpha_\theta < 1$ and πi if $\alpha_\theta = 1$. In this case, banks will always choose $d_\theta \in [\pi - (1 - \rho_0/R_\theta), \pi]$. The continuation scale in state θ when pledging debt $d_\theta i$ is given by $j_\theta = i (\pi - d_\theta) / (1 - \rho_0/R_\theta)$ and the investment scale by[30]

$$(8) \qquad i = \frac{A}{1 - \int_0^1 [(1 - \alpha_\theta) d_\theta + \alpha_\theta (\pi + \rho_0)] d\theta}.$$

The banking entrepreneur's expected payoff is $(\rho_1 - \rho_0) \int_0^1 [\alpha_\theta i + (1 - \alpha_\theta) j_\theta + (1 - \alpha_\theta)i] d\theta$.

Proposition 3 *(i) (general structure of strict equilibria) All the strict equilibria have the following properties: (a) there exists a set of crisis states $\Theta^{crisis} \subseteq [0, 1]$ of measure $1 - \alpha$ such that $R_\theta < 1$ if $\theta \in \Theta^{crisis}$ and $R_\theta = 1$ otherwise, (b) $\alpha_\theta = 0$ if $\theta \in \Theta^{crisis} \subseteq [0, 1]$ and $\alpha_\theta = 1$ otherwise, and (c) $d_\theta = \pi - (1 - \rho_0/R_\theta)$ and i is given by equation (8).*
(ii) (particular class of strict equilibria) To every set of crisis states $\Theta^{crisis} \subseteq [0, 1]$ of measure $1 - \alpha$ and solution R^{nc} of equation (3) corresponds a strict equilibrium where: (a) for $\theta \in \Theta^{crisis}$, we have $\alpha_\theta = 0$, $R_\theta = R^{nc}$, (b) for $\theta \notin \Theta^{crisis}$, we have $\alpha_\theta = 1$, $R_\theta = 1$, and (c) $d = \pi - (1 - \rho_0/R^{nc})$ and $i = i(R^{nc})$.

Proof. Assumption 2 implies that $d_\theta = \pi - (1 - \rho_0/R_\theta)$ when $\alpha_\theta < 1$. The results then follow easily from the fact that the derivative with respect to α_θ of the objective function obtained by replacing these values of d_θ in the objective function of banking entrepreneurs is higher, the higher is R_θ. ∎

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[30]The investors' date-0 breakeven condition is $i - A = \int_0^1 [(1 - \alpha_\theta)d_\theta i + \alpha_\theta \pi i + \rho_0 i] d\theta$.

Banks want to fail when the largest possible number of other banks are failing and correlate their risks with those of other banks. Because interest rate policy is non-targeted, bailouts take place in states of the world where a large number of banks are in distress, making it cheaper to refinance in these states. Proposition 3 illustrates the presence of strategic complementarities in correlation choices. In equilibrium, banks coordinate on a given set of crisis states Θ^{crisis} which is completely indeterminate up to the constraint that it be of measure $1 - \alpha$. This proposition also validates our choice of focusing on aggregate shocks as opposed to idiosyncratic shocks: this is the stochastic structure that prevails when correlation choices are endogenized.

It is important to contrast Proposition 3 with the standard prescriptions of the CAPM. In a CAPM world, the cost of capital associated with an investment project is negatively related to the correlation of its cash flows with the market return. As a result, a bank would always choose a minimal correlation with the aggregate risk in the economy. In our economy, just the opposite occurs. Banks maximize their correlations in order to fail when all the other banks are failing and the central bank lowers interest rates.

IV. Welfare and regulation

The time inconsistency of policy introduces a soft budget constraint problem and creates moral hazard on the banks' side. In this context, banks' leverage and liquidity hoarding choices at time 0 can be inefficient.

Welfare. The equilibria in $\{R^{nc}\}$ can be ranked in terms of welfare. Indeed, under our assumptions, ex ante welfare $W^{\text{ex ante}}(R)$ is increasing in R. As a result, equilibria with a higher interest rate R^{nc} feature higher welfare. The equilibrium with the highest welfare is the equilibrium that prevails under commitment with no bailout and the interest rate equal to 1. Moreover, the banking entrepreneurs' perspective is exactly the opposite: the lower the interest rate R^{nc}, the better the equilibrium for banks.

Role for regulation. In this context, regulation of banks' leverage and liquidity hoarding choices at time 0 can be welfare improving. Indeed, consider putting a cap on short-term debt: $d \leq \pi - (1 - \rho_0)$, or equivalently regulating liquidity hoarding by imposing $x \geq 1 - \rho_0$.[31] At $t = 1$, there is then no incentive for the central bank to proceed to a bailout: there would

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[31]In our simple model, no matter what interest rate is expected at the contracting stage, this is equivalent to regulating leverage by imposing that $i/A \leq A/[1 - \alpha + 1 - \pi - \rho_0]$.

be a distortionary cost and no benefit to lower the interest rate below 1 since all banks are able to continue at full scale when $R = 1$. Therefore, this regulation reduces the set of equilibrium interest rates $\{R^{nc}\}$ down to a singleton $\{1\}$, i.e. the no bailout, commitment solution.

Proposition 4 *With limited commitment, the optimal regulation of banks' choices at $t = 0$ takes the form of a liquidity requirement $x \geq 1 - \rho_0$ or equivalently of a maximum short-term debt $d \leq \pi - (1 - \rho_0)$. With this regulation, there is only one equilibrium, which coincides with the commitment solution $R^{nc} = 1$. By contrast, there is no role for such regulation under commitment.*

Remark 1 *Subsidizing liquidity, a form of intervention that is sometimes put forward, would be counterproductive in our model. It would only allow banks to increase their scale and aggravate the time-inconsistency problem of policy, rendering bailouts more likely.*[32]

Remark 2 *Regulations are hard to enforce and banks try to circumvent them. In Farhi-Tirole (2009), we used $\pi = 0$ so that banks' liquidity came exclusively in the form of hoarded assets. We introduced the possibility for banks to engage in regulatory arbitrage to fool the regulator by purchasing toxic assets instead of safe assets to fulfill their liquidity requirement. These toxic assets are cheaper but run the risk of a bad performance. We showed that in this context, there are strategic complementarities in regulatory arbitrage. The more banks engage into regulatory arbitrage, the lower the interest rate set by the central bank in the event where toxic assets are worth 0, and the more each bank can afford to hoard toxic assets. The very insights gleaned with respect to the quantity of liquidity hoarded hold just the same with respect to their quality.*

Macroprudential versus microprudential regulation. This soft budget constraint rationale for regulation is also present in microeconomic principal-agent models when the principal lacks commitment. The difference in our setting is that the actions of the central bank (the interest rate) affect all banks at the same time. If one bank were to take idiosyncratic risks that would materialize only when none of the other banks are in distress, there

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[32]To formalize this insight, assume that the government subsidizes the short-term hoarding of liquidity so that it returns $q > 1$ per unit in the bad state. So $q(\pi - d) = 1 - \frac{\rho_0}{R}$. The analysis of Section III carries over. In particular the characterization of equilibria is literally identical. The only difference is that $i(R)$ is now given by $i(R) = A/[1 - \pi - \alpha\rho_0 + (1/q - \alpha/q)/(1 - \rho_0/R)]$. It is then easy to verify that the set $\{R^{nc}\}$ of equilibrium interest rates is expanding in the return of liquidity q. Note that by having a corner solution we shut down a possible channel through which subsidizing liquidity hoarding may help (a substitution effect).

would be no soft budget constraint problem. Because the only policy instrument, interest rate policy, is not targeted, the central bank would not be tempted to lower interest rates to bail out this individual bank when its individual risk is realized. As a result, our framework suggests that the focus of regulation should be on aggregate leverage and liquidity hoarding and not only on individual risk-taking. In other words, in our model, the optimal regulation is macroprudential and not only microprudential.

It is important to stress that in our model, breaking down banks into smaller banks would be ineffective. The set of equilibria would be unaffected. The problem here is not so much that banks are too big to fail, but that the financial sector as a whole might take on too much correlated risk and too much short-term debt. This irrelevance result would break down if big banks (with a high A) carried a higher welfare weight (β) than small banks per unit of investment, say because big banks' failures have bigger systemic consequences, or because the bankruptcy of a large bank is disproportionately reported in the media, creating pressure for a bailout.

The pecking order of regulation. So far, we have assumed that regulation is costless and that banks are homogenous. We now relax those assumptions. Banks are allowed to differ on size A and weight β in the central bank's objective function. These characteristics are assumed to be distributed according to an arbitrary distribution $F(\beta, A)$. We assume that the costs $c(i)$ of regulating a bank increase with the scale i of the bank, where c is homogenous of degree $\lambda : c(i) = ci^\lambda$ with $c > 0$ and $\lambda \geq 0$. In this context, regulation involves a trade-off and the regulator might find it optimal to regulate certain banks but not others. In order to analyze this trade-off formally, we characterize the minimal amount of aggregate resources devoted to regulation

$$K = \int n(\beta, A) c \left(\frac{A}{1 - \pi - \alpha \rho_0 + (1 - \alpha)(1 - \rho_0)} \right)^\lambda dF(\beta, A)$$

required to ensure that $\{R^{nc}\} = \{1\}$. Here the authorities regulate a fraction $n(\beta, A) \in [0, 1]$ of banks of size A and weight β.

Proposition 5 *Suppose that Assumptions 2, 6 and 7 hold for every β in the support of F. Then the minimal cost of insuring that $\{R^{nc}\} = \{1\}$ is achieved by regulating banks for which $[\beta - (1 - \rho_0)] A^{1-\lambda}$ is greater than a certain threshold Λ.*

Proof. See the appendix. ∎

When regulation is costly, optimal regulation is characterized by a pecking order determined by a summary statistic $[\beta - (1 - \rho_0)] A^{1-\lambda}$ which combines their size A and their weight β in the central bank's objective function. This summary statistic can be thought of as a cost-of-regulation-adjusted systemic importance. For a given size A, the higher β, the higher the bank in the regulatory pecking order (recall that β is the sum of the weights placed on the different categories of bank stakeholders, i.e. banking entrepreneurs, firm managers, and workers). Whether the rank of the bank in the regulatory pecking order increases or decreases with its size A for a given β depends on the returns to scale in regulation λ. With increasing returns to scale in regulation ($\lambda < 1$), the rank of the bank in the regulatory pecking order increases with A, and the opposite holds true with decreasing returns to scale in regulation ($\lambda > 1$). With constant returns to scale in regulation ($\lambda = 1$), size per se is irrelevant: the costs and benefits of regulation scale up exactly at the same rate with the size of bank.

V. Sowing the seeds of the next crisis

In Section I, we gave some examples of immediate deadweight losses L associated with low interest rates. This section focuses on deferred costs, associated with the incentive for new borrowers to lever up and increase maturity mismatch, or with the central bank's loss of reputation. We derive alternative microfoundations for the distortions associated with an interest rate bailout. We extend the model to an overlapping generations structure with two successive generations G_{-1} and G_0 of banking entrepreneurs. We derive a loss function L from an interest rate bailout of generation G_{-1} at date 0. This loss function originates in the perverse consequences on the subsequent generation G_0 of banking entrepreneurs, who end up levering up more and hoarding less liquidity, resulting in an interest rate bailout at date 1 (if a crisis occurs). In order to present these extensions, it is useful to first first extend our basic setup by allowing policy to also determine the date-0 interest rate R_0. Throughout Section V, we assume that there are no immediate date-0 distortions from lowering R_0, so that any fixed cost from an interest rate bailout at date 0 will be a deferred one. In Section V.A, we allow for immediate distortions from lowering the interest at date 1, but in Sections V.B and V.C, we assume them away.

A. Date-0 Interest Rate

We assume that the government can tax the rate of return on the storage technology between dates 0 and 1. The proceeds are rebated lump sum to consumers. This is rigorously equivalent to assuming the government can subsidize investment in the banks at date 0, a subsidy financed by lump sum taxes on consumers. We show that it is optimal, both under commitment and under no commitment, to set the date-0 interest rate R_0 to 1, even without assuming any distortion from lowering R_0.[33]

The borrowing capacity of banking entrepreneurs at date 0 is now given by

$$(9) \qquad i = \frac{A}{1 - \frac{\pi}{R_0} - \frac{\alpha \rho_0}{R_0} + \frac{(1-\alpha)}{R_0}x}$$

where x, as earlier, denotes the liquidity ratio. Banking entrepreneurs hoard liquidity $x = 1 - \rho_0/R$ at date 0 if and only if it expects a date-1 interest rate R satisfying

$$(10) \qquad \pi + \alpha - R_0 \le \alpha \rho_0 \left(\frac{1}{R} - 1 \right),$$

and hoards no liquidity ($x = 0$) otherwise. This generalizes our analysis of Section II to arbitrary $R_0 \le 1$. Denote by $i(R, R_0)$ the corresponding investment scale chosen by banking entrepreneurs—obtained by replacing the optimal liquidity choice x of banking entrepreneurs in equation (9), a choice governed by equation (10). Similarly, let $j(R, R_0)$ denote the corresponding reinvestment scale in case of a crisis.

Ex-ante welfare under commitment is given by

$$(11) \qquad \begin{aligned} W^{\text{ex ante}}(R, R_0) \equiv\ & \beta i(R, R_0) - (1-\alpha)\frac{1-R}{R}\rho_0 j(R, R_0) - (1-\alpha)L(R) \\ & - [i(R, R_0) - A](1 - R_0). \end{aligned}$$

The last term on the right-hand side is new and reflects the implicit subsidy to bank investment at date-0 (consumers invest a total amount $i(R, R_0) - A$). The following proposition shows that under the assumptions that we have maintained throughout the paper, the optimal interest rate policy under commitment is passive both at date 1 and at date 0.

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[33]These results would only be reinforced if we were to introduce a loss function $L_0(R_0)$ as we did for the date-1 interest rate.

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Proposition 6 *The optimal interest rate policy under commitment features $R_0^c = R^c = 1$.*

The intuition is straightforward. Assumption 6 guarantees that redistributing resources from consumers to banking entrepreneurs is welfare reducing. Lowering the interest rate R_0 or R below 1 induces such a redistribution of resources (with additional distortions in the case of R). Setting these interest rates equal to 1 is therefore optimal.

Analyzing the no-commitment solution requires solving a dynamic game. We focus on subgame-perfect equilibria. We will need to impose some refinement. We know from our previous analysis that the set of equilibrium interest rates of the continuation game is expanding in the date-0 investment scale. We therefore find it natural to focus on subgame perfect equilibria that satisfy the following monotonicity requirement:

Assumption 8 *The continuation equilibrium of the no-commitment game is such that the date-1 interest rate is non-decreasing in the date-0 interest rate.*

This assumption would be automatically satisfied if for example, we always chose the worst possible continuation equilibrium (the one with the lowest R) for any value of R_0.

Corollary 4 *In every equilibrium of the no-commitment game, $R_0^{nc} = 1$.*

This corollary follows directly from Proposition 6. Indeed, lowering the interest rate R_0 below 1 only entails larger costs compared with the commitment solution, because it leads banking entrepreneurs to hoard less liquidity anticipating a lower interest rate R at date 1.

B. Leverage decisions, going forward

Consider a longer-horizon model, say the overlapping-generations version of this model, in which banking entrepreneurs live, like in this model, for three periods. A bailout at date $t + 1$ of those banks that borrowed at date t then also affects the financing decisions of the next generation of banks, which borrow at date $t + 1$. Interestingly, this operates through two channels: increased leverage and increased maturity mismatch.[34]

To preview the results, bailing out generation G_t induces leverage and maturity mismatch for generation G_{t+1}, sowing the seeds for a date-$t + 2$ crisis and bailout. Indeed, an interest

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[34]Arguably, both channels seem to have operated during the long episode of very low interest rates in the wake of the 2000 Internet bubble crash.

rate bailout of generation G_t makes generation G_{t+1} (a) more willing to take an illiquid position by loading up on short-term debt and (b) increase the size of its investment. These two effects—the increased maturity mismatch and the increased leverage channels respectively— distort generation G_{t+1}'s incentives and generate a social cost that is a fixed cost (in the sense that it does not depend on generation G_t's size or illiquidity) when contemplating a rescue of generation G_t.

The logic of the argument can be grasped by appending to the model of Section V.A a prior generation of entrepreneurs, generation G_{-1}, living at dates $-1, 0$ and 1.[35] Generation G_{-1} is in all respects similar to generation G_0 born at date 0 and studied in this paper, except that its short-term (date-0) profit π_0 is sufficiently large that not hoarding any liquidity is a dominant strategy (for any $R_0 \in [\rho_0, 1]$), which boils down to the condition that $\pi_0 + \alpha > 1 + \alpha (1 - \rho_0)$. This assumption merely shortens the analysis by ensuring that generation G_{-1} in case of a date-0 crisis is unable to withstand the shock unless the date-0 interest rate is brought down to $R_0 = \rho_0$.

The analysis of the generation G_0 born at date 0 is exactly as in Section V.A. We make the following assumption, which is consistent with our previous assumptions.[36]

Assumption 9 *(no liquidity hoarded when $R_0 = \rho_0$):* $\pi > (1 - \alpha)\rho_0$.

We rule out any deadweight loss from inefficient investments, so as to starkly illustrate that L can come from impaired incentives for the subsequent generation of entrepreneurs. We maintain this assumptions throughout Sections V.B and V.C.

From our previous analysis and Assumption 8, we know that the optimal date-0 policy if one ignores the welfare of generation G_{-1} consists in setting $R_0 = 1$. Suppose that $R_0 = 1$ is actually chosen. There are a number of possible continuation equilibria, corresponding to different expectations regarding the interest rate R. Because we have assumed away any current distortion, all interest rates $R \in [\rho_0, 1]$ correspond to a possible continuation equilibrium, where banking entrepreneurs hoard liquidity $x = 1 - \rho_0/R$. Using the definition in

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[35]The model can be extended to infinite-horizon overlapping generations; we offer here the simplest illustration.

[36]The analysis remains valid when this assumption is violated, but the cost of bailing out generation G_{-1} in terms of generation G_0's incentives then comes solely from an increased investment scale, not from an increased maturity mismatch.

equation ([11](#)), this yields an ex-ante welfare for generation G_0

$$W^{\text{ex ante}}(R,1) = \frac{\left[\beta - (1-\alpha)(1-R)\frac{\rho_0}{R}\right]A}{1 - \pi - \alpha\rho_0 + (1-\alpha)\left(1 - \frac{\rho_0}{R}\right)}$$

where $W^{\text{ex ante}}(R,1)$ increases with R. In our analysis, we do not need to take a stand on which equilibrium is actually selected.

Any $R_0 \in (\rho_0, 1)$ is dominated: by Assumption [8](#), such interest rates reduce welfare from generation G_0 and do nothing to help generation G_{-1}. Therefore, we only have to analyze whether $R_0 = \rho_0$ is preferred to $R_0 = 1$. When $R_0 = \rho_0$, no liquidity is hoarded by generation G_0, and because there are no immediate distortions from interest rate policy, a full interest rate bailout with $R = \rho_0$ follows. Using the definition in equation ([11](#)), welfare from the date-0 generation becomes

$$W^{\text{ex ante}}(\rho_0, \rho_0) = \frac{\left[\beta - (1-\alpha)(1-\rho_0)\right]A}{1 - \pi - \alpha\rho_0} - (1-\rho_0)\left[\frac{1}{1 - \frac{\pi}{\rho_0} - \alpha} - 1\right]A.$$

Turning now to the decision of whether to rescue generation G_{-1} at date 0, and remembering that choices outside $\{\rho_0, 1\}$ are dominated, we conclude that there exists a fixed cost of bailing out generation G_{-1} by setting $R_0 = \rho_0$. Depending on which continuation equilibrium is selected when $R_0 = 1$, with corresponding welfare $W^{\text{ex ante}}(R,1)$, this fixed cost is given by $L(\rho_0) \equiv W^{\text{ex ante}}(R,1) - W^{\text{ex ante}}(\rho_0, \rho_0)$. It is maximized when following $R_0 = 1$, the continuation equilibrium with $R = 1$ is selected, and minimized when the continuation equilibrium with $R = \rho_0$ is selected. It can be verified that even in the latter case, we have $L(\rho_0) > 0$ as long as Assumption [6](#) holds. The selection of the continuation equilibrium with $R = \rho_0$ isolates the increased leverage channel mentioned above. By contrast, the selection of the continuation equilibrium with $R = 1$ combines both the increased leverage and the increased maturity-mismatch channels.

Proposition 7 *When contemplating whether to rescue the generation G_{-1} banks, the government faces a fixed cost equal to $L(\rho_0) = W^{ex\ ante}(R,1) - W^{ex\ ante}(\rho_0,\rho_0) > 0$, where $R \in [\rho_0, 1]$ corresponds to the selected continuation equilibrium when $R_0 = 1$. This cost is fixed in that it does not depend on generation G_{-1}'s investment to be rescued.*

C. Central bank's reputation

Yet another deferred cost of bailouts is the loss of reputation by the central bank. This can be modelled by introducing a tough type and a soft type. A bailout then reveals the type of the central bank to be soft, raising the likelihood of future bailouts and pushing banks to take on more risk, hoard less liquidity and lever up, resulting in increased economywide maturity mismatch and in turn larger bailouts. Even a central bank of the soft type internalizes this reputation cost and is reluctant to engage in a bailout in the first place.

To show how reputation concerns generate yet another fixed cost, we follow Section V.B but introduce uncertainty about the central banker's preferences. With prior (date-0) probability $1 - z$, the central banker is "bailout-prone" as he puts weight $\beta > 1 - \rho_0$ (so $w > 0$) on investment as earlier. With probability z, the central bank is "tough" as he puts no or little weight on investment and therefore always chooses interest rates equal to 1.

The situation is otherwise the same as in Section V.B: Generation G_{-1} optimally hoards no liquidity; when it faces a crisis, the rational choice for a bailout-prone central banker is again between $R_0 = 1$ and $R_0 = \rho_0$. A choice of $R_0 = \rho_0$ reveals that he is bailout-prone.

Suppose first that the central banker sets $R_0 = \rho_0$ to bail out generation G_{-1}. The equilibrium for generation G_0 is then as in Section V.B. Welfare is $W^{\text{ex ante}}(\rho_0, \rho_0)$ for that generation. By choosing $R_0 = 1$ by contrast, the central banker creates posterior beliefs $\hat{z} \geq z$.[37] If $\hat{z}$ is large enough ($\hat{z} \geq \underline{z}$), then hoarding liquidity is a dominant strategy for a generation-G_0 bank and welfare for generation G_0 is $W^{\text{ex ante}}(1, 1)$.[38]

Proposition 8 *There exists $\underline{z} < 1$ such that for $z \geq \underline{z}$, the bailout-prone central banker faces a fixed reputation-loss cost equal to $L(\rho_0) \equiv W^{ex\ ante}(1, 1) - W(\rho_0, \rho_0) > 0$ when rescuing the generation G_{-1} banks. This cost is fixed in that it does not depend on generation G_{-1}'s investment to be rescued.*

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[37]$\hat{z} = z$ in a pooling equilibrium, $\hat{z} \geq z$ in a partially revealing one, including a separating equilibrium for which $\hat{z} = 1$.

[38]The threshold $\underline{z}$ is the solution of $\frac{z\alpha + [1 - z]}{1 - \pi - \alpha\rho_0} = \frac{1}{1 - \pi - \alpha\rho_0 + (1 - \alpha)(1 - \rho_0)\underline{z}}$. From Assumption 2, $\underline{z} < 1$. To understand this condition, note that the tough central banker (who has posterior probability $\hat{z}$) will not bail out banks in case of a crisis, and so the probability of continuation is at most $\hat{z}\alpha + [1 - \hat{z}]$ in the absence of liquidity hoarding. Conversely, hoarding liquidity $(1 - \rho_0)i$ is needed in order to be able to continue in a crisis when the central banker is tough; but if the central banker turns out to be bailout-prone and lowers the interest rate at date 1 in case of crisis to $R = \rho_0$ (this is the most optimistic hypothesis, which arises if other bankers hoard no liquidity), then this unneeded liquidity can be returned to investors. The cost of liquidity is therefore only $(1 - \alpha)(1 - \rho_0)\hat{z}$ on average, which explains the term on the right-hand side.

The derivation of equilibrium behavior is then straightforward. For $z \geq \underline{z}$, the bailout-prone central banker pools with the tough one if and only if $w_{-1}i_{-1} \leq L(\rho_0)$ is verified, where $w_{-1} = \beta_{-1} - (1 - \rho_0)$ and β_{-1} and i_{-1} are the weight on, and the investment of generation G_{-1}. If $w_{-1}i_{-1} > L(\rho_0)$, the equilibrium is separating. Finally, for $z < \underline{z}$, the details of equilibrium behavior depend on equilibrium selection, but the overall pattern (a trade-off between the benefit of bailout and a cost of reputation loss) remains the same.

VI. Optimal ex-post bailouts

So far, we have restricted the set of policy instruments to the interest rate. We emphasized that this instrument was not targeted. Together with time inconsistency, this feature was a key ingredient in generating strategic complementarities in leverage choices. In this section, we relax the assumption of an exogenously specified policy instrument set. Instead, we follow a mechanism design approach and characterize the optimal ex-post bailout where policy tools are endogenous to the constraints of the economic environment.

As we noted in the introduction, one may wonder whether interest rate bailouts, which involve both an implicit subsidy to banks and various distortions, are still desirable when other interventions are possible. For example, purchases of legacy assets, liquidity support and recapitalizations also involve direct transfers from consumers to banks—boosting their net worth and allowing them to refinance at a larger scale. However, these transfers do not reduce borrowing costs at the margin, and do not generate similar distortions.

Interestingly, we show that interest rate policy still plays an important role within the optimal bailout scheme: It is always part of the optimal ex-post package, and over a range of parameters, bailouts boil down to pure interest rate policy. Furthermore, we recover a key insight from our previous analysis, the existence of strategic complementarities: to some extent, optimal bailouts are themselves untargeted.

Setup. We allow the fraction γ of banks that are distressed in a crisis to be less than 1. Denoting the probability of a crisis by $1 - \hat{\alpha}$, we maintain the convention that $\alpha \equiv \hat{\alpha} + (1 - \hat{\alpha})(1 - \gamma)$ represents the probability of being intact. In what follows the probability $\hat{\alpha}$ of a crisis is kept constant and the dependence of α with respect to γ is left implicit.

Moreover, we introduce an informational friction: the central bank can observe which banks are distressed, but the underlying auditing technology is imperfect. More precisely, we assume that the probability of generating a false positive when assessing if a bank is

distressed is equal to ν.[39] As a result, in a crisis, a fraction $\nu(1-\gamma)$ of banks are mistaken by the authorities as banks that need liquidity. These banks are aware that they belong to the false positive group.

We assume that banking entrepreneurs and their investors form perfect coalitions, and that banks have full bargaining power in these coalitions.

Instruments. We assume that the government cannot directly hold bank securities. The only available instruments are the interest rate (the borrowing cost of banks) and direct transfers to banks perceived as being distressed. We have already commented on the possible interpretations of interest rate policies as policies that lower the borrowing cost of banks. Direct transfers capture policies used in practice to boost the net worth of banks, such as the purchase of legacy assets at inflated prices.

The assumption that the government cannot directly hold bank securities deserves some comments, as it rules out some forms of government intervention that are used in practice. For example, a recapitalization involves a transfer from the government in exchange of shares or warrants. However in most practical cases, the government usually sells its stake relatively quickly. What remains is a transfer from the government to the bank. At a theoretical level, this assumption limits the ability of the government to screen between intact and distressed banks. In Farhi-Tirole (2009) we analyzed the case where the government could hold stakes in banks and use this as a screening device. We showed that our insights were robust in this environment. However the analysis was quite involved. The assumption that the government cannot directly hold a stake in banks allows us to considerably streamline the analysis.

If direct transfers could be perfectly targeted to distressed banks, they would dominate interest rates as a policy instrument. However, the government cannot perfectly recognize if a given bank is distressed and some banks might engage in rent seeking by successfully portraying themselves as distressed. Moreover, low interest rates benefit distressed banks comparatively more than intact ones. As a result, there is a non-trivial policy tradeoff between interest rate policy and direct transfers to institutions perceived as being distressed.[40]

Although we will consider only symmetric equilibria, we analyze the general case where the authorities face an arbitrary distribution $F(i, x)$ of banks with scale and liquidity (i, x).

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[39]Note that for simplicity, we assume that there are no false negatives.

[40]The possibility of false positives is crucial for the following reason. If $\nu = 0$ or $\gamma = 1$, then the authorities do not face any information extraction problem. As long as Assumption 7 holds, then banks in distress are always rescued through a direct transfer and are allowed to continue at full scale even if banks are completely illiquid. In equilibrium, banks would then choose to be completely illiquid, interest policy would not be used.

A bailout specifies an interest rate R, and for every scale and liquidity (i, x) a transfer $t(i, x) i \geq 0$ for banks perceived as being distressed. This implies the following reinvestment scale for distressed banks

$$j(i, x) = \min \left\{ \frac{x + t(i, x)}{1 - \frac{\rho_0}{R}}, 1 \right\} i.$$

Timing. The timing within period 1 is as follows: (1) the government announces a rescue scheme $\{R, t(i, x)\}$; (2) each banking entrepreneur accepts the plan if and only it makes him better off given that investors must be as well off as they would in the absence of participation in the scheme.

To simplify the expressions, we assume that the government places no Pareto weight on banking entrepreneurs $\beta_b = 0$. Transfers to banks that are perceived to be distressed but are in fact intact therefore represent a pure welfare loss.

Planning problem. In order to solve for the optimal ex-post bailout, we use the variables $\{R, j(i, x)\}$ rather than $\{R, t(i, x)\}$. The transfers $t(i, x)$ can be inferred as follows: $t(i, x) = (1 - \rho_0/R) j(i, x) / i - x \geq 0$ since without loss of generality $x \leq 1 - \rho_0/R$.

Up to a constant, the optimal ex-post bailout maximizes[41]

(12)
$$-L(R) + \int \gamma \left(\beta - (1 - R) \frac{\rho_0}{R} \right) j(i, x) \, dF(i, x)$$

$$- [\gamma + (1 - \gamma) \nu] \int \left[\left(1 - \frac{\rho_0}{R} \right) j(i, x) - xi \right] dF(i, x)$$

s.t.

$$i \geq j(i, x) \geq \frac{x}{1 - \frac{\rho_0}{R}} i.$$

To obtain equation (12), note that for distressed banks, the total subsidy is given by $t(i, x) + (1 - R) \rho_0 j(i, x) / R$. For intact banks that are perceived as distressed, the subsidy is $t(i, x)$ as those banks do not need to borrow more funds. The first two terms in equation (12) are exactly as in Section III. The third term corresponds to the additional implicit subsidy associated with the direct transfer $t(i, x)$. This is a transfer to distressed firms and intact firms that are perceived as distressed. Consumers get a rate of return equal to 0 (and not R as when they reinvest in the banks) on this transfer.

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[41]Because of the linearity of the objective and the constraints in $t(i, x)$, we need to take a stand on government actions when the government is indifferent. We assume that it sets $t(i, x)$ equal to its minimal possible value in case of indifference.

The government could achieve a given pattern of continuation scales $j(i, x)$ entirely through direct transfers by setting $t(i, x) = j(i, x) - x$. This would economize on interest rate distortions. However, it increases undesirable transfers to intact banks perceived to be distressed. Lowering the interest rate R increases the collateral value $\rho_0 j(i, x)/R$ of distressed firms, allowing them to continue at a larger scale, while at the same time reducing the funds transferred to intact firms perceived to be distressed. By affecting the terms at which banks can borrow on the market, the government is able to better target firms that are actually distressed. However, it pays a cost in terms of distortions $L(R)$. We need to ensure that the latter is convex enough.

Assumption 10 (*enough convexity*). *The function* $-R^2 L'(R)$ *is decreasing in* $R \in [\rho_0, 1]$.

Optimal ex post bailout. The optimal bailout policy is summarized in Figure 1. Due to the linearity of the program, there can be two equilibrium configurations. We discuss them informally here and refer the reader to the appendix for the full details.

(i) *No liquidity hoarding* ($x = 0$ for all banks). In this equilibrium configuration, banks load up on short-term debt as they expect to be rescued in case of a crisis. Given the absence of liquidity in the banking sector, the least-cost rescue policy for the government is a mix of transfer and interest rate policies. To see this, note that low interest rates reduce the wasteful transfer t to intact banks given that distressed ones require a unit refinancing from both sources: $1 = t + \rho_0/R$. Basically, distressed banks can lever up the direct transfer to refinance at a greater scale. However low interest rates also increase the deadweight loss $L(R)$; the marginal distortion is 0 at $R = 1$, and increases as the interest rate is reduced. When the severity (γ) of the crisis increases, undue transfers to intact banks become less of a concern, and the optimal interest rate $\bar{R}(\gamma)$ increases to reach 1 when there are no false positives ($\gamma = 1$).[42]

It must also be the case that the government wants to bail out the banks, that is that there is not too much adverse selection. Thus if the crisis is not very severe ($\gamma < \underline{\gamma}$ in the figure), the no-liquidity-hoarding equilibrium disappears.

(ii) *Liquidity hoarding.* In this equilibrium configuration, banks hoard just enough liquidity (issue just as little short-term debt) as to be able to continue at full scale in case of a crisis. This requires two conditions.

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[42]The function $\bar{R}(\gamma)$ is defined by the equation $-\bar{R}(\gamma)^2 L'(\bar{R}(\gamma)) = \frac{A(1-\gamma)\nu\rho_0}{1-\pi-\alpha\rho_0}$.

First, given the expected date-1 interest rate, an individual bank must not expect that the government will make up for insufficient liquidity through a targeted transfer. Otherwise it would hoard no liquidity at all. Put differently, liquidity hoarding relies on the expectation of a pure interest rate policy bailout. Whether the government is willing to bail out a bank with insufficient liquidity depends on the trade-off, discussed in (i), between wasteful transfers to intact banks and rescues of distressed ones but in reverse: when R increases, the needed transfer $t = 1 - \rho_0/R$ increases and so bailouts become less attractive. This implies that R has to be greater than or equal to $\underline{R}(\gamma)$, where $\underline{R}(\gamma)$ is an increasing function of γ as the government is more tempted to rescue banks as the crisis becomes more severe. It reaches 1 when $\gamma = \bar{\gamma}$.[43]
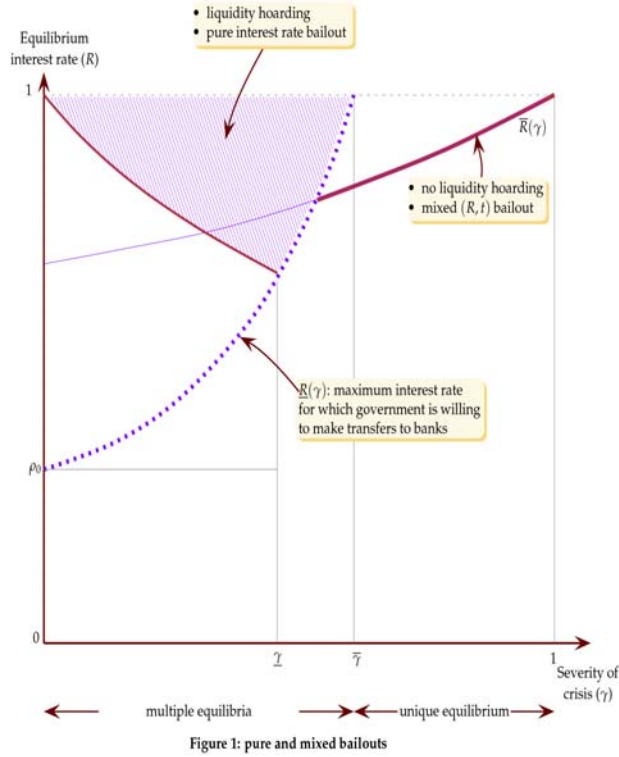

Figure 1: pure and mixed bailouts

Second, because the bailout is a pure interest rate intervention, the equilibrium must be an equilibrium when no transfers are feasible; that is, it must satisfy equation (3). So the equilibrium set (the shaded area in Figure 1) is the set of interest rates in Proposition 2 that satisfy $R \geq \underline{R}(\gamma)$.

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[43]A simple inspection of equation (12) shows that this threshold $\underline{R}(\gamma)$ is defined as the solution of the following equation in R: $\frac{1-\frac{\rho_0}{R}}{w+1-\frac{\rho_0}{R}} = \frac{\gamma}{\gamma+(1-\gamma)\nu}$. The threshold $\underline{R}(\gamma)$ is increasing in γ with $\underline{R}(0) = \rho_0$ and $\underline{R}(\bar{\gamma}) = 1$ where $\bar{\gamma} \in [0,1]$ is the solution of the following equation in γ: $\frac{\gamma}{\gamma+(1-\gamma)\nu} = \frac{1-\rho_0}{w+1-\rho_0}$.

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Once again banks' leverage and liquidity choices are strategic complements, and there can be multiple equilibria. More generally, in this region, the insights gleaned from our analysis of pure interest rate bailouts in Sections III and IV carry over to optimal bailouts. There is one difference in the form of an additional lower bound $\underline{R}(\gamma) \geq \rho_0$ on the interest rate. For $\gamma > \bar{\gamma}$ these equilibria disappear.

The function $\bar{R}(\gamma)$ is increasing in γ. For γ close to $\bar{\gamma}$, $\bar{R}(\gamma)$ is below $\underline{R}(\gamma)$ and the opposite is true for γ close to 0 provided that $\bar{R}(0) > \rho_0$. The following assumption ensures that these two functions cross only once. We denote by $\underline{\gamma}$ the crossing point.

Assumption 11 *The function $\bar{R}(\gamma)$ defined by the solution of the equation $-R^2 L'(R) = A(1-\gamma)\nu\rho_0 / [1 - \pi - \alpha\rho_0]$ crosses the function $\underline{R}(\gamma)$ once from above on the interval $[0, \bar{\gamma}]$.*

Proposition 9 *The symmetric equilibria of the no-commitment economy are as follows:*

(i) if $\gamma > \underline{\gamma}$ then there is an equilibrium with $R = \bar{R}(\gamma)$; it features $i/A = 1/(1 - \pi - \alpha\rho_0)$, $x = 0$, and $d = \pi$;

(ii) if $\gamma \leq \bar{\gamma}$ then there is an equilibrium associated with each fixed point of the equation $R \in \mathcal{R}(R) \cap [\underline{R}(\gamma), 1]$; it features $i/A = 1/[1 - \pi - \alpha\rho_0 + (1-\alpha)(1 - \rho_0/R)]$, $x = 1 - \rho_0/R$, and $d = \pi - (1 - \rho_0/R)$.

To sum up, interest rate policy is always used in equilibrium; indeed direct transfers are not even used in some regions of the parameter space; in the remaining regions, interest rate policy and direct transfers are used in conjunction. Interest rate policy is a market-driven solution, in that it benefits primarily those institutions with actual borrowing needs. Direct transfers better focus on strategic actors, but they entail a greater waste of resources by supporting entities that have no need for, or should not engage in refinancing. When direct transfers are not used (for equilibria of type (ii)), the insights from Sections III and IV carry over to optimal bailouts: multiple equilibria, macroprudential regulation, and endogenous macroeconomic uncertainty.[44]

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[44]Marvin Goodfriend and Robert G. King (1988) argue that the central bank should just provide liquidity anonymoulsy through open market operations and leave it to the market to allocate this liquidity efficiently. They argue against providing liquidity to specific institutions. Our interpretation of interest rate policy is broader than open market operations, complicating the comparison with their view. Nevertheless, in our framework, optimal policy under commitment is entirely passive and can therefore be seen as vindicating their view. Even under limited commitment, the central bank finds it optimal ex post to use only interest rate policy for equilibria of type (ii). However for equilibria of type (i), a combination of interest rate policy and direct transfers to (perceived to be) distressed institutions is used. Direct transfers are used only when

VII. Conclusion

We have built a simple framework to jointly analyze leverage and maturity mismatch in the banking sector, bailouts, and optimal regulation. In order to derive a simple yet general model, we have made a number of simplifying assumptions. Refining our analysis would require a richer model.

For example, we have argued that one interpretation of interest rate policy in our framework was as a reduced form for conventional monetary policy. We could introduce an explicit nominal structure with sticky prices and imperfect competition to model conventional monetary policy as in New-Keynesian models. The framework could be further enriched to study the consequences of bailouts in a setup where interactions between the central bank's balance sheet, inflation, and the government budget are non trivial. We could also introduce the possibility of sovereign default. We have also maintained the assumption of risk neutrality. Introducing risk aversion would allow studying how different policy interventions can impact banks net worth and borrowing costs by affecting risk premia. Finally, in our model, liquidity is costless in the sense that there are no liquidity premia. If liquidity were costly instead, interest rate policy under commitment would not necessarily be passive: it could be optimal for the government to provide liquidity in bad times. However, our insights would carry over: authorities would provide too much liquidity in the time-consistent outcome.[45] We leave these questions for future research.

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the losses from mistaken transfers to intact institutions are smaller than the distortions arising from interest rate policy. Note that these policies are not Pareto improving ex post since they transfer resources away from consumers, but they do lead to an increase overall ex post welfare. These policies reduce ex ante welfare.

[45]This could be formalized along the lines of Holmström-Tirole (1998). Imagine that consumers cannot commit at $t = 0$ to provide funds to banks at $t = 1$. Banks then have to hoard liquidity by purchasing assets at $t = 0$ and selling them at $t = 1$ in the even of a crisis. Banking entrepreneurs are willing to pay a liquidity premium (in the form of a lower return) on these assets, which can happen if there is a scarcity of stores of value. Then, the lack of commitment of consumers is a form of market incompleteness which can be alleviated through government intervention (even under the commitment solution). For example, through a commitment to a loose interest rate policy in crisis. See Farhi-Tirole (2009) for a discussion.

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Appendix

Proof of Proposition 5

Ex-post (date-1) welfare $W^{\text{ex post}}(R; R^*)$ is given by

$$W^{\text{ex post}}(R; R^*) = -L(R) + \int \frac{\left[\beta - (1-R)\frac{\rho_0}{R}\right] \frac{1-\frac{\rho_0}{R^*}}{1-\frac{\rho_0}{R}} A(1 - n(\beta, A))}{1 - \pi - \alpha \rho_0 + (1-\alpha)\left(1 - \frac{\rho_0}{R^*}\right)} dF(\beta, A)$$

and the equilibrium correspondance $\mathcal{R}$ is defined accordingly. The corresponding planning problem is

(1)
$$\underline{K} \equiv \min_{\{n(\beta, A)\}} K$$

s.t.

$$0 \leq n(\beta, A) \leq 1$$

and

$$R^* \notin \mathcal{R}(R^*) \text{ for all } R^* \in [\rho_0, 1).$$

The condition for $R^* \in [\rho_0, 1)$ not to be an equilibrium is that there exists $R \in (R^*, 1]$ such that
$$W^{\text{ex post}}(R; R^*) - W^{\text{ex post}}(R^*; R^*) > 0$$

or equivalently

$$\int_{R^*}^{R} \frac{\partial W^{\text{ex post}}\left(\hat{R}; R^*\right)}{\partial \hat{R}} d\hat{R} > 0.$$

For any given set of values $\Delta\left(R, R^*\right)$ and consider the following subproblem

(2)
$$\underline{K}\left(\left\{\Delta\left(R, R^*\right)\right\}\right) \equiv \min_{\left\{n(\beta, A)\right\}, \left\{\Delta(R, R^*)\right\}} K$$

s.t.

$$0 \leq n\left(\beta, A\right) \leq 1$$

and

$$\int_{R^*}^{R} \frac{\partial W^{\text{ex post}}\left(\hat{R}; R^*\right)}{\partial \hat{R}} d\hat{R} \geq \Delta\left(R, R^*\right) \text{ for all } R^* \in [\rho_0, 1) \text{ and } R \in [R^*, 1].$$

Then the original planning problem (1) and the subproblem (2) are related in the following way:

(3)
$$\underline{K} = \min_{\left\{\Delta(R, R^*)\right\}} \underline{K}\left(\left\{\Delta\left(R, R^*\right)\right\}\right)$$

s.t. the constraint that for all $R^* \in [\rho_0, 1)$, there exists $R \in (R^*, 1]$ such that $\Delta\left(R, R^*\right) > 0$. Moreover the solution $\left\{n\left(\beta, A\right)\right\}$ of (1) coincides with the solution of (2) when $\left\{\Delta\left(R, R^*\right)\right\}$ is set as the solution of (3).

Turning back to (2) and take $\left\{\Delta\left(R, R^*\right)\right\}$ to be the solution of (3), the constraint set and the objective function are linear in $\left\{n\left(\beta, A\right)\right\}$ so the first order conditions are necessary and sufficient for optimality. Let $\mu_{R^*, R} \geq 0$ be the multiplier on the constraint

$$\int_{R^*}^{R} \frac{\partial W^{\text{ex post}}\left(\hat{R}; R^*\right)}{\partial \hat{R}} d\hat{R} \geq \Delta\left(R, R^*\right).$$

Let $\bar{\nu}_{\beta, A} dF\left(\beta, A\right)$ be the multiplier on the constraint $n\left(\beta, A\right) \leq 1$ and $\underline{\nu}_{\beta, A} dF\left(\beta, A\right)$ be the multiplier on the constraint $n\left(\beta, A\right) \geq 0$. Finally, let

$$\mu = \sum \mu_{R^*, R} \int_{R^*}^{R} \frac{1 - \frac{\rho_0}{R^*}}{1 - \pi - \alpha\rho_0 + (1 - \alpha)\left(1 - \frac{\rho_0}{R^*}\right)} \frac{\frac{\rho_0}{R^2}}{\left(1 - \frac{\rho_0}{R}\right)^2} d\hat{R}.$$

2

The first-order condition for $n(\beta, A)$ is

$$c(m(1))^\lambda A^\lambda = \underline{\nu}_{\beta,A} - \bar{\nu}_{\beta,A} + A(\beta + \rho_0 - 1)\mu.$$

The result follows directly from this first-order condition and the complementary slackness conditions $\underline{\nu}_{\beta,A} n(\beta, A) = \bar{\nu}_{\beta,A}[1 - n(\beta, A)] = 0$.

Proof of Proposition 6

Let us first focus on values of (R_0, R) such that banking entrepreneur choose to hoard enough liquidity to continue at full scale in case of a crisis, i.e. values that satisfy equation (??). For such values, we have

$$j(R_0, R) = i(R_0, R) = \frac{A}{1 - \frac{\pi}{R_0} - \frac{\alpha\rho_0}{R_0} + \frac{(1-\alpha)}{R_0}\left(1 - \frac{\rho_0}{R}\right)}.$$

Plugging these expressions into equation (??), we verify that that $W^{\text{ex ante}}(R, R_0)$ increases in R if and only if Assumption ?? holds. For any couple (R_0, R) satisfying equation (??), so does $(R_0, 1)$. We therefore have that

$$W^{\text{ex ante}}(R_0, R) \leq W^{\text{ex ante}}(R_0, 1).$$

It is then easy to verfy that $W^{\text{ex ante}}(R_0, 1)$ is increasing in R_0 as long as Assumption ?? holds.

Let us now turn to values of (R_0, R) such that banking entrepreneurs choose to hoard no liquidity and instead load up on short-term debt, i.e. values such that equation (??) is violated. We only have to consider two values for R: ρ_0 and 1. We have

$$j(R_0, \rho_0) = i(R_0, 1) = i(R_0, \rho_0) = \frac{A}{1 - \frac{\pi}{R_0} - \frac{\alpha\rho_0}{R_0}}$$

and

$$j(R_0, 1) = 0.$$

Let us first consider the case where $R = 1$. Plugging these expressions in equation (??), it can be verified that $W^{\text{ex ante}}(R_0, 1)$ is increasing in R_0 if and only if $\alpha\beta \leq 1 - \pi - \alpha\rho_0$. This condition is implied by Assumptions ?? and ??. Turning now to the case where $R_0 = \rho_0$, we

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find that $W^{\text{ex ante}}(R_0, \rho_0)$ is increasing in ρ_0 if and only if Assumption **??** holds.

Proof of Proposition 9

The proposition follows easily from the following two lemmas. The first one characterizes the form of the optimal bailout given an interest rate R. The second one derives the optimal ex-ante liquidity choices of banks. The first lemma follows easily from the linearity of the ex-post bailout program in $j(i, x)$. The second lemma follows from a simple calculation of the banking entrepreneur's welfare given his liquidity choice x.

Lemma 1 *(optimal bailout). Under the optimal bailout:*

(i) if $R \geq \underline{R}(\gamma)$, then $t(i, x) = 0$ and $j(i, x) = x/(1 - \rho_0/R)$;

(ii) if $R < \underline{R}(\gamma)$ then $t(i, x) = (1 - x - \rho_0/R)i$ and $j(i, x) = i$.

Lemma 1 shows that given R, whether direct transfers are used depends on whether or not $R < \underline{R}(\gamma)$. When $R < \underline{R}(\gamma)$, it is optimal to transfer funds to banks that claim to be distressed, even though a fraction will end up in banks that are truly intact. Enough funds are then transferred so that distressed banks can continue at full scale. When $R \geq \underline{R}(\gamma)$, it is preferrable not to engage in direct transfers because too high a fraction would end up in banks that are truly intact. This is intuitive: when the interest rate R is low, distressed banks can lever up the direct transfers more (and intact banks perceived as distressed cannot), which makes direct transfers more attractive. That the threshold interest rate $\underline{R}(\gamma)$ for direct transfers increases with γ makes sense: The asymmetric information problem is worse when γ is low so that the proportion of false positives $(1 - \gamma)\nu$ is high.

Lemma 2 *(liquidity choice). The optimal scale and liquidity choice of banks when they expect the interest rate to be R in the event of a crisis is:*

(i) if $R \geq \underline{R}(\gamma)$, then $i/A = 1/[1 - \pi - \alpha\rho_0 + (1 - \alpha)(1 - \rho_0/R)]$, $x = 1 - \rho_0/R$ and $d = \pi - (1 - \rho_0/R)$;

(ii) if $R < \underline{R}(\gamma)$, then $i/A = 1/[1 - \pi - \alpha\rho_0]$, $x = 0$ and $d = \pi$.

Lemma 2 shows that the ex-ante liquidity choices also depends on whether or not $R < \underline{R}(\gamma)$. When $R < \underline{R}(\gamma)$, the government provides a big enough direct transfer to banks that claim to be distressed so that they can continue at full scale if they are truly

distressed. Therefore, hoarding liquidity is useless. It only reduces the investment scale and total leverage i/A. As a result, banks opt to be completely illiquid ($x = 0$) and choose a maximal level of short-term debt ($d = \pi$). When $R \geq \underline{R}(\gamma)$, the government does not provide any direct transfer. Continuation scale therefore increases with liquidity and decreases with the amount of short-term debt. Banks choose to hoard enough liquidity ($x = 1 - \rho_0/R$) and take on only as much short-term debt ($d = \pi - (1 - \rho_0/R)$) so as to be able to continue at full scale in case of a crisis.

Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts

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Abstract

The paper shows that time-consistent, imperfectly targeted support to distressed institutions makes private leverage choices strategic complements. When everyone engages in maturity mismatch, authorities have little choice but intervening, creating both current and deferred (sowing the seeds of the next crisis) social costs. In turn, it is profitable to adopt a risky balance sheet. These insights have important consequences, from banks choosing to correlate their risk exposures to the need for macro-prudential supervision.

Keywords: monetary policy, funding liquidity risk, strategic complementarities, macro-prudential supervision

JEL numbers: E44, E52, G28.

One of the many striking features of the recent financial crisis is the extreme exposure of economically and politically sensitive actors to liquidity needs and market conditions:

- Subprime borrowers were heavily exposed to interest rate conditions, which affected their monthly repayment (for those with adjustable rate mortgages) and conditioned their ability to refinance (through their impact on housing prices).

- Commercial banks, which traditionally engage in transformation, had increased their sensitivity to market conditions. First, and arbitraging loopholes in capital adequacy regulation, they pledged substantial amounts of off-balance-sheet liquidity support to the conduits they designed.[1] These conduits had almost no equity on their own and rolled over commercial paper with an average maturity under one month. For many large banks, the ratio of asset backed commercial paper to the bank's equity was substantial (for example, in January 2007, 77.4% for Citibank and 201.1% for ABN Amro, the two largest conduit administrators).[2] Second, on the balance sheet, the share of retail deposits fell from 58% of bank liabilities in 2002 to 52% in 2007.[3] Third, going forward commercial banks counted on further securitization to provide new cash. They lost an important source of liquidity when the market dried up.

- Broker-dealers (investment banks) gained market share and became major players in the financing of the economy. Investment banks rely on repo and commercial paper funding much more than commercial banks do. An increase in investment banks' market share mechanically resulted in increased recourse to market financing.[4]

The overall picture is one of a wide-scale maturity mismatch. It is also one of substantial systematic-risk exposure, as senior CDO tranches, a good share of which was held by commercial banks, amounted to "economic catastrophe bonds".[5]

This paper argues that this wide-scale transformation is closely related to the unprecedented intervention by central banks and treasuries.[6] As described more in detail when we map out the interpretation of our model in terms of actual policies, roughly two categories of

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[1]See e.g. Figure 2.3 (page 95) in Acharya-Richardson (2009), documenting the widening gap between total assets and risk-weighted assets.

[2]See Table 2.1 (page 93) in Acharya-Richardson for the numbers for the 10 largest administrators.

[3]Source: Federal Reserve Board's Flow of Funds Accounts.

[4]Source: Federal Reserve Board's Flow of Funds accounts.

[5]To use Coval et al (2007)'s expression.

[6]Strikingly, by March 2009, the Fed alone had seen its balance sheet triple in size (to $ 2.7 trillion) relative to 2007.

interventions were pursued in order to facilitate financial institutions' access to refinancing. For lack of better words, we term them respectively interest rate and transfer policies. By interest rate policies, we have in mind various forms of government intervention which effectively lower borrowing costs for banks: lowering the Fed Funds rate to zero, extending debt guarantees to a wide range of financial institutions, accepting low-quality assets as collateral with low haircuts in loans or repurchasing agreements, purchasing commercial paper in the primary market etc. By transfer policies, we refer to interventions that primarily boost the net worth of financial institutions without lowering their borrowing cost: recapitalizations, the purchase of legacy assets at inflated prices etc. The distinction between these two categories is sometimes blurred in practice. From our perspective, the key distinguishing feature is whether the intervention under consideration reduces banks' borrowing costs or simply acts to boost their net worth.

In a nutshell, the central argument of the paper is that private leverage choices depend on the anticipated policy reaction to the overall maturity mismatch. Difficult economic conditions call for public policy to help financial institutions weather the shock. Policy instruments however are only imperfectly targeted to the institutions they try to rescue. For example, the archetypal non-targeted policy, lowering the Fed Funds rate, benefits financial institutions engaging in maturity mismatch, but its effects apply to the entire economy. An accommodating interest rate policy involves (a) an invisible subsidy from consumers to banks (the lower yield on savings transfers resources from consumers to borrowing institutions), (b) current costs, such as the (subsidized) financing of unworthy projects by unconstrained entities, and (c) differed costs (the sowing of seeds for the next crisis, both through incentives for maturity mismatch, going forward, and the authorities' loss of credibility).

While the first cost is proportional to the volume of refinancing, the other two are not and are instead akin to a fixed cost. This generates strategic complementarities in balance-sheet riskiness choices. It is ill-advised to be in a minority of institutions exposed to the shock, as policymakers are then reluctant to incur the "fixed cost" associated with active interest rate policy. By contrast, when everybody engages in maturity transformation, the central bank has little choice but intervening. Refusing to adopt a risky balance sheet then lowers banks' rate of return. It is unwise to play safely while everyone else gambles.[7]

The same insight applies when some players expose themselves to liquidity risk either

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[7]As Charles Prince, then CEO of Citigroup, famously stated in the summer of 2007: "*as long as the music is playing, you have to get up and dance*".

because they are unsophisticated[8] or because they engage in regulatory arbitrage.[9] Strategic complementarities then manifest themselves in the increased willingness of other actors to take on more liquidity risk due to the presence of unsophisticated players or regulatory arbitrageurs. A reinterpretation of our analysis is thus in terms of an amplification mechanism.

The paper's first objective is to develop a simple framework that is able to capture and build on these insights. Corporate entities (called "banks") choose their level of short-term debt (or, equivalently, whether to hoard liquid instruments in order to meet potential liquidity needs). In the basic model liquidity shocks are correlated, and so there is macroeconomic uncertainty. Maturity transformation is intense in the economy when numerous institutions take on substantial short-term debt. The issuance of short-term debt enables banks to increase their leverage and investment, but exposes them to a potential refinancing problem in case of a shock. When privileging leverage and scale, bankers thereby put at risk "banking stakeholders", a designation regrouping those agents who would be hurt in case banks have to delever: bankers themselves, industrial companies that depend on bank loans for their financing, and employees of those banks and industrial companies.[10]

Authorities maximize a weighted average of consumer surplus and banking stakeholders' welfare. Focusing in a first step on interest rate policy, they can, in case of an aggregate shock, facilitate troubled institutions' refinancing by lowering the effective interest rate at which banking entrepreneurs borrowing. However, loose interest rate policy, besides transferring resources from consumers to banks with refinancing needs, might for example facilitate the financing of unworthy projects (in the basic version) or entails future costs (future illiquidity of institutions or loss of credibility). This distortion is akin to a fixed cost, which is worth incurring only if the size of the troubled sector is large enough. We obtain the following insights:

- Excessive maturity transformation. The central bank supplies too much liquidity in the time-consistent outcome. Our theory therefore brings support to the view that authorities in the recent crisis had few options when confronted with the fait accompli, and that the crisis should have been contained ex ante through more careful prudential policies. While prudential supervision is traditionally concerned with the solvency of

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[8]Such players may for instance miscalibrate the risk involved in relying on funding liquidity or on securitization to cover their future needs, and thereby mistakenly engage in maturity mismatch.

[9]As was the case with largely underpriced liquidity support to conduits.

[10]Note that consumers may have multiple incarnations: As taxpayers/savers, they should oppose an intervention, while as employees of these corporate entities, they might welcome it. All these effects are taken into account in our welfare analysis.

individual institutions, our framework suggests the potential value of a new, macro-prudential approach, in which prudential regulators consider not only the individual institutions' transformation activities, but also the overall transformation of the strategic institutions.[11],[12]

- Optimal regulation. In our model, optimal regulation takes the form of a liquidity requirement or equivalently of a cap on short-term debt. Importantly, breaking down banks into smaller banks would achieve no benefit in our framework. The basic problem here is not too big too fail, but rather that the banks as a whole are doing too much maturity mismatch, and are taking on too much correlated risk.

- Regulatory pecking order. If regulation is costly, our model suggests that regulation should be confined to a subset of key institutions, the ones that authorities are the most tempted to bail-out ex post.[13]

- Endogenous macroeconomic uncertainty. We relax the correlated-shock assumption and let banks choose the correlation of their shock with that of other banks. We find that they actually choose to maximize the correlation of their shocks due to the nature of the policy response. This result runs counter to conventional wisdom. Financial theory (CAPM) predicts that, when faced with a choice among activities, a firm will want to take as much risk as possible in those states of nature in which the economy is doing well. That is, it will strive to be as negatively correlated as possible with the market portfolio.

- Sowing the seeds of the next crisis. Loose interest rate policy today increases the likelihood of future crises. First, they signal the central bank's willingness to accommodate maturity mismatches, and deprive it of future credibility. Second, they stimulate new

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[11]Although extremely imperfect, liquidity regulation does exist at the micro level (both through stress tests under Basel II, and through the definition of country-specific liquidity ratios).

[12]These questions are at the forefront of the regulatory reform agenda. The Financial Stability Forum (2009) calls for "a joint research program to measure funding and liquidity risk attached to maturity transformation, enabling the pricing of liquidity risk in the financial system" (Recommendation 3.2) and recommends that "the BIS and IMF could make available to authorities information on leverage and maturity mismatches on a system-wide basis" (Recommendation 3.3).

[13]These strategic institutions correspond to large retail banks (where size matters indirectly because of the disruption in the payment and credit systems, or because of the greater coverage in the media), or to other large financial institutions that are deeply interconnected with them through opaque transactions (as was the case recently with AIG or the large investment banks). They also include those with close connections with the central bank; in the latter respect, while starting with Barro-Gordon (1983) the literature on central bank independence as a response to time-inconsistency has emphasized political independence, our analysis stresses the need for independence with respect to the financial industry.

maturity mismatches through a price effect: They make short-term debt cheaper, encouraging maturity mismatches; and they provide a subsidy to capital, encouraging overall leverage.

Interest rate policies are rough instruments because they entail distortions. By contrast transfer policies do not entail similar distortions and instead involve only a subsidy. One might therefore conjecture that interest rate policy is a dominated instrument when such transfer policies are available. Relatedly, we need to check the robustness of the insights stated above to optimal policy interventions.

Accordingly, the second objective of the paper is to analyze the optimal bailout mix using a mechanism design approach. We allow authorities to operate direct transfers to institutions. However when implementing transfer policies, they face an asymmetry of information (they are unsure which banks are distressed or intact); consequently, direct transfers entail a different set of distortions, associated with wasted-support costs. We characterize the optimal policy intervention (interest rate and transfers) given informational constraints. We show that:

- Interest rate policy is actually always used in equilibrium; indeed transfers are not even used unless the crisis affects a large fraction of the banks, in which case interest rate policy and transfers are used in conjunction. The key insight is that interest rate policy is a market-driven solution, in that it benefits primarily those institutions with actual borrowing needs; put more technically, it helps screen out opportunistic institutions with limited refinancing needs. While transfers better focus on strategic actors, they entail a greater waste of resources by supporting entities that have no need for, or should not engage in refinancing.

- The insights gleaned for pure interest rate bailouts carry over to optimal bailouts: strategic complementarities in the size and quality of liquidity positions, excessive maturity transformation, pecking order of regulation and endogenous macroeconomic uncertainty.

The paper is organized as follows. Section I sets up the model. Section II analyses the commitment benchmark, where the central bank can announce and stick to an interest-rate policy. Section III performs the same exercise for the time-consistent outcome. Section IV draws the implications for regulation. Section V provides the two foundations for the hazards

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of low interest rate policies as sowing the seeds for the next crisis. Section VI allows for the full range of policy instruments and derives the optimal bailout policy. Finally, Section VII concludes.

Relationship to the literature. Our paper is related to several disjoint bodies of literature. The importance of keeping interest low in recessions is classic in macroeconomic theory. Our paper contributes to this literature first by pointing out a new channel through which interest rate policy suffers from time inconsistency (of the kind emphasized by Finn E. Kydland and Edward Prescott 1977), and second by viewing interest rate policy in a broader bailout context in which support to institutions and asset prices are alternative instruments.

Potential macroeconomic shortages of liquidity exist if corporations are net lenders (Michael Woodford 1990) or if corporations are net borrowers and face macroeconomic shocks (Bengt Holmström and Jean Tirole 1998). The literature on aggregate liquidity has emphasized the role of governments in providing (possibly contingent) stores of value that cannot be created by the private sector. Like in Holmström and Tirole, liquidity support is viewed here as redistribution from consumers to firms in bad states of nature; it however is an ex post redistribution rather than a planned one, and it emphasizes the role of interest rates and more generally borrowing costs in enabling refinancing.

Time-inconsistency from rescuing banks and the resulting moral hazard problems in a single-bank context have been emphasized by numerous works, starting with Walter Bagehot (1873).

Through its emphasis on strategic complementarities, our paper is reminiscent of the wide body of literature on multiple equilibria in macroeconomics, starting with Peter A. Diamond (1982) and Russell W. Cooper and Andrew John (1988) (see, e.g., Cooper 1999 for a review). Our paper emphasizes the idea that strategic complementarities stem from the government's policy response. In that, it is particularly related to Stephen Morris and Hyun Song Shin (1998), Martin Schneider and Aaron Tornell (2004) and Romain Rancière, Aaron Tornell and Frank Westermann (2008). Morris and Shin, and Schneider and Tornell are concerned with exchange rates while Rancière, Tornell and Westermann focus on a risky technological choice. These papers posit that the government accommodates private agents once the latter have reached some exogenous threshold of private involvement (in speculation, currency mismatch or realized returns). This threshold gives rise to strategic complementarities. An important difference in our model is that the incentives to bail out and hence the policy reaction function are endogenized. This puts the time-inconsistency of

policy at the center stage and has important positive implications for comparative statics, as well as normative consequences by allowing us to study the optimal design of regulation.

Viral Acharya and Tanju Yorulmazer (2007, 2008) study the incentives for banks to correlate the risks inherent in their investment choices. In Acharya-Yorulmazer (2007), the possibility for one bank to acquire the other pushes banks to minimize their correlation. However, they assume that when both banks fail, both banks are bailed out. If the bailout guarantee when both banks fail is worth more than the rent obtained by the surviving bank when only one bank fails and is sold to the other, then banks seek to maximize their correlation. Acharya-Yorulmazer (2008) introduces a richer model with fire-sales and makes the point that from an ex-post perspective, bailing out failed banks and subsidizing intact banks to take over failed banks have similar effects, but that the latter is preferable ex ante because it induces banks to differentiate their risks. There are important differences with our paper: first, in our setup, bank managers are indispensable to the project, so that intact banks are at no comparative advantage over outside investors when refinancing failed banks; second, these papers do not emphasize the role of untargeted policy instruments; third, they do not allow banks to vary the amount of risk that they take.[14]

Humberto N. Ennis and Todd Keister (2009, 2010) study a modification of the model of Douglas W. Diamond and Phillip H. Dybvig (1983) where a policy maker can only choose policies (such as deposit freezes) that are contingent on the fraction of agents who have already withdrawn their deposits, and that are efficient ex post. They point out that bank run equilibria can exist together with the efficient equilibrium. Our paper shares the idea that policy responses (under no commitment) can generate multiplicity. However, while they focus on the incentives of depositors to run, we analyze instead the ex ante choices of banks.

Diamond and Raghuram Rajan (2009) emphasize, as we do, that interest rate policy is time inconsistent and that low-interest rate policies may encourage excessive leverage. Interestingly, in their framework, because of an assumed form of market incompleteness (non-contingent deposits) absent in our model, optimal interest rate policy under commitment involves both low interest rates in bad times and high interest rates in good times.

Varadarajan V. Chari and Patrick J. Kehoe (2010) study a model in which inefficient ex-post liquidations are required for ex-ante efficiency, but the possibility of ex-post bailouts introduces a time-inconsistency problem. They show, as we do, that regulation in the form of specific ex-ante restrictions on private contracts can increase welfare. Interestingly, in

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[14]In our model, this occurs through the choice of short term debt, leverage, and maturity mismatch.

their setup, the ex-post cost of bailouts is that they trigger a bad continuation equilibrium of the policy game. In the best equilibrium, any deviation triggers the worst equilibrium, introducing a fixed cost from bailouts. This reputational mechanism is therefore similar to ours. We elaborate on this analogy in Section V.

Finally, the optimal regulation in our model bears some resemblance to Anil K. Kahsyap, Rajan and Jeremy Stein (2008). They propose replacing capital requirements by mandatory capital insurance policy, whereby banks are forced to hoard liquidity, in the form of T-bills.

I. The model

A. Banks

There are three periods, $t = 0, 1, 2$. Banking entrepreneurs have utility function $U = c_0 + c_1 + c_2$, where c_t is their date-t consumption. They are protected by limited liability and their only endowment is their wealth A at date 0. Their technology set exhibits constant returns to scale. At date 0 they choose their investment scale i and a level of short-term debt (see below). At date 1, a safe cash flow πi accrues, that can be used to pay back the short-term debt. Uncertainty bears on the investment project: It is intact with probability α, and distressed with probability $1 - \alpha$. Whether the project is intact or distressed depends on the realization of an aggregate shock—a "crisis". In other words, the shocks impacting the different banking entrepreneurs are perfectly correlated.[15]

If the project is intact, the investment delivers at date 1; it then yields, besides the safe cash flow πi, a payoff of $\rho_1 i$, of which $\rho_0 i$ is pledgeable to investors.[16] If the project is distressed, the project yields no payoff at date 1, except for the safe cash flow πi. It yields a payoff at date 2 if fresh resources j are reinvested. The project can be downsized to any level $j \leq i$. It then delivers at date 2 a payoff of $\rho_1 j$, of which $\rho_0 j$ is pledgeable to investors.[17] The

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[15]Later we will allow entrepreneurs to choose the correlation of their shock with those faced by other entrepreneurs.

[16]As usual, the "agency wedge" $\rho_1 - \rho_0$ can be motivated in multiple ways, including limited commitment, private benefits or incentives to counter moral hazard (see Section I.B; see also Holmström and Tirole 2010).

[17]Note that we are assuming that the manager is indispensable to the project. As a result, intact banks are at no advantage over consumers in buying or operating distressed banks. This assumption turns off a channel that could generate strategic substituabilities, whereby some institutions (banks, or other specialist buyers) overhoard liquidity to secure available resources when a lot of banks are distressed and attractive opportunities arise. On the other hand if banks are expected to be rescued (as is the case in the paper), specialist buyers have no incentive to hoard liquidity. See Acharya-Yorulmazer (2007, 2008), and chapter 7

following assumption will guarantee that the projects are attractive enough that banking entrepreneurs will always invest all their net worth.[18]

Assumption 1 *(high return)* $\rho_1 > 1 - \pi + 1 - \alpha$.

The interest rate is a key determinant of the collateral value of a project. It plays an important role in determining the initial investment scale i as well as the reinvestment scale j. We explain how interest rates are determined in Section I.B. In sum, the gross rate of interest is equal to 1 between dates 0 and 1. Between dates 1 and 2, the interest rate is equal to 1 in the absence of a crisis, and to $R \leq 1$ otherwise.[19] For the rest of the paper, we adopt the convention that R refers to the interest rate between dates 1 and 2 if there is a crisis.

At the core of the model is a maturity mismatch issue, where a long-term project requires occasional reinvestments. The bank has to compromise between initial investment scale i and reinvestment scale j in the event of a crisis. Maximizing initial scale i requires loading up on short-term debt and exhausting reserves of pledgeable income. This in turn forces the bank to downsize and delever in the event of a crisis. Conversely, limiting the amount of short-term debt to mitigate maturity mismatch requires sacrificing initial scale i.

The bank issues state-contingent short-term debt. It is always optimal to set short-term debt in event of no crisis equal to πi. We denote di (where $d \leq \pi$) the amount of short-term debt in the event of a crisis; we refer to it simply as short-term debt throughout the paper. The excess $xi \equiv (\pi - d)i$ of the safe cash flow πi over debt payments di represents cash available at date 1 in the event of a crisis (x is the analog of a liquidity ratio). We assume that any potential surplus of cash over liquidity needs for reinvestment—$\max\{(\pi - d)\,i - j\,(1 - \rho_0/R)\,,0\}$ —is consumed by banking entrepreneurs. The policy of pledging all cash that is unneeded for reinvestment is always weakly optimal. Pledging less is also optimal (and leads to the same allocation) if the entrepreneur has no alternative use of the unneeded cash to distributing to investors. However, if the entrepreneur can divert (even an arbitrarily small) fraction of the extra cash for her own benefit, then pledging the entire unneeded cash is *strictly* optimal.

At date 1, in the adverse state, the bank can issue new securities against the date-2

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in Holmström-Tirole (2010).

[18]This condition is intuitive: investing 1 at $t = 0$ and 1 at date 1 if a crisis occurs yields a return $\rho_1 + \pi$ and costs $1 + (1 - \alpha)$.

[19]In all the cases that we consider, it is always optimal for the central bank to set the interest rate to 1 at date 0 (see Section V), and also at date 1 if there is no crisis, but to some $R \leq 1$ at date 1 if there is a crisis.

pledgeable income $\rho_0 j$, and so its continuation $j \in [0, i]$ must satisfy:

$$j \le (\pi - d)i + \frac{\rho_0 j}{R}$$

yielding continuation scale:

$$j = \min \left\{ \frac{x}{1 - \frac{\rho_0}{R}}, 1 \right\} i.$$

This formula captures the fact that lower interest rates facilitate refinancing. A banking entrepreneur would never choose to have excess liquidity and so we restrict our attention to $d \in [\pi - (1 - \rho_0/R), \pi]$ or equivalently $x \in [0, 1 - \rho_0/R]$.

The banks needs to raise $i - A$ from outside investors at date 0. Because the bank returns $di + (\pi - d)i + \rho_0 i$ to these investors in the good state and only di in the bad one, its borrowing capacity at date 0 is given by:

$$i - A = \alpha (\pi i + \rho_0 i) + (1 - \alpha) di$$

i.e.

$$i = \frac{A}{1 - \pi - \alpha \rho_0 + (1 - \alpha)x}.$$

The banking entrepreneur therefore maximizes over $d \in [\pi - (1 - \rho_0/R), \pi]$ or equivalently $x \in [0, 1 - \rho_0/R]$:

$$(\rho_1 - \rho_0) [\alpha i + (1 - \alpha)j] = (\rho_1 - \rho_0) \left[\frac{\alpha + (1 - \alpha)\frac{x}{1 - \rho_0/R}}{1 - \pi - \alpha \rho_0 + (1 - \alpha)x} \right] A$$

The banking entrepreneur loads up on short-term debt ($x = 0$, i.e., $d = \pi$) if and only if

$$\alpha + \pi > 1 + \alpha \rho_0 \left(\frac{1}{R} - 1 \right),$$

and takes on just enough short-term debt to be able to continue full scale ($x = 1 - \rho_0/R$, i.e., $d = \pi - 1 + \rho_0/R$) otherwise.

We assume that the banking entrepreneurs prefer to limit the amount of short-term debt to have enough liquidity to continue at full scale in the adverse state of nature in the relevant range of interest rates (which will be $[\rho_0, 1]$):[20]

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[20]Note that Assumption 2 implies that $1 - \pi - \alpha \rho_0 > 0$ which guarantees that investment is finite.

Assumption 2 *(demand for liquidity): $\alpha + \pi < 1$.*

B. Rest of the economy

Consumers born at date $t \in \{0, 1\}$ consume at date $t + 1$; so their utility is $u_t = c_{t+1}$. They are endowed with a large amount or resources (savings) s when born.

A short-term *storage technology* yields 1 in the next period for 1 invested today. In particular the natural rate of interest (the marginal rate of transformation) between dates 1 and 2 is $R = 1$. For the date-1 interest rate to be $R \neq 1$, the storage technology must be taxed at rate $1 - R$ (see later for interpretations). The proceeds are rebated lump sum to consumers at date 2. Throughout the paper, we assume that s is large enough to finance all the necessary investments in the projects of banking entrepreneurs at each date t. As a result, consumers always invest a fraction of their savings in the short-term storage technology.

This modelling device is a way to capture a range of policy interventions that reduce borrowing costs for banks. For instance, taxing the short-term storage technology and rebating the proceeds lump-sum to consumers is essentially equivalent to subsidizing investment in the banks and financing this subsidy by a lump-sum tax on consumers. We elaborate on this analogy below and propose several interpretations for which this way of modelling interest rate policy could be a convenient reduced form. For now, we do not introduce any other instrument. In Section VI, we allow for the full range of policy instruments and derive the optimal bailout policy.

Assumption 3 *(interest rate distortion): The set of feasible interest rates is $[\rho_0, 1]$. Furthermore, there exists a fixed distortion or deadweight loss $L(R) \geq 0$ when the interest rate R diverges from its natural rate: $L(1) = L'(1) = 0$, and L is decreasing on $[\rho_0, 1]$.*

The upper bound at 1 for the interest rate R is not crucial but simplifies the analysis. As we shall see below, it will be used to normalize the optimal interest rate under commitment to $R = 1$. One can justify this assumption by positing arbitrage (foreigners or some long-lived consumers would take advantage of $R > 1$) or by assuming that marginal distortions $L'(R)$ are very high beyond 1. But again, we want to emphasize that this particular assumption only simplifies the exposition and plays no economically substantive role in the analysis.

The lower bound at ρ_0 for the interest rate R is also without loss of generality. Indeed, as will become apparent, the central bank will never find it optimal to lower the interest rate in

times of crisis below ρ_0. At that interest rate, projects can continue at full scale even when no liquidity has been hoarded. Lowering the interest rate below ρ_0 would only increase the distortion associated with interest policy at no gain.

Assumption 4 *(consumers): Suppose that date-0 investment is equal to i, that banks hoard liquidity x and so can salvage $j = xi/\left(1 - \rho_0/R\right)$ in case of crisis. Then*

(i) if there is a crisis at date 1, date-1 consumer welfare is $V = -L(R) - (1 - R)\rho_0 j/R$;

(ii) if there is no crisis at date 1, date-1 consumer welfare is $V = -L\left(1\right) = 0$.

In (i), the second term in V stands for the implicit subsidy from savers to borrowing banks. Indeed date-1 consumers' return on their savings $\widetilde{s}$ is $R\widetilde{s} + (1 - R)\left(\widetilde{s} - \rho_0 j/R\right)$ (the last term representing the lump-sum rebate on the $\widetilde{s} - (\rho_0 j/R)$ invested in the storage technology), or $\widetilde{s} - (1 - R)\rho_0 j/R$.[21] Note that welfare V does not include the additional indirect benefits that firms' managers and workers might derive from banks functioning at high scale. More on this below. Finally, we ignore the welfare of date-0 consumers as they have constant utility $u_0 = s$.

Our modelling of interest rate policy deserves some comments. It is a stylized representation of some actual interest rate policies. Their common feature is to reduce borrowing costs for banks. We list a few of those below.

Interpretation 1. One case in point is unconventional monetary policy.[22] Extended debt guarantees by the government reduce the rate (R) paid both by constrained institutions that the government wants to help (the "banks") and by other borrowers. The subsidy is paid by taxpayers who end up bearing the risk of debt.[23] Similarly, accepting assets as collateral at low haircuts in loans or repurchasing agreements and directly purchasing commercial paper at favorable terms lower the effective interest rate faced by borrowers. Such interventions

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[21] Note that we use the notation $\tilde{s}$ instead of s for the savings of date-1 consumers. This is because under our Interpretation 1 below, some of the savings s of date-1 consumers are invested in alternative wasteful investment projects. As a result, only a part $\tilde{s}$ of their savings are split between reinvestment in banks and the short-term storage technology.

[22] See, e.g., Gertler-Karadi (2009) and Gertler-Kiyotaki (2009) for models with both conventional and unconventional monetary policies.

[23] One way to formalize this is as follows. Imagine that in case of a crisis, the pledgeable part of the return ρ_0 from reinvesment is the expectation of a random variable realized at $t = 2$ that takes the value $\rho_0^+ > \rho_0$ with probability λ and 0 with probability $1 - \lambda$. Banking entrepreneurs can issue (defaultable) debt with nominal value ρ_0^+. A guarantee from the government to deliver a fraction ϕ of the value of every debt contract in case of default then reduces the (gross) interest rate demanded by creditors by a proportion factor $\lambda/\left[\lambda + (1 - \lambda)\phi\right] < 1$.

involve (in expectation) a subsidy from savers to borrowers, reduce the marginal borrowing cost of banks, and are therefore captured by our model.

With this in mind, the deadweight loss may for instance result from the date-1 financing of projects that have negative net present value at the natural rate. Suppose that there is a distribution of financially unconstrained firms with projects that have unit cost and return κ with cumulative distribution function $H(\kappa)$. Then the deadweight loss if consumers and project owners are weighted equally is $L(R) = \int_R^1 (1 - \kappa) dH(\kappa)$. If the projects' owners receive welfare weight $\beta_u \leq 1$ relative to consumers instead, then $L(R) = (1 - R)[H(1) - H(R)] - \beta_u \int_R^1 (\kappa - R) dH(\kappa)$ still satisfies our assumptions.[24]

Moreover, with our assumed preferences for consumers, a tax on the storage technology between dates 1 and 2 combined with a lump sum rebate is exactly equivalent to a subsidy on reinvestment in banks between dates 1 and 2. The distortion behind $L(R)$ arises because these projects are subsidized at the same rate as reinvestment in banks (this implicitly assumes that the government cannot screen out these projects from genuine positive net present value bank projects).

Interpretation 2. Another interest rate policy captured in a stylized way by the model is conventional monetary policy. An interpretation closely related to Interpretation 1 relies on a view of the monetary transmission mechanism whereby higher reserves allow banks to lever more through access to cheap retail deposits, as deposit insurance tends to be underpriced (at least during hard times since it is not indexed on the banks' riskiness).[25] This involves an implicit subsidy to banks since this deposit insurance is backed by taxes on consumers. Increasing reserves (or reducing reserve requirements) therefore both reduces the borrowing cost of banks (i.e. lowers the effective interest rate face by banks) and involves a subsidy from taxpayers to borrowing banks.

Interpretation 3. The deadweight loss function L can also be interpreted as a re-duced form of a more standard distortion associated with conventional monetary policy, as emphasized in the New-Keynesian literature. Here we have in mind not a short-term inter-vention, but a prolonged reduction of interest rates (a year to several years, think of Japan). Even though our model is entirely without money balances, sticky prices or imperfect com-petition, it captures a key feature of monetary policy in New-Keynesian models routinely used to discuss and model monetary policy. In New-Keynesian models, the nominal interest

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[24]Note that in this case, project owners are lumped with consumers, and their welfare is included in consumer welfare.

[25]See Stein (2010) for a detailed exposition of this view.

rate is controlled by the central bank. Prices adjust only gradually according to the New-Keynesian Phillips Curve, and the central bank can therefore control the real interest rate. The real interest rate regulates aggregate demand through a version of the consumer Euler equation—the dynamic IS curve. Without additional frictions, the central bank can achieve the allocation of the flexible price economy by setting nominal interest rates so that the real interest rate equals to the "natural" interest rate. Deviating from this rule introduces variations in the output gap together with distortions by generating dispersion in relative prices. To the extent that these effects enter welfare separately and additively from the effects of interest rates on banks' balance sheets—arguably a strong assumption—our loss function $L(R)$ can be interpreted as a reduced form for the loss function associated with a real interest rate below the natural interest rate in the New-Keynesian model.[26][27] Under this interpretation, monetary policy works both through the usual New-Keynesian channel and through its effects on banks via a version of the "credit channel".[28]

C. Welfare and policy-making

The authorities (the "central bank") control the date-1 real rate of interest.

Assumption 5 *(welfare function): At date 1, the central bank's objective function is a weighted average W of consumer welfare V and continuation scale j ($j = i$ if there is no crisis): $W = V + \beta j$. At date 0, the central bank's objective function is the expectation of its date-1 objective function.*

The second term βj in the social welfare function deserves some comments. One possible interpretation is as follows. Imagine that, say, three categories of banking stakeholders' benefit from the banks' ability to continue. First, and most obviously the banking entrepreneurs themselves: They receive rent $s_b j$, where s_b is the banks' stake in continuation. Second, the

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[26]Yet another cost, absent in cashless New Keynesian models, is the so called inflation tax and arises when money demand is elastic.

[27]Because they are not our focus, we imagine here that the traditional time-inconsistency problems associated with monetary policy in the New-Keynesian model have been resolved. As is well known, this is the case if a sales subsidy is available to eliminate the monopoly price distortion.

[28]There are two versions of the credit channel (see Bernanke-Gertler 1995 for a review): the "balance sheet channel" and the "bank lending channel". Our model is consistent with the former in its emphasis on the effect of interest rates on collateral value. It is consistent with the latter in that low interest rates boost the real economy by facilitating bank refinancing and thereby increasing the volume of loanable funds to the economy.

higher j, the better off their borrowers. Third, the workers working in banks and industrial companies; to the extent that they are better off employed (e.g., they receive an efficiency wage) and that preserved employment is related to j, then workers' welfare grows with j. Thus if s_f and s_w denote the stakes of the industrial firms and the workers, and if β_b, β_j and β_w denote the three categories of stakeholders' welfare weights or political influence, then $\beta j = \left(\beta_b s_b + \beta_f s_f + \beta_w s_w\right) j$. Assumptions 1 and 6 (see below) imply that $\beta_b \leq 1$: this guarantees that the central bank never seeks to lower interest rates more than what is necessary to guarantee reinvestment at full scale since the freed-up cash is appropriated by banking entrepreneurs and hence represents an unattractive transfer to them from consumers.

This can be formalized further along the lines of Holmström-Tirole (1997). We only sketch it here: At date 0, the bank makes an investment in knowledge/staff so as to be able to monitor a mass i of firms at date 1. These firms enter in a relationship with the bank at date 0; from then on, they share available resources in coalition with the banks. Independently of whether or not firms succeed, they each produce short-term profits π at date 1 (hence the mass i of firms produces short-term profits πi). Firms succeed (return r per firm) or fail (return 0). Success is guaranteed if the bank monitors, and the firm managers as well as the workers in the firm do not shirk. Otherwise success accrues with probability 0. Shirking on monitoring yields benefit b, shirking for a firm manager brings benefit f, and shirking for a firm worker brings benefit w. Therefore incentive payments b, f and w per firm are required to discipline the bank, the firm manager and the workers. For simplicity, we assume that workers are cashless. Firms are cashless at date 0; at date 1, each firm has resources π. If there is no crisis, then the return of firms (r or 0 per firm) occurs in period 1. A crisis means that firms to be monitored must invest 1 each at date 1 and the return on reinvested funds occurs in period 2. This model is summarized by the equations in Sections I.A and I.B with $\rho_0 = r - (b + f + w)$, $\rho_1 = r - (f + w)$, $s_b = b$, $s_f = f$ and $s_w = w$. The only difference is that the total return on a unit of successful investment is $r > \rho_1$. This difference, however, makes no difference to our analysis, since only a fraction ρ_0 of the return can be pledged to outside investors, and banking entrepreneurs get to keep the difference between ρ_1 and the fraction of the return pledged to outside investors.

A roadmap. We will analyze two situations: one where the central bank can commit at date 0 to a specific contingent policy at date 1, and the (probably more likely) alternative where the central bank lacks commitment and instead determines its policy at $t = 1$ with no regard for previous commitments. In both cases, banking entrepreneurs and consumers form expectations regarding the interest rate $R \in [\rho_0, 1]$ that will be set if a crisis occurs.

Note that we have not included the interest rate R_0 between dates 0 and 1 in the set of policy instruments. In Section V.A we relax this assumption and allow for the storage technology between dates 0 and 1 to be taxed, with the proceeds rebated lump sum to consumers. We show that both under commitment and under no commitment, $R_0 = 1$ is always chosen. This justifies proceeding under the assumption that $R_0 = 1$.

II. Commitment solution

This Section analyzes the equilibrium when the date-1 interest rate is chosen at date 0. Ex-ante welfare is

$$W^{\text{ex ante}}(R) \equiv \alpha[V(1) + \beta i(R)] + (1-\alpha)[V(R) + \beta j(R)]$$

where

$$V(R) \equiv -[L(R) + (1-R)\frac{\rho_0 j(R)}{R}]$$

and

$$j(R) = i(R) = \frac{A}{1 - \pi - \alpha\rho_0 + (1-\alpha)(1 - \frac{\rho_0}{R})}.$$

Using $V(1) = 0$, we can write

$$W^{\text{ex ante}}(R) = \left[\beta - (1-\alpha)\frac{1-R}{R}\rho_0\right] i(R) - (1-\alpha)L(R).$$

An increase in the interest rate $R \in [\rho_0, 1]$ in case of crisis reduces the distortion $L(R)$. It also reduces the banks' leverage and therefore investment $i(R)$, which involves a redistribution from banking stakeholders to the rest of the population. We assume that $\beta \leq 1 - \alpha + 1 - \pi - \rho_0$, which implies that $[\beta - (1-\alpha)(1-R)\rho_0/R] i(R)$ is non-decreasing in R and hence that the optimum under commitment is $R = 1$.

Note that this condition is equivalent to that making socially undesirable a date-0 unit lump-sum transfer from consumers to banks in the absence of interest rate policy ($R = 1$): such a transfer would have social welfare cost $1 - \beta [i(1)/A] = 1 - \beta/(1 - \alpha + 1 - \pi - \rho_0)$, where the term in brackets is the leverage multiplier.

Assumption 6 *(No ex-ante wealth transfer):* $\beta \leq 1 - \alpha + 1 - \pi - \rho_0$.

Proposition 1 *The optimal interest rate policy under commitment features $R^c = 1$.*

III. No-commitment solution

Let us now assume that the interest rate is set at date 1, without commitment. At date 0, investors and banking entrepreneurs form an expectation for the interest rate $R^* \in [\rho_0, 1]$ that the central bank will set if a crisis occurs. Based on this expectation, the representative bank invests at scale $i(R^*)$ and hoards just enough liquidity $x^* i(R^*)$ to be able to reinvest at full scale in the event of a crisis, where $x^* = 1 - (\rho_0/R^*)$.

The central bank's decision. At date 1, the central bank is not bound by any previous commitment and is free to set the interest rate to maximize welfare from date 1 on. We have to distinguish two cases depending on whether or not the economy is in a crisis.

If there is no crisis, it is optimal to set $R = 1$. There is no point in lowering the interest rate since all banks are intact. If there is a crisis, the central bank is confronted with the following trade-off. By setting a low interest rate, it can limit the amount of downsizing that banks have to undergo. But this comes at the cost of a large interest rate distortion. An additional cost comes together with the implicit subsidy to banks, in the form of a redistribution of resources from consumers to banking stakeholders.

Because $\beta_b \leq 1$, the central bank would never set $R < R^*$. Indeed, lowering R below R^* does not increase continuation scale but merely redistrbutes resources from consumers to banking entrepreneurs, and comes at the cost of a greater interest rate distortion. However, the central bank might be tempted to set $R > R^*$. In this case, banks are forced to downsize. The reinvestment scale is determined according to:

$$j = \frac{x^*}{1 - \frac{\rho_0}{R}} i(R^*) \iff j = \frac{1 - \frac{\rho_0}{R^*}}{1 - \frac{\rho_0}{R}} i(R^*).$$

Proceeding as in Section II, we can compute ex-post (date-1) welfare $W^{\text{ex post}}(R; R^*)$ in case of crisis when the central bank sets the interest rate to $R \geq R^*$ and agents contracted at date 0 anticipating an interest rate of R^*:

(1)
$$W^{\text{ex post}}(R; R^*) = -L(R) + \left[\beta - (1 - R)\frac{\rho_0}{R}\right] \frac{1 - \frac{\rho_0}{R^*}}{1 - \frac{\rho_0}{R}} i(R^*).$$

At date 1, in the event of a crisis, the central bank sets $R \in [R^*, 1]$ so as to maximize $W^{\text{ex post}}(R; R^*)$. Denote by $\mathcal{R}(R^*)$ the set correspondence defined by

$$(2) \qquad\qquad \mathcal{R}(R^*) \equiv \arg \max_R W^{\text{ex post}}(R; R^*)$$

and let

$$w \equiv \beta - (1 - \rho_0).$$

The first term on the right-hand side of (1) is increasing in R with $L'(1) = 0$. The behavior of the second term on the right-hand side of (1) depends crucially on the sign of w. If $w \leq 0$, then it is increasing in R with a positive derivative at $R = 1$. In this case, $\mathcal{R}(R^*) = \{1\}$: There is no commitment problem. If $w > 0$ on the other hand, then this term is strictly decreasing in R with a negative derivative at $R = 1$ so that $\mathcal{R}(R^*) \subseteq [R^*, 1]$. We will focus on this latter case in the rest of the paper.

Assumption 7 *(Ex-post bailout temptation):* $w = \beta - (1 - \rho_0) > 0$.

Note that this assumption is consistent our previous assumptions, and in particular with Assumption 6: it is more tempting to transfer wealth to banks ex post than ex ante.

Equilibria. We are now in position to describe the set of equilibria of the no-commitment economy, parametrized by the interest rate R^{nc} set by the central bank in the event of a crisis. The equilibrium set $\{R^{nc}\}$ corresponds to the set of fixed points of:

$$(3) \qquad\qquad R^{nc} \in \mathcal{R}(R^{nc}).$$

Proposition 2 *To every solution R^{nc} of equation (3) corresponds an equilibrium where investors and banking entrepreneurs correctly anticipate that the central bank will set $R = R^{nc}$ if a crisis occurs, invest at scale $i(R^{nc})$, and issue short-term debt $(\pi - 1 + \rho_0/R^{nc}) i(R^{nc})$. Moreover, there exists $\chi > 0$ such that $[1 - \chi, 1] \subseteq \{R^{nc}\}$.*

The equilibrium of the commitment economy $R^{nc} = 1$ is always an equilibrium of the no-commitment economy. However, there are always other equilibria with $1 > R^{nc} \geq \rho_0$. The condition for R^{nc} to be an equilibrium, namely $R^{nc} \in \mathcal{R}(R^{nc})$, is equivalent to the following condition

$$(4) \qquad\qquad \frac{w\rho_0}{1 - \frac{\rho_0}{R}} \left(\frac{1}{R^{nc}} - \frac{1}{R} \right) i(R^{nc}) \geq L(R^{nc}) - L(R) \text{ for all } R \in [R^{nc}, 1].$$

The left-hand side of equation (4) represents the cost in terms of a lower reinvestment scale of setting a higher interest rate $R > R^{nc}$. The right-hand side of equation (4) represents the gain in terms of a lower interest rate distortion of setting such a higher interest rate. The interest rate R^{nc} is an equilibrium if and only if the cost exceeds the gain for all interest rates $R > R^{nc}$. The fact that a neighborhood $[1 - \chi, 1]$ of 1 is always part of the equilibrium set $\{R^{nc}\}$ follows direction from the fact that $L'(1) = 0$. Intuitively, the right-hand side of equation (4) is small compared to the left-hand side for R^{nc} close enough to 1.

It is illuminating to examine the necessary and sufficient condition for $R^{nc} = \rho_0$ to be an equilibrium. In this case, the banks hoard no liquidity and the optimal policy is either to let the banks fail (and set $R = 1$) or to make continuation self-financing ($R = \rho_0$). A bailout is chosen if $-L(\rho_0) - (1 - \rho_0)i(\rho_0) + \beta i(\rho_0) \geq 0$, or:

$$(5) \qquad \frac{wA}{1 - \pi - \alpha \rho_0} \geq L(\rho_0).$$

Corollary 1 *Suppose that condition (5) holds. Then $R^{nc} = 1$ and $R^{nc} = \rho_0$ are equilibria of the no-commitment economy.*

In words, if agents expect the central bank to adopt a tough stance by setting $R = 1$ in case of crisis, then banks choose a small scale $i(1)$ and hoard enough liquidity $(1 - \rho_0)i(1)$ to withstand the shock even if the central bank sets $R = 1$. In turn, the central bank has no incentive to lower the interest rate below 1. Conversely, if agents expect the central bank to adopt a soft stance by setting $R = \rho_0$ in case of a crisis, then banks choose a large scale $i(\rho_0)$ and hoard no liquidity. Then if a crisis occurs, banks can continue at a positive scale only if the central bank sets the interest rate at its lowest possible level $R = \rho_0$ and engineers an extreme bailout. In turn, this extreme bailout is the optimal course of action for the central bank.

Strategic complementarities. The possibility of multiple equilibria illustrates that banks' leverage decisions are strategic complements. These strategic complementarities result from the interaction of three ingredients: imperfect pledgeability on the banks' side, untargeted instruments and time inconsistency on the policy side. Each bank's leverage decision has an effect on the other banks through the policy reaction function in case of a crisis.

To see this more formally, let $x \in [0, 1 - \rho_0]$ be the liquidity choice of a particular bank and $\bar{x} \in [0, 1 - \rho_0]$ the choice of other banks. The central bank would never choose an

interest rate R lower than $\rho_0/(1-\bar{x})$. It sets the interest rate $R \in [\rho_0/(1-\bar{x}), 1]$ in order to maximize

$$(6) \qquad -L(R) + \frac{\beta + \rho_0 - \frac{\rho_0}{R}}{1 - \frac{\rho_0}{R}} \frac{A\bar{x}}{1 - \pi - \alpha\rho_0 + (1-\alpha)\bar{x}}.$$

Without further hypotheses, the objective function in this equation is not necessarily concave in R. For the sake of this discussion, assume that there is enough convexity in the loss function $L(R)$ so that the objective function (6) is indeed strictly concave in R for all values of $\bar{x}$. This guarantees the existence of a unique maximizer $R^*(\bar{x}) \in [\rho_0/(1-\bar{x}), 1]$. Because the objective function (6) has a negative cross partial derivative between R and $\bar{x}$, and that $\rho_0/(1-\bar{x})$ is increasing in $\bar{x}$, there exists $\tilde{x} \in [0, 1-\rho_0]$ such that: for $\bar{x} < \tilde{x}$ we have $R^*(\bar{x}) > \rho_0/(1-\bar{x})$ and $R^*(\bar{x})$ is decreasing in $\bar{x}$; for $\bar{x} \geq \tilde{x}$, we have $R^*(\bar{x}) = \rho_0/(1-\bar{x})$ and $R^*(\bar{x})$ is increasing in $\bar{x}$.

The particular bank under consideration chooses $x \in [0, 1 - \rho_0/R^*(\bar{x})]$, where its objective function is

$$U(x, \bar{x}) = (\rho_1 - \rho_0) \frac{A\left[\alpha + (1-\alpha)\frac{x}{1 - \frac{\rho_0}{R^*(\bar{x})}}\right]}{1 - \pi - \alpha\rho_0 + (1-\alpha)x}.$$

The best response of this particular bank is therefore given by $x(\bar{x}) \equiv 1 - \rho_0/R^*(\bar{x})$. It has the following properties: for $\bar{x} < \tilde{x}$, we have $x(\bar{x}) > \bar{x}$ and so $x(\bar{x})$ is decreasing in $\bar{x}$; for $\bar{x} \geq \tilde{x}$, we have $x(\bar{x}) = \bar{x}$ and so $x(\bar{x})$ is increasing in $\bar{x}$. The best response $x(\bar{x})$ is not increasing over the whole range of liquidity choices $[0, 1-\rho_0]$. However, because it is increasing over $[\tilde{x}, 1-\rho_0]$, there are strategic complementarities in liquidity choices over that range. Note also that $[\tilde{x}, 1-\rho_0]$ is the relevant range, since all equilibria are in that range.[29]

Comparative statics. There are two ways to perform comparative statics when there are multiple equilibria. One possibility is to use a selection criterion. For example, one could select the bank's preferred equilibrium, i.e., the one associated with the lowest interest rate $\min\{R^{nc}\}$. Another, more ambitious approach, which we pursue here, is to establish the set monotonicity (with respect to the inclusion order) of $\{R^{nc}\}$ with respect to parameters.

Below, we will establish a number of results of the following form: "the set of equilibrium interest rates $\{R^{nc}\}$ is expanding with respect to some parameter μ". By this we mean

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[29]In fact, with the extra assumptions that we have made for this discussion—that the the objective function in equation (6) is strictly concave—the set of possible equilibrium liquidity choices is simply $[\tilde{x}, 1-\rho_0]$, and the set of equilibrium interest rates is $[\rho_0/(1-\tilde{x}), 1]$. The set of equilibria need not be an interval when this extra assumption is not verified.

that for $\mu < \mu'$, the set of equilibrium interest rates associated with μ is included in the one associated with μ'. The minimum of the set of equilibrium interest rates $\min\{R^{nc}\}$ is weakly decreasing in μ. By contrast, the maximum of the set of equilibrium interest rates $1 = \max\{R^{nc}\}$ is invariant to μ.

Corollary 2 *The set $\{R^{nc}\}$ of equilibrium interest rates is expanding in the relative weight β of banking stakeholders in the central bank's objective function as well as with the size (as measured by A) of banks.*

Proof. Suppose that $R^{nc} \in \mathcal{R}(R^{nc})$ - i.e. that (4) holds- when β and A are set to some initial value. Then if β and A are increased to β' and A', (4) still holds. As a result it is still the case that $R^{nc} \in \mathcal{R}(R^{nc})$. ∎

Strategic complementarities associated with bigger, more powerful and more strategic banks are stronger. The fixed cost of a bailout is independent of the characteristics or choices of banks. By contrast, for any given interest rate anticipated by the banks, the benefits of a bailout increase with the size (as measured by A), the influence and the importance (as measured by β) of banks.

It is also interesting to perform comparative statics on the set of equilibria with respect to the severity of the crisis. To this end, we now consider an extension of the basic model where only a fraction γ of banks are distressed in the event of a crisis. The parameter γ indexes the severity of the crisis. We keep the probability $1 - \hat{\alpha}$ of a crisis constant and let the probability of being intact $\alpha \equiv \hat{\alpha} + (1 - \hat{\alpha})(1 - \gamma)$ adjust. The logic of the model is essentially unchanged. The only difference is that date-1 welfare $W^{\text{ex post}}(R; R^*)$ is now given by

$$(7) \quad W^{\text{ex post}}(R; R^*) = -L(R) + \left[\beta - (1-R)\frac{\rho_0}{R}\right]\frac{1 - \frac{\rho_0}{R^*}}{1 - \frac{\rho_0}{R}}\frac{\gamma A}{1 - \pi - \alpha\rho_0 + (1-\alpha)(1 - \frac{\rho_0}{R^*})}.$$

Corollary 3 *The set $\{R^{nc}\}$ of equilibrium interest rates is expanding in the severity of the crisis γ.*

One interpretation of Corollary 3 (whose proof is similar to that of Corollary 2) is that one could observe banks increasing their leverage i/A and decreasing their liquidity hoarding x as the severity of a crisis increases. This is particularly interesting since exactly the opposite would happen in a model with fixed or pre-committed interest rate. Indeed, the opposite

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conclusion is usually obtained in corporate finance. Strategic complementarities can therefore result in perverse comparative statics.

Endogenous correlation. We have so far assumed that the correlation of distress shocks across banks was exogenous. We now relax this assumption and allow banks to choose the correlation of their distress risk with other banks' distress risk. There is a continuum of states of the world indexed by $\theta \in [0,1]$. The distribution of θ is uniform on $[0,1]$. Each bank faces a given probability of distress $1 - \alpha$ but can decide how to spread this distress risk across the states of the world. That is, each bank can choose a probability of being intact α_θ in state θ subject to the constraint that $\alpha \geq \int \alpha_\theta d\theta$. Let $R_\theta \in [\rho_0, 1]$ be the interest rates that banking entrepreneurs expect the central bank to set in state θ.

The banking entrepreneur commits to repay $d_\theta i$ in state θ if $\alpha_\theta < 1$ and πi if $\alpha_\theta = 1$. In this case, banks will always choose $d_\theta \in [\pi - (1 - \rho_0/R_\theta), \pi]$. The continuation scale in state θ when pledging debt $d_\theta i$ is given by $j_\theta = i (\pi - d_\theta) / (1 - \rho_0/R_\theta)$ and the investment scale by[30]

$$(8) \qquad i = \frac{A}{1 - \int_0^1 [(1 - \alpha_\theta) d_\theta + \alpha_\theta (\pi + \rho_0)] d\theta}.$$

The banking entrepreneur's expected payoff is $(\rho_1 - \rho_0) \int_0^1 [\alpha_\theta i + (1 - \alpha_\theta) j_\theta + (1 - \alpha_\theta)i] d\theta$.

Proposition 3 *(i) (general structure of strict equilibria) All the strict equilibria have the following properties: (a) there exists a set of crisis states $\Theta^{crisis} \subseteq [0,1]$ of measure $1 - \alpha$ such that $R_\theta < 1$ if $\theta \in \Theta^{crisis}$ and $R_\theta = 1$ otherwise, (b) $\alpha_\theta = 0$ if $\theta \in \Theta^{crisis} \subseteq [0,1]$ and $\alpha_\theta = 1$ otherwise, and (c) $d_\theta = \pi - (1 - \rho_0/R_\theta)$ and i is given by equation (8).*
(ii) (particular class of strict equilibria) To every set of crisis states $\Theta^{crisis} \subseteq [0,1]$ of measure $1 - \alpha$ and solution R^{nc} of equation (3) corresponds a strict equilibrium where: (a) for $\theta \in \Theta^{crisis}$, we have $\alpha_\theta = 0$, $R_\theta = R^{nc}$, (b) for $\theta \notin \Theta^{crisis}$, we have $\alpha_\theta = 1$, $R_\theta = 1$, and (c) $d = \pi - (1 - \rho_0/R^{nc})$ and $i = i(R^{nc})$.

Proof. Assumption 2 implies that $d_\theta = \pi - (1 - \rho_0/R_\theta)$ when $\alpha_\theta < 1$. The results then follow easily from the fact that the derivative with respect to α_θ of the objective function obtained by replacing these values of d_θ in the objective function of banking entrepreneurs is higher, the higher is R_θ. ∎

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[30]The investors' date-0 breakeven condition is $i - A = \int_0^1 [(1 - \alpha_\theta)d_\theta i + \alpha_\theta \pi i + \rho_0 i] d\theta$.

Banks want to fail when the largest possible number of other banks are failing and correlate their risks with those of other banks. Because interest rate policy is non-targeted, bailouts take place in states of the world where a large number of banks are in distress, making it cheaper to refinance in these states. Proposition 3 illustrates the presence of strategic complementarities in correlation choices. In equilibrium, banks coordinate on a given set of crisis states Θ^{crisis} which is completely indeterminate up to the constraint that it be of measure $1 - \alpha$. This proposition also validates our choice of focusing on aggregate shocks as opposed to idiosyncratic shocks: this is the stochastic structure that prevails when correlation choices are endogenized.

It is important to contrast Proposition 3 with the standard prescriptions of the CAPM. In a CAPM world, the cost of capital associated with an investment project is negatively related to the correlation of its cash flows with the market return. As a result, a bank would always choose a minimal correlation with the aggregate risk in the economy. In our economy, just the opposite occurs. Banks maximize their correlations in order to fail when all the other banks are failing and the central bank lowers interest rates.

IV. Welfare and regulation

The time inconsistency of policy introduces a soft budget constraint problem and creates moral hazard on the banks' side. In this context, banks' leverage and liquidity hoarding choices at time 0 can be inefficient.

Welfare. The equilibria in $\{R^{nc}\}$ can be ranked in terms of welfare. Indeed, under our assumptions, ex ante welfare $W^{\text{ex ante}}(R)$ is increasing in R. As a result, equilibria with a higher interest rate R^{nc} feature higher welfare. The equilibrium with the highest welfare is the equilibrium that prevails under commitment with no bailout and the interest rate equal to 1. Moreover, the banking entrepreneurs' perspective is exactly the opposite: the lower the interest rate R^{nc}, the better the equilibrium for banks.

Role for regulation. In this context, regulation of banks' leverage and liquidity hoarding choices at time 0 can be welfare improving. Indeed, consider putting a cap on short-term debt: $d \leq \pi - (1 - \rho_0)$, or equivalently regulating liquidity hoarding by imposing $x \geq 1 - \rho_0$.[31] At $t = 1$, there is then no incentive for the central bank to proceed to a bailout: there would

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[31]In our simple model, no matter what interest rate is expected at the contracting stage, this is equivalent to regulating leverage by imposing that $i/A \leq A/[1 - \alpha + 1 - \pi - \rho_0]$.

be a distortionary cost and no benefit to lower the interest rate below 1 since all banks are able to continue at full scale when $R = 1$. Therefore, this regulation reduces the set of equilibrium interest rates $\{R^{nc}\}$ down to a singleton $\{1\}$, i.e. the no bailout, commitment solution.

Proposition 4 *With limited commitment, the optimal regulation of banks' choices at $t = 0$ takes the form of a liquidity requirement $x \geq 1 - \rho_0$ or equivalently of a maximum short-term debt $d \leq \pi - (1 - \rho_0)$. With this regulation, there is only one equilibrium, which coincides with the commitment solution $R^{nc} = 1$. By contrast, there is no role for such regulation under commitment.*

Remark 1 *Subsidizing liquidity, a form of intervention that is sometimes put forward, would be counterproductive in our model. It would only allow banks to increase their scale and aggravate the time-inconsistency problem of policy, rendering bailouts more likely.[32]*

Remark 2 *Regulations are hard to enforce and banks try to circumvent them. In Farhi-Tirole (2009), we used $\pi = 0$ so that banks' liquidity came exclusively in the form of hoarded assets. We introduced the possibility for banks to engage in regulatory arbitrage to fool the regulator by purchasing toxic assets instead of safe assets to fulfill their liquidity requirement. These toxic assets are cheaper but run the risk of a bad performance. We showed that in this context, there are strategic complementarities in regulatory arbitrage. The more banks engage into regulatory arbitrage, the lower the interest rate set by the central bank in the event where toxic assets are worth 0, and the more each bank can afford to hoard toxic assets. The very insights gleaned with respect to the quantity of liquidity hoarded hold just the same with respect to their quality.*

Macroprudential versus microprudential regulation. This soft budget constraint rationale for regulation is also present in microeconomic principal-agent models when the principal lacks commitment. The difference in our setting is that the actions of the central bank (the interest rate) affect all banks at the same time. If one bank were to take idiosyncratic risks that would materialize only when none of the other banks are in distress, there

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[32]To formalize this insight, assume that the government subsidizes the short-term hoarding of liquidity so that it returns $q > 1$ per unit in the bad state. So $q(\pi - d) = 1 - \frac{\rho_0}{R}$. The analysis of Section III carries over. In particular the characterization of equilibria is literally identical. The only difference is that $i(R)$ is now given by $i(R) = A / [1 - \pi - \alpha\rho_0 + (1/q - \alpha/q) / (1 - \rho_0/R)]$. It is then easy to verify that the set $\{R^{nc}\}$ of equilibrium interest rates is expanding in the return of liquidity q. Note that by having a corner solution we shut down a possible channel through which subsidizing liquidity hoarding may help (a substitution effect).

would be no soft budget constraint problem. Because the only policy instrument, interest rate policy, is not targeted, the central bank would not be tempted to lower interest rates to bail out this individual bank when its individual risk is realized. As a result, our framework suggests that the focus of regulation should be on aggregate leverage and liquidity hoarding and not only on individual risk-taking. In other words, in our model, the optimal regulation is macroprudential and not only microprudential.

It is important to stress that in our model, breaking down banks into smaller banks would be ineffective. The set of equilibria would be unaffected. The problem here is not so much that banks are too big to fail, but that the financial sector as a whole might take on too much correlated risk and too much short-term debt. This irrelevance result would break down if big banks (with a high A) carried a higher welfare weight (β) than small banks per unit of investment, say because big banks' failures have bigger systemic consequences, or because the bankruptcy of a large bank is disproportionately reported in the media, creating pressure for a bailout.

The pecking order of regulation. So far, we have assumed that regulation is costless and that banks are homogenous. We now relax those assumptions. Banks are allowed to differ on size A and weight β in the central bank's objective function. These characteristics are assumed to be distributed according to an arbitrary distribution $F(\beta, A)$. We assume that the costs $c(i)$ of regulating a bank increase with the scale i of the bank, where c is homogenous of degree $\lambda : c(i) = ci^\lambda$ with $c > 0$ and $\lambda \geq 0$. In this context, regulation involves a trade-off and the regulator might find it optimal to regulate certain banks but not others. In order to analyze this trade-off formally, we characterize the minimal amount of aggregate resources devoted to regulation

$$K = \int n(\beta, A) c\left(\frac{A}{1 - \pi - \alpha\rho_0 + (1 - \alpha)(1 - \rho_0)}\right)^\lambda dF(\beta, A)$$

required to ensure that $\{R^{nc}\} = \{1\}$. Here the authorities regulate a fraction $n(\beta, A) \in [0, 1]$ of banks of size A and weight β.

Proposition 5 *Suppose that Assumptions 2, 6 and 7 hold for every β in the support of F. Then the minimal cost of insuring that $\{R^{nc}\} = \{1\}$ is achieved by regulating banks for which $[\beta - (1 - \rho_0)] A^{1-\lambda}$ is greater than a certain threshold Λ.*

Proof. See the appendix. ∎

When regulation is costly, optimal regulation is characterized by a pecking order determined by a summary statistic $[\beta - (1 - \rho_0)] A^{1-\lambda}$ which combines their size A and their weight β in the central bank's objective function. This summary statistic can be thought of as a cost-of-regulation-adjusted systemic importance. For a given size A, the higher β, the higher the bank in the regulatory pecking order (recall that β is the sum of the weights placed on the different categories of bank stakeholders, i.e. banking entrepreneurs, firm managers, and workers). Whether the rank of the bank in the regulatory pecking order increases or decreases with its size A for a given β depends on the returns to scale in regulation λ. With increasing returns to scale in regulation ($\lambda < 1$), the rank of the bank in the regulatory pecking order increases with A, and the opposite holds true with decreasing returns to scale in regulation ($\lambda > 1$). With constant returns to scale in regulation ($\lambda = 1$), size per se is irrelevant: the costs and benefits of regulation scale up exactly at the same rate with the size of bank.

V. Sowing the seeds of the next crisis

In Section I, we gave some examples of immediate deadweight losses L associated with low interest rates. This section focuses on deferred costs, associated with the incentive for new borrowers to lever up and increase maturity mismatch, or with the central bank's loss of reputation. We derive alternative microfoundations for the distortions associated with an interest rate bailout. We extend the model to an overlapping generations structure with two successive generations G_{-1} and G_0 of banking entrepreneurs. We derive a loss function L from an interest rate bailout of generation G_{-1} at date 0. This loss function originates in the perverse consequences on the subsequent generation G_0 of banking entrepreneurs, who end up levering up more and hoarding less liquidity, resulting in an interest rate bailout at date 1 (if a crisis occurs). In order to present these extensions, it is useful to first first extend our basic setup by allowing policy to also determine the date-0 interest rate R_0. Throughout Section V, we assume that there are no immediate date-0 distortions from lowering R_0, so that any fixed cost from an interest rate bailout at date 0 will be a deferred one. In Section V.A, we allow for immediate distortions from lowering the interest at date 1, but in Sections V.B and V.C, we assume them away.

A. Date-0 Interest Rate

We assume that the government can tax the rate of return on the storage technology between dates 0 and 1. The proceeds are rebated lump sum to consumers. This is rigorously equivalent to assuming the government can subsidize investment in the banks at date 0, a subsidy financed by lump sum taxes on consumers. We show that it is optimal, both under commitment and under no commitment, to set the date-0 interest rate R_0 to 1, even without assuming any distortion from lowering R_0.[33]

The borrowing capacity of banking entrepreneurs at date 0 is now given by

$$(9) \qquad i = \frac{A}{1 - \frac{\pi}{R_0} - \frac{\alpha \rho_0}{R_0} + \frac{(1-\alpha)}{R_0}x}$$

where x, as earlier, denotes the liquidity ratio. Banking entrepreneurs hoard liquidity $x = 1 - \rho_0/R$ at date 0 if and only if it expects a date-1 interest rate R satisfying

$$(10) \qquad \pi + \alpha - R_0 \leq \alpha \rho_0 \left(\frac{1}{R} - 1\right),$$

and hoards no liquidity ($x = 0$) otherwise. This generalizes our analysis of Section II to arbitrary $R_0 \leq 1$. Denote by $i(R, R_0)$ the corresponding investment scale chosen by banking entrepreneurs—obtained by replacing the optimal liquidity choice x of banking entrepreneurs in equation (9), a choice governed by equation (10). Similarly, let $j(R, R_0)$ denote the corresponding reinvestment scale in case of a crisis.

Ex-ante welfare under commitment is given by

$$(11) \qquad \begin{aligned} W^{\text{ex ante}}(R, R_0) \equiv\ & \beta i(R, R_0) - (1 - \alpha)\frac{1 - R}{R}\rho_0 j(R, R_0) - (1 - \alpha)L(R) \\ & - [i(R, R_0) - A](1 - R_0). \end{aligned}$$

The last term on the right-hand side is new and reflects the implicit subsidy to bank investment at date-0 (consumers invest a total amount $i(R, R_0) - A$). The following proposition shows that under the assumptions that we have maintained throughout the paper, the optimal interest rate policy under commitment is passive both at date 1 and at date 0.

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[33]These results would only be reinforced if we were to introduce a loss function $L_0(R_0)$ as we did for the date-1 interest rate.

Proposition 6 *The optimal interest rate policy under commitment features $R_0^c = R^c = 1$.*

The intuition is straightforward. Assumption 6 guarantees that redistributing resources from consumers to banking entrepreneurs is welfare reducing. Lowering the interest rate R_0 or R below 1 induces such a redistribution of resources (with additional distortions in the case of R). Setting these interest rates equal to 1 is therefore optimal.

Analyzing the no-commitment solution requires solving a dynamic game. We focus on subgame-perfect equilibria. We will need to impose some refinement. We know from our previous analysis that the set of equilibrium interest rates of the continuation game is expanding in the date-0 investment scale. We therefore find it natural to focus on subgame perfect equilibria that satisfy the following monotonicity requirement:

Assumption 8 *The continuation equilibrium of the no-commitment game is such that the date-1 interest rate is non-decreasing in the date-0 interest rate.*

This assumption would be automatically satisfied if for example, we always chose the worst possible continuation equilibrium (the one with the lowest R) for any value of R_0.

Corollary 4 *In every equilibrium of the no-commitment game, $R_0^{nc} = 1$.*

This corollary follows directly from Proposition 6. Indeed, lowering the interest rate R_0 below 1 only entails larger costs compared with the commitment solution, because it leads banking entrepreneurs to hoard less liquidity anticipating a lower interest rate R at date 1.

B. Leverage decisions, going forward

Consider a longer-horizon model, say the overlapping-generations version of this model, in which banking entrepreneurs live, like in this model, for three periods. A bailout at date $t + 1$ of those banks that borrowed at date t then also affects the financing decisions of the next generation of banks, which borrow at date $t + 1$. Interestingly, this operates through two channels: increased leverage and increased maturity mismatch.[34]

To preview the results, bailing out generation G_t induces leverage and maturity mismatch for generation G_{t+1} , sowing the seeds for a date-$t + 2$ crisis and bailout. Indeed, an interest

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[34]Arguably, both channels seem to have operated during the long episode of very low interest rates in the wake of the 2000 Internet bubble crash.

rate bailout of generation G_t makes generation G_{t+1} (a) more willing to take an illiquid position by loading up on short-term debt and (b) increase the size of its investment. These two effects—the increased maturity mismatch and the increased leverage channels respectively—distort generation G_{t+1}'s incentives and generate a social cost that is a fixed cost (in the sense that it does not depend on generation G_t's size or illiquidity) when contemplating a rescue of generation G_t.

The logic of the argument can be grasped by appending to the model of Section V.A a prior generation of entrepreneurs, generation G_{-1}, living at dates $-1, 0$ and 1.[35] Generation G_{-1} is in all respects similar to generation G_0 born at date 0 and studied in this paper, except that its short-term (date-0) profit π_0 is sufficiently large that not hoarding any liquidity is a dominant strategy (for any $R_0 \in [\rho_0, 1]$), which boils down to the condition that $\pi_0 + \alpha > 1 + \alpha (1 - \rho_0)$. This assumption merely shortens the analysis by ensuring that generation G_{-1} in case of a date-0 crisis is unable to withstand the shock unless the date-0 interest rate is brought down to $R_0 = \rho_0$.

The analysis of the generation G_0 born at date 0 is exactly as in Section V.A. We make the following assumption, which is consistent with our previous assumptions.[36]

Assumption 9 *(no liquidity hoarded when $R_0 = \rho_0$)*: $\pi > (1 - \alpha)\rho_0$.

We rule out any deadweight loss from inefficient investments, so as to starkly illustrate that L can come from impaired incentives for the subsequent generation of entrepreneurs. We maintain this assumptions throughout Sections V.B and V.C.

From our previous analysis and Assumption 8, we know that the optimal date-0 policy if one ignores the welfare of generation G_{-1} consists in setting $R_0 = 1$. Suppose that $R_0 = 1$ is actually chosen. There are a number of possible continuation equilibria, corresponding to different expectations regarding the interest rate R. Because we have assumed away any current distortion, all interest rates $R \in [\rho_0, 1]$ correspond to a possible continuation equilibrium, where banking entrepreneurs hoard liquidity $x = 1 - \rho_0/R$. Using the definition in

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[35]The model can be extended to infinite-horizon overlapping generations; we offer here the simplest illustration.

[36]The analysis remains valid when this assumption is violated, but the cost of bailing out generation G_{-1} in terms of generation G_0's incentives then comes solely from an increased investment scale, not from an increased maturity mismatch.

equation (11), this yields an ex-ante welfare for generation G_0

$$W^{\text{ex ante}}(R, 1) = \frac{\left[\beta - (1 - \alpha)(1 - R)\frac{\rho_0}{R}\right]A}{1 - \pi - \alpha\rho_0 + (1 - \alpha)\left(1 - \frac{\rho_0}{R}\right)}$$

where $W^{\text{ex ante}}(R, 1)$ increases with R. In our analysis, we do not need to take a stand on which equilibrium is actually selected.

Any $R_0 \in (\rho_0, 1)$ is dominated: by Assumption 8, such interest rates reduce welfare from generation G_0 and do nothing to help generation G_{-1}. Therefore, we only have to analyze whether $R_0 = \rho_0$ is preferred to $R_0 = 1$. When $R_0 = \rho_0$, no liquidity is hoarded by generation G_0, and because there are no immediate distortions from interest rate policy, a full interest rate bailout with $R = \rho_0$ follows. Using the definition in equation (11), welfare from the date-0 generation becomes

$$W^{\text{ex ante}}(\rho_0, \rho_0) = \frac{\left[\beta - (1 - \alpha)(1 - \rho_0)\right]A}{1 - \pi - \alpha\rho_0} - (1 - \rho_0)\left[\frac{1}{1 - \frac{\pi}{\rho_0} - \alpha} - 1\right]A.$$

Turning now to the decision of whether to rescue generation G_{-1} at date 0, and remembering that choices outside $\{\rho_0, 1\}$ are dominated, we conclude that there exists a fixed cost of bailing out generation G_{-1} by setting $R_0 = \rho_0$. Depending on which continuation equilibrium is selected when $R_0 = 1$, with corresponding welfare $W^{\text{ex ante}}(R, 1)$, this fixed cost is given by $L(\rho_0) \equiv W^{\text{ex ante}}(R, 1) - W^{\text{ex ante}}(\rho_0, \rho_0)$. It is maximized when following $R_0 = 1$, the continuation equilibrium with $R = 1$ is selected, and minimized when the continuation equilibrium with $R = \rho_0$ is selected. It can be verified that even in the latter case, we have $L(\rho_0) > 0$ as long as Assumption 6 holds. The selection of the continuation equilibrium with $R = \rho_0$ isolates the increased leverage channel mentioned above. By contrast, the selection of the continuation equilibrium with $R = 1$ combines both the increased leverage and the increased maturity-mismatch channels.

Proposition 7 *When contemplating whether to rescue the generation G_{-1} banks, the government faces a fixed cost equal to $L(\rho_0) = W^{ex\ ante}(R, 1) - W^{ex\ ante}(\rho_0, \rho_0) > 0$, where $R \in [\rho_0, 1]$ corresponds to the selected continuation equilibrium when $R_0 = 1$. This cost is fixed in that it does not depend on generation G_{-1}'s investment to be rescued.*

C. Central bank's reputation

Yet another deferred cost of bailouts is the loss of reputation by the central bank. This can be modelled by introducing a tough type and a soft type. A bailout then reveals the type of the central bank to be soft, raising the likelihood of future bailouts and pushing banks to take on more risk, hoard less liquidity and lever up, resulting in increased economywide maturity mismatch and in turn larger bailouts. Even a central bank of the soft type internalizes this reputation cost and is reluctant to engage in a bailout in the first place.

To show how reputation concerns generate yet another fixed cost, we follow Section V.B but introduce uncertainty about the central banker's preferences. With prior (date-0) probability $1 - z$, the central banker is "bailout-prone" as he puts weight $\beta > 1 - \rho_0$ (so $w > 0$) on investment as earlier. With probability z, the central bank is "tough" as he puts no or little weight on investment and therefore always chooses interest rates equal to 1.

The situation is otherwise the same as in Section V.B: Generation G_{-1} optimally hoards no liquidity; when it faces a crisis, the rational choice for a bailout-prone central banker is again between $R_0 = 1$ and $R_0 = \rho_0$. A choice of $R_0 = \rho_0$ reveals that he is bailout-prone.

Suppose first that the central banker sets $R_0 = \rho_0$ to bail out generation G_{-1}. The equilibrium for generation G_0 is then as in Section V.B. Welfare is $W^{\text{ex ante}}(\rho_0, \rho_0)$ for that generation. By choosing $R_0 = 1$ by contrast, the central banker creates posterior beliefs $\hat{z} \geq z$.[37] If $\hat{z}$ is large enough ($\hat{z} \geq \underline{z}$), then hoarding liquidity is a dominant strategy for a generation-G_0 bank and welfare for generation G_0 is $W^{\text{ex ante}}(1, 1)$.[38]

Proposition 8 *There exists $\underline{z} < 1$ such that for $z \geq \underline{z}$, the bailout-prone central banker faces a fixed reputation-loss cost equal to $L(\rho_0) \equiv W^{ex\ ante}(1, 1) - W(\rho_0, \rho_0) > 0$ when rescuing the generation G_{-1} banks. This cost is fixed in that it does not depend on generation G_{-1}'s investment to be rescued.*

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[37]$\hat{z} = z$ in a pooling equilibrium, $\hat{z} \geq z$ in a partially revealing one, including a separating equilibrium for which $\hat{z} = 1$.

[38]The threshold $\underline{z}$ is the solution of $\frac{z\alpha + [1 - z]}{1 - \pi - \alpha\rho_0} = \frac{1}{1 - \pi - \alpha\rho_0 + (1 - \alpha)(1 - \rho_0)\underline{z}}$. From Assumption 2, $\underline{z} < 1$. To understand this condition, note that the tough central banker (who has posterior probability $\hat{z}$) will not bail out banks in case of a crisis, and so the probability of continuation is at most $\hat{z}\alpha + [1 - \hat{z}]$ in the absence of liquidity hoarding. Conversely, hoarding liquidity $(1 - \rho_0)i$ is needed in order to be able to continue in a crisis when the central banker is tough; but if the central banker turns out to be bailout-prone and lowers the interest rate at date 1 in case of crisis to $R = \rho_0$ (this is the most optimistic hypothesis, which arises if other bankers hoard no liquidity), then this unneeded liquidity can be returned to investors. The cost of liquidity is therefore only $(1 - \alpha)(1 - \rho_0)\hat{z}$ on average, which explains the term on the right-hand side.

The derivation of equilibrium behavior is then straightforward. For $z \geq \underline{z}$, the bailout-prone central banker pools with the tough one if and only if $w_{-1} i_{-1} \leq L(\rho_0)$ is verified, where $w_{-1} = \beta_{-1} - (1 - \rho_0)$ and β_{-1} and i_{-1} are the weight on, and the investment of generation G_{-1}. If $w_{-1} i_{-1} > L(\rho_0)$, the equilibrium is separating. Finally, for $z < \underline{z}$, the details of equilibrium behavior depend on equilibrium selection, but the overall pattern (a trade-off between the benefit of bailout and a cost of reputation loss) remains the same.

VI. Optimal ex-post bailouts

So far, we have restricted the set of policy instruments to the interest rate. We emphasized that this instrument was not targeted. Together with time inconsistency, this feature was a key ingredient in generating strategic complementarities in leverage choices. In this section, we relax the assumption of an exogenously specified policy instrument set. Instead, we follow a mechanism design approach and characterize the optimal ex-post bailout where policy tools are endogenous to the constraints of the economic environment.

As we noted in the introduction, one may wonder whether interest rate bailouts, which involve both an implicit subsidy to banks and various distortions, are still desirable when other interventions are possible. For example, purchases of legacy assets, liquidity support and recapitalizations also involve direct transfers from consumers to banks—boosting their net worth and allowing them to refinance at a larger scale. However, these transfers do not reduce borrowing costs at the margin, and do not generate similar distortions.

Interestingly, we show that interest rate policy still plays an important role within the optimal bailout scheme: It is always part of the optimal ex-post package, and over a range of parameters, bailouts boil down to pure interest rate policy. Furthermore, we recover a key insight from our previous analysis, the existence of strategic complementarities: to some extent, optimal bailouts are themselves untargeted.

Setup. We allow the fraction γ of banks that are distressed in a crisis to be less than 1. Denoting the probability of a crisis by $1 - \hat{\alpha}$, we maintain the convention that $\alpha \equiv \hat{\alpha} + (1 - \hat{\alpha})(1 - \gamma)$ represents the probability of being intact. In what follows the probability $\hat{\alpha}$ of a crisis is kept constant and the dependence of α with respect to γ is left implicit.

Moreover, we introduce an informational friction: the central bank can observe which banks are distressed, but the underlying auditing technology is imperfect. More precisely, we assume that the probability of generating a false positive when assessing if a bank is

distressed is equal to ν.[39] As a result, in a crisis, a fraction $\nu\left(1-\gamma\right)$ of banks are mistaken by the authorities as banks that need liquidity. These banks are aware that they belong to the false positive group.

We assume that banking entrepreneurs and their investors form perfect coalitions, and that banks have full bargaining power in these coalitions.

Instruments. We assume that the government cannot directly hold bank securities. The only available instruments are the interest rate (the borrowing cost of banks) and direct transfers to banks perceived as being distressed. We have already commented on the possible interpretations of interest rate policies as policies that lower the borrowing cost of banks. Direct transfers capture policies used in practice to boost the net worth of banks, such as the purchase of legacy assets at inflated prices.

The assumption that the government cannot directly hold bank securities deserves some comments, as it rules out some forms of government intervention that are used in practice. For example, a recapitalization involves a transfer from the government in exchange of shares or warrants. However in most practical cases, the government usually sells its stake relatively quickly. What remains is a transfer from the government to the bank. At a theoretical level, this assumption limits the ability of the government to screen between intact and distressed banks. In Farhi-Tirole (2009) we analyzed the case where the government could hold stakes in banks and use this as a screening device. We showed that our insights were robust in this environment. However the analysis was quite involved. The assumption that the government cannot directly hold a stake in banks allows us to considerably streamline the analysis.

If direct transfers could be perfectly targeted to distressed banks, they would dominate interest rates as a policy instrument. However, the government cannot perfectly recognize if a given bank is distressed and some banks might engage in rent seeking by successfully portraying themselves as distressed. Moreover, low interest rates benefit distressed banks comparatively more than intact ones. As a result, there is a non-trivial policy tradeoff between interest rate policy and direct transfers to institutions perceived as being distressed.[40]

Although we will consider only symmetric equilibria, we analyze the general case where the authorities face an arbitrary distribution $F\left(i,x\right)$ of banks with scale and liquidity (i,x).

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[39]Note that for simplicity, we assume that there are no false negatives.

[40]The possibility of false positives is crucial for the following reason. If $\nu=0$ or $\gamma=1$, then the authorities do not face any information extraction problem. As long as Assumption 7 holds, then banks in distress are always rescued through a direct transfer and are allowed to continue at full scale even if banks are completely illiquid. In equilibrium, banks would then choose to be completely illiquid, interest policy would not be used.

A bailout specifies an interest rate R, and for every scale and liquidity (i, x) a transfer $t(i, x) i \geq 0$ for banks perceived as being distressed. This implies the following reinvestment scale for distressed banks

$$j(i, x) = \min \left\{ \frac{x + t(i, x)}{1 - \frac{\rho_0}{R}}, 1 \right\} i.$$

Timing. The timing within period 1 is as follows: (1) the government announces a rescue scheme $\{R, t(i, x)\}$; (2) each banking entrepreneur accepts the plan if and only it makes him better off given that investors must be as well off as they would in the absence of participation in the scheme.

To simplify the expressions, we assume that the government places no Pareto weight on banking entrepreneurs $\beta_b = 0$. Transfers to banks that are perceived to be distressed but are in fact intact therefore represent a pure welfare loss.

Planning problem. In order to solve for the optimal ex-post bailout, we use the variables $\{R, j(i, x)\}$ rather than $\{R, t(i, x)\}$. The transfers $t(i, x)$ can be inferred as follows: $t(i, x) = (1 - \rho_0/R) j(i, x) /i - x \geq 0$ since without loss of generality $x \leq 1 - \rho_0/R$.

Up to a constant, the optimal ex-post bailout maximizes[41]

(12)
$$-L(R) + \int \gamma \left(\beta - (1 - R)\frac{\rho_0}{R} \right) j(i, x) \, dF(i, x)$$

$$- [\gamma + (1 - \gamma)\nu] \int \left[\left(1 - \frac{\rho_0}{R} \right) j(i, x) - xi \right] dF(i, x)$$

s.t.

$$i \geq j(i, x) \geq \frac{x}{1 - \frac{\rho_0}{R}} i.$$

To obtain equation (12), note that for distressed banks, the total subsidy is given by $t(i, x) + (1 - R) \rho_0 j(i, x) /R$. For intact banks that are perceived as distressed, the subsidy is $t(i, x)$ as those banks do not need to borrow more funds. The first two terms in equation (12) are exactly as in Section III. The third term corresponds to the additional implicit subsidy associated with the direct transfer $t(i, x)$. This is a transfer to distressed firms and intact firms that are perceived as distressed. Consumers get a rate of return equal to 0 (and not R as when they reinvest in the banks) on this transfer.

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[41]Because of the linearity of the objective and the constraints in $t(i, x)$, we need to take a stand on government actions when the government is indifferent. We assume that it sets $t(i, x)$ equal to its minimal possible value in case of indifference.

The government could achieve a given pattern of continuation scales $j(i,x)$ entirely through direct transfers by setting $t(i,x) = j(i,x) - x$. This would economize on interest rate distortions. However, it increases undesirable transfers to intact banks perceived to be distressed. Lowering the interest rate R increases the collateral value $\rho_0 j(i,x)/R$ of distressed firms, allowing them to continue at a larger scale, while at the same time reducing the funds transferred to intact firms perceived to be distressed. By affecting the terms at which banks can borrow on the market, the government is able to better target firms that are actually distressed. However, it pays a cost in terms of distortions $L(R)$. We need to ensure that the latter is convex enough.

Assumption 10 (*enough convexity*). *The function* $-R^2 L'(R)$ *is decreasing in* $R \in [\rho_0, 1]$.

Optimal ex post bailout. The optimal bailout policy is summarized in Figure 1. Due to the linearity of the program, there can be two equilibrium configurations. We discuss them informally here and refer the reader to the appendix for the full details.

(i) *No liquidity hoarding* ($x = 0$ for all banks). In this equilibrium configuration, banks load up on short-term debt as they expect to be rescued in case of a crisis. Given the absence of liquidity in the banking sector, the least-cost rescue policy for the government is a mix of transfer and interest rate policies. To see this, note that low interest rates reduce the wasteful transfer t to intact banks given that distressed ones require a unit refinancing from both sources: $1 = t + \rho_0/R$. Basically, distressed banks can lever up the direct transfer to refinance at a greater scale. However low interest rates also increase the deadweight loss $L(R)$; the marginal distortion is 0 at $R = 1$, and increases as the interest rate is reduced. When the severity (γ) of the crisis increases, undue transfers to intact banks become less of a concern, and the optimal interest rate $\bar{R}(\gamma)$ increases to reach 1 when there are no false positives ($\gamma = 1$).[42]

It must also be the case that the government wants to bail out the banks, that is that there is not too much adverse selection. Thus if the crisis is not very severe ($\gamma < \underline{\gamma}$ in the figure), the no-liquidity-hoarding equilibrium disappears.

(ii) *Liquidity hoarding.* In this equilibrium configuration, banks hoard just enough liquidity (issue just as little short-term debt) as to be able to continue at full scale in case of a crisis. This requires two conditions.

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[42]The function $\bar{R}(\gamma)$ is defined by the equation $-\bar{R}(\gamma)^2 L'(\bar{R}(\gamma)) = \frac{A(1-\gamma)\nu\rho_0}{1-\pi-\alpha\rho_0}$.

First, given the expected date-1 interest rate, an individual bank must not expect that the government will make up for insufficient liquidity through a targeted transfer. Otherwise it would hoard no liquidity at all. Put differently, liquidity hoarding relies on the expectation of a pure interest rate policy bailout. Whether the government is willing to bail out a bank with insufficient liquidity depends on the trade-off, discussed in (i), between wasteful transfers to intact banks and rescues of distressed ones but in reverse: when R increases, the needed transfer $t = 1 - \rho_0/R$ increases and so bailouts become less attractive. This implies that R has to be greater than or equal to $\underline{R}(\gamma)$, where $\underline{R}(\gamma)$ is an increasing function of γ as the government is more tempted to rescue banks as the crisis becomes more severe. It reaches 1 when $\gamma = \bar{\gamma}$.[43]
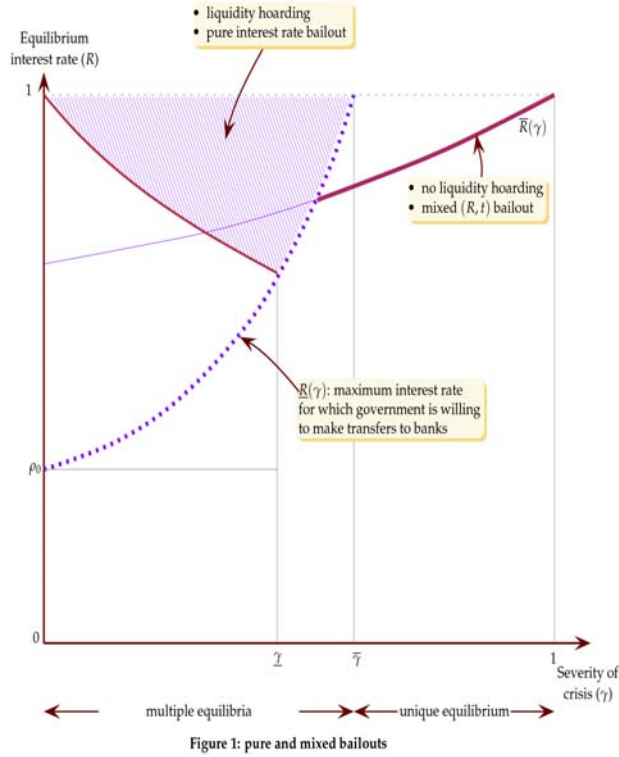

Figure 1: pure and mixed bailouts

Second, because the bailout is a pure interest rate intervention, the equilibrium must be an equilibrium when no transfers are feasible; that is, it must satisfy equation (3). So the equilibrium set (the shaded area in Figure 1) is the set of interest rates in Proposition 2 that satisfy $R \geq \underline{R}(\gamma)$.

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[43]A simple inspection of equation (12) shows that this threshold $\underline{R}(\gamma)$ is defined as the solution of the following equation in R: $\frac{1-\frac{\rho_0}{R}}{w+1-\frac{\rho_0}{R}} = \frac{\gamma}{\gamma+(1-\gamma)\nu}$. The threshold $\underline{R}(\gamma)$ is increasing in γ with $\underline{R}(0) = \rho_0$ and $\underline{R}(\bar{\gamma}) = 1$ where $\bar{\gamma} \in [0,1]$ is the solution of the following equation in γ: $\frac{\gamma}{\gamma+(1-\gamma)\nu} = \frac{1-\rho_0}{w+1-\rho_0}$.

Once again banks' leverage and liquidity choices are strategic complements, and there can be multiple equilibria. More generally, in this region, the insights gleaned from our analysis of pure interest rate bailouts in Sections III and IV carry over to optimal bailouts. There is one difference in the form of an additional lower bound $\underline{R}(\gamma) \geq \rho_0$ on the interest rate. For $\gamma > \bar{\gamma}$ these equilibria disappear.

The function $\bar{R}(\gamma)$ is increasing in γ. For γ close to $\bar{\gamma}$, $\bar{R}(\gamma)$ is below $\underline{R}(\gamma)$ and the opposite is true for γ close to 0 provided that $\bar{R}(0) > \rho_0$. The following assumption ensures that these two functions cross only once. We denote by $\underline{\gamma}$ the crossing point.

Assumption 11 *The function $\bar{R}(\gamma)$ defined by the solution of the equation $-R^2 L'(R) = A(1-\gamma)\nu\rho_0 / [1-\pi-\alpha\rho_0]$ crosses the function $\underline{R}(\gamma)$ once from above on the interval $[0, \bar{\gamma}]$.*

Proposition 9 *The symmetric equilibria of the no-commitment economy are as follows:*

(i) if $\gamma > \underline{\gamma}$ then there is an equilibrium with $R = \bar{R}(\gamma)$; it features $i/A = 1/(1-\pi-\alpha\rho_0)$, $x = 0$, and $d = \pi$;

(ii) if $\gamma \leq \bar{\gamma}$ then there is an equilibrium associated with each fixed point of the equation $R \in \mathcal{R}(R) \cap [\underline{R}(\gamma), 1]$; it features $i/A = 1/[1-\pi-\alpha\rho_0 + (1-\alpha)(1-\rho_0/R)]$, $x = 1 - \rho_0/R$, and $d = \pi - (1 - \rho_0/R)$.

To sum up, interest rate policy is always used in equilibrium; indeed direct transfers are not even used in some regions of the parameter space; in the remaining regions, interest rate policy and direct transfers are used in conjunction. Interest rate policy is a market-driven solution, in that it benefits primarily those institutions with actual borrowing needs. Direct transfers better focus on strategic actors, but they entail a greater waste of resources by supporting entities that have no need for, or should not engage in refinancing. When direct transfers are not used (for equilibria of type (ii)), the insights from Sections III and IV carry over to optimal bailouts: multiple equilibria, macroprudential regulation, and endogenous macroeconomic uncertainty.[44]

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[44]Marvin Goodfriend and Robert G. King (1988) argue that the central bank should just provide liquidity anonymoulsy through open market operations and leave it to the market to allocate this liquidity efficiently. They argue against providing liquidity to specific institutions. Our interpretation of interest rate policy is broader than open market operations, complicating the comparison with their view. Nevertheless, in our framework, optimal policy under commitment is entirely passive and can therefore be seen as vindicating their view. Even under limited commitment, the central bank finds it optimal ex post to use only interest rate policy for equilibria of type (ii). However for equilibria of type (i), a combination of interest rate policy and direct transfers to (perceived to be) distressed institutions is used. Direct transfers are used only when

VII. Conclusion

We have built a simple framework to jointly analyze leverage and maturity mismatch in the banking sector, bailouts, and optimal regulation. In order to derive a simple yet general model, we have made a number of simplifying assumptions. Refining our analysis would require a richer model.

For example, we have argued that one interpretation of interest rate policy in our framework was as a reduced form for conventional monetary policy. We could introduce an explicit nominal structure with sticky prices and imperfect competition to model conventional monetary policy as in New-Keynesian models. The framework could be further enriched to study the consequences of bailouts in a setup where interactions between the central bank's balance sheet, inflation, and the government budget are non trivial. We could also introduce the possibility of sovereign default. We have also maintained the assumption of risk neutrality. Introducing risk aversion would allow studying how different policy interventions can impact banks net worth and borrowing costs by affecting risk premia. Finally, in our model, liquidity is costless in the sense that there are no liquidity premia. If liquidity were costly instead, interest rate policy under commitment would not necessarily be passive: it could be optimal for the government to provide liquidity in bad times. However, our insights would carry over: authorities would provide too much liquidity in the time-consistent outcome.[45] We leave these questions for future research.

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the losses from mistaken transfers to intact institutions are smaller than the distortions arising from interest rate policy. Note that these policies are not Pareto improving ex post since they transfer resources away from consumers, but they do lead to an increase overall ex post welfare. These policies reduce ex ante welfare.

[45]This could be formalized along the lines of Holmström-Tirole (1998). Imagine that consumers cannot commit at $t = 0$ to provide funds to banks at $t = 1$. Banks then have to hoard liquidity by purchasing assets at $t = 0$ and selling them at $t = 1$ in the even of a crisis. Banking entrepreneurs are willing to pay a liquidity premium (in the form of a lower return) on these assets, which can happen if there is a scarcity of stores of value. Then, the lack of commitment of consumers is a form of market incompleteness which can be alleviated through government intervention (even under the commitment solution). For example, through a commitment to a loose interest rate policy in crisis. See Farhi-Tirole (2009) for a discussion.

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Appendix

Proof of Proposition 5

Ex-post (date-1) welfare $W^{\text{ex post}}(R; R^*)$ is given by

$$W^{\text{ex post}}(R; R^*) = -L(R) + \int \frac{\left[\beta - (1-R)\frac{\rho_0}{R}\right] \frac{1-\frac{\rho_0}{R^*}}{1-\frac{\rho_0}{R}} A(1-n(\beta, A))}{1-\pi - \alpha\rho_0 + (1-\alpha)\left(1-\frac{\rho_0}{R^*}\right)} dF(\beta, A)$$

and the equilibrium correspondance $\mathcal{R}$ is defined accordingly. The corresponding planning problem is

(1)
$$\underline{K} \equiv \min_{\{n(\beta, A)\}} K$$

s.t.

$$0 \leq n(\beta, A) \leq 1$$

and

$$R^* \notin \mathcal{R}(R^*) \text{ for all } R^* \in [\rho_0, 1).$$

The condition for $R^* \in [\rho_0, 1)$ not to be an equilibrium is that there exists $R \in (R^*, 1]$ such that

$$W^{\text{ex post}}(R; R^*) - W^{\text{ex post}}(R^*; R^*) > 0$$

or equivalently

$$\int_{R^*}^{R} \frac{\partial W^{\text{ex post}}\left(\hat{R}; R^*\right)}{\partial \hat{R}} d\hat{R} > 0.$$

For any given set of values $\Delta(R, R^*)$ and consider the following subproblem

(2)
$$\underline{K}\left(\{\Delta(R, R^*)\}\right) \equiv \min_{\{n(\beta, A)\}, \{\Delta(R, R^*)\}} K$$

s.t.

$$0 \leq n(\beta, A) \leq 1$$

and

$$\int_{R^*}^{R} \frac{\partial W^{\text{ex post}}\left(\hat{R}; R^*\right)}{\partial \hat{R}} d\hat{R} \geq \Delta(R, R^*) \text{ for all } R^* \in [\rho_0, 1) \text{ and } R \in [R^*, 1].$$

Then the original planning problem (1) and the subproblem (2) are related in the following way:

(3)
$$\underline{K} = \min_{\{\Delta(R, R^*)\}} \underline{K}\left(\{\Delta(R, R^*)\}\right)$$

s.t. the constraint that for all $R^* \in [\rho_0, 1)$, there exists $R \in (R^*, 1]$ such that $\Delta(R, R^*) > 0$. Moreover the solution $\{n(\beta, A)\}$ of (1) coincides with the solution of (2) when $\{\Delta(R, R^*)\}$ is set as the solution of (3).

Turning back to (2) and take $\{\Delta(R, R^*)\}$ to be the solution of (3), the constraint set and the objective function are linear in $\{n(\beta, A)\}$ so the first order conditions are necessary and sufficient for optimality. Let $\mu_{R^*, R} \geq 0$ be the multiplier on the constraint

$$\int_{R^*}^{R} \frac{\partial W^{\text{ex post}}\left(\hat{R}; R^*\right)}{\partial \hat{R}} d\hat{R} \geq \Delta(R, R^*).$$

Let $\bar{\nu}_{\beta, A} dF(\beta, A)$ be the multiplier on the constraint $n(\beta, A) \leq 1$ and $\underline{\nu}_{\beta, A} dF(\beta, A)$ be the multiplier on the constraint $n(\beta, A) \geq 0$. Finally, let

$$\mu = \sum \mu_{R^*, R} \int_{R^*}^{R} \frac{1 - \frac{\rho_0}{R^*}}{1 - \pi - \alpha\rho_0 + (1 - \alpha)\left(1 - \frac{\rho_0}{R^*}\right)} \frac{\frac{\rho_0}{R^2}}{\left(1 - \frac{\rho_0}{R}\right)^2} d\hat{R}.$$

The first-order condition for $n(\beta, A)$ is

$$c(m(1))^\lambda A^\lambda = \underline{\nu}_{\beta,A} - \bar{\nu}_{\beta,A} + A(\beta + \rho_0 - 1)\mu.$$

The result follows directly from this first-order condition and the complementary slackness conditions $\underline{\nu}_{\beta,A} n(\beta, A) = \bar{\nu}_{\beta,A}[1 - n(\beta, A)] = 0$.

Proof of Proposition 6

Let us first focus on values of (R_0, R) such that banking entrepreneur choose to hoard enough liquidity to continue at full scale in case of a crisis, i.e. values that satisfy equation (??). For such values, we have

$$j(R_0, R) = i(R_0, R) = \frac{A}{1 - \frac{\pi}{R_0} - \frac{\alpha\rho_0}{R_0} + \frac{(1-\alpha)}{R_0}\left(1 - \frac{\rho_0}{R}\right)}.$$

Plugging these expressions into equation (??), we verify that that $W^{\text{ex ante}}(R, R_0)$ increases in R if and only if Assumption ?? holds. For any couple (R_0, R) satisfying equation (??), so does $(R_0, 1)$. We therefore have that

$$W^{\text{ex ante}}(R_0, R) \leq W^{\text{ex ante}}(R_0, 1).$$

It is then easy to verfy that $W^{\text{ex ante}}(R_0, 1)$ is increasing in R_0 as long as Assumption ?? holds.

Let us now turn to values of (R_0, R) such that banking entrepreneurs choose to hoard no liquidity and instead load up on short-term debt, i.e. values such that equation (??) is violated. We only have to consider two values for R: ρ_0 and 1. We have

$$j(R_0, \rho_0) = i(R_0, 1) = i(R_0, \rho_0) = \frac{A}{1 - \frac{\pi}{R_0} - \frac{\alpha\rho_0}{R_0}}$$

and

$$j(R_0, 1) = 0.$$

Let us first consider the case where $R = 1$. Plugging these expressions in equation (??), it can be verified that $W^{\text{ex ante}}(R_0, 1)$ is increasing in R_0 if and only if $\alpha\beta \leq 1 - \pi - \alpha\rho_0$. This condition is implied by Assumptions ?? and ??. Turning now to the case where $R_0 = \rho_0$, we

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find that $W^{\text{ex ante}}(R_0, \rho_0)$ is increasing in ρ_0 if and only if Assumption **??** holds.

Proof of Proposition 9

The proposition follows easily from the following two lemmas. The first one characterizes the form of the optimal bailout given an interest rate R. The second one derives the optimal ex-ante liquidity choices of banks. The first lemma follows easily from the linearity of the ex-post bailout program in $j(i, x)$. The second lemma follows from a simple calculation of the banking entrepreneur's welfare given his liquidity choice x.

Lemma 1 *(optimal bailout). Under the optimal bailout:*

(i) if $R \geq \underline{R}(\gamma)$, then $t(i, x) = 0$ and $j(i, x) = x/(1 - \rho_0/R)$;

(ii) if $R < \underline{R}(\gamma)$ then $t(i, x) = (1 - x - \rho_0/R)i$ and $j(i, x) = i$.

Lemma 1 shows that given R, whether direct transfers are used depends on whether or not $R < \underline{R}(\gamma)$. When $R < \underline{R}(\gamma)$, it is optimal to transfer funds to banks that claim to be distressed, even though a fraction will end up in banks that are truly intact. Enough funds are then transferred so that distressed banks can continue at full scale. When $R \geq \underline{R}(\gamma)$, it is preferrable not to engage in direct transfers because too high a fraction would end up in banks that are truly intact. This is intuitive: when the interest rate R is low, distressed banks can lever up the direct transfers more (and intact banks perceived as distressed cannot), which makes direct transfers more attractive. That the threshold interest rate $\underline{R}(\gamma)$ for direct transfers increases with γ makes sense: The asymmetric information problem is worse when γ is low so that the proportion of false positives $(1 - \gamma)\nu$ is high.

Lemma 2 *(liquidity choice). The optimal scale and liquidity choice of banks when they expect the interest rate to be R in the event of a crisis is:*

(i) if $R \geq \underline{R}(\gamma)$, then $i/A = 1/[1 - \pi - \alpha\rho_0 + (1 - \alpha)(1 - \rho_0/R)]$, $x = 1 - \rho_0/R$ and $d = \pi - (1 - \rho_0/R)$;

(ii) if $R < \underline{R}(\gamma)$, then $i/A = 1/[1 - \pi - \alpha\rho_0]$, $x = 0$ and $d = \pi$.

Lemma 2 shows that the ex-ante liquidity choices also depends on whether or not $R < \underline{R}(\gamma)$. When $R < \underline{R}(\gamma)$, the government provides a big enough direct transfer to banks that claim to be distressed so that they can continue at full scale if they are truly

distressed. Therefore, hoarding liquidity is useless. It only reduces the investment scale and total leverage i/A. As a result, banks opt to be completely illiquid ($x = 0$) and choose a maximal level of short-term debt ($d = \pi$). When $R \geq \underline{R}(\gamma)$, the government does not provide any direct transfer. Continuation scale therefore increases with liquidity and decreases with the amount of short-term debt. Banks choose to hoard enough liquidity ($x = 1 - \rho_0/R$) and take on only as much short-term debt ($d = \pi - (1 - \rho_0/R)$) so as to be able to continue at full scale in case of a crisis.