Credit, Wages and Bankruptcy Laws

Bruno Biais† Thomas Mariotti‡

May 15, 2008

Abstract

We analyze how bankruptcy laws affect the general equilibrium interactions between credit and wages. Soft laws reduce the frequency of liquidations and thus ex-post inefficiencies, but they worsen credit rationing ex-ante. This hinders firm creation and thus depresses labor demand. Rich agents who need little outside funds can invest even if creditor rights are weak. Hence, they favor soft laws that exclude poorer agents from the credit market and reduce the competition for labor. Such laws can generate greater utilitarian welfare than under perfect contract enforcement: by barring access to credit to some agents, soft laws lower wages, which increases the pledgeable income of richer agents and decreases the liquidation rates they must commit to. Yet, when they induce strong credit rationing, soft laws are Pareto dominated by tougher laws combined with subsidies to entrepreneurs.

Keywords: Bankruptcy Law, Credit Rationing, Financial Contracts.

JEL Classification: D82, G33, K22.

†Toulouse School of Economics (CNRS/GREMAQ, IDEI).
‡Toulouse School of Economics (CNRS/GREMAQ, IDEI).

*We thank the Editor, Patrick Bolton, and two anonymous referees for insightful comments. We have benefited from discussions with Stéphane Auray, Alberto Bennardo, Patrick Bolton, Catherine Casamatta, Georges Casamatta, Anne Epaulard, Denis Gromb, Roman Inderst, Steven Kaplan, Nobuhiro Kiyotaki, Jean-Jacques Laffont, David Martimort, Marco Pagano, Fausto Panunzi, Jean-Charles Rochet, Howard Rosenthal, Gilles Saint-Paul, Klaus Schmidt, Andrei Shleifer, Jean Tirole and Luigi Zingales. We also thank conference participants at the Banca d’Italia/CEPR Conference on Money, Banking and Finance, at the CEPR/Princeton University Conference on the Political Economy of Financial Markets and at the 55ème Congrès de l’Association Française de Science Economique, as well as seminar participants at Saïd Business School, Stanford University, Università Bocconi, Université du Québec à Montréal and Université Toulouse I for useful comments.
1 Introduction

Should contracts be enforced? If they were not, agents would fail to commit resources to meet their contractual obligations. This would jeopardize economic activity. Yet, bankruptcy laws often entail violations of clauses stated in financial contracts. As stated by La Porta et al. (1998), “The most basic right of a senior collateralized creditor is the right to repossess [...] collateral when a loan is in default [...] In some countries, law makes it difficult for such creditors to repossess collateral, in part because such repossession leads to the liquidation of firms, which is viewed as socially undesirable.” The goal of this paper is to offer a theoretical investigation of the causes and consequences of such violations of contractual rights.

Bankruptcy laws vary quite significantly across countries. The US Constitution gave Congress large powers to create bankruptcy laws interfering with the application of contracts (Bergl¨of and Rosenthal 2000). The current US law, in particular the Chapter 11 procedure, allows to maintain distressed firms in operation. Whenever creditors disagree with the reorganization plan, the judge can decide to use the “cram down” procedure to implement the plan in spite of their opposition. The French bankruptcy law goes even further in this direction, as bankruptcy judges can unilaterally write-off the creditors’ rights (Biais and Malécot 1996). According to La Porta et al. (1998), “The French-civil-law countries offer creditors the weakest protections.” Russian courts also have significant discretion in bankruptcy procedures (Lambert-Mogiliansky, Sonin and Zhuravskaya 2007). These laws contrast with those prevailing in the UK or Germany. Franks and Sussman (2005a) show that the English bankruptcy procedure was mainly developed by lenders and borrowers, exercising their right to contract freely. State intervention in this process was relatively limited, and largely confined to enforcing the contracts signed by private parties. Similarly, under the German law, companies that default on their debt repayment obligations are usually liquidated, and the proceeds distributed to debtholders (Davydenko and Franks 2008). As stated by La Porta et al. (1998), “German-civil-law countries are very responsive to secured creditors.”

While debtor-oriented (soft) bankruptcy laws can avoid inefficient liquidations ex-post, they have adverse effects ex-ante. Anticipating the violation of creditors’ rights, banks are reluctant to grant loans. This amplifies credit rationing. Consistent with this, La Porta et al. (1997, 1998) and Giannetti (2003) find that access to debt financing is lower in countries with

---

3See Franks and Sussman (2005b) for an empirical study of the bankruptcy process in the UK.
soft bankruptcy codes. As we argue in this paper, soft laws also indirectly affect the labor market. By restricting access to credit, they reduce investment. This lowers the demand for labor, and thus the opportunities of wage earners. Thus both entrepreneurs, interested in access to credit, and workers, interested in job creation and high wages, should reject bankruptcy laws that restrict the freedom of contracting. This suggests that the optimal bankruptcy law should simply enforce contracts, and avoid interfering with their application. This paper provides some foundations, as well as some challenges, to these conjectures.

In Section 2, we present a simple general equilibrium model that allows us to analyze the interactions between the credit market and the labor market. There is a population of risk-neutral agents, who differ only in terms of their initial wealth. These agents choose whether to become employees or entrepreneurs. The latter invest in a business project and hire the former in their firms. Workers incur some disutility to supply labor and are compensated by wages. Entrepreneurs must exert costly efforts to make the investment project profitable and are compensated by profits (net of wages and reimbursements) and non-transferable private benefits. In the benchmark case of perfect markets, agents are indifferent between becoming workers or entrepreneurs. The aggregate level of investment is independent of the distribution of wealth across agents and only reflects the disutility of labor and the profitability of investment.

In Section 3, we turn to the case of imperfect financial markets.\footnote{For the sake of simplicity, we maintain the assumption that the labor market is frictionless. While unrealistic, this assumption enables us to focus on one aspect of the problem. We leave the important issues raised by labor market imperfections to further research.} Entrepreneurial effort is unobservable, as in Holmström and Tirole (1997), which raises a moral hazard problem. After the realization of the cash-flow, a firm can be liquidated or maintained in operation, as in Bolton and Scharfstein (1990). We consider the case where ex-post efficiency goes against liquidation, as private benefits from continuation exceed liquidation proceeds. Nevertheless, an ex-ante optimal financial contract can involve the liquidation of the firm when the cash-flow from the project is low. Indeed, the threat of liquidation enhances the entrepreneur’s incentives to exert effort, and thus reduces agency rents. Furthermore, since liquidation proceeds are allocated to investors, liquidation increases their willingness to fund the project. Hence, the income that entrepreneurs can pledge to outside investors is increasing in the liquidation rate in case of failure. It is also decreasing in the wages paid to the workers. In equilibrium, agents with low initial wealth cannot obtain a loan, as their need for outside funds exceeds their pledgeable income. They thus have no other choice than to become workers. By contrast, very wealthy agents need little outside financing and can therefore
raise funds without committing to liquidation in case of failure. This corresponds to equity financing. Agents with intermediate levels of wealth need greater outside financing, and thus must promise greater repayments to outside investors. To raise their pledgeable income, they must commit to higher liquidation rates in case of failure, and thus issue risky debt.\footnote{In our analysis financial contracts are optimal. Agents who issue risky debt, and thus face the risk of inefficient liquidation, would not have been able to rely on equity financing.} Yet, while borrowers and lenders would like to choose financial arrangements which are, from their point of view, ex-ante Pareto optimal, a soft bankruptcy law can preclude such contracts if they lead to high liquidation rates. This in turn prevents relatively poor entrepreneurs from accessing the credit market.

In Section 4, we analyze the preferences of different agents towards bankruptcy laws, and the political process through which these laws can emerge. Agents with intermediate wealth favor laws that are tough enough to enable them to access credit. By contrast, rich agents who can finance their investment projects irrespective of the law are in favor of restricting the freedom of contracting. Indeed, weak creditor rights increase exclusion from the credit market. This reduces the competition for labor, lowers wages, and thus raises the profits of the rich. This is in line with the finding of Rajan and Zingales (2003) that incumbents are opposed to efficient financial systems, which facilitate entry and thus lower their profits. Our analysis thus predicts that bankruptcy laws should tend to be soft in countries where the economic elite strongly influences the political process. As an illustration, the very soft 1841 US bankruptcy law was pushed by the Whigs, which represented the economic elite in 19th century America. When this law was repealed by the Congress, the New England Whigs, clearly the richest people in the country, still voted in favor of it (Bergløf and Rosenthal 2000). Our analysis also implies that, when the moral hazard problem is severe, middle-class voters should favor rather tough laws, which would help them access credit, and would in turn result in relatively high aggregate leverage.

Finally, in Section 5, we switch from a positive to a normative viewpoint. We first show that soft laws can generate greater utilitarian social welfare than tough laws. Indeed, when an agent opts for entrepreneurship, this raises wages and thus reduces the pledgeable income for the other entrepreneurs. This makes higher liquidation rates necessary, which lowers utilitarian welfare. By contrast, soft laws reduce the pressure on wages by excluding some agents from the credit market. This reduces inefficient liquidations for those who remain entrepreneurs, which can raise utilitarian welfare.

However, when it prevents many agents from accessing credit, a soft law can be Pareto improved upon by a tougher law combined with a redistribution scheme. A tougher law leads
to higher investment by reducing credit rationing, while subsidies to entrepreneurs mimic the
effect of soft laws by increasing pledgeable income and thus reducing inefficient liquidations.
Part of the wage increase induced by higher investment can be taxed away from workers to
fund the subsidies to entrepreneurs. The amount that can be raised in this way is limited,
since workers must be at least as well off as under the softer law. Yet, when credit rationing
is high, this redistribution scheme is budget balanced.

Our paper builds on the large literature on the design of bankruptcy procedures. There
are several differences between our approach and that literature. First, we emphasize the
difference between laws and contracts and study how agents take into account the bankruptcy
law when writing financial contracts. Second, we consider a general equilibrium setting,
where, because of the interactions between the credit market and the labor market, the
fact that some agents have access to credit affects the other agents. Third, we study the
political underpinnings of the bankruptcy law, and thus analyze how different laws can
emerge. Fourth, we compare the different laws in terms of aggregate social welfare.

Our focus on the interactions between financial decisions and politics or legislation in
general equilibrium is in line with the insightful paper by Bolton and Rosenthal (2002). A
key difference is that voting on moratoria occurs ex-post in their analysis, while in ours the
bankruptcy law is set up ex-ante. Furthermore, in their model soft laws complete contracts by
making their application contingent on macro-shocks, while in ours the soft law generates a
form of contractual incompleteness by precluding the enforcement of some financial contracts.
Our emphasis on the interactions between imperfect credit markets and the labor market is
in line with Pagano and Volpin (2005). Their analysis focuses on a different instrument to
discipline managers, namely takeovers. While we emphasize the classical conflict between
managers and workers over wages, they identify a situation where the interests of managers
and workers can be aligned. Finally, our general equilibrium analysis of credit rationing in a
context where some agents can seek to become entrepreneurs is in the spirit of Aghion and
Bolton (1997). In their model, however, the fraction of agents who become entrepreneurs
determines the cost of capital, while in ours it determines the wage rate. Besides, our focus
on the potential inefficiencies of liquidations and the violation of creditors’ rights induced
by soft bankruptcy laws is a distinctive feature of our analysis.

2 Model and First-Best Benchmark

\footnote{See for instance Bebchuck (1988), White (1989), Aghion, Hart and Moore (1992), Berkovitch, Israel and
Zender (1997), and Berkovitch and Israel (1999).}
2.1 The Environment

Our basic model is in line with Holmström and Tirole (1997). There is a continuum of mass one of risk-neutral agents, with limited liability. Each agent has an investment project, requiring initial investment $I$. While all investment projects are identical, agents differ in their initial wealth $A \in [0, I]$. We denote by $F$ the cumulative distribution function of wealth, which is assumed to be twice differentiable on $[0, I]$, with a density $f$ that is bounded away from zero over this interval. To undertake his investment project, and thus become an entrepreneur, an agent with initial wealth $A$ needs to raise outside funds $I - A$. The supply of funds is provided by international financial markets, and for simplicity we assume perfect capital mobility. Competitive risk-neutral outside investors are willing to lend if they break even on average, and we normalize their required rate of return to zero. Once undertaken, a project can succeed, delivering a revenue $R$, or fail, delivering no revenue. If an entrepreneur exerts effort, at a disutility cost $e$, the probability of success is $p_H$, while if he does not exert effort, the probability of success is lowered to $p_L = p_H - \Delta p$. Success or failure are independent across projects.

Our model departs in two crucial ways from Holmström and Tirole (1997). First, besides the investment $I$, each project also requires one unit of labor, which is purchased at price $w$ on a perfectly competitive labor market. The workers are agents that chose, or possibly had no other choice than to become entrepreneurs. Supplying $l$ units of labor entails a disutility $C(l)$ for a worker. We assume that the function $C$ is thrice differentiable, and that is satisfies the usual monotonicity and convexity conditions $C' > 0$ and $C'' > 0$, as well as the Inada conditions $C(0) = 0$, $C'(0) = 0$ and $\lim_{l \to \infty} C''(l) = \infty$. Second, after the cash-flow realization, the project can be continued or liquidated. In the latter case, observable and contractible liquidation proceeds $L$ are obtained. By contrast, if the project is continued, the entrepreneur obtains non-transferable private benefits $B$. A natural interpretation is that these benefits stem from the private use of the firm’s assets by the entrepreneur. Non-transferable private benefits from continuation would also arise in a dynamic extension of our model. In that context, they would reflect the present value of the agency rents to be obtained by the entrepreneur in the future. Our parameter $B$ can be interpreted as a reduced form representation of these future rents.

We assume that liquidation is ex-post inefficient, that is

---

7By convention, wages are paid ex-post by the entrepreneur whenever his project is successful, and not upfront by the investors. This does not affect our results given that all agents are risk-neutral.

8See Biais et al. (2007) for an example of a dynamic agency model with endogenous non-transferable managerial benefits from continuation.
\[ B > L. \] (1)

Condition (1) captures the idea that liquidation often fails to allocate the firm’s assets to the party valuing them the most. The entrepreneur who created the firm is often key to its profitability, because he has the necessary skills and information.\(^9\) Hence, the firm’s assets are typically worth less to outsiders than to insiders. Furthermore, these assets often have to be sold quickly in case of liquidation, which increases the risk that they do not end up in the hands of the most efficient outsider.

Next, we assume that each project generates a negative net value if the entrepreneur does not exert effort, even if the project is continued in any case:

\[ p_L R + B - I < 0, \] (2)

and that each project generates a positive net value if the entrepreneur exerts effort and the project is not liquidated except in case of failure:

\[ p_H(R + B) + (1 - p_H)L - e - I > 0. \] (3)

Finally, we also assume that

\[ \frac{e}{\Delta p} \geq B. \] (4)

According to (4), the magnitude of the moral hazard problem, as measured by \( \frac{e}{\Delta p} \), is large relative to the private benefit \( B \) from continuation. As we will see, this limits the income that can be pledged to outside investors.

Remark. We have also analyzed the case in which \( B \leq L \) for some firms. It is then efficient to transfer ownership of these firms’ assets to the investors. Yet, soft bankruptcy laws can reduce their ability to commit to this policy and thus impair access to credit. Our positive analysis, which relies on the fact that weak creditor rights worsen credit rationing, is unaffected by this alternative assumption. However, it modifies our normative analysis, which hinges on the efficiency gains resulting from less frequent liquidations. If the fraction of firms for which \( B \leq L \) is not too large, our qualitative results are upheld.

2.2 Efficiency and Equilibrium without Moral Hazard

As a benchmark, we characterize the efficient allocation of agents into entrepreneurs and workers when entrepreneurial effort is contractible, so that there is no moral hazard problem.

\(^9\)This is consistent with the empirical findings of Sraer and Thesmar (2007). In their sample of French listed firms over the 1994–2000 period, 31\% were managed by their founder/owner. Moreover, these firms outperformed the other firms in the sample.
It follows from (1)–(3) that for each project that is undertaken, it is efficient to exert high effort and never to liquidate. The first-best net value created by a project is then

\[ S^{FB} = p_H R + B - e - I. \]  

(5)

Without moral hazard, only the total mass of workers, not their identity, is relevant for efficiency. Moreover, because the disutility of labor is strictly convex, efficiency requires that all workers supply the same amount of labor. One then has the following result, whose proof is in the Appendix.

**Proposition 1** An efficient allocation is reached when there is a mass \( m^{FB} \) of workers, and each worker supplies \( l^{FB} \) units of labor, where \( m^{FB} \) and \( l^{FB} \) are related by

\[ m^{FB} l^{FB} = 1 - m^{FB}, \]  

(6)

\[ S^{FB} + C(l^{FB}) = \frac{C'(l^{FB})}{m^{FB}}. \]  

(7)

Absent moral hazard constraints, each efficient allocation can be decentralized in a competitive equilibrium.

Condition (6) requires that the aggregate labor supply be equal to the total mass of entrepreneurs, while condition (7) equalizes the marginal social cost and the marginal social benefit of an extra worker. Proposition 1 implies that the efficient proportion of workers, and thus the level of aggregate investment, do not depend on the distribution of wealth among agents.\(^\text{10}\) However, it will be helpful for future reference to consider the case in which the agents who become workers are those with wealth below some cutoff \( \hat{A} \), to be determined in equilibrium. Given wage \( w \), individual labor supply \( \ell^*(w) \) satisfies the first-order condition

\[ p_H w = C'(\ell^*(w)), \]  

(8)

while labor market clearing requires that

\[ F(\hat{A})\ell^*(w) = 1 - F(\hat{A}). \]  

(9)

Therefore individual labor supply is \( \frac{1}{F(\hat{A})} - 1 \), and the resulting wage is \( \frac{1}{p_H} C'\left(\frac{1}{F(\hat{A})} - 1\right) \). Focusing on the utility agents obtain through economic interactions on top of their initial wealth, we obtain that the utility of a worker, as a function of the wealth \( \hat{A} \) of the marginal agent, is given by

\(^{10}\)As shown in the next section, this property of first-best allocations no longer holds in the second-best environment.
\[ U_W(\hat{A}) = C'(\frac{1}{F(\hat{A})} - 1)\left[\frac{1}{F(\hat{A})} - 1\right] - C\left(\frac{1}{F(\hat{A})} - 1\right), \]  

while the utility of an entrepreneur, as a function of \( \hat{A} \), is given by

\[ U_E^{FB}(\hat{A}) = S^{FB} - C'(\frac{1}{F(\hat{A})} - 1). \]

It follows from the strict convexity of the disutility of labor \( C \) that \( U_W(\hat{A}) \) and \( U_E^{FB}(\hat{A}) \) are respectively decreasing and increasing in \( \hat{A} \). This reflects that the more workers there are, the lower is the wage rate. In the first-best, the wealth \( A^{FB} \) of the marginal agent is such that \( U_W(A^{FB}) = U_E^{FB}(A^{FB}) \). This competitive equilibrium is illustrated on Figure 1.

![Figure 1](image.png)

**Figure 1** A competitive equilibrium without moral hazard

The figure plots the utility of the workers and that of the entrepreneurs, as functions of the wealth of the marginal agent. The two curves intersect at the equilibrium point where agents are indifferent between the two occupational choices. Again, while the equilibrium threshold of wealth below which agents become workers depends on the distribution of wealth, the total mass of workers does not.

### 3 Moral Hazard and Soft Bankruptcy Laws

When entrepreneurial effort is not observable, agents cope with the resulting moral hazard problem by designing optimal financial contracts. These contracts must ensure that investors are ready to lend and entrepreneurs to exert effort. To this end, they rely on two instruments. First, a minimal amount of initial wealth may be required in order to grant funds, as in Holmström and Tirole (1997). Second, inefficient ex-post liquidation in case of failure may be used as an incentive to exert effort, as in Bolton and Scharfstein (1990). However, as
discussed in the Introduction, bankruptcy laws in many countries do not strictly enforce financial contracts. Instead of this, they frequently force continuation of activity in cases where the existing contract requested liquidation. In this section, we study the impact of such soft bankruptcy laws on optimal financial contracting, taking into account the general equilibrium interactions between the credit market and the labor market.

3.1 The Credit Market

For each agent with wealth \( A \), a financial contract first stipulates whether or not his project can be financed, that is, whether or not he can become an entrepreneur. In the former case, the contract specifies a transfer \( \tau \) to the entrepreneur whenever his project succeeds, and a liquidation probability \( \lambda \) whenever his project fails.\(^{11}\) Equivalently, \( \lambda \) can be interpreted as the fraction of the firms’ assets to be liquidated.\(^{12}\)

As we shall see below, the optimal financial contract specifies in some cases a positive liquidation rate in case of failure. Under a soft bankruptcy law, however, courts can interfere with the application of the contract and impose that the project be continued instead of being liquidated. To model this process in the simplest possible way, we assume that, in the states in which the contract prescribes that the project should be liquidated, the project is effectively liquidated with probability \( \pi \) only, while with probability \( 1 - \pi \) the court overrules the contract and imposes continuation. As a result, when the financial contract states a nominal liquidation rate \( \lambda \) in case of failure, the actual liquidation rate is \( \lambda a = \lambda \pi \).

To counter this effect, the investors can insist on a higher nominal liquidation rate. But since the latter cannot exceed one, a soft bankruptcy law constrains actual liquidation rates to be at most equal to \( \pi \). The parameter \( \pi \) can thus be interpreted as a measure of the toughness of the law: the closer \( \pi \) is to one, the tougher is the law. We will say that the law is tough whenever financial contracts are perfectly enforced, that is \( \pi = 1 \).

Given wage \( w \) and law toughness \( \pi \), a financial contract \((\tau, \lambda)\) is incentive compatible if the following holds:

\[
p_H(\tau + B - w) + (1 - p_H)(1 - \lambda \pi)B - e \geq p_L(\tau + B - w) + (1 - p_L)(1 - \lambda \pi)B.
\]

\(^{11}\)One can verify that it is never optimal to liquidate the project following a success, as doing so would result in a tighter incentive constraint for the entrepreneur. To simplify the exposition, we do not allow for contracts in which the financing decision itself is randomly taken. While such contracts could increase efficiency, we have checked that if the liquidation cost \( B - L \) is low enough, our main conclusions still hold even when such random contracts are enforceable.

\(^{12}\)In practice, when borrowing firms enter financial distress, their files are managed by a special department of the lending bank, that has its own staff and procedures. Franks and Sussman (2005b) offer an empirical analysis of the workings of such recovery units in several UK banks. Committing to a given liquidation rate can be achieved by an appropriate specification of the objectives and procedures of the recovery unit.
The left-hand side of this inequality is the expected utility the entrepreneur derives from the project if he exerts effort, while the right-hand side is his expected utility without effort. This incentive compatibility constraint requires that the payoff $\tau$ to the entrepreneur in case of success be at least
\[ \tau = \frac{e}{\Delta p} - \lambda \pi B + w. \]

Given a nominal liquidation rate $\lambda$, the highest income in case of success that can be pledged to outside investors without jeopardizing the entrepreneur’s incentives is thus
\[ R - \tau = R - \frac{e}{\Delta p} + \lambda \pi B - w. \]

Hence, the total expected amount that can be pledged to outside investors is
\[ p_H (R - \tau) + (1 - p_H) \lambda \pi L = p_H \left( R - \frac{e}{\Delta p} \right) + \lambda \pi [p_H B + (1 - p_H) L] - p_H w. \]

This expected pledgeable income is decreasing in $\frac{e}{\Delta p}$, which measures the severity of the moral hazard problem. It is increasing in $\lambda$ and $\pi$ because an increase in the liquidation rate or in the toughness of the law raises the investors’ revenue in case of failure, and strengthens the incentives of the entrepreneur to exert effort in order to avoid liquidation.

To ensure that some income can be pledged without liquidating the firm, we assume that
\[ R > \frac{e}{\Delta p}. \]

We also assume that the maximum ex-wages expected pledgeable income is less than the investment expenditures, even if the law is tough, so that some initial wealth $A > 0$ is required for investing:
\[ p_H \left( R - \frac{e}{\Delta p} \right) + p_H B + (1 - p_H) L < I. \]

For investors to break even on average, the expected pledgeable income must exceed the investors’ commitment:
\[ p_H \left( R - \frac{e}{\Delta p} \right) + \lambda \pi [p_H B + (1 - p_H) L] - p_H w \geq I - A. \] (12)

It follows that, given the wage rate $w$, an agent can obtain a loan with nominal liquidation rate $\lambda$ if and only if his initial wealth $A$ is above the threshold level
\[ A(w, \lambda, \pi) = I - p_H \left( R - \frac{e}{\Delta p} \right) - \lambda \pi [p_H B + (1 - p_H) L] + p_H w. \] (13)

The fact that a minimum amount of wealth is required to obtain funding is in line with Holmström and Tirole (1997). This required amount of wealth decreases with the promised
liquidation rate, which reflects that more frequent liquidations raise the pledgeable income. However, since liquidation is ex-post inefficient, it is optimal to keep the liquidation rate as low as possible. Hence, when the optimal contract requires that part of the firm’s assets be liquidated in case of failure, the optimal actual liquidation rate is such that the investors’ participation constraint (12) is binding:

$$\lambda^a(A, w) = \frac{I - A - p_H\left(R - \frac{e}{\Delta p}\right) + p_H w}{p_H B + (1 - p_H) L}.$$  \hspace{1cm} (14)

The higher the initial wealth of an entrepreneur, the lower the optimal liquidation rate.\textsuperscript{13}

For a fixed wage $w$, the actual liquidation rate is the same under a soft law and under the tough law. Thus, holding wages constant, the only impact of a soft bankruptcy law is to increase the minimum amount of wealth required to obtain a loan, $\mathcal{A}(w, 1, \pi)$.

Agents with wealth below $\mathcal{A}(w, 1, \pi)$ have very limited initial wealth. In order to invest, they would need to raise large amounts of outside funds. But even if they set the actual liquidation rate to its maximal value $\lambda^a = \pi$, their pledgeable income would remain below their outside funding need. Hence they cannot become entrepreneurs. By contrast, agents with wealth above $\mathcal{A}(w, 0, \pi)$ are very rich. They only need small amounts of outside funds, which they can raise while setting an actual liquidation rate $\lambda^a = 0$. In this case, whereas outside investors are granted a share of the revenue in case of success, they cannot impose liquidation in case of failure. This corresponds to equity financing by minority shareholders. Finally, agents with initial wealth between $\mathcal{A}(w, 0, \pi)$ and $\mathcal{A}(w, 1, \pi)$ can raise funds, but must promise an actual liquidation rate between 0 and $\pi$, as given by (14). This corresponds to a debt contract. These financing regimes are summarized in the following proposition.

\textbf{Proposition 2} Given wage $w$ and law toughness $\pi$, only agents with wealth $A \geq \mathcal{A}(w, 1, \pi)$ can raise outside funds. Agents with wealth $A \geq \mathcal{A}(w, 0, \pi)$ have access to equity financing, while agents with wealth $\mathcal{A}(w, 1, \pi) \leq A < \mathcal{A}(w, 0, \pi)$ only have access to debt financing.

\subsection{3.2 Competitive Equilibrium}

Given wage $w$ and law toughness $\pi$, the utility of an agent with wealth $A \geq \mathcal{A}(w, 1, \pi)$ who decides to become an entrepreneur is given by

$$S^{FB} - p_H w - \lambda^a(A, w)(1 - p_H)(B - L),$$  \hspace{1cm} (15)

\textsuperscript{13}This is reminiscent of the result by Bernanke and Gertler (1989) that the greater the net worth of a borrower, the lower the agency cost implied by the optimal loan contract.
where $S^{FB}$ is the first-best net value creation defined in (5). Since $B > L$, liquidations are inefficient and the utility of the entrepreneur is decreasing in the liquidation rate. Thus, since $\lambda^a(A, w)$ is a decreasing function of $A$, the utility of the entrepreneur is an increasing function of his wealth, in contrast with the first-best. Moreover, only agents with wealth higher than $A(w, 1, \pi)$ can be financed. Thus, in equilibrium, those who choose or are forced to become workers are the poorest agents.

To solve for a competitive equilibrium, one needs to determine the minimum amount of wealth required to become an entrepreneur. To do so, one has to take into account the following circular causation chain. First, credit rationing constraints determine who can become an entrepreneur and therefore the mass of potential entrepreneurs. The latter in turn affects supply and demand on the labor market and hence wages. Finally, wages reduce the pledgeable income and thus influence credit rationing. The following lemma is proven in the Appendix.

**Lemma 1** Define $A(\pi) \in (0, I)$ as the solution to

$$p_H \left( R - \frac{e}{\Delta p} \right) + \pi[p_H B + (1 - p_H)L] - C'' \left( \frac{1}{F(A(\pi))} - 1 \right) = I - A(\pi).$$

(16)

Then an agent can obtain funding in equilibrium only if his initial wealth is at least $A(\pi)$.

$A(\pi)$ is the level of wealth for the marginal agent at which the maximum expected pledgeable income, corresponding to an actual liquidation rate $\pi$, is just equal to the required funding. It is easy to check from (16) that $A(\pi)$ is decreasing in $\pi$: the tougher the law, the lower the minimum amount of wealth required to become an entrepreneur.

To complete our analysis, we still have to determine whether the wealth $\hat{A}$ of the marginal agent is equal to or greater than $A(\pi)$ in equilibrium. To answer this question, we need to compare his utility with that of a worker. To compute the former, observe that, since the equilibrium wage is $\frac{1}{p_H} C' \left( \frac{1}{F(A)} - 1 \right)$ by (8)–(9), the liquidation rate that the marginal agent needs to promise to obtain funding is

$$\Lambda^a(\hat{A}) = \lambda^a \left( \hat{A}, \frac{1}{p_H} C' \left( \frac{1}{F(A)} - 1 \right) \right),$$

(17)

where the function $\lambda^a$ is defined as in (14), with $\lambda^a(A, w) = 0$ if $A \geq A(w, 0, \pi)$.\footnote{It should be noted that, by construction, $\Lambda^a(A(\pi)) = \pi$.} As a result, the utility of the marginal agent is

$$U_{E}^{SB}(\hat{A}) = U_{E}^{FB}(\hat{A}) - \Lambda^a(\hat{A})(1 - p_H)(B - L).$$

(18)
It follows from (14) and (17) that $\Lambda^a(\hat{A})$ is decreasing in $\hat{A}$, and thus from (11) and (18) that $U_E^B(\hat{A})$ is increasing in $\hat{A}$.

Relying on the above analysis, we can now characterize equilibrium credit and wages. In the second-best environment, it may still be the case that, as in the first-best benchmark, the marginal agent is indifferent between becoming a worker or an entrepreneur. Yet, if the moral hazard problem is severe or the bankruptcy law very soft, this indifference condition will not be satisfied. In that case, there is credit rationing, in the sense that agents with wealth slightly below that of the marginal agent would strictly prefer to become entrepreneurs, but are constrained to become workers. This is stated in the following proposition, where a worker’s utility is defined as in (10).

**Proposition 3** There exists a unique competitive equilibrium, with the following properties:

(i) If $U_E^B(\bar{A}(\pi)) \leq U_W(\bar{A}(\pi))$, there is no credit rationing in equilibrium. The agents who become workers are those with initial wealth below $A^{SB}$, where $U_E^B(A^{SB}) = U_W(A^{SB})$.

(ii) If $U_E^B(\bar{A}(\pi)) > U_W(\bar{A}(\pi))$, there is credit rationing in equilibrium, and agents with initial wealth below $\bar{A}(\pi)$ must become workers, while those with greater initial wealth prefer to become entrepreneurs.

In case (i), the marginal agent has more wealth than the minimum required to access credit, that is, $A^{SB} \geq \bar{A}(\pi)$. Hence there is no credit rationing. This occurs when the maximum liquidation rate that would arise in equilibrium under the tough law is lower than $\pi$. In this case, the constraint imposed by the soft law does not bind, and equilibrium and welfare are the same than under the tough law. Since $\bar{A}(\pi)$ is decreasing in $\pi$, this scenario is more likely to happen when the law is relatively tough.

In case (ii), some agents with wealth below $\bar{A}(\pi)$ would prefer to become entrepreneurs, but they are not rich enough to obtain funding. The marginal agent has initial wealth $\bar{A}(\pi)$. His utility is strictly higher than if he were a worker, and workers with slightly lower initial wealth are strictly worse off than him. Without credit market imperfections, this would be corrected by a decrease in the number of workers, an increase in the number of entrepreneurs, and an increase in wages. However, under moral hazard, agents with initial wealth below $\bar{A}(\pi)$ cannot become entrepreneurs, and hence wages cannot adjust so as to make the marginal agent indifferent between becoming a worker or an entrepreneur. Such credit rationing occurs when the maximum liquidation rate that would prevail under the tough law is higher than $\pi$. Agents with wealth below $\bar{A}(\pi)$ would have to set the actual
liquidation rate above \( \pi \) to become entrepreneurs, but this is precluded by the soft law. The softer is the law, the more likely is credit rationing, and the lower is the equilibrium wage

\[
w(\pi) = \frac{1}{p_H} C' \left( \frac{1}{F(A(\pi))} - 1 \right).
\]

(19)

A competitive equilibrium with credit rationing is illustrated on Figure 2.

![Figure 2](image)

**Figure 2** A competitive equilibrium with moral hazard and credit rationing

The figure plots the utility of the workers and that of the entrepreneurs, as functions of the wealth of the marginal agent. The utility of entrepreneurs is defined only above \( A(\pi) \), the minimum amount of wealth required to become an entrepreneur.

The following corollary states an immediate consequence of Proposition 3.

**Corollary 1** Compared to the first-best benchmark, moral hazard reduces the fraction of agents who become entrepreneurs and thus depresses investment and wages. Whenever \( U_E^{SB}(A(\pi)) > U_W(A(\pi)) \), this effect is stronger under a soft law than under the tough law.

The corollary reflects that moral hazard raises the minimum amount of wealth required to become an entrepreneur. Two effects are at work here. First, ex-post inefficient liquidations make entrepreneurship less attractive for the agents relative to the first-best, as can be seen from (18). Second, credit rationing may exclude further agents with relatively low initial wealth from the credit market. The latter effect is magnified under a soft law.

### 4 The Political Economy of Soft Bankruptcy Laws

In this section, we examine the political determinants of bankruptcy laws. To this end, we first characterize the preferences of agents over the toughness of the law, as a function of their initial wealth. Next, we take a median voter approach to illustrate how changes in
the fundamentals of the economy affect the bankruptcy law and the structure of financial contracts in equilibrium. Finally, we investigate the impact of a shift in political power towards the richest agents.

4.1 Political Preferences

The tough bankruptcy law states that contracts will be strictly enforced. We shall focus on the case in which no credit rationing occurs under such a law, as in case (i) of Proposition 3. Thus, under the tough law, the marginal agent with wealth $A_{SB}$ is indifferent between becoming a worker and becoming an entrepreneur. We suppose that his firm is liquidated at a positive rate $\lambda_{SB}$ in case of failure, so that debt and equity coexist in equilibrium. Any law $\pi \geq \lambda_{SB}$ is outcome equivalent to the tough law. Hence there is no loss of generality in restricting the policy space to the interval $[0, \lambda_{SB}]$. Soft bankruptcy laws set a maximum liquidation rate precluding the strict enforcement of financial contracts. Thus, under a soft law $\pi < \lambda_{SB}$, and there is credit rationing. Agents with wealth below $A(\pi) > A_{SB}$ are then constrained to become workers, and earn a wage $w(\pi)$ as given by (19).

**Poor Agents** Consider first the case of an agent with wealth $A < A_{SB}$. Irrespective of the law, this agent has no other choice than to become a worker. His utility $U_W(A(\pi))$ is increasing in $\pi$. This reflects that workers benefit from tough laws, which facilitate firm creation and investment, and result in higher labor demand and higher wages. Thus these agents vote for the toughest possible law.

**Rich Agents** Consider next the case of an agent with wealth $A \geq A(0)$. Irrespective of the law, this agent becomes an entrepreneur and is never liquidated in equilibrium. His utility $S_{FB} - p_H w(\pi)$ is decreasing in $\pi$. This reflects that agents who are never credit constrained benefit from soft laws, which hinder firm creation and investment, and result in lower labor demand and lower wages. Thus these agents want the law to be as soft as possible.

**Intermediate Agents** Consider finally the case of an agent with wealth $A_{SB} \leq A < A(0)$. There exists a law $\pi_A \in (0, \lambda_{SB})$ such that this agent is just rich enough to have access to credit, that is, $A = A(\pi_A)$. If a softer law is enacted, this agent is forced to become a worker, with a utility $U_W(A(\pi))$ that is increasing in $\pi$ over $[0, \pi_A]$. By contrast, if a tougher law is enacted, he becomes an entrepreneur, thereby obtaining utility

$$S_{FB} - p_H w(\pi) - \lambda^a(A, w(\pi))(1 - p_H)(B - L),$$

Formally, we impose that $U_{EB}^B(A(1)) \leq U_W(A(1))$ and $A_{SB} < A(0)$. One then has $\lambda_{SB} = \Lambda^a(A_{SB}) > 0$, where the function $\Lambda^a$ is as defined in (17).
which is decreasing in $\pi$ over $[\pi^A, \lambda^B)$. Thus, conditional on becoming an entrepreneur, he prefers that as few as possible other agents be entrepreneurs, in order to benefit from lower wages. His utility is maximum whenever $\pi = \pi_A$, so that he is in effect the marginal agent.

These results are in line with Rajan and Zingales (2003), who argue that incumbent firms that do not rely much on external capital markets to finance their projects extract a rent from an underdeveloped financial system that does not strictly enforce financial contracts. In our analysis, a tough bankruptcy law enhances the competition for labor and thus increases wages, leading to lower profits for those agents who are rich enough to enjoy a privileged access to finance. This implies that these agents should oppose tough bankruptcy laws which would allow newcomers to enter the credit market.

**Remark** In order to abstract from any externality on third parties that would not be directly linked to financial contracting, we focused in our analysis on the case where the liquidation costs are entirely borne by entrepreneurs. A natural question is whether taking into account similar costs for workers would affect their political preferences. It turns out that this depends to which extent wages that are set on the labor market reflect these costs. When this is the case, firms with different initial funds, and therefore different liquidation rates ex-post, will typically pay different wages in equilibrium, so as to make the workers indifferent between working in any firm. To see this point, suppose that for each unit of labor that a worker supplies to a given firm, he incurs a disutility $k$ in case this firm is liquidated after a failure. For workers to be indifferent among firms, it must be that the wages $w_A$ paid by firms with different levels of initial funds $A$ are such that the expected wage net of the expected liquidation cost for the workers,

$$p_H w_A - (1 - p_H)k\lambda^a(A, w_A),$$

is a constant independent of $A$. This constant must be positive, which requires $k$ not to be too high. It is straightforward to check from (14) that this makes $w_A$ a decreasing function of $A$. Indeed, firms with low initial funds must commit to higher liquidation rates to raise outside funds, and must therefore compensate workers by setting higher wages. This mechanism naturally amplifies credit rationing, by raising the minimal amount of wealth required to become an entrepreneur. Because workers are perfectly compensated for liquidation costs, their political preferences are unchanged relative to when $k = 0$: they still prefer a tough bankruptcy law that enhances investment and firm creation, and results in higher labor demand and higher wages.

Consider by contrast the case in which workers cannot be fully compensated for these
liquidation costs. As suggested by Pagano and Volpin (2001), this may occur because workers must invest in firm-specific human capital. For instance, suppose that there is a fixed liquidation cost $K$ for workers, and that wages are set in the labor market under a veil of ignorance, that is, before workers know the specific liquidation rates of the firms for which they will eventually work. Then, given a law $\pi$ and the corresponding equilibrium wage $w(\pi)$, a typical worker chooses his labor supply $l$ so as to maximize

$$p_H w(\pi) l - C(l) - (1 - p_H)K \frac{\int_{A(\pi)}^{A(w(\pi),0,\pi)} \lambda^a(A, w(\pi)) dF(A)}{1 - F(A(\pi))}.$$  

The last term in this objective function is the expected liquidation cost for the worker. Since it is independent of $l$, it does not affect individual labor supply. Whenever $K > 0$, the attractiveness of becoming a worker is reduced. Hence, if there is no rationing in equilibrium, less agents become workers, and wages are higher than when $K = 0$. Because of higher wages, the liquidation rates must increase, and the proportion of entrepreneurs who finance their projects with equity decreases. If $K$ is high, workers will typically favor an intermediate law that trades off the liquidation costs and the positive wage impact of firm creation. If $K$ is low, they will still favor a tough bankruptcy law, and our analysis is unaffected.

4.2 Voting on the Bankruptcy Law

The upshot of the previous subsection is that, under our assumptions, all agents have single-peaked preferences with respect to the toughness of the law. This implies that the median voter theorem applies, and thus the policy favored by the median agent cannot be defeated under majority voting by any other alternative.

Denote by $A^M = F^{-1}\left(\frac{1}{2}\right)$ the initial wealth of the median agent, and denote by $\pi^M$ the bankruptcy law favored by this agent. We assume throughout that $A^{SB} < A^M < A(0)$. Thus the median agent is richer than the agent who would be the marginal agent under the tough law, but he still cannot be fully equity financed in equilibrium. The law $\pi^M$ grants the median agent access to credit, but denies it to those with wealth below $A^M$. Hence, in line with equation (16), we can write that

$$p_H \left(R - \frac{e}{\Delta p}\right) + \pi^M[p_H B + (1 - p_H)L] - C'(1) = I - F^{-1}\left(\frac{1}{2}\right).$$

Since $A^M > A^{SB}$, some credit rationing takes place in equilibrium when voting gives rise to the law favored by the median agent. Several implications can be drawn from (20).

- An increase in the magnitude of the moral hazard problem, as measured by $\frac{e}{\Delta p}$, reduces the pledgeable income of the marginal agent. In response to this, the median agent
wants to raise his liquidation rate so as to maintain his access to funding. To make this possible, the median agent favors an increase in the toughness of the law.

- By contrast, an increase in the profitability of the project, as measured by $R$ or $p_H$, or an increase in the private benefit $B$ or in the liquidation proceeds $L$, raises the pledgeable income and thus facilitates the access to funding. In response to this, the median agent favors a decrease in the toughness of the law.

- Changes in the magnitude of the moral hazard problem or in the profitability of the project leave the identity of the median voter unaffected. By contrast, a shift in the wealth distribution modifies it. Suppose that distribution $F_2$ dominates distribution $F_1$ in the monotone likelihood ratio order. 16 Aggregate wealth in the economy is larger under $F_2$ than under $F_1$, and the median agent is richer. Since richer agents have lower outside financing needs and thus can afford lower liquidation rates, the median agent votes for a softer law under $F_2$ than under $F_1$.

Equation (20) also yields implications for the structure of financial contracts in the political equilibrium. Given a law $\pi$ and the corresponding equilibrium wage $w(\pi)$, the aggregate leverage ratio in the economy is given by

$$\frac{\int_{A(w(\pi),0,\pi)}^{A(\pi)} (I - A) dF(A)}{\int_{A(w(\pi),0,\pi)}^{A(\pi)} (I - A) dF(A)}$$

(21)

that is, the ratio of the total value of debt to the total value of outside equity. One then has the following result, whose proof is in the Appendix.

**Proposition 4** When agents vote over the bankruptcy law, an increase in the magnitude of the moral hazard problem leads to a tougher law and a higher aggregate leverage, while an increase in the profitability of the project or a positive shift of the wealth distribution in the monotone likelihood ratio order leads to a softer law and a lower aggregate leverage.

### 4.3 Shifts in Political Power

While the previous analysis focuses on the preferences of the median voter, it is unclear that majority voting adequately reflects the procedure by which bankruptcy laws are chosen in practice. As pointed out for instance by Bénabou (2000), relatively poor citizens have less

---

16 That is, the densities $f_1$ and $f_2$ are such that the likelihood ratio $\frac{f_2(A)}{f_1(A)}$ is increasing in $A$. 

---

18
influence on the political process than relatively rich citizens. Indeed, it takes resources to influence political outcomes, be it by lobbying or activism. In our setup, investing such resources in the political process is particularly costly for small entrepreneurs whose marginal valuation for cash is high, since initial wealth is needed to reduce the frequency of inefficient liquidations. Therefore it is likely that richer agents will invest more in the political process than poorer ones.

To model this link between wealth and political influence, we follow Bénabou (2000). Given an increasing weight function $\gamma$, let the proportion of votes cast by agents with wealth less than $\hat{A}$ be given by

$$G(\hat{A}) = \int_0^{\hat{A}} \gamma(A) dF(A),$$

where $\gamma$ is normalized in such a way that $G(I) = 1$. Thus $G$ dominates $F$ in the monotone likelihood ratio order. Under $G$ the political influence of richer agents is greater than under the original distribution $F$. Denote by $A^G = G^{-1}(1/2)$ the wealth of the median agent under $G$. This agent is richer than the median agent corresponding to $F$. Since the preferred policy is monotonic in wealth, with wealthier agents preferring a lower level of $\pi$, a shift in political power towards the richest agents leads to a softer bankruptcy law. This aggravates credit rationing, which in turn reduces the level of aggregate investment and hence wages. This decrease in the wage bill benefits the richest agents, who find it easier to finance their projects with equity, with no need to commit to costly liquidation in case of failure. As a result, the aggregate leverage ratio tends to decrease.

5 What Is the Optimal Bankruptcy Law?

5.1 Soft Laws can Generate Higher Utilitarian Welfare than the Tough Law

A soft law typically induces more credit rationing than the tough law in which contracts are perfectly enforced. This does not mean, however, that the tough law generates greater utilitarian welfare than any soft law. Indeed, soft laws reduce wages, and thus relax the pressure on entrepreneurs with intermediate levels of wealth by reducing their equilibrium rates of liquidation in case of failure. This in turn limits the efficiency losses from liquidation.

To see this, suppose as above that there is no rationing under the tough law. Under that

---

\footnote{Drawing from the empirical findings of Rosenstone and Hansen (1993), Bénabou (2000) notes that the poorest 16% account for only 12.2% of the votes and 4% of the number of campaign contributors. By contrast, the richest 5% account for 6.4% of the votes and 16.3% of the contributors. For campaign contributions the figures understate the bias, since the data reflects only the number of contributions and not their amounts.}
law, the marginal agent with wealth $A^{SB}$ is indifferent between being a worker and being an entrepreneur, that is $U^E_W(A^{SB}) = U_W(A^{SB})$. Also, we suppose that his firm is liquidated at a positive rate $\lambda^{SB}$ in case of failure.

How does this situation compare to that arising under a soft law, specifying a maximum liquidation rate $\pi < \lambda^{SB}$? The utilitarian welfare under law $\pi$ is equal to

$$S(A(\pi)) = [1 - F(A(\pi))]S^{FB} - F(A(\pi)) C\left(\frac{1}{F(A(\pi))} - 1\right)$$

$$- \int_{\underline{A}(\pi)}^{\underline{A}(\pi)} \lambda^a(A, w(\pi))(1 - p_H)(B - L) dF(A).$$

The first two terms on the right-hand side of (22) represent the first-best value creation when the marginal agent has wealth $\underline{A}(\pi)$. The third term represents the average cost of liquidation for entrepreneurs with wealth between $\underline{A}(\pi)$ and $\underline{A}(\pi)$ who finance their projects by issuing debt. Using the definitions of the functions $U_W$, $U^E_W$ and $\underline{A}$ along with the fact that $\lambda^a(\underline{A}(\pi), 0, \pi) = 0$, one obtains that

$$S'(A(\pi)) = -f(A(\pi))[U^E_W(A(\pi)) - U_W(A(\pi))]$$

$$+ [F(A(w(\pi), 0, \pi)) - F(A(\pi)))] \frac{(1 - p_H)(B - L)}{p_H B + (1 - p_H)L} C''\left(\frac{1}{F(A(\pi))} - 1\right) \frac{f(A(\pi))}{F^2(A(\pi))}.$$ (23)

Recalling that a reduction of $\pi$ leads to an increase in $\underline{A}(\pi)$, this expression has a natural interpretation. The first term on the right-hand side of (23) represents the welfare loss incurred because of the soft law. This loss is equal to the difference between the utility of the marginal agent with wealth $\underline{A}(\pi)$ and that of a worker, which is positive as $\pi < \lambda^{SB}$, weighted by the density of wealth at $\underline{A}(\pi)$. Because the disutility of labor $C$ is convex with $C'' > 0$, the second term on the right-hand side of (23) is positive and represents the welfare gain generated by a soft law. This gain is proportional to $F(A(w(\pi), 0, \pi)) - F(A(\pi))$, the mass of entrepreneurs who finance their projects with debt and are thus liquidated at a positive rate in case of failure. For these entrepreneurs, a decrease in $\pi$, and thus an increase in $\underline{A}(\pi)$, has a positive impact on their utility since it lowers the wage and thus their liquidation rates. The corresponding effect on wages is

$$p_H \frac{dw(A^{-1}(A))}{dA} \bigg|_{A=\underline{A}(\pi)} = -C''\left(\frac{1}{F(A(\pi))} - 1\right) \frac{f(A(\pi))}{F^2(A(\pi))}. $$ (24)

The marginal agent thus exerts a pecuniary externality on all debt issuers by raising wages, which makes their moral hazard problem more severe and compels them to commit to higher
liquidation rates.\textsuperscript{18}

We are now ready to analyze the welfare impact of making the bankruptcy law soft. From the expression for $S'(A(\pi))$, it is clear that setting a maximum liquidation rate $\pi$ slightly below $\lambda^{SB}$ only entails a second-order welfare loss since the absence of credit rationing under the tough law implies that $U_{EB}^{SB}(A(\lambda^{SB})) = U_W(A(\lambda^{SB}))$. This reflects that, under the tough law, the contribution to social welfare of the marginal agent is negligible as long as there is no rationing. By contrast, there is a first-order welfare gain of slightly lowering $\pi$ from $\lambda^{SB}$, as this allows one to reduce the liquidation rates of all entrepreneurs who finance their project by issuing debt. It thus follows that $S'(A(\lambda^{SB})) > 0$. Symmetrically, it is easy to see that $S'(A(0)) < 0$, which reflects the fact that if liquidation is infeasible, the positive impact of a soft law on social welfare vanishes as debt financing is no longer an option.\textsuperscript{19} Hence the following proposition.

**Proposition 5** Whenever the tough law generates no credit rationing and debt and equity coexist in that situation, the bankruptcy law that maximizes utilitarian welfare is soft and calls for some credit rationing in equilibrium.

The two key ingredients for this result are the existence of a moral hazard problem in the credit market and the endogeneity of wages. It should also be noted that the positive impact of soft bankruptcy laws would be even more pronounced if workers also incurred ex-post costs in case of liquidation. By setting those costs to zero, we have put ourselves in the worst possible scenario for the optimality of a soft law from a utilitarian viewpoint.

**Remark** The above analysis hinges on the assumption that there is no credit rationing under the tough law. By continuity, the conclusion of Proposition 5 remains true if there is little rationing under the tough law, that is, if the difference $U_{EB}^{SB}(A(1)) - U_W(A(1))$ is small. If this is not the case, then the comparison between the positive and negative impacts of a soft law becomes ambiguous, because the social welfare loss associated to making the marginal agent a worker is no longer negligible. It can be shown that the welfare maximizing law is tough whenever the marginal disutility of labor is low enough. In that case, the pecuniary externality generated by the marginal agent on debt financed entrepreneurs is of small magnitude because the cost of labor is low, and perfect enforcement of financial contracts is therefore optimal.

\textsuperscript{18}This pecuniary externality bears some analogy with the incomplete markets literature (Stiglitz 1982, Geanakoplos and Polemarchakis 1986). In both cases, the idea is that agents do not internalize the impact of their decisions on prices and thus on other agents’ welfare. We depart from this literature by focusing on how legal restrictions on contracting can improve social welfare.

\textsuperscript{19}This last point remains true no matter the nature of equilibrium under the tough law.
5.2 Are Soft Laws Pareto Dominated by Tougher Laws Combined with Redistribution?

The cost of soft laws is that they lower investment. Their benefit is that, by lowering wages, they increase the pledgeable income and thus reduce liquidation rates. Is there a way to reap this benefit without incurring that cost? In this subsection, we study whether, starting from a soft law, it is possible to achieve a Pareto improvement by combining a tougher law that stimulates investment with a redistribution scheme that raises the pledgeable income of entrepreneurs.\(^{20}\)

Our starting point is a soft law with maximum liquidation rate \(\pi < \lambda^{SB}\). Under this law, agents with wealth \(A < A(\pi)\) are workers and earn wage \(w(\pi)\). Agents with wealth \(A(\pi) \leq A < A(w(\pi), 0, \pi)\) are debt financed entrepreneurs and are liquidated at a rate

\[
\lambda(A, w(\pi)) = \frac{I - A - p_H (R - \frac{e}{\Delta p}) + p_H w(\pi)}{p_H B + (1 - p_H)L}
\]  

between 0 and \(\pi\). Finally, agents with wealth \(A \geq A(\pi, w(\pi), 0, \pi)\) are fully equity financed entrepreneurs.

Now consider a tougher bankruptcy law, where the maximum liquidation rate is set to \(\pi' \in (\pi, \lambda^{SB})\). Under this law, the wealth of the marginal agent goes down to \(A(\pi')\) and investment rises. Higher investment induces an increase in wages to \(w(\pi')\). This benefits the workers, but hurts the entrepreneurs. To ensure that this new outcome leads to a Pareto improvement, one must subsidize the entrepreneurs to compensate them for greater labor costs. These subsidies are funded by taxing the workers. The initial law can be Pareto improved upon if this redistribution scheme is budget balanced. Such balance can be difficult to strike. Indeed, subsidies to entrepreneurs must be high enough to maintain their utility level as well as their pledgeable income, while taxes on workers must be limited for them to remain at least as well off as under the initial law. As shown below, these conflicting constraints may or may not be consistent, depending on parameter values.

Agents with wealth \(A \geq A(\pi)\), who are thus entrepreneurs under the initial law, remain so under the new law. The minimum subsidy they must receive is equal to the difference in expected wages,

\[
S_E = p_H [w(\pi') - w(\pi)].
\]  

Fully equity financed entrepreneurs are by construction as well off under this new scheme as under the initial law. For debt financed entrepreneurs, the increase in wages has two

---

\(^{20}\)We are grateful to one of the referees for pointing to this interesting issue.
consequences. The direct effect is a reduction in profits. The indirect effect is a reduction in their pledgeable income, which can lead to an increase in inefficient liquidations. Upon receiving the subsidy $S_E$, the optimal course of action for these entrepreneurs is to invest it in their own firm along with their initial wealth. This ensures that they obtain the same profits and the same liquidation rates as under the initial law, since $\lambda(A + S_E, w(\pi')) = \lambda(A, w(\pi))$ by (25)–(26).

Consider next the agents with wealth $A < A(\pi)$, who are thus workers under the initial law. Among them, those with wealth $A < A(\pi')$ still remain workers under the new law, as the increase in the maximum allowed liquidation rate is not large enough to grant them access to credit. Their utility under the initial law $\pi$ is $U_W(A(\pi))$, while their utility under the tougher law $\pi'$ is $U_W(A(\pi'))$. Since $A(\pi') < A(\pi)$, the latter is larger than the former. Indeed, the increase in labor income is larger than the increase in the disutility of labor, since expected wages are equal to the marginal disutility of labor and the disutility of labor is convex. Hence, the maximum tax that can be levied on these workers, while leaving them as well off as under the previous regime, is

$$T_W = U_W(A(\pi')) - U_W(A(\pi)).$$

The situation is different for agents with wealth $A(\pi') \leq A < A(\pi)$. These agents, who have no other choice than to become workers under the initial law, can now change status and become entrepreneurs under the new law, thanks to the higher allowed liquidation rate. Their utility under the new law is strictly higher than under the initial law, since they are no longer credit constrained. Thus, there is no need to subsidize them, unlike richer agents who can afford to be entrepreneurs under the initial law. The following lemma, whose proof is in the Appendix, shows that it would in fact be impossible to indiscriminately subsidize all entrepreneurs.

**Lemma 2** The policy combining a bankruptcy law $\pi' > \pi$ with subsidies $S_E$ to agents with wealth $A \geq A(\pi')$ and taxes $T_W$ from agents with wealth $A < A(\pi')$ is not budget feasible:

$$[1 - F(A(\pi'))]S_E > F(A(\pi'))T_W.$$  

In view of Lemma 2, a Pareto improvement is budget feasible only if the agents with wealth $A(\pi') \leq A < A(\pi)$ who form the class of new entrepreneurs do not benefit from the subsidy $S_E$. To ease the exposition, consider next the case where these agents are neither taxed nor subsidized. The proposed policy can then be summarized as follows:

- Increase the maximum liquidation rate from $\pi$ to $\pi'$;
• Give subsidy $S_E$ to all agents with initial wealth above $A(\pi)$;
• Raise tax $T_W$ from all agents with initial wealth below $A(\pi')$;
• Neither subsidize nor tax agents with initial wealth between $A(\pi')$ and $A(\pi)$.

This policy makes all agents at least as well off as under the initial law, and it strictly increases the utility of agents with wealth $A(\pi') \leq A < A(\pi)$.\(^{21}\) Hence, it leads to a feasible Pareto improvement if and only if the redistribution scheme is budget balanced:

$$F(A(\pi')) T_W \geq [1 - F(A(\pi))] S_E.$$  \hfill (29)

Unfortunately, for arbitrary $\pi$ and $\pi'$, it is difficult to determine whether (29) holds without relying on involved conditions on the shape of the wealth distribution. To avoid these technical problems, we perform a local analysis, focusing on a small increase $\Delta \pi$ in the toughness of the law. The following proposition is proven in the Appendix.

**Proposition 6** For $\Delta \pi > 0$ small, the policy combining a bankruptcy law $\pi' = \pi + \Delta \pi$ with subsidies $S_E$ to agents with wealth $A \geq A(\pi)$ and taxes $T_W$ from agents with wealth $A < A(\pi')$ Pareto dominates the bankruptcy law $\pi$ and is budget feasible if and only if

$$F(A(\pi)) > \frac{1}{2}.$$  \hfill (30)

An equivalent condition is that

$$S_E > T_W$$  \hfill (31)

for $\pi'$ close to but greater than $\pi$.

To understand this result, and in particular the somewhat puzzling condition (31), let us examine the impact of a small increase $\Delta \pi$ in the toughness of the law. By making access to credit easier, such a change reduces by $\Delta A$ the amount of wealth necessary to become an entrepreneur. This intensifies the competition for labor and thus leads to an increase $\Delta w$ in wages and to an increase $\Delta U_W$ in the workers’ utility. This in turn translates into subsidies $S_E^\Delta = p_H \Delta w$ to the entrepreneurs and taxes $T_W^\Delta = \Delta U_W$ from the workers. A first-order approximation of the budget balance is then

$$F(A(\pi)) \Delta U_W - [1 - F(A(\pi))] p_H \Delta w.$$  \hfill (32)

\(^{21}\)By contrast, it should be noted that the reverse policy that consists in making the bankruptcy law softer excludes a fringe of agents from the credit market, making them strictly worse off than under the initial law. Thus, even if workers are subsidized and entrepreneurs taxed for lower wages, this policy does not lead to a Pareto improvement.
To evaluate this, let us express $\Delta U_W$ as a function of $\Delta w$. Applying the envelope theorem to the workers’ labor supply decision yields, up to a first-order approximation,

$$\Delta U_W = p_H \ell^*(w(\pi)) \Delta w,$$

(33)

where $\ell^*(w(\pi))$ is the optimal individual labor supply given wage $w(\pi)$. Substituting this envelope condition into (32) along with the labor market clearing condition

$$F(A(\pi))\ell^*(w(\pi)) = 1 - F(A(\pi)),$$

(34)

we obtain that the first-order effect of the proposed policy on the budget balance is zero. To determine whether this policy is budget feasible, one thus needs to determine its second-order effect on the budget balance. This effect corresponds to the new entrepreneurs with wealth $A(\pi) - \Delta A \leq A < A(\pi)$ who are neither subsidized nor taxed under the proposed redistribution scheme. Compared to the too generous scheme considered in Lemma 2, the latter saves the cost of subsidizing these agents. Yet, increasing their mass reduces tax revenue. Cumulating these gains and losses over all the new entrepreneurs yields

$$\int_{A(\pi) - \Delta A}^{A(\pi)} (S_E - T_W) dF(A),$$

(35)

which, up to a second-order approximation, is equal to

$$\frac{1}{2} f(A(\pi)) \Delta A (S_E^A - T_W^A).$$

(36)

According to (36), the second-order effect of the proposed policy on the budget balance is positive if and only if the benefit $S_E^A$ of not subsidizing the marginal agent with wealth $A(\pi) - \Delta A$ under the new law exceeds the cost $T_W^A$ of not taxing him. For $\Delta \pi$ small enough, this is exactly what (31) asserts. Using the envelope condition (33) and the labor market clearing condition (34), it is easy to check that this is the case if and only if (30) holds. This corresponds to a situation in which investment is relatively limited and credit rationing relatively severe under the initial law. This scenario is likely to prevail if, as argued in Section 4.3, rich agents who favor soft bankruptcy laws have strong political influence so that the initial law is softer than the preferred law of the median agent.

The policy examined above leaves new entrepreneurs unaffected by redistribution. Yet, these agents obtain greater utility than under the initial law. When condition (30) fails to

---

22It should be noted that the integrand $S_E - T_W$ in (35) is implicitly a function of $A$ that represents the net gain from neither subsidizing nor taxing a new entrepreneur with wealth $A$.

23This approximation reflects that the net gain from neither subsidizing nor taxing the new entrepreneurs is equal to $\frac{1}{2} (S_E^A - T_W^A)$ on average, and that the mass of new entrepreneurs is approximately equal to $f(A(\pi)) \Delta A$. An exact derivation of formula (36) is provided in the Appendix.
hold and relatively few agents are credit constrained under the initial law, it would thus seem natural to tax new entrepreneurs so as to relax the budget constraint. However, the extent to which this can be done is limited. Indeed, these agents are credit constrained under the initial law. They can therefore take advantage from the increase $\Delta \pi$ in the maximum allowed liquidation rate to become entrepreneurs only if the amount of resources that is taxed away from them is low enough. Specifically, for a new entrepreneur with wealth $A$, the tax can be at most $A - A(\pi) + \Delta A$, so that he becomes in effect the marginal agent with wealth $A(\pi) - \Delta A$ and is liquidated at a rate $\pi + \Delta \pi$ under the new law.\(^{24}\) The overall tax revenue generated in this way is

$$\int_{A(\pi) - \Delta A}^{A(\pi)} [A - A(\pi) + \Delta A] dF(A),$$

which, up to a second-order approximation, is equal to\(^{25}\)

$$\frac{1}{2} f(A(\pi)) \Delta A^2.$$

Combining (36) and (38) yields the total effect of this policy on the budget balance,

$$\frac{1}{2} f(A(\pi)) \Delta A(S_E - T_W + \Delta A).$$

Proceeding as for (36), one can check that this effect is positive if and only if

$$F(A(\pi)) > \frac{1}{2 + \frac{2^2 A(\pi)}{\int f(A(\pi))}},$$

a weaker condition than (30). It should be noted that this final redistribution scheme maximizes taxes and minimizes subsidies, under the constraint that the agents’ utilities and the pledgeable income are not decreased. Therefore, if condition (40) does not hold, it is impossible to Pareto improve upon the initial law by relying on a locally tougher law combined with a budget balanced redistribution scheme.

### 6 Conclusion

This paper studies the impact of bankruptcy laws on investment and welfare when the credit market is imperfect. Our analysis reveals a two-way link between the credit market and the

\(^{24}\)It should be noted that, for this agent to be as well off under the new regime as under the initial law, his total utility, including that which he derives from his initial wealth, must not decrease. That is, one must have $U_E^{SB}(A(\pi) - \Delta A) + A(\pi) - \Delta A \geq U_W(A(\pi)) + A$. This inequality is satisfied for all agents with wealth $A(\pi) - \Delta A \leq A < A(\pi)$ as long as $\Delta A$, or equivalently $\Delta \pi$, is small enough, because the presence of credit rationing under the initial law ensures that $U_E^{SB}(A(\pi)) > U_W(A(\pi))$.

\(^{25}\)As for (36), this approximation reflects that the tax on new entrepreneurs is equal to $\frac{1}{2} \Delta A$ on average, and that the mass of new entrepreneurs is approximately $f(A(\pi)) \Delta A$. An exact derivation of formulas (38) and (40) is provided in the Appendix.
labor market. On one hand, the credit market influences the labor market: moral hazard constraints on financial contracting depress investment, and thus labor demand and wages. On the other hand, the labor market influences the credit market: higher wages reduce the income that entrepreneurs can pledge to outside investors, which makes higher liquidation rates necessary, and thus increases the incidence of ex-post inefficient liquidations.

In this context, we show that soft bankruptcy laws can generate greater utilitarian welfare than tough laws. Indeed, soft laws exclude some of the relatively poor agents from the credit market, which lowers investment and thus wages. For the entrepreneurs who still have access to credit, this increases the pledgeable income, which in turn makes inefficient liquidations less frequent. Yet, when they generate severe credit rationing, soft laws can be Pareto improved upon by tougher laws combined with redistribution schemes. Tough laws raise investment and wages. In the optimal redistribution scheme, part of the wage increase is taxed away from workers and redistributed to entrepreneurs. This involves subsidies that are large enough to reduce liquidations, and taxes that are low enough to leave workers as well off as under the soft law.

For simplicity we have abstracted from worker-specific liquidation costs and labor market imperfections. These frictions are likely to be significant in practice and interrelated, due for instance to the lack of mobility of the work force or to firm-specific investments. It would be interesting, in future research, to take into account these costs and imperfections and to study how they affect the efficiency of different bankruptcy laws and the political preferences of different constituencies.

Appendix

Proof of Proposition 1. Let $\mu$ be the probability measure corresponding to the cumulative distribution function $F$. An efficient allocation is given by a measurable set $W \subset [0, I]$ of workers’ wealth levels and a measurable allocation of labor $\ell$ that maximize the social surplus

$$[1 - \mu(W)]S^{FB} - \int_W C(\ell(A)) \, d\mu(A)$$

subject to the aggregate resource constraint

$$\int_W \ell(A) \, d\mu(A) = 1 - \mu(W).$$

Let $(W, \ell)$ be an efficient allocation. Suppose that $\ell$ is not constant over $W$, and let $\hat{\ell}$ be the allocation of labor that requires from each agent with wealth in $W$ to supply $\hat{\ell} = \frac{1}{\mu(W)} - 1$
units of labor. Since $C$ is strictly convex, Jensen’s inequality implies that

$$- \int_W C(\ell(A)) \, d\mu(A) < -\mu(W) C \left( \frac{1}{\mu(W)} \int_W \ell(A) \, d\mu(A) \right) = -\mu(W) C(\hat{\ell}),$$

so that the allocation $(W, \hat{\ell})$ strictly dominates the allocation $(W, \ell)$, a contradiction. Hence, in an efficient allocation, all workers supply the same amount of labor $l = \frac{1}{m} - 1$, where $m$ is the total mass of workers. This yields (6). The optimal work force is obtained by solving

$$\max_{m \in [0,1]} \left\{ (1 - m) S^{FB} - m C \left( \frac{1}{m} - 1 \right) \right\}.$$  

Given (6), (7) is simply the first-order condition for this problem. The strict convexity of $C$ guarantees that the second-order condition is satisfied at $m^{FB}$.

To complete the proof, it remains to show that, absent moral hazard constraints, efficient allocations can be decentralized in a competitive equilibrium. Let $\ell^*(w)$ be the optimal individual labor supply given wage $w$. Equilibrium requires that expected wages equal the marginal disutility of labor:

$$p_H w = C'(\ell^*(w)). \tag{41}$$

The second equilibrium condition relates to occupational choices, and requests that the utility from becoming a worker equal that from becoming an entrepreneur:

$$p_H w \ell^*(w) - C(\ell^*(w)) = S^{FB} - p_H w. \tag{42}$$

Finally, the labor market clearing condition implies that, at the competitive equilibrium wage $w^{CE}$, individual labor supply satisfies

$$m^{CE} \ell^*(w^{CE}) = 1 - m^{CE}, \tag{43}$$

where $m^{CE}$ is the total mass of workers in equilibrium. Using (41)–(43), we obtain that

$$S^{FB} + C(\ell^*(w^{CE})) = \frac{C'(\ell^*(w^{CE}))}{m^{CE}}. \tag{44}$$

Equations (43)–(44) form the clear counterpart of (6)–(7). Thus $m^{CE} = m^{FB}$, as required. The total mass of workers in equilibrium is independent of the distribution of wealth. This reflects that gains from trade in (42) do not depend on initial wealth. \[\square\]

**Proof of Lemma 1.** Let $\hat{A}$ be the wealth of the marginal agent in equilibrium, and let $w$ be the equilibrium wage. It follows from the definition of the function $\mathcal{A}$ that one must have $\hat{A} \geq \mathcal{A}(w, 1, \pi)$, since otherwise the marginal agent could not be funded. In equilibrium,
\[ w = \frac{1}{p_H} C'(\frac{1}{F(A)} - 1) \] by (8)–(9), and hence
\[ \hat{A} \geq \mathcal{A}\left(\frac{1}{p_H} C'\left(\frac{1}{F(A)} - 1\right), 1, \pi\right). \] (45)

It remains to show that there exists a threshold \( \mathcal{A}(\pi) \in (0, 1) \) such that (45) holds if and only if \( \hat{A} \geq \mathcal{A}(\pi) \). Using the expression (13) for \( \mathcal{A} \) along with the convexity of \( C \), one can check that the right-hand side of (45) is decreasing in \( \hat{A} \). The existence and uniqueness of the threshold \( \mathcal{A}(\pi) \) is then a consequence of the positivity of the minimum ex-wages pledgeable income \( R - \frac{e}{\Delta p} \) and of the Inada conditions on \( C \). \( \blacksquare \)

**Proof of Proposition 4.** When the bankruptcy law is determined by the median agent, the equilibrium wage is \( w(\pi^M) = \frac{C'(1)}{p_H} \). Changes in the magnitude of the moral hazard problem or in the profitability of the project have no impact on \( \mathcal{A}(\pi^M) = F^{-1}(\frac{1}{\pi}) \), but they affect
\[ \mathcal{A}(w(\pi^M), 0, \pi^M) = I - p_H\left(R - \frac{e}{\Delta p}\right) + C'(1). \] (46)

The result then follows from the definition (21) of the aggregate leverage ratio. Consider next a positive shift of the wealth distribution from \( F_1 \) to \( F_2 \) in the monotone likelihood ratio order. Denote by \( \pi_1^M \) and \( \pi_2^M \) the laws preferred by the median agent under \( F_1 \) and \( F_2 \). By (46), \( \mathcal{A}(w(\pi_1^M), 0, \pi_1^M) = \mathcal{A}(w(\pi_2^M), 0, \pi_2^M) \). Furthermore, since \( F_1(\mathcal{A}(\pi_1^M)) = F_2(\mathcal{A}(\pi_2^M)) = \frac{1}{\pi} \) and dominance in the monotone likelihood ratio order implies first-order stochastic dominance, one also has \( \mathcal{A}(\pi_1^M) < \mathcal{A}(\pi_2^M) \). Therefore
\[ \int_{\mathcal{A}(w(\pi_1^M), 0, \pi_1^M)}^{\mathcal{A}(w(\pi_2^M), 0, \pi_2^M)} (I - A) \, dF_2(A) < \int_{\mathcal{A}(w(\pi_1^M), 0, \pi_1^M)}^{\mathcal{A}(w(\pi_2^M), 0, \pi_2^M)} (I - A) \, dF_1(A) \]
where the second inequality reflects the fact that the likelihood ratio \( \frac{f_2(A)}{f_1(A)} \) is increasing in \( A \). The result follows again from the definition (21) of the aggregate leverage ratio. \( \blacksquare \)

**Proof of Lemma 2.** Suppose by way of contradiction that this policy is budget feasible. Then (28) does not hold, that is \( T_W \geq \left[\frac{1}{\mathcal{A}(\pi)}\right] - 1 S_E \). Substituting the values of \( S_E \) and \( T_W \) given by \( (26)–(27) \) and using \( (10) \) and \( (19) \), this inequality can be rewritten as:
\[ \left[ p_H w(\pi^*) \ell^*(w(\pi^*)) - C(\ell^*(w(\pi^*))) \right] - \left[ p_H w(\pi) \ell^*(w(\pi)) - C(\ell^*(w(\pi))) \right] \geq p_H \ell^*(w(\pi^*)) [w(\pi^*) - w(\pi)], \]

29
where \( \ell^*(w(\pi)) = \frac{1}{F(A(\pi))} - 1 \) and \( \ell^*(w(\pi')) = \frac{1}{F(A'(\pi'))} - 1 \) are the equilibrium individual labor supplies under the laws \( \pi \) and \( \pi' \). Simplifying this expression yields
\[
p_H w(\pi)[\ell^*(w(\pi')) - \ell^*(w(\pi))] \geq C(\ell^*(w(\pi')))) - C(\ell^*(w(\pi)))).
\]
By the first-order condition (8) for individual labor supply, this is equivalent to
\[
C'(\ell^*(w(\pi)))[\ell^*(w(\pi')) - \ell^*(w(\pi))] \geq C(\ell^*(w(\pi')))) - C(\ell^*(w(\pi))).
\]
This, however, is a contradiction, since \( C \) is strictly convex and \( \ell^*(w(\pi')) > \ell^*(w(\pi)) \). Hence the result. \[\square\]

**Proof of Proposition 6.** Using (26)–(27), the budget balance condition (29) can be rewritten as \( \beta(A(\pi')) \geq 0 \), where the function \( \beta \) is defined by
\[
\beta(A) = F(A)[U_W(A) - U_W(A(\pi)) - p_H[1 - F(A(\pi))[w(A^{-1}(A)) - w(A)]]
\]
for all \( A \in [A(\lambda^{SB}), A(\pi)] \). By construction, \( \beta(A(\pi)) = 0 \). The problem is to determine under which circumstances, if any, \( \beta(A(\pi + \Delta \pi)) \) is positive when \( \Delta \pi > 0 \) is small. Since \( A \) is decreasing and continuous, this amounts to study the sign of \( \beta \) in a left-neighborhood of \( A(\pi) \). The first-order Taylor expansion
\[
\beta(A) = \beta'(A(\pi))(A - A(\pi)) + o(A - A(\pi))
\]
gives no information on this matter, since, as can be checked from (10) and (24),
\[
\beta'(A(\pi)) = F(A(\pi)) U_W'(A(\pi)) - p_H[1 - F(A(\pi))][\frac{d w(A^{-1}(A))]}{d A}]
\]
is small. Hence
\[
\beta'(A(\pi)) = 0.
\]
This intuitively reflects that the proposed policy generates no gain in the first-order sense. Consider then the second-order Taylor expansion
\[
\beta(A) = \frac{1}{2} \beta''(A(\pi)) [A - A(\pi)]^2 + o([A - A(\pi)]^2).
\]
Using again (10) and (24), one can verify that
\[
\beta''(A(\pi)) = 2 f(A(\pi)) U_W'(A(\pi)) + F(A(\pi)) U_W''(A(\pi)) - p_H[1 - F(A(\pi))] \frac{d^2 w(A^{-1}(A))]}{d A^2}]
\]
\[
= C''\left(\frac{1}{F'(A(\pi))} - 1\right) \frac{f^2(A(\pi))}{F^2(A(\pi))} \left[ 2 - \frac{1}{F(A(\pi))} \right].
\]
Since \( C'' > 0 \), \( \beta''(A(\pi)) > 0 \) if and only if (30) holds, in which case \( \beta > 0 \) in a left-neighborhood of \( A(\pi) \). This implies the first part of the result.

30
To complete the proof, it remains to show that conditions (30) and (31) are equivalent. Condition (31) can be rewritten as \( \gamma(A) > 0 \), where the function \( \gamma \) is defined by

\[
\gamma(A) = p_H[w(A^{-1}(A)) - w(\pi)] - [U_W(A) - U_W(\pi)]
\]

for all \( A \in [A(\lambda^{sB}), A(\pi)] \). By construction, \( \gamma(A(\pi)) = 0 \). As for \( \beta \), the problem is to determine the sign of \( \gamma \) in a left-neighborhood of \( \pi \). Unlike for \( \beta \), it is enough to consider the first-order Taylor expansion

\[
\gamma(A) = \gamma'(A(\pi))[A - A(\pi)] + o(A - A(\pi)).
\]

Indeed, by (10) and (24),

\[
\gamma'(A(\pi)) = p_H \frac{dw(A^{-1}(A))}{dA} \bigg|_{A=\pi} - U_W'(\pi)
\]

\[
= -C''(\frac{1}{F'(\pi)} - 1) \frac{f'(\pi)}{F''(\pi)} \left[ 2 - \frac{1}{F'(\pi)} \right].
\]

Since \( C'' > 0 \), \( \gamma'(A(\pi)) < 0 \) if and only if (30) holds, in which case \( \gamma > 0 \) in a left-neighborhood of \( A(\pi) \). The result follows. \[\blacksquare\]

**Derivation of (36).** Let \( \Delta A = A - A(\pi) + \Delta \pi \). From the proof of Proposition 6, the second-order effect of the proposed policy on the budget balance is

\[
\frac{1}{2} \beta''(A(\pi))\Delta A^2 = \frac{1}{2} C'' \left( \frac{1}{F'(\pi)} - 1 \right) \frac{f'(\pi)}{F''(\pi)} \left[ 2 - \frac{1}{F'(\pi)} \right] \Delta \pi^2.
\]

Using (24) and (34), this may be rewritten as:

\[
\frac{1}{2} \beta''(A(\pi))\Delta A^2 = -\frac{1}{2} f(A(\pi)) p_H \frac{dw(A^{-1}(A))}{dA} \bigg|_{A=\pi} [1 - f^*(w(\pi))] \Delta \pi^2.
\]

To simplify this expression, let \( \Delta \pi = w(\pi + \Delta \pi) - w(\pi) \). Then

\[
\frac{dw(A^{-1}(A))}{dA} \bigg|_{A=\pi} = -\frac{\Delta w}{\Delta \pi} + o(1).
\]

Next, let \( \Delta U_W = U_W(\pi + \Delta \pi) - U_W(\pi) \). Since \( U_W(\pi) = \max_{l \in R^+} \{p_H w(l) - C(l)\} \) and \( f^*(w(\pi)) = \arg \max_{l \in R^+} \{p_H w(l) - C(l)\} \), it follows from the envelope theorem that

\[
\Delta U_W = p_H f^*(w(\pi)) \Delta w + o(\Delta w).
\]

Plugging (50) in (49) and using (51) to eliminate \( f^*(w(\pi)) \) then yields

\[
\frac{1}{2} \beta''(A(\pi))\Delta A^2 = \frac{1}{2} f(A(\pi)) \Delta A(\Delta w - \Delta U_W) + o(\Delta w \Delta A) + o(\Delta A^2).
\]
Neglecting terms of order higher than two in (52) finally leads to (36) given that, by definition, $S_E^\Delta = p_H \Delta w$ and $T_W^\Delta = \Delta U_W$.

It remains to show that (36) is a second-order approximation of (35). Defining the function $\gamma$ as in the proof of Proposition 6, one has

$$\int_{A(\pi)-\Delta A}^{A(\pi)} (S_E - T_W) dF(A) = \int_{A(\pi)-\Delta A}^{A(\pi)} \gamma(A) dF(A).$$

Using Leibniz’s rule along with the fact that $\gamma(A(\pi)) = 0$ then yields the second-order Taylor expansion

$$\int_{A(\pi)-\Delta A}^{A(\pi)} \gamma(A) dF(A) = -\frac{1}{2} \gamma'(A(\pi)) f(A(\pi)) \Delta A^2 + o(\Delta A^2)$$

$$= \frac{1}{2} \beta''(A(\pi)) \Delta A^2 + o(\Delta A^2),$$

$$= \frac{1}{2} f(A(\pi)) \Delta A (S_E^\Delta - T_W^\Delta) + o(\Delta w \Delta A) + o(\Delta A^2),$$

where the second equality follows from (47)–(48), and the third from (52) and the definitions of $S_E^\Delta$ and $T_W^\Delta$. Hence the result.

**Derivation of (38) and (40).** Using Leibniz’s rule yields the second-order Taylor expansion

$$\int_{A(\pi)-\Delta A}^{A(\pi)} [A - A(\pi) + \Delta A] dF(A) = \frac{1}{2} f(A(\pi)) \Delta A^2 + o(\Delta A^2).$$

(53)

Neglecting terms of order higher than two in (53) then leads to (38).

It remains to show that the effect (39) is positive if and only if (40) holds. Defining the function $\gamma$ as in the proof of Proposition 6, one has

$$S_E^\Delta - T_W^\Delta + \Delta A = \gamma(A(\pi) - \Delta A) + \Delta A = [1 - \gamma'(A(\pi))] \Delta A + o(\Delta A),$$

where the second equality follows from the fact that $\gamma(A(\pi)) = 0$. Observing from (48) that $1 > \gamma'(A(\pi))$ if and only if (40) holds then implies the result.

**References**


