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**Essays on Corporate Finance Theory and Behavioral  
Asset Pricing**

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UNIVERSITE TOULOUSE 1 CAPITOLE

# *Abstract*

Toulouse School of Economics

Finance Department

Doctor of Philosophy

## **Essays on Corporate Finance Theory and Behavioral Asset Pricing**

by Jieying Hong

This thesis consists of three self-contained papers. The first two papers study how firms should be structured to facilitate their access to funds in the face of agency conflicts between borrowers (firms) and lenders (investors). The last paper analyzes the formation and evolution of asset bubbles with boundedly rational traders.

Chapter 1 studies the relationship between firm scope and financial constraints. Conventional wisdom suggests that large firms have greater financial flexibility due to diversification. Bringing projects under the same top management, however, can increase the level of correlation and reduce the level of diversification, by exposing the projects to the same manager-specific shock. I challenge the conventional wisdom and show that such positive correlation enhances the firm's ability to relax financial constraints. This is because correlation can mitigate ex-post agency problem. Thus, when credit rationing is the main concern, it is optimal to put multiple projects under the control of a big firm rather than different small firms. I also find that when credit rationing is not an issue, large firms can create value only if the likelihood of large shocks to small ones is large. These predictions are consistent with empirical observations.

Chapter 2 uses an optimal contracting approach to analyze the development of an innovative product through strategic alliance by an entrepreneur and an incumbent. The entrepreneur has limited endowment and the development of the innovation affects the profit of the incumbent because of externalities. When the externalities are positive, an increase in the entrepreneurs endowment increases the outside option of the incumbent. This tightens the participation constraint of the incumbent, which reduces and can sometimes offset the positive effect in relaxing financial constraints of an increase in endowment. The incentive compatible financial claims and the optimal organizational structure are consistent with empirical observations.

Chapter 3<sup>1</sup> analyzes whether traders experience reduce their propensity to speculate? This paper studies a financial market populated by adaptive traders. Following Camerer and Ho (1999)'s Experience-Weighted Attraction learning model, these traders are assumed to adjust their behavior according to actions past performance: according to the law of actual effect, traders reinforce actions that were actually successful in the past; according to the law of simulated effect, traders also reinforce actions that would have been successful if they had been chosen. In our economic environment, because there is a cap on the maximum price that can be achieved, no rational bubbles can form. In the long run, the market converges to the unique no bubble equilibrium. However we show that learning initially increases traders propensity to speculate. In the short run, more experienced traders thus create more bubbles. Moreover, we show that this effect is stronger when traders are more sophisticated (that is, when they use the law of simulated effect) and when the price cap is higher.

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<sup>1</sup>This is a joint work with Sophie Moinas and Sebastien Pouget.

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## Chapter 1

# Financial Constraints, Managerial Incentives and the Scope of the Firm

### 1.1 Introduction

That bringing multiple projects under the same roof can relax financial constraints has been empirically documented, in terms of higher leverage (Berger and Ofek [1]), greater investment scale (Hubbard and Palia [2]), lower cost of capital (Hann et al. [3]), and better dealing with credit crunch in the recent financial crisis (Kuppuswamy and Villalonga [4]). The conventional wisdom, at least since Lewellen [5], is that bringing projects within a firm generates more financial leeway because diversification leads to coinsurance among projects and reduces volatility. It's not clear, however, that bringing projects under the same roof generates diversification. On the contrary, in Gabaix [6], idiosyncratic shocks of firms matter in aggregate, and large firms don't have lower volatility. This is consistent with the evidence of approximate independence of firm volatility to size in Stanley et al. [7].

If projects within the same firm inherit some common, firm-specific shocks, then big firms do not generate diversification as simply bundling the cash flows of these projects. As noted in Gabaix [6]: "If Walmart doubles its number of supermarkets [...] the newly acquired supermarkets inherit the Walmart shocks." Furthermore, as noted by Gabaix [6] an important source of common shocks is "the firm's chief executive officer." Indeed, a growing empirical literature, e.g. Bertrand and Schoar [8] and Bloom and Van Reenen

[9], underscores managers as a key driver of productivity.<sup>1</sup> This suggests that bringing several projects under the same top manager within a big firm could increase the correlation across these projects. One might fear such increased correlation would reduce their ability to relax financial constraints. The novel contribution of this paper is to show that, on the contrary, financial constraints can be relaxed by the positive correlation between the projects within a firm.

To analyze these issues, we consider a three-period model with two types of risk neutral players, managers and investors, as well as two independent and symmetric projects. The managers are penniless and protected by limited liability and the investors have deep pockets. At period 0, the investors choose the organizational structure, i.e., running the two projects in two separate small firms, with two different managers, or within a big firm, under a single manager. Both projects require the same amount of initial investment from the investors. At period 1, each project can be subject to a shock, requiring additional funding. This shock is assumed to be manager-specific. The manager-specific productivity shocks can stem from the CEO's death, divorce, sudden change in his belief in managing the firm, and etc.. In the case of two small firms, the two projects are subject to independent shocks, corresponding to the productivities of two different managers. In the case of one big firm, the two projects are subject to a common shock, corresponding to the productivity of a single manager. Thus, bringing projects under the same roof increases their correlation of shocks. The manager-specific shock is unknown to both parties at period 0, but is revealed publicly at period 1. For each project, after observing the shock, the investors decide whether or not to inject new funds to withstand the shock and continue the project. At period 2, any continued project is subject to moral hazard. Its manager privately chooses between exerting effort and shirking, as in Holmstrom and Tirole [10]. Credit constraints arise for the basic reason that the manager must be granted a minimum incentive rent which reduces income pledgeability and thus make it unprofitable for the investors.

At period 2, if one manager is in charge of both projects, optimally reducing the incentive rent involves a reward only when both projects are successful. This mechanism, referred to as "cross-pledging" in Tirole [11], implies that the rent left to the manager in charge of two projects is smaller than twice the rent left to the manager in charge of a single project. Actually, the benefits of cross pledging are enhanced by the correlation among projects, thus the increase in the correlation among projects relaxes, rather than tightens, financial constraints.

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<sup>1</sup>Bertrand and Schoar [8] find that manager fixed effects can explain a significant extent of the heterogeneity in investment, financial and organizational practices and firm performance. Bloom and Van Reenen [9] use an innovative survey tool to score the management quality of firms in the United States, France, Germany, and the United Kingdom, and find that the management scores are strongly associated with productivity, profitability, Tobin's Q, and survival rates.

To see the rational for this result, consider the following, ultra-simplified, version of our model: The manager-specific shock can be, with equal probability, 0 or  $\infty$ . In the latter case, the project must be abandoned; it would be too expensive to continue. At period 1, each project generates an expected value  $Y$  if it is continued and 0 otherwise. In addition, denote the rent given to the manager in charge of one project  $r_1$  and the rent to the manager in charge of two projects  $r_2$ . Due to cross pledging,  $r_2 < 2r_1$ . In the case of one big firm, the two projects are subject to a common shock. Hence, both projects are continued or liquidated together with equal probability. At period 0, the expected pledgeable income for the investors is  $\frac{1}{2}(2Y - r_2)$ . In the case of two small firms, the two projects are subject to two independent shocks. Thus, we have both projects continued with probability  $\frac{1}{4}$ , one continued and the other liquidated with probability  $\frac{1}{2}$ , and both liquidated with probability  $\frac{1}{4}$ . In order to take advantage of cross-pledging, the two projects are merged at period 1 if they are continued together. As a result, at period 0, the expected pledgeable income for the investors is  $\frac{1}{4}(2Y - r_2) + \frac{1}{2}(Y - r_1)$ . Since  $r_2 < 2r_1$ , the investors obtain a larger expected income if projects are within a big firm than in two separate small firms. The driving force is that, because of the cross-pledging effect, pledgeable income is increasing and convex in the number of viable projects. The increase in the correlation among shocks induces a mean-preserving spread in the distribution of the number of viable projects, and therefore increases income pledgeability and relaxes financial constraints. Thus, putting projects under the same roof can relax financial constraints.

Now, we turn to study the effect of organizational structure on total value. In our model, the total value is the sum of the investors' pledgeable income and the managerial rent. The advantage of big firms have in relaxing their financial constraint does not necessarily imply that they generate more value. We show that, whether big firms increase or decrease value depends on the distribution of manager-specific shock. To our knowledge, this paper is one of the first to study the relation between optimal organizational structure and managerial characteristics.

Consider the case where the manager-specific shock is continuously distributed over  $[0, \infty)$ . In our previous simple case, the shock was either 0 or  $\infty$ , hence the project was always optimal to be liquidated when the shock was  $\infty$ . The only inefficiency which arises in that case was ex-ante credit rationing, i.e., the project cannot get financing at period 0. However, with a general distribution of manager-specific shock, another type of inefficiency, ex-post inefficient liquidation, may occur since at period 1 the shock can turn out to be lower than the full value of the project while greater than the pledgeable income.

Whether big firms create or destroy value depends on their relative abilities in mitigating both ex-ante credit rationing and ex-post inefficient liquidation. If the initial outside financing requirement is large, ex-ante credit rationing is the main concern. In this case, the projects can only be financed if they are operated in a large firm. Hence, big firms always dominate small firms. If the initial outside financing requirement is small, ex-ante credit rationing is not an issue. Thus, the projects can always get financing regardless of organizational structure. In this case, whether big firms or small firms are better in terms of total value depends on their relative advantage in alleviating ex-post inefficient liquidation.

Due to the cross-pledging effect, the continuation of one project not only depends on its own shock but also on the other's. If the project is operated in a big firm, the shock of the other project always has the same magnitude. However, in the case of two small firms, the other shock can be small or large. If the other shock turns out to be small, the project is more likely to be continued. If the other shock turns out to be large, we obtain an opposite result. It implies that small firms are more likely to dominate large firms if small shocks are more likely to occur than large shocks, i.e., the likelihood of small shocks to large ones increases.

The predictions of our theory are consistent with the dramatic reversal in the assessment of conglomerate mergers, which is positive during the 1960s and 1970s and then became negative in the 1980s and 1990s.<sup>2</sup> Bhide [17] argues that due to technological, economic and regulatory changes during 1970s and 1980s, information asymmetries became less of an issue in corporate financing. Hence, credit constraints were more of a concern during the 1960s and 1970s. Our theory implies that in this context, conglomerate mergers would create value. Credit constraint, however, became less of an issue in the 1980s and 1990s. Furthermore, it is likely that, during this period, the increased the competition in the managerial labor market (Murphy and Zabochnik [18]) and the improvement in CEO education (Palia [19]) reduced the proportion of large manager-specific shocks. In this context, our model predicts that conglomerate mergers were less likely to be efficient.

This paper contributes to the literature on the relation between financing constraints and organizational structure. One segment of literature is based on the tradeoff theory of capital structure (Lewellen [5], Higgins [20], Scott [21], Sarig [22], Leland [23], and Banal-Estanol et al. [24]). The other segment is the internal capital market literature, based on agency conflicts (Gertner et al. [25], Stein [26], Scharfstein and Stein [27], Rajan et al. [28], and Inderst and Muller [29]). The present paper also underscores

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<sup>2</sup>Negative view refers to Elgers and Clark [12], Schipper and Thomson [13] and Matsusaka [14]; Positive view refers to Lang and Stulz [15], Berger and Ofek [1] and Morck et al. [16].

agency conflicts, but by taking account of manager-specific shocks, we obtain the new finding that correlation can help big firms relax financial constraints.

This paper is also associated with the studies on the relation between managerial characteristics and organizational structure. Van den Steen [30] shows that a manager with strong beliefs about the right course of action will attract, through sorting in the labor market, employees with similar beliefs. Dessein [31] provides a formal theory of the firm in which managerial direction and bureaucratic decision-making play a key role. The key difference in the present paper is that rather than focusing on managerial vision or direction, we focus on manager-specific shocks.

This paper also complements the literature on managers' span of control (Calvo and Wellisz [32] and Rosen [33]). Rajan and Wulf [34] document the increase of managers' span of control in past decades. They attribute the change of managers' span of control to three possible factors: the development of information technology, the improvement in corporate governance and the increased competition in product markets. This paper complements this literature by arguing that financial constraints are another important driver of managers' span of control.

This paper contributes to the literature regarding how firms deal with liquidity shocks. During the recent financial crisis, lack of liquidity has been regarded as one of the main factors contributing to the propagation of the initial shock (Brunnermeier and Pedersen [35]). Holmstrom and Tirole [10] analyze it by focusing on whether private assets provide sufficient liquidity and discussing the role of government in supplying additional liquidity. The manager-specific shocks in our model can be interpreted as liquidity shocks. With this interpretation in mind, our analysis sheds light on how firms should be structured to withstand liquidity shocks.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 studies different organizational structures in the case without moral hazard. Section 4 analyzes the case with moral hazard. Section 5 discusses the robustness of the results. Section 6 presents the empirical implications and the conclusion is in Section 7. All formal proofs are in the appendix.

## 1.2 The Model

There are two types of players, investors and managers, as well as two independent and symmetric projects,  $A$  and  $B$ . Both types of players are risk neutral. The investors have deep pockets, but do not have the necessary skills to operate any project. In contrast,

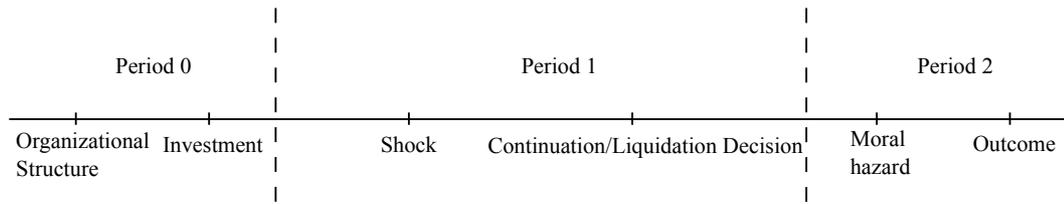


FIGURE 1.1: Timeline

the managers are penniless and protected by limited liability. They are able to manage the projects.

We consider a three period model,  $t = 0, 1, 2$ . The timeline is summarized as in Figure 1.1. At period 0, the two projects can be managed in two separate small firms or within a big firm. In the former case, the two projects are operated by two different managers. In the latter case, the two projects are operated by the same manager. Each project requires an initial investment  $I$ .

Once the manager starts overseeing the firm, things may go wrong. The manager brings a random shock to all the projects under his management at date 1. Its magnitude is unknown to all parties at period 0 but revealed to the public at period 1. The manager-specific shock  $\rho$  is distributed according to a c.d.f  $F(\cdot)$  over  $[0, +\infty)$  (with a p.d.f  $f(\cdot)$ ).<sup>3</sup> In the case of two small firms, the two projects are managed by two different managers, hence the shocks of the two projects are independent. In the case of one big firm, the two projects are managed by a single manager, thus the shocks are perfectly correlated.

To continue the project and reap the final cash-flow, the investors must inject additional investment  $\rho$  to cover the shock. Otherwise, the project is liquidated, the additional expense  $\rho$  is avoided, but the final cash-flow will be lost. The shock can be interpreted as a cost overrun or a shortfall in cash flows to finance operating expenses. After observing the two shocks, the investors need to make the continuation and liquidation decisions. In other words, they determine which project to continue and which to liquidate.

At period 2, any continued project is subject to moral hazard in that its manager privately chooses between effort and shirking à la Holmstrom and Tirole [10]. If the manager exerts effort, the probability of success is  $P$ ; if he shirks, this probability is lowered to  $P - \Delta$  but he enjoys a non-transferable private benefit  $B$ .<sup>4</sup> The project matures at period 2, delivering a revenue  $R$  if it succeeds and no revenue if it fails.

<sup>3</sup>Our results are robust in a more general setup where  $\rho$  can be either positive or negative. If it is positive, additional liquidity needs to be injected, otherwise, the project receives a positive interim revenue. This general consideration would not affect our results at all, since what matters for financial constraints and value is the intermediate continuation or liquidation of the project, while the positive interim income has no impact on this decision.

<sup>4</sup>One possible explanation is that private benefits stem from the private use of the firm's assets by the manager. It is equivalent to cast the model in terms of cost of effort.

Our model departs from Holmstrom and Tirole [10] in two crucial aspects.<sup>5</sup> First, the liquidity shock in our model is manager-specific rather than project-specific. This is very crucial in the sense that the correlation of the shocks across projects differs according to whether the two projects are separately or jointly managed at the initial stage. Since Bertrand and Schoar [8], a growing empirical literature underscores the importance of manager-specific productivity shocks.<sup>6</sup> The manager-specific productivity shocks can stem from the CEO's death, divorce, sudden change in his belief in managing the firm, and etc.<sup>7</sup> Second, in Holmstrom and Tirole [10], the managers are assumed to have all the bargaining power. In contrast, we assume that the investors have all the bargaining power. With this assumption, our contracting problem is simplified. In addition, under this assumption, there is no role for hedging policies that firms can apply to deal with liquidity shocks, such as credit lines or liquidity hoarding. Hence, we can concentrate our study on organizational design.<sup>8</sup>

### 1.3 Equilibrium without Moral Hazard

In this section, we consider a benchmark case where managers' effort is observable and contractable.

#### 1.3.1 Two Small Firms

In the case of two small firms, one manager is only in charge of one project. At period 1, the shocks of the two projects,  $\rho_A$  and  $\rho_B$ , are independent because of the combination of the two managers. After observing  $\rho_A$  and  $\rho_B$ , the investors need to determine which project to be continued and which to be liquidated, by comparing the continuation benefit with the cost of withstanding the shock. At period 1, the continuation value for each project is always  $PR$  and the cost to withstand the cost overrun is  $\rho_i$ , where  $i = A, B$ . If  $\rho_i \leq PR$ , the investor will provide liquidity to continue project  $i$ . Otherwise,

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<sup>5</sup>The original model in Holmstrom and Tirole [10] is a variable investment model. With that model, each manager has an initial endowment of  $A$ . If the investors delegate the two projects to two different managers, the total initial endowment for the two projects is  $2A$ . If the investors delegate the two projects to a single manager, the total initial endowment for the two projects is only  $A$ . This asymmetry of endowment in different organizational structures is unappealing for modeling. Hence, we set  $A = 0$ . In the variable investment model, based on equation (6) in Holmstrom and Tirole [10], the optimal investment is always 0. In this paper, to deal with this issue, I turn to the fixed investment model as in Tirole [11] and Holmstrom and Tirole [36], which is another benchmark model to discuss the liquidity issues.

<sup>6</sup>See Adams et al. [37], Malmendier and Tate [38], Bloom and Van Reenen [9] and Kaplan et al. [39].

<sup>7</sup>The death of Steven Jobs can be a good example for the manager-specific productivity shock.

<sup>8</sup>See the details for why there is no role of liquidity hoarding or credit lines in the discussion section.

it will be liquidated. Thus, the expected value of each project is

$$F(PR)PR - \int_0^{PR} \rho f(\rho) d\rho - I, \quad (1.1)$$

where  $F(PR)$  is the expected continuation probability and  $\int_0^{PR} \rho f(\rho) d\rho$  is the expected liquidity injection to cover the shock. This expected value can be simplified as  $\int_0^{PR} F(\rho) d\rho - I$ .<sup>9</sup>

### 1.3.2 One Big Firm

In the case of one big firm, one manager is in charge of both projects. At period 1, the shocks of the two projects  $\rho_A$  and  $\rho_B$ , affected by the same manager, are exactly the same, i.e.,  $\rho_A = \rho_B$ . After observing the two shocks, the investors decide which project to be continued and which to be liquidated. Project  $i$  is optimal to be continued if and only if  $\rho_i \leq PR$ .

However, due to the difference in the correlation of the two shocks, the ex-post continuation of the two projects in the case of one big firm is different from that of two small firms, as in Figure 1.2. In the case of one big firm, the two shocks are perfectly correlated, hence the two projects are either continued or liquidated together. By contrast, in the case of two small firms, the two shocks are independent, thus there are three possible situations: the two projects are continued together, one project is continued and the other is liquidated, and both are liquidated.

The difference in the ex-post continuation between the two organizational structures, however, does not imply the divergence in the ex-ante value. At period 0, the project is expected to face the same manager-specific shock regardless of the organizational structure. In addition, without moral hazard, the project is continued if and only if its continuation value is greater than the magnitude of its shock. The continuations of the two projects are not interdependent. As a result, organizational structure is irrelevant for value.

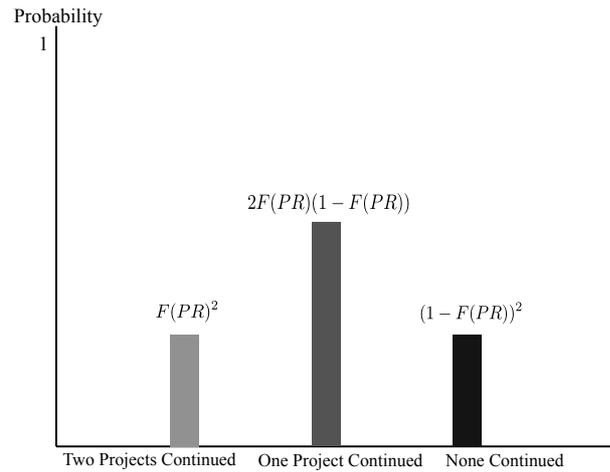
The main results are summarized as follows.

#### **Proposition 1.1.**

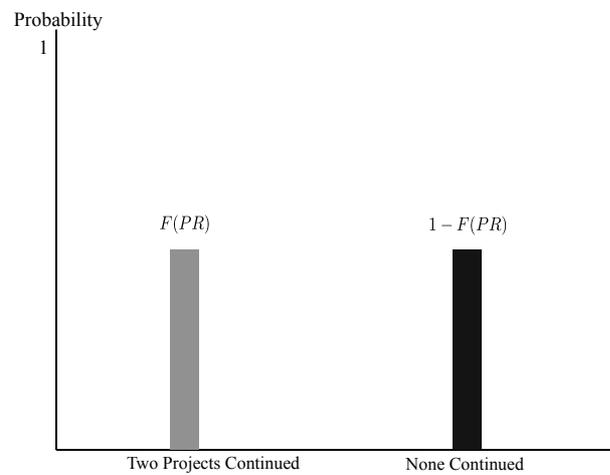
*In the case without moral hazard,*

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<sup>9</sup>If the manager-specific shock is distributed according to another c.d.f.  $G(\cdot)$ , which is second-order stochastically dominated by  $F(\cdot)$ , we obtain  $\int_0^{PR} G(\rho) d\rho \geq \int_0^{PR} F(\rho) d\rho$ . It indicates that the value of the project increases with the risk of the manager-specific shock. It is because the problem faced by each investor is actually a real option problem and the value of option is an increasing function of the risk.



(a) Two Small Firms



(b) One Big Firm

FIGURE 1.2: In the case of two small firms, since the two shocks are independent, there are three possible situations: with probability  $F(PR)^2$  both are continued, with probability  $2F(PR)(1 - F(PR))$  one is continued and the other is liquidated, and with probability  $(1 - F(PR))^2$  both are liquidated. In the case of one big firm, since the two shocks are perfectly correlated, with probability  $F(PR)$  both projects are continued and with probability  $1 - F(PR)$  both are liquidated.

*i) the project is continued if and only if its shock  $\rho$  is lower than its continuation value  $PR$ .*

*ii) organizational structure is irrelevant for value.*

In the frictionless environment, the traditional argument, including Myers [40], Levy and Sarnat [41], and Adler and Dumas [42], is that putting projects under the same roof does not alter the total value. This argument is based on the irrelevance of diversification on value since the investors can achieve the diversification by themselves. One important assumption for this irrelevance theorem is that organizational structure has no impact on the real cash flows of projects.

In this paper, the real ex-post cash flows of a project can be different in different organizational structures, since putting projects under the same roof leads to a change in the correlation of manager-specific shocks across projects. However, we show that the correlation is irrelevant for value. This is because, without frictions, the continuations of the two projects are not interdependent. Therefore, the correlation of the shocks would not matter for each project's continuation decision and thereby its ex-ante expected value.

Other research, e.g., Lewellen [5], Diamond [43], Leland [23] and Banal-Estanol et al. [24], find that if there are frictions, i.e., bankruptcy cost or agency problem, putting projects under the same roof can add firm value by reducing default risk or agency costs. Similarly, in the following we study the impact of organizational structure on financial constraints and value in the case with moral hazard.

## 1.4 Equilibrium with Moral Hazard

In this section, we turn to the moral hazard case where managers privately choose between exerting effort and shirking. In this case, the income of each project that can be pledged to the investors is strictly lower than its full value. Hence, two types of inefficiency may arise: i) ex-post inefficient liquidation at period 1 if the shock of the project is greater than its pledgeable income but lower than its full value; and ii) ex-ante credit rationing, i.e., the investors are not willing to contribute the initial investment at period 0 even if it is optimal to do so. The main goal of the present paper is to study which organizational structure is better at dealing with these inefficiencies. However, before the analysis, we first need to study how the final payoffs of the project are split between the investors and the manager in order to incentivize the manager.

### 1.4.1 Managerial Compensation and Income Pledgeability

At period 2, the project is subject to moral hazard in that the manager privately chooses between effort and shirking. In order to induce effort, the manager must be granted a positive rent. The income of the project cannot be totally pledged to the investors.

In our model, there are two possible cases at period 2: i) one manager only operates one project; ii) one manager operates both projects. In the first case, the manager is granted with  $R_b$  in case of success and 0 in case of failure. The incentive compatibility constraint which guarantees that the manager prefers exerting effort rather than consuming private benefits is

$$\Delta R_b \geq B. \tag{1.2}$$

It implies that, to be incentivized, the expected gain from exerting effort for the manager must be greater than the private benefit that he can consume by shirking. Hence, the manager is rewarded  $\frac{B}{\Delta}$  in case of success and 0 in case of failure. The managerial compensation is linear in performance. The maximum pledgeable income to the investors is  $P(R - \frac{B}{\Delta})$ , denoted  $a$ .

In the second case, the manager receives a reward  $\hat{R}_b$  when both projects are successful and 0 otherwise. This sharing rule for two independent projects is an optimal incentive scheme.<sup>10</sup> The condition for the manager to prefer exerting effort on both projects rather than on one is

$$P^2 \hat{R}_b \geq P(P - \Delta) \hat{R}_b + B, \quad (1.3)$$

and the condition which guarantees that the manager works on both rather than on neither is

$$P^2 \hat{R}_b \geq (P - \Delta)^2 \hat{R}_b + 2B. \quad (1.4)$$

It is easy to show that condition(1.3) is redundant given condition(1.4). Thus, the incentive compatible bonus that the manager obtains in case of two successes satisfies

$$\hat{R}_b \geq \frac{2B}{(2P - \Delta)\Delta}. \quad (1.5)$$

In this case, the manager is granted  $\frac{2B}{(2P - \Delta)\Delta}$  if both projects succeed and 0 otherwise. The managerial compensation is increasing and convex in performance. The maximum pledgeable income to the investors per project is  $P(R - \frac{P - \Delta}{2P - \Delta} \frac{B}{\Delta})$ , denoted  $b$ .

The optimal incentive schemes in the above two cases indicate the following proposition.

**Proposition 1.2.** *The managerial compensation is more convex in performance if the manager operates both projects than if he operates one project.*

Proposition 1.2 implies that the convexity of managerial compensation is positively related to the number of projects under his management, i.e., the level of diversification. In other words, the convexity of managerial compensation would be negatively correlated with the volatility of the firm's final outcome. The literature has two opposite arguments. On one hand, Jensen and Meckling [45] and Haugen and Senbet [46] argue that, a convex compensation scheme leads to more risk taking behavior of the manager and hence increases the volatility of the firm. On the other hand, Smith and Stulz [47], Starks [48] and Carpenter [49] argue that the risk-averse manager, who cannot perfectly hedge his risk, may mitigate the risk of the outcome to reduce his own risk exposure. Thus, the convexity of managerial compensation results in less volatility of the firm. We

<sup>10</sup>See Laffont and Martimort [44] p203 and Tirole [11] p159.

obtain a similar result as the second group. However, in the literature, the managerial compensation scheme is exogenously given, while in the present paper it is endogenously determined with risk neutral managers. Hence, Proposition 1.2 provides another mechanism to explain the negative relation between the convexity of managerial compensation and the firm's volatility.

Now we turn to the analysis on the pledgeable income. In both cases, the income that can be pledged to the investors per project, i.e.,  $a$  or  $b$ , is always lower than the full value of the project  $PR$ . At period 1, inefficient liquidation occurs if the shock is greater than the pledgeable income but lower than the full value. In addition, we also obtain that  $a < b$ , i.e., the manager can pledge more income to the investors per project if he operates both projects than if he only operates one project. The intuition is that, when the two projects are jointly managed, the manager can use one project as a collateral to raise financing for the other to mitigate agency conflicts, referred to "cross-pledging" as in Tirole [11].

**Proposition 1.3.** *In the case with moral hazard, after the occurrence of the two manager-specific shocks,*

- i) the income that can be pledged to the investors is always lower than the full value of the project.*
- ii) the pledgeable income per project is larger in the case where the manager is in charge of both projects than in the case where he is in charge of one project.*

The first part of Proposition 1.3 is consistent with the argument in Jaffee and Russell [50] and Stiglitz and Weiss [51] that credit rationing is an equilibrium phenomenon if there is information asymmetry between borrowers and lenders. The second part is consistent with Diamond [43] and Tirole [11] that the cross-pledging of independent projects can mitigate agency problem and increase income pledgeability. This cross-pledging effect is also similar to the coinsurance effect in Lewellen [5], Leland [23] and Banal-Estanol et al. [24]. However, the underlying mechanisms are different. In their papers, the combination of two independent cash flows reduces default cost, while in this paper it mitigates agency conflicts.

Generally speaking, financial constraint is loosened if more income can be pledged to the investor. Hence, in the following we will study the ex-ante pledgeable income that the investors can obtain at period 0 in the two organizational structures.

### 1.4.2 Two Small Firms

In the case of two small firms, the two projects are operated separately by two different managers at period 0. At period 1, the two shocks  $\rho_A$  and  $\rho_B$  are independent. On observing the two shocks, the investors need to decide which project to be continued and which to be liquidated. There are four possible choices for the investors at period 1: i) continue both projects; ii) continue project  $A$  while liquidating project  $B$ ; iii) continue project  $B$  while liquidating project  $A$ ; iv) liquidate both. If both projects are continued together, it is preferable they be merged due to the cross-pledging benefit.<sup>11</sup> However, if only one project or none is continued, there is no scope for cross-pledging.

Here, we ignore any specific sharing rule among the investors, and only consider the total profit to them. This is due to the fact that as long as the action is profitable, there always exists some specific rule to split the cost and the income to benefit all investors. The total profit to the investors at period 1 is  $2b - \rho_A - \rho_B$  in case i),  $a - \rho_A$  in case ii),  $a - \rho_B$  in case iii), and 0 in case iv). Denote  $c = 2b - a = P(R - \frac{\Delta}{2P-\Delta} \frac{B}{\Delta})$ . If both projects are bundled, the pledgeable income per project is  $b$ , while  $a$  is the marginal pledgeable income for the first project and  $c$  is the marginal pledgeable income for the second project, where  $a < b < c$ .

By comparing the net profit that the investors obtain in the four cases, we can see that if both projects can be continued alone ( $\rho_A, \rho_B \leq a$ ), they are always continued together. If one project can be continued alone ( $\rho_i \leq a$ , where  $i = A, B$ ) while the other cannot ( $\rho_{-i} > a$ ), the other is saved only when its shock is not greater than the marginal pledgeable income for the second project ( $\rho_{-i} \leq c$ ), otherwise, it is liquidated. If neither project can be continued alone ( $\rho_A, \rho_B > a$ ), both projects can be continued together only when the total shock is lower than the total pledgeable income of bundling the two projects ( $\rho_A + \rho_B \leq 2b$ ), otherwise, both are liquidated.

In summary, the continuation and liquidation conditions of the two projects, as in Figure 1.3, are as follows.<sup>12</sup>

**Lemma 1.4.**

- i) The two projects are merged and continued together if  $\rho_A + \rho_B \leq 2b$  and  $\rho_A, \rho_B \leq c$ .*
- ii) Project  $A$  is continued while project  $B$  is liquidated if  $\rho_A \leq a$  and  $\rho_B > c$ .*
- iii) Project  $B$  is continued while project  $A$  is liquidated if  $\rho_B \leq a$  and  $\rho_A > c$ .*
- iv) Both projects are liquidated if  $\rho_A + \rho_B > 2b$  and  $\rho_A, \rho_B > a$ .*

<sup>11</sup>Given merged, one manager is in charge of both projects, while the other is fired.

<sup>12</sup>See proof in Appendix.

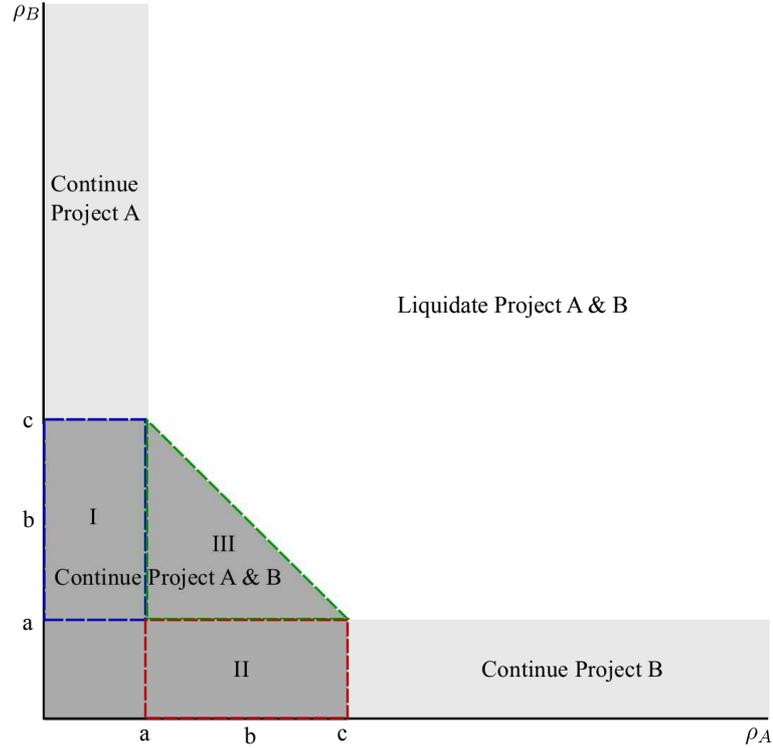


FIGURE 1.3: In the case of two small firms, the two projects are merged and continued together in the dark grey area, i.e.,  $\rho_A + \rho_B \leq 2b$  and  $\rho_A, \rho_B \leq c$ ; one project is continued and the other is liquidated in the light grey area, i.e.,  $\rho_i \leq a$  and  $\rho_{-i} > c$ , where  $i = A, B$ ; both are liquidated in the white area, i.e.,  $\rho_A + \rho_B > 2b$  and  $\rho_A, \rho_B > a$ .

Without moral hazard, the continuation of one project only depends on its own shock rather than on the other's. However, Lemma 1.4 implies that, with moral hazard, the two projects' continuation become interdependent due to the cross-pledging effect.

Based on the interim continuation and liquidation conditions, we can simply obtain the continuation probability per project. Since the two projects are symmetric, we consider project  $A$  as an example. In Figure 1.3, project  $A$  is continued with project  $B$  in the dark grey area. This probability is

$$q_1 = F(a)F(c) + \int_a^c \int_0^{2b-\rho_A} f(\rho_A)f(\rho_B)d\rho_Bd\rho_A. \quad (1.6)$$

In the upper light grey area, project  $A$  is continued alone. This probability is

$$q_2 = F(a)(1 - F(c)). \quad (1.7)$$

Thus, the total probability of continuation for project  $A$  is  $q_1 + q_2$ . The corresponding expected liquidity injected to withstand the shock is

$$E\rho = \int_0^a \rho_A f(\rho_A)d\rho_A + \int_a^c \int_0^{2b-\rho_A} \rho_A f(\rho_A)f(\rho_B)d\rho_Bd\rho_A. \quad (1.8)$$

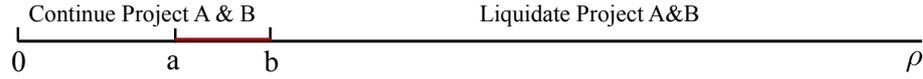


FIGURE 1.4: In the case of one big firm, both projects are continued if  $\rho_A = \rho_B \leq b$ , while liquidated if  $\rho_A = \rho_B > b$ .

Due to the symmetry, project  $B$  has the same continuation probability and expected liquidity injection at period 1. The distribution of the continuation of the two projects are in Figure 1.5(a).

Therefore, the ex-ante expected value per project at period 0 is

$$(q_1 + q_2)PR - E\rho - I, \quad (1.9)$$

and the ex-ante expected return to the investors per project is

$$q_1b + q_2a - E\rho - I. \quad (1.10)$$

The pledgeable income to the investors is  $b$  if the project is continued with the other, while it is  $a$  if the project is continued alone.

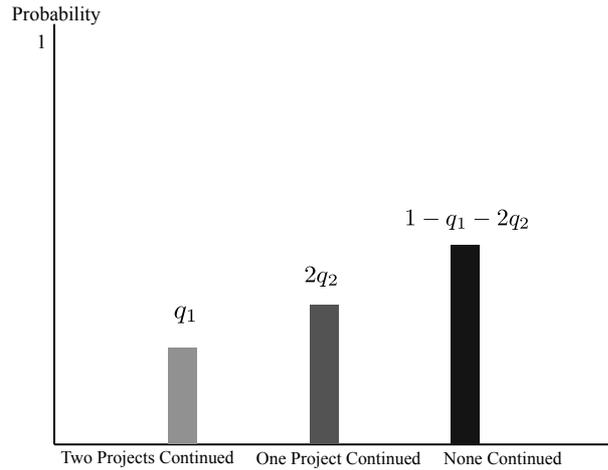
### 1.4.3 One Big Firm

In the case of one big firm, at period 0 both projects are managed by the same manager. At period 1, the shocks are perfectly correlated, thus the two projects are either continued or liquidated together. It is never optimal to spin off the two projects ex-post, since the cross-pledging benefit only exists when the two projects are jointly operated by the same manager. The pledgeable income per project is  $b$  when both projects are continued together. Therefore, the interim continuation conditions of the two projects, as in Figure 1.4, are as follows.

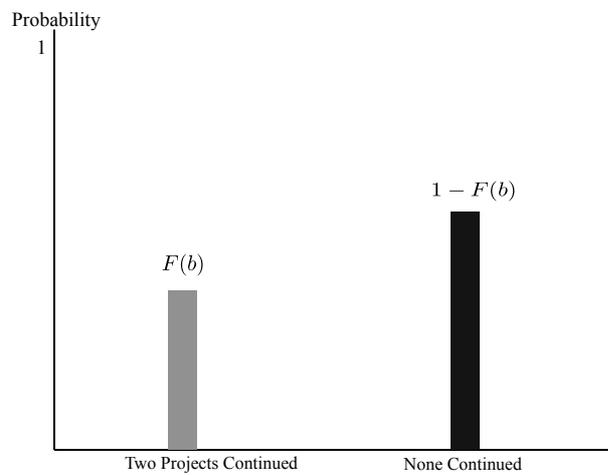
**Lemma 1.5.**

*In the case of one big firm, the two projects are continued together if  $\rho_A = \rho_B \leq b$ , otherwise, both are liquidated.*

The distribution of the continuation of the two projects is in Figure 1.5(b). The interim continuation in the case with moral hazard seems similar to the case without moral hazard in that the two projects are either continued or liquidated together. However, the difference between the two still hinges on the existence of the interdependent continuation between the two projects. With moral hazard, the two projects depend on each other to be continued, when the shock is greater than  $a$  while lower than  $b$ . In this case,



(a) Two Small Firms



(b) One Big Firm

FIGURE 1.5: In the case of two firms, both projects are continued with probability  $q_1$ , one project is continued while the other is liquidated with probability  $2q_2$ , both are liquidated with probability  $1 - q_1 - 2q_2$ . In the case of one big firm, both projects are continued with probability  $F(b)$ , and liquidated with probability  $1 - F(b)$ .

both projects cannot be continued alone, but can be continued together if they use each other as collateral to raise financing.

Therefore, in the case of one big firm, the ex-ante expected value per project at period 0 is

$$F(b)PR - \int_0^b \rho f(\rho) d\rho - I, \quad (1.11)$$

where  $F(b)$  is the continuation probability and  $\int_0^b \rho f(\rho) d\rho$  is the expected liquidity injection to cover the shock.

The ex-ante expected return to the investors per project is

$$F(b)b - \int_0^b \rho f(\rho) d\rho - I. \quad (1.12)$$

For the investors, they obtain pledgeable income  $b$  per project if both projects are continued and 0 otherwise.

#### 1.4.4 Ex-ante Credit Rationing

With moral hazard, the income of the project cannot be totally pledged to the investors. Credit rationing may occur at period 0 when the project initiates. In this subsection, we want to study which organizational structure is better at relaxing the financial constraints by mitigating the ex-ante credit rationing problem. In general, ex-ante credit rationing is less likely to arise if the income that can be pledged to the investors is larger. Hence, we compare the ex-ante pledgeable income generated in these two organizational structures.

In our setup, on observing the shock at period 1, the investors need to determine whether to inject liquidity to continue the project. The problem faced by the investors at period 0 is actually a real option problem. The pledgeable income obtained by the investors is equivalent to the option value. In the case without moral hazard, at period 1, the continuation of each project only depends on its own shock. Hence, the option value that the investors obtain for each project is also determined by its own random shock. As a result, organizational structure is irrelevant for value. However, in the case with moral hazard, the interdependent continuation between the two projects arises. The continuation of each project depends not only on its own shock but also on the other's. Hence, the option value that the investors obtain for each project is determined by both shocks. In this case, correlation matters for the option value. When the two shocks are more correlated, the risk the investors face is larger, thereby leading to a larger option value. As a consequence, compared to small firms, big firms have an advantage in generating pledgeable income to the investors and therefore relaxing financial constraints.<sup>13</sup>

Denote  $PI_b$  as the ex-ante expected pledgeable income per project to the investors in the case of one big firm, and  $PI_s$  as the ex-ante expected pledgeable income per project in the case of two small firms, where  $PI_b = F(b)b - \int_0^b \rho f(\rho)d\rho$ ,  $PI_s = q_1b + q_2a - E\rho$  and  $PI_b > PI_s$ .

**Proposition 1.6.**

- i) If  $PI_s \geq I$ , the projects can always obtain financing at period 0 regardless of organizational structure.*
- ii) If  $PI_s < I \leq PI_b$ , the projects can only obtain financing at period 0 if they are operated within a big firm.*

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<sup>13</sup>Rigorous proof is in Appendix.

iii) If  $PI_b < I$ , the projects can never be financed at period 0 regardless of organizational structure.

Proposition 1.6 implies that large firms are better at mitigating ex-ante credit rationing than small firms. In this paper, the relaxation of the financial constraint stems from cross-pledging. However, in the presence of ex-post merger option at period 1, small firms also can take advantage of the cross-pledging benefit. As a result, the advantage of large firms relative to small firms does not come from the fact that cross pledging only exists in large firms, but from the fact that more correlated shocks lead to better exploitation of cross pledging.

Proposition 1.6 also indicates that if the initial investment need is large, the projects can only be initiated in a large firm and ex-ante credit rationing is the main concern. In this case, large firms dominate small firms due to their advantage in relaxing financial constraints. Nevertheless, if the initial investment need is small, the projects can always be initiated regardless of organizational structure. In this case, ex-ante credit rationing is not an issue. Which organizational structure is better depends on their relative abilities in mitigating ex-post inefficient liquidations.

#### 1.4.5 Ex-post Inefficient Liquidation

With moral hazard, ex-post inefficient liquidation occurs at period 1 if the shock of the project is lower than its full value but greater than the income that can be pledged to the investors. In this subsection, we consider the case where  $PI_s \geq I$ , i.e., the projects are always initiated regardless of organizational structure. In this case, we study which organizational structure is better at alleviating ex-post inefficient liquidation.

Due to symmetry, we take project  $A$  as an example, and study how its continuation depends on the organizational structure. When its shock is too low  $\rho_A \leq a$  (too high  $\rho_A > c$ ), the project is always continued (liquidated) regardless of organizational structure. However, when  $a < \rho_A \leq c$ , the continuation of project  $A$  depends on whether it is managed in a big firm or in a small firm. If  $a < \rho_A \leq b$ , project  $A$  is always continued in the case of one big firm, while it is liquidated in the case of two small firms if the other shock turns out to be large, i.e.,  $\rho_B > 2b - \rho_A$ . In this case, project  $A$  is less likely to be continued in the case of two small firms than in the case of one big firm. The decrease in the probability of continuation is  $\int_a^b \int_{2b-\rho_A}^{+\infty} f(\rho_A) f(\rho_B) d\rho_B d\rho_A$  as the light grey area in Figure 1.6. If  $b < \rho_A \leq c$ , the project is always liquidated in the case of one big firm, while it is continued in the case of two small firms if the other shock turns out to be small, i.e.,  $\rho_B \leq 2b - \rho_A$ . In this case, project  $A$  is more likely to be continued in the

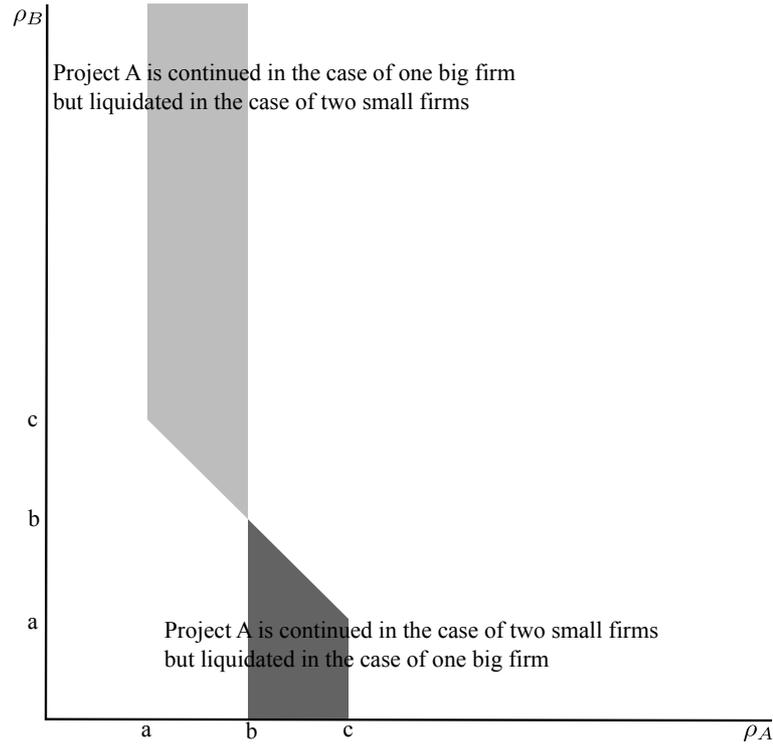


FIGURE 1.6: In the light grey area, i.e.,  $\rho_A \in [a, b]$  and  $\rho_B \in (2b - \rho_A, +\infty)$ , project A is liquidated in the case of two small firms while continued in the case of one big firm. It represents the relative cost if the two projects are managed in two separate firms. In the dark grey area, i.e.,  $\rho_A \in (b, c]$  and  $\rho_B \in [0, 2b - \rho_A]$ , project A is continued in the case of two small firms while liquidated in the case of one big firm. It represents the relative benefit if the two projects are managed in two separate firms.

case of two small firms than in the case of one big firm. The increase in the probability of continuation is  $\int_b^c \int_0^{2b-\rho_A} f(\rho_A)f(\rho_B)d\rho_B d\rho_A$  as the dark grey area in Figure 1.6.

**Proposition 1.7.**

*The benefit in ex-post continuation of small firms relative to big firms is that the project with shock  $\rho_i \in (b, c]$  is saved by the other project with shock  $\rho_{-i} \leq 2b - \rho_i$ ; The cost in ex-post continuation is that the project with shock  $\rho_i \in (a, b]$  is dragged down by the other project with shock  $\rho_{-i} > 2b - \rho_i$ .*

The rationale behind Proposition 1.7 is that if the project is operated together with the other project within a big firm, the shock of other project is exactly the same. However, if the project is operated in a separate firm, the magnitude of the other project's shock can be small or large. If the other shock turns out to be small, the project is more likely to be continued when it is managed in a small firm than in a big firm. If the other shock turns out to be large, we obtain an opposite result. In our model, the project can be continued in big firms if only if its shock is lower than  $b$ . In small firms, the project, even with a shock larger than  $b$ , can be saved by the other with a small shock. Alternatively, the project, even with a shock lower than  $b$ , can be dragged down by the other with a

large shock. As a result, if the likelihood of small shocks relative to large ones increases, big firms will be less able to mitigate ex-post inefficient liquidation than small firms.

*Example: the manager-specific shock is uniformly distributed over  $[0, \phi]$ .*

The difference between the light grey area and the dark grey area in Figure 1.6 represents the difference of the continuation probability per project between the big firm case and the small firm case, which is written as following.

$$dp = \int_a^b \int_{2b-\rho_A}^{+\infty} f(\rho_A)f(\rho_B)d\rho_Bd\rho_A - \int_b^c \int_0^{2b-\rho_A} f(\rho_A)f(\rho_B)d\rho_Bd\rho_A. \quad (1.13)$$

If the manager-specific shock is uniformly distributed over  $[0, \phi]$ , i.e.,  $f(\cdot) = \frac{1}{\phi}$ , by computing equation (1.13) we find that i) if  $\phi \leq b$ , i.e., the maximum shock is sufficiently small, both projects can always be continued regardless of organizational structure; ii) if  $b < \phi \leq 2b$ , i.e., the likelihood of large shocks relative to small ones is small, the projects are more likely to be continued in small firms than in big firms. iii) if  $\phi > 2b$ , i.e., the likelihood of large shocks relative to small ones is large, the projects are less likely to be continued in small firms than in big firms.<sup>14</sup>

Moreover, we can also compare the value difference per project between the two organizational structures, which is represented in the following equation.

$$dv = \int_a^b \int_{2b-\rho_A}^{+\infty} (PR - \rho_A)f(\rho_A)f(\rho_B)d\rho_Bd\rho_A - \int_b^c \int_0^{2b-\rho_A} (PR - \rho_A)f(\rho_A)f(\rho_B)d\rho_Bd\rho_A. \quad (1.14)$$

By computing the above equation when  $f(\cdot) = \frac{1}{\phi}$ , we find that i) if  $\phi \leq b$ , small firms and big firms generate the same value per project; ii) if  $b < \phi \leq \phi^*$ , where  $\phi^* \in (b, 2b)$ , small firms generate a larger value per project than big firms; iii) if  $\phi > \phi^*$ , small firms generate a smaller value per project than big firms. Generally speaking, more continuation leads to larger value. Hence, the qualitative argument on value is similar to that on continuation probability. However,  $\phi^* < 2b$ . It indicates that the range for small firms dominating big firms in terms of value is smaller than in terms of continuation probability. This is because, the expected cost to withstand shocks is larger in the small firm case than in the big firm case.

**Proposition 1.8.**

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<sup>14</sup>See proof in Appendix.

If the shock is uniformly distributed over  $[0, \phi]$ ,

*i) relative to big firms, the continuation probability for each project in small firms is the same if  $\phi \in [0, b]$ , larger if  $\phi \in (b, 2b]$ , and smaller if  $\phi \in (2b, +\infty)$ .*

*ii) relative to big firms, the value per project generated by small firms is the same if  $\phi \in [0, b]$ , larger if  $\phi \in (b, \phi^*)$ , and smaller if  $\phi \in (\phi^*, +\infty)$ , where  $\phi^* \in (b, 2b)$ .*

This result tells us that big firms can generate a larger value than small firms if  $\phi$  is sufficiently high, i.e.,  $\phi > \phi^*$ , i.e., the likelihood of large shocks to small ones is large.

Our results show that in terms of relaxing ex-ante financial constraints, big firms are always better than small firms, while in terms of total value, big firms can either be better or worse than small firms depending on the distribution of manager-specific shock. If the likelihood of small shocks relative to large ones increases, big firms are more likely to destroy value.

## 1.5 Discussion

### 1.5.1 Firm-specific Shocks

The main element driving the results in the present paper is the fact that projects are subject to manager-specific shocks. Nevertheless, our results are robust in a much broader environment.

The main implication for manager-specific shocks is that the shocks of the two projects become more correlated if they are managed within the same firm. In addition to manager-specific shocks, there are other productivity shocks leading to an increase in the correlation across projects in big firms, such as, quoted in footnote 5 in Gabaix [6], the shocks coming from “a decision of the firm’s research department, of the firm’s chief executive officer, of how to process shipments, inventories.....” or the shocks stemming from “changes in capacity utilization, and, particularly, strikes.” In fact, as long as the shock is company-specific, our results are robust. This is because firm-specific shocks always make projects more correlated when they are managed within the same firm than in separate firms.

In our setup, for simplicity we consider a case where the projects are subject to manager-specific shocks at the intermediate date. However, in reality, in addition to manager-specific shocks, projects may be also subject to project-specific shocks. In this case, the shocks of projects within a big firm are not perfectly correlated but we should still

observe an increase in the correlation of shocks when projects are put under the same roof. This increase in the correlation leads to a larger option value (pledgeable income) to the investors. Our results are still robust.

### 1.5.2 Hedging

In the case with moral hazard, the outside investors may not be willing to provide liquidity to continue the project when the shock occurs, even if it is optimal. This naturally raises the question as to whether or not it is best for firms to hedge ex-ante, by hoarding liquidity or using credit lines, to deal with the shortage in liquidity ex-post. This issue was addressed in Holmstrom and Tirole [10]. Their main assumption is that managers have all the bargaining power, and this, in turn, generates the need for hedging.

We turn to the simple one-project case in Tirole [11]<sup>15</sup> to discuss the intuition of hedging policies in Holmstrom and Tirole [10]. The project is optimal to be continued at intermediate date if and only if  $\rho < \rho^*$ . The manager's expected payoff is

$$U_M = F(\rho^*)PR - \int_0^{\rho^*} \rho f(\rho) d\rho - I. \quad (1.15)$$

The break-even condition for the investors is

$$F(\rho^*)a - \int_0^{\rho^*} \rho f(\rho) d\rho - I = 0. \quad (1.16)$$

We can show that  $a < \rho^* \leq PR$ . The optimal contract is such that the investors should provide liquidity as long as  $\rho \leq \rho^*$ . However, at date 1, the maximum pledgeable income the investors can obtain is  $a$ . They are not willing to provide liquidity if  $\rho > a$ . The conflict of interest between ex-ante and ex-post decisions of the investors results in a need for firms to hedge against the shortage of liquidity ex-post.

In this paper, however, we assume that the investors have all the bargaining power, thus the conflict of interest between ex-ante and ex-post decisions of the investors no longer exist. The problem faced by the investors is actually a real option problem. Any hedging policies would reduce the option value. With this assumption, we can focus the discussion on how firms should be structured to deal with liquidity shocks. In the following, we will show that our results on organizational design are robust even if we consider a competitive capital market where managers have all the bargaining power.

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<sup>15</sup>See Chapter 5 in Tirole [11]

### 1.5.3 Competitive Capital Market

In a competitive capital market, the manager maximizes his profit subject to the investors' break-even constraint. The break-even constraint is such that the expected pledgeable income for the investors is equal to their initial investment, i.e.,  $PI = I$ . This break-even constraint ensures that the manager obtains the total profit of the project. In this case, the manager wants to continue the project as long as its shock is not greater than the total continuation value  $PR$  as the first-best. However, this optimal solution is not attainable if this continuation condition violates the investors' break-even constraint.

We study the maximum investment that the investors can provide, which is equal to the maximum pledgeable income that they can obtain. In our initial setup, we minimize the rents to the managers and maximize the income to the investors. In other words, we have already obtained the maximum pledgeable income for the investors. Hence, the maximum investment per project that the investors can provide is  $PI_s$  in the case of two small firms, and  $PI_b$  in the case of one big firm.

First, consider the case where  $PI_s < I \leq PI_b$ . The projects can only be initiated if they are managed within the same firm. In this case, the maximum investment  $PI_b$  that can be provided by the investors is strictly greater than the required investment. The maximum shock that can be withstood is  $b$  when the investors obtain the maximum pledgeable income  $PI_b$ . However, in a competitive capital market we need to maximize the total value of the projects, thus the pledgeable income for the investors is reduced to the initial investment by increasing the maximum shock that can be withstood. Denote the maximum shock that can be withstood  $\rho'$ , which satisfies the investors' break-even condition.

$$F(\rho')b - \int_0^{\rho'} \rho f(\rho) d\rho - I = 0. \quad (1.17)$$

We can show that  $b < \rho' \leq PR$ .

Second, consider the case where  $PI_s \geq I$ . The projects are always initiated regardless of organizational structure. In this case, ex-ante credit rationing is not a concern. Similarly, in a competitive capital market, to maximize the total profit of the projects, the investors break even and the pledgeable income for them is less than the maximum one. Thus, the maximum shock that can be withstood is greater than that in the case where the investors have all the bargaining power.

Even though firms can withstand larger shocks in a competitive capital market than in our initial case, the main conclusion that big firms can better deal with the ex-ante credit rationing problem is still robust. In addition, that the advantage in relaxing financial

constraints does not indicate big firms create value is also robust in a competitive capital market. In the following, I will illustrate this point through a simple example.

Consider the case where the shock can either be 0 or  $\varrho$  with equal probability. Assume that  $a < 2I < b$  and  $2(b - I) < \varrho < \max\{3b - 4I, PR\}$ , implying that the two projects must be liquidated when both encounter the shocks.<sup>16</sup> If the two projects are operated within a big firm at period 0, the projects are either continued or liquidated together with equal probability. The total expected payoff is  $PR - 2I$ . If the two projects are operated in two small firms at period 0, the two shocks are independent. Hence, the projects are continued together except the case where both are hit by shocks. The probability of continuation per project is  $\frac{3}{4}$ . The probability of liquidity injection per project is  $\frac{1}{4}$ . Thus, the expected total payoff is  $\frac{3}{2}PR - \frac{1}{2}\varrho - 2I$ . It is easy to see that the expected payoff in the case of two small firms is larger than that in the case of one big firm. Thus, big firms destroy value. In this case, managers prefer small firms.

#### 1.5.4 The cost of changing management

In our basic setup, the two projects are always merged if they are continued together at period 1. It is impossible to have two small firms coexist after the shocks. One way to address this problem is to assume a fixed cost  $C$  of changing the management. If the two projects are operated within a big firm, the manager stays the same at different periods. Hence, the introduction of the cost of changing management has no impact on the results in this case. However, if the two projects are operated in two separate small firms, the manager of one project is changed after the ex-post merger. In the following, we study the continuation and liquidation decisions.

If  $C \geq 2(b - a)$ , the cost of changing the management is higher than the gain of the pledgeable income from the ex-post merger. The two small firms would never be merged at period 1. Each project is continued if and only if its shock is lower than  $a$  and two small firms coexist after the shocks.

If  $C < 2(b - a)$ , the cost is lower than the gain. At period 1, the pledgeable income that the investors can obtain from the ex-post merger is scaled down to  $2b - C$ . The optimal continuation and liquidation decision is that i) continue and merge both projects

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<sup>16</sup>In the case of one big firm, both projects are either hit by a shock together or not with equal probability. If the investors withstand the shocks, their maximum profit is  $2(b - \frac{1}{2}\varrho - I)$ . When  $\varrho > 2(b - I)$ , the break-even constraint is violated. In this case, the two projects are liquidated when both are hit by shocks. In the case of two small firms, the two shocks are independent. If the investors absorb shocks in all situations, their maximum profit is  $2(b - \frac{1}{2}\varrho - I)$ . If the investors absorb shocks only when one project is distressed, their maximum profit is  $\frac{1}{4}2b + \frac{1}{2}(2b - \varrho) - 2I = \frac{3}{2}b - \frac{1}{2}\varrho - 2I$ . If the investors absorb shocks only when both are distressed, their maximum profit is  $\frac{1}{4}2b + \frac{1}{2}a + \frac{1}{4}(2b - 2\varrho) - 2I = b + \frac{1}{2}a - \frac{1}{2}\varrho - 2I$ . When  $a < 2I < b$  and  $2(b - I) < \varrho < \max\{3b - 4I, PR\}$ , only the intermediate case does not violate the investors' break-even constraint. As a result, the projects are liquidated when both are hit by shocks.

if  $\rho_A, \rho_B \leq c - C$  and  $\rho_A + \rho_B \leq 2b - C$ ; ii) continue project  $i$  and liquidate project  $-i$  if  $\rho_i \leq a$  and  $\rho_{-i} > c - C$  ( $i = A, B$ ); or iii) liquidate both if  $\rho_A + \rho_B > 2b - C$  and  $\rho_A, \rho_B > a$ .

We can see that, due to the additional cost in changing the management, small firms become less able to exploit the cross-pledging benefit. This reduces the probability of continuation and the pledgeable income to the investors. Hence, the ability for small firms to relax financial constraints is deteriorated with the introduction of this additional cost. The conclusion, that big firms are better at relaxing the initial financial constraint, is always robust. Furthermore, the conclusion that relaxing financial constraints for big firms does not imply they create value is still robust, as long as the cost of switching the management is small. In this case, the comparable advantage of small firms in dealing with ex-post inefficient liquidation still dominates.

### 1.5.5 Endogenous Investment

Our previous analysis focuses on the fixed investment model. In this subsection, we extend the analysis to the endogenous investment case and study the robustness of our results. We assume a convex investment cost function for each project  $c(I)$ , where  $c$  is continuous and twice differentiable, satisfying the monotonicity and the convexity conditions  $c' > 0$  and  $c'' > 0$ .<sup>17</sup> The follow-up shocks, payoffs and private benefits are scaled at per unit of investment. The optimal investment must be such that its marginal cost is equal to its marginal benefit to the investors.

With moral hazard, the optimal investment per project in the case of two small firms is

$$c'(I) = q_1b + q_2a - E\rho. \quad (1.18)$$

while in the case of one big firm, the optimal investment per project is

$$c'(I) = F(b)b - \int_0^b \rho f(\rho) d\rho = \int_0^b F(\rho) d\rho. \quad (1.19)$$

The larger the benefit to the investors, the more they are willing to invest in the project at the initial stage. Since  $\int_0^b F(\rho) d\rho > q_1b + q_2a - E\rho$ , big firms always generate a larger marginal benefit to the investors than small firms. Therefore, large firms are better at increasing initial investment than small firms. In other words, big firms are better at mitigating ex-ante credit rationing than small firms. In the fixed investment case, the advantage of big firms in mitigating ex-ante credit rationing is reflected in

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<sup>17</sup>As explained in footnote 5, it is not appealing to use the variable investment model. In order to generate a reasonable investment, we assume a convex cost function of investment.

that projects are more likely to be initiated. In the endogenous investment case, it is reflected in that projects have larger scales in big firms. With respect to the ex-post inefficient liquidation, the results are quite similar to the fixed investment case, due to the fact that the follow-up shocks, payoffs and private benefits are scaled at per unit of investment.

### 1.5.6 Ex-post Increasing Returns to Scale

In this paper, another important element driving our results is the ex-post cross-pledging benefit, i.e., the incentive rent to the manager is reduced by merging the two projects at period 1. Hence, the pledgeable income to the investors at date 1 is increasing and convex in the number of viable projects. Due to the convexity, the increase in the correlation across projects in big firms helps exploit this cross-pledging benefit and thus relaxes financial constraint.

In addition to cross pledging, our results are robust with other types of ex-post increasing return to scale, such as economies of scale, market power, complementarity in research and technology. The idea is that with these positive synergies, mergers at period 1 also boost pledgeable income and thus pledgeable income is increasing and convex in the number of viable projects. This convexity indicates that correlation is good and it helps large firms relax more financial constraints.

In our initial setup, for simplicity we assume that at date 2, projects are only subject to project-specific shocks. In reality, projects may also be subject to manager-specific shocks date 2. Our results are still robust in this case. The idea is that when projects are subject to both project-specific and manager-specific shocks, their final payoff are not perfectly correlated when they merged. In this case, cross-pledging benefit always exists.

## 1.6 Empirical Implications

The first implication characterizes the relationship between the managerial compensation and the number of projects the manager operates. Proposition 1.2 can be easily extended to the case with more than two projects. Thus we have the following implication.

*IMPLICATION 1: The managerial compensation is more convex in performance if the manager has more projects under his management.*

The convexity of the managerial compensation can be reflected in the use of equity and options in a manager's compensation package. Hence, we expect more diversified firms use more equity or options to reward managers. May [52] finds a positive relation between firm diversification and the proportion of personal wealth vested in firm equity, while Denis et al. [53] find a negative relation between the two. As argued in Aggarwal and Samwick [54], these two tests are misleading since they treat the level of managerial incentives as exogenously determined. Aggarwal and Samwick [54] use a setup where incentives are set in equilibrium and find a significant positive relation. They show that the negative relation in Denis et al. [53] is the result of unobserved, firm-specific factors. We see the same positive association between the convexity of managerial compensation and firm diversification. However, according to our setup, this positive relation should be more significant when the credit constraint is the main concern. Hence, one possible way to distinguish our hypothesis from others is to investigate how this relation changes with the financial constraint.

The second implication characterizes the relationship between organizational structure and the correlation of liquidity shocks. We know that, if the two projects are within the same firm, they are subject to common manager-specific shocks, thus their liquidity shocks become more correlated than if they are in two separate firms.

*IMPLICATION 2: The correlation of external financing needs across projects is larger if they are within the same firm than in two separate small firms.*

To our knowledge, this implication has not been directly tested yet. However, this implication is indirectly confirmed by some other empirical observations.

Bringing multiple projects under the same roof increases the correlation across projects and therefore enhances the aggregate volatility. In other words, the distribution of firm size matters for the level of aggregate volatility. If the ratio of large firms relative to small ones in society increases, the aggregate volatility goes up. Gabaix [6] shows that when the distribution of firm size is fat-tailed, idiosyncratic shocks to individual firms do not average out. He finds that the idiosyncratic shocks of the largest 100 firms in the United States can explain about one-third of aggregate shocks in output growth. This empirical observation is consistent with implication 2.

Based on Proposition 1.6, we have the following implication.

*IMPLICATION 3: Putting projects under the same roof can relax more financial constraints than in separate firms.*

The relaxation of financial constraints may be reflected in different ways. Berger and Ofek [1] find that conglomerates are significantly more leveraged than their comparable stand-alone firms. In contrast, Comment and Jarrell [55] find no significant association between leverage and firm diversification. These mixed observations do not necessarily indicate that conglomerates have no advantage in relaxing the financial constraint. In fact, relaxing financial constraints may be reflected in the reduction in the cost of capital rather than the increase in the leverage. Hann et al. [3] find that, on average, conglomerates have a lower cost of capital than comparable portfolios of stand-alone firms. In addition, the benefit of conglomerates may be more evident in the environment where credit rationing is the main concern for the firm as in our setup. Kuppuswamy and Vilalunga [4] treat the 2007–2009 crisis as an exogenous shock of credit rationing for firms and find that conglomerates have significantly lower cash ratios, better credit ratings, and are more leveraged relative to comparable focused firms.

The traditional theory attributes the advantage of conglomerates in relaxing financial constraints to diversification. However, bringing different projects under the same roof does not imply more “diversification” than in several separate firms. If one conglomerate can generate more or at least equal diversification than two stand-alone firms, large firms cannot explain aggregate volatility at all, which contradicts the observation in Gabaix [6]. Our theory argues that correlation is the reason why putting projects within a firm can better relax financial constraints than in separate firms. In order to identify this mechanism, we need to test the following implication.

*IMPLICATION 4: Market value of the firm increases with the correlation of future external financing needs across projects.*

To our knowledge, this implication has not been tested either. To test this implication, we need to measure external financing needs of different projects. According to Rajan and Zingales [56], external finance need is defined as capital expenditures minus cash flows from operations divided by capital expenditures. Since 1978, the SEC in the United States required public firms to disclose accounting data for their main business lines. Hence, we can test how the market value of the firm relates to the correlation of external financing needs across different business lines. According to our theory, we should observe a positive relation between the two and this relation should be more significant in a financial constraint environment.

The next implication states the relationship between organizational structure and total value.

*IMPLICATION 5: When credit constraint is the main concern, putting projects within the same firm can create value. However, when credit constraint is not an issue, putting projects within the same firm can destroy value if the ratio of small manager-specific (firm-specific) shocks to big ones is sufficiently large.*

This implication is consistent with the dramatic reversal in the empirical view towards conglomerate mergers; positive during the 1960s and 1970s while negative in the 1980s and 1990s. Bhidé [17] argues that due to technological, economic and regulatory changes during 1970s and 1980s, information asymmetries become less of an issue in corporate financing. Hence, credit constraint is a significant concern for firms during the 1960s and 1970s. Our theory predicts that conglomerates can create value in a credit constrained environment, which is consistent with the positive view towards conglomerate mergers during this period. Credit constraint, however, becomes less important in the 1980s and 1990s. Our theory implies that conglomerate mergers are more likely destroy value if the likelihood of small shocks to big ones is sufficiently large. During this period, the increased competition in the managerial labor market (Murphy and Zbojnik [18]) and the improvement in CEO education (Palia [19]) would reduce the likelihood of big manager-specific shocks. In this context, my model predicts that conglomerate mergers are more likely to destroy value, which is consistent with the negative view during this period.

## 1.7 Conclusion

That bringing multiple projects under the same roof relaxes financial constraints has been empirically well documented. The conventional wisdom, at least since Lewellen [5], is that bringing projects within a firm generates more financial leeway because of diversification. By putting several projects under the same top manager, however, large firms can increase their correlation by exposing them to the same manager-specific shock. I challenge the conventional wisdom and show that this positive correlation enhances projects' ability to obtain outside financing. In addition, by taking account of manager-specific shocks, our theory predicts that when credit rationing is the main concern, putting projects within the same firm always creates value. However, when credit rationing is not an issue, whether it creates or destroys value depends on the distribution of manager-specific shock. It is more likely to destroy value if the likelihood of small shocks relative to large ones increases. These predictions fit quite well with the parallel evolution of the managerial labor market and the empirical view toward conglomerate mergers from the 1960s to the 1990s in the US.

## Chapter 2

# Optimal Financial Contracting in Strategic Alliances

### 2.1 Introduction

Strategic alliance has become an increasingly common and favorable vehicle for companies to speed up innovation in recent decades. As can be seen in Figure 2.1, the number of newly established strategic alliances each year increases dramatically in the past twenty years, from almost 0 at year 1985 to around 2800 at year 2005. The popularity of strategic alliances draws extensive attention from economists, whose research mainly focused on the specific investment or hold-up problem.<sup>1</sup> Baker et al. [62], however, find that rather than the hold-up problem, the externalities generated by the joint project on the parent firms are the main issue emphasized by practitioners. Externalities can stem from many sources, e.g., product market competition between the two products, knowledge transfers, cross-market synergies and etc. These spillover effects are important considerations when firms decide to form and structure strategic alliances.

We use an optimal contracting approach to study the development of an innovative product through strategic alliance by an entrepreneur and an incumbent. This innovative product generates externalities on the existing product of the incumbent. The main aim of the present paper is to study how the externalities affect the optimal financial contracting of alliances. We consider a reduced form of externalities as in Hellmann [63]. In the model, both the entrepreneur and the incumbent privately exert effort, which determines the probability of success of the innovation. With double moral hazard, the total output is affected by the efforts of both agents while we only observe the final output. In order to deal with the free riding problem, we need an outside investor to be

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<sup>1</sup>See Aghion and Tirole [59], Elfenbein and Lerner [60] and Malmendier and Lerner [61].

a budget breaker as in Holmstrom [64] to punish both agents. In addition to effort, the development of the innovation product also requires financial investment.

The entrepreneur only has limited endowment, thus he needs to obtain financing from the incumbent and the outside investor. In standard models, an increase in the entrepreneur's endowment would relax his financial constraint. This argument, however, does not always hold if the innovation generates externalities on the financier, i.e., the incumbent in the present paper. With the externalities, the development of the innovation affects the profit of the incumbent even when the incumbent does not participate in the strategic alliance. The impact of the endowment on the financing constraint also depends on how the endowment affects this outside option value. If the success of the innovation harms the incumbent, an increase in the entrepreneur's endowment deteriorates the incumbent's outside option. This loosens his participation constraint, which increases the positive impact in relaxing financial constraints of an increase in endowment. In contrast, if the success of the innovation benefits the incumbent, an increase in the endowment raises the incumbent's outside option. This tightens his participation constraint, which reduces and can sometimes offset the positive effect in relaxing financial constraints of an increase in endowment. The offsetting scenario happens if the marginal effect of the endowment on the incumbent's outside option is greater than 1.

The incumbent can not only contribute initial investment but also provide effort, as the venture capitalist in Casamatta [65]. One main result in her paper is that financing and advising must go hand in hand. In contrast, in the present paper, financing does not need go hand in hand with advising for the incumbent. The intuition is that, in this paper, the incumbent and the outside investor are deep-pocket, thus they have flexibility in making transfers ex-post between them. This ex-post flexibility in making transfers ensures that the identity of the agent providing the outside financing ex-ante is irrelevant for value. This result is consistent with the empirical observation of Robinson and Stuart [57] that up-front payments from pharmaceutical firms to biotechnology *R&D* firms are sometimes not requested.

In addition to the financial investment, the efforts of the entrepreneur and the incumbent also are important inputs to produce the innovative product. To induce effort, both the entrepreneur and the incumbent must be given proper incentives through cash-flow rights over the total income generated by the innovation. The total income includes not only the revenue in the entrepreneur firm but also external effects on the incumbent firm. Therefore, the cash flow rights should be also contingent on the externalities. The optimal cash-flow rights can be implemented by granting proper financial claims to different agents. In order to generate more realistic financial claims, we impose a monotonicity constraint, i.e., the revenue of each agent must be nondecreasing in the

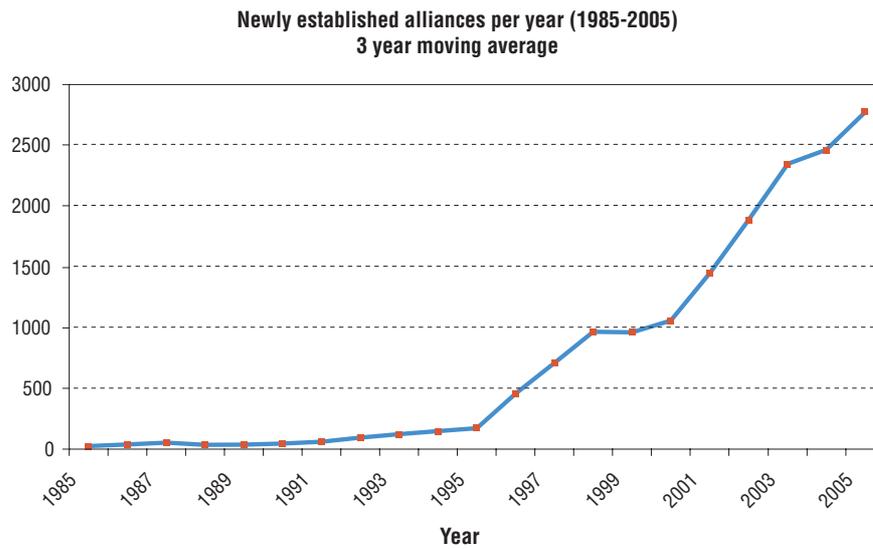


FIGURE 2.1: Number of newly established alliances (source: the second state of alliance management study (2007) in association of strategic alliance professionals)

firm's profit, as in Innes [66]. The monotonicity constraint ensures that the entrepreneur or the incumbent has no incentive to revise their profit reports upward with hidden borrowing. With the monotonicity constraint, the outside investor works as a budget breaker if and only if the externality is negative, in which case the outside investor is able to obtain a higher payoff in case of failure through a claim on the incumbent firm.

If the externality is negative, i.e., the success of the innovation harms the incumbent, the revenue difference between the two states is positive in the entrepreneur firm while negative in the incumbent firm. The effort of each agent is only determined by his revenue difference between the two states. Thus, with a negative externality, the revenue in the entrepreneur firm is sufficient to incentivize the entrepreneur while the revenue in the incumbent firm cannot induce effort from the incumbent. In this case, the incumbent must hold preferred equity or equity in the entrepreneur firm while not the reverse.

If the externality is positive but not very large, the incumbent can be induced to exert some effort with the revenue in his own firm but not enough. Hence, as before, to align incentives, the incumbent own equity or preferred equity in the entrepreneur firm. In contrast, if the positive externality is very large, the incumbent can be incentivized through a claim in his own firm while the entrepreneur cannot exert sufficient effort given the revenue in her own firm. In this case, the entrepreneur must hold equity in the incumbent firm while the incumbent holds either debt or nothing in the entrepreneur firm.

The specific financial instruments for the incumbent holding in the entrepreneur firm are determined by the amount of investment contributed by the entrepreneur. With

the same expected final income, equity provides more powerful incentives than preferred equity, while debt does not provide any incentives. If the entrepreneur contributes a large amount of initial investment, the incumbent's investment will be small, so is his expected income. In order to induce enough effort, the incumbent must be given higher-powered incentives. Thus, he is granted either equity or nothing in the incumbent firm depending on the magnitude of externality. If the entrepreneur provides a small amount of investment, the incumbent's contribution will be large, so is his expected income. In this case, if the incumbent is given equity, this would make him exert too much effort. Therefore, in order to recoup his investment without distorting incentives, the incumbent must be granted preferred equity or debt depending on the magnitude of externality.

The above results rationalize the use of preferred equity in the entrepreneur firm for the incumbent, which is consistent with the empirical observations in corporate venture capital contracts by Cumming [67] and in biotech strategic alliances by Robinson and Stuart [57]. Meanwhile, comparing with venture capital contracts in Casamatta [65], pure debt securities are more likely to occur if the innovation generates externalities. It is because with a positive externality, the incumbent is already incentivized to some extent given a claim on his own firm and only needs to hold securities which provides less powerful incentives, such as debt, in the entrepreneur firm. This phenomenon is also evidenced by Cumming [67]. He finds that Canadian corporate venture capitalists are more likely to use non-convertible debt than Canadian limited partnership venture capitalists.

In addition, if the entrepreneur is more financially constrained, reciprocal holdings between firms are less popular, and it is usually the case for the incumbent to hold equity in the entrepreneur firm while not the reverse. In general, small firms are more financially constrained while big firms are less financially constrained.<sup>2</sup> It implies that, if the strategic alliance is formed by one small firm and one big firm, it is common for the big established firm to hold equity in the small entrepreneur firm, which is consistent with the observation in the United States (Allen and Philips [69]). In contrast, if the strategic alliance is formed by two big firms, cross holdings between the two are more common, which is in line with the phenomenon of financial keiretsu in Japan, whose main features are extensive inter-firm trading and cross-holdings of debt and equity.

Our study also sheds light on the optimal organizational structure of innovation. The innovation can be operated either by the entrepreneur alone, through strategic alliance or within the incumbent by hiring the entrepreneur. Strategic alliances always dominate stand-alone operations, since they benefit from i) joint effort support; ii) internalization

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<sup>2</sup>Internal capital market, such as Gertner et al. [25], Stein [26] and Stein [68], indicates that big firms are less financially constrained.

of the externality. In addition, comparing with innovating within the incumbent, strategic alliance is better in case of negative externality, since the intervention of a budget breaker only exists if the entrepreneur and the incumbent are separate entities. The two structures, however, are indifferent in case of positive externality, since the role of a budget breaker is ruled out. It predicts that on average, agents are less incentivized to do the innovation within the incumbent, which is consistent with the finding of Seru [70] that firms acquired in mergers are less innovative, and the acquirers move R&D activity outside the boundary of the firm via the use of strategic alliances.

The remainder of the paper is organized as follows. Section 2 introduces the basic model. Section 3 studies the stand-alone case. Section 4 analyzes the strategic alliance case. Section 5 focuses on how to implement the optimal contract in strategic alliance by proper financial claims. Section 6 sheds new light on the optimal organizational structure of innovation. Section 7 concludes. All formal proofs are in the Appendix.

## 2.2 The Model

There are three players: an entrepreneur, an incumbent and an outside investor. The entrepreneur, with an endowment  $E_A$ , has an innovative idea to develop a new product and is protected by limited liability. The incumbent has a mature product with a revenue  $Y$ . Both the incumbent and the outside investor are deep-pocket. All three players are risk neutral.

We consider a two period model,  $t = 0, 1$ . The new product requires an initial investment  $I$  at date 0. After the investment, the innovation is operated either by the entrepreneur alone or through strategic alliance by the entrepreneur and the incumbent. At date 1, the innovation matures, delivering a revenue to the entrepreneur and an externality to the incumbent. Both the revenue of the entrepreneur and the externality affecting the incumbent depend on the actions of the players. When the project is operated by the entrepreneur alone, it succeeds with probability  $a \in [0, 1]$ , in which case it generates revenue  $R_u^s$  for the entrepreneur and externality  $Y_u^s$  for the incumbent, and it fails with probability  $1 - a$ , generating  $R_d^s$  for the entrepreneur and  $Y_d^s$  for the incumbent. The probability of success is affected by the effort of the entrepreneur. For simplicity,  $a$  is both the probability of success and the level of effort. When the project is operated with a strategic alliance, the probability of success is  $\min\{a + b, 1\}$ , where  $b$  is the effort of the incumbent. In addition, the revenues and externalities are  $R_u, R_d, Y_u$  and  $Y_d$ .

The externalities are a reduced form of potentially very complicated interactions between the entrepreneur and the incumbent, including product market competition, licensing,

acquisitions and so on. This reduced form consideration of externalities is similar to that in Hellmann [63]. In this paper, we focus our analysis on how the externalities affect the optimal financial contract.

We allow the revenues and externalities to be different if the innovation is operated by the entrepreneur alone v.s. through strategic alliance. This is because, through strategic alliance, the interaction between firms can change the payoff structure. For example, the entrepreneur may need to design the new product to be more compatible with the existing product of the incumbent, the incumbent may expropriate the entrepreneur ex-post after he gains access to the new technology, or both can benefit the knowledge spillover. If  $Y_u^s - Y_d^s > 0$  ( $Y_u - Y_d > 0$ ), the innovation generates a positive externality on the incumbent in case of stand-alone operation (strategic alliance). Otherwise, the innovation generates a negative externality.

Both efforts are costly.  $C_A(a)$  and  $C_B(b)$  denote the entrepreneur's and the incumbent's disutility of effort, where

$$C_A(a) = \frac{1}{2}Aa^2, \quad (2.1)$$

and

$$C_B(b) = \frac{1}{2}Bb^2. \quad (2.2)$$

The interest rate is normalized to zero. In addition,  $0 < A < B$ , indicating that the entrepreneur is more efficient in developing this new product than the incumbent.<sup>3</sup>

This setup is similar to the model in Casamatta [65], but it departs from it in two crucial aspects. First, in Casamatta [65], the venture capitalist only cares about the financial returns. In contrast, in the present paper, due to externalities, the incumbent also takes account of the strategic consequences of the innovation on his own business. Second, in Casamatta [65], the outside investor only provides initial funding, but, in the present model, the outside investor also serves as a budget breaker, which is key for incentives as in Holmstrom [64].

## 2.3 Stand-alone Operation

We first study the financial contracting in the stand-alone case where the entrepreneur develops the new product alone without the help from the incumbent.

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<sup>3</sup>Lerner et al. [71] find that strategic alliances, which assign less control rights to R&D firms and more control rights to the big established firm, are significantly less successful. This result is consistent with our assumption that the entrepreneur firm is more efficient in producing the product.

### 2.3.1 No Moral Hazard

In the case without moral hazard, the entrepreneur's effort is observable and the value of this innovation to the entrepreneur is

$$V(a) = aR_u^s + (1 - a)R_d^s - \frac{1}{2}Aa^2 - I. \quad (2.3)$$

By maximizing  $V(a)$ , the optimal level of effort is

$$a = \frac{1}{A}(R_u^s - R_d^s). \quad (2.4)$$

Assume that  $R_u^s - R_d^s < A$ , to ensure that the optimal level of effort has an internal solution.

The value of the innovation to the entrepreneur is

$$R_d^s + \frac{1}{2A}(R_u^s - R_d^s)^2 - I. \quad (2.5)$$

The externality the incumbent passively receives is

$$Y_d^s + \frac{1}{A}(Y_u^s - Y_d^s)(R_u^s - R_d^s). \quad (2.6)$$

### 2.3.2 Moral Hazard

With moral hazard, the entrepreneur's effort is not observable. At date 0, the entrepreneur makes a take-it-or-leave-it offer to the outside investor. The contract specifies: i) the initial investment  $I_A$  and  $I_o$  from the entrepreneur and the outside investor respectively, where  $I_A + I_o = I$ ; ii) the split of final payoff at date 1:  $R_i^A$  to the entrepreneur and  $R_i^s - R_i^A$  to the outside investor in state  $i$ , where  $i = u, d$ .

The entrepreneur chooses his effort level to maximize his expected utility. His incentive compatibility constraint ( $IC_A^s$ ) is

$$a \in \arg \max_a \quad aR_u^A + (1 - a)R_d^A - \frac{1}{2}Aa^2 - I_A, \quad (2.7)$$

According to the first-order condition of (2.7), the level of effort  $a$  is

$$a = \frac{1}{A}(R_u^A - R_d^A). \quad (2.8)$$

Effort  $a$  is increasing in the revenue difference between the two states. In our two-state setup, the entrepreneur can increase the probability of success by exerting more effort.

Hence, if the entrepreneur is granted more revenue in case of success relative to failure, he has incentive to exert more effort.

In order to ensure that the outside investor is willing to participate in the innovation, he must be granted a nonnegative profit. His participation constraint ( $PC_o^s$ ) is

$$a(R_u^s - R_u^A) + (1 - a)(R_d^s - R_d^A) - I_o \geq 0. \quad (2.9)$$

The contract is chosen to maximize the entrepreneur's profit, given his incentive constraint, the participation constraint of the outside investor and other feasibility constraints. The program is

$$\begin{aligned} \max_{R_u^A, R_d^A, I_A, I_o} \quad & aR_u^A + (1 - a)R_d^A - \frac{1}{2}Aa^2 - I_A \\ \text{s.t.} \quad & IC_A^s \\ & PC_o^s \\ & I_A + I_o = I \\ & I_A \leq E_A \\ & R_u^A, R_d^A \geq 0 \end{aligned} \quad (2.10)$$

where the last condition is the feasibility constraints reflecting the limited liability protection for the entrepreneur.

Assume that  $R_d^s + \frac{1}{4A}(R_u^s - R_d^s)^2 > I$ , which implies that it is always profitable for the entrepreneur to do the innovation as long as  $E_A \geq 0$ .

**Proposition 2.1.**

*If  $E_A \geq I - R_d^s$ , the entrepreneur exerts the same level of effort as without moral hazard. In the optimal contract, the outside investor holds safe debt with face value not greater than  $R_d^s$  and the entrepreneur holds the equity.*

*If  $E_A < I - R_d^s$ , the entrepreneur exerts less effort than that in the case without moral hazard. In the optimal contract, the outside investor holds risky debt with face value greater than  $R_d^s$  and the entrepreneur holds the equity.*

The intuition of Proposition 1 is as follows. According to equation (2.8), the effort level of the entrepreneur is determined by his revenue difference between the two states. If the entrepreneur has enough endowment, i.e.,  $E_A \geq I - R_d^s$ , the outside investor only needs to invest  $I - E_A$ , which is less than  $R_d^s$ . Thus, the outside investor is granted a safe debt with face value lower than  $R_d^s$ . In this case, the equity held by the entrepreneur

delivers him a revenue difference  $R_u^s - R_d^s$ , leading to the same effort level as the case without moral hazard. On the other hand, if the entrepreneur is financially constrained, i.e.,  $E_A < I - R_d^s$ , the outside investor contributes an investment  $I - E_A$ , which is larger than  $R_d^s$ . He can only recoup it back by holding a risky debt with face value greater than  $R_d^s$ . In this case, the equity held by the entrepreneur would generate a revenue difference lower than  $R_u^s - R_d^s$ , resulting in less effort than the case without moral hazard.

In the stand-alone case, the incumbent passively receives the externality of the innovation. The expected externality for the incumbent is

$$U_B = Y_d^s + a(Y_u^s - Y_d^s), \quad (2.11)$$

where

$$a = \begin{cases} \frac{R_u^s - R_d^s}{A} & E_A \geq I - R_d^s \\ \frac{R_u^s - R_d^s + \sqrt{(R_u^s - R_d^s)^2 + 4A(E_A - I + R_d^s)}}{2A} & E_A < I - R_d^s. \end{cases} \quad (2.12)$$

If  $Y_u^s - Y_d^s < 0$ , i.e., the innovation generates a negative externality, the value to the incumbent is a decreasing function of the entrepreneur's financing strength  $E_A$ ; if  $Y_u^s - Y_d^s > 0$ , i.e., the innovation generates a positive externality, the value to the incumbent is an increasing function of the entrepreneur's financing strength  $E_A$ . This is intuitive. With a negative externality, successful innovations impose costs on the incumbent. Thus, the incumbent would prefer the entrepreneur to be financially weak so as to reduce the probability of success. Otherwise, the reverse holds.

## 2.4 Strategic Alliance

In this section, we turn to the case of strategic alliance through which both the entrepreneur and the incumbent exert effort to develop the new product.

### 2.4.1 No Moral Hazard

We first consider the benchmark case where efforts are observable and contractible, and externalities are internalized. The total income generated by the innovation is  $R_u + Y_u$  in case of success and  $R_d + Y_d$  in case of failure.

The expected total income of the innovation is

$$TV(a, b) = \min\{a + b, 1\}(R_u + Y_u) + \max\{1 - (a + b), 0\}(R_d + Y_d) - \frac{1}{2}Aa^2 - \frac{1}{2}Bb^2 - I. \quad (2.13)$$

The levels of effort maximizing  $TV(a, b)$  are

$$a^* = \frac{1}{A} [(R_u - R_d) + (Y_u - Y_d)], \quad (2.14)$$

and

$$b^* = \frac{1}{B} [(R_u - R_d) + (Y_u - Y_d)]. \quad (2.15)$$

Assume that  $0 < (R_u - R_d) + (Y_u - Y_d) < \frac{AB}{A+B}$ , so the efforts are always positive, and  $\min\{a + b, 1\} \leq 1$  is not binding at first best. Note that the effort of the entrepreneur is more efficient than the effort of the incumbent, the optimal level of effort  $a^*$  is greater than  $b^*$ . In addition, both optimal levels of effort are determined by not only the revenue difference in the entrepreneur firm but also the externality difference in the incumbent firm. If the innovation has a positive external effect on the incumbent firm, the success of the innovation can benefit both firms. In this case, both the entrepreneur and the incumbent would like to exert more effort than the case of negative external effect.

The expected total value of the innovation is

$$TV^* = R_d + Y_d + \frac{1}{2} \left( \frac{1}{A} + \frac{1}{B} \right) [(R_u - R_d) + (Y_u - Y_d)]^2 - I. \quad (2.16)$$

Assume that

$$I \leq R_d + Y_d + \frac{1}{2} \left( \frac{1}{A} + \frac{1}{B} \right) [(R_u - R_d) + (Y_u - Y_d)]^2, \quad (2.17)$$

so that the project is profitable in the case without moral hazard.

In the case without moral hazard, efforts  $a$  and  $b$  must be provided by the entrepreneur and the incumbent respectively. However, regarding the initial investment  $I$ , the identity of the financing agent is irrelevant for value. The same total value can be attained no matter whether the outside investor, the incumbent, or the entrepreneur himself provides the financial investment. Thus, the Modigliani and Miller theorem holds. However, the results are quite different in the case with moral hazard.

### 2.4.2 Moral Hazard

In the case with moral hazard, both the entrepreneur's and the incumbent's efforts are unobservable. At date 0, the entrepreneur makes a take-it-or-leave-it offer to the incumbent and the outside investor,<sup>4</sup> which specifies: i) the initial investment  $I_A$ ,  $I_B$  and  $I_o$  from the entrepreneur, the incumbent and the outside investor respectively, where  $I_A + I_B + I_o = I$ ; ii) the split of total final payoff  $R_i + Y_i$  of both the entrepreneur and incumbent firm:  $R_i^A$ ,  $R_i^B$  and  $R_i + Y_i - R_i^A - R_i^B$  to the entrepreneur, the incumbent and the outside investor respectively at state  $i$ , where  $i = u, d$ .

Contrary to the case without moral hazard, the way the cash flow is shared determines how much effort is provided by each agent. For the entrepreneur, the level of effort is given by his incentive compatibility constraint  $IC_A$ .

$$a \in \arg \max_a (a+b)R_u^A + (1-(a+b))R_d^A - \frac{1}{2}Aa^2 - I_A, \quad (2.18)$$

which means that the entrepreneur chooses his effort to maximize his expected profit, given the contract established, his rational expectation of the effort level of the incumbent, and his cost of effort. Similarly, the incentive compatibility constraint of the incumbent  $IC_B$  is

$$b \in \arg \max_b (a+b)R_u^B + (1-(a+b))R_d^B - \frac{1}{2}Bb^2 - I_B. \quad (2.19)$$

According to the first-order conditions of  $IC_A$  and  $IC_B$ , the optimal levels of effort  $a$  and  $b$  are

$$a = \frac{1}{A}(R_u^A - R_d^A), \quad (2.20)$$

and

$$b = \frac{1}{B}(R_u^B - R_d^B). \quad (2.21)$$

For each agent, the level of effort increases with the revenue difference between the two states. Indeed,  $a$  ( $b$ ) is increasing in  $R_u^A$  ( $R_u^B$ ) but decreasing in  $R_d^A$  ( $R_d^B$ ).

The entrepreneur also needs to ask for financial support from the incumbent and the outside investor. In order to make them willing to provide financing, the participation constraints must ensure that they recoup their investment in expectation.

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<sup>4</sup>This assumption simplifies our computation without altering the qualitative results. In addition, Allen and Philips [69] find a significant increase in stock prices of the entrepreneur firm while not for the incumbent firm when they form strategic alliances, which indicates that all the benefit goes to the entrepreneur firm. This observation is consistent with our assumption.

The participation constraint for the incumbent  $PC_B$  is

$$R_d^B + (a+b)(R_u^B - R_d^B) - \frac{1}{2}Bb^2 - I_B \geq U_B. \quad (2.22)$$

The left hand side represents the expected profit of the incumbent in the case of strategic alliance. The right hand side represents the reservation utility the incumbent obtains in the case of stand-alone operation. Note that  $U_B$  is endogenous, since it depends on the action of the entrepreneur in the case of stand-alone operation.

The participation constraint for the outside investor  $PC_o$  is

$$\begin{aligned} -I_o + R_d + Y_d - R_d^A - R_d^B \\ + (a+b)(R_u - R_d + Y_u - Y_d - (R_u^A - R_d^A) - (R_u^B - R_d^B)) \geq 0. \end{aligned} \quad (2.23)$$

The reservation income for the outside investor is 0.

The financial contract is chosen to maximize the expected profit of the entrepreneur given the incentive constraints, participation constraints and other feasibility constraints:

$$\begin{aligned} \max_{R_u^A, R_d^A, R_u^B, R_d^B, I_A, I_B, I_o} \quad & R_d^A + (a+b)(R_u^A - R_d^A) - \frac{1}{2}Aa^2 - I_A \\ \text{s.t.} \quad & IC_A \\ & IC_B \\ & PC_B \\ & PC_o \\ & I = I_o + I_A + I_B \\ & I_A \leq E_A \\ & R_u^A, R_d^A \geq 0. \end{aligned} \quad (2.24)$$

Denote  $U = R_u^A - R_d^A$  and  $V = R_u^B - R_d^B$ . The program can be written as

$$\max_{U, V} \quad R_d + Y_d + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d) - \frac{1}{2A}U^2 - \frac{1}{2B}V^2 - I - U_B \quad (2.25)$$

$$\text{s.t.} \quad R_d + Y_d - R_d^A + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d - U) - \frac{1}{2B}V^2 - U_B \geq I - E_A, \quad (1)$$

$$R_d^A, U \geq 0. \quad (2)$$

Assume that  $I \leq R_d + Y_d + \frac{1}{2}\left(\frac{1}{A} + \frac{1}{B}\right)(R_u - R_d + Y_u - Y_d)^2 - U_B = \bar{I}$ , which ensures that the entrepreneur obtains a nonnegative profit given the effort levels as the case without moral hazard.

### 2.4.2.1 Provision of Financing and Effort

In the case without moral hazard, the financing and effort provisions are separated. The value of the innovation is independent of the financial structure, i.e., the identity of the financing agent. In contrast, under moral hazard, since the entrepreneur only has limited endowment and is protected by limited liability, the financial structure matters for value. The following proposition states the relationship between the value and the financial participation of the entrepreneur.

**Proposition 2.2.** *There exists a threshold  $I^* = R_d + Y_d - \frac{1}{2B}(R_u - R_d + Y_u - Y_d)^2 - U_B$  such that the financial participation of the entrepreneur increases the value of the innovation if  $I > I^*$ , while it is neutral if  $I \leq I^*$ .*

$I^*$  represents the maximum outside investment that can be provided by the incumbent and the outside investor together given the levels of effort in the case without moral hazard. Under moral hazard, in order to induce the same effort from the entrepreneur as without moral hazard, the minimum income that the entrepreneur must obtain is  $R_u - R_d + Y_u - Y_d$  in case of success and 0 in case of failure. Hence, the maximum expected income left for the incumbent and the outside investor is  $R_d + Y_d$ . As the incumbent participates in the strategic alliance, it also costs him a disutility of effort  $\frac{1}{2B}(R_u - R_d + Y_u - Y_d)^2$  and an outside option value  $U_B$ . Therefore, the maximum total expected income for the incumbent and the outside investor is  $R_d + Y_d - \frac{1}{2B}(R_u - R_d + Y_u - Y_d)^2 - U_B$ , corresponding to their maximum investment.

If the initial investment is small ( $I \leq I^*$ ), the project can be entirely financed by outside capital. The financial participation of the entrepreneur is neutral for incentives and hence for value. If the initial investment is large ( $I > I^*$ ), in order to preserve the incentives for the entrepreneur and the incumbent, the financing provided by the entrepreneur cannot be lower than  $I - I^*$ . Otherwise, the incentives are distorted and the value of the innovation is reduced. The next proposition states in detail the relationship between the entrepreneur's endowment and the effort choices of both agents.

**Proposition 2.3.** *In the case where  $I > I^*$ , the entrepreneur and the incumbent exert the same effort as without moral hazard if  $E_A \geq I - I^*$ , otherwise, both exert less effort than without moral hazard.*

In the case where  $I > I^*$ , if  $E_A \geq I - I^*$ , the entrepreneur can at least contribute  $I - I^*$  amount of investment. It preserves the incentives of both agents as the case without moral hazard. However, if  $E_A < I - I^*$ , the incumbent and the outside investor must provide financing  $I - E_A$ , which is greater than  $I^*$ .

In this case,  $R_d^A = 0$ . The two participation constraints are always binding. The total outside investment is

$$R_d + Y_d + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d - U) - \frac{1}{2B}V^2 - U_B. \quad (2.26)$$

Intuitively, recouping the investment for the entrepreneur and the incumbent requires a reduction in the entrepreneur's stake in the innovation. According to equation (2.26), this reduction will increase the stake for the incumbent and the outside investor but simultaneously has a counter effect, i.e., reducing the probability of success. This cost is increasing with a reduction in  $U$ . Hence, to increase the outside investment from  $I^*$  to  $I - E_A$ , we should also reduce the incumbent's stake ( $V$ ) in the innovation. Even though a reduction in  $V$  leads to less success, it saves the effort cost of the incumbent. This marginal cost saving benefit is big when  $V$  is large. As a result, both the entrepreneur's and the incumbent's reduction in their stakes results in less effort than the case without moral hazard.

Until now, we have illustrated that under moral hazard, the financial participation of the entrepreneur has a crucial impact on the real decisions and the value of the innovation. The basic intuition is that, agency costs reduce the maximum outside financing that can be raised by the entrepreneur. In this case, his own endowment can help mitigate this agency problem and enhance the value of innovation. The following proposition turns to the impact of the financial participations of the incumbent and the outside investor.

**Proposition 2.4.** *The incumbent and the outside investor must provide outside financing not smaller than  $I - E_A$ . However, the identity of the agent providing outside financing is irrelevant for value.*

The remaining financing  $I - E_A$  can be provided by the incumbent, the outside investor or both. Since the incumbent and the outside investor are deep-pocket, they can make transfers to each other when the innovation matures. Hence, the income for the incumbent in case of failure  $R_d^B$  can take any value. With a different  $R_d^B$ , to make the incumbent willing to participate in the strategic alliance, we just need adjust his initial investment  $I_B$ . Hence, the ex-post flexibility in making transfers between the incumbent and the outside investor ensures that the identity of the agent providing the outside financing ex-ante is irrelevant for value.

This irrelevance result is in contrast to Casamatta [65]. Casamatta [65] argues that financing and advising must go hand in hand for Venture Capitalists. However, in this paper, financing does not need go hand in hand with advising for corporate investors. The reason is that the incumbent and the outside investor are deep-pocket and not

protected by limited liability ex-post. In contrast, in Casamatta [65], all agents are protected by limited liability regardless of their financial situations. The present irrelevance result is consistent with the empirical observation of Robinson and Stuart [57] that up-front payments from pharmaceutical firms to biotechnology *R&D* firms are sometimes not requested.

As a result, the identity of the agent providing the outside financing is irrelevant while the provision of internal financing from the entrepreneur is crucial in affecting the real decisions. The entrepreneur's endowment affects the real decisions through his financial constraints. Next, we turn to study in detail the impact of the entrepreneur's endowment  $E_A$ .

### 2.4.2.2 Financial Constraints

In the presence of agency problems, access to outside financing is limited by the combination of the incentive and participation constraints. To measure the extent to which the firm is financially constrained, I propose to use the shadow value of this constraint, i.e., the lagrange multiplier of the outside investment constraint (1) in Program (2.25).

**Proposition 2.5.** *The shadow value  $\lambda$  satisfies*

$$f(\lambda, A, B)(R_u - R_d + Y_u - Y_d)^2 = I - E_A - (R_d + Y_d) + U_B, \quad (2.27)$$

where  $f(\lambda, A, B) = \frac{(1 + \lambda)(2A^2\lambda - AB(\lambda - 1)^2(\lambda + 1) + 2B^2\lambda(1 + \lambda)^2)}{2A(-A\lambda^2 + B(1 + 3\lambda + 2\lambda^2))^2}$ , increasing in  $\lambda$ .

The shadow value of the outside investment constraint  $\lambda$  reflects the conflict between the incentives and the investment constraint. If it is more difficult for the entrepreneur to get an amount  $I - E_A$  of investment from the incumbent and the outside investor,  $\lambda$  will become larger. Hence,  $\lambda$  actually measures the tightness of the entrepreneur's financial constraint.

According to equation (2.27), we can simply do comparative statics to study the impact of different parameters on the tightness of the financial constraint. Based our previous analysis, the entrepreneur's endowment is very important in determining the tightness of financial constraints. In the following, we focus on the impact of the entrepreneur's endowment on the tightness of firms' financial constraints.<sup>5</sup>

**Proposition 2.6.**

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<sup>5</sup>See the impact of other parameters on  $\lambda$  in Appendix.

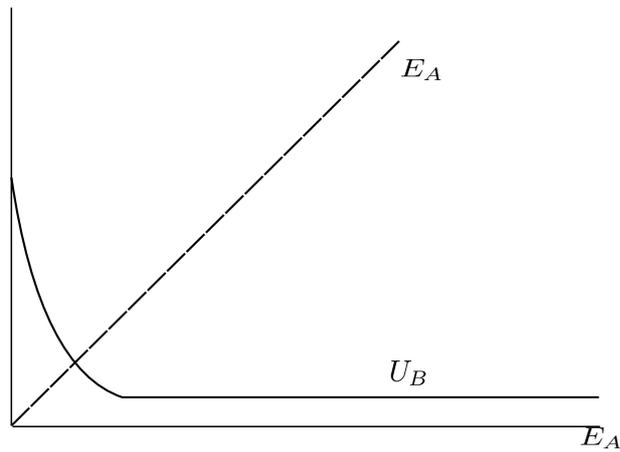
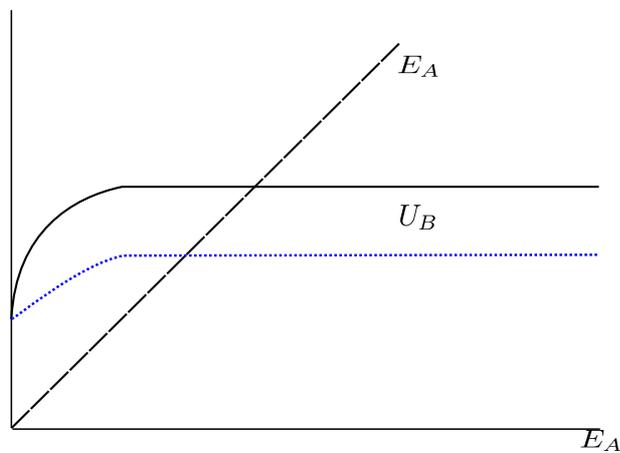
(a)  $Y_u^s - Y_d^s \leq 0$ (b)  $Y_u^s - Y_d^s > 0$ 

FIGURE 2.2: The solid depicts the outside option value  $U_B$  the incumbent receives in the case of stand-alone operation. The dashed line is the entrepreneur's endowment  $E_A$ . In (b), the dotted line also represents the outside option value of the incumbent  $U_B$ . For the dotted line, the marginal effect of the endowment  $E_A$  on  $U_B$  is always lower than 1, while for the solid line, this marginal effect is first greater than 1 and then decreases to 0.

If  $Y_u^s - Y_d^s \leq 0$ , an increase in the endowment of the entrepreneur  $E_A$  reduces the outside option value  $U_B$  and relaxes the financial constraint.

If  $Y_u^s - Y_d^s > 0$ , an increase in the endowment  $E_A$  raises the incumbent's outside option  $U_B$ . The financial constraint is tightened with the endowment if  $\frac{\partial U_B}{\partial E_A} > 1$ . Otherwise, the financial constraint is loosened.

If  $Y_u^s - Y_d^s \leq 0$ , the success of the innovation harms the incumbent. An increase in the endowment leads to an increase in the probability of success in the stand-alone case. Hence, as the endowment increases, the outside option value  $U_B$  the incumbent receives declines and he would like to provide more investment, leading to a relaxation in the financial constraint. This is depicted in Figure 2.2(a).

In contrast, if  $Y_u^s - Y_d^s > 0$ , the outside option value  $U_B$  increases with  $E_A$ . The change in the financial constraint depends on the marginal effect of the endowment on the outside option value  $U_B$  the incumbent receives in the stand-alone case. If this marginal effect is greater than 1, the outside option value  $U_B$  increases more quickly than the endowment. In this case, the financial constraint is tightened with the endowment. Otherwise, the financial constraint is loosened.

**Corollary 2.7.** *When the development of the innovation has a positive external effect on the profits of the incumbent ( $Y_u^s - Y_d^s > 0$ ), the financial constraint is relaxed by an increase in the entrepreneur's endowment if  $Y_u^s - Y_d^s$  is small, i.e.  $Y_u^s - Y_d^s \leq \sqrt{(R_u^s - R_d^s)^2 - 4A(I - R_d^s)}$ . Otherwise, the financial constraint is tightened by an increase in the entrepreneur's endowment, if the latter is low in the sense that  $E_A \in [0, \min\{\frac{(Y_u^s - Y_d^s)^2 - (R_u^s - R_d^s)^2}{4A}, 0\} + I - R_d^s)$ .*

In Figure 2.2(b), the dotted line indicates the financial constraint is loosened if the endowment goes up, since the marginal effect is always lower than 1. As indicated in the Corollary 2.7, this happens when the externality that the incumbent receives is sufficiently low. Otherwise, the financial constraint is initially tightened and then loosened as implied in the solid line. This is because the marginal effect is initially greater than 1 and eventually decreases to 0.

This result challenges the traditional view that the increase in the internal capital relaxes financial constraints. Our result indicates that if the firm generates externality on the financier, the impact of the internal capital on financial constraints can be non-monotonic. The next proposition illustrates the impact of the entrepreneur's endowment on effort.

**Proposition 2.8.** *The entrepreneur's effort and the probability of success of the innovation increases with  $E_A$  when  $\frac{\partial U_B}{\partial E_A} < 1$ , and decreases when  $\frac{\partial U_B}{\partial E_A} > 1$ .*

According to Program 2.25, the entrepreneur's endowment  $E_A$  affects the effort choices only through the outside investment constraint. When  $\frac{\partial U_B}{\partial E_A} > 1$ , the increase in the endowment  $E_A$  tightens the financial constraint  $\lambda$ . The maximization program (2.25) puts less weight on the value of innovation and more weight on the investment constraint. Hence, we should reduce the stake for the entrepreneur to boost the outside investment. In this case, the entrepreneur exerts less effort. The probability of success also declines

when the conflict between incentives and the investment constraint becomes larger.<sup>6</sup> On the other hand, when  $\frac{\partial U_B}{\partial E_A} < 1$ , the increase in the endowment  $E_A$  loosens the financial constraint and thereby increases the entrepreneur's effort and the probability of success.

### 2.4.2.3 Budget Breaker

In this double moral hazard framework, both the entrepreneur's and the incumbent's efforts are unobservable. The total output is observable and determined by the efforts of both agents. As in Holmstrom [64], in order to deal with the free riding problem, we need a third party to punish both agents in case of failure.

Denote  $W$  the revenue difference between the two states for the outside investor.

**Proposition 2.9.** *The outside investor works as a budget breaker by obtaining a higher payoff in case of failure than in case of success, i.e.,  $W < 0$ .*

In our setup, without the outside investor, the payoffs for the entrepreneur and the incumbent must satisfy the budget constraint, i.e.,  $U + V = R_u - R_d + Y_u - Y_d$ . However, introducing the outside investor, we have  $U + V + W = R_u - R_d + Y_u - Y_d$ . It is shown that  $W$  is optimally to be negative, thus  $U + V > R_u - R_d + Y_u - Y_d$ . It indicates that the introduction of the outside investor breaks the initial budget constraint while increase the value of the innovation. The intuition is that, the outside investor works as budget breaker by taking more income in case of failure. In this case, both the entrepreneur and the incumbent are punished when the innovation fails. In turn, this punishment in case of failure will give them better incentives to exert effort. Hence, in our setup, the outside investor not only work as a financier but also a budget breaker as in Holmstrom [64].

## 2.5 Implementation of Optimal Financial Contracts

The objective of this section is to design financial claims to provide right incentives for both the entrepreneur and the incumbent. To generate more realistic financial claims, in this section we restrict our analysis to the case where the payoffs of all players are

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<sup>6</sup>The probability of success is the sum of both agents' effort. Hence, if the entrepreneur's effort and the probability of success are known, we can directly obtain the incumbent's effort. The entrepreneur's effort is monotonic with  $E_A$  given  $\frac{U_B}{E_A} > 1$  or  $\frac{U_B}{E_A} < 1$ . However, the incumbent's effort may not be monotonic, depending on the specific value of  $\frac{A}{B}$ . Nevertheless, this non-monotonicity does not affect the monotonic characteristics of the probability of success. Hence, we only report the result on the entrepreneur's effort and the probability of success for simplicity. The results of the incumbent's effort are in Appendix.

constrained to be nondecreasing in the firm's profit. As argued in Innes [66], this monotonicity constraint ensures that the entrepreneur or the incumbent has no incentive to revise their profit reports upward with hidden borrowing.

### 2.5.1 Monotonicity Constraint

In this subsection, we turn to study the robustness of our results given the monotonicity constraint. To be a budget breaker, the outside investor should obtain a higher payoff in case of failure. However, with monotonicity constraint, the payoff of each agent should be nondecreasing in the firm's payoff. Hence, the monotonicity constraint must have a critical impact on the role of the outside investor as a budget breaker.

**Proposition 2.10.** *With monotonicity constraint, the outside investor can work as a budget breaker if  $Y_u - Y_d < 0$ , otherwise, he is a pure financier.*

The final outcome of the entrepreneur firm is  $R_u$  in case of success and  $R_d$  in case of failure. Its externality on the incumbent firm is  $Y_u$  and  $Y_d$  respectively. The outside investor, as a budget breaker, should obtain a higher revenue in case of failure. Since the revenue difference in the entrepreneur firm is always positive, i.e.,  $R_u - R_d > 0$ , whether the outside investor can work as a budget breaker only depends on the sign of the externality difference  $Y_u - Y_d$  in the incumbent firm. If the innovation generates a positive external effect in the incumbent, i.e.,  $Y_u - Y_d \geq 0$ , the outside investor can only work as a pure financier without violating the monotonicity constraint. If the innovation has a negative external effect, i.e.,  $Y_u - Y_d < 0$ , the outside investor can work as a budget breaker by granted a share in the incumbent firm.

In this case, the maximization program 2.25 should incorporate the monotonicity constraint as following.

$$U + V \leq \begin{cases} R_u - R_d + Y_u - Y_d & Y_u - Y_d \geq 0 \\ R_u - R_d & Y_u - Y_d < 0. \end{cases} \quad (2.28)$$

If  $Y_u - Y_d \geq 0$ , the outside investor cannot work as a budget breaker, i.e.,  $W \geq 0$ . Hence,  $U + V = R_u - R_d + Y_u - Y_d - W \leq R_u - R_d + Y_u - Y_d$ . If  $Y_u - Y_d < 0$ , the outside investor can work as a budget breaker, but his revenue difference cannot be lower than that provided in the incumbent firm, i.e.,  $W \geq Y_u - Y_d$ . Thus,  $U + V = R_u - R_d + Y_u - Y_d - W \leq R_u - R_d$ .

With the monotonicity constraint, the incentives can be different for the agents since the total revenue they obtain in case of success exceeding in case of failure is constrained.

However, all the propositions without the monotonicity constraint have not been qualitatively changed, while the quantitative results are much more complex with monotonicity constraints. Please refer to Appendix for the detailed discussion on the robustness.

### 2.5.2 Financial Claims

The objective of this subsection is to study the financial claims that provide right incentives for both the entrepreneur and the incumbent without violating the monotonicity constraint. The following proposition states that the design of the financial claims depends on the spillover effect on the incumbent.

**Proposition 2.11.**

1) If  $Y_u - Y_d < 0$ , the incumbent holds preferred equity or equity in the entrepreneur firm while the entrepreneur does not hold equity in the incumbent firm. The outside investor holds equity in the incumbent firm to work as a budget breaker.

2) If  $Y_u - Y_d \geq 0$ ,

2.1) If  $U < R_u - R_d$ , the entrepreneur and the incumbent hold the same type of financial claims as in 1).<sup>7</sup>

2.2) If  $U \geq R_u - R_d$ , the incumbent holds debt or nothing in the entrepreneur firm, and the entrepreneur holds equity in the incumbent firm when  $U > R_u - R_d$  while does not when  $U = R_u - R_d$ .

2.3) The outside investor can be excluded since he cannot be a budget breaker.

Proposition 2.11 indicates that the externality has a pivotal impact on the optimal financial claims held by the entrepreneur and the incumbent. In the model, the level of effort of each agent is determined by the revenue difference he receives between success and failure. When the externality is negative, the revenue difference in the entrepreneur firm  $R_u - R_d > 0$  while that in the incumbent firm  $Y_u - Y_d < 0$ . In this case, the revenue in the entrepreneur firm is sufficient to incentivize the entrepreneur while the revenue in the incumbent firm cannot induce effort from the incumbent. Thus, the incumbent must hold equity or preferred equity in the entrepreneur firm. Nevertheless, the entrepreneur does not need to hold equity in the incumbent firm since it will dilute his incentives.

When the externality is positive,  $U + V = R_u - R_d$ . If  $U < R_u - R_d$ , the entrepreneur can be still fully incentivized through a share of revenue in his own firm while it is not the case for the incumbent. Thus, we obtain the same result as in the case of negative

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<sup>7</sup> $U$  is the optimal rent for the entrepreneur in the case with monotonicity constraint.

externality. If  $U \geq R_u - R_d$ ,  $V \leq Y_u - Y_d$ , i.e., a share of revenue in the incumbent firm can induce sufficient effort from the incumbent. Hence, the incumbent holds either debt or nothing in the entrepreneur firm. For the entrepreneur, if  $U > R_u - R_d$ , the revenue in the entrepreneur firm is not enough to incentivize him, thus, he should hold equity in the incumbent firm, while not the case when  $U = R_u - R_d$ .

The specific financial instruments the incumbent hold in the entrepreneur firm, such as preferred equity or common equity when  $U < R_u - R_d$ , debt or nothing when  $U \geq R_u - R_d$ , depend on the total outside investment contribution from the incumbent and the outside investor. Denote  $\mathcal{S}'$  as this outside investment, and  $\mathcal{S}$  as the maximum outside investment given the effort levels in the case where the entrepreneur is not financially constrained.

To be a budget breaker, the outside investor cannot hold equity in the entrepreneur firm, since it harms his incentives. It is possible for the outside investor to hold debt in the entrepreneur firm. Nevertheless, this has nothing to do with incentives. Without loss of generality, we neglect the possibility for the outside investor to hold debt in the entrepreneur firm. The revenue in the entrepreneur firm is only shared by the entrepreneur and the incumbent.

**Proposition 2.12.**

- 1) *In the case where  $U < R_u - R_d$ , there exists a threshold  $\mathcal{S}^* < \mathcal{S}$ , such that if  $\mathcal{S}' \leq \mathcal{S}^*$ , the entrepreneur holds preferred equity while the incumbent holds common equity in the entrepreneur firm. If  $\mathcal{S}' > \mathcal{S}^*$ , the entrepreneur holds common equity while the incumbent holds preferred equity in the entrepreneur firm.*
- 2) *In the case where  $U \geq R_u - R_d$ , there exists another threshold  $\mathcal{S} - R_d$ , such that if  $\mathcal{S}' \leq \mathcal{S} - R_d$ , the entrepreneur holds equity while the incumbent holds nothing in the entrepreneur firm. If  $\mathcal{S}' > \mathcal{S} - R_d$ , the entrepreneur holds equity while the incumbent holds debt in the entrepreneur firm.*

In the model, with the same expected final income, equity provides more powerful incentives than preferred equity, while debt does not provide any incentives. If the amount of outside investment is small and so is the expected income. In order to induce enough effort, the incumbent must be given claims with higher-powered incentives. Thus, if  $U < R_u - R_d$ , the incumbent is not incentivized sufficiently given the income in his own firm and must be granted with equity in the entrepreneur firm. If  $U \geq R_u - R_d$ , the incumbent already exerts enough effort given the income from his own firm and does not need to hold any claim in the entrepreneur firm. If the amount of outside financing is large and so is the expected income. In this case where  $U < R_u - R_d$ , if the incumbent

is given equity, this would make him exert too much effort. Thus, in order to recoup his investment without distorting incentives, the incumbent must be granted with preferred equity. Similarly, in the case where  $U \geq R_u - R_d$ , the incumbent should hold debt.

Proposition 2.11 and 2.12 rationalizes the use of preferred equity or convertible debt in the entrepreneur firm for the incumbent.<sup>8</sup> It is consistent with the empirical observation of widely-used convertible claims in corporate venture capital contracts by Cumming [67] and in biotech strategic alliances by Robinson and Stuart [57]. Comparing with the results for venture capital contracts in Casamatta [65], we obtain that pure debt securities are more likely to occur if the incumbent rather than the venture capitalist participates in the innovation. It is because with a positive externality, the incumbent is already incentivized to some extent given a claim in his own firm and only needs to hold securities which provides less powerful incentives, such as debt, in the entrepreneur firm. This phenomenon is also evidenced by Cumming [67]. He finds that Canadian corporate venture capitalists are more likely to use non-convertible debt than Canadian limited partnership venture capitalists.

Proposition 2.11 also implies that when the entrepreneur is more financially constrained, his stake  $U$  is reduced. In this case, it is more common for the incumbent to hold equity in the entrepreneur firm and not the reverse. Nevertheless, when the entrepreneur becomes less financially constrained, reciprocal holdings between firms become more popular. We use the firm size to indicate the tightness of its financial constraint. Generally speaking, big firms are less financially constrained while small firms are more financially constrained.<sup>9</sup> In the United States, big established firms often hold equity in small entrepreneur firms while reciprocal shareholding is very rare.<sup>10</sup> Nevertheless, in Japan, most big firms are affiliated with a financial keiretsu. The main features of the financial keiretsu are extensive inter-firm trading and cross-holdings of debt and equity. Thus, the implication of our theory is in line with the empirical observation on the difference of cross-holdings among firms in the United States and Japan.

## 2.6 Organizational Structure

This section sheds some light on how to structure the organization to facilitate innovation. The innovation can be operated either by the entrepreneur alone, through strategic alliance or within the incumbent. The first two organizational structures have already

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<sup>8</sup>In this paper, there is no difference between preferred equity and convertible debt, just as Casamatta [65].

<sup>9</sup>Internal capital market, such as Gertner et al. [25], Stein [26] and Stein [68], indicates that big firms are less financially constrained.

<sup>10</sup>See Allen and Philips [69]

been analyzed. The third one means that the incumbent hires the entrepreneur and develops this new product within the boundary of the incumbent.

**Proposition 2.13.**

*If  $R_u = R_u^s$ ,  $R_d = R_d^s$ ,  $Y_u = Y_u^s$  and  $Y_d = Y_d^s$ , strategic alliances always generate larger value than stand-alone operations.*

*If  $Y_u - Y_d < 0$ , developing innovation through strategic alliances dominates within the incumbent. Nevertheless, if  $Y_u - Y_d \geq 0$ , there is no difference for innovation between through strategic alliances and within the incumbent.*

We first compare stand-alone operations and strategic alliances. In the case with the same payoff structure in strategic alliances and stand-alone operations, i.e.,  $R_u = R_u^s$ ,  $R_d = R_d^s$ ,  $Y_u = Y_u^s$  and  $Y_d = Y_d^s$ , strategic alliances generate a larger value than stand-alone operations, since the former benefits from i) joint effort support; ii) internalization of the externality. The joint effort support benefit of strategic alliances accords with the resource-based explanation of strategic alliance in Eisenhardt and Schoonhoven [72] and Das and Teng [73], which emphasizes the pooling of resources of different firms to speed up innovation or gain competitive advantage. It also has a similar spirit to the advising role of venture capitalists in Casamatta [65], where the joint provision of effort improves the productivity of an investment project. In addition, the internalization of externalities is in line with the work of Clayton and Jorgensen [74], Gilo et al. [75], Mathews [76], and Foros et al. [77], which argue that the partial ownership among firms takes account of externalities and reduces ex-post competition. Despite the two benefits for strategic alliances, it does not imply that strategic alliances always make the innovation more successful, especially when  $Y_u - Y_d$  is negative. Thus, the instability of strategic alliances can be a misleading indicator of their failure. This theoretical result is in line with the discussion of the difference between instability and failure in strategic alliances. <sup>11</sup>

We now turn to the comparison between innovating through strategic alliances and within the incumbent. In both cases, the entrepreneur and the incumbent collaborate. The difference is that through strategic alliances, there are two separate entities, while within the incumbent, there is only one entity. When the innovation has a positive external effect on the incumbent, monotonicity constraint precludes the intervention of a budget breaker. Nevertheless, when the innovation has a negative external effect, the intervention of a budget breaker is consistent with the monotonicity constraint, only when the incumbent and the entrepreneur are separate entities. Therefore, the optimal design rules out developing innovation within the incumbent if the innovation has a

<sup>11</sup>see e.g. Kogut [78], Kogut [79] and Park and Russo [80].

negative external effect, while does not if the innovation has a positive external effect. It predicts that, on average, agents are less incentivized to do the innovation within the incumbent. Thus, it is less profitable to innovate within the incumbent than strategic alliances, which is consistent with the finding of Seru [70] that firms acquired in mergers are less innovative, and the acquirers move R&D activity outside the boundary of the firm via the use of strategic alliances and joint-ventures.

## Chapter 3

# Learning to Speculate

### 3.1 Introduction

Do traders learn to avoid participating in speculative bubbles? This question is the object of a long-standing debate in the financial economics literature. Smith et al. [81] propose an experimental design to study speculative bubbles and show that bubbles are less likely but do not disappear with experience. This result is confirmed by King et al. [82]. Dufwenberg et al. [83] further show that bubbles also diminish when only part of the traders are experienced. On the contrary, using an alternative experimental design, Moinas and Pouget [84] show that traders' propensity to speculate do not decrease after several rounds of play.

The present paper studies whether traders learn to speculate in the context of the bubble game designed by Moinas and Pouget [84]. In this game, trading proceeds sequentially, traders' position in the sequence is random, and prices increase exponentially. When there is a price cap, there is no bubble at the dominance-solvable Bayesian Nash equilibrium: confronted with the highest potential price, a rational trader refuses to buy. Anticipating this behavior, a rational trader receiving the second highest price should also refuse to buy. Backward induction thus rules out the formation of bubbles (the higher the price cap, the higher the number of iterated reasoning steps needed to reach equilibrium). However, when traders are boundedly rational, bubbles can emerge. We study whether experience reduces the propensity to speculate.

We capture traders' learning process using Camerer and Ho [85]'s Experience-Weighted Attraction model. This adaptive learning model is general in the sense that it nests belief-based learning and reinforcement learning. A crucial parameter in this model is the imagination parameter. When it is equal to 0, agents only reinforce chosen actions,

as it is the case in reinforcement learning. When the imagination parameter is greater than 0, agents also reinforce actions that were not actually chosen, as it is implicitly assumed in belief-based learning. Using Camerer and Ho [85]’s model is useful because it enables us to study whether adaptive traders’ speculating behavior depends on the learning process.

To obtain our result, we simulate traders’ behavior with 1,000 independent trials that each include 1,000 successive runs. Traders’ attraction towards the various actions are transformed into choice probabilities via a logistic function with a given payoff responsiveness parameter. When this parameter is 0, players choose each action with the same probability, while when it is infinite, players choose with probability one the action with the highest attraction.

Our results show that, in the long run, the market converges to the unique no bubble equilibrium. This is to be expected given that the no-bubble equilibrium is unique and dominance solvable. However, we show that learning initially increases traders’ propensity to speculate. In the short run, more experienced traders thus create more bubbles. Moreover, we show that this effect is stronger when traders are more sophisticated (that is, when they have a higher degree of imagination) and when the price cap is higher. Our results are robust if i) the exogenous price path is more or less explosive, ii) traders are randomly assigned positions at each run, iii) price caps are different, and iv) we allow traders to choose the price at which they propose to sell.

Overall, our results reconcile the findings of the experimental literature: when a few steps of reasoning are needed due a short experiment (in the setting of Smith et al. [81]) or to a low price cap (in the setting of Moinas and Pouget [84]), learning shuts down speculation rapidly, in line with results of King et al. [82]. On the contrary, when a lot of steps of reasoning are needed, learning does not reduce speculation (at least in the short and medium run), in line with the results of Moinas and Pouget [84].

The rest of the paper is organized as follows. The next section presents the setting we use to study speculation. Section 3 presents the baseline results. Section 4 offers robustness checks.

## 3.2 The bubble game and the learning model

Our baseline setting derives from Moinas and Pouget [84]. Consider a valueless financial asset that can be traded in a sequential market. Traders are equally likely to be in each position in the market sequence. If a trader is proposed to buy the asset, he can choose

whether to accept or to refuse. If he refuses, trading stops. If he accepts, he proposes to sell to the next trader. We focus on the case with three traders.

In the baseline setting, prices are exogenous. The first trader is offered a price  $10^n$ , where  $n$  is random and follows a geometric distribution:  $P(n = j) = \frac{1}{2}^{j+1}$ . Each following trader is (potentially) offered a price that is ten times higher than the previous price. This setting is such that no trader can ever be sure to be last in the market sequence despite prices revealing some information regarding traders' position. When rationality is common knowledge, Moinas and Pouget [84] show that, when there is a price cap, the unique dominance solvable equilibrium involves no-trade (and thus no bubble): at equilibrium all traders refuse to buy the asset (when there is no cap on prices, bubbles can arise at equilibrium).

To study how speculation decisions depend on previous experiences, we consider that traders adopt an adaptive behavior and adjust their choices according to past performance. We capture adaptive behavior according to Camerer and Ho [85]' Experience-Weighted Attraction model that nests reinforcement and belief-based learning. Specifically, the attraction of action  $a_i^j$  for agent  $i$  at time  $t$  is governed as follows.

$$A_i^j(t) = \frac{\phi N(t-1)A_i^j(t-1) + [\delta + (1-\delta)1_{a_i^j=a_i(t)}]\pi(a_i^j, a_{-i}(t))}{N(t)} \quad (3.1)$$

$N(t)$  is the experience parameter:

$$N(t) = \rho N(t-1) + 1 \quad (3.2)$$

where  $\rho$  is the depreciation parameter for the pervious-period experience.

In equation(3.1),  $\phi$  controls the depreciation of previous attractions;  $\pi(a_i^j, a_{-i}(t))$  is the profit for agent  $i$  to choose action  $a_i^j$  given other agents choose action  $a_{-i}(t)$ ;  $1_{a_i^j=a_i(t)}$  is an indicator function, which is equal to 1 if  $a_i^j = a_i(t)$  and 0 otherwise; when  $\delta = 0$ , agent  $i$  reinforce the profit of action  $a_i^j$  only if it is selected at date  $t$ , while when  $\delta > 0$ , agent  $i$  reinforce the profit of action  $a_i^j$  no matter whether it is actually chosen or not.  $\delta$  is the imagination parameter that controls how much agents are able to display counterfactual reasoning.

According to the attraction, agent  $i$  decides the probability to choose action  $a_i^j$  as following:

$$Pr_i^j(t+1) = \frac{e^{\lambda A_i^j(t)}}{\sum_k e^{\lambda A_i^k(t)}} \quad (3.3)$$

where  $\lambda$  represents the sensitivity of agents to attractions. Equation(3.3) indicates that the probability for agent  $i$  to choose action  $a_i^j$  is determined by its relative attraction in the previous period.

This adaptive learning model captures both the law of actual effect and the law of simulated effect. The law of actual effect means that the attraction of an action is adjusted only if this action has been selected ( $\delta = 0$ ). If the action generates a positive profit, this action will be more attractive, otherwise, it will be less attractive. This law is at the core of reinforcement learning (see, for example, Roth and Erev [86]). The law of simulated effect indicates that the attraction of an action is adjusted according to the profit it could have generated even if it has not been selected ( $\delta > 0$ ). This law is at the core of belief-based learning (see, for example, Fudenberg and Levine [87]).

In this trading game, at any given price, traders need to determine whether to buy the asset or not. Thus, the actions for any trader is to buy or not conditional on a price. According to equation(3.3), the probability to buy for each trader and for a given price  $P$  at date  $t + 1$  is determined by:

$$Pr_i^B(t + 1|P) = \frac{e^{\lambda A_i^B(t|P)}}{e^{\lambda A_i^B(t|P)} + e^{\lambda A_i^\Phi(t|P)}} = \frac{1}{1 + e^{-\lambda A_i^B(t|P)}}, \quad (3.4)$$

where  $A_i^B(t|P)$  and  $A_i^\Phi(t|P)$  are the attractions for agent  $i$  of accepting or refusing to buy respectively given price  $P$  at date  $t$ , and  $A_i^\Phi(t|P) = 0$ .

To close the system, the initial values  $N(0)$  and  $A_i^j(0)$  need to be specified. We set  $N(0) = 1$  and  $A_i^j(0) = 0$ . When  $A_i^j(0) = 0$ , traders initially choose each action with the same probability. In addition, we also set  $\rho = 0$  and  $\phi = 1$ . These parameter values are identical to the ones used by Pouget [88]. Finally, our baseline simulations uses a  $2 \times 2$  design with  $\delta$  being equal to 0 or 1, and  $\lambda$  being equal to 1 or 1,000. Our simulation proceeds as follows: for a given set of parameters, each simulation contains 1,000 independent trials; each trial contains 1,000 runs, where each run represents one trading session. For each trial, at the beginning each trader is randomly assigned a position in the trading game and this position is fixed with all the future runs.

### 3.3 Adaptive traders and speculation: a simple case

In this section, we look at a simple case where the probabilities to buy for the first and last traders are always fixed and study the speculative behavior of the second trader. Denote the probability to buy for the first, the second and the last trader are  $p_b$ ,  $p_t$  and

$p_s$  respectively. The second trader is always assigned a price  $P$ . He can sell the asset at price  $10P$ .

### 3.3.1 Case 1: $\delta = 1$

We first consider the case where  $\delta = 1$  and study the evolution of the attraction and the probability to buy for the second trader.

At date  $t$ , with probability  $p_b p_s$ , the first and third traders buy the asset. The second trader gains  $9P$  if he chooses to buy the asset. Thus, his attraction of buying the asset at date  $t$  is

$$A_2^B(t|P) = A_2^B(t-1|P) + 9P. \quad (3.5)$$

With probability  $p_b(1-p_s)$ , the first trader buys the asset while the third one does not. The second trader gains  $-P$  if he buys the asset. Thus, his attraction of buying the asset is

$$A_2^B(t|P) = A_2^B(t-1|P) - P. \quad (3.6)$$

With probability  $1-p_b$ , the first trader does not buy the asset. The second trader gain nothing if he buys the asset. Thus,

$$A_2^B(t|P) = A_2^B(t-1|P). \quad (3.7)$$

Denote  $X_t = A_2^B(t|P) - A_2^B(t-1|P)$ , which is a random variable satisfying

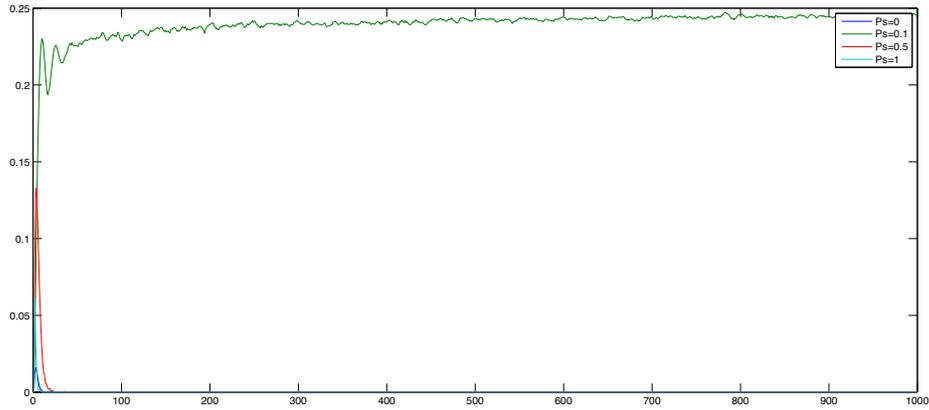
$$X_t = \begin{cases} 9P, & \text{w.p. } p_b p_s \\ 0, & \text{w.p. } 1 - p_b \\ -P, & \text{w.p. } p_b(1 - p_s) \end{cases}.$$

$X_t$  represents the incremental attraction of buying at date  $t$ .  $X_t$  is i.i.d. Thus,  $A_2^B(t|P) = \sum_{i=1}^t X_i$  is a random walk.

Hence, the expected attraction of buying at date  $t$  for the second trader is

$$E(A_2^B(t|P)) = \sum_{i=1}^t E(X_i) = P p_b (10 p_s - 1) t. \quad (3.8)$$

Denote  $\mu = P p_b (10 p_s - 1)$ .

FIGURE 3.1: Volatility of  $p_t$  when  $\lambda = 1$ ,  $\delta = 1$  and  $p_b = 0.6$ 

**Proposition 3.1.** *If  $p_b > 0$  and  $p_s < \frac{1}{10}$ , the probability to buy for the second trader  $p_t$  converges to 0 in probability. If  $p_b > 0$  and  $p_s > \frac{1}{10}$ , the probability to buy for the second trader  $p_t$  converges to 1 in probability.*

*Proof:* If  $p_b > 0$  and  $p_s < \frac{1}{10}$ , the probability to buy for the second trader  $p_t$  converges to 0 in probability.

Denote  $\bar{X} = \frac{1}{t} \sum_{i=1}^t X_i$ . According to Hoeffding's inequality,  $\forall \epsilon > 0$ , we obtain that

$$Pr(|\bar{X} - \mu| \geq \epsilon) \leq 2e^{-\frac{2t^2\epsilon^2}{(10P)^2t}}. \quad (3.9)$$

In other words,

$$Pr(|A_2^B(t|P) - \mu t| \geq \epsilon t) \leq 2e^{-\frac{2t^2\epsilon^2}{(10P)^2t}}. \quad (3.10)$$

Thus,

$$Pr(A_2^B(t|P) \geq (\mu + \epsilon)t) \leq 2e^{-\frac{2t^2\epsilon^2}{(10P)^2t}}. \quad (3.11)$$

Since  $\lim_{t \rightarrow +\infty} 2e^{-\frac{2t^2\epsilon^2}{(10P)^2t}} = 0$ ,  $A_2^B(t|P)$  converges in probability to  $-\infty$ . Denote  $f(x) = \frac{1}{1+e^{-\lambda x}}$ .  $f(x)$  is a continuous function of  $x$ . If  $A_2^B(t|P)$  converges in probability to  $-\infty$ , according to continuous mapping theorem,  $f(A_2^B(t|P))$  converges in probability to  $f(-\infty) = 0$ . That is, the probability to buy for the second trader converges in probability to 0 when  $p_s < 0.1$  and  $p_b > 0$ .

*Proof:* If  $p_b > 0$  and  $p_s > \frac{1}{10}$ , the probability to buy for the second trader  $p_t$  converges to 1 in probability.

Similar to the previous case.

If  $p_b = 0$  or  $p_s = \frac{1}{10}$ , the expected attraction of buying for the second trader is always 0. However, his probability to buy does not converge to a constant in probability. This can be seen in Figure 3.1, where the volatility of  $p_t$  in this case does not converge to 0.

**Proposition 3.2.** *If  $p_b = 0$ ,  $E_t(p_{t+1}) = p_t$ . If  $p_b > 0$ ,  $E_t(p_{t+1}) > p_t$  iff  $p^* < p_s \leq 1$ , where  $p^* = \frac{1+p_t(e^{9\lambda P}-1)}{\sum_{i=0}^9 e^{i\lambda P}}$ .*

*Proof:*

At date  $t$ , the incremental attraction of buying for the second trader have three possible cases. Hence, his expected probability to buy at  $t + 1$  is

$$E_t(p_{t+1}) = p_b p_s \frac{1}{1 + e^{-\lambda(A_2^B(t-1|P)+9P)}} + p_b(1-p_s) \frac{1}{1 + e^{-\lambda(A_2^B(t-1|P)-P)}} + (1-p_b) \frac{1}{1 + e^{-\lambda A_2^B(t-1|P)}}. \quad (3.12)$$

According to equation (3.4), we can show that

$$A_2^B(t-1|P) = -\frac{1}{\lambda} \ln\left(\frac{1}{p_t} - 1\right). \quad (3.13)$$

Plug equation (3.13) into equation (3.12), we obtain that

$$E_t(p_{t+1}) = p_b p_s \frac{1}{1 + \left(\frac{1}{p_t} - 1\right)e^{-9\lambda P}} + p_b(1-p_s) \frac{1}{1 + \left(\frac{1}{p_t} - 1\right)e^{\lambda P}} + (1-p_b)p_t, \quad (3.14)$$

where  $p_0 = \frac{1}{2}$ .

If  $p_b = 0$ ,  $E_t(p_{t+1}) = p_t$ .

If  $p_b > 0$ , we find that

$$E_t(p_{t+1}) - p_t = p_b \left\{ p_s \frac{1}{1 + \left(\frac{1}{p_t} - 1\right)e^{-9\lambda P}} + (1-p_s) \frac{1}{1 + \left(\frac{1}{p_t} - 1\right)e^{\lambda P}} - p_t \right\} > 0. \quad (3.15)$$

holds iff  $p^* < p_s \leq 1$ , where  $p^* = \frac{1+p_t(e^{9\lambda P}-1)}{\sum_{i=0}^9 e^{i\lambda P}}$ .

When  $t = 0$ , we find that  $p^* = \frac{1+\frac{1}{2}(e^{9\lambda P}-1)}{\sum_{i=0}^9 e^{i\lambda P}} = \frac{1}{2} \frac{1+e^{9\lambda P}}{\sum_{i=0}^9 e^{i\lambda P}} < \frac{1}{2}$ .

### 3.3.2 Case 2: $\delta = 0$

In the case where  $\delta = 0$ , the attraction of buying for the second trader is updated if and only if he has chosen to buy the asset, otherwise, it will not be updated. Hence, at

date  $t$ , the attraction of buying for the second trader is updated only with probability  $p_t$ . The incremental attraction  $X_t$  satisfies

$$X_t = \begin{cases} 9P, & \text{w.p. } p_b p_s p_t \\ 0, & \text{w.p. } 1 - p_b p_t \\ -P, & \text{w.p. } p_b(1 - p_s)p_t \end{cases} .$$

With probability  $p_b p_s p_t$ , all the three traders choose to buy the asset, hence the second trader obtains  $9P$ . With probability  $p_b(1 - p_s)p_t$ , the first and the second trader buy the asset while the last does not buy. Thus, the second trader obtains  $-P$ . With probability  $1 - p_b p_t$ , the first trader does not buy or the first trader buys but the second does not buy, the second trader always obtains 0.

According to equation (3.4), and  $A_2^B(t|P) = A_2^B(t-1|P) + X_t$ , the probability to buy for the second trader at date  $t+1$  is

$$p_{t+1} = \begin{cases} \frac{1}{1 + (\frac{1}{p_t} - 1)e^{-9\lambda P}}, & \text{w.p. } p_b p_s p_t \\ p_t, & \text{w.p. } 1 - p_b p_t \\ \frac{1}{1 + (\frac{1}{p_t} - 1)e^{\lambda P}}. & \text{w.p. } p_b(1 - p_s)p_t \end{cases} \quad (3.16)$$

Hence, the probability to buy for the second trader at date  $t+1$  is

$$E_t(p_{t+1}) = p_b p_s p_t \frac{1}{1 + (\frac{1}{p_t} - 1)e^{-9\lambda P}} + (1 - p_b p_t)p_t + p_b(1 - p_s)p_t \frac{1}{1 + (\frac{1}{p_t} - 1)e^{\lambda P}}, \quad (3.17)$$

which yield the following proposition

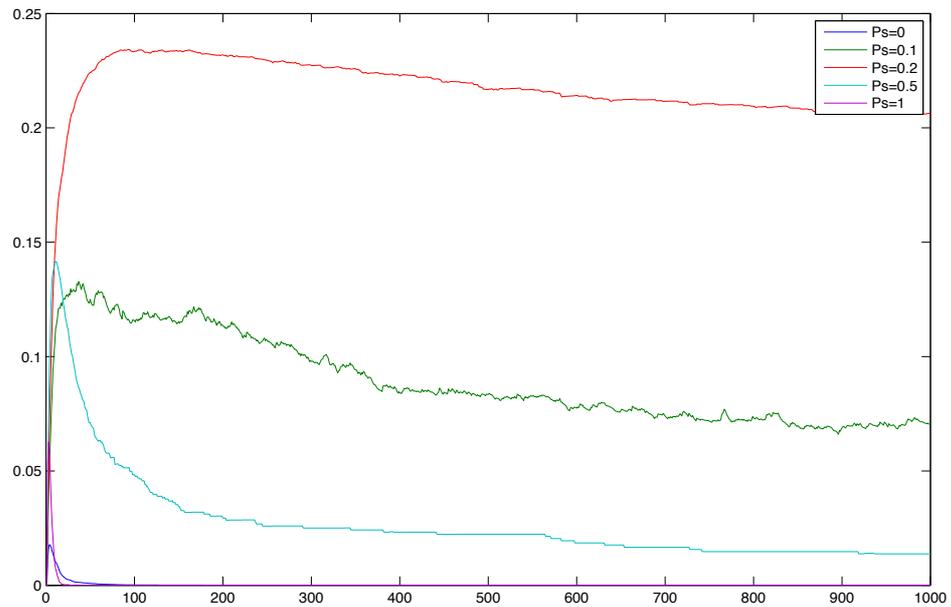
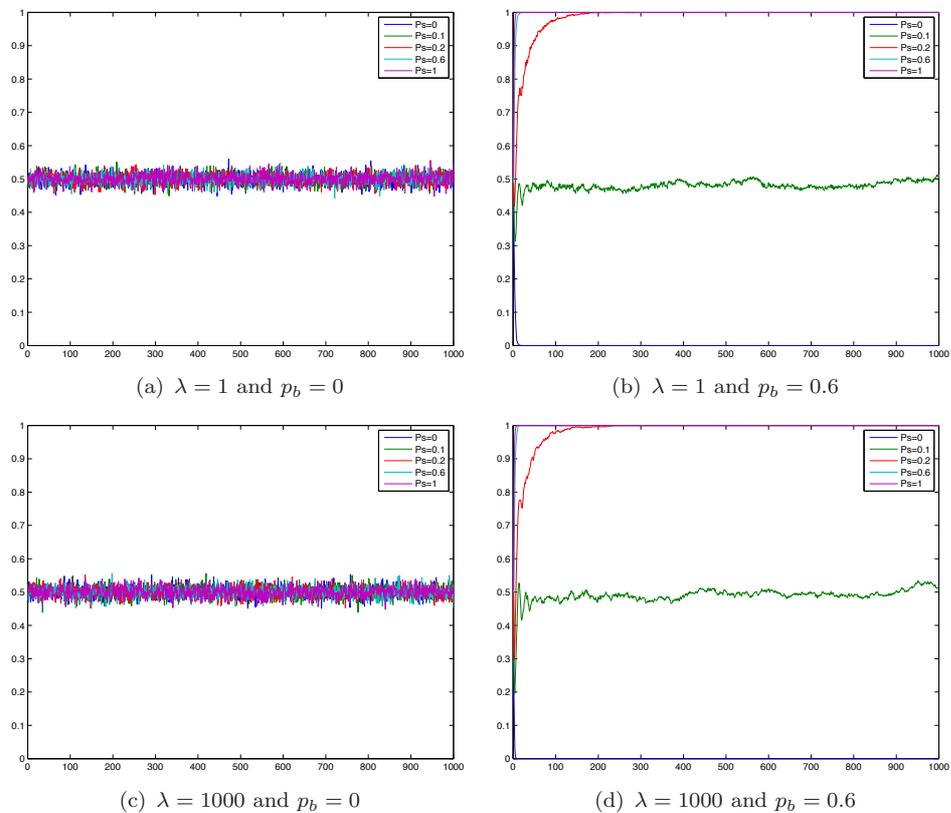
**Proposition 3.3.** *If  $p^* \leq p_s \leq 1$ ,  $E_t(p_{t+1}) \geq p_t$ , i.e.,  $p_1, p_2, p_3, \dots$  is a submartingale. If  $0 \leq p_s < p^*$ ,  $E_t(p_{t+1}) \leq p_t$ , i.e.,  $p_1, p_2, p_3, \dots$  is a supermartingale.*

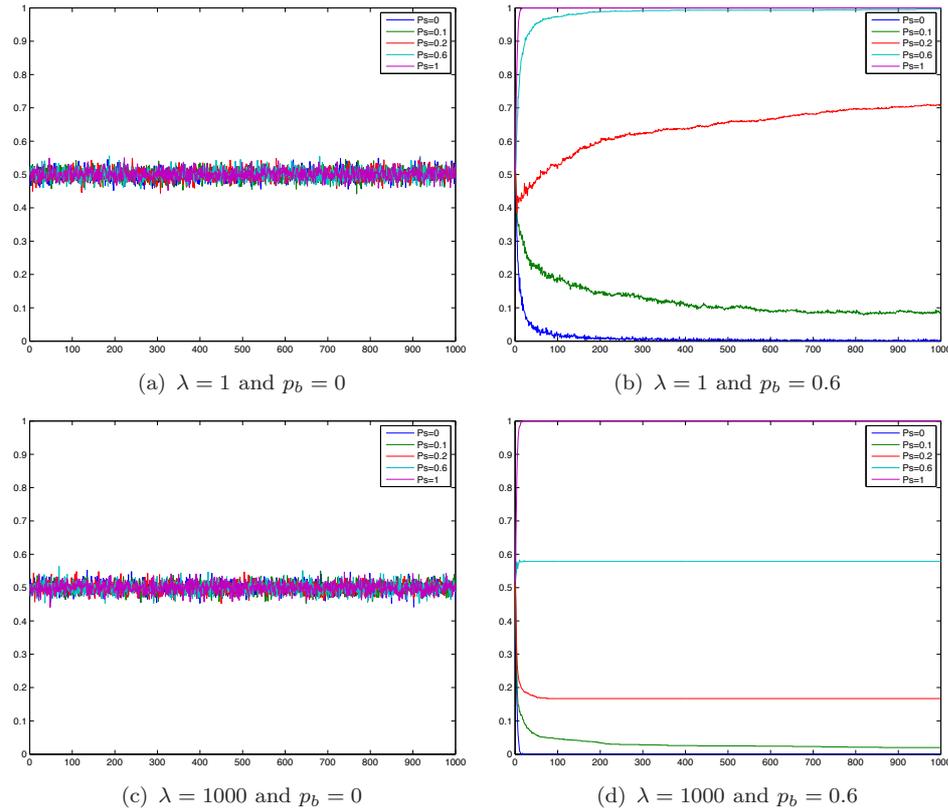
**Proposition 3.4.** *When  $t \rightarrow +\infty$ ,  $p_t$  converges almost surely. (Converge to a random variable rather than a constant)*

*Proof:* Refer to Doob's first martingale convergence theorem.

The volatility of  $p_t$  for the second trader is in Figure 3.2.

According to the simulation results in Figure 3.3 and 3.4, we can see that the probability to sell  $p_s$  has an important impact on the evolution of traders' speculation. When  $p_s$  is high, learning can lead to more speculation, since traders can benefit from it. In this section, we use the simplest framework to analyze the mechanism under which experience can lead to more speculation. In the following, we will come back to our general model and try to study speculation when all traders strategically choose their actions.

FIGURE 3.2: Volatility of  $p_t$  when  $\lambda = 1$ ,  $\delta = 1$  and  $p_b = 0.6$ FIGURE 3.3: The probability to buy for the second trader given  $p_b$  and  $p_s$ : case  $\delta = 1$

FIGURE 3.4: The probability to buy for the second trader given  $p_b$  and  $p_s$ : case  $\delta = 0$ 

## 3.4 Adaptive traders and speculation

### 3.4.1 Individual trading behavior

This section presents the individual speculative behavior of our game when the price cap is 1. Hence, the price assigned to the first, second and third trader is 1, 10 and 100 respectively. We will analyze whether and how the no bubble equilibrium can be reached when traders learn from past experience.

We first look at the case in which  $\delta = 1$  and  $\lambda = 1$ . Figure 3.5 depicts the probability to buy for individuals at different prices. Panel (a) refers to the case in which learning initially leads to more speculation for the first two traders and the last trader monotonically learns not to speculate. Since the last trader can never sell the asset, he cannot gain if he chooses to buy and therefore he learns not to participate in trading this asset. Due to lack of experience, the last trader initially chooses to buy with a high probability (50% given the initial attractions). In this case, the second trader can sell the asset at 10 times higher price with a high probability. He can gain from trading, thus leading to more speculation. This is consistent with the result in the previous section that traders are more likely to speculate with a high probability to sell the asset. As time

goes by, the last trader learns not to speculate and the second trader is then less prone to speculate. The same pattern occurs for the first trader, the only difference being that the first trader is more inclined to buy the asset due to the higher speculation of the second trader. As a result, the convergence to no speculation is very slow for the first trader (after 200 runs). Note that the probability to buy of the last trader does converge to zero, since traders early in the market sequence learn not to speculate, the last trader is very rarely offered the opportunity to buy (and thus to learn). Based on our quantitative result in the previous section, we know as long as the previous trader buys the asset with a positive probability, the trader learns not to speculate if the latter trader chooses to buy the asset with a probability lower than 0.1, which is consistent with our results here.

Speculative behavior changes when traders have a lower imagination ( $\delta = 0$ ) as illustrated in Figure 3.5, Panel (c). When they reinforce actions that were actually chosen, the first two traders initially learn less quickly to speculate. This is because when initially speculating can be very profitable, traders start a trial with a high tendency to speculate. However, when  $\delta = 0$ , the traders only reinforce the actual actions, and thus they learn less to take advantage of this profit. This implies that the propensity to speculate of the first traders reverts back to zero pretty fast. In contrast, the last trader learns less quickly not to speculate.

An increase in traders' responsiveness to attractions,  $\lambda$ , reduces (but does not eliminate) speculation when  $\delta = 0$  (see Figure 3.5, Panel (d)) but has no significant effect when  $\delta = 1$  (see Figure 3.5, Panel (b)).

These results on individual behavior shed some light on the aggregate market behavior. The first (respectively, last) traders in the market sequence initially learn to (respectively, not to) speculate indicates that the likelihood of bubbles initially increase and then takes some time to converge to zero. Second, imagination induces the first traders in the market sequence to learn more strongly to speculate. This explains that, when traders have imagination, bubbles become more frequent and more rapidly.

### 3.4.2 Bubble evolution

In this subsection, rather than individual trading behavior, we turn to the evolution of bubbles when the trading game is repeated many times and traders learn from past experience. We compute the probability that a large, medium, or small bubble arises or that no bubble emerges. The magnitude of bubbles is referred to as large if all three traders choose to buy the asset, medium if the first two traders buy, and small if only

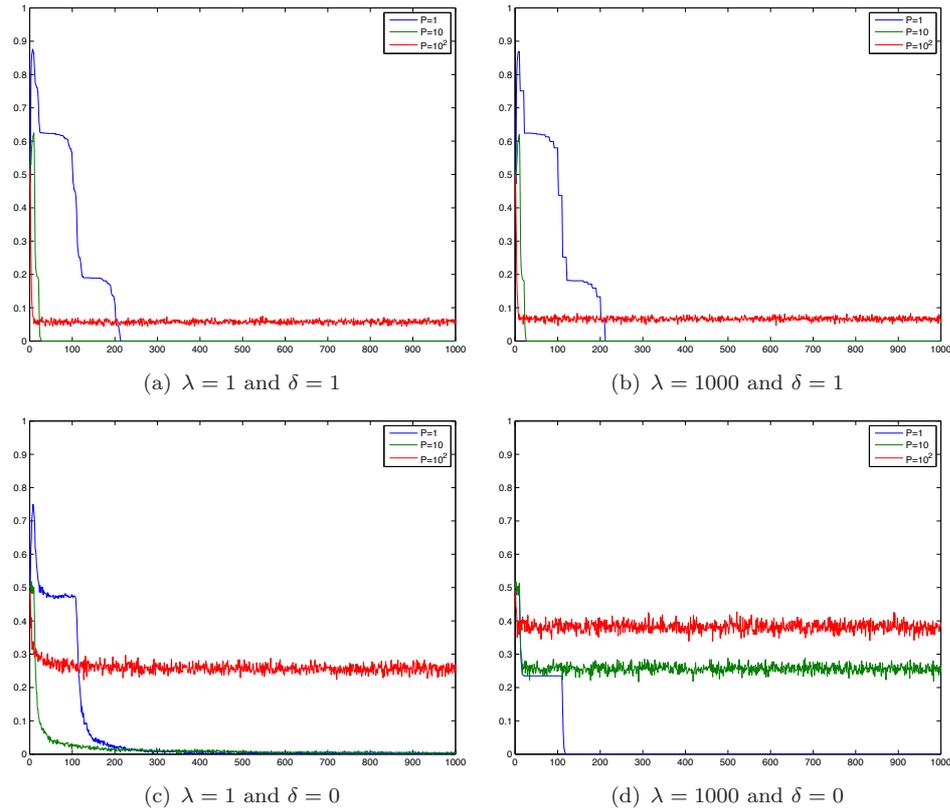


FIGURE 3.5: The individual trading behavior

the first trader buys. We consider that there is no bubble if the first trader refuses to buy. The simulation results are shown in Figure 3.6.

Our simulations display three main results. First, the likelihood of no bubbles initially decreases: bubbles become more frequent when traders gain more experience. This decrease stops after the first several runs, and the likelihood of no bubbles increases steadily towards 1, a level that is reached after 200 runs. Second, the likelihood of large bubbles decreases with traders' experience pretty rapidly. Third, the likelihood of other two types of bubbles initially increases with traders' experience and this effect is more pronounced for small bubbles: the likelihood of medium bubbles increases for the first 10 runs while the likelihood of small bubbles increases for a longer time (up to around 100 runs).

Let us now look at the effect of an increase in  $\delta$  and  $\lambda$ . When the imagination parameter  $\delta$  equals 1 instead of 0, the likelihood of bubbles increases much more with experience. Indeed, after a few runs, medium bubbles occur with probability 60%, instead of around 20% or 35% when  $\delta$  equals 0. In addition, after a few additional runs, small bubbles also occur with probability around 60%, instead of 25% or 45% when  $\delta = 0$ . We conclude that sophistication fosters rather than impedes bubble formation when traders learn from past experience.

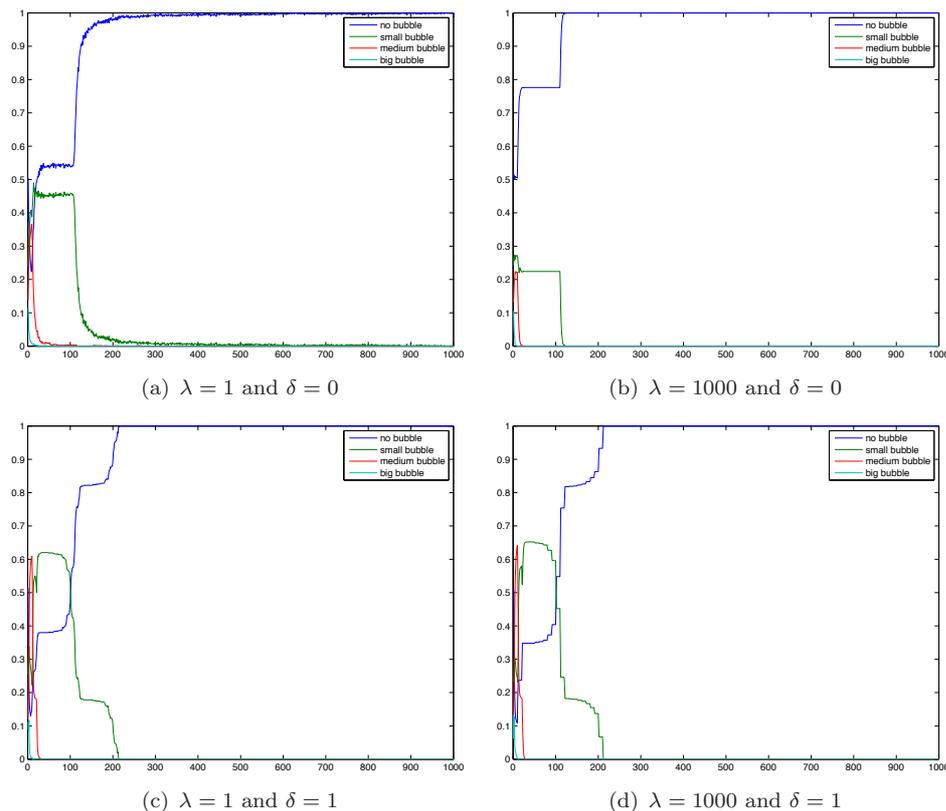


FIGURE 3.6: Evolution of bubbles

When  $\delta$  equals 0, traders only reinforce actions that were actually chosen, the increase of  $\lambda$ , the sensitivity of choices to attractions, from 1 to 1,000 reduces the likelihood of bubbles (particularly the small ones) but does not qualitatively affect the results. Moreover, when  $\delta$  equals 1, increasing  $\lambda$  has no effect on bubble formation. As argued by Camerer and Ho (1999), the sensitivity to attractions is likely to be high when agents are highly motivated. We conclude that traders' level of motivation is not an important factor in bubble formation.

As a result, the simulations show that the evolution of bubbles is consistent with our analysis of individual trading behavior.

### 3.5 Robustness

Until now, we have studied the baseline case of our game and find that learning initially leads to more speculation and sophistication boosts the bubbles. In the following, we are interested the robustness of our results. Analysis in each of the following subsections differs from the baseline case by only one feature: positions are random, price cap increases, price explosiveness is modified, or price is made endogenous.

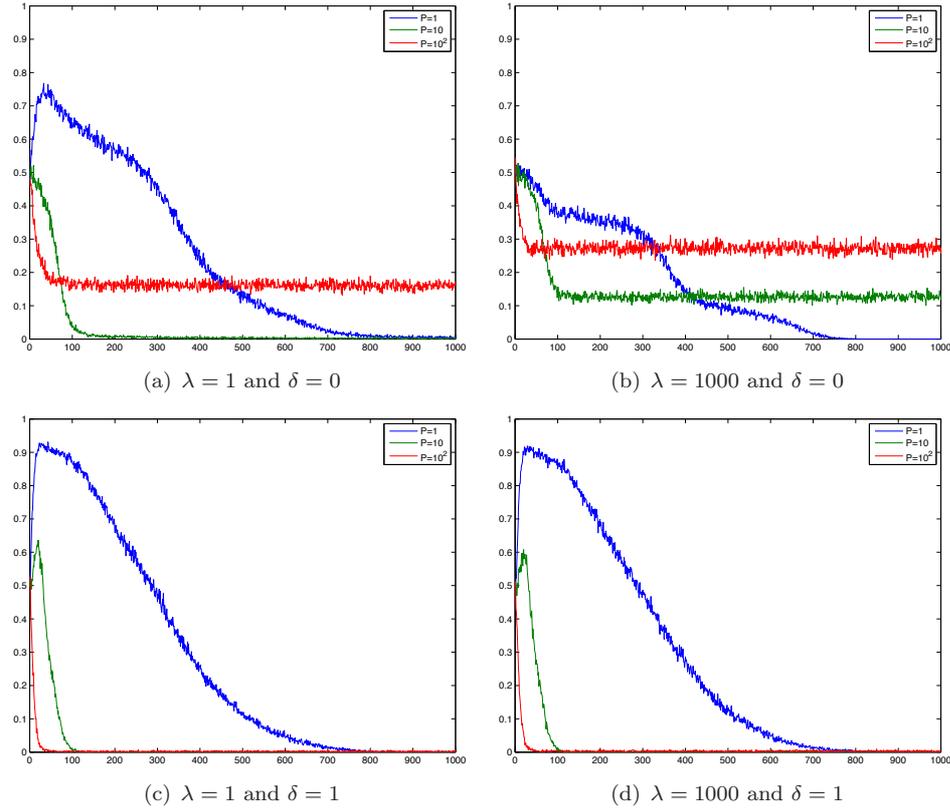


FIGURE 3.7: Individual trading behavior when trading positions are random

### 3.5.1 Bubbles with random trading positions

In this subsection, we assume that each trader is assigned a random position in every run, in line with Moinas and Pouget (2012). We check whether our basic results are still consistent with this modification. Figures 3.7 display the individual trading behavior. We can see that the qualitative results are robust: i) learning initially leads to more speculation and trading converges to no bubble slowly; ii) an increase in the sophistication  $\delta$  boosts the initial speculation, and iii) an increase in traders responsiveness to attractions  $\lambda$  reduces speculation only when  $\delta = 0$ . The difference in this case is that the bubble converges more slowly. This is because traders learn more slowly not to speculate when their positions in the trading sequence are randomly assigned in each run.

### 3.5.2 Bubbles with different caps

In this subsection, we turn to study the speculative behavior and bubble formation if we increase the price caps. Figure 3.8 and 3.9 display the speculative trading behavior and bubble evolution when the cap on the first price is  $10^6$ . We find that, raising the cap on the first price fosters traders' speculation: with price cap 1, no bubble arises after

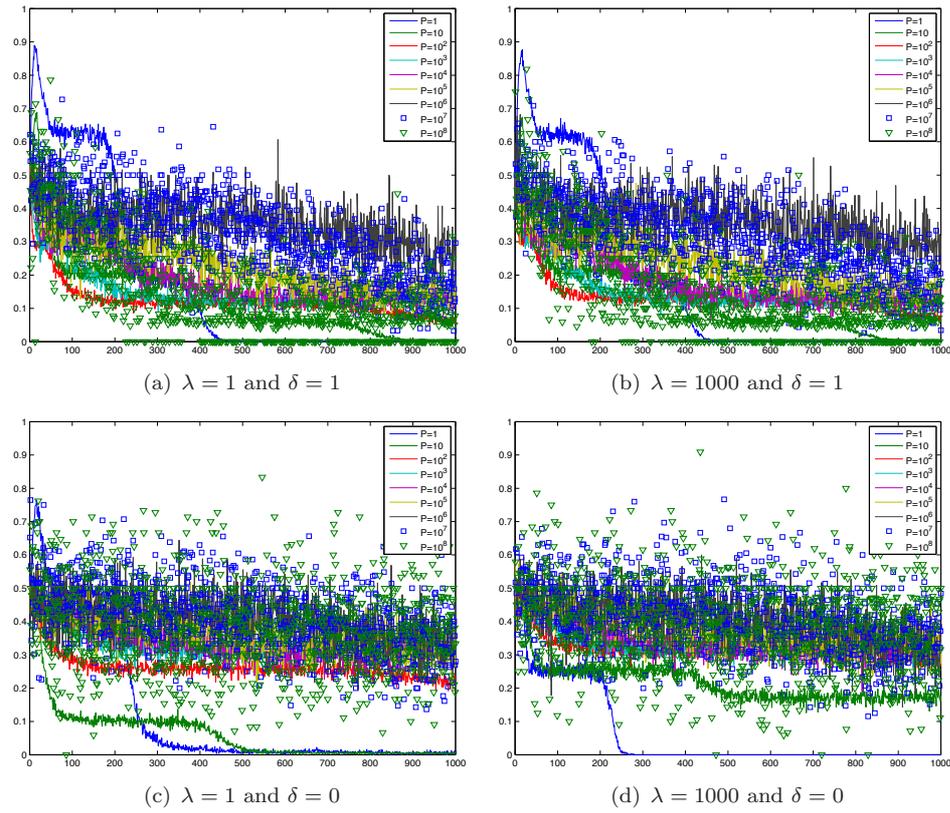


FIGURE 3.8: The individual trading behavior when price cap is  $10^6$

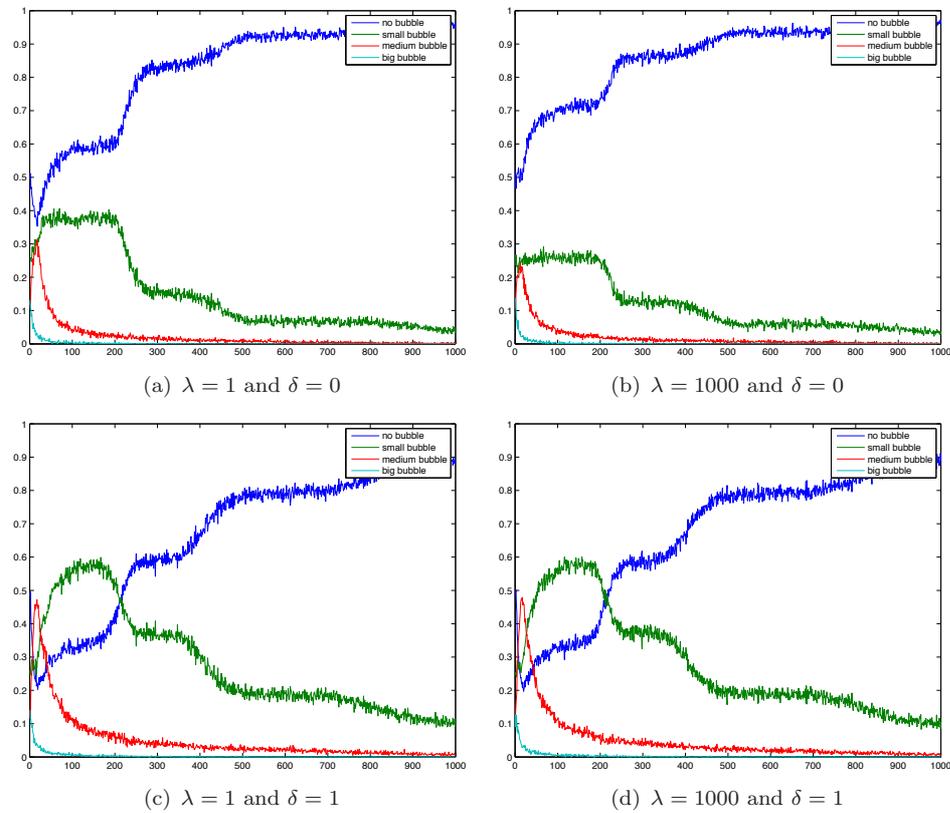


FIGURE 3.9: Evolution of bubbles

around 200 runs, but with price cap  $10^6$ , the bubble still emerges even after 1000 runs (with probability 5 – 10%). However, the qualitative results are the same: the increase of  $\delta$  leads to more speculation at lower prices. Similarly, the increase of sensitivity to attractions,  $\lambda$ , reduces speculation when traders reinforce only the actions that were actually chosen but not when they have more imagination.

### 3.5.3 Bubbles with different stakes

In this subsection, we check whether and how different stakes may affect the traders' speculation and bubble formation. In the initial study, the price for the first trader is assumed to be  $10^n$  and the following trader is offered a price which is tenfold of the previous one. Rather with stake 10, we assume that the first price is  $2^n$  ( $20^n$ ) and the following price is twice (twentyfold) of the previous one. Here, we consider the case where  $n = 0$ . In figures 3.10, a decrease in stake from 10 to 2 will lead to a faster convergence to no bubble equilibrium, while an increase in stake from 10 to 20 will lead to a slower convergence to no bubble equilibrium as in figures 3.11. This is because that an increase (a decrease) in stake results in more (less) profitable trading and therefore interferes (facilitates) the convergence to no bubble equilibrium. In addition, the effects of  $\lambda$  and  $\delta$  are the same as in the initial study.

### 3.5.4 Bubbles with endogenous prices

In the previous analysis, we consider an exogenous price path and check whether the market converges to the no bubble equilibrium as traders learn from past experiences. In this section, we relax the assumption of exogenous prices: in addition to choose whether or not to buy the asset, each trader is free to choose a price at which he proposes to sell to subsequent traders. In this setup, for a given price, each trader has three potential actions: not buying, buying and proposing to sell at a price which is ten times the previous price, and buying and proposing to sell at a price which is half of the previous price.

Potential price paths are displayed in Figure 3.12 for the case in which the first price is always 1. The first trader decides whether to buy the asset at price 1 or not. If he buys, he can propose to sell this asset at price of 10 or 0.5. When the selling price of 10, the second trader needs to decide whether to buy the asset or not. If he buys, he can propose to sell back the asset at price of 100 or 5. When the selling price is 0.5 for the first trader, the second trader also determines whether to buy the asset or not. If he buys, he can propose to sell the asset at a price of 5 or 0.25. The third trader can

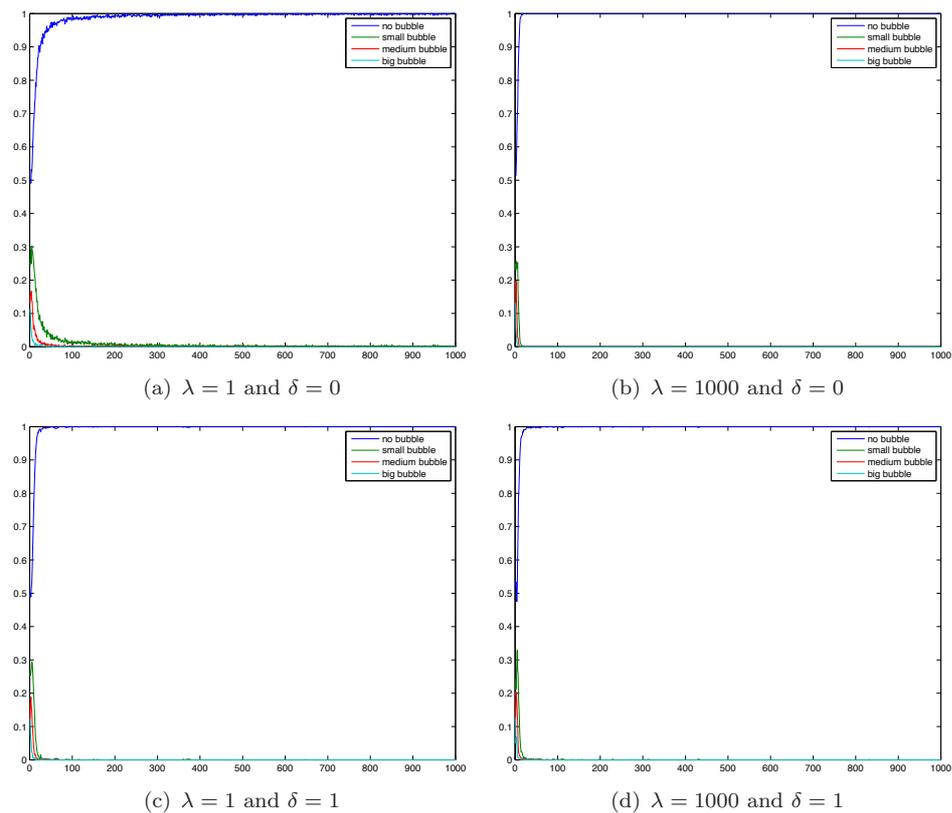


FIGURE 3.10: Evolution of bubbles with stake 2

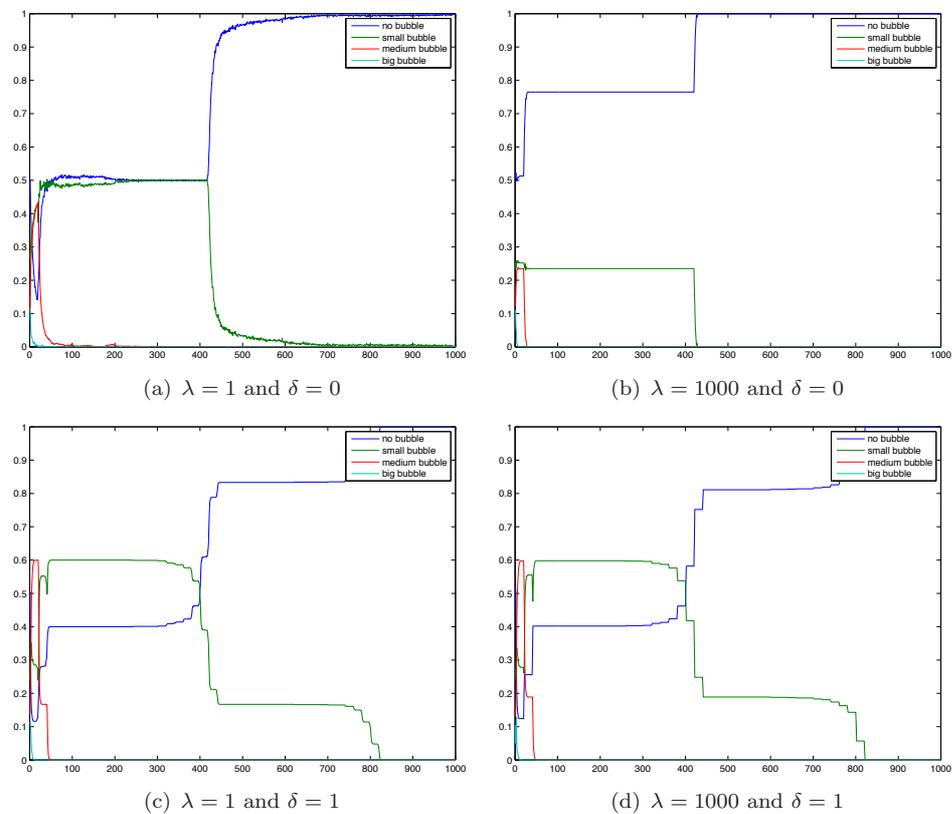


FIGURE 3.11: Evolution of bubbles with stake 2

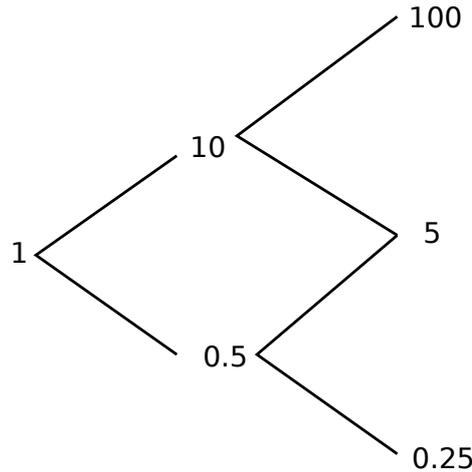


FIGURE 3.12: Price Path

decide whether to buy the asset and resell at two potential prices. However, regardless of the selling price, he does not find any buyer.

Figure 3.13 depicts the frequency with which each of the eight potential price paths is realized (including the no bubble price path that corresponds to a price of 0 all the time). This Figure shows that it is rare for traders to sell at a price which is half of the given price, since the proportion of trials with a price path that includes 0.25, 0.5 or 5 drops very quickly to almost 0%. The most likely price paths are those that include 1, 10 and 100, which is consistent with our exogenous assumption on prices when the cap on is 1. Figure 3.13 also shows that bubbles emerge even when the price is endogenous: indeed, the likelihood of bubbles, especially small and medium bubbles, initially rises when traders learn. This is in line with the results we found when the price was exogenously determined. The level of the imagination parameter,  $\delta$ , has no influence on bubble formation while responsiveness to attractions,  $\lambda$ , reduces but does not eliminate speculation.

### 3.6 Conclusion

In this paper, we study whether traders' experience reduce their propensity to speculate? This paper studies a financial market populated by adaptive traders. Following Camerer and Ho [85]'s Experience-Weighted Attraction learning model, these traders are assumed to adjust their behavior according to actions' past performance: according to the law of actual effect, traders reinforce actions that were actually successful in the past; according to the law of simulated effect, traders also reinforce actions that would have been successful if they had been chosen. In our economic environment, because there is a cap on the maximum price that can be achieved, no rational bubbles can form.

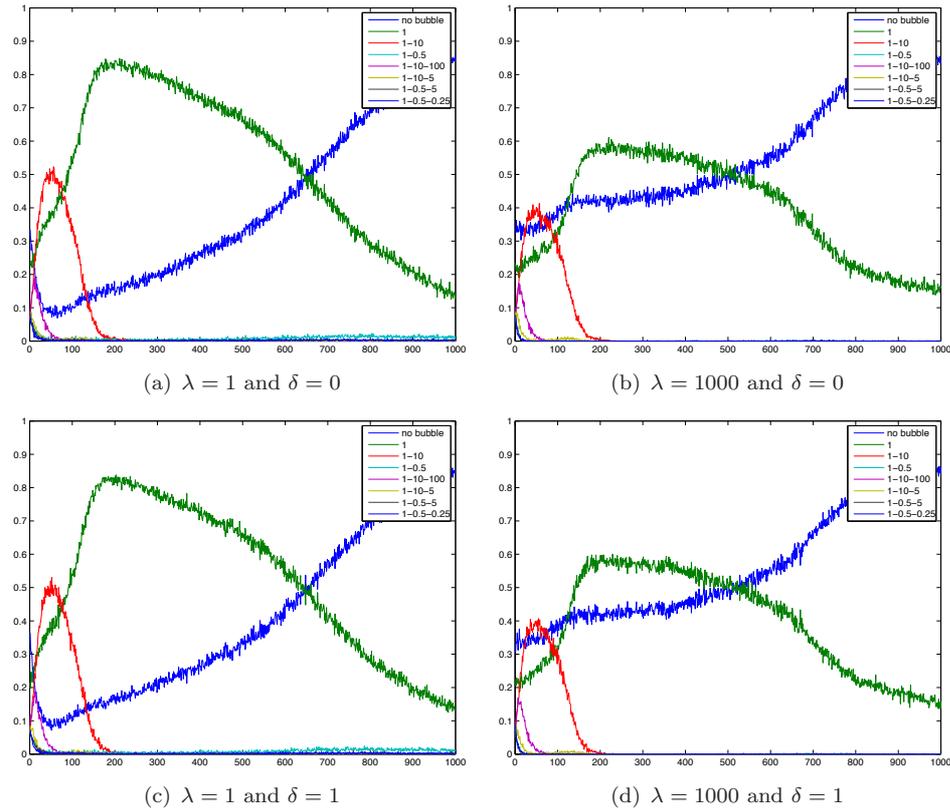


FIGURE 3.13: Evolution of endogenous bubbles

In the long run, the market converges to the unique no bubble equilibrium. However we show that learning initially increases traders' propensity to speculate. In the short run, more experienced traders thus create more bubbles. Moreover, we show that this effect is stronger when traders are more sophisticated (that is, when they use the law of simulated effect) and when the price cap is higher.

# Appendix A

## Appendix A: Chapter 1

### A.1 Continuation conditions in the case of one big firm

Continue both projects if  $2b - \rho_A - \rho_B \geq a - \rho_A$ ,  $2b - \rho_A - \rho_B \geq a - \rho_B$  and  $2b - \rho_A - \rho_B I \geq 0$ . These three inequalities hold when  $\rho_A + \rho_B \leq 2b$  and  $\rho_A, \rho_B \leq c$ .

Continue project  $A$  while liquidate project  $B$  if  $a - \rho_A > 2b - \rho_A - \rho_B$ ,  $a - \rho_A > a - \rho_B$  and  $a - \rho_A \geq 0$ . Hence,  $\rho_A \leq a$  and  $\rho_B > c$ .

Similarly, continue project  $B$  while liquidate project  $A$  if  $\rho_B \leq a$  and  $\rho_A > c$ .

Liquidate both projects if  $2b - \rho_A - \rho_B < 0$ ,  $a - \rho_A < 0$  and  $a - \rho_B < 0$ . Hence  $\rho_A + \rho_B > 2b$  and  $\rho_A, \rho_B > a$ . Q.E.D.

## A.2 Pledgeable income per project in big firms v.s. in small firms

In the case of one big firm, the expected return or the expected pledgeable income to investors can be transformed as

$$\begin{aligned}
F(b)b - \int_0^b \rho f(\rho) d\rho - I &= \int_0^b (b - \rho) f(\rho) d\rho - I \\
&= \int_0^b \int_0^{+\infty} (b - \rho_A) f(\rho_A) f(\rho_B) d\rho_B d\rho_A - I \\
&= \underbrace{\int_0^a \int_0^{+\infty} (b - \rho_A) f(\rho_A) f(\rho_B) d\rho_B d\rho_A}_{\textcircled{1}} \\
&\quad + \underbrace{\int_a^b \int_0^{+\infty} (b - \rho_A) f(\rho_A) f(\rho_B) d\rho_B d\rho_A - I}_{\textcircled{2}}
\end{aligned} \tag{A.1}$$

In the case of two small firms, the expected return to investors can be transformed as

$$\begin{aligned}
q_1 b + q_2 a - E\rho - I &= \int_0^a \int_0^c (b - \rho_A) f(\rho_A) f(\rho_B) d\rho_B d\rho_A + \int_0^a \int_c^{+\infty} (a - \rho_A) f(\rho_A) f(\rho_B) d\rho_B d\rho_A \\
&\quad \underbrace{\hspace{10em}}_{\textcircled{1}} \\
&+ \underbrace{\int_a^b \int_0^{2b-\rho_A} (b - \rho_A) f(\rho_A) f(\rho_B) d\rho_B d\rho_A}_{\textcircled{2}} \\
&+ \underbrace{\int_b^c \int_0^{2b-\rho_A} (b - \rho_A) f(\rho_A) f(\rho_B) d\rho_B d\rho_A - I}_{\textcircled{3}}
\end{aligned} \tag{A.2}$$

It is easy to see that  $\textcircled{1} > \textcircled{1}$ ,  $\textcircled{2} > \textcircled{2}$  and  $\textcircled{3} < 0$ , hence the investors obtain a larger expected return in the case of one big firm. Q.E.D

## A.3 Continuation probability per project in big firms v.s. in small firms: example

The shock of each project is uniformly distributed according to  $[0, \phi]$ . The density function is  $\frac{1}{\phi}$ .

1) If  $\phi \leq b$ ,  $dp = 0 - 0 = 0$ .

2) If  $b < \phi \leq c$ ,  $dp = \int_{2b-\phi}^b \int_{2b-\rho_A}^{\phi} \frac{1}{\phi^2} d\rho_B d\rho_A - \int_b^{\phi} \int_0^{2b-\rho_A} \frac{1}{\phi^2} d\rho_B d\rho_A = \frac{(\phi-2b)(\phi-b)}{\phi^2} < 0$ .

3) If  $\phi > c$ ,  $dp = \int_a^b \int_{2b-\rho_A}^{\phi} \frac{1}{\phi^2} d\rho_B d\rho_A - \int_b^c \int_0^{2b-\rho_A} \frac{1}{\phi^2} d\rho_B d\rho_A = \frac{(b-a)(\phi-2b)}{\phi^2}$ . We obtain that  $dp \leq 0$  if  $c < \phi \leq 2b$ , otherwise,  $dp > 0$ .

Thus, if  $\phi \leq b$ ,  $dp = 0$ ; if  $b < \phi \leq 2b$ ,  $dp \leq 0$ ; if  $\phi > 2b$ ,  $dp > 0$ . Q.E.D.

#### A.4 The value per project in big firms v.s. in small firms: example

1) If  $\phi \leq b$ ,  $dv = 0 - 0 = 0$ .

2) If  $b < \phi \leq c$ ,

$$\begin{aligned} dv &= \int_{2b-\phi}^b \int_{2b-\rho_A}^{\phi} (PR - \rho_A) \frac{1}{\phi^2} d\rho_B d\rho_A - \int_b^{\phi} \int_0^{2b-\rho_A} (PR - \rho_A) \frac{1}{\phi^2} d\rho_B d\rho_A \\ &= \frac{(\phi-b)(\phi-2b)}{\phi^2} PR - \frac{(\phi-b)(\phi^2 + b\phi - 8b^2)}{6\phi^2} \\ &= \frac{\phi-b}{6\phi^2} (-\phi^2 + (6PR-b)\phi + 8b^2 - 12bPR) \end{aligned}$$

If  $a^2 + 6aPR - 5ab - 2b^2 \geq 0$ , we can show that  $dv \leq 0$  when  $\phi \in [b, c]$ . Otherwise, there exists  $\phi^* = \frac{1}{2}[(6PR-b) - \sqrt{3(11b^2 - 20bPR + 12(PR)^2)}]$ , such that when  $\phi \in [b, \phi^*]$ ,  $dv \leq 0$ , and when  $\phi \in [\phi^*, c]$ ,  $dv > 0$ .

3) If  $\phi > c$ ,

$$\begin{aligned} dv &= \int_a^b \int_{2b-\rho_A}^{\phi} (PR - \rho_A) \frac{1}{\phi^2} d\rho_B d\rho_A - \int_b^c \int_0^{2b-\rho_A} (PR - \rho_A) \frac{1}{\phi^2} d\rho_B d\rho_A \\ &= \frac{(b-a)(\phi-2b)}{\phi^2} PR - \frac{-4a^3 + 3a^2(4b-\phi) + b^2(-8b+3\phi)}{6\phi^2} \\ &= \frac{b-a}{6\phi^2} \{(6PR - 3(a+b))\phi - (12bPR + 4a^2 - 8b^2 - 8ab)\} \end{aligned}$$

If  $a^2 + 6aPR - 5ab - 2b^2 \geq 0$ , we can show that there exists a  $\phi^*$ , where  $\phi^* = \frac{12bPR+4a^2-8b^2-8ab}{6PR-3(a+b)} \geq c$ , such that when  $\phi \in (c, \phi^*]$ ,  $dv \leq 0$  and when  $\phi \in (\phi^*, +\infty)$ ,  $dv > 0$ . Otherwise, we can show that  $dv > 0$  when  $\phi \in (c, +\infty)$ .

Thus, if  $\phi \in [0, b]$ ,  $dv = 0$ ; if  $\phi \in (b, \phi^*]$ ,  $dv \leq 0$ ; and if  $\phi \in (\phi^*, +\infty)$ ,  $dv > 0$ , where  $\phi^* \in (b, 2b)$ . Actually, if  $a^2 + 6aPR - 5ab - 2b^2 \geq 0$ ,  $\phi^* = \frac{12bPR+4a^2-8b^2-8ab}{6PR-3(a+b)}$

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and  $\phi^* \in [c, 2b)$  , otherwise  $\phi^* = \frac{1}{2}[(6PR - b) - \sqrt{3(11b^2 - 20bPR + 12(PR)^2)}]$  and  $\phi^* \in (b, c)$ . Q.E.D.

## Appendix B

# Appendix B: Chapter 2

### B.1 Optimal Contracts in Stand-alone Operation: Moral Hazard

The participation constraint of the outside investor  $PC_o$  is always binding. If it were not, increasing  $I_o$  would increase the entrepreneurs expected income without affecting the entrepreneur's incentives. Program (2.10) can be transformed to:

$$\begin{aligned}
 \max_{R_u^A, R_d^A, I_A, I_o} \quad & R_d^s + a(R_u^s - R_d^s) - \frac{1}{2}Aa^2 - I \\
 \text{s.t.} \quad & IC_A^s \\
 & PC_o^s \\
 & I_A + I_o = I \\
 & I_A \leq E_A \\
 & R_u^A \geq 0 \\
 & R_d^A \geq 0,
 \end{aligned} \tag{B.1}$$

Denote  $U = R_u^A - R_d^A$ . Based on the incentive compatibility constraint of  $IC_A$ ,  $a = \frac{1}{A}U$ . Replacing  $a$ ,  $I_A$  and  $I_o$ , the above program can be rewritten as

$$\begin{aligned}
 \max_{U, R_d^A} \quad & R_d^s + \frac{1}{A}U(R_u^s - R_d^s) - \frac{1}{2A}U^2 - I \\
 \text{s.t.} \quad & R_d^s - R_d^A + \frac{1}{A}U(R_u^s - R_d^s - U) \geq I - E_A \\
 & R_d^A, U \geq 0,
 \end{aligned} \tag{B.2}$$

According to  $I_o = R_d^s - R_d^A + \frac{U}{A}(R_u^s - R_d^s - U)$ , the maximum investment that can be provided by the outside investor  $I_o^{\max} = R_d^s + \frac{1}{4A}(R_u^s - R_d^s)^2$  when  $R_d^A = 0$  and  $U = \frac{1}{2}(R_u^s - R_d^s)$ .

Assume that  $R_d^s + \frac{1}{4A}(R_u^s - R_d^s)^2 > I$ , which implies the outside investor is still willing to finance the project even if the entrepreneur has no endowment.

The Lagrangian  $L$  of the Program (B.2) is

$$L = R_d^s + \frac{1}{A}U(R_u^s - R_d^s) - \frac{1}{2A}U^2 - I + \lambda(R_d^s - R_d^A + \frac{1}{A}U(R_u^s - R_d^s - U) - I + E_A) \quad (\text{B.3})$$

**Proof of Proposition 2.1:** First, consider the case where  $\lambda = 0$ . First-order conditions of Lagrangian  $L$  give that  $U = R_u^s - R_d^s$ , which is exactly the same as the case without moral hazard. The maximum investment that provided by the outside investor without distorting the entrepreneur's effort is  $R_d^s$  when  $R_d^A = 0$ . The minimum investment from the entrepreneur without distorting the incentives is  $I - R_d^s$ . Hence, the solution that  $U = R_u^s - R_d^s$  and  $\lambda = 0$  is feasible if and only if

$$E_A \geq I - R_d^s. \quad (\text{B.4})$$

In this case, the optimal contract can be implemented by: the outside investor holds debt with face value not greater than  $R_d^s$  and the entrepreneur holds the equity.

If  $E_A < I - R_d^s$ ,  $\lambda > 0$ . The level of effort in the case without moral hazard is not attainable. The entrepreneur always invests  $E_A$  and the outside investor invests  $I - E_A$ . The constraint  $R_d^s - R_d^A + \frac{U}{A}(R_u^s - R_d^s - U) = I - E_A$  is always binding. The first-order conditions of Lagrangian  $L$  give  $R_d^A = 0$  and

$$U = \frac{R_u^s - R_d^s + \sqrt{(R_u^s - R_d^s)^2 + 4A(E_A - I + R_d^s)}}{2}. \quad (\text{B.5})$$

In this case, the optimal contract can be implemented by: the outside investor holds a risky debt with face value  $R_u^s - U$ , which is greater than  $R_d^s$ , and the entrepreneur holds the equity. Q.E.D.

## B.2 Optimal Contracts in Strategic Alliances: Moral Hazard

Assume that  $R_d + Y_d + \frac{B}{2A(2B-A)}(R_u - R_d + Y_u - Y_d)^2 - U_B \geq I$ , which implies that the incumbent and the outside investors are still willing to provide financing even if the entrepreneur has no endowment.

The Lagrangian  $L$  of Program (2.25) is<sup>1</sup>

$$\begin{aligned} L = & R_d + Y_d + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d) - \frac{1}{2A}U^2 - \frac{1}{2B}V^2 - I - U_B + \\ & \lambda\{R_d + Y_d - R_d^A + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d - U) - \frac{1}{2B}V^2 - U_B - I + E_A\}, \end{aligned} \quad (\text{B.6})$$

where  $\lambda$  is the shadow value of the outside investment from the incumbent and the outside investor.

First, consider the case where  $\lambda = 0$ . The first-order conditions of the Lagrangian  $L$  yield that  $U = V = R_u - R_d + Y_u - Y_d$ , which are exactly the same as in the case without moral hazard. In this case, the maximum investment that can be provided by the incumbent and the outside investor together without distorting incentives is  $I^* = R_d + Y_d - \frac{1}{2B}(R_u - R_d + Y_u - Y_d)^2 - U_B$  when  $R_d^A = 0$ .

***Proof of Proposition 2.2:*** In order to study the impact of the entrepreneur's financial participation  $I_A$  on value, we need to check whether the entrepreneur's limited liability constraints hold  $\forall I_A \in [0, +\infty)$  given  $U = V = R_u - R_d + Y_u - Y_d$ . If the limited liability constraints always hold,  $I_A$  is neutral for value, otherwise, it is not. Since  $U > 0$ , we only study  $R_d^A$ .

We first consider the case where  $I \leq I^*$ , the maximum outside investment from incumbent and the outside investor is sufficient to initiate the project. The actual amount of outside financing  $I - I_A = I^* - R_d^A$ .  $\forall I_A \in [0, +\infty)$ ,  $R_d^A = I^* - I + I_A \geq 0$ . As a result, the financial participation of the entrepreneur is neutral for value.

Now turn to the case where  $I > I^*$ , the maximum outside investment without distorting the incentives is not enough to finance the project. If  $I_A \geq I - I^*$ ,  $R_d^A = I^* - I + I_A \geq 0$ . The incentives as in the case without moral hazard are preserved. Otherwise,  $R_d^A < 0$ , the incentives as in the case without moral hazard are not attainable. The value of the innovation is reduced. Consequently, in the case where  $I > I^*$ , the financial participation of the entrepreneur can enhance the value of the innovation. Q.E.D.

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<sup>1</sup>We first omit the limited liability constraints and check later whether they are satisfied.

**Proof of Proposition 2.3:** In the following, we focus on the case where  $I < I^*$ . The minimum investment from the entrepreneur by preserving the incentives as without moral hazard is  $I - I^*$ . Hence, the solutions that  $\lambda = 0$  and  $U = V = R_u - R_d + Y_u - Y_d$  are feasible if and only if

$$E_A \geq I - I^*. \quad (\text{B.7})$$

If  $E_A < I - I^*$ ,  $\lambda > 0$ . The first-order conditions of the Lagrangian L give that  $R_d^A = 0$ , as well as yield the following results:

$$\begin{aligned} \frac{\partial L}{\partial U} &= \frac{1}{A}(R_u - R_d + Y_u - Y_d) - \frac{1}{A}U \\ &+ \lambda \left\{ \frac{1}{A}(R_u - R_d + Y_u - Y_d - U) - \left( \frac{U}{A} + \frac{V}{B} \right) \right\} = 0, \end{aligned} \quad (\text{B.8})$$

$$\begin{aligned} \frac{\partial L}{\partial V} &= \frac{1}{B}(R_u - R_d + Y_u - Y_d) - \frac{1}{B}V \\ &+ \lambda \left\{ \frac{1}{B}(R_u - R_d + Y_u - Y_d - U) - \frac{V}{B} \right\} = 0, \end{aligned} \quad (\text{B.9})$$

and

$$R_d + Y_d + \left( \frac{U}{A} + \frac{V}{B} \right) (R_u - R_d + Y_u - Y_d - U) - \frac{1}{2B}V^2 - U_B = I - E_A. \quad (\text{B.10})$$

From equations (B.8) and (B.9), we obtain that

$$U = g_1(\lambda, A, B)(R_u - R_d + Y_u - Y_d), \quad (\text{B.11})$$

and

$$V = g_2(\lambda, A, B)(R_u - R_d + Y_u - Y_d). \quad (\text{B.12})$$

where  $g_1(\lambda, A, B) = \frac{(1+\lambda)(B-(A-B)\lambda)}{B(1+3\lambda+2\lambda^2)-A\lambda^2} > 0$  and  $g_2(\lambda, A, B) = \frac{B(1+\lambda)^2}{B(1+3\lambda+2\lambda^2)-A\lambda^2} > 0$ .

Since

$$g_1(\lambda, A, B) - 1 = \frac{-\lambda(A+B+B\lambda)}{B(1+3\lambda+2\lambda^2)-A\lambda^2} < 0, \quad (\text{B.13})$$

and

$$g_2(\lambda, A, B) - 1 = \frac{-\lambda(B+(B-A)\lambda)}{B(1+3\lambda+2\lambda^2)-A\lambda^2} < 0, \quad (\text{B.14})$$

$U, V < R_u - R_d + Y_u - Y_d$ . In other words, both the entrepreneur and the incumbent exert less effort than the case without moral hazard. Q.E.D.

**Proof of Proposition 2.4:** Since  $I_A \leq E_A$ ,  $I_B + I_o = I - I_A \geq I - E_A$ . According to the participation constraints of the incumbent  $PC_B$ , to preserve the optimal incentives of the entrepreneur and the incumbent,  $I_B - R_d^B$  is fixed. For a given  $I_B$ , we can always

find a  $R_d^B$  to keep the difference between the two fixed. As a result, the ex-post flexibility in making transfers between the incumbent and the outside investor ( $R_d^B$ ) ensures that the identity of the agent providing the outside financing ex-ante is irrelevant for value. Q.E.D.

**Proof of Proposition 2.5:** Substituting equations (B.11) and (B.12) into equation (B.10), we obtain that

$$f(\lambda, A, B)(R_u - R_d + Y_u - Y_d)^2 = I - E_A - (R_d + Y_d) + U_B, \quad (\text{B.15})$$

$$\text{where } f(\lambda, A, B) = \frac{(1 + \lambda)(2A^2\lambda - AB(\lambda - 1)^2(\lambda + 1) + 2B^2\lambda(1 + \lambda)^2)}{2A(-A\lambda^2 + B(1 + 3\lambda + 2\lambda^2))^2}.$$

Since  $f_\lambda(\lambda, A, B) > 0$ ,  $f(\lambda, A, B)$  is increasing in  $\lambda$ .

$$f(0, A, B) = -\frac{1}{2B} \text{ and } \lim_{\lambda \rightarrow \infty} f(\lambda, A, B) = \frac{B}{2A(2B-A)}, \text{ thus } f(\lambda, A, B) \in \left(-\frac{1}{2B}, \frac{B}{2A(2B-A)}\right).$$

$E_A < I - I^*$  can be rewritten as  $I - E_A - (R_d + Y_d) + U_B > -\frac{1}{2B}(R_u - R_d + Y_u - Y_d)^2$ . The assumption that  $R_d + Y_d + \frac{B}{2A(2B-A)}(R_u - R_d + Y_u - Y_d)^2 - U_B > I$  implies that  $I - E_A - (R_d + Y_d) + U_B < \frac{B}{2A(2B-A)}(R_u - R_d + Y_u - Y_d)^2$ .

As a result, there always exists a unique  $\lambda \in (0, +\infty)$  given  $-\frac{1}{2B}(R_u - R_d + Y_u - Y_d)^2 < I - E_A - (R_d + Y_d) + U_B < \frac{B}{2A(2B-A)}(R_u - R_d + Y_u - Y_d)^2$ . Q.E.D.

**Proof of Proposition 2.6:** Differentiate equation (2.27) with respect to  $E_A$ , we obtain

$$\frac{\partial \lambda}{\partial E_A} = \frac{-1 + \frac{\partial U_B}{\partial E_A}}{f_\lambda(\lambda, A, B)(R_u - R_d + Y_u - Y_d)^2}. \quad (\text{B.16})$$

Since  $f_\lambda(\lambda, A, B) > 0$ , the sign of  $\frac{\partial \lambda}{\partial E_A}$  is determined by the sign of  $-1 + \frac{\partial U_B}{\partial E_A}$ . If  $\frac{\partial U_B}{\partial E_A} > 1$ , the increase in the entrepreneur's endowment tightens his financial constraint, otherwise, it loosens his financial constraint. Based on equation 2.11, we can obtain  $\frac{\partial U_B}{\partial E_A}$  as

$$\frac{\partial U_B}{\partial E_A} = \begin{cases} 0 & E_A \geq I - R_d^s \\ \frac{Y_u^s - Y_d^s}{\sqrt{(R_u^s - R_d^s)^2 + 4A(E_A - I + R_d^s)}} & E_A < I - R_d^s. \end{cases} \quad (\text{B.17})$$

Hence, if  $Y_u^s - Y_d^s \leq 0$ ,  $\frac{\partial U_B}{\partial E_A} \leq 0$ . The increase in  $E_A$  reduces the incumbent's outside option value and thus relaxes the entrepreneur's financial constraint. If  $Y_u^s - Y_d^s > 0$ ,  $\frac{\partial U_B}{\partial E_A} > 0$ . The increase in  $E_A$  enhances the incumbent's outside option value. It relaxes the entrepreneur's financial constraint if  $0 < \frac{\partial U_B}{\partial E_A} \leq 1$ . Otherwise, it tightens the entrepreneur's financial constraint. Q.E.D.

**Proof of Corollary 2.7:** Consider the case where  $Y_u^s - Y_d^s > 0$ . We know that if  $E_A \geq I - R_d^s$ ,  $\frac{\partial U_B}{\partial E_A} = 0$ .

In the following, we focus on the case where  $E_A < I - R_d^s$ ,  $\frac{\partial U_B}{\partial E_A}$  reaches its maximum when  $E_A = 0$ .

$$\max \frac{\partial U_B}{\partial E_A} = \frac{Y_u^s - Y_d^s}{\sqrt{(R_u^s - R_d^s)^2 + 4A(-I + R_d^s)}}, \quad (\text{B.18})$$

and approaches its minimum when  $E_A = I - R_d^s$

$$\min \frac{\partial U_B}{\partial E_A} = \frac{Y_u^s - Y_d^s}{R_u^s - R_d^s}. \quad (\text{B.19})$$

If  $0 < Y_u^s - Y_d^s \leq \sqrt{(R_u^s - R_d^s)^2 - 4A(I - R_d^s)}$ ,  $\frac{\partial U_B}{\partial E_A} \leq 1$  always hold.

If  $Y_u^s - Y_d^s \geq R_u^s - R_d^s$ ,  $\frac{\partial U_B}{\partial E_A} > 1$  holds if  $\forall E_A \in [0, I - R_d^s)$ , otherwise,  $\frac{\partial U_B}{\partial E_A} = 0$ .

If  $\sqrt{(R_u^s - R_d^s)^2 - 4A(I - R_d^s)} < Y_u^s - Y_d^s < R_u^s - R_d^s$ ,  $\frac{\partial U_B}{\partial E_A} > 1$  holds if and only if

$$Y_u^s - Y_d^s > \sqrt{(R_u^s - R_d^s)^2 + 4A(E_A - I + R_d^s)}. \quad (\text{B.20})$$

In other words,  $\frac{\partial U_B}{\partial E_A} > 1$  iff

$$E_A < I - R_d^s + \frac{(Y_u^s - Y_d^s)^2 - (R_u^s - R_d^s)^2}{4A}, \quad (\text{B.21})$$

Otherwise,  $\frac{\partial U_B}{\partial E_A} \leq 1$ . Q.E.D.

**Proof of Proposition 2.8:** We first study the impact of  $E_A$  on the entrepreneur's effort choice  $a$ .

$$\frac{\partial a}{\partial E_A} = \frac{\partial a}{\partial \lambda} \frac{\partial \lambda}{\partial E_A} \quad (\text{B.22})$$

Since we have already studied  $\frac{\partial \lambda}{\partial E_A}$ , we now only need to look at  $\frac{\partial a}{\partial \lambda}$ .

Based on equation (B.11) and  $a = \frac{U}{A}$ , we obtain that

$$\frac{\partial a}{\partial \lambda} = \frac{1}{A} \frac{\partial g_1(\lambda, A, B)}{\partial \lambda} (R_u - R_d + Y_u - Y_d). \quad (\text{B.23})$$

where  $\frac{1}{A} \frac{\partial g_1(\lambda, A, B)}{\partial \lambda} = \frac{-(A^2 - AB + B^2)\lambda^2 - 2B^2\lambda - (AB + B^2)}{A(B(1 + 3\lambda + 2\lambda^2) - A\lambda^2)^2} < 0$

We now turn to study the impact of  $E_A$  on the probability of success  $a + b$ .

$$\frac{\partial a + b}{\partial E_A} = \frac{\partial a + b}{\partial \lambda} \frac{\partial \lambda}{\partial E_A}. \quad (\text{B.24})$$

Similarly, we focus on  $\frac{\partial a+b}{\partial \lambda}$  and obtain that

$$\frac{\partial a+b}{\partial \lambda} = \left[ \frac{1}{A} \frac{\partial g_1(\lambda, A, B)}{\partial \lambda} + \frac{1}{B} \frac{\partial g_2(\lambda, A, B)}{\partial \lambda} \right] (R_u - R_d + Y_u - Y_d), \quad (\text{B.25})$$

where  $\frac{1}{A} \frac{\partial g_1(\lambda, A, B)}{\partial \lambda} + \frac{1}{B} \frac{\partial g_2(\lambda, A, B)}{\partial \lambda} = \frac{-(B^2 - A^2)\lambda^2 - (2B^2 + 2AB - 2A^2)\lambda - (2AB + B^2)}{A(B(1 + 3\lambda + 2\lambda^2) - A\lambda^2)^2} < 0$ .

As a result, if  $\frac{\partial U_B}{\partial E_A} < 1$ , the increase in  $E_A$  leads to an increase in  $a$  and  $a+b$ . If  $\frac{\partial U_B}{\partial E_A} > 1$ , it leads to a decrease in  $a$  and  $a+b$ . Q.E.D.

*The Impact of  $E_A$  on the Incumbent's Effort  $b$ :* In the following, we will study the impact of  $E_A$  on the incumbent's effort  $b$ .

$$\frac{\partial b}{\partial E_A} = \frac{\partial b}{\partial \lambda} \frac{\partial \lambda}{\partial E_A} \quad (\text{B.26})$$

Similarly, we only need to study  $\frac{\partial b}{\partial \lambda}$ .

$$\frac{\partial b}{\partial \lambda} = \frac{1}{B} \frac{\partial g_2(\lambda, A, B)}{\partial \lambda} (R_u - R_d + Y_u - Y_d), \quad (\text{B.27})$$

where  $\frac{1}{B} \frac{\partial g_2(\lambda, A, B)}{\partial \lambda} = \frac{(1+\lambda)((2A-B)\lambda - B)}{(B(1+3\lambda+2\lambda^2) - A\lambda^2)^2}$ .

If  $0 < \frac{A}{B} \leq \frac{1}{2}$ ,  $\frac{\partial b}{\partial \lambda} < 0$ , the impact of  $E_A$  on the incumbent's effort  $b$  is similar to on  $a$  and  $a+b$ . In other words, if  $\frac{\partial U_B}{\partial E_A} < 1$ , the increase in  $E_A$  leads to an increase in  $b$ . If  $\frac{\partial U_B}{\partial E_A} > 1$ , it leads to a decrease in  $b$ .

However, if  $\frac{1}{2} < \frac{A}{B} < 1$ , we can obtain that  $\frac{\partial b}{\partial \lambda}$  is negative when  $\lambda \in (0, \frac{B}{2A-B}]$  while positive in  $\lambda \in (\frac{B}{2A-B}, +\infty)$ . Hence, even given  $\frac{\partial U_B}{\partial E_A} < 1$  or  $\frac{\partial U_B}{\partial E_A} > 1$ , the impact of  $E_A$  on  $b$  can be non-monotonic.

***Proof of Proposition 2.9:***

$$\begin{aligned} W &= R_u - R_d + Y_u - Y_d - (U + V) \\ &= R_u - R_d + Y_u - Y_d - (g_1(\lambda, A, B) + g_2(\lambda, A, B))(R_u - R_d + Y_u - Y_d) \\ &= (1 - g_1(\lambda, A, B) - g_2(\lambda, A, B))(R_u - R_d + Y_u - Y_d) \\ &= -\frac{B + (B - A)\lambda}{B(1 + 3\lambda + 2\lambda^2) - A\lambda^2} < 0. \end{aligned} \quad (\text{B.28})$$

Q.E.D.

### B.3 Optimal Financial Contracts in Strategic Alliances: Moral Hazard and Monotonicity Constraint

In this case, the maximization program becomes

$$\begin{aligned}
\max_{U,V} \quad & R_d + Y_d + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d) - \frac{1}{2A}U^2 - \frac{1}{2B}V^2 - I - U_B \\
s.t. \quad & R_d + Y_d - R_d^A + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d - U) \\
& - \frac{1}{2B}V^2 - U_B \geq I - E_A, \\
& R_d^A, U \geq 0, \\
& U + V \leq \begin{cases} R_u - R_d + Y_u - Y_d & Y_u - Y_d \geq 0 \\ R_u - R_d & Y_u - Y_d < 0. \end{cases}
\end{aligned} \tag{B.29}$$

We add the monotonicity constraint to Program (2.25).

If  $Y_u - Y_d < 0$ , the monotonicity constraint is  $U + V \leq R_u - R_d$ . Hence, the lagrangian  $L$  of the maximization program is

$$\begin{aligned}
L = & R_d + Y_d + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d) - \frac{1}{2A}U^2 - \frac{1}{2B}V^2 - I + \\
& \lambda \left\{ R_d + Y_d - R_d^A + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d - U) - \frac{1}{2B}V^2 - U_B - I + E_A \right\} \\
& + \eta \{R_u - R_d - U - V\}
\end{aligned} \tag{B.30}$$

#### B.3.1 *Case 1:* $Y_u - Y_d \leq -\frac{1}{2}(R_u - R_d)$

Without monotonicity constraint, we have show that  $U + V \leq 2(R_u - R_d + Y_u - Y_d) \leq R_u - R_d$ . The rents already satisfy the monotonicity constraint. Hence, the monotonicity constraint has no impact on the initial results. ***In this case, all the propositions are exactly the same regardless of the monotonicity constraint.***

#### B.3.2 *Case 2:* $-\frac{1}{2}(R_u - R_d) < Y_u - Y_d < 0$

First, we look back the case without the monotonicity constraint.

- 1) If  $E_A \geq I - I^*$ ,  $U + V = 2(R_u - R_d + Y_u - Y_d) > R_u - R_d$ .
- 2) If  $E_A < I - I^*$ ,

$$\begin{aligned} U + V &= (g_1(\lambda, A, B) + g_2(\lambda, A, B))(R_u - R_d + Y_u - Y_d) \\ &= \frac{(1 + \lambda)(2B + (2B - A)\lambda)}{B(1 + 3\lambda + 2\lambda^2) - A\lambda^2}(R_u - R_d + Y_u - Y_d). \end{aligned} \quad (\text{B.31})$$

We differentiate  $U + V$  with respect to  $\lambda$  and obtain that

$$\frac{\partial(U + V)}{\partial\lambda} = \frac{-(2B - A)(B - A)\lambda^2 - 2B(2B - A)\lambda - (AB + 2B^2)}{(B(1 + 3\lambda + 2\lambda^2) - A\lambda^2)^2} < 0, \quad (\text{B.32})$$

Hence,  $U + V$  decreases with  $\lambda$  and takes value in  $[R_u - R_d + Y_u - Y_d, 2(R_u - R_d + Y_u - Y_d)]$ .

Since  $R_u - R_d + Y_u - Y_d < R_u - R_d < 2(R_u - R_d + Y_u - Y_d)$ , there exists a unique  $\lambda'$  such that  $U + V = R_u - R_d$ . Thus,  $U + V \leq R_u - R_d$  if  $\lambda \geq \lambda'$ , otherwise,  $U + V > R_u - R_d$ .

Denote

$$I' = R_d + Y_d + f(\lambda', A, B)(R_u - R_d + Y_u - Y_d)^2 - U_B > I^*, \quad (\text{B.33})$$

According to equation (2.27), we obtain that if  $E_A \leq I - I'$ ,  $U + V \leq R_u - R_d$  while  $I - I' < E_A < I - I^*$ ,  $U + V > R_u - R_d$ .

Second, we turn to the case with the monotonicity constraint.

- 1) If  $E_A \leq I - I'$ ,  $U + V \leq R_u - R_d$ , i.e., the rents always satisfy the monotonicity constraint. Hence, the monotonicity constraint has no impact on the results.
- 2) If  $E_A > I - I'$ , the rents violate the monotonicity constraint. In this case, the monotonicity constraint must be binding, i.e.,  $\eta > 0$ .

We first consider the case where  $\lambda = 0$ . The first-order conditions of Lagrangian L yield that

$$U = \frac{B(R_u - R_d) + (B - A)(Y_u - Y_d)}{A + B}, \quad (\text{B.34})$$

and

$$V = \frac{A(R_u - R_d) + (A - B)(Y_u - Y_d)}{A + B}. \quad (\text{B.35})$$

In this case, the maximum outside investment should be

$$I'' = R_d + Y_d + \frac{A^2 + 2B^2}{2B(A + B)^2}(R_u - R_d)^2 + \frac{4A^2 - 2AB + 6B^2}{2B(A + B)^2}(R_u - R_d)(Y_u - Y_d) + \frac{3(A - B)^2}{2B(A + B)^2}(Y_u - Y_d)^2 - U_B. \quad (\text{B.36})$$

Hence, if  $E_A \geq I - I''$ ,  $\lambda = 0$ , and the rents for the entrepreneur and the incumbent are determined by equations (B.34) and (B.35).

If  $I - I' < E_A < I - I''$ ,  $\lambda > 0$  and  $R_d^A = 0$ . In this case, the investment constraint must also be binding. Hence,  $U$  and  $V$  should satisfy the following two constraints.

$$U + V = R_u - R_d, \quad (\text{B.37})$$

and

$$R_d + Y_d + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d - U) - \frac{1}{2B}V^2 - U_B = I - E_A. \quad (\text{B.38})$$

Based on the above two conditions, we can show that

$$U < \frac{B(R_u - R_d) + (B - A)(Y_u - Y_d)}{A + B}, \quad (\text{B.39})$$

and

$$V > \frac{A(R_u - R_d) + (A - B)(Y_u - Y_d)}{A + B}. \quad (\text{B.40})$$

By summarizing the above results, in the following we check the robustness of all the propositions in the case where  $-\frac{1}{2}(R_u - R_d) < Y_u - Y_d < 0$ .

**Robustness of Proposition 2.2:** There exists a  $I''$ , the financial participation of the entrepreneur increases the value of innovation if  $I > I''$ , otherwise, his financial participation is neutral for value.

The qualitative result of Proposition 2.2 is robust, however, the threshold investment  $I''$  is greater than  $I^*$ . If  $I \leq I''$ , the outside investment is sufficient to generate the optimal results given the monotonicity constraint. The total value is constant regardless of  $E_A$ . However, if  $I > I''$ , the total value of the innovation is an increasing function of  $E_A$ .

**Robustness of Proposition 2.3:** In the case where  $I > I''$ , the entrepreneur and the incumbent always exert effort  $a = \frac{1}{A} \frac{B(R_u - R_d) + (B - A)(Y_u - Y_d)}{A + B}$  and  $b = \frac{1}{B} \frac{A(R_u - R_d) + (A - B)(Y_u - Y_d)}{A + B}$  if  $E_A \geq I - I''$ . Otherwise, the entrepreneur exerts less effort, while the incumbent may exert more or less effort. However, the probability of success is always smaller.

The qualitative result of Proposition 2.3 is robust. The quantitative results have some difference: i) the efforts exerted by the entrepreneur and the incumbent when  $E_A \geq I - I''$  are different from that without moral hazard. ii) the incumbent may exert more effort, since the monotonicity constraint is binding in the case where  $I - I' < E_A < I - I''$ .

**Robustness of Proposition 2.4:** The incumbent and the outside investor must provide outside financing not smaller than  $I - E_A$ . However, the identity of the agent providing outside financing is irrelevant for value.

This Proposition is exactly the same due to the fact that the incumbent and the outside investor can freely choose ex-ante and ex-post transfers between them.

**Robustness of Proposition 2.5:** If  $E_A < I - I''$ , the entrepreneur is financially constrained. The tightness of his financial constraint  $\lambda$  satisfies

$$I - E_A - (R_d + Y_d) + U_B = \begin{cases} f(\lambda, A, B)(R_u - R_d + Y_u - Y_d)^2 & E_A \leq I - I' \\ q(\lambda, A, B, R_u - R_d, Y_u - Y_d) & I - I' < E_A < I - I'' \end{cases} \quad (\text{B.41})$$

where

$$q(\lambda, A, B, R_u - R_d, Y_u - Y_d) = \mathcal{A}(R_u - R_d)^2 + \mathcal{B}(R_u - R_d)(Y_u - Y_d) + \mathcal{C}(Y_u - Y_d)^2, \quad (\text{B.42})$$

and

$$\begin{aligned} \mathcal{A} &= \frac{B^2(2B - A)\lambda^2 + 2B^2(A + B)\lambda + A(A^2 + 2B^2)}{2AB(A + B + (2B - A)\lambda)^2}, \\ \mathcal{B} &= \frac{B^2(2B - A)\lambda^2 + 2B^2(A + B)\lambda + A(2A^2 - AB + 3B^2)}{AB(A + B + (2B - A)\lambda)^2}, \\ \mathcal{C} &= \frac{(B - A)^2(1 + \lambda)(3A + (2B - A)\lambda)}{2AB(A + B + (2B - A)\lambda)^2}. \end{aligned}$$

Proof: consider the case where  $I - I' < E_A < I - I''$ . The first-order conditions of the lagrangian  $L$  are

$$\begin{aligned} \frac{\partial L}{\partial U} &= \frac{1}{A}(R_u - R_d + Y_u - Y_d) - \frac{1}{A}U \\ &+ \lambda \left[ \frac{1}{A}(R_u - R_d + Y_u - Y_d - U) - \left( \frac{U}{A} + \frac{V}{B} \right) \right] - \eta = 0, \end{aligned} \quad (\text{B.43})$$

$$\begin{aligned} \frac{\partial L}{\partial V} &= \frac{1}{B}(R_u - R_d + Y_u - Y_d) - \frac{1}{B}V \\ &+ \lambda \left[ \frac{1}{B}(R_u - R_d + Y_u - Y_d - U) - \frac{V}{B} \right] - \eta = 0, \end{aligned} \quad (\text{B.44})$$

and equations (B.39) and (B.40).

From equations (B.43), (B.44) and (B.39), we obtain that

$$U = \frac{B + (B - A)\lambda}{A + B + (2B - A)\lambda}(R_u - R_d) + \frac{(B - A)(1 + \lambda)}{A + B + (2B - A)\lambda}(Y_u - Y_d), \quad (\text{B.45})$$

$$V = \frac{A + B\lambda}{A + B + (2B - A)\lambda}(R_u - R_d) + \frac{(A - B)(1 + \lambda)}{A + B + (2B - A)\lambda}(Y_u - Y_d), \quad (\text{B.46})$$

and

$$\eta = \frac{B(1+\lambda) - A\lambda}{B(A+B+(2B-A)\lambda)}(R_u - R_d) + \frac{2B(1+\lambda)^2 - A\lambda(1+\lambda)}{B(A+B+(2B-A)\lambda)}(Y_u - Y_d). \quad (\text{B.47})$$

Now plug equations (B.45) and (B.46) into equation (B.40), we can obtain the expression of financial constraint as the function  $q$ .

**Robustness of Proposition 2.6:** If  $Y_u^s - Y_d^s \leq 0$ , an increase in the endowment of the entrepreneur  $E_A$  reduces the outside option value  $U_B$  and relaxes the financial constraint. If  $Y_u^s - Y_d^s > 0$ , an increase in the endowment  $E_A$  raises the incumbent's outside option  $U_B$ . The financial constraint is tightened with the endowment if  $\frac{\partial U_B}{\partial E_A} > 1$ . Otherwise, the financial constraint is loosened.

This Proposition is always robust.

Proof: for Proposition 2.6, we differentiate  $\lambda$  with respect to  $E_A$ . According to equation (B.41), we know that this proposition holds if  $E_A \leq I - I'$ . In the following, we focus on the case where  $I - I' < E_A < I - I''$  and obtain that

$$\frac{\partial \lambda}{\partial E_A} = \frac{-1 + \frac{\partial U_B}{\partial E_A}}{\frac{\partial q}{\partial \lambda}}. \quad (\text{B.48})$$

Since

$$\frac{\partial q}{\partial \lambda} = \frac{[(A^2 - AB + B^2)(R_u - R_d) + (2A^2 - 3AB + B^2)(Y_u - Y_d)]^2}{AB(A+B+(2B-A)\lambda)^3} > 0 \quad (\text{B.49})$$

Hence, Proposition 2.6 is also robust when  $I - I' < E_A < I - I''$ . In addition, we can find that  $\lambda$  is a continuous function, hence Proposition 2.6 is robust for the whole range of  $E_A$ .

**Robustness of Proposition 2.8:** The entrepreneur's effort and the probability of success of the innovation increases with  $E_A$  when  $\frac{\partial U_B}{\partial E_A} < 1$ , and decreases when  $\frac{\partial U_B}{\partial E_A} > 1$ .

This Proposition is always robust.

Proof: we need to differentiate  $a$  and  $a + b$  decrease with  $E_A$  in the case when  $I - I' < E_A < I - I''$ .

According to equations (B.39) and (B.40), we can obtain that

$$\begin{aligned} & -\left(\frac{1}{A} - \frac{1}{2B}\right)U^2 + \left(\frac{1}{A} - \frac{1}{B}\right)(R_u - R_d + Y_u - Y_d)U + \frac{1}{2B}(R_u - R_d)(R_u - R_d + 2(Y_u - Y_d)) \\ & = I - E_A - (R_d + Y_d) + U_B. \end{aligned} \quad (\text{B.50})$$

Thus,

$$\begin{aligned}
\frac{\partial a}{\partial E_A} &= \frac{1}{A} \frac{\partial U}{\partial E_A} \\
&= \frac{1}{A} \frac{-1 + \frac{\partial U_B}{\partial E_A}}{-2(\frac{1}{A} - \frac{1}{2B})U + (\frac{1}{A} - \frac{1}{B})(R_u - R_d + Y_u - Y_d)} \\
&= \frac{1}{A} \frac{-1 + \frac{\partial U_B}{\partial E_A}}{\frac{2B-A}{AB}(\frac{B-A}{2B-A}(R_u - R_d + Y_u - Y_d) - U)}.
\end{aligned} \tag{B.51}$$

When  $U = \frac{B-A}{2B-A}(R_u - R_d + Y_u - Y_d)$ , the investment from the incumbent and the outside investor is maximized. Hence,  $U \geq \frac{B-A}{2B-A}(R_u - R_d + Y_u - Y_d)$ . The denominator is negative.

$$\begin{aligned}
\frac{\partial(a+b)}{\partial E_A} &= \frac{1}{A} \frac{\partial U}{\partial E_A} + \frac{1}{B} \frac{\partial V}{\partial E_A} \\
&= \frac{1}{A} \frac{\partial U}{\partial E_A} + \frac{1}{B} \frac{\partial(R_u - R_d - U)}{\partial E_A} \\
&= \left(\frac{1}{A} - \frac{1}{B}\right) \frac{\partial U}{\partial E_A}.
\end{aligned} \tag{B.52}$$

As a result, if  $\frac{\partial U_B}{\partial E_A} \geq 1$ ,  $\frac{\partial a}{\partial E_A} \leq 0$  and  $\frac{\partial(a+b)}{\partial E_A} \leq 0$ . If  $\frac{\partial U_B}{\partial E_A} < 1$ ,  $\frac{\partial a}{\partial E_A} > 0$  and  $\frac{\partial(a+b)}{\partial E_A} > 0$ . Since  $a$  and  $b$  are continuous, this proposition is robust for the whole range of  $E_A$ .

### B.3.3 Case 3: $Y_u - Y_d \geq 0$

If  $Y_u - Y_d \geq 0$ , the monotonicity constraint is  $U + V \leq R_u - R_d + Y_u - Y_d$ . The Lagrangian  $L$  is changed to

$$\begin{aligned}
L &= R_d + Y_d + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d) - \frac{1}{2A}U^2 - \frac{1}{2B}V^2 - I + \\
&\quad \lambda \left\{ R_d + Y_d + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d - U) - \frac{1}{2B}V^2 - U_B - I + E_A \right\} \\
&\quad + \eta \{R_u - R_d + Y_u - Y_d - U - V\}.
\end{aligned} \tag{B.53}$$

In the case without monotonicity constraint, since  $U + V > R_u - R_d + Y_u - Y_d$ . The rents always violate the monotonicity constraint. Therefore, in this program the monotonicity constraint must always be binding, i.e.,  $\eta > 0$ .

First consider the case where  $\lambda = 0$ . The first-order conditions yield:

$$U = \frac{B}{A+B}(R_u - R_d + Y_u - Y_d), \quad (\text{B.54})$$

$$V = \frac{A}{A+B}(R_u - R_d + Y_u - Y_d). \quad (\text{B.55})$$

In this case, the maximum outside investment provided by the incumbent and the outside investor together is

$$I''' = R_d + Y_d + \frac{(A^2 + 2B^2)}{2B(A+B)^2}(R_u - R_d + Y_u - Y_d)^2 - U_B. \quad (\text{B.56})$$

If  $E_A \geq I - I'''$ ,  $\lambda = 0$  and the rents are determined by the above equations.

However, if  $E_A < I - I'''$ ,  $\lambda > 0$ . The rents satisfy the following two conditions

$$U + V = R_u - R_d + Y_u - Y_d, \quad (\text{B.57})$$

and

$$R_d + Y_d + \left(\frac{U}{A} + \frac{V}{B}\right)(R_u - R_d + Y_u - Y_d - U) - \frac{1}{2B}V^2 - U_B = I - E_A. \quad (\text{B.58})$$

We can show that  $U < \frac{B}{A+B}(R_u - R_d + Y_u - Y_d)$  and  $V > \frac{A}{A+B}(R_u - R_d + Y_u - Y_d)$ .

**Robustness of Proposition 2.2:** There exists a  $I'''$ , the financial participation of the entrepreneur increases the value of innovation if  $I > I'''$ , otherwise, his financial participation is neutral for value.

**Robustness of Proposition 2.3:** In the case where  $I > I'''$ , the entrepreneur and the incumbent always exert effort  $a = \frac{1}{A} \frac{B}{A+B}(R_u - R_d + Y_u - Y_d)$  and  $b = \frac{1}{B} \frac{A}{A+B}(R_u - R_d + Y_u - Y_d)$  if  $E_A \geq I - I'''$ . Otherwise, the entrepreneur exerts less effort and the incumbent exerts more effort. However, the probability of success is always smaller.

**Robustness of Proposition 2.4:** The proposition is exactly the same.

**Robustness of Proposition 2.5:** If  $E_A < I - I'''$ , the entrepreneur is financially constrained. The tightness of his financial constraint  $\lambda$  satisfies

$$h(\lambda, A, B)(R_u - R_d + Y_u - Y_d)^2 = I - E_A - (R_d + Y_d) + U_B, \quad (\text{B.59})$$

where  $h(\lambda, A, B) = \frac{A^3 + 2B^3\lambda(1+\lambda) + AB^2(2+2\lambda-\lambda^2)}{2AB(A+B+(2B-A)\lambda)^2}$ , increasing in  $\lambda$ .

Proof: the first-order conditions of the lagrangian are

$$\frac{\partial L}{\partial U} = \frac{1}{A}(R_u - R_d + Y_u - Y_d) - \frac{1}{A}U + \lambda\left[\frac{1}{A}(R_u - R_d + Y_u - Y_d - U) - \left(\frac{1}{A}U + \frac{1}{B}V\right)\right] - \eta = 0, \quad (\text{B.60})$$

$$\frac{\partial L}{\partial V} = \frac{1}{B}(R_u - R_d + Y_u - Y_d) - \frac{1}{B}V + \lambda\left[\frac{1}{B}(R_u - R_d + Y_u - Y_d - U) - \frac{1}{B}V\right] - \eta = 0, \quad (\text{B.61})$$

and equations (B.57) and (B.58).

From equations (B.60), (B.61) and (B.57), we obtain

$$U = \frac{B + (B - A)\lambda}{A + B + (2B - A)\lambda}(R_u - R_d + Y_u - Y_d), \quad (\text{B.62})$$

and

$$V = \frac{A + B\lambda}{A + B + (2B - A)\lambda}(R_u - R_d + Y_u - Y_d). \quad (\text{B.63})$$

Substitute the above two equations into equation (B.58), we obtain that equation (B.59).

We differentiate  $h(\lambda, A, B)$  with respect to  $\lambda$  and obtain that

$$\frac{\partial h(\lambda, A, B)}{\partial \lambda} = \frac{(A^2 - AB + B^2)^2}{AB(A + B + (2B - A)\lambda)^3} > 0. \quad (\text{B.64})$$

**Robustness of Proposition 2.6:** The proposition is exactly the same.

Proof: We differentiate  $\lambda$  with respect to  $E_A$ . According to equation (B.59), we obtain that

$$\frac{\partial \lambda}{\partial E_A} = \frac{-1 + \frac{\partial U_B}{\partial E_A}}{h_\lambda(\lambda, A, B)(R_u - R_d + Y_u - Y_d)^2}. \quad (\text{B.65})$$

Since the denominator is positive, Proposition 2.6 holds.

**Robustness of Proposition 2.8:** This proposition is exactly the same.

Proof: We differentiate  $a$  and  $a + b$  with respect to  $E_A$ .

$$\frac{\partial a}{\partial E_A} = \frac{1}{A} \frac{\partial U}{\partial \lambda} \frac{\partial \lambda}{\partial E_A}. \quad (\text{B.66})$$

$$\begin{aligned} \frac{\partial(a + b)}{\partial E_A} &= \left(\frac{1}{A} \frac{\partial U}{\partial \lambda} + \frac{1}{B} \frac{\partial V}{\partial \lambda}\right) \frac{\partial \lambda}{\partial E_A} \\ &= \left(\frac{1}{A} - \frac{1}{B}\right) \frac{\partial U}{\partial \lambda} \frac{\partial \lambda}{\partial E_A}. \end{aligned} \quad (\text{B.67})$$

According to equation (B.62), we obtain that

$$\frac{\partial U}{\partial \lambda} = -\frac{B(B - A) + A^2}{(A(\lambda - 1) - B(1 + 2\lambda))^2}(R_u - R_d + Y_u - Y_d) < 0. \quad (\text{B.68})$$

In addition, based on the results of Proposition 2.6, we can obtain that  $a$  and  $a + b$  increase with  $E_A$  if  $\frac{\partial U_B}{\partial E_A} < 1$ , while decrease with  $E_A$  if  $\frac{\partial U_B}{\partial E_A} > 1$ . Hence, Proposition 2.8 holds.

## B.4 Implementation of Optimal Financial Contracts

In order to be a budget breaker, the outside investor cannot hold equity in the entrepreneur firm, since it harms his incentives.<sup>2</sup> Hence, the revenue in the entrepreneur firm is only shared by the entrepreneur and the incumbent. In the following, we consider how the revenue of the entrepreneur firm is splitted between the two agents.

### B.4.1 $U < R_u - R_d$

Let  $\alpha$  be the fraction of equity the entrepreneur holds and  $1 - \alpha$  be the fraction of equity the incumbent holds.<sup>3</sup> Define  $D$  is the revenue the entrepreneur receives in case of failure. There are two possible incomes for the entrepreneur firm:  $R_u$  in case of success and  $R_d$  in case of failure. According to the definition of preferred equity and common equity, in case of failure, it is impossible to remunerate common equity with the same dividend as preferred equity, while in case of success, both types of stocks generate the same dividend.

CASE 1: the incumbent holds preferred equity while the entrepreneur holds common equity in the entrepreneur firm.

Denote  $\mathcal{I}$  the maximum outside investment given the effort levels in the case where the entrepreneur is not financially constrained.

$$\mathcal{I} = \begin{cases} I^* & Y_u - Y_d \leq -\frac{1}{2}(R_u - R_d) \\ I'' & -\frac{1}{2}(R_u - R_d) < Y_u - Y_d < 0 \\ I''' & Y_u - Y_d \geq 0, \end{cases} \quad (\text{B.69})$$

If  $E_A < I - \mathcal{I}$ , the entrepreneur is financially constrained. In this case,  $D = R_d^A = 0$ ,  $\alpha = \frac{U}{R_u}$ . The entrepreneur obtains 0 in case of failure and  $U$  in case of success, while the incumbent receives  $R_d$  and  $R_u - U$  respectively. It is easy to show that  $(1 - \alpha)R_d < R_d < (1 - \alpha)R_u$ , indicating that the dividend the incumbent receives is greater than his share

<sup>2</sup>It is possible for the outside investor to hold debt in the entrepreneur firm. Nevertheless, this has nothing to do with incentives. Without loss of generality, we neglect the possibility to hold debt.

<sup>3</sup>Equity may be common equity or preferred equity.

of equity in case of failure and is equal to his share of equity in case of success. Thus, the incumbent firm holds preferred equity while the entrepreneur firm holds common equity.

In this case, the outside investment provided by the incumbent and the outside investor  $\mathcal{I}' > \mathcal{I}$ .

If  $E_A \geq I - \mathcal{I}$ , the entrepreneur is not financially constrained. In this case, the entrepreneur obtains  $D$  in case of failure and  $D+U$  in case of success, while the incumbent receives  $R_d - D$  and  $R_u - (D + U)$  respectively.

In this case,  $R_d^A = D$ . Hence, the outside investment provided by the incumbent and the outside investor is

$$\mathcal{I}' = \mathcal{I} - D. \quad (\text{B.70})$$

If the incumbent holds preferred equity while the entrepreneur holds common equity, it indicates that

$$(1 - \alpha)R_d < R_d - D < (1 - \alpha)R_u, \quad (\text{B.71})$$

and

$$R_d - D \leq R_d. \quad (\text{B.72})$$

where  $\alpha = \frac{D+U}{R_u}$ . The first inequality is due to the definition of preferred equity and the second inequality is from the limited liability condition of the entrepreneur.

Plug equation (B.70) into the above two inequalities, we can obtain that there exists a  $\mathcal{I}^*$ , where  $\mathcal{I}^* = \mathcal{I} - \frac{R_d}{R_u - R_d}U$ , such that if  $\mathcal{I}^* < \mathcal{I}' \leq \mathcal{I}$ , the incumbent holds preferred equity while the entrepreneur holds common equity.

Consequently, the incumbent holds preferred equity while the entrepreneur holds common equity if  $\mathcal{I}' > \mathcal{I}^*$ .

CASE 2: the incumbent holds common equity while the entrepreneur holds preferred equity in the entrepreneur firm.

If the incumbent holds common equity while the entrepreneur holds preferred equity, it indicates that

$$\alpha R_d < D < \alpha R_u. \quad (\text{B.73})$$

This is only possible when the entrepreneur is not financially constrained, i.e.,  $E_A \geq I - \mathcal{I}$ . Similarly, plug equation (B.70) into the above inequality, we can obtain that if  $\mathcal{I}' \leq \mathcal{I}^*$ , the incumbent holds common equity while the entrepreneur holds preferred equity.

In conclusion, there exists a  $\mathcal{J}^* < \mathcal{J}$ , the incumbent holds common equity while the entrepreneur holds preferred equity if  $\mathcal{J}' \leq \mathcal{J}^*$ , nevertheless, the incumbent holds preferred equity while the entrepreneur holds common equity if  $\mathcal{J}' > \mathcal{J}^*$ .

#### B.4.2 $U \geq R_u - R_d$

If  $E_A < I - \mathcal{J}$ , the entrepreneur is financially constrained. The split of the revenue in the entrepreneur firm is: the entrepreneur obtains 0 in case of failure and  $R_u - R_d$  in case of success, while the incumbent receives a fixed income  $R_d$  regardless of the state. Hence, the incumbent holds debt while the entrepreneur holds equity. In this case,  $\mathcal{J}' > \mathcal{J}$ .

If  $E_A \geq I - \mathcal{J}$ , the entrepreneur is not financially constrained. The split of the revenue in the entrepreneur firm is: the entrepreneur obtains  $D$  in case of failure and  $D + R_u - R_d$  in case of success, while the incumbent receives a fixed income  $R_d - D$  regardless of the state. If  $D < R_d$ , i.e.,  $\mathcal{J}' > \mathcal{J}^* - R_d$ , the incumbent holds debt while the entrepreneur holds equity. If  $D \geq R_d$ , i.e.,  $\mathcal{J}' \leq \mathcal{J} - R_d$ , the incumbent holds nothing while the entrepreneur holds full equity.

In conclusion, if  $\mathcal{J}' \leq \mathcal{J} - R_d$ , the incumbent holds nothing while the entrepreneur holds full equity. If  $\mathcal{J}' > \mathcal{J} - R_d$ , the incumbent holds debt while the entrepreneur holds equity.

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