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Mortality Decline and Aggregate Wealth Accumulation

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Abstract

The paper discusses the impact of longevity extension on aggregate wealth accumulation, accounting for changes in individual behaviors as well as changes in population age structure. It departs from the standard literature by adopting a formulation of individual preferences that accounts for temporal risk aversion. Human impatience is then closely related to mortality rates and aggregate wealth accumulation appears to be much more sensitive to demographic factors than with the traditional approach. Illustrations are provided using historical mortality data from different countries.

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1 Introduction

Recent human history is characterized by rapid changes in mortality that are likely to have major economic consequences. The paper discusses the impact of longevity extension on aggregate wealth accumulation, accounting for changes in individual behaviors as well as changes in population age structure. As such, it contributes to an expanding body of literature that includes Blanchard (1985), Lee, Mason and Miller (2002 and 2003), Bloom, Canning and Graham (2003) and Sheshinsky (2005).

The originality of the present paper is that it relies on a formulation of individual preferences that accounts for temporal risk aversion. It therefore departs from the mainstream literature by considering a class of preferences that is larger than the one introduced by Yaari (1965). Instead of assuming that agents have a von Neumann-Morgenstern utility function such that a life of length T with a consumption profile c(.) provides a utility:

$$U^{\rm yaari}(c,T) = \int_0^T \alpha(t) u(c(t)) dt$$

it is assumed that their von Neumann-Morgenstern utility function is given by:

$$U(c,T) = \phi\left(\int_0^T \alpha(t)u(c(t))dt\right)$$
(1)

where ϕ is an increasing function.

The function ϕ that enters into this formulation of individual utility has a straightforward interpretation in terms of temporal risk aversion, or more generally, in terms of risk aversion. Individuals have positive temporal risk aversion if and only if ϕ is concave (Richard, 1975). Two individuals who only differ by their functions ϕ have identical ordinal preferences but different degrees of risk aversion (Kihlstrom and Mirman, 1974). Therefore, considering specification (1) makes it possible to study the role of risk aversion; this is clearly a natural line of research since mortality is undoubtedly a risk. Accounting for temporal risk aversion is crucial for understanding life cycle consumption smoothing of human (and therefore mortal) beings, since the combination of temporal risk aversion and lifetime uncertainty generates impatience (Bommier, 2006). The intuition is that temporally risk averse individuals rationally consume a lot when young in order to avoid the particularly bad outcome which consists in having a life that is both short and lacking in fun. In other words, "*Carpe diem*" is a rational precept for individuals with positive temporal risk aversion.

Impatience being related to mortality, it is naturally found that mortality decline induces changes in human impatience. Thus in addition to the various effects that are documented in studies that are based on Yaari's model, the present paper highlights and calibrates a novel effect: that of a change in impatience.

The origin of this impatience effect is carefully explained in Bommier (2008). In particular it is shown that, although there is a strong theoretical relation between mortality risks, temporal risk aversion and human impatience, it is not necessarily true that lower mortality implies lower impatience. The story is more complex since mortality contributes to several terms that impact human impatience in opposite directions. Whether mortality decline eventually leads to an increase or a decrease in human impatience depends on how mortality at young ages falls compared to mortality at old ages. Thus, although the theory does unambiguously support the idea that there may be a substantial impatience effect, applications are still needed to evaluate the magnitude of this effect.

The present paper makes contributions in two directions: firstly it provides a simple method for estimating the impact of mortality decline on aggregate wealth accumulation when temporal risk aversion is taken into account. The method makes it possible to break down the impact of mortality decline into several components, reflecting aggregation, income dilution and behavioral effects. Secondly, this method is implemented with realistic historical mortality data taken from different countries. Computations are derived with different assumptions on individual preferences. The discussion then highlights the qualitative and quantitative contributions of temporal risk aversion. It is found that accounting for temporal risk aversion may induce a significant shift in the assessment of the impact of mortality decline.

The paper is structured as follows. In Section 2 we present the life-cycle model of individual behavior. Section 3 deals with the aggregation of individuals' wealth. We suggest a breakdown of the impact of mortality changes on aggregate wealth accumulation into three components reflecting aggregating, income dilution and impatience effects. Section 4 develops and discusses illustrations based on mortality rates observed over the period 1950-2000 in different countries. Concluding comments are set forth in Section 5.

2 Life cycle savings

2.1 Individual preferences

In order to model intertemporal choice under uncertain lifetime, one has to consider preferences allowing lotteries involving lives of different lengths to be compared. Since Yaari's seminal paper, the usual strategy involves assuming that agents are expected utility maximizers with a utility function such that a consumption profile c(.) and a life of length T provides a utility:

$$U^{\rm yaari}(c,T) = \int_0^T \alpha(t) u(c(t)) dt$$

The function u is called instantaneous utility, and α the subjective discount function. Some recent contributions (for example Halevy, 2007 and Drouhin, 2006) did explore extensions of Yaari's model to non expected utility maximizers but, surprisingly, very little exploration has been achieved within the expected utility framework. It is this latter approach that we adopt here: we maintain the assumption that agents have von Neumann-Morgenstern preferences, but extend Yaari's model by considering that agents have a utility function of the form:

$$U(c,T) = \phi\left(\int_0^T \alpha(t)u(c(t))dt\right)$$

where ϕ is an increasing (but not necessarily linear) function. By normalization it will be assumed that $\phi(0) = 0$. The interest of dealing with such a specification is that it makes it possible to explore the role of risk aversion. As is known from the seminal contribution of Kihlstrom and Mirman (1974), playing with the concavity of the function ϕ , while holding the functions u and α unchanged, is indeed the formal way of discussing comparative risk aversion within the expected utility framework. The greater the concavity of the function ϕ the greater the agent's risk aversion.

The particular case considered by Yaari, where ϕ is linear, corresponds to an assumption of temporal risk neutrality, or equivalently, to an assumption of risk neutrality with respect to life duration, when defined in a way that control for time preferences¹. To highlight the consequences of assuming temporal risk neutrality we may compare the following lotteries:

$$L_1 \begin{cases} (T = 40, c(t) = 1 \text{ for all } t); \text{ with } p = \frac{1}{2} \\ (T = 75; c(t) = 2 \text{ for all } t); \text{ with } p = \frac{1}{2} \end{cases}$$

and

$$L_2 \begin{cases} (T = 40, c(t) = 2 \text{ for all } t); \text{ with } p = \frac{1}{2} \\ (T = 75; c(t) = 1 \text{ for all } t \le 40 \text{ and } c(t) = 2 \text{ for } t > 40); \text{ with } p = \frac{1}{2} \end{cases}$$

In both the above lotteries, which are illustrated in Figure 1, there is a 0.5 probability of dying at age 40 and a 0.5 probability of dying at age 75. There is also an even chance to have a low or high consumption during the 40 first years of life. Both lotteries assume that, in the case of survival after age 40, consumption between ages 40 and age 75 equals 2. The difference between L_1 and L_2 is that

¹See Bommier (2006).

bad luck in instantaneous consumption during the first part of life is associated with bad luck in life duration in lottery L_1 , while it is associated with good luck in life duration in lottery L_2 . It is straightforward to check that when ϕ is linear, both lotteries provide the same expected utility. However, when ϕ is concave then L_2 is preferred to L_1 . When ϕ is concave, the individual chooses L_2 , the lottery that avoids the risk of having a life which is both short and with a low level of instantaneous consumption.

Considering temporal risk aversion is of particular interest when considering endogenous choice of consumption under uncertain lifetime. Consider the case of a random length of life described by a distribution of the age at death d(T). The expected utility associated with a consumption profile c(.) is given by:

$$EU(c) = \int_0^{+\infty} d(T)\phi\left(\int_0^T \alpha(t)u(c(t))\right)dt$$
(2)

When mortality is exogenous, an agent's rational behavior involves choosing the consumption profile that maximizes EU(c) among those affordable given his budget constraints.

Similarly to what is done in Yaari (1965), an alternative formulation of expected utility can be obtained after integrating (2) by parts. Denoting $s(t) = 1 - \int_0^t d(T) dT$, the survival function, the following is obtained

$$EU(c) = \left[-s(T)\phi\left(\int_0^T \alpha(t)u(c(t))\right) \right]_0^{+\infty} + \int_0^{+\infty} s(t)\alpha(t)u(c(t))\phi'\left(\int_0^t \alpha(\tau)u(c(\tau))d\tau\right)dt$$

and then

$$EU(c) = \int_0^{+\infty} s(t)\alpha(t)u(c(t))\phi'\left(\int_0^t \alpha(\tau)u(c(\tau))d\tau\right)dt$$
(3)

In the case where ϕ is linear (i.e. when ϕ' is a constant) one can recognize Yaari's formulation of expected utility which is taken as the starting point for most (if

not all) papers that discuss the impact of mortality decline. However, when ϕ is not linear, equation (3) makes it appear an endogenous discount function, which generates a strong relation between mortality, risk aversion and impatience. We refer to Bommier (2008) for a detailed discussion of such a relation. Below we will only explore the consequences when agents maximize their expected utility under specific budget constraints involving exogenous mortality patterns and perfect intertemporal markets.

2.2 Life cycle behavior

Assume that a survival function s(t) and an age specific income profile y(t) are exogenously given. Assume also that intertemporal markets are perfect. In presence of a perfect annuity market the return on individual's wealth is the sum of the rate of interest, denoted r, and the mortality rate, $\frac{-s'(t)}{s(t)}$. Thus, individual's wealth has the following dynamic:

$$w'(t) = \left(r - \frac{s'(t)}{s(t)}\right)w(t) + y(t) - c(t)$$
(4)

We assume that individuals have no initial wealth. The budget constraints impose:

$$w(0) = 0 \text{ and } w(\infty) \ge 0 \tag{5}$$

Form (4) and (5) the budget constraint can be rewritten in its integral form:

$$\int_0^{+\infty} s(t)c(t)e^{-rt}dt \le \int_0^{+\infty} s(t)y(t)e^{-rt}dt$$
(6)

Rational consumers aim at maximizing their expected utility:

$$EU(c) = \int_0^{+\infty} d(T)\phi\left(\int_0^T \alpha(t)u(c(t))dt\right)dT$$
(7)

under the constraint (6).

The first order conditions are:

$$\alpha(t)u'(c(t))\int_{t}^{+\infty}d(T)\phi'\left(\int_{0}^{T}\alpha(\tau)u(c(\tau))d\tau\right)dT = \lambda s(t)e^{-rt}$$
(8)

where λ is the Lagrangian multiplier.

Taking the logarithmic derivative and using d(t) = -s'(t), one obtains:

Lemma 1 The optimal consumption profile is such that $\forall t > 0$:

$$\frac{c'(t)}{c(t)} = \sigma(c(t)) \left[r + \frac{\alpha'(t)}{\alpha(t)} - \frac{s'(t)}{s(t)} - \frac{s'(t)\phi'\left(\int_0^T \alpha(\tau)u(c(\tau))d\tau\right)}{\int_t^{+\infty} s'(T)\phi'\left(\int_0^T \alpha(\tau)u(c(\tau))d\tau\right)} \right]$$

where $\sigma(c) = -\frac{u'(c)}{cu''(c)}$ is the intertemporal elasticity of substitution.

An immediate consequence is:

Corollary 1 When ϕ is linear, the optimal consumption profile is such that $\forall t > 0$:

$$\frac{c'(t)}{c(t)} = \sigma(c(t)) \left[r + \frac{\alpha'(t)}{\alpha(t)} \right]$$

In particular, in the case where intertemporal elasticity of substitution is constant, the consumption growth rate is independent of the mortality pattern, as has been known since Yaari (1965). In such a case, an explicit solution can be given to the consumption problem:

$$c^{\text{yaari}}(t) = K\widetilde{c}(t) \text{ with } \widetilde{c}(t) = \alpha(t)^{\sigma} e^{r\sigma t} \text{ and } K = \frac{\int_0^{+\infty} s(t)y(t)e^{-rt}dt}{\int_0^{+\infty} s(t)\widetilde{c}(t)e^{-rt}dt}$$
(9)

When ϕ is not linear, however, equation (8) does not provide an explicit solution to the consumer problem, since the right hand side of the equality depends on consumption. Still, it does suggest a fairly simple way to derive the optimum consumption profile through a simple iterative process, especially when $\sigma(c)$ the intertemporal elasticity of substitution is constant (which is assumed hereafter).

At stage 1, start with an arbitrary consumption profile c_0 . Then, iteratively compute c_1 , c_2 , c_3 , etc. with the recursive formula:

$$c_n(t) = K_n \widetilde{c}_n(t)$$

with $\widetilde{c}_n(t) = \alpha(t)^{\sigma} e^{r\sigma t} \left[\int_t^{+\infty} \frac{s'(T)}{s(t)} \phi' \left(\int_0^T \alpha(\tau) u(c_{n-1}(\tau)) d\tau \right) \right]^{\sigma}$
and $K_n = \frac{\int_0^{+\infty} s(t) y(t) e^{-rt} dt}{\int_0^{+\infty} s(t) \widetilde{c}_n(t) e^{-rt} dt}$

The iterative process involves firstly computing what would be the discount function, if the consumption profile was exogenously given by c_{n-1} ; secondly, determining the consumption profile c_n that maximizes expected utility, given that discount function. It is formally shown in Bommier (2008) that when the concavity of ϕ is small enough, that iterative process converges towards the optimum consumption profile as $n \to +\infty$. In practice this procedure is extremely efficient when numerically implemented, convergence being extremely quick, typically obtained in a fraction of second.

It is also possible to derive analytic but approximate solutions by considering the case where the difference in welfare between life and death is much larger than the difference between high and low of consumption (that is when the value of life is extremely large). The strategy first involves determining a plausible range $[c_{\min}, c_{\max}]$ for the levels of instantaneous consumption, picking an arbitrary reference level $c^* \in [c_{\min}, c_{\max}]$ and writing:

$$u(c) = u(c^*)[1 + \varepsilon v(c)]$$

with $\varepsilon = \frac{u(c_{\max})-u(c_{\min})}{u(c^*)}$, $v(c) = \frac{u(c)-u(c^*)}{u(c_{\max})-u(c_{\min})}$. Then, an approximation can be obtained by assuming that $\varepsilon \ll 1$ or, in other words that the difference in instantaneous utility between consuming c_{\max} or consuming c_{\min} is much smaller than the difference between consuming c^* or being dead. The idea is to approximate the expected utility function (3) by a first order approximation in ε . From (3) one obtains

$$EU(c) \simeq u(c^*) \int_0^{+\infty} s(t)\alpha(t)\phi'\left(u(c^*) \int_0^t \alpha(\tau)d\tau\right) dt +\varepsilon u(c^*) \int_0^{+\infty} s(t)v(c(t))\alpha(t)\phi'\left(u(c^*) \int_0^t \alpha(\tau)d\tau\right) dt +\varepsilon \left[u(c^*)\right]^2 \int_0^{+\infty} s(t)\alpha(t) \left(\int_0^t \alpha(\tau_1)v(c(\tau_1))d\tau_1\right)\phi''\left(u(c^*) \int_0^t \alpha(\tau)d\tau\right) dt$$

After a few operations (a switch in integration order and an integration by parts in the third term) one obtains:

$$EU(c) \simeq u(c^*) \int_0^{+\infty} s(t)\alpha(t)\phi'\left(u(c^*) \int_0^t \alpha(\tau)d\tau\right) dt + \varepsilon u(c^*) \int_0^{+\infty} s(t)\beta(t)v(c(t))dt$$
(10)

where $\beta(t)$ is given by

$$\beta(t) = \frac{\alpha(t)}{s(t)} \int_{t}^{+\infty} d(\tau) \phi' \left(u(c^*) \int_{0}^{\tau} \alpha(\tau_1) d\tau_1 \right) d\tau \tag{11}$$

When mortality is exogenous, the first term of the right hand side of (10) is a constant and therefore does not affect individuals' choice. Agents, behave then as if they were maximizing:

$$EU^{\text{lin app}} = \int_0^{+\infty} s(t)\beta(t)v(c(t))dt$$

an additive utility function with the discount function $\beta(t)$ given by (11). This problem is of the same nature as the one obtained when ϕ is linear. The solution is given by

$$c^{\text{lin app}}(t) = K\widetilde{c}(t)$$

with $\widetilde{c}(t) = \alpha(t)^{\sigma} e^{r\sigma t} \left[\frac{1}{s(t)} \int_{t}^{+\infty} d(T) \phi' \left(u(c^{*}) \int_{0}^{T} \alpha(\tau) d\tau \right) \right]^{\sigma}$
and $K = \frac{\int_{0}^{+\infty} s(t) y(t) e^{-rt} dt}{\int_{0}^{+\infty} s(t) \widetilde{c}(t) e^{-rt} dt}$

It is easy to check that this approximate solution coincides with the exact one

given in (9) in the case where ϕ is linear. Moreover $c^{\text{lin app}}$ corresponds to c_1 , which is obtained in the first step of the iterative process described above, when c_0 is taken constant and equal to c^* .

When ϕ is not linear, a noteworthy feature of the exact or approximate solutions is that the shape of the consumption profile depends on the survival function. In fact, the consumption growth rate at a given age depends on the mortality rate at that age and also on mortality rates at greater ages. From Lemma 1, we can see that the mortality rate at age t tends to decrease the consumption growth rate at age t while mortality rates at age greater than t have the opposite impact. The reasons for these opposing effects of mortality on the consumption growth rate are discussed in Bommier (2008). Basically, at age t, mortality involves a potential loss, whose likelihood of occurrence is positively related to the mortality rate at age t but whose magnitude (roughly speaking, remaining life expectancy) is negatively related to mortality rates at greater ages.

3 Aggregate wealth

Consider now a population composed of individuals of different ages. More precisely, denote by N(x) the density of individuals of age x, so that there are N(x)dx individuals of age between x and x + dx in the population, the whole population size being normalized to 1. In the case of a steady-state population, N(x) would be proportional to $e^{-nx}s(x)$ where n is the population growth rate. Still, in order to be able to consider realistic demographic data on population age structure, we do not make such an assumption. In what follows N(x) can be any distribution with compact support.

From (4) and (5) we can compute individual's wealth at age x:

$$w(x) = \frac{1}{s(x)} \int_{x}^{+\infty} s(t) e^{-r(t-x)} (c(t) - y(t)) dt$$

The aggregate wealth in the population is:

$$W = \int_0^{+\infty} N(x)w(x)dx = \int_0^{+\infty} dx \int_x^{+\infty} \frac{N(x)e^{rx}}{s(x)}s(t)e^{-rt}(c(t) - y(t))dt$$

After a switch in integration order, we get:

$$W = \int_0^{+\infty} \Omega_{N,s}(t) (c(t) - y(t)) dt$$
(12)

with

$$\Omega_{N,s}(t) = \int_0^t \frac{s(t)}{s(x)} N(x) e^{r(x-t)} dx$$

Note that $\Omega_{N,s}(.)$ is a function that depends on survival probabilities and population age structure, but which is independent of individual preferences and budget constraints. $\Omega_{N,s}(.)$ can therefore be computed from demographic data alone. As we will see in Section 3.1 below, equation (12) then proves to be quite practical to compute and discuss the determinants of aggregate wealth accumulation, disentangling what is due to purely demographic factors from what is related to saving behaviors.

The ratio of aggregate wealth over aggregate income is:

$$\frac{W}{Y} = \frac{\int_{0}^{+\infty} \Omega_{N,s}(t)(c(t) - y(t))dt}{\int_{0}^{+\infty} N(t)y(t)dt}$$
(13)

This is the variable on which we will focus in order to assess the impact of mortality decline. The choice to focus on W/Y rather than on W was guided by the fact that in the case where aggregate capital equals aggregate wealth (no asset bubbles or capital owned by foreigners), the ratio $\frac{W}{Y}$ equals the capital/labor income ratio.

3.1 Impact of mortality changes

Compare now two demographic states A and B which are characterized by different survival functions (s_A and s_B) and different population structures (N_A and N_B). We consider a partial equilibrium (or a small open economy), so that wealth accumulation have no effect on labor income and the rate of interest. We denote by r and y(.) the rate of interest and age specific income profile which, by assumptions, are the same in states A and B. We denote by $\frac{W_A}{Y_A}$ and $\frac{W_B}{Y_B}$ the ratio of aggregate wealth over aggregate income in states A and B. Using (13), we may break down the variation of this ratio into three terms:

$$\frac{W_B}{Y_B} - \frac{W_A}{Y_A} = I_1 + I_2 + I_3$$

with

$$I_{1} = \frac{\int_{0}^{+\infty} \Omega_{N_{B},s_{A}}(t)(c_{A}(t) - y(t))dt}{\int_{0}^{+\infty} N_{B}(t)y(t)dt} - \frac{\int_{0}^{+\infty} \Omega_{N_{A},s_{A}}(t)(c_{A}(t) - y(t))dt}{\int_{0}^{+\infty} N_{A}(t)y(t)dt}$$

$$I_{2} = \frac{\int_{0}^{+\infty} (\widehat{c_{A}}(t) - y(t))\Omega_{N_{B},s_{B}}(t)dt}{\int_{0}^{+\infty} N_{B}(t)y(t)dt} - \frac{\int_{0}^{+\infty} (c_{A}(t) - y(t))\Omega_{N_{B},s_{A}})dt}{\int_{0}^{+\infty} N_{B}(t)y(t)dt}$$

$$I_{3} = \frac{\int_{0}^{+\infty} \Omega_{N_{B},s_{B}}(t)(c_{B}(t) - \widehat{c_{A}}(t))dt}{\int_{0}^{+\infty} N_{B}(t)y(t)dt}$$

where

$$\widehat{c}_A(t) = \kappa c_A(t) \text{ with } \kappa = \frac{\int_0^{+\infty} s_B(t) y(t) e^{-rt} dt}{\int_0^{+\infty} s_B(t) c_A e^{-rt} dt}$$

The first term, I_1 , is an aggregating effect. It shows how the ratio of aggregate wealth over aggregate income would have shifted, if the only factor to change was the population age structure. The second, I_2 , shows how $\frac{W}{Y}$ would have changed if the only consequence of longevity extension was a shift from consumption c_A to \hat{c}_A , that is a simple rescaling of individual consumption in order to match the new budget constraint. This terms therefore represents an income dilution effect associated with the fact that, when longevity increases, agents have to lower their instantaneous consumption in order to cover a greater life duration. The last term is an impatience effect: it measures the consequences of the changes in the shape of the life cycle consumption profiles following a change in mortality rates. This term equals zero, according to Yaari's model, but this is no longer the case when temporal risk aversion is introduced.

4 Applications

We illustrate the above framework using realistic demographic data provided by the Human Mortality Database, covering the second half of the twentieth century for twenty-two countries². These data gather life tables as well as accurate data on the population age structure. It is thus possible to implement the above computation with accuracy.

4.1 Demographic facts

Tables 1 and 2 provide information on mortality for all twenty-two countries. As mortality at young and old ages may play different roles, we reported information on adulthood and old age mortality. More precisely, Table 1 provides life expectancies at ages 20 and 60, when computed according to the 1950 and 2000 life tables. Table 2 respectively reports $_{30}q_{50}$ and $_{70}q_{80}$ the mortality ratios between ages 30 and 50 and between ages 70 and 80 according to the same life tables. For all countries but Bulgaria, all these indicators unanimously indicate a decline in mortality between years 1950-2000. This corresponds to the well documented trend of longevity extension that has been observed in developed countries. The decline is substantial on average, but the data report quite significant variations. At the bottom-end we find countries like Bulgaria, where mortality has hardly declined (this is mainly due to a deterioration in the last two decades of the century) or, more surprisingly, a country like Denmark, where mortality did decline, but relatively little compared to what happened in other countries. At the other extreme we find Japan, which is characterized by a huge decline in mortality. The fall is particularly spectacular for middle aged adults, since $_{30}q_{50}$, the mortality ratio between ages 30 and 50, was almost divided by

²Country selection was determined by data availability. More precisely, we considered all countries for which the Human Mortality Database provided 1950 and 2000 life tables.

5 between 1950 and 2000. As a consequence, Japan that initially had the lowest life expectancy at age 20 (and the greatest $_{30}q_{50}$) in 1950 became the country of our dataset with the greatest life expectancy (and the lowest $_{30}q_{50}$) in 2000.

Table 3 reports data on population age structure. More precisely, what are shown are the "elderly ratios" that were observed in years 1950 and 2000. By "elderly ratio", we mean the ratio of the population of age greater than 60 to the ratio of the population of age 20-60. In all countries, the elderly ratio increased between 1950 and 2000. However, mortality is just one determinant of this elderly ratio, which also depends on past birth rate and on migration. As a result, the pattern that arises from Table 3, with respect to the elderly ratio, does not closely replicate the patterns found in Tables 1 and 2 relating to mortality. Japan, which was characterized by a huge mortality decline is characterized by a huge increase of the elderly ratio (+148%). But a comparable increase (+122%) is also found in Bulgaria, although there were only minor mortality changes in that country. Meanwhile, the elderly ratio only changed slightly in New Zealand, although mortality did substantially decline.

4.2 Assumptions and model calibration

In order to implement the method developed in section 3, we make a list of simplifying assumptions that we detail below.

4.2.1 Institutional and demographic assumptions

The aim of this paper is to illustrate the potential role of mortality decline but definitely not to provide an accurate and complex representation of what may have been going on in the countries studied. For our objective, we thought it to be more judicious to stick to a stylized representation of reality, rather than using a complex model based on assumptions that would have to be country-specific in order to account for the variety of institutional settings. Thus, we focused on demographic heterogeneity, and deliberately decided to ignore all other (economic, cultural, etc.) aspects that may differ between countries or that may have changed between 1950 and 2000. In order to avoid confusing over-interpretation of the results, one should simply consider that country names such as "Australia" or "Denmark" that appear in the discussion and tables that follow, do not refer to actual countries with specific institutions, but simply to different patterns of demographic changes.

We assume that, as far as savings are concerned, individuals' economic life begins at age 20. In other words, individuals do not save before that age. Such an assumption can be viewed as corresponding to the case where children have stringent liquidity constraints that compel them to consume all they receive from their parents until they reach age 20.

Labor income is supposed to be exogenous, constant up to age 60, and equalling zero afterwards. We therefore rule out the existence of Social Security systems as well as the endogeneity of retirement age. Social Security and retirement regulations being extremely heterogenous across countries, it would have been quite hazardous to suggest a universal model that would have covered all the countries under consideration. Age specific variations in productivity are also ignored.

The rate of interest is assumed to be exogenous and equal to 3%. Of course, this may be open to discussion, especially when encountering changes in aggregate wealth of substantial magnitude. In a close economy, an increase of accumulated wealth should push down the rate of interest: such a general equilibrium adjustment is not taken into account in our illustrations.

As for demographic data, we consider cross-sectional agents, and imagine societies where agents would live according to these cross-sectional mortality rates. This is of course a thought experiment since cross-sectional mortality data do not reflect longitudinal mortality data. A 50 year old agent alive in year 2000 did not face the 2000 mortality rates in his youth, and certainly does not expect to face these same 2000 rates in the future. The use of historical cross-sectional mortality data does not aim therefore at reproducing the life of real agents, but simply at providing demographic patterns that are fairly reasonable.

4.2.2 Model calibration

The papers aims at emphasizing the role of temporal risk aversion when studying the relation between mortality and aggregate wealth accumulation. A possibility would be to look at the role played by the function ϕ keeping everything else constant. But when changing the function ϕ , while keeping other parameters constant, one also changes the rate of time discounting. With such a strategy, it is then difficult to conclude whether what is emphasized is the role of temporal risk aversion or that of human impatience. The other possibility, that is pursued of the paper, is to consider different combination for the functions ϕ and α providing identical rates of time discounting (for a reference mortality pattern) and see what happens when mortality is changed. The idea, for proceeding in such a way, is to consider that the empirical literature does provide us with some information on the shape of age-specific consumption profiles but that this can be explained by different combinations of the functions ϕ and α , with different implications regarding the impact of mortality decline.

For simplicity sake, the results will be presented for only two specifications of individual preferences suggesting two orthogonal explanations to human impatience. The first, called the "additive model" assumes that agents are temporally risk neutral (ϕ is linear). This specification was suggested by Yaari (1965), and is now found in almost all economic papers that discuss the impact of mortality decline. In that case, human impatience is almost exogenous (mortality playing a minor role), and is governed by the shape of the subjective discount function α . The second specification, called the "time neutral model", assumes that agents have no pure time preferences (α is constant) but are temporally risk averse. According to the time neutral model, agents' impatience exclusively results from lifetime uncertainty, whose impact is magnified by temporal risk aversion. The additive and the time neutral models can be viewed as polar cases in the set of possible explanations for human impatience. Intermediate positions where human impatience would result both from pure time preferences and from temporal risk aversion would involve choosing a specification with both a decreasing function α and a concave function ϕ . Results obtained with such intermediate specifications typically fall in between those of the additive and time neutral cases and are not reported in the present paper.

In both specifications, we assume that the intertemporal elasticity of substitution is constant and relatively close to one, consistent with the results of empirical studies such as Blundell, Browning and Meghir (1994). To avoid having to deal with the limit case where this elasticity equals 1 and instantaneous utility is logarithmic we took an intertemporal elasticity of substitution equal to 0.9. We then have $u(c) = 1 + \lambda^{1-\gamma} \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma = \frac{1}{0.9}$, where λ is a constant. The constant λ (which matters as soon as temporal risk aversion is taken into account) is what determines the value of life. Now, u(c) can obviously be rewritten as $u(c) = 1 + \frac{(\lambda c)^{1-\gamma}}{1-\gamma}$ which makes it clear that changing λ is equivalent to choosing the consumption measurement scale. There are therefore two possible strategies at the calibration stage: either choose an arbitrary λ and calibrate the measurement scale of the income profile y(.) (which is what indirectly determines the consumption level) in order to obtain values of statistical lives that are reasonable; or choose an arbitrary income profile y(.) and then calibrate λ . We followed this latter approach. Calibration was performed so that with r = 3%and with 1950 US mortality rates, the value of a statistical life of a 40 year old individual is about 250 times its annual income. A 40 year old individual earning 20,000 dollars per year would then have a value of statistical life of about 5 million dollars, in the range of what is suggested by empirical estimates derived from US data (see Viscusi and Aldy 2004). There is of course some possible dispute about taking 5 million dollar for the value of a statistical life, since empirical studies provide a broad range of estimates. However, we shall emphasize that our results are not very sensitive to that assumption. In particular, the additive approximation, which involves assuming an infinite value for the value of a statistical life provides quite similar results. In fact, as soon as we assume a value of statistical life that is significantly larger than remaining lifetime consumption (which is what is typically reported by empirical studies), we are not far from the limit case where the value of life is infinite, and the choice of a particular value has little importance.

In order to have models that are reasonably comparable, we chose the function α in the additive model and the function ϕ in the time neutral model, so that both models would predict a fairly close pattern of consumption smoothing. More precisely, in the time neutral specification, the function ϕ is given by:

$$\phi(x) = \frac{1 - e^{-kx}}{k}$$

assuming therefore constant absolute risk aversion with respect to life duration. The value of k has been chosen so that we obtain a reasonable inverse U-shape consumption profile, qualitatively similar to what is found in empirical studies such as Fernández-Villaverde and Krueger (2007), when r = 3% and mortality corresponds to that of the 1950 US life table. This consumption profile is shown in Figure 2. To get an order of magnitude of the degree of temporal risk aversion associated with such a specification, one may consider the coefficient of absolute risk aversion with respect to life duration when considering constant consumption profiles. We found a coefficient of 0.76 % per year, when using the constant consumption profile that satisfies the budget constraint. With this degree of risk aversion, individuals would be indifferent between (i) living 73.1 years for sure and (ii) having 50 percent chances to live 65 years and 50 percent chances to live 85 years.

As for the additive model, the function α is chosen so that both models give exactly the same optimal consumption profile when mortality rates are those of the 1950 US life table. Thus, by construction, both the additive and multiplicative models provide the same prediction on wealth accumulation when using the mortality pattern observed in 1950 in the USA. The differences obtained when looking at the impact of mortality decline are therefore not the consequences of diverging beliefs about the strength of human impatience but result from diverging views on the causes of human impatience.

4.3 The ratio of aggregate wealth over aggregate income

For each of the twenty-two countries, we computed the (theoretical) ratio of aggregate wealth over aggregate income W/Y, using either the additive or the time neutral model of individual preferences. We also computed the results obtained with the linear approximation of the time neutral model detailed in Section 2.2. Results are reported in Table 4. The first two columns give the ratio in year 1950 and 2000. The third column, computes the relative increase. This latter is then broken down into three components, representing aggregating, income dilution and behavioral effects, respectively. For example, in Australia, according to the time neutral model, the ratio of aggregate wealth over aggregate (yearly) income would have been 5 in 1950 and 6.81 in year 2000 (columns 1 and 2). It would thus have increased by 62.4% (column 3). Out of these 62.4%, there are 8.3 percentage points that come from the aggregating effect (and hence from the change in population age structure), 27.9 from the income dilution effect, and 28.1 from the behavioral effect (columns 4, 5 and 6).

For all countries, the ratio of aggregate wealth over aggregate income is found to be greater in year 2000 than in year 1950. But results significantly differ depending on the countries and the model assumed.

The additive model constrains the behavioral effect to equal zero since the shape of the optimal consumption profile c^{yaari} is independent of mortality rates. The time neutral model, which introduces a link between mortality and impatience generates a behavioral effect, which is anything but negligible. In all countries but Bulgaria and Hungary, this behavioral effect happens to be the largest of the three reported effects. In most cases, its size is comparable to the sum of the other two effects, indicating that accounting for this behavioral effect would be as important as taking into account the other aspects together. Quantitatively speaking, the standard approach based on the additive model, which focuses on the aggregating and income dilution effects, might have led half of the story to be forgotten. This sometimes amounts to more, as in Japan and Spain, sometimes to less, as in Denmark and the Netherlands, but in all cases it represents a non

negligible part.

The strongest behavioral effect is found in Japan, the country where the most important decline of mortality was observed. It is clear that in such a case the predicted increase in aggregate wealth is so large that our assumption of a constant rate of interest becomes problematic. In general equilibrium, with an endogenous rate of interest, one would find a significant decrease in the rate of interest and a smaller increase in wealth accumulation. Thus, the numbers provided in Table 4 should not be directly compared to what was observed in Japan during the same period. Still, it is interesting to mention that the interest rate has strongly fallen in Japan during the second half of the twentieth century (see for example Braun, Ikeda and Joines, 2006) as would predict our model in a general equilibrium. Our paper highlights therefore another channel through which demographic evolutions may end having an impact on savings and wealth accumulation in Japan, adding to those highlighted in Horioka, Suzuki and Hatta (2007).

One important point is that the behavioral effect cannot be assessed correctly without looking at the age-specific changes in mortality. Bulgaria provides an interesting example. In that country, life expectancy at age 20 was roughly at the same level in years 1950 and 2000. Still, the behavioral effect is found to be substantial. In fact, the stagnation of life expectancy in Bulgaria results from the combination of a decrease of mortality at young ages and an increase at old ages (see Table 1). Although the mean age at death did not radically change, the distribution did change quite a lot, inducing changes in impatience. This therefore provides an example where characterizing mortality by life expectancy (as is often done in empirical studies) may be a poor strategy.

The last point worth noticing in Table 4 is that linear approximation of the time neutral model does a reasonably good job. The predicted levels for W/Y are slightly different when using the linear approximation than when using the time neutral model itself. But, when looking at the impact of mortality changes, the linear approximation provides results that are close to the true ones. For example,

in the USA the time neutral model predicts an increase of 54.2% for W/Y when going from 1950 to 2000 mortality rates. According to the linear approximation we would have found a pretty close result, with a predicted increase of 54.8%. In other words, the linear approximation does introduce a small bias, but this bias is found to be nearly steady and therefore to vanish when looking at differences.

5 Discussion

The paper discussed the impact of longevity extension on aggregate wealth accumulation. It has highlighted the potential role of an aspect of individual preferences that has hitherto been ignored: that of temporal risk aversion. When temporal risk aversion is introduced, a novel interesting relation emerges linking mortality and time discounting. Mortality changes may then lead individuals to significantly modify their saving behaviors, with sizable consequences on the aggregate wealth.

Illustrations based on historical mortality data show that accounting for temporal risk aversion may radically change predictions about the impact of decline in mortality. Firstly, extension of longevity may have had a much greater impact than is usually thought. Secondly, the impact has to be evaluated by carefully looking at the changes in the whole distribution of the age at death, and not by using a single synthetic indicator, such as life expectancy. For a given increase in life expectancy, radically different results may be obtained depending on whether this increase in life expectancy is due to a fall in mortality at young or old ages.

Our results naturally raise a burning issue: to what extent are humans temporally risk averse? Could it be the case that human impatience, which is usually taken as being mostly exogenous and independent of risk aversion, is in fact endogenous, closely related to mortality rates and risk aversion? Are individuals closer to the additive formulation than to the time neutral model? Answering these questions is crucial to understand the impact of decline in mortality.

These issues are discussed in greater length in Bommier (2008), where it is

shown that up to now, there is no evidence that would suggest the superiority of the additive specification over the time neutral one. Indeed, as long as heterogeneity in mortality is not considered, the time neutral model can reproduce all the predictions of the additive model. Consequently, most empirical evidence that provided the additive formulation with some credibility, is in fact unable to discriminate between the time neutral and the additive models. Moreover, considerations about observed heterogeneity in time preference between men and women, or between individuals from different socioeconomic groups, would actually argue in favor of a strong link between mortality and impatience and therefore against the additive formulation.

The economic literature has therefore focused on the model assuming temporal risk neutrality, while alternative specifications, with at least as much empirical support, would lead to radically different conclusions. There is no doubt that, technically speaking, assuming temporal risk neutrality is a very convenient choice. In that case, individuals' expected utilities are additively separable and analytic computation is fairly simple. The temptation to focus on the additive model is fairly understandable. But is this a reasonable choice? A few lines written by George Stigler, although dating back to 1950 are amazingly topical. In a discussion about preferences over several goods, Stigler wrote:

"Manageability, should mean the ability to bring the theory to bear on specific economic problems, not ease of manipulation. The economist, has no right to expect of the universe he explores that its laws are discoverable by the indolent and the unlearned. The faithful adherence for so long to the additive utility function strikes one as showing at least a lack of enterprise. I think it showed also a lack of imagination: no economic problem has only one avenue of approach" (Stigler, 1950, p394).

The same statement holds for the analysis of intertemporal choice under uncertain lifetime: the *faithful adherence to the additive utility function* is no less questionable. Just as risk aversion plays a key role in Finance, temporal risk aversion is likely to become a central element in the economics of aging. Death is an event with long-lasting consequences; the rational response to the risk of death has thus to crucially depend on temporal risk aversion.

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Country	Life ex	pectancy	at age 20	Life expectancy at age 60			
	in year	in year	Variation	in year	in year	Variation	
	1950	2000	1950-2000	1950	2000	1950-2000	
Australia	52.4	61.3	16.9~%	17.4	24.0	38.0~%	
Austria	51.5	59.5	15.7~%	17.0	22.6	33.1~%	
Belgium	51.6	59.1	14.5~%	17.2	22.5	30.6~%	
Bulgaria	52.0	53.6	3.2~%	18.4	18.2	-1 %	
Canada	53.0	60.6	14.4~%	18.0	23.4	29.7~%	
Czech	50.9	56.2	10.5~%	16.4	19.9	21.2%	
Denmark	53.9	58.1	7.8~%	18.0	21.2	18.2~%	
England	52.5	59.5	13.2~%	17.3	22.2	28.8~%	
Finland	49.1	58.8	19.8~%	15.9	22.3	40.1~%	
France	51.8	60.4	16.7~%	17.5	23.9	36.5~%	
Hungary	50.5	53.2	5.3~%	16.9	18.8	10.9~%	
Iceland	54.3	60.8	12.0~%	19.5	23.4	20.0~%	
Italy	52.8	60.9	15.3~%	18	23.4	30.4%	
Japan	47.0	62.4	32.7~%	15.8	24.9	58.1%	
Netherlands	54.9	59.6	8.7~%	18.6	22.2	19.5~%	
New Zealand	53.0	60.3	13.7~%	17.9	23.2	30.0~%	
Norway	55.3	59.9	8.4~%	19.2	22.7	18.1~%	
Slovakia	51.0	54.7	7.2~%	17.2	18.9	9.6~%	
Spain	50.6	60.5	19.7~%	17.4	23.5	34.7%	
Sweden	54.0	60.8	12.6~%	18.1	23.2	28.1~%	
Switzerland	52.8	61.2	16.0~%	17.6	23.8	35.6~%	
USA	51.9	58.6	13.0~%	17.7	22.3	26.3~%	

Table 1: Changes in life expectancy

Country	Mortality between 30 and 50 $(\%)$			Mortality between 70 and 80 (%)			
	in year	in year	Ratio	in year	in year	Ratio	
	1950	2000	1950/2000	1950	2000	1950/2000	
Australia	6.3	2.8	2.25	50.9	25.9	1.97	
Austria	7.2	3.2	2.25	52.7	29.5	1.79	
Belgium	7.3	3.6	2.05	51.4	30	1.72	
Bulgaria	7.7	5.9	1.3	48.3	46.9	1.03	
Canada	6.4	2.8	2.32	47.2	27.8	1.7	
Czech	7.3	4.2	1.74	55.6	39.3	1.41	
Denmark	5.3	3.5	1.5	49.1	35	1.4	
England	6	3	2	51.5	32	1.61	
Finland	9.1	4	2.28	57.4	30.4	1.88	
France	7.8	4	1.95	49.9	24.7	2.02	
Hungary	8.5	8	1.06	52.5	42.8	1.23	
Iceland	7.4	2.5	2.98	41.9	27	1.55	
Italy	6.7	2.6	2.6	49.1	27.3	1.8	
Japan	12.2	2.5	4.83	57	22.8	2.5	
Netherlands	4.4	2.7	1.63	46.5	30.9	1.5	
New Zealand	6	2.9	2.04	49.1	28.3	1.74	
Norway	4.8	2.9	1.64	43.7	29.3	1.49	
Slovakia	7.9	5.2	1.52	51.8	42.9	1.21	
Spain	9	3.3	2.74	51.2	26.9	1.9	
Sweden	5	2.4	2.13	48.4	27.4	1.77	
Switzerland	6.1	2.6	2.35	50.4	25.1	2.01	
USA	7.7	4.2	1.83	48.2	30.9	1.56	

Table 2: Changes in mortality rates

Country	Elderly Ratio				
	in year	in year	Variation		
	1950	2000	1950-2000		
Australia	0.23	0.30	27.9~%		
Austria	0.28	0.35	27.6~%		
Belgium	0.28	0.4	41.6~%		
Bulgaria	0.18	0.40	122.1~%		
Canada	0.22	0.29	31.2~%		
Czech	0.22	0.31	41.4~%		
Denmark	0.25	0.35	$41.5 \ \%$		
England	0.28	0.39	37.1~%		
Finland	0.19	0.36	84.1~%		
France	0.30	0.38	26.3~%		
Hungary	0.21	0.35	62.2~%		
Iceland	0.22	0.28	29.2~%		
Italy	0.23	0.43	87.9~%		
Japan	0.17	0.41	148.4~%		
Netherlands	0.22	0.32	42.0~%		
New Zealand	0.26	0.29	10.1~%		
Norway	0.24	0.35	44.6~%		
Slovakia	0.19	0.27	44.1~%		
Spain	0.2.0	0.38	87.3~%		
Sweden	0.26	0.41	56.1~%		
Switzerland	0.25	0.36	42.4~%		
USA	0.22	0.30	32.0~%		

Table 3: Changes in population age structure

	1	2	3	4	5	6
		TTT / TT				
Country + model	. W/Y	W/Y	T 7 •			
	in year	in year	Variation	Aggreg.	Dilution	Behav.
	1950	2000	1950-2000	Effect	Effect	Effect
Australia additive	5.00	6.81	36.3~%	8.5~%	27.9~%	0.0~%
Australia time neutral	5.49	8.92	62.4~%	8.3~%	26.0~%	28.1~%
Australia t.n. (lin. app.)	6.08	9.89	62.6~%	8.0 %	23.8~%	30.9~%
Austria additive	5.56	7.05	26.7~%	3.2~%	23.5~%	0.0~%
Austria time neutral	5.69	9.2	61.7~%	3.0~%	23.6~%	35.2~%
Austria t.n. (lin. app.)	6.26	10.18	62.6~%	2.7~%	21.6~%	38.3~%
Belgium additive	5.54	7.41	33.7~%	10.2~%	23.5~%	0.0~%
Belgium time neutral	5.63	9.32	65.5~%	10.3~%	23.7~%	31.6~%
Belgium t.n. (lin. app.)	6.17	10.27	66.5~%	9.8~%	21.7~%	35.0~%
Bulgaria additive	4.60	6.56	42.4~%	44.5~%	-2.0 %	0.0~%
Bulgaria time neutral	3.87	7.51	94.2~%	54.0~%	-1.9 %	42.1~%
Bulgaria t.n. (lin. app.)	4.12	8.25	100.3~%	51.0~%	-1.6 %	50.9~%
Canada additive	4.88	6.7	37.5~%	14.4 %	23.1~%	0.0~%
Canada time neutral	5.28	9.02	70.8~%	14.3~%	22.1~%	34.4~%
Canada t.n. (lin. app.)	5.82	9.99	71.6~%	13.9~%	20.3~%	37.5~%
Czech additive	4.88	6.22	27.4~%	10.7~%	16.7~%	0.0~%
Czech time neutral	4.93	7.80	58.3~%	11.0~%	16.9~%	30.4~%
Czech t.n. (lin. app.)	5.48	8.61	57.1~%	10.1~%	15.4~%	31.6~%
Denmark additive	5.36	6.83	27.3~%	13~%	14.3~%	0.0~%
Denmark time neutral	6.38	8.88	39.2~%	12.4~%	13.1~%	13.7~%
Denmark t.n. (lin. app.)	7.14	9.79	37.1~%	11.9~%	12.1~%	13.0~%
England additive	5.52	7.22	30.8~%	8.9~%	21.9~%	0.0~%
England time neutral	6.18	9.63	55.9~%	8.5~%	20.4~%	27~%
England t.n. (lin. app.)	6.86	10.67	55.5~%	8~%	18.8~%	28.7~%
Finland additive	4.38	7.29	66.6~%	30.1~%	36.5~%	0.0~%
Finland time neutral	3.72	8.98	141.2~%	34.7~%	42.4~%	64.1~%
Finland t.n. (lin. app.)	4.09	9.88	141.8~%	32.2~%	38.0~%	71.6~%
France additive	5.70	7.48	31.2~%	5.2~%	26~%	0.0~%
France time neutral	5.61	9.06	61.5~%	5.2~%	27.1~%	29.3~%
France t.n. (lin. app.)	6.09	9.84	61.8~%	5.0~%	24.8~%	32~%
Hungary additive	4.85	6.17	27.2~%	18.5~%	8.7~%	0.0~%
Hungary time neutral	4.34	6.44	48.1~%	20.9~%	9.9~%	17.4~%
Hungary t.n. (lin. app.)	4.72	6.84	44.9~%	19.4~%	8.9~%	16.6~%
(continued)						

Table 4: Ratio of aggregate wealth other aggregate income

Table 4: (continued)

	1	2	3	4	5	6
Country I model	\mathbf{W}/\mathbf{V}					
Country + model	$\frac{VV}{I}$	VV / I	Variation	Aggrog	Dilution	Bohow
	1050	$\frac{111}{2000}$	1050 2000	Fffoct	Effect	Effoct
Teele 1 - 11'4' -	1900	2000	1950-2000			Ellect
Iceland additive	5.13	0.39			14.4 %	
Iceland time neutral	4.93					50.8 %
Iceland t.n. (lin. app.)	5.29	9.75	84.4 %	10.3 %	14.9 %	59.2 %
Italy additive	5.03	7.80	55.1 %	28.5 %	26.6 %	
Italy time neutral	5.20	10.58	103.4 %	29.3 %	27.2 %	46.9 %
Italy t.n. (lin. app.)	5.74	11.75	104.5 %	27.9 %	24.9 %	51.8 %
Japan additive	3.93	8.32	111.5 %	53.3 %	58.2 %	0.0 %
Japan time neutral	2.35	11.23	378.7 %	81.7 %	94.3 %	202.7 %
Japan t.n. (lin. app.)	2.38	12.39	419.6 %	77.6 %	86.4 %	255.6 %
Netherlands additive	5.16	6.75	30.9 %	15.6 %	15.3~%	0.0 %
Netherlands time neutral	6.41	9.46	47.6 %	15 %	13.7 %	18.9 %
Netherlands t.n. (lin. app.)	7.19	10.53	46.6 %	14.7 %	12.8~%	19.1 %
New Zealand additive	5.27	6.56	24.4~%	3.3~%	21.1~%	0.0~%
New Zealand time neutral	5.80	8.52	46.8~%	3.4 %	20.1~%	23.3~%
New Zealand t.n. (lin. app.)	6.42	9.42	46.8~%	3.3~%	18.4~%	25~%
Norway additive	5.51	6.94	26.1~%	12.2~%	13.9~%	0.0~%
Norway time neutral	6.53	9.22	41.2~%	12.0~%	13.1~%	16.1~%
Norway t.n. (lin. app.)	7.28	10.27	41.0~%	11.6~%	12.2~%	17.2~%
Slovakia additive	4.55	5.52	21.5~%	13.8~%	7.8~%	0.0~%
Slovakia time neutral	4.09	6.61	61.5~%	15.5~%	9.0~%	36.9~%
Slovakia t.n. (lin. app.)	4.47	7.27	62.5~%	14.6~%	8.1~%	39.8~%
Spain additive	4.63	7.18	55.1~%	26.0~%	29.1~%	0.0~%
Spain time neutral	3.73	9.25	148.2~%	31.1~%	36.6~%	80.5~%
Spain t.n. (lin. app.)	3.95	10.22	159~%	29.2~%	33.5~%	96.3~%
Sweden additive	5.57	7.71	38.6~%	16.2~%	22.4~%	0.0~%
Sweden time neutral	6.60	10.87	64.8~%	15.6~%	20.6~%	28.6~%
Sweden t.n. (lin. app.)	7.38	12.1	63.9~%	14.9~%	19.1~%	29.9~%
Switzerland additive	5.34	7.41	38.9~%	12.0 %	26.9~%	0.0 %
Switzerland time neutral	5.89	10.01	70.0~%	11.7~%	25.7~%	32.6~%
Switzerland t.n. (lin. app.)	6.54	11.12	70.1~%	11.2~%	23.6~%	35.3~%
USA additive	4.97	6.44	29.5~%	9.1 %	20.4~%	0.0~%
USA time neutral	4.97	7.66	54.2~%	9.1 %	20.4~%	24.7~%
USA t.n. (lin. app.)	5.39	8.35	54.8~%	8.6~%	18.6~%	27.6~%







Figure 2: Consumption Profile Used at Calibration Stage

Consumption

Age (years)