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l'Université n'entend ni approuver, ni désapprouver les opinions particulières du candidat.

For my beloved mother and father

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Chapter 1

Introduction

Just after the beginning of my master courses in "Financial Market and Intermediaries" at the Toulouse School of Economics, the worst moment of the 2007-09 financial crisis had been reached. Bear Stern had disappeared from the stage of the Wall Street six months before, and Lehman Brother filed for bankruptcy while Bank of America announced the purchase of Merrill Lynch. Wall Street investment banks were completely annihilated and commercial banks were hit hard. Hence, I got my master's degree with a curious and anxious mood . Curious because my personal understanding of the financial crisis was only a partial view from some newspapers or television programs, having no clue about why it happened and how it was devastating. However, I was also anxious just like most students in the finance faculty during that time, unrealistically expecting that there would be a negative correlation between the job market and the financial market at least until my own graduation. But obviously, this is not an idea that a rational agent would have. Fortunately, both because of this curiosity and anxiety, I started my Ph.D thesis. Of course, I am not going to say what I was anxious about, but I would better say what I was curious about - what is the financial crisis?

The financial crisis is a very large concept to describe a situation in which some financial assets suddenly lose a large part of their nominal value. Until 1930s, many financial crises were associated with "banking crisis".[1] Diamond and Dybvig (1983) has very well explained why this kind of panics may happen in the banking sector. Since depositors are not insured, they are willing to run the banks if they realize that the other depositors are going to run as well. This

[1] Alternatively speaking, banking panics.

bank run may trigger a liquidity problem in the banks, eventually turning into a solvency problem. Then the classic banking crises have been prevented by the Federal Deposit Insurance Corporation (FDIC) in U.S., which guarantees depositor's accounts. However, the modern banking crises are characterized by the wholesale investors run on the shadow banking system. See chapter 2 for details.

The financial crisis is also linked to a stock market crash. These crashes can be observed as a sudden dramatic decline of stock prices across a significant cross-section of stock markets, resulting in a significant loss of monetary wealth. Note that they often follow speculative stock market bubbles. An example is the so called "Black Monday", which refers to Monday October 19, 1987, when stock markets around the world crashed, shedding a huge value in a very short time. The crash began in Hong Kong and spread west to Europe, hitting the United States after other markets had already declined by a significant margin. Another example is that the stock market crash of 2000-2002, which caused the loss of 5 trillion dollars in the market value of companies between March 2000 and October 2002. Brunnermeier and Nagel (2004) document that the sophisticated market participants, like hedge funds, did not exert a correcting force on stock prices during the technology bubble; instead they rode bubbles and questioned the efficient market hypothesis that rational speculators always stabilize prices.

Sovereign crisis is becoming an important factor for financial crisis. When a country fails to pay back its sovereign debt, this is called a sovereign default. It is well known that from May 2010, the southern European countries have suffered such a sovereign crisis. As the probability of default increases, the yield of government bonds increases as well, which increases the funding cost of the country and worsens the initial situation, which in turn increase the yield further and so on. The sovereign crisis is not just an economic problem, but also a political issue. On one hand, these European countries follow the same monetary policy but different fiscal policy, which limits the macroeconomic tools available to deal with their domestic problem,[2] while the fiscal policy serves more as a "presidential election" tool than as a stabilizing force. On the other hand, during a quite long a time, the Euro area has been implementing a low interest rate policy, which allowed countries like Greece to benefit from cheap funding cost to stimulate their economic growth, but they cover the structural problems of low productivity and high labor cost. The global financial crisis uncovered these

[2] The monetary policy is implemented by European central bank.

problems, which triggered massive defaults.[3]

The current financial crisis (late 2000s financial crisis) has shown its huge negative impact on the real economy. This is because this financial crisis is an interplay of a banking crisis, a bubble burst which led to the financial market crash and a sovereign crisis. It is well known that the subprime crisis was triggered by the burst of US housing bubble and shortly swept the whole financial system. Furthermore, its destructive capacity might eventually be seen to have been systemic rather than merely specific to one failed financial institution. This new feature of financial crisis has given rise to the research on this phenomenon by many economists and this is also the main topic of my Ph.D thesis - **systemic risk**.

Systemic risk is the risk that is posed to the financial system or/and the economy, as opposed to the risk that is faced by an investor or a portfolio.

Systemic risk is most commonly discussed in relation to the risks posed by banks and financial institutions. The failure of a major financial institution can have serious consequences. If a bank fails, not only will its depositors lose money, but it is also likely to renege on obligations to other financial institutions. Both the depositors and the other institutions are then likely to be under financial pressure, which can lead to further failures of both banks and other businesses. The resulting ripple effect can bring down an economy. Controlling this systemic risk is a major concern for regulators after the current financial crisis, particularly given that consolidation in banking industry has led to the creation of some extremely large banks. Even though governments usually bail out those banks that are "too big to fail" (TBTF) or "too connected to fail", the cost of doing so would have its own effects: the encouragement it gives them to be less cautious because they know that they are going to be bailed out. This creates a problem similar to the conflicts of interest between debt and equity holders. In this case the shareholders of big institutions take the gain if the risk pays off, government pick up the cost if it goes wrong.

However, **systematic risk** is sometimes confused with systemic risk, but their distinction is crucial for both risk quantification and economic interpretation. As Hansen (2013) has also pointed out, systematic risk was well studied throughout almost 50 years. This is the macroeconomic or aggregate risk that cannot be avoided through diversification. In the perspective of risks, it is **exogenous**, since the sources of systematic risk are the exposures to the

[3] The financial crisis was associated with other crises such as currency crisis etc.

macro/aggregate shocks which are out of control by individual market participants. Investors and risk managers often use financial econometric models to calculate the potential losses and charge the related reserve to hedge this risk. However, systemic risk has just gained the attention of world-wide economists and regulators during the recent financial crisis.[4] Unlike the former, systemic risk is **endogenous**, since one financial institution's risk contribution to the whole system (or to the other financial institutions) can be controlled by itself ex-ante. According to Lorenzoni (2008), the fire-sales externalities can be reduced if institutions borrow less short-tern debt ex-ante or take less risk on the balance-sheet when they make their investment decisions. This leaves a room for regulators to stabilize the financial system as a whole, and this new policy is called - **Macroprudential policy**. Before the late 2000s financial crisis, regulators paid most attention on the traditional microprudential policy that seeks to enhance the safety and soundness of individual financial institutions, as opposite to the macroprudential view which focuses on welfare of the financial system as a whole. Several aspects of Basel III reflect a macroprudential approach to financial regulation. More concretely, under Basel III banks' capital requirements have been strengthened and new liquidity requirements (liquidity coverage ratio and net stable funding ratio), a leverage cap and a countercyclical capital buffer have been introduced. Also, the systemically important financial institutions (SIFIs) are required to hold more and higher-quality capital, which is consistent with the cross sectional aspect of macroprudential approach. In chapter 2, I will illustrate why there are systemic risks in the financial system and what kinds of regulatory view we should have to cope with these risks.

In order to efficiently implement the macroprudential tools, regulators need to use the risk measures to accurately estimate the systemic risk for each financial institution. The common measure of risk used by financial institutions is the Value at Risk (VaR), which focuses on the risk of an individual institution in isolation. The $q\%$-VaR is the maximum dollar loss within the 1- $q\%$ confidence interval. However, such a measure of a single institution's risk does not necessarily reflect the risk that the stability of the financial system as a whole is threatened. Several measures of systemic risk have been put forward, such as the CoVaR,[5] in an attempt to capture externalities inflicted by some institutions on others. For example Goldman Sachs could affect JP Morgan or the investment banking sector could affect the insurance sector as a whole institutions. Some economists have

[4]Rochet and Tirole (1996), Freixas, Parigi, and Rochet (2000), Allen and Gale (2000) and Borio (2003) among others had noticed the systemic risk before the current financial crisis.

[5]The Value at Risk (VaR) of the financial system conditional on institutions being under distress. See section 2.5 for details.

also argued that regulatory capital requirements could be based on the Δ-CoVaR, capturing an institution's (marginal) contribution to systemic risk. Regulation should take into account institutions' characteristics such as leverage, maturity mismatch and size that predict systemic risk contributions. As mentioned before, there are some measures which targeted at measuring systematic risk, such as Marginal Expected Shortfall (MES),[6] in an attempt to capture the one financial institution's exposure to an aggregate shock on the system/market. I propose a new measure which is based on CoVaR measure and Shapley value methodology in chapter 3.

Many papers have identified the over-borrowing on short term debt as being the root of the externality in the banking sector. But why do banks have a funding structure so relied on short term debt? My coauthor and I developed a theoretical banking model in order to have a better understanding on the funding structure of banks in chapter 4. In this chapter we have also mentioned that when using short term debt, banks' refinancing need is triggered by an exogenous macro productivity shock. While by using long term debt, banks do not need to refinance, yet, they may misbehave due to a lack of interim discipline. Banks choose short term maturity when they expect a macro shock to occur with a small probability. From a social perspective, the externalities caused by over borrowing in short term debt exist only in the case where the probability of a macro shock is large; otherwise, the social optimum coincides with the market equilibrium. This suggests that regulators should be "prudent" in implementing liquidity regulations.

Below, I provide a timing of the crisis.

04/22/2007 Second-largest subprime lender, New Century Financial, declares bankruptcy.

06/22/2007 2 hedge funds run by Bear Sterns have trouble meeting their margin calls : Bear Sterns injects \$3.2bn to protect one of them but does not support another.

08/09/2007 BNP Paribas suspends calculation of asset values of three money market funds exposed to subprime and halts redemptions. AXA had earlier announced support for its funds.

08/09/2007 European Central Bank (ECB) injects €95 billion overnight to improve liquidity. Injections by other central banks.

[6] See Acharya, Pederson, Phillipon, and Richardson (2010) and Brownlees and Engle (2011).

09/14/2007 Bank of England announces it has provided a liquidity support facility to Northern Rock.

09/14/2007 Moody's announces it will re-estimate capital adequacy ratio of U.S. mono-line insurers/financial guarantors.

01/24/2008 Société Générale reveal trading losses resulting from fraudulent trading by a single trader.

03/14/2008 JP Morgan Chase &Co. announces that it has agreed, in conjunction with the Federal Reserve Bank of New York, to provide secured funding to Bear Sterns for an initial period of up to 28 days.

06/16/2008 Lehman Brothers confirms a net loss of US$2.8bn in Q2.

09/15/2008 Lehman Brothers files for bankruptcy. Bank of America announces purchase of Merrill Lynch.

09/16/2008 U.S. government provides emergency loan to AIG of US$85bn in exchange for a 79.9 percent stake and right to veto dividend payments.

09/21/2008 The Federal Reserve approves transformation of Goldman Sachs and Morgan Stanley into bank holding companies.

11/09/2008 Chinese authorities declares an economics stimulus plan for 4000 billions yuans (around $650bn).

01/11/2009 Citigroup confirms a net loss of US$8.29bn in Q4 2008. Fed injects $20bn into the capital of Bank of America.

02/11/2009 UBS and Credit Suisse confirm a net loss of 13.3bn euros and 5.5bn Euros in 2008 respectively.

03/11/2009 Freddie Mac confirms a net annual loss of US$50.1bn.

03/07/2009 Société Générale registers a net loss of 278 millions euros in Q1.

05/08/2009 Commerzbank confirms a net loss of 861 millions euros in Q1.

09/24/2009 The 2009 G-20 Pittsburgh Summit.

10/02/2009 The publication of the results of European stress test for 28 big banks.

03/24/2010 Fitch downgraded Portugal's credit rating (from AA to AA-).

04/09/2010 Fitch downgraded Greece's credit rating (from BBB+ to BBB-).

04/22/2010 Moody's downgraded Greece's credit rating (from A2 to A3).

04/27/2010 Standard & Poor's downgraded Greece's credit rating (from BBB+ to BB+). Standard & Poor's downgraded Portugal's credit rating (from A+ to A-).

04/28/2010 Standard & Poor's downgraded Spain's credit rating (from AA+ to AA).

05/18/2010 Greece receives the first aid plan for 14.5 billion euros.

05/28/2010 Fitch downgraded Spain's credit rating (from AAA to AA+).

06/14/2010 Moody's downgraded Greece's credit rating (from A3 to Ba1).

06/27/2010 The 2009 G-20 Pittsburgh Summit. An agreement has been made to increase the banks capital ratio (Basel III).

07/12/2010 The Dodd-Frank Act has been promulgated.

07/13/2010 Moody's downgraded Portugal's credit rating (from Aa2 to A1).

08/24/2010 Standard & Poor's downgraded Ireland's credit rating (from AA to AA-).

09/30/2010 Moody's downgraded Spain's credit rating (from Aaa to Aa1).

12/09/2010 Fitch downgraded Ireland's credit rating (from A+ to BBB+).

12/23/2010 Fitch downgraded Portugal's credit rating (from AA- to A+).

01/14/2011 Fitch downgraded Greece's credit rating (from BBB- to BB+).

02/02/2011 Standard & Poor's downgraded Ireland's credit rating (from AA- to A-).

03/07/2011 Moody's downgraded Greece's credit rating (from Ba1 to B1).

03/10/2011 Moody's downgraded Spain's credit rating (from Aa1 to Aa2).

03/15/2011 Moody's downgraded Portugal's credit rating (from A1 to A3).

03/29/2011 Standard & Poor's downgraded Greece's credit rating (from BB+ to BB-) and downgraded Portugal's credit rating (from BBB to BBB-).

04/01/2011 Standard & Poor's downgraded Ireland's credit rating (from A- to BBB+). Fitch downgraded Portugal's credit rating (from A- to BBB-).

04/05/2011 Moody's downgraded Portugal's credit rating (from A3 to Baa1).

04/15/2011 Moody's downgraded Ireland's credit rating (from Baa1 to Baa3).

05/08/2011 Standard & Poor's downgraded Greece's credit rating (from BB- to B).

05/20/2011 Fitch downgraded Greece's credit rating (from BB+ to B+).

06/01/2011 Moody's downgraded Greece's credit rating (from B1 to Caa1).

06/13/2011 Standard & Poor's downgraded Greece's credit rating (from B to CCC).

06/30/2011 Fed launched QE2.

07/05/2011 Moody's downgraded Portugal's credit rating (from A3 to Baa3).

07/13/2011 Fitch downgraded Greece's credit rating (from B+ to CCC).

07/25/2011 Moody's downgraded Greece's credit rating (from Caa1 to Ca).

07/27/2011 Standard & Poor's downgraded Greece's credit rating (from CCC to CC).

08/05/2011 Standard & Poor's downgraded United State's credit rating to AA+ for the first time in history.

09/19/2011 Standard & Poor's downgraded Italy's credit rating (from A+ to A).

10/04/2011 Moody's downgraded Italy's credit rating (from Aa2 to A2).

10/07/2011 Fitch downgraded Italy's credit rating (from AA- to A+) and Spain's credit rating (from AA+ to AA-).

10/13/2011 Standard & Poor's downgraded Spain's credit rating (from AA to AA-).

10/14/2011 Standard & Poor's downgraded BNP Paribas's credit rating (from AA to AA-).

10/18/2011 Moody's downgraded Spain's credit rating (from Aa2 to A1).

11/11/2011 Standard & Poor's confirms the France credit rating (AAA).

Chapter 2

Systemic Risk and Macroprudential Policy: A Survey

2.1 Introduction

The integration of global financial markets has delivered large welfare gains through improvements in static and dynamic efficiency - the allocation of real resources and the rate of economic growth. However, these achievements have come at the cost of increased systemic fragility, as experienced during the recent 2007-2009 financial crisis. We must now face the challenge of re-designing the regulatory overlay of the global financial system in order to make it more resilient without crippling its ability to innovate and spurring economic growth. Recent financial crises have also revealed the failure of current financial regulatory framework, which pushes towards public authorities paying the more attention to the financial stability in a *macroprudential* perspective (Borio (2003)). This is because when large financial institutions encounter financial problems a negative externality can arise in the financial system, since they expect to be bailed out by the public authorities. This leads two serious consequences: 1) the global risk in the financial system is underestimated by many market participants, who have no incentives to reduce systemic risk, since this has not been written in the regulatory framework yet; 2) once a bail-out occurs, the tax payer will pay the bill for those bank managers who misbehaved and decided for the wrong balance between risk and profit; but if the public should not pay for these gamblers, they have to do so, since these institutions are too big to fail (TBTF) . How to correct the incentives of these market participants has been put on the top of the agenda of public authorities. To deal with these issues, we need to gain a deep understanding on why and how a systemic event could occur in the financial system through specific mechanisms and what adverse consequences would happen to the financial system and the real economy.

In this article I try to identify the sources of the fragility of the financial system as a whole by surveying the growing literature that studies systemic risk and macroprudential policy. This is important, because it is just like diagnosing a patient: the problem can only be efficiently addressed once the pathology is determined. The objective of this survey is to present the mechanisms that may trigger a systemic event and also introduce the possible regulation framework to cope with this issue - the macroprudential policy. As we are standing at the research frontier on the topic, this article is far away from a definitive version on the field.

Like any contagious disease it always starts from an individual patient, we observed during the 2007-2009 financial crisis that the individual liquidity problem of a financial institution plays an important role in the vulnerability of the financial system. More specifically, this liquidity problem is compounded with the solvency problem in the financial institution. A financial institution is insolvent when its 'going concern' value does not exceed the expected value of its liability. During normal times, it is fairly easy to identify insolvent financial institution, since financial markets are strong. However, during crisis period, it is difficult to do so, because insolvency can be triggered by liquidity issues (Rochet and Vives (2004) and Morris and Shin (2009)). A crucial characteristic of modern liquidity problem is that the traditional mechanism in the model of Diamond and Dybvig (1983) is no suitable anymore to explain the current financial crisis. This is because the deposit base of financial institution can be fully covered by the deposit insurance provided by the authorities, and also because the interbank lending market is now well developed. But why is the liquidity problem still there in the financial system? This is because nowadays the more and more financial institutions' debts are funded by wholesale investors, who usually have a very short maturity and are not covered by any insurance. Therefore, the modern bank run is mainly embodied in the wholesale market where investors are not willing to rollover the short-term debt of financial institutions. In this case, the financial institutions have to liquidate their assets on the market to raise funds, which may generate huge losses, potentially harmful to social welfare.

The short-term debt holders have to sell their asset in order to meet their obligations, the price of assets become disconnected from the estimates of expected cash flow. Since, in bad times, the short-term creditors observe the losses on financial institutions, which in turn decreases the the likelihood of a short-term debt creditor roll over their funds (funding illiquidity[1]). Due to the market illiquidity[2], they can only sell their assets at a fire sale price, this leads to a loss on existing positions on the asset side of institutions' balance sheets, then worsens the situation of funding liquidity. On the other hand, due to the presence of adverse selection on the secondary market, when financial institutions anticipate a low price, which is very possible during the crisis period, it may lead to a market freeze because there are only "lemons" in the market. This worsens the market illiquidity further. This loss spiral will not stop until relevant authorities intervene in this situation, and this is exactly what happened during the recent financial crisis. This is an illustration from an individual institution's perspective, however, it is clear that the level of market and funding liquidity is not exogenously given but determined in the economy as a whole and thus, important adverse feedback effects might arise. This requires a more systemic view of liquidity crises, but identifying the sources of individual liquidity fragility is indispensable (Allen and Gale (1994), Allen and Gale (1998), Brunnermeier and Pedersen (2009) and Malherbe (2012)).

Macroprudential policy is a complement to *microprudential* policy, which focuses on the stability of a single financial institution, and is meant to interact with other types of public policy that have an impact on financial stability (e.g. monetary policy, fiscal policy, etc.). Its goals are to address two dimensions of system-wide risk: first, the evolution of the risk over time – the "*time dimension*" - and second, the distribution of risk in the financial system at a given point in time – the "*cross-sectional dimension*".

A simple explanation of the time-dimension aspect of the macroprudential policy is to deal with the procyclicality issue. Because of asymmetric information (borrower's human capital is inalienable), when investors lend their money, a credit constraint is always imposed in the economy, therefore small shocks to the economy might be amplified into a large credit fluctuation. This is because the borrower's assets play a role as the collateral against the amount borrowed, in the present case, the credit ceiling of each financial institution positively depends on the value of their asset. When a negative shock occurs, the value of the asset decreases and financial institutions will borrow less as a response to the initial

[1]This also can be seen as the financial institutions have some difficulties to borrow more funds or raising funds is becoming highly costly.

[2]It is difficult to raise money by selling assets at reasonable prices.

shock, which in turn reduces the credit volume in the economy. The credit volume is high during the boom period and becomes low during the recession, which lowers the pace of the recovery. On the one hand, the procyclicality is triggered by the over-borrowing ex ante along with the over-investment, which generate an externality that financial institutions will not internalize when they make their decisions. On the other hand, an "only-risk-targeting" regulation, such as Basel II, could be another reason why such a procyclicality may take shape. Normally, the purpose of bank capital regulation is to minimize the market failure and make the financial system more stable. From this perspective, when the economic environment becomes bad, it is quite natural for a social planner to increase the capital. However, this is not the only important factor in a social planner's objective function; he should also put some weight on those objectives of banks that were properly taken into account in the first place (e.g. making positive net present value (NPV) loans). Therefore by imposing the same level of capital regulation during booms and recessions would generate procyclicality and it is suboptimal.

Empirically, the evidences show that financial intermediaries adjust their balance sheets actively, and do so in such a way that leverage is high during booms and low during busts. That is, leverage is procyclical. Procyclical leverage can be seen as a consequence of active management of balance sheet by financial intermediaries who respond to changes in prices and measured risk. For financial intermediaries, their models of risk and economic capital dictate active management of their overall Value-at-Risk (VaR) through adjustments of their balance sheets. One main consequence of procyclical leverage is that, during a crisis period, for a fall in the price of an asset widely held by hedge funds and banks, the net worth of such institutions falls faster than the rate at which the value of assets falls, eroding its equity cushion. One way for the bank to restore its equity cushion is to sell some of its assets, and use the proceeds to pay down its debt. Note the importance of marked-to-market. By synchronizing the actions of market participants, the feedback effect is amplified.

The Basel Committee on Banking Supervision (BCBS) has introduced a counter-cyclical capital requirement *à la* Basel III in September 2010 whose purpose is to cope with the procyclicality. Different from the "only-risk-targeting" capital requirement, this new regulation can been seen as the cyclical-adjusted capital requirement, which does consider the trade-off between the stability of financial institutions and the financial activity in the system.

The cross-sectional dimension of macroprudential policy focuses on the interbank contagions and fire-sale externalities as well as the common exposure of financial institutions to an aggregate uncertainty. As mentioned above, the interbank lending can reduce banks' individual liquidity problem, but it exposes banks to a systemic risk. That is to say, if a single bank is about to fail, due to the interbank linkage, the balance sheet of the other banks, which have lent to it, will be weakened and thus in a danger of failure. Usually interbank lending provides a peer monitoring among banks in order to reduce the asymmetric information effect; however, the potential bailout by the central bank in the crisis period may destroy this peer-monitoring and give a chance of systemic crisis to occur. The fragility may also come from an insufficient capital buffer of a representative bank. The fact that interbank market may reduce the liquidity problem of an idiosyncratic liquidity shock in a financial institution is well known by economists, but, in case of an aggregate liquidity shock, if the banks' capital buffer is insufficient to cover the liquidity needs, a contagion effect will arise in this market; furthermore, if the aggregate shock is large enough the whole system could be swept out.

Fire sales externality is at the core of recent financial crisis. When banks face the liquidity demands, sometimes they have to liquidate part of their long term assets (illiquid assets), and given that the market's demand for long term assets is less than perfect elastic, marked-to-market asset prices will decrease. This low level of prices may endogenously generate another round of asset sales, which would depress prices further and induce further sales. Since the assets are marked-to-market, a small initial shock may generate contagious failures in the whole system. The basic idea of fire sale externality is as follows. An increase in ex-ante investment will imply an increase in ex-post sale of assets during a crisis, therefore lowering the price of asset. If the seller of asset has a larger marginal utility than the buyer and the insurance market is missing, this ex-ante investment generates a negative pecuniary externality. This implies that equilibrium is constrained inefficient and there is an over-borrowing and over-investment ex-ante, which provides a rationale for appropriate regulation.

Two approaches have been proposed to deal with the liquidity problem in order to make individual banks internalize this externality ex-ante. The first is a *price-based* approach, by imposing a Pigouvian tax on short-term debt, which aims at equating private and social liquidity cost in order to reach the social optimal allocation. The second is a *quantity-based* approach with instruments such as liquidity coverage ratio and net stable funding ratio introduced in Basel

III. Both liquidity ratios have the objective to lower the probabilities that a bank runs into liquidity difficulties.

However, there is a sector which is not subject to financial regulation - the shadow banking system. The shadow banks can reduce their idiosyncratic risk by trading or securitizing the loans to diversify their portfolio. Consequently, this behavior exposes their portfolio to systematic shocks. It decomposes the simple process of deposit-funded, hold-to-maturity lending conducted by banks into a more complex, wholesales-funded, securitization-based lending process. Through this intermediation process, the shadow banking system transforms risky, long-term loans (e.g. sub-prime mortgages) into seemingly credit-risk free, short-term, money like instruments, etc. By doing so, it makes the financial stability become more unstable and fragile in the sense that not only make FIs expose to an aggregate uncertainty but also leave an opportunity for financial institutions to evade prudential regulation. Whether to regulate the shadow banking system or not is still under debate among economists and policy makers.

On all account, the appropriate risk measures are indispensable to help policy makers propose the relevant regulation framework to stabilize the financial system. Based on their characteristic I divide these risk measures into two categories - *systematic risk measure* and *systemic risk measure*. The former has received a lot of attention after the crisis; although the fall in US property prices (which is probably the fundamental cause of the crisis) was widely predicted, its huge impact on banks and financial markets were not. This is because financial intermediaries are highly exposed to the financial system. Before the crisis, the most popular measure of risk used by the financial profession is the Value at Risk (VaR), which focuses on the risk of an individual institution in isolation. There was no measure which could provide some information about an institution's exposure to the whole financial system when an extreme event happened. From a theoretical point of view, it is akin to a Beta computed within the Capital Asset Pricing Model (CAPM). The difference rests on the fact that the MES measure, so-called "tail-Beta", is a projection of institution's return to a space where the market return is lower than a specific threshold. By contrast, in the CAPM, the Beta is simply the projection of an institution's return to the market return, without any threshold.

However, these measures do not provide any information about the contagion risk among financial institutions. The latter category of risk measures - systemic risk measures - which allows to measure the contagion risk to which each bank contributes with respect to the financial but also with respect to other banks bilaterally. These measures can capture the systemic importance of individual banks and provide useful information for regulators to design the macroprudential policy.

This survey mainly focuses on the *micro mechanism* with which the systemic risk formed and what can be done by policy makers to stabilize the financial system by providing an efficient regulatory framework. The structure of the paper is organized as follows: the liquidity fragility of financial institutions is studied firstly in section 2.2. In section 2.3 the time-dimension of systemic risk is analyzed and is followed by the cross-sectional dimension of systemic risk in section 2.4. In section 2.5 several empirical risk measures are presented and Section 2.6 gives the relationship of macroprudential policy and monetary policy. Section 2.7 concludes. Given the focus oh this survey, I do not consider many topics in the financial stability literature such as the financial asset "bubbles", the "monetary policy" implemented by central banks. For a more comprehensive literature on bubble I refer to Tirole (1985) and Brunnermeier (2008), and on monetary policy I refer to Clarida, Gali, and Gertler (1999) and Blinder, Fratzscher, Haan, and Jansen (2008). As this survey focuses mostly on the microeconomic models to illustrate the instability of the system as a whole, it leaves apart the macroeconomic perspective on the field, for this reason I refer to Brunnermeier, Eisenbach, and Sannikov (2012) to fill in the gap this missing part.

2.2 Liquidity

The 2007-2009 financial crisis shows that the liquidity issue plays a crucial role in the (in)stability of the financial system. If one wants to mitigate the impact of liquidity problems on the financial system, it is worth to that by identifying the determinants of liquidity problems.[3]

[3] In this section, I only talk about the individual liquidity problems. However, a financial institution's individual liquidity problem has also contagion effects to other financial institutions. This secondary effect will be presented in section 4.

2.2.1 Bank runs

Preference shocks. Creditor's/investor's preference shocks may create bank run as illustrated by Diamond and Dybvig (1983) who build on Bryant (1980) (hereafter BDD). In BDD, investors may face a sudden preference shock, then a bank run may destroy the value of the banking sector. More precisely, they consider a framework with a fractional reserve system, where banks finance profitable but illiquid projects. Lenders are uncertain about their future liquidity preference. Compared to the market economy, banks can efficiently pool resources together and insure the liquidity risk thus attain the social optimum. However, there is also another possibility: a lender may expect that all the others may withdraw their funds together, thus the rational behavior for her is also to withdraw immediately. If all lenders no matter their actual liquidity preference are doing so, the bank goes bankrupt.

With the uncertain future consumption needs, investors allocate their savings in long-term to gain a higher return against in short-term to guarantee the availability of their money. The model has three date $t = 0, 1, 2$, and a continuum of ex-ante identical agents, each of them is endowed with one unit good only at $t = 0$. At interim date $t = 1$, impatient agents need to consume early, whose utility function is $u(C_1)$; patient agents consume late, whose utility function is $u(C_2)$. At $t = 0$, each agent with probability λ of being impatient and with probability $1 - \lambda$ of being patient, so each agent's expected utility function is $U = \lambda u(C_1) + (1 - \lambda)u(C_2)$.

There are two investment technologies: at $t = 0$, investors can allocate their consumption good either to a storage technology, which stores the consumption good from one period to the next without cost and gives a return of 1; or to a long-run technology, which delivers a return $R > 1$ at $t = 2$ but only pays a salvage value $l \leq 1$ if liquidated early at $t = 1$.

To obtain Pareto-optimal allocation, we need to find the amount of investment I in illiquid technology such that

$$\max_I \lambda u(C_1) + (1 - \lambda)u(C_2), \tag{2.1}$$

subject to $\lambda C_1 = 1 - I$ and $(1 - \lambda)C_2 = RI$. The optimal allocation

(C_1^*, C_2^*) satisfies the following condition,

$$u'(C_1^*) = Ru'(C_2^*). \tag{2.2}$$

First, consider the allocation in an autarky case where there is no trade between agents, each investor invests I in the long-run investment and stores the remaining $1 - I$. If some of investors early liquidate their investment yields in $C_1 = 1 - I + lI$, while the others end up with $C_2 = 1 - I + RI$. In this case, the autarky allocation is inefficient because $\lambda C_1 + \frac{(1-\lambda)C_2}{R} < 1$. Now, consider a bond market is open at $t = 1$ allowing agents to sell their claims in the long-run technology at price p without it have to be liquidating. In this case, The consumption levels $C_1 = 1-l+pRI$ and $C_2 = RI+\frac{1-l}{p}$ can be achieved. The non-arbitrage condition between long-run technology and storage technology implies that the interior equilibrium price satisfies $pR = 1$, and equilibrium allocation with a bond market is $C_1^m = 1$, $C_2^m = R$ and $I^m = 1 - \lambda$. Note that, the market allocation is not efficient because $u'(C_1^m) > Ru'(C_2^m)$.[4]

The main contribution of BDD is that they show that the financial intermediaries (henceforth FIs) can efficiently pool resources together and insure the liquidity risk thus attain the social optimal solution in the following way: the Pareto-Optimal allocation (C_1^*, C_2^*) can be implemented by a FI who offers a deposit contract stipulating that in exchange for a deposit one unit at $t = 0$, individuals can get either C_1^* at $t = 1$ or C_2^* at $t = 2$. To fulfill its debt obligation, the FI stores λC_1^* and invest $I = 1 - \lambda C_1^*$ in the long-run technology.

It is natural to ask the question: "is this fractional reserve system stable"? The answer depends very much on the behavior of patient consumers. Note that the optimal allocation is a Nash equilibrium since $C_2^* > C_1^*$ and it is optimal for a patient agent not to withdraw early if other patient agents don't withdraw early. However, the patient agent will withdraw early if she anticipates that all other patient agents want to withdraw early, thus it generates another inefficient Nash equilibrium corresponding to a *bank run* where all agents withdraw early. In this case bank will be forced to liquidate its long-run investment with the total value $\lambda C_1^* + (1 - \lambda C_1^*)l < 1 < C_1^*$, and bank will be insolvent, so the optimal strategy for the patient agents is to withdraw, which implies an inefficient *bank run*.

Signals on fundamental. However, the traditional bank run triggered by the

[4]This result holds under the assumption that $\frac{-Cu''(C)}{u'(C)} > 1$, i.e., the function $C \to Cu'(C)$ is decreasing.

early withdrawal of uninformed depositors is prevented by imposing a deposit insurance. The modern form of bank run is mostly characterized by the case where large well-informed wholesale investors refuse to renew their credit on the money market (repo for example). 2007-2009 financial crisis is mainly suffered from this type of bank run. Even several years before the crisis, Rochet and Vives (2004) (Hereafter RV04) had shown that there is a coordination problem between investors, and even in a sophisticated financial market, a bank run will take place if the bank is solvent, therefore they claim that Bagehot's doctrine is still hold in nowadays. More specifically, in RV04, each investor's decision of renew the credit is based on two factors: i) his own opinion about the bank's fundamental risk, which is captured by the signal on fundamental he received is whether larger than some threshold; ii) his expectation of what others do, that is a higher threshold used by others induces a manager to use a higher threshold also and there is a strategy complementarity between investors' decision. The intuition behind is that a large withdrawal by other investors might force the bank to to liquidate some of its assets at a loss ("fire sales") or for for an amount strictly lower than the value of the assets that the bank can offer as collateral ("high margin/haircut"). As a consequence, liquidity problems might provoke the initial solvent bank become insolvent, there is an inefficient bank run.[5][6]

Goldstein and Pauzner (2005) presents a modified version of BDD model, in which the fundamentals determine which equilibrium occurs. Morris and Shin (2009) use the similar set up and technology show that liquidity risk is the probability of a default due to a run when the institution would otherwise have been solvent. He and Xiong (2012) presents a model where the coordination problem between investors has been dynamically studied. Unlike in BDD model, they derive a unique monotone equilibrium, in which the investors coordinate their asynchronous rollover decisions based on the publicly observable time-varying bank fundamental.

[5]By using global games techniques as in Morris and Shin (2001), RV04 show that in the imperfection information case, where investors have different opinions about the bank's solvency, there is a unique equilibrium.

[6]Another main contribution of RV04 is that they show that the Bagehot's Lend of Last Resort might increase social welfare by avoiding inefficient closures of solvent banks. But, the main purpose of this sector is to identify the sources of liquidity fragility, so I don't mention this in the main body. For details, see Rochet and Vives (2004).

2.2.2 Self-fulfilling liquidity dry up

Unlike the bank run literature, in which the liquidity problem mainly results from a coordination problem between creditors. Malherbe (2012) shows that, due to the adverse selection problem in the secondary market, liquidity dry-up can endogenously arise because FIs self-insure against it (self-fulfilling liquidity dry-up). This is because hoarding behavior and adverse selection may reinforce each other. The intuition behind this logic is that, i) if FIs anticipate that secondary market will be illiquid then they will hoard liquidity; ii) next, this hoarding behavior will worsen the adverse selection problem in the secondary market (only lemons in the market); iii) with limited participation, the secondary market is indeed illiquid, which provides a rational for hoarding ex-ante. The liquidity problem generally has a negative impact on social welfare. In BDD, the liquidity problem comes from an early withdrawal of patient consumers, force FIs to liquidate their productive investment, generating a loss. However, in Malherbe (2012), liquidity problem will break FIs' incentive to invest in the long-term investment, therefore reducing social welfare. The model is summarized below.

There are three dates, $t = 1, 2, 3$, and a measure one of ex-ante identical FIs[7], who are initially endowed with one unit of consumption good and maximize the expected utility function $\mathbb{E}[\ln C_1 + \ln C_2]$, where C_t is their consumption at date t. FIs can invest in two technologies ex-ante ($t = 0$): a storage technology as in BDD, and a risky long-term technology, which yields R_H per unit invested with probability π if it is a good project (good asset) or R_L per unit invested with probability $1 - \pi$ if it is a bad project (bad asset) at date $t = 2$. By assumption, the expected return of long-term technology is better than storage technology, and in case of failure, the storage technology is better.[8] At interim date, $t = 1$, the type of the project is privately revealed by FIs. They may issue claims to the payoff of their project in a competitive market with price q, and this is the only way to raise fund at interim date. There is also a measure one of risk-neutral "deep pocket" buyers, who ensures that the market clears at the expected value of the underlying payoff and only have access to storage and the interim market. Due to the adverse selection (Akerlof (1970)), the market unit price q is given by:

$$q(\alpha) = R_L + \alpha(R_H - R_L), \tag{2.3}$$

where α denotes the proportion of good assets in the market. The interim market

[7] More generally, this can be seen as a measure one of ex-ante identical investors, or banks or SIV (Structured investment vehicle).

[8] That is characterized by $R_L < 1 < \pi R_H + (1 - \pi) R_L$.

is a source of liquidity provision, since for a unit invested in long-term asset may yield q units of consumption good at date 1. Therefore the market liquidity is measure by this price p, and the equation (2.3) implies that adverse selection undermines liquidity provision. The market is said to be illiquid if $q < 1$, which take place when $\alpha < \frac{1-R_L}{R_H-R_L}$.

From ex-ante perspective, FIs need to choose $y \in [0, 1]$ as the initial investment decision, they also have to choose $x_i \in [0, y]$ as the amount of long-term asset to sell at market price q at interim date as well as how much to store until date 2 s_i. Note that $i \in \{L, H\}$ for different types of FIs. Therefore the FIs seek to maximize

$$\mathbb{E}[\ln(C_{1i}) + \ln(C_{2i})],$$

subject to

$$\begin{cases} C_{1i} = 1 - y + qx_i - s_i \\ C_{2i} = (y - x_i)R_i + s_i \end{cases}, \tag{2.4}$$

the budget constraints (2.4) state the following: date-1 resources consist of storage from date-0, plus the revenue from asset sales. This resources can be consumed or stored until date 2. At date 2, resources available for consumption consist of the output of the remaining share of long-term asset, plus the storage from date 1. This problem is solved by backward induction.

At interim date, the types of the projects are privately revealed to FIs, y is predetermined and q is taken as given. FIs maximize $\ln(C_{1i}) + \ln(C_{2i})$, subject to the budget constraints(2.4). Now, let's discuss the behavior of different types of FIs. L-type FIs will obviously sell all their assets, since they can always gain more by selling their bad assets on the market and store, instead of waiting the long-term project matured.[9] Given the couple (q, y), the optimal asset sale for L-type FIs is

$$x_L(q, y) = y. \tag{2.5}$$

To equate their marginal utility of consumption over time, they then set $s_L(q, y)$ so as to split their resources equally across two dates. Hence, their optimal consumption plan is: $C_{1L}(q, y) = C_{2L}(q, y) = \frac{1-y+qy}{2}$. For H-type FIs, it is not optimal to have both $x_H > 0$ and $S_H > 0$, since $q \leq R_H$ and store the sold assets

[9]Note that from equation (2.3), $p \in [R_L, R_H]$. By assumption when $p = R_L$, FIs is still willing to sell their assets.

will generate a loss. The first order conditions yield:

$$\begin{cases} x_H(q,y) = \max \quad \{0, \frac{qy-1+y}{2q}\} \\ s_H(q,y) = \max \quad \{0, \frac{1-y-yR_H}{2}\} \end{cases}, \tag{2.6}$$

Note that, the quantity of good assets to be sold, $x_H(q,y)$, increases with the amount of initial investment in long-term project y and strictly increases if $y \geq \frac{1}{1+q}$. This is because that the less cash they have at hand, the more they need to sell assets.

The optimal investment level is chosen at ex-ante, at this stage, FIs don't know their type of projects. Define $U_i(q,y) \equiv \ln[C_{1i}(q,y)] + \ln[C_{2i}(q,y)]$, the optimal level investment level given q corresponds to: $y(q) = \arg\max_y \pi U_H(q,y) + (1-\pi)U_L(q,y)$, which yields:

$$y(q) \in \begin{cases} [0, \frac{1}{2}) & ; q < 1 \\ [\frac{1}{2}, 1] & ; q = 1 \\ \{1\} & ; q > 1 \end{cases}.$$

Now, plug this optimal investment level to equations (2.5) and(2.6). This gives,

$$x_L(q, y(q)) = y;$$

$$x_H(q, y(q)) \in \begin{cases} \{0\} & ; q < 1 \\ [0, \frac{1}{2}] & ; q = 1 \\ \{\frac{1}{2}\} & ; q > 1 \end{cases}. \tag{2.7}$$

FIs are said to be hoarding when they decide to fully cover date-1 consumption needs by storing a part of their initial endowment, rather than issuing claims to the payoff of their project on the interim market. That is, when $C_{1i} \leq 1 - y$, for $i = H$, L. After some algebra, it is straightforward to conclude that when FIs anticipate an illiquid market ($q < 1$) leads to hoarding. Given a q, the proportion of good assets at the optimal investment level is:

$$\alpha(q, y(q)) \equiv \frac{\pi x_H(q, y(q))}{\pi x_H(q, y(q)) + (1-\pi)x_L(q, y(q))} \in \begin{cases} \{0\} & ; q < 1 \\ [0, \frac{\pi}{2-\pi}] & ; q = 1 \\ \{\frac{\pi}{2-\pi}\} & ; q > 1 \end{cases} \tag{2.8}$$

The rational expectation equilibrium is characterized by a triple (y^*, α^*, q^*) which satisfies: i) y^* is an optimal investment level given q^*, ii) α^* is the proportion of good assets in the market given q^* and y^*, iii) $q^* = R_L + \alpha^*(R_H - R_L)$. The equilibrium can be found by combining the buyer's no-arbitrage condition (2.3) with equation (2.8).

In this economy, there are multiple equilibrium that differ by the level of market liquidity which is measured by the anticipated market price q.[10] There is a liquid equilibrium when FIs anticipate $q > 1$, which characterize the market is liquid. Accordingly, they invest all their endowment in the long-term asset, $y^* = 1$. Consequently, they have to go to the market to raise funds to satisfy the interim consumption needs. This ensures that there is also a relative high proportion of good asset in the market, α^* relatively high, which indeed guarantee that the market is liquid (equilibrium price $q^* > 1$). However, if FIs anticipate an illiquid market ($q < 1$), they will hoard liquidity (self-insurance and $y^* < \frac{1}{2}$). Furthermore, this hoarding behavior worsen the adverse selection problem in the market, and only lemons are available in the market ($\alpha^* = 0$), which implies the market is illiquid ($q^* = R_L < 1$). The second equilibrium is said to be a *self-fulfilling liquidity dry-up*.

A very similar paper is presented by Plantin (2009), in which the author assumes that long-term assets will distribute proprietary information to their holders in the future, because of this "learning by holding" behavior, a lemons problem is created in the market. The fear of such lemons problem deters participation in the market, thus current and future illiquidity reinforce each other and generate a self-fulfilling liquidity problem even with a large pool of potential investors. Eisfeldt (2004) also develops a model in which long-term assets are illiquid due to adverse selection problem, and this illiquidity is less severe when the productivity is high.

2.2.3 Cash-in-the-market pricing

The relationship between liquidity and asset prices plays a crucial role in the fragility of FIs. Allen and Gale (1994) and Allen and Gale (1998) first illustrate how asset prices depend on the liquidity of the market participants' portfolios and then Allen and Gale (2005) (hereafter AG05) provide a review of the literature that explores the relation between asset price and financial fragility when markets

[10]Here, I ignore the unstable equilibrium when $p = 1$.

and contracts are incomplete. If an aggregate shock requires several FIs to sell assets at the same time, as the assets sales push asset prices lower, FIs are forced to sell even more assets, which exacerbates further the decline in prices further. The idea relies on the fact that supply and demand for liquidity are inelastic in the short-run. This concept is called "*cash-in-the-market pricing*".

More precisely, in AG05 there are two basic elements about liquidity. On the demand side (liability side of balance sheet), following Diamond and Dybvig (1983), liquidity preference is represented by uncertainty about future time preferences. On the supply side (asset side of balance sheet), there are two assets exhibit a trade-off between asset return and maturity. There are three dates indexed by $t = 0, 1, 2$, a single all-purpose good, and two assets, short-term asset (invested in storage technology) and long-term asset (invested in long-term risky technology). Both technologies are the same as in BDD, for one unit invested ex-ante, storage technology yields unit return at each date, and long-term technology yields $R > 1$ at final date. Note that the trade-off is the short-term asset offers immediate but lower return; the long-term asset offers higher but delayed return. There is a continuum of ex-ante identical consumers, each with an endowment of one unit of good at date 0. At interim date, consumer receives a preference shock. With probability λ, he becomes an early consumer who only values consumption at date 1 and with probability $1 - \lambda$ he becomes a late consumer who only values consumption at date 2. The expected utility is the same as in formula (2.1). The only aggregate uncertainty concerns the demand for liquidity, in which the number of consumers who desire to consume early is binomial. In different states of world, λ takes two values $0 < \lambda_L < \lambda_H < 1$ with equal probabilities. At interim date, the true value of λ is realized.

For the purpose of illustration, suppose at $t = 0$, consumers invest their endowments in a portfolio consisting of y units of short-term assets and $1 - y$ units of long-term assets. Let q_s denote the price of the long-term asset at $t = 1$ in states $s = H$, L. Market clearing at interim date requires that late consumers are willing to hold the long-term asset, which strictly dominate short-term asset, this implies that $q_s < R$. In a given state s, the fraction of early consumers, λ_s, is known with certainty and the only uncertainty is the consumer's liquidity preference. As in BDD, FIs can efficiently pool resources together and insure the liquidity risk thus attain the social optimality.

At $t = 0$, consumers deposit their resources with a FI that offer them a deposit contract in exchange. If the FI offers them a fully contingent contract, there is no need for or possibility of default. If FIs use (non-contingent) deposit contracts, however, the ability of FIs to meet its obligation will highly depends on the price of long-term assets, for some prices sufficiently low, default may be unavoidable.

Suppose the deposit contract commit to pay a fixed amount d if consumers withdraw early at interim date and the residual value of the portfolio date final date. Since a late consumer has the option to withdraw at $t = 1$ and storing the good until final date, he must receive as much as an early consumer. Then the FI is solvent at $t = 1$ if the present value of consumption promised to consumers does not exceed the present value of assets:

$$\lambda d + \frac{q_s}{R}(1-\lambda)d \leq y + q_s(1-y),$$

where $\frac{q_s}{R}$ is the present value of one unit of the good at $t = 2$. The FI is insolvent if this inequality is violated, therefore it must be liquidate all its assets and give each depositor $y + q_s(1-y)$. If the FI is solvent, in case of $\lambda d > y$ (short-term asset can't fully cover the early consumption), it supplies S units of assets so that $q_s S + y = \lambda d$, it needn't to liquidate any long-term asset in the case of $\lambda d \leq y$ (we don't consider this simple case in the model); if FI is insolvent, it supplies $S = 1 - y$. Thus, the supply of asset is

$$S(q) = \begin{cases} \frac{\lambda d - y}{P} & \text{if } q \geq q^* \equiv \frac{y-\lambda d}{(1-\lambda)d/R-(1-y)} \\ 1-y & \text{if } q < q^* \equiv \frac{y-\lambda d}{(1-\lambda)d/R-(1-y)} \end{cases}.$$

This supply function is a "backward bending supply curve" and has three important features: i) there is a discontinuity at $q = q^*$; ii) for values of $q > q^*$, the FI is solvent and the supply of asset is decreasing in price P, since the amount of asset to be sold as the price of asset decreases; iii) when $q < q^*$, the supply of asset is constant, since the price is too low and FI defaults and has to liquidate all its long-term asset. In an interesting case where FI has to sell off $(\lambda d - y)/q$ units of long-term asset to pay for $\lambda d - y$ units of consumption. The lower the asset price q, the more of the asset must be sold. Some of the long-term asset have to be hold to meet $(1-\lambda)d$ units of date 2 consumption, if the price q falls far enough, it is impossible to satisfy both early and late consumers. Any price lower than q^*, will cause default and FI has to liquidate all its entire stock of the long-term asset $1 - y$.

In summary, FIs can pool liquidity and offer non-contingent contract deposit contract. However, the lower the resale price, the more long-term assets the

FI needs to be liquidated in order to honor its commitment towards consumers. If the resale price is low enough, the FI goes bankrupt and then its entire holding of long-term assets are sold on the market, creating a failure and even a "crisis". **?** have also stressed that long-term assets are particularly subject to market liquidity risk. Bolton, Santos, and Scheinkman (2011) studied this phenomenon combine with adverse selection of long-term assets in the secondary market, they show that the adverse selection may inefficiently accelerate asset liquidation.

2.2.4 Liquidity spiral

In Diamond and Dybvig (1983), the main liquidity problem comes from the instability of the fractional reserve system. More specifically, this is because the patient depositors may withdraw early and cause a funding liquidity problem for FIs. Malherbe (2012) present a "restricted" BDD model, where FI cannot pool the liquidity, he mainly focus on the market liquidity for FIs. However, an important feature of modern counterpart run is characterized by collateral run, based on this, Brunnermeier and Pedersen (2009) illustrates that a FI's funding liquidity can reinforce each other with market liquidity, due to margin spiral and loss spiral. Note that the cash-in-the-market pricing phenomenon (Allen and Gale (1994), Allen and Gale (1998)) also stresses the link between these two types of liquidity problem, but it's presented in a static way.

In Brunnermeier and Pedersen (2009), when FIs buy an asset, they can use this asset as collateral and borrow against it. However, the collateral value is lower than the price of asset, the difference is called margin/haircut, m, which have to be financed by FIs' own capital. Similarly, capital is used as margin for short-selling as well. Therefore, the margins cannot exceed the capital at any time:

$$\Sigma_j(x_t^{j+}m_t^{j+} + x_t^{j-}m_t^{j-}) \leq C_t. \tag{2.9}$$

Each risk-neutral FI finance $x_t^{j+}m_t^{j+}$ with his own capital on the long-position of asset j, whose total value is $q_t^i x_t^{j+}$. It is the same for short-position $x_t^{j-}m_t^{j-}$.

Asset j's conditional expected fundamental value v_t^j is assumed to follow an ARCH process, which is described as follow:

$$v_t^j = v_{t-1}^j + \Delta v_t^j = v_{t-1}^j + \sigma_t^j \epsilon_t^j, \tag{2.10}$$

$$\sigma_{t+1}^j = \underline{\sigma}^j + \theta^j \mid \Delta v_t^j \mid, \tag{2.11}$$

where $\epsilon_t^j \sim^{iid} N(0,1)$, and $\underline{\sigma}^j, \theta^j \geq 0$. A positive θ^j implies that the future volatility is positive correlated with the shocks to fundamental. Denote ω_t^j the deviation of asset j's price at time t, q_t^j, from the fundamental value. Define the market illiquidity measure is the absolute value of this deviation, $\mid \omega_t^j \mid = \mid q_t^j - v_t^j \mid$. Note that this gap is zero when the market is perfect liquid, however this phenomenon can be hardly realized in a world with credit constraint, since it limits the market liquidity provision. Then FI invests all his capital such that his credit constraint is binding and optimally trades only in asset with the highest expected profit per dollar used, that is the ratio of the deviation from fundamental over the unit margin. The measurement of FI's funding liquidity is denoted by ϕ_t, which is equal to the marginal value of an extra dollar.

Investors finance FIs with collateralized financing. They set the margins based on Value-at-Risk of the adverse price variation. More precisely, the margin m_t is set large enough to cover the position's α-VaR ($\alpha$is set based on the investor's risk aversion):

$$\alpha = \Pr(-\Delta q_{t+1}^j > m_t^{j+} | \text{information set}_t), \tag{2.12}$$

with formula (2.12), an explicit expression for margins m_t^{j+} can be determined, with the short-position case is in the same logic behind. If the investors confound with fundamental shocks and instantaneous shocks[11] (very similar in the real world), they will generate "destabilizing margins". In this case, margins are increasing in market illiquidity, the intuition is that these investors may translate price volatility into fundamental volatility and this increases margins.

As FI's wealth falls, which can be triggered by an independent shock η_t or a low realization of fundamental value, the price discontinuously declines. This continuity is referred as fragility of liquidity. On top of this liquidity fragility, the price is also very sensitive to a negative shock on FI's wealth ($\eta_t < 0$) due to two liquidity spirals: the margin spiral and loss spiral that leads to deleverage. This give a procyclical effect in the economy, and Adrian and Shin (2011) shows the evidence on this issue based on the large U.S. investment banks' data set. This is because an initial loss may worsen the FI's funding problem, then the position is reduced (formula (2.9)), further the prices move away from fundamentals (market illiquidity increases/$\mid \omega_t^j \mid$ large), investors set a higher margin (formula (2.12)), in turn worsen the funding illiquidity further (*margin spiral*); meanwhile when market illiquidity increases, FI also suffer a big loss from existing positions, which will further limit FI's credit constraint (*loss spiral*), and so on.

[11] Only the price process of asset is observable by the investors.

2.3 Time dimension of macroprudential policy

In the previous section we have seen the sources that may trigger a financial fragility to an individual FI. If the analysis just ends up at this stage, our knowledge about the financial fragility will be at the same level as before the 2007-2009 financial crisis. This recent financial crisis has highlighted the need to go beyond a purely micro approach to maintain the financial stability and has intensified the theoretical and political interest in strengthening the *macroprudential* orientation of the current regulatory framework. Macroprudential policy is a complement to *microprudential* policy and it interacts with other types of public policy that have an impact on financial stability (e.g. Monetary policy, fiscal policy, etc.). Its goals are to address two dimensions of system-wide risk: first, the evolution of risk over time – the "*time dimension*;" and second, the distribution of risk in the financial system at a given point in time – the "*cross-sectional dimension*." In this section I present the time dimension of macro-prudential policy, and the cross-sectional dimension will be presented in the next section.

The key issue in the time dimension is to mitigate or dampen financial system procyclicality. A common explanation for the procyclicality of the financial system has its roots in the informational asymmetries between borrowers and lenders. When economic conditions are depressed and collateral values are low (high margin/haircut - this is very well explained by Brunnermeier and Pedersen (2009) in the previous section), informational asymmetries can mean that even borrowers with profitable projects find it difficult to obtain a funding. When economic conditions improve and collateral values rise, these firms are able to gain access to external finance and this adds to the economic stimulus. This explanation of economic and financial cycles is often known as the "financial accelerator". Moreover, another source of procyclicality could be the risk-sensitive regulation. As a result of this risk-sensitive regulation, a widespread concern about Basel II is that it might amplify business cycle fluctuations, allowing banks to expand their credit volume in the booming period and forcing banks to restrict their lending when the economy falls into recession.

2.3.1 Asymmetric information channel

Bernanke and Gertler (1989) introduce imperfect information into a real-business-cycle framework which extends to a two-period overlapping generation model, in which the project outcomes are only observable by risk-neutral entrepreneurs. They borrow funds from lenders who suffer from a costly state verification problem (Townsend (1979)). A negative shock on entrepreneurs net worth may increase the financing friction and decrease the current real capital investments. The opposite results can be obtained if entrepreneurs receive a positive fundamental shock.

Kiyotaki and Moore (1997) (hereafter KM97) show how small shocks to the economy might be amplified into a large output fluctuations by credit constraint. In KM97 model, in the presence of credit contract with limited enforcement[12], durable asset (land) plays two distinct roles: (i) the role of collateral for debt and (ii) the role of productive input. In such an economy, the level of credit limit to each firm positively depends upon the value of land, while the demand for land is increasing in credit provided to each firm.

More specifically, in the model of KM97, two types of households with different time preference rates are assumed: "patient" (Gatherers) and "impatient" (Farmers). As the impatient households are not satisfied with the market interest rates, they borrow from the patient household. When borrowing money, they have to provide durable asset (land) as collateral. As the value of durable asset (land) declines, so does the amount of debt they can acquire. This feeds back into the land market, driving the price of land further down.

These two types of households are risk neutral and infinitely lived with population 1 for Farmers and m for Gatherers. Farmer are productive agents that characterized by (i) a constant-return-to-scale production function $y_{t+1}(k_t)$ which yields tradable output ak_t and nontradable output ck_t in period $t+1$ for an input of k_t of assets in period t, and (ii) a discount factor $\beta < 1$. Gatherers are unproductive agents that characterized by (i) a decreasing-return-to-scale production technology which yields output $G(k_t^{'})$ in period $t+1$ for an input of $k_t^{'}$ of assets in period t, where $G^{'} > 0$ and $G^{''} < 0$, and (ii) a discount factor $\beta^{'} \in (\beta, 1)$.

As mentioned before, productive agents will want to borrow from unproductive agents, but each productive agent's technology is idiosyncratic[13] and he cannot precommit to work, so lenders will call for the land as collateral against the money they lend to the borrowers. By assuming there is no uncertainty for

[12] See Hart and Moore (1994).

[13] In the sense that, once his production has started at date t with land k_t, only he has the skill necessary for the land to be harvest at date $t+1$.

the future asset price, the productive agents' credit constraint is given by:

$$Rb_t \leq q_{t+1}k_t. \tag{2.13}$$

In equilibrium, when there is no shocks on output, productive agents prefer to borrow up to the maximum and invest in land, consuming no more than their current nontradable output and the credit constraint (2.13) binding. This gives the following demand function for assets k_t in period t:

$$k_t = \frac{1}{q_t - \frac{q_{t+1}}{R}}[(a + q_t)k_{t-1} - Rb_{t-1}]. \tag{2.14}$$

The term $(a + q_t)k_{t-1} - Rb_{t-1}$ is the productive agent's net worth given by value of tradable output and current asset holdings from the previous period, net of debt repayment. This net worth is levered up by the inverse of difference between price of land, and the amount the farmer can borrow against each unit of land as collateral. $(q_t - q_{t+1}/R) = u_t$ is the margin requirement implied by the credit constraint.

The unproductive agents' technology is not idiosyncratic, that say, if one decide to quite the project there is always another one can continue to implement the project. Therefore, the productive agents are not credit constraint and equilibrium interest is equal to their discount rate, $R = 1/\beta'$. An unproductive agent's demand for land is determined at the point at which the present marginal benefit of land using is equal to the opportunity cost of holding land u_t:

$$\frac{G'(k_t')}{R} = u_t. \tag{2.15}$$

Denoting aggregate quantities by capital letters and the total supply of assets by $\overline{K}$, market clearing in the assets market at t requires the total demand of assets of productive and unproductive agents equals the total supply, $K_t + mk_t' = \overline{K}$. Given the unproductive agent's first order condition (2.15) this implies

$$u_t = q_t - \frac{1}{R}q_{t+1} = \frac{1}{R}G'[\frac{1}{m}(\overline{K} - K_t)] \equiv F(K_t). \tag{2.16}$$

In equilibrium, the productive agents' margin requirement u_t is positively related to their demand for assets K_t, since the unproductive agent's technology is concave. A less K_t gives that there is more assets being used in the unproductive agents' technology which implies a lower marginal product. In equilibrium, this lower marginal product is associated with a lower opportunity cost of holding assets.

In the steady state, the margin requirement per unit of land is equal to the productive agents' marginal productivity of tradable output. This implies the asset price in the steady state is

$$q^* = a\frac{R}{R-1}, \tag{2.17}$$

which implies that the total demand of assets K^* for productive agents is given by,

$$G'[\frac{1}{m}(\overline{K} - K^*)] = aR. \tag{2.18}$$

Note that the input allocation is not efficient at steady state. Given the linearity of productive agents' production function, their marginal production is always equal to $a+c$, larger than the marginal production for unproductive agents is aR[14]. Therefore, around K^*, an increase for K_t causes a Pareto improvement in the total output, since $G'' < 0$.

The main contribution of KM97 model is that then they introduce an unexpected shock to productivity and illustrate the dynamics of the economy. This is also the main effect that I emphasize in this subsection that how the procyclicality may appear in the economy. The response to this unexpected shock is studied by the reaction of the model log-linearized around steady state. Before all, suppose at period $t-1$ the economy is in the steady state and then there is a unexpected one-period shock that increases the production of both agents by a factor $1+\Delta$[15] at period t.

Denote $\widehat{K_t}$ the percentage change in the asset holdings of productive agent K_t relative to its steady state K^* and for a given percentage change in asset price $\widehat{q}_t$. We have,

$$\widehat{K_t} = (\frac{1+\xi}{\xi})^{-1}(\Delta + \frac{R}{1+R}\widehat{q}_t). \tag{2.19}$$

where $\xi > 0$ denotes the elasticity of the unproductive agents' residual asset

[14]Given the assumption that $aR < a/\beta < a+c$.

[15]Here, Δ is taken to be positive. To illustrate an adverse shock in the economy, one can take Δ to be negative as well.

supply with respect to the opportunity cost at the steady state.[16] The second bracket of (2.19) shows that there is two positive effects to productive agents' asset holdings. The direct effect of the productivity shock Δ; and the indirect effect by an unexpected rise in price $\widehat{q}_t$ and an increase in the value of asset holdings from the previous period. Crucially, the second effect is scaled up by the factor $R/(1+R)$ because of leverage.[17] Finally, the sum of these two effects is scaled down by the factor $\xi/(1+\xi)$, as the marginal production of unproductive agents increase, in the market clear condition, which increase the opportunity cost that limit the demand of asset for productive agents. For the following periods $t+1$, $t+2$, ... we have

$$\widehat{K}_{t+s} = (\frac{1+\xi}{\xi})^{-1}\widehat{K}_{t+s-1}. \tag{2.20}$$

This shows that the initial productivity shock persist and affect asset holdings in the following periods.

Then, the percentage change in asset price $\widehat{q}_t$ for given percentage change in asset holdings $\widehat{K}_t$, $\widehat{K}_{t+1}$, and expression (2.16) tells us that the asset price is the discounted sum of future opportunity cost, we have

$$\widehat{q}_t = \frac{1}{\xi}\frac{R-1}{R}\sum_{s=0}^{\infty} R^{-s}\widehat{K}_{t+s} \tag{2.21}$$

This shows that all changes in future asset holdings have an impact on today's asset price.

The percentage changes in $\widehat{K}_t$ and $\widehat{q}_t$ can be expressed as a function the size of the shock Δ:

$$\begin{aligned} \widehat{K}_t &= [1+\frac{1}{(R-1)(1+\xi)}]\Delta \\ \widehat{q}_t &= \frac{1}{\xi}\Delta \end{aligned}$$

We see that for the variation of asset holdings, the amplification effect appears from the term in the squared bracket is greater than one. In particular, the lower interest rate R and elasticity ξ, the larger the amplification effect. In terms of asset price, the initial shock Δ implies the percentage change of the same order of magnitude and again the effect is negatively related to the elasticity.

[16] $\frac{1}{\xi} = \frac{dlogF(K)}{dlogK}|_{K=K^*}$

[17] This is easily seen from expression (2.17).

The comparison between static and dynamic multiplier effects can be done by artificially pegging q_{t+1}, the price of asset at period$t+1$, at the steady state level q^*. By doing so, the dynamic responses between asset holdings and asset price are cut off. The proportional changes with respect to the shock Δ are summarized as follows:

	Static	Dynamic
$\widehat{K}_t$	1	$\frac{1}{(R-1)(1+\xi)}$
$\widehat{q}_t$	$\frac{R-1}{R}\frac{1}{\xi}$	$\frac{1}{R}\frac{1}{\xi}$

The comparison shows clearly that the dynamic effect has a greater importance than static effect for both $\widehat{K}_t$ and $\widehat{q}_t$. Next, the output and productivity response to the shocks is expressed as follow:

$$\widehat{Y}_{t+s} = \frac{a+c-Ra}{a+c}\frac{(a+c)K^*}{Y^*}\widehat{K}_{t+s-1} \qquad \text{for } s \geq 1 \tag{2.22}$$

With the formula (2.22), a procyclicality effect is clearly presented. A productivity positive (negative) shock leads to a growth (reduction) of output and of the value of collateral and increase (reduction) in borrowing, which increases (reduces) output further; the multiplier effect amplify gains (losses). Note that the effects of shocks in KM97 are completely symmetric.

One of the key sources of the procyclicality effect is that the borrower's human capital is inalienable, which is identified by Hart and Moore (1994). Therefore, a credit constraint is imposed in the economy, which makes the net worth of the borrower's project important. In contrast, in an unconstrained economy, the changes in the net worth of borrower do not have a significant effect on the value of collateral and productivity. As a result, productivity shocks will have an effect on output only in the period when they occur.

Beyond KM97, Gersbach and Rochet (2011) shows that credit cycles can be harmful to social welfare even if there is no financial crisis. In Gersbach and Rochet (2011), the interplay of three ingredients: i) moral hazard, ii) high exposure to aggregate shocks and iii) possibility to reallocate capital across sectors, characterizing the activities of modern banks, which gives the explanation that when productivity shocks take place, the banking sector tends to exacerbate them generating excessive fluctuations of credit, output and asset prices. More precisely, the moral hazard issue raised from that the banks are not willing to shirk on their monitoring activities only if they have received sufficient information rents. These rents limit the size of the investment, since investors participation constraint is sensitive to it.[18] High exposure to aggregate shocks suggest that banks' assets returns are positively correlated. Last, flexibility of investment decision guarantees that banks can reallocate capital between lines of business. Myerson (2012) have also introduced a model of credit cycles driven by moral hazard in FI. Furthermore, Gersbach and Rochet (2012) (hereafter GR12) shows that, even in a complete market, there is an excessive credit fluctuation in banking sector, and then provide a rational for imposing counter-cyclical capital ratios in this sector.

More specifically, in GR12 model, it has three dates $t = 0, 1, 2$ and two goods, consumption and capital goods; the total physical capital good is normalized to 1. There is a continuum of bankers, each of them is endowed with wealth e and $E < 1$ for aggregate level, they can access to a lending technology with constant return to scale. Denote R the banks' assets expected return. Bankers can borrow from outside investors, whose endowment is $1-E$, due to the financial friction[19], but they cannot fully pledge their future cash flow to its creditor. Like in Holmstrom and Tirole (1997), the rent have to be kept by banker is a multiple ρk of the size k of investment. Apart from banking sector, there is a traditional sector which has a decreasing-returns-to-scale technology with production function $F(\cdot)$. All agents are risk neutral. The aggregate uncertainty is at interim date. The return on banking sector is R_s with probability λ_S, where $s = \{h, l\}$, for R_h is called s "boom" and for R_s is called a "recession". The traditional sector is independent on this uncertainty.

Note that, there is a contingent markets for capital at date 0, therefore, bankers can obtain a state contingent capital endowment e_s. Once the state realized at date 1, bankers invest k_s units of capital with an amount of outside funding $k_s - e_s$ from investors. In exchange, bankers promise to pay $q_s(k_s - e_s)$ units of consumption to investors at date 2, so q_s can be interpreted as the deposit rate/price of capital. The total output in the economy is sum of the outputs in tow sectors: $F(1 - K_s) + R_s K_s$, for K_s is the aggregate level of credit volume in the economy for state s.

The complete markets assumption holds due to the fact that modern banks have access to complex financial instruments that allow them to hedge against macro-shocks. Therefore, it is possible for the bank to swap $e_h - e$ unit of capital in "boom" against $e - e_s$ units of capital in "recession" at a swap rate

[18]See Holmstrom and Tirole (1997) for details.
[19]The specific form of this financial friction does not matter.

must be equal to $\frac{\lambda_h q_h}{\lambda_l q_s}$. Thus the budget constraint of the bank writes as

$$\mathbb{E}[q_s(e_s - e)] = 0. \tag{2.23}$$

From the investor's participation constraint we know that the promised expected repayment $q_s(k_s - e_s)$ does not excess the bank's maximum pledgeable income $(R_s - \rho)k_s$. Alternatively speaking, investors are willing to borrow their money to banks if and only if their stake of cash flow is not less than their outside return. As bankers take on as much leverage as they can, only equality holds, which gives

$$e_s = k_s(1 - \frac{R_s - \rho}{q_s}), \tag{2.24}$$

the bank will maximize the expected rent $\rho\mathbb{E}[k_s]$ by choosing the contingent credit volumes (k_h, k_l) subject to the constraint obtained by plugging (2.24) into (2.23):

$$\mathbb{E}[k_s(q_s + \rho - R_s) - q_s e] = 0. \tag{2.25}$$

Since both objective function and constraint (2.25) are linear in (k_h, k_l). The only possible interior solution in equilibrium is such that the coefficient of k_s in constraint (2.25) is the same in both states.[20] Denoting by $q \equiv \mathbb{E}[q_s]$ the expected price of capital and $R \equiv \mathbb{E}[R_s]$ the expected return on asset, the constraint (2.25) can be simplified:

$$\mathbb{E}[k_s](q + \rho - R) = qe.$$

By aggregating this condition over all banks, we obtain the expected demand of capital by bank.

$$\mathbb{E}[K_s] = \frac{E}{1 - \frac{R-\rho}{q}}. \tag{2.26}$$

The non-arbitrage condition guarantees that for marginal productivity of capital in traditional sector should be equal to the price of capital q_s, in state of s: $q_s = F^{'}(1 - K_s)$, which can be written as $K_s = S(q_s)$. By the property of the coefficient of k_s in equation (2.25) is the same, the expected supply of capital is: $K_s = S(R_s - R + q)$. Therefore, the expected price of capital q can be determined by equaling expected supply and demand:

$$\mathbb{E}[S(R_s - R + q)] = \frac{E}{1 - \frac{R-\rho}{q}}. \tag{2.27}$$

The intersection of demand and supply implies that, the volume of credit increases with banking sectors' aggregated initial capital E and decrease with the

[20] Since there is no evidence have shown that in any state the credit volume will be zero, the corner solution can be ignored in this case.

non transparency in banking sector ρ. So, when financial markets are complete, there is a unique equilibrium denoted by $K^c = (K_h^c, K_l^c)$ with prices (q_h^c, q_l^c). This market equilibrium is procyclical, since $K_h^c > K_l^c$ and $q_h^c > q_l^c$; moreover, this equilibrium is generally constraint inefficient. To illustrate this inefficiency, consider a social planner who has the interest of maximizing the total expected under an aggregate form of constraint (2.25) :

$$\mathbb{E}[K_s(q_s - R_s + \rho)] \leq \mathbb{E}[q_s]E, \tag{2.28}$$

which yields the optimal allocation (K_h^o, K_l^o), and $K_h^o < K_h^c$ and $K_l^o > K_l^c$.[21] This is said that the total expected output in market equilibrium can be ameliorated by reducing the aggregate credit volume in "boom" and increasing the aggregate credit volume in "recession". The result shows that the complete market do not sufficiently stabilize credit fluctuations, as banks allocate too much borrowing capacity to good states and too little to bad states.

At last, GR12 propose that, under mild conditions, by imposing an upper bound on credit in the good state, the regulator can restore the optimality in the market. They also argue that the term "counter-cyclical capital buffer" should be banned, since the standard notion of minimum capital as a buffer is for absorbing losses and it is in the perspective of micro-prudential. Therefore, the right term would be "Counter-cyclical Capital Ratio", as its role is no longer absorb losses but must be influencing the volume of bank lending. Furthermore, the micro-prudential policy such as Basel II doesn't work in this context because it is procyclical by nature, we will investigate this effect in section 3.2. Shleifer and Vishny (2010) also introduces a theoretical model to explain the cyclical behavior of credit and investment and identifies this is one of the key factors of the instability of universal banking system.

2.3.2 Risk-sensitive regulation

Basel II introduced a menu of approaches to determine capital requirements, which depends on the sophistication of a bank's activities and on its internal risk management capabilities. The standardized approach is based on external credit ratings to refine the risk weights of the 1988 Accord (Basel I), but leaves the capital charges for loans to unrated firms essentially unchanged. The internal-rating based (IRB) approach allows banks to assign an exposure to different

[21] Note that $q_s = F^{'}(1 - K_s)$.

asset classes; within each class, banks assign a different internal grade to the creditworthiness of borrowers. This implies that banks need to estimate the one-year ahead probability of default (PD) of each borrower in foundation IRB approach, and in the advanced IRB approach, they need to estimate the loss given default (LGD), the exposure at default (EAD) and maturity (M) for each borrower (for foundation IRB approach, these three variables are taken directly from supervisors).

The issue of risk-sensitive regulation (Basel II) is that it may also amplify the fluctuation of credit volume through the business cycle (Borio, Furfine, and Lowe (2001) and Kashyap and Stein (2003)), which sows the seeds of the crisis in the boom and impairs the lending capacity in downturn prolonging the crisis period. For example, under the IRB approach of Basel II, capital requirements are an increasing function of the four variables mentioned above. During the booming period, these variables are low and determine a low capital requirement, generating a massive credit growth and asset credit bubbles; however, during the recession, these variables sharply increase along with a high requirement of capital and lead to a limitation in the supply of credit. Note that, in downturn, it is difficult or costly for bank to raise new external capital, by maintaining the same capital requirement ratio,[22] it will be forced to reduce its lending activity, thereby amplify the initial downturn. Some other papers also point out the potential importance of the procyclical effects of risk-sensitive capital requirements before the financial crisis, for example, Danielsson, Embrechts, Goodhart, Keating, Muennich, Renault, and Shin (2001) and Gordy and Howells (2006). Adrian and Shin (2011) shows that there is a strong positive link between the leverage growth and assets growth in investment banking sector. This result is the consequence of the countercyclical nature of the individual risk measure and the relevant regulatory framework is based on this risk measure.[23]

In order to go into the details of shortcomings of this risk-sensitive capital regulation, it is worth starting to point out the underlying economic goals of bank capital regulation. As with any form of financial regulation, the purpose of bank capital regulation is to minimize the market failure and make the financial system more stable. From this perspective, for a social planner, it is quite reasonable to increase the capital when the economic environment becomes bad. However, this is not the only aspect that the social planner should care about, he should also continue to put weight on those objectives of banks that were properly take

[22]The set of risk weights assigned to loans is unchanged.

[23]The risk measure mentioned here is the Value-at-Risk.

into account at first place, for example, making positive net present value (NPV) loans. This demonstrates a trade-off between bank's riskiness and bank's profit, therefore, for any capital regulation only focus on the riskiness of bank would be suboptimal. In contrast to this trade-off type logic, Basel II approach can be seen as a single "risk curve",[24] which relates the capital charge solely to the riskiness of loan portfolio. The problem with such a single once-and-for-all curve is that this essentially takes the market failure as the only factor in the social planner's objective function and completely ignoring the importance of bank lending activity. This is the root that make capital requirements may inefficiently exacerbate cyclical fluctuation in credit volume.

Why this "single risk curve" may lead to a inefficient procyclical effect? For instance, consider the economy is in downturn, and this will generate two effects. First, there is a loss on existing loan positions, thereby eroding bank's capital. Second, the existing non-default loans are likely to become riskier, which says the variables in the IRB approaches have sharply increased for these loans. Thus, the capital requirement under this "single risk curve" will go up. This will further tighten the overall capital constraint, putting additional downward pressure on lending activity. In terms of economic efficiency, the key factor of interest is the shadow value of bank capital. As in Repullo (2012), this shadow value of bank capital is the Lagrange multiplier of capital availability constraint λ, which measures the variation of social welfare (including the positive NPV lending opportunity) resulting from a marginal variation in the scarcity of capital. A higher λ indicates that the bank capital becomes more scarce and, hence, more severe issues of underinvestment in terms of lending, and ultimately in terms of social welfare. If a time-fixing capital requirement ratio has implemented through the whole business cycle; during the crisis, λ will increase[25], it leads to a excessive low lending activities generating an inefficient procyclicality effect. From the perspective of the tradeoff mentioned above, this risk-sensitive capital regulation would be suboptimal.

Cyclical adjustment of capital requirements. Repullo (2012) explicitly introduces a social planner whose objective function has indeed considered the tradeoff mentioned above. This paper shows that the optimal capital requirements are greater than the capital that banks are willing to hold voluntarily if there is no capital requirement imposed on them. This is because when bank failures entails a social cost, and this effect won't be taken into account when banks'

[24]In Basel II, capital requirement ratio is time-fixing.

[25]Since, during the crisis, a negative shock will take place at the supply side of bank capital, and implies a higher scarcity of bank capital relative to positive BNP lending opportunities.

managers make their decisions. Moreover, when a negative shock to the aggregate bank capital supply, which could be interpreted as the result of a downturn in the economy that produces losses to the banks' loan portfolio and consequently reduces their capital. This leads to a higher shadow value of bank capital, which implies a shrink of banks' lending in order to economize on scarce bank capital; moreover, the optimal capital requirement is deceasing in λ, the intuition is that the optimal way to accommodate the shock in the aggregate supply of bank capital is to reduce capital requirements in order to avoid the sharp reduction in aggregate investment. Therefore, the optimal capital requirements should be cyclical adjustable, to avoid the bank lending to bear the entire brunt of the adjustment. The paper also offers a comparison of cyclical adjustment of capital requirements and time-fixing capital requirements facing a 25% decrease of aggregate bank capital supply:

	Cyclical adjustment		Time-fixing
Aggregate investment	I^a	$>$	I^f
Social welfare	S^a	$>$	S^f

The compassion table clearly states that in terms of aggregate investment and social welfare, the cyclical adjustment capital requirements is more efficient than the one for time-fixing capital requirement à la Basel II. This concludes that the countercyclical capital requirements in Basel III is a good start for dealing the procyclical effects in the economy.

Repullo and Suarez (2012) presents a dynamic equilibrium model and business cycle is characterized by a Markov process that determines probability of default. In their model, they restrict that banks are unable to access the equity markets every period. As in Kashyap and Stein (2003) and Repullo (2012), they introduce a social cost of bank failure,[26] and claim that when this social cost is small, even the Basel II is significantly more procyclical then Basel I, but, is still more efficient in social welfare sense. When this cost is becoming high, again, they show that Basel III points in the right direction, with higher but less cyclically-varying capital requirements.

[26]Note that if there is no social cost of bank failure, it is unnecessary to introduce capital requirement, which is shown by Repullo (2012)

2.4 Cross-sectional dimension of macroprudential policy

2.4.1 Interbank activities and contagion

It is well known that the interbank lending can reduce individual FI's liquidity problem in the whole financial system (Battacharya and Gale (1987)). Decentralized interbank lending usually implies that banks have incentives to monitor each other. However, due to the implicit government guarantees (Lender of Last Resort), a moral hazard issue may arise in this market and destroy peer-monitoring among banks. Based on these effect, Rochet and Tirole (1996), one of the first papers (see Goldstein and Pauzner (1996)), take into account the contagion effect between banks. That is to say if a single bank is about to fail, due to the interbank linkage, the balance sheet of the other banks, which lent to it, will be weakened and thus in a danger of failure.

Allen and Gale (2000) presents a model where the optimal allocation can be decentralized by a competitive banking sector and then explain how a small shock on a few banks spread to the rest of the financial system and provide the microeconomic foundations for financial contagion. More precisely, the model is based on the Diamond and Dybvig (1983), Allen and Gale (1994) and Allen and Gale (1998) (in section 2), the key difference is that there are 4 different banks A, B, C and D constitute as a financial system. For each bank, it receives different liquidity shocks, and there are two equally likely states S_1 and S_s. The corresponding realization of the liquidity preference shocks are given in the table below:

$$\begin{array}{ccccc} & A & B & C & D \\ S_1 & \lambda_H & \lambda_L & \lambda_H & \lambda_L \\ S_2 & \lambda_L & \lambda_H & \lambda_L & \lambda_H \end{array}.$$

The average fraction of early consumers be denoted by $q = (\lambda_H + \lambda_L)/2$, a_l and a_s are the per capital amount invested in the long and short asset respectively. So, the representative bank i holds an investment portfolio $(a_s^i, a_l^i) = (a_s, a_l)$ and offers a deposit contract $(C_1^i, C_2^i) = (C_1, C_2)$. In the same logic with BDD model, a planner choose to make each bank has $a_s = qC_1$ units of the short asset, which provide qC_1 units early consumption. If the state S_1 is realized at interim date, so banks A and C each have an excess demand for $(\lambda_H - q)C_1$ units of consumption and banks B and D each have an excess supply of $(q - \lambda_L)C_1 = (\lambda_H - q)C_1$

units of consumption. By reallocating this consumption, each bank's needs can be satisfied. At date 2, an opposite direction transaction will be taken place in order to satisfy the needs for the late consumption.

Suppose a complete markets and an exchange of deposits is allowed at $t = 0$.[27] Assume every bank i holds an aggregate deposit $3(\lambda_H - q)/2$ in others banks $j \neq i$, since this aggregate deposit is equally divided in other banks and every bank i holds $d_i = (\lambda_H - q)/2 > 0$ deposits in each of the bank $j \neq i$.

Consider the budget constraint of a bank which face a high liquidity demand. It must pay C_1 to the fraction λ_H of early consumers and $d_j = (\lambda_H - q)/2$ units of deposit to the other high demand bank. So, the total demand for repayment is $[\lambda_H + (\lambda_H - q)/2]C_1$. On the other side, it has a_s units of the short asset and aggregate deposit $3(\lambda_H - q)/2$ in the other banks. Thus, the budget constraint must be satisfied is

$$[\lambda_H + (\lambda_H - q)/2]C_1 = a_s + 3(\lambda_H - q)/2,$$

which can be simplified as planner's choice $qC_1 = a_s$. Banks with low demand for liquidity must pay C_1 to a fraction λ_L to satisfy early consumer and the deposits of 2 high demand banks $(\lambda_H - q)$. It has a_s units of short asset at hand, so the budget constraint that must be satisfied is

$$[\lambda_L + (\lambda_H - q)]C_1 = a_s.$$

Since $\lambda_H - q = q - \lambda_L$, this equation simplifies to the planner's constraint $qC_1 = a_s$. In both cases, the interbank activities allow banks to meet the demands of their early consumption needs without liquidating long asset. At the final date, all the banks liquidate their remaining asset and it is easy to show that if the budget constraints at interim date are satisfied, the budget constraints at the final date are automatically satisfied too, $(1 - q)C_2 = Ra_l$. Thus, by reallocating deposits among the different banks, it is possible for banks to satisfy their budget constraints in each state S and at each date $t = 0, 1, 2$ while providing their deposits with the first-best consumption allocation through a standard deposit contract.

Since the complete market assumption may not be realistic in some cases, an incomplete market structure case may raise in this case. Specifically, each bank

[27] The complete market refers to the each bank has an access to the other banks in the system, which means that the deposits can be circulated between banks without any difficulties.

holds deposits only in one adjacent bank (see figure below). That is said that bank A can hold deposits in bank B, bank B can hold deposits in bank and so on. By assuming that the bank holds $d_i = (\lambda_H - q)$ deposits in the adjacent bank at the first date, that is, the bank A holds $(\lambda_H - q)$ deposits in bank B, and so on. Note that, the market structure has the property that every bank with a high liquidity shocks has deposits in a bank with low liquidity shocks, and vice versa. In this scenario, the results show that the first best allocation can be reached by shuffling deposits through the interbank market, even if the interbank deposit market is incomplete.

$$\begin{array}{ccc} \text{A} & \rightarrow & \text{B} \\ \uparrow & & \downarrow \\ \text{D} & \leftarrow & \text{C} \end{array}$$

Now, a state S_3 has been introduced to "perturb" the model. In this state, the aggregate demand for liquidity is greater than the system's ability to supply liquidity. The market structure is the same in the previous incomplete markets case. Note that, the probability of state S_3 converges to zero, so it will not change the allocation at date 0. The liquidity shocks among banks is summarized as follow:

$$\begin{array}{ccccc} & \text{A} & \text{B} & \text{C} & \text{D} \\ S_3 & q+\epsilon & q & q & q \end{array}.$$

In state S_3, every bank has the previous average demand for liquidity q except for bank A where the demand for liquidity is higher $q+\epsilon$. The crucial fact is that the expected demand for liquidity across all for banks is slightly higher than in state S_1 and S_2. This perturbation can be interpreted as a banking crisis in bank A. At the equilibrium, consumers will optimally decide whether to withdraw early or late, early consumers always withdraw at interim date; late consumers will withdraw at interim date or final date depending on which gives them the larger amount of consumption. Bank may liquidate their long asset in order to meet the demand for liquidity. The following analysis is focused on state S_3, assuming the allocations consistent with the first best at date 0.

If a bank can meet the demands of every depositor who wants to withdraw (including other banks) by using only its liquidity asset, this bank is said to be *solvent*. If a bank can meet the demands of its depositors but only by liquidating some of the long asset, this bank is said to be *insolvent*. If a bank cannnot meet the demands of its depositors by liquidating all its long assets, this bank is said to be *bankrupt*. On the left side of bank's balance sheet, each bank has three different assets, short asset, deposit as adjacent bank and long asset. However, the cost of obtaining current consumption (interim date) in terms of future consumption (final date) is different. The cheapest is short term asset, since one unit short asset is worth one unit consumption all the time, so the cost of obtaining liquidity by selling short asset is 1. Similarly, by liquidating one unit of deposits, the bank give up C_2 units of future consumption and obtains C_1 units of present consumption. So, the cost of obtaining by liquidating deposits is C_2/C_1. From the equation (2.2), we know that $C_2/C_1 > 1$. Finally, cost of obtaining liquidity by liquidating long asset is R/l.[28] Therefore, the liquidation pecking order is short assets will be liquidated before it liquidates deposits in other bank before liquidate its long assets:[29]

$$1 < \frac{C_2}{C_1} < \frac{R}{l}. \tag{2.29}$$

Let v_i denotes the value of representative bank's deposits in bank i at interim date. If v_i is lower then the value of deposits at interim date, C_1, if the bank is not bankrupt; then all the depositors will withdraw as much as then can at interim date. In particular, the other banks will be seeking to withdraw their claims on the bank at the same time, then the value is of v_i must be determined simultaneously

$$v_i = \frac{a_s + la_l + (\lambda_H - q)v_{i-1}}{1 + (\lambda_H - q)}, \tag{2.30}$$

where v_{i-1} is the value of the adjacent bank's deposit. The denominator of equation (2.30) is consumers in bank i hold 1 deposit and the bank which take bank i as a adjacent bank hold $(\lambda_H - q)$ deposits. The sum of the two multiply by the bank i's deposit value, v_i, is the value of liabilities of bank i. The value of liabilities must be equal to the value of assets, which is the sum of the value of short asset, the liquidation value of long assets and the value of deposits in adjacent bank $i - 1$. Note that the assets value is the same as the numerator of equation (2.30). v_i can be determined if $v_{i-1} = C_1$, but if $v_{i-1} < C_1$, we need another equation which include the value of v_{i-2}, and so on.[30]

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[28] This is the same as in Diamond and Dybvig (1983), since early liquidation of long asset only yields l per unit invested ex ante.

[29] Without loss of generality, $\frac{C_2}{C_2} < \frac{R}{l}$, is held by assumption, since one can always choose l sufficiently small.

[30] Note that this value of v_{i-2} is the deposit value of the bank $i - 2$. This is the adjacent bank of bank $i - 1$.

Suppose that a bank is insolvent and the late consumers will not run the bank since they can obtain at least C_1 at final date. So, the bank has to keep at least $(1-\lambda)C_1/R$ units of long asset satisfy the late consumers. Therefore, the amount of consumption obtained by liquidating the long asset without causing a run is

$$b(\lambda) \equiv l(a_l - (1-\lambda)C_1/R), \tag{2.31}$$

where $b(\lambda)$ is called as the bank's buffer.

The bank A can meet its demand for liquidity without help from the other banks if and only if its buffer can cover the additional withdraw:

$$b(q+\epsilon) \geq \epsilon C_1. \tag{2.32}$$

However, this condition can be satisfied only in the case $\epsilon > 0$ is small enough. For large value of ϵ, the condition in (2.32) will be violated, and bank A will go bankrupt. The condition of bank A will go bankrupt is

$$b(q+\epsilon) < \epsilon C_1. \tag{2.33}$$

Although they have deposits in adjacent bank B, these deposit are of no use as long as the value of deposits in bank A is $v_A = C_1$. Once the bank A bankrupt, there will be a contagion effect on bank D. From equation (2.30), bank D will suffer a loss when cross holdings of deposits are liquidated. If ϵ is not too large, the bank D may be insolvent to meet its demands of liquidity by liquidating some of long assets. If ϵ is large enough, the contagion effect will devour bank D's buffer, then bank D will go bankrupt too. The liquidation of long assets in bank D will causes a loss in bank C, and with initial ϵ, the contagion effect will make bank C go bankrupt too. The initial losses has a snow ball effect as the contagion matters from bank to bank and the more losses have accumulated from liquidating the long assets.

Freixas, Parigi, and Rochet (2000) analyze the risk of contagion runs through the payment system and focus on the interbank market exposes the system to a *coordination failure*. However, Allen and Gale (2000) shows that interbank markets provide optimal liquidity insurance when banks are subject to idiosyncratic shocks and focus on the *aggregate liquidity shocks* may lead to a contagion among these banks. Both of them have the similar summing-up, they show that each bank's level of buffer is a key determinant of contagion and the failure of one bank has an impact on the propagation of systemic crisis. At last, they also show that the systemic crisis is triggered by cross-holding of assets and liabilities among banks.

Interbank network model. One of the most pervasive aspects of the contemporary financial system is the rich network of interconnections among financial institutions. Although the financial liabilities owed by one firm to another are usually modeled as unidirectional obligations dependent only on the financial health of the issuing bank, in reality, the liability structure of corporate obligations is invariably much more intricate. The value of most banks is dependent on the payoffs they receive from their claims on other banks. The value of these claims depends, in turn, on the financial health of yet other banks in the system. Moreover, linkages between banks can be cyclical. A default by bank A on its obligations to bank B may lead B to default on its obligations to bank C. A default by C may, in turn, have a feedback effect on A. This example illustrates a general feature of financial system architectures, which is cyclical interdependence. In Eisenberg and Noe (2001) (Hereafter EN01), the paper has considered the problem of finding a clearing mechanism in cases in which this sort of cyclical interdependence is present. Unlike Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000), they focus on the liquidity issue of banks, this paper is mainly focus on the solvency problem of the banks.

More precisely, in EN02 model, there are N nodes. Each of these nodes is to be thought of as a bank, participating in the clearing network. The structure of liabilities is represented as n×n liabilities matrix L, where $L_{ij} \geqslant 0$ represents the nominal liability of bank i to node j. A bank has no liabilities to itself, so $L_{ii} = 0$, for all $i = 1, \dots, N$. Let e_i be the exogenous operating cash flow received by bank i. A financial system is thus a pair (L, e) consisting of a nominal matrix L and an operating cash flow vector e. Let q_i represent the total dollar payment by bank i to the other banks in the system. Let $q = (q_1, q_2, \dots, q_n)$ represent the vector of total payments made by the banks. Let d_i represent total nominal obligation of bank$i$ to all other banks, that is,

$$d_i = \sum_{j=1}^{n} L_{ij},$$

Let $d = (d_1, d_2, \dots, d_n)$ represent the associated vector, which term the total obligation vector. Let

$$\Pi_{ij} = \begin{cases} \frac{L_{ij}}{d_i} & \text{if } d_i > 0 \\ 0 & \text{otherwise} \end{cases},$$

and let Π[31] represent the corresponding matrix, which is the relative liabilities matrix. This matrix captures the nominal liability of one bank to another in the system as a proportion of debtor bank's total liabilities. Due to the assumption of that all debt claims have equal priority, the total payments received by i are equal to $\sum_{j=1}^{n} \Pi_{ji} q_j$. Then the terminal wealth of bank i is

$$\sum_{j=1}^{n} \Pi_{ij}^{T} q_j + e_i - q_i,$$

or in matrix notation a vector of terminal wealth is

$$(\Pi^T - I)p + e,$$

and a financial system (L, e) can be described by the corresponding triple (Π, d, e).

Intuitively, a clearing vector should satisfy these three criteria, a) *limited liability*, which requires that the total payments made by a bank must never exceed the cash flow available in the bank; b) *the priority of debt claims*, which requires that stockholders in the bank receive no value until the bank is able to completely pay off all of its outstanding liabilities; c) *proportionality*, which requires that if default occurs, all claimant banks are paid by the defaulting bank in proportion to the size of their nominal claim on firms assets. Therefore, we have the following definition.

Definition 1. *A clearing payment vector for the system* (Π, d, e) *is a vector* q^* *such that for all* $i \in N$

$$q_i^* = \min[d_i,\ \max(\sum_{j=1}^{n} \pi_{ji} q_j^* + e_i,\ 0)].$$

Thus, the clearing payment vector directly gives us two important insights: for a given structure of liabilities and bank values (Π, d, e) it tells us which banks in the system are insolvent ($q_i^* < d_i$) and it tells us the recovery rate for each defaulting bank ($\frac{q_i^*}{d_i}$).

EN01 has proved that under mild regularity condition this clearing payment vector is unique and always exists. They have also developed a fictitious

[31]By the construction of matrix Π, we have $\forall i$, $\sum_{j=1}^{n} \Pi_{ij} = 1$.

default algorithm to find this clearing vector. Note that, this paper does not tell us which bank is the most systemically important bank and only take into account the solvency problem in the model, meanwhile to obtain a rich interbank data will not be a easy task. Therefore, all these issues limit the usefulness of the model. Demange (2012) elaborates the network model further, in the paper she identifies a threat index which may help to assess the contributions of individual banks to the risk in the system. Gautier, He, and Souissi (2010) combines Morris and Shin (2009) and EN01, they incorporate the liquidity problem into a network model. See also Gourieroux, Heam, and Monfort (2012).

Numerical example.(EN01 - Fictitious default algorithm) To illustrate the key concepts of the network model, consider a banking system composed three hypothetical banks in which liabilities matrix is given by:

$$L = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

The columns of L refer to the claims each bank has on each of the remaining two banks. For example, bank 3 holds claim of 1 with bank 1 and a claim of 1 with bank 2. The rows of L represents the liabilities vis-à-vis the other banks in the system. In this example, bank 1 has liabilities of 1 against bank 1 and bank 2. Total interbank liabilities of banks toward the rest of the system are therefore given by the vector $d = (2,\ 1,\ 1)$, where the three components correspond to total liabilities of bank 1, 2 and 3 respectively. Assume that we can summarize the net wealth of the banks that is generated from all other activities by a vector $e = (\frac{1}{2},\ 0,\ 0)$.

The normalized liability matrix Π is given by

$$\Pi = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Note that $\Pi' d$ provides each bank's total interbank assets. To determine the clearing vector associated with the above structure on the interbank market, let us first assume that all banks fulfill all their interbank liabilities. Under this assumption, the net value of a given bank can be derived as the sum of its full interbank income plus its outside net wealth, minus its total promised interbank payment to other banks:

$$\begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ 1 \end{pmatrix}$$

We arrive at a negative net value for bank 1 which is therefore insolvent. We refer to this as a *fundamental default*.

Lets assume that other banks fulfill their obligations towards bank 1 which implies that bank 1's net value before any interbank payment is $\frac{3}{2}$. This amount, given the assumed proportional sharing rule, is allocated to bank 2 and 3, then the following banks' net wealth:

$$\begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & 0 \end{pmatrix} \times \begin{pmatrix} \frac{3}{2} \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{3}{2} \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1}{4} \\ \frac{3}{4} \end{pmatrix}$$

The resulting net value of bank 2 is negative. The insolvency of bank 1 reduces the interbank claim of bank 2 to such an extent that it fails to keep its interbank promises (bank 2 is only able to pay $\frac{3}{4}$ rather than 1), and become insolvent as well. That is what we call a *contagious default*. Applying again the proportional sharing rule we obtain

$$\begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & 0 \end{pmatrix} \times \begin{pmatrix} \frac{3}{2} \\ \frac{3}{4} \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{3}{2} \\ \frac{3}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}$$

The outcome of the network model is a clearing vector which makes all interbank claims consistent. In this example, this vector is $q^* = (\frac{3}{2},\ \frac{3}{4},\ 1)'$ and is unique.

2.4.2 Fire-sales externalities

Cifuentes, Ferrucci, and Shin (2005), building on EN02, incorporate the required capital ratio on banks to show the limited capacity of the financial markets to absorb sales of assets, where the price repercussion of asset sales have important adverse welfare consequences (in line with the cash-in-the-market pricing (Allen and Gale (1994) and Allen and Gale (1998)) presented in the section 2.3). The mechanism is that when banks have to satisfy their capital ratio, sometimes they have to liquidate part of their long term assets, given that the market's demand for long tern assets (illiquid assets) is less than perfect elastic, henceforth marked-to-market asset prices decrease. This low level of prices may endogenously generate another round of asset sales, which depresses prices further and induce further sales. Since the assets are marked-to-market, an small initial shock may generate contagious failures in the whole system. Diamond and Rajan (2005) study the fact that the inefficient liquidation of long term assets may leads to an analogous effect, different from Diamond and Dybvig (1983), they find that bank runs are triggered by the asset side of the bank's balance sheet. The shortage of aggregate liquidity that such liquidations bring about can generate contagious failures in the banking system. Note that, in the first paper, banks liquidate their long term assets is in the purpose of satisfying the required capital ratio, however, the second one is subject to a bank run.

Fire sales externality. From GR12 we have seen that banks may issue excessive loans during boom periods. However, this excessive loans issued is directly followed by an over-borrowing ex-ante, as shown in Lorenzoni (2008), which generate a pecuniary externality since the atomistic banks do not take into account the effect of asset sales on prices, and this pecuniary externality has a negative effect on social welfare. The basic idea of fire sale externality is as follow. An increase in ex-ante investment will imply an increase in ex-post sale of assets during crisis, therefore lower the price of asset. If the seller of asset has a larger marginal utility than the buyer and the insurance market is missing, this ex-ante investment generates a negative pecuniary externality. This implies that equilibrium is constrained inefficient and there is an over-borrowing and over-investment ex-ante, which provide a rationale for appropriate regulation - macroprudential regulation.

Building on KM97, in the model of Lorenzoni (2008), there are two sectors, banking sector and traditional sector which are similar as described in GR12 and there are three dates $t = 0, 1, 2$. Investors receive an large endowment, w, of dollars at all dates and have the preference $C_0 + C_1 + C_2$. banks are endowed of dollars, e, at date 0 and have the preference C_2. They can borrow or save between dates 0 and 1 subject to constraints below. For simplicity there is no borrowing-saving between dates 1 and 2. At date 1, the states $s \in \{h, \underbrace{l}_{\text{crisis}}\}$ is realized with probability λ_h and λ_l respectively.

Banks invest k_0 units of capital good at date 0, which yields cash flow R_h, R_l at date 1. At interim date, bank chooses a continuation size, k_{1s}, depends on the realization of the state; if $k_{1s} - k_0 > 0$, then bank makes new investment, if $k_{1s} - k_0 < 0$, then bank sells his capital on the market. At date 2, the continuation size yields Ak_{1s} dollars, where $A > 1$. The capital is also subject to maintenance costs γ at date 1 (as in Holmstrom and Tirole (1998)), that is said the capital is productive only if this cost is paid, otherwise it will be fully depreciated. Denote q_S as the capital price at interim date on the market, assume $q_s > \gamma$, so one always has the willingness to pay this maintenance cost. Therefore, the net cash flow is denoted as $f_s = R_s - \gamma$, and by assumption, $f_l < 0$ and $f_h > 0$ and large, so that there are asset sales in crisis but not in good state.

At date 1, investor can also convert capital to consumption good with technology $F(\bar{k}_{1s})$ and assume $F^{'}(0) = 1 < A$ and $F^{'}(\bar{k}_{1s})$ is bounded below by $\underline{q} > \gamma$. As described above, capital traded in a competitive market at price q_s. If $k_{1s} > k_0$, we have $\bar{k}_{1s} = 0$ and the price $q_s = 1$; if $k_{1s} < k_0$, we have $q_s = F^{'}(\bar{k}_{1s})$ and $\bar{k}_{1s} = k_0 - k_{1s}$. Note that the market clearing condition is

$$q_s = F^{'}(\underbrace{[k_0 - k_{1s}]_+}_{\text{nonnegative part}}). \tag{2.34}$$

Bank borrows from investors using a contingent and safe debt contracts $(b_{1s}k_0)$, which is normalized by capital for simplicity. $b_{1s}k_0$ is the promised repayment in state s and total amount borrowed is $\Sigma_s \lambda_s b_{1s} k_0$. Therefore, bank's budget constraints are

$$\begin{cases} \text{Date } 0: & k_0 \leq w + \Sigma_s \lambda_s b_{1s} k_0 \\ \text{Date } 1: & q_s k_{1s} \leq w_{1s} \equiv (q_s + f_s - b_{1s}) k_0 \end{cases}. \tag{2.35}$$

Due to the presence of financial friction on both sides of bank (limited access to external funds for banks) and investor (limited ability for banks to insure ex-ante aggregate liquidity shocks), these imply

$$b_{1s} \in [0, q_s - \gamma]. \tag{2.36}$$

Therefore, banks have an optimization problem: $\max_{b_{1s}} \Sigma_s \lambda_s A k_{1s}$, subject to constraints (2.35)-(2.36) for each state s. Since this optimization problem is linear in net worth, and the problem can be reformulated as

$$z_0 \equiv \max_{\{b_{1s}\in[0,q_s-\gamma]\}} \underbrace{\frac{1}{1-\Sigma_s\lambda_s b_{1s}}}_{k_0 \text{ per } w} \Sigma_s\lambda_s z_{1s} \underbrace{(q_s + f_s - b_{1s})}_{w_{1s} \text{ per } k_0}, \tag{2.37}$$

where $z_{1s} = \frac{A}{q_s}$ is the date 1's marginal return on net worth, since at date 1, bank can buy capital at the price q_s, one dollar of internal funds can thus be leveraged by a factor $\frac{1}{q_s}$, at date 2 this investment gives A per unit of capital. Similarly at date 0, one extra dollar of internal funds can be leveraged by a factor of $1/(1-\Sigma_s\lambda_s b_{1s})$ and capital invested gives a random net payoff $(q_s + f_s - b_{1s})$ at date 1, which will be reinvested at rate of return z_{1s}. Averaging across states we obtain expression (2.37). The optimal debt choice is given by the first-order-condition with respect to b_{1s}, when $z_0 > z_{1s}$ the bank will choose $b_{1s} = (q_s - \gamma)$, when $z_0 < z_{1s}$ the bank will choose $b_{1s} = 0$, and an interior choice will only arise if $z_0 = z_{1s}$.

Now, consider the asset market at date 1. The net investment by the banks is derived the date 1's budget constraint from equation (2.35)

$$k_{1s} - k_0 = \frac{f_s - b_{1s}}{q_s} k_0. \tag{2.38}$$

Note that, as $f_l < 0$ and $f_h > 0$ and large, in good state, banks are asset buyer because $k_{1h} > k_0$ and $q_h = 1$; however, in crisis state, banks are asset seller because $k_{1l} < k_0$ and $q_l < 1$. Under mild condition, in state of crisis, two equations (2.34) and (2.38) ensure a unique intersection between these two curve. Since the equation (2.38) characterize a negative relationship between asset prices, q_l and the net supply of asset, $k_0 - k_{1l}$, we obtain again the same result as in Allen and Gale (2005). Importantly, the date 0's budget constraint in equation (2.35) gives an explanation for an increase in debt increases the initial investment

$$k_0 = \frac{w}{1-\Sigma_s\lambda_s b_{1s}},$$

and leads to an increase in ex-ante borrowing increases a loss in crisis period, $-(f_l - b_{1l})k_0$ and to a fall in equilibrium price. This is the *fire sales* in the crisis state.

In competitive equilibrium the price of asset is lower in crisis state than good state, $q_l^c < q_h^c = 1$. Define the debt capacity in high state as, $\hat{\rho} = \lambda_h(1-\gamma)$ and the ratio of outside borrowing to total investment as, $\rho = \Sigma_s\lambda_s b_{1s}$. Note that only in the case that $\rho > \hat{\rho}$, the bank will borrow against profit in the crisis state.

Now turn to the social planner's perspective. Different from market equilibrium, the social planner takes into account the relation between the financial contract and the equilibrium price on the capital market. That is the social planner's problem will include the constraint (2.34). On top of that, planner will put another minimum utility constraint for investor to attain the constrained optimality. That is to make banks better off keeping investors as good as in equilibrium. So, the main difference from market equilibrium is that the planner takes into account effect of b_{1s} and k_0 on q_s. Denote $\rho^c = \Sigma_s \lambda_s b_{1s}$ the date 0 borrowing in equilibrium and $\rho^* = \Sigma_s \lambda_s b_{1s}$ the date 0 borrowing under social planner's program. Then $\rho^* \leq \rho^c$, and there is a overborrowing effect in market equilibrium caused by the pecuniary externalities. The intuition is as follows. For a reduction in k_0 by a corresponding reduction in initial borrowing leads to a reduction in fire sales in the crisis and $dq_l > 0$. This implies a transfer of $(k_0 - k_{1l})dq_l > 0$ from investors to banks, in order to keep investors have the same utility, banks have to compensate investors ex-ante for this loss with a date 0 transfer $\lambda_l(k_0 - k_{1l})dq_l$. So the banks will loose a part of marginal return on net worth at date 0 for an amount of $z_0\lambda_l(k_0 - k_{1l})dq_l$, but they will gain a marginal return on net worth at date 1 for an amount $z_{1l}\lambda_l(k_0 - k_{1l})dq_l$. In case of $z_{1l} < z_0$, the net gain from less borrowing ex-ante is $(z_{1l} - z_0)\lambda_l(k_0 - k_{1l})dq_l$.

Other papers have also stressed this point such as in Korinek (2011) and Bianchi (2011). The former shows that the banks can buy an insurance against the aggregate uncertainty but this insurance is costly. The latter provides a quantitative assessment of macroeconomic and welfare implications of overborrowing and allows for the evaluation of the benefits of policy measures to correct these externalities. All these suggest a macroprudential regulation tool to deal with the banks liquidity in order to prevent fire sales externalities.

Liquidity regulation. A lesson we learnt from the recent financial crisis is that the rapid expansion of credit has been largely funded by short-term funding, and this exposes banks to liquidity risk. Some economists have proposed a price rule as liquidity regulation to make the banks to internalize the fire sales externalities in the market. Perotti and Suarez (2011) propose a Pigouvian approach to liquidity regulation, which aimed at equating private and social liquidity cost by imposing a tax on short-term debt. The model compares the first order condition of market case and social optimal case, in order to make individual banks internalize the externalities issued on others, the paper propose a Pigouvian tax which is equal to extra term in the first order condition in social optimal case.[32] Alternatively speaking, this tax imposed on individual banks can make the private marginal cost equals to the social marginal cost to help to attain the efficiency in the market case. By imposing such a tax, market equilibrium can be corrected to coincide with the social optimal. This is what so called *price-based* approach in liquidity regulation.

In addition to this, there are also two *quantity-based* instruments such as liquidity coverage ratio and net stable funding ratio have been introduced in Basel III. Both liquidity ratios have the objective to lower the probabilities that a bank runs into a liquidity difficulties. More precisely, net stable funding ratio is aiming at the liability side of a bank's balance sheet, which impose an upper bound on short-term debt to reduce overall liquidity risk. Liquidity coverage ratio requires bank to maintain a level of high-quality assets that can be converted into cash needs to meet its liquidity needs for on month. So, the overall purpose is to encourage the banks to hold higher liquidity buffers (+ higher quality) and lower maturity risk. An open question will be raised here, according to Malherbe (2012), should the liquidity coverage ratio also be set in a counter-cyclical manner? The argument is that when one hold liquidity buffer during the crisis (during which banks will anticipate a low price of asset) will worsen the adverse selection on the secondary market, which generate negative externality on other banks; however during the booming period, holding a fractional liquidity buffer may affect the credit volume and also absorb the losses without fire saling the asset to satisfy the capital requirement (Cifuentes, Ferrucci, and Shin (2005)).

2.4.3 Shadow banking system, securitization and regulatory arbitrage

Shadow banking system. The shadow banking system played a crucial role during the 2007-2009 financial crisis. Pozsar, Adrian, Ashcraft, and Boesky (2010) provide an excellent detailed description of the shadow banking system. Unlike the traditional banking system which has the guarantee of the public-sector, the shadow banking system was presumed to be safe due to the liquidity and credit support provided by the private sector. This view underpinned the perceived risk-free, highly liquid nature of most AAA-rated assets that collateralized credit repos and shadow banks' liabilities more broadly. However, once the solvency of private sector support providers started to be questioned, the confidence that

[32] This extra term is just the externality term.

underpinned the stability of the shadow banking system vanished. This failure of the support from private sector comes mainly from the underestimation of asset price correlations This is because the shadow banking system can reduce their idiosyncratic risk by trading or securitizing the loans to diversify their portfolio. Consequently, this behavior exposes the portfolio to systematic shocks. So what is shadow banking system? A shadow bank is defined as a credit intermediation includes all the activities in the traditional credit intermediation that are not directly subject to the prudential regulation of banks; therefore they do not directly and explicitly benefited from official guarantees. The shadow banking system decomposes the simple process of deposit-funded, hold-to-maturity lending conducted by banks into a more complex, wholesales-funded, securitization-based lending process. Through this intermediation process, the shadow banking system transforms risky, long-term loans (e.g. sub-prime mortgages) into seemingly risk-free and short-term, money like instruments, etc. However, it makes the financial system become more unstable and fragile in the sense that not only FIs are expose to an aggregate uncertainty but there also have an opportunity for FIs to evade prudential regulation.

Securitization and asset diversification. Gennaioli, Shleifer, and Vishny (2011) explicitly present a shadow banking model where FIs can securitize their risky assets in order to diversify their portfolio to reduce the idiosyncratic risk. By doing so, more fixed return can be pleadged by FIs and they can issue more safe debt to infinite risk averse investors, whose wealth is large enough, to expand their balance sheet size. Note that these debts are backed on the risk-free return and these fix repayment projects can be used as collateral. By assuming a rational expectation framework, the paper shows that the shadow banking system is stable and favorable for the economy which improves the social welfare. The model explains the famous funding by Adrian and Shin (2011) as mentioned above, that the leverage and assets of intermediaries grow together; and explains how diversification of idiosyncratic through securitization is accompanied by the concentration of systemic risks on the book of FIs as in Acharya, schnabl, and Suarez (2010). When investors and FIs are local thinker, that is to say that both agents neglect the worst scenario in the states of world, the expansion of risky assets and concentration of risks in the FIs create financial fragility and make the whole system unstable. Note that the neglected risks can be interpreted as the widely perceived risk free nature of highly rated structure credit product, such as the AAA tranches of ABS (Asset Backed Security). Coval, Jurek, and Stafford (2009) point out that these AAA tranches behave like catastrophe bonds that load on a systemic risk state. In such a systemic risk state, on one hand, the underestimation of assets correlation enable FIs to hold insufficient amount of liquidity and capital against the supports that underpinned the stability of the shadow banking system. On the other hand, investors may overestimate the value of private liquidity and credit support, the result is an excess supply of cheap credit (In the model is presented as the more safe debts issued by FIs.).

Regulatory arbitrage. Since the cost of capital is very high, the FIs have a strong incentive to evade the capital regulation. Kane (1988) states that the financial innovation is mostly spurred by financial regulation for regulatory arbitrage purposes, which gives an explanation for a big build-up in shadow banking system, since such capital regulation can be bypassed by the shadow banking system as in Acharya, schnabl, and Suarez (2010). This paper focuses on the economics of ABCP (Asset Backed Commercial Paper) and documents that commercial banks set up conduits to securitize assets while insuring the newly securitized assets using credit guarantees structured to reduce bank capital requirement. The intuition is that the traditional banks may get rid of high risk assets by securitizating them through shadow banking system, only get back the high-rated tranches such as AAA tranches and the rest of them are sold on the market, usually these AAA tranches are assigned a risk weight which is much lower than the initial assets, therefore these banks have been required a relatively low capital level. Plantin (2012) establish the optimal regulation in response to the regulatory arbitrage. The model shows that if it is impossible to regulate shadow banking system at all, then relaxing capital requirement for commercial banks so as to shrink shadow activities may be more desirable then tightening them. Again, this is because if a high level of capital requirement is imposed in banking sector, it will drive banks to go to the opaque shadow banking activities, which in turn increase the financial fragility.

2.5 Systemic and systematic Risk Measures

In this section, I will present the different systemic risk measures. Here, I distinguish the measures by two categories. The first one measures the exposure of FIs given a systematic shock in the system. From a theoretical point of view, it is akin to a Beta computed within the Capital Asset Pricing Model (CAPM). The other category measures the contagion risk of one FI to the whole financial system or the contagion risk among FIs in the financial system. For a complete survey of risk measures, see Bisias, Flood, Lo, and Valavanis (2012); and an interesting comparison of systemic risk measures, see Benoit, Colletaz, Hurlin, and Perignon (2012).

2.5.1 Systemic risk measures

To begin this chapter, I first present a risk measure that is very popular in academia and also in the industry, called Value-at-Risk (VaR). Recall that $VaR_q^i$is implicitly defined as the q quantile, i.e.,

$$\mathbb{P}(X^i \leq VaR_q^i) = q,$$

where X^i is the variable of intermediation i for which the $VaR_q^i$is defined. Note that VaR_q^i is typically a negative number. In the industry, this measure helps risk manager to ensure that risks are not taken beyond the level at which the intermediation can absorb the losses of a probable worst outcome. Therefore, it's not difficult to see that, this VaR only captures the individual risk and ignores the risks that a given intermediation may issue to the whole financial system. After the 2007-09 financial crisis, many economists and policy makers have turned their attention from individual risks to systemic risk.[33]

2.5.1.1 Contagion to the whole system

CoVaR. **Adrian and Brunnermeier (2011)** (Hereafter AB11) propose a systemic risk measure called $CoVaR$ and this measure allow to measure what's an intermediation's risk contribution to the whole financial system. In AB11, authors use $\Delta CoVaR$, which is the difference between the VaR of the financial system conditional on a given FI being in a tail event and the VaR of the financial system conditional on this FI in a median state, to capture the marginal contribution of systemic risk.

Definition 2. *We denote by $CoVaR_q^{sys|i}$the VaR of the financial system conditional on some event $C(X^i)$ of intermediation i. That is $CoVaR_q^{sys|i}$ is implicitly defined by the q-quantile of conditional probability distribution:*

$$\mathbb{P}(X^{sys} \leq CoVaR_q^{sys|C(X^i)} | C(X^i)) = q.$$

[33] Another very popular individual risk measure is Expected Shortfall (ES). Unlike VaR, ES tells you how much the intermediation will loss given the variable of intermediation is already under its VaR.

We denote intermediation i's contribution to j by

$$\Delta CoVaR_q^{sys|i} = CoVaR_q^{sys|X^i=VaR_q^i} - CoVaR_q^{sys|X^i=Median^i}.$$

Later on, we focus on the conditioning event $\{X^i = VaR_q^i\}$ and simplify the notation to $CoVaR_q^{sys|i}$. Note that, the direction of conditioning matters for this measure of systemic risk. In AB11, they calculate the intermediation i's systemic risk contribution by making financial system conditional on one specific intermediation being under distress. The intuition by doing that is that we want to see what's the impact has been given to the system when one intermediation is under distress. The variable of intermediation i, X^i, is defined as follow.

As mentioned earlier, we are in a financial system where balance sheets are marked to market, changes in assets prices show up immediately on balance sheets. Therefore, it is preferred to use market data to stress this question, an alternative objective by using market data is to try to make an early warning through market[34]. The paper focuses the analysis on the VaR_q^i and $CoVaR_q^i$ of returns of market-valued total financial assets. More formally, denote by ME_t^i the market value of an intermediation i's total equity, and by LEV_t^i the ratio of total assets to book equity[35]. We define the return of market valued total asset, X^i, by

$$X_t^i = Log(ME_t^i * LEV_t^i) - Log(ME_{t-1}^i * LEV_{t-1}^i),$$

and the financial system's return is given by

$$X_t^{system} = \Sigma_i \frac{A_{t-1}^i}{\Sigma_j A_{t-1}^j} X_t^i.$$

Therefore, the return of financial market is the asset weighted sum of return of each intermediation in the system.

In AB11, they have proposed 7 state variables to estimate time-varying $CoVaR_t$ and VaR_t. Those are:

[34] However, one could use supervisory data to compute the VaR_q^i and $CoVaR_q^i$ from a broader definition of total assets which could include off-balance sheet items, exposure from derivative contracts, and other claims that are not properly captured by the accounting value of total asset. Another issue is that, accounting rule can be different across countries, and this makes regulation more difficult for supervises.

[35] It is the leverage of intermediation i. Note that the leverage doesn't have same frequency as market capitalization on market data, therefore, one can do a linear interpolation to make the leverage has the same frequency as market capitalization, then make the product of these two data to obtain total asset. Actually the total asset, here, is quasi-total asset. Alternatively, one can keep the leverage unchanged between the dates of publishing the new data.

i) VIX, which captures the implied volatility in the stock market.

ii) A short term liquidity spread, defined as the difference between the three-month repo rate and the three-month bill rate, which measures short-term liquidity risk. The three-month general collateral repo rate is available on Bloomberg, and The three-month Treasury rate is obtained from the Federal Reserve Bank of New York.

iii) The change in the three-month Treasury bill rate from the Federal Reserve Boards H.15. By using the change in the three-month Treasury bill rate, not the level, since the change is most significant in explaining the tails of financial sector market-valued asset returns.

iv) The change in the slope of the yield curve, measured by the yield spread between the ten-year Treasury rate and the three-month bill rate obtained from the Federal Reserve Boards H.15 release.

v) The change in the credit spread between BAA-rated bonds and the Treasury rate (with the same maturity of ten years) from the Federal Reserve Boards H.15 release.

Note that iv) and v) are two fixed-income factors that capture the time variation in the tails of asset returns.

Then introduce two variables to control for the following equity market returns:

vi) The weekly equity market return.

vii) The one-year cumulative real estate sector return is the value weighted average of real estate companies (SIC code 65-66) from CRSP.

These state variables are well specified to capture time variation in conditional moments of asset returns, and are liquid and easily traded. However, a big shortcoming by using these variables is that it makes the results less robust. Imagine, if we add or delete one variable may affect the final results of quantile regression, then the results of measure.

They are many different ways to calculate VaR, one is to use $GARCH$ family to estimate the conditional volatility of the return of underlying variable[36].

[36] see Ergun and Girardi (2012).

Another method to compute VaR is the quantile regression. To capture time variation in the joint distribution of X^i and X^{system}, we can estimate the conditional distribution as a function of state variables. Indicating the time-varying VaR_t with a subscript t and estimate it conditional on a vector of state variables M_t. The one week lag of the state variables is denoted by M_{t-1}. Then run the following quantile regressions in the weekly data (where i is the intermediation i):

$$X_t^i = \alpha^i + \beta^i M_{t-1} + \epsilon_t^i, \tag{2.39}$$

where ϵ_t^i is the idiosyncratic shock for intermediation i at time t. Then, generate the predicted values from this regression to obtain:

$$VaR_t^i = \alpha_q^i + \beta_q^i M_{t-1}, \tag{2.40}$$

where α_q^i and β_q^i are two estimated coefficients by the q-quantile regression. Particularly, we can set $q = 1\%$.

As VaR, there are many different ways to do the estimation. Here, again, the quantile regression methodology has been proposed to calculate $CoVaR$. Run the following quantile regression:

$$X_t^{system} = \alpha^i + \beta^i M_{t-1} + \gamma^i X_t^i + \epsilon_t^{i,system}, \tag{2.41}$$

replacing the estimated coefficients and the individual Value-at-risk, we obtain:

$$CoVaR_t^i[X^i = VaR_t^i] = \alpha_q^i + \beta_q^i M_{t-1} + \gamma_q^i VaR_q^i,$$

and therefore $\Delta CoVaR$ can be obtained as follow

$$\Delta CoVaR_t^i = CoVaR_t^i[X^i = VaR_t^i] - CoVaR[X^i = Median[X^i]].$$

An attractive feature of $CoVaR$ is that it can be easily adopted for other "co-risk measures". One of them is co-expected-shortfall, $Co - ES$. Expected Shortfall has a number of advantages relative to VaR and can be calculated as a sum of $VaRs$. I denote the $CoES_q^i$, the Expected Shortfall of the financial system conditional on $X^i \leq VaR_q^i$ of intermediation i. That is, $CoES_q^i$ is defined by the expectation over the q-tail of the conditional probability distribution:

$$\mathbb{E}[X^{system}|X^{system} \leq CoVaR_q^i].$$

Expected Shortfall is a coherent risk measure, since it satisfies 4 axioms, positive homogeneity, sub-additivity, monotonicity, translation invariance[37].

Multi-CoVaR. Cao (2012) extends this standard $CoVaR$ measure by proposing a multi-$CoVaR$ measure. The intuition behind this extension is that during the financial crisis, several FIs may have being in financial distress at the same time, therefore the author can put several FIs in the conditional part of standard $CoVaR$ definition. This measure is so called "Multi-$CoVaR$".

Definition 3. *Denote by $CoVaR_{q,t}^{1,...,S}$ the VaR of financial system conditional on some event $\{C(X_t^1),...,C(X_t^S)\}$ of a set of intermediations $\{1,...,S\}$ at time t. That is $CoVaR_{q,t}^{1,...,S}$ for financial system and confidence level q when the set of intermediations $\{1,...,S\}$ is on some event $\{C(X_t^1),...,C(X_t^S)\}$ at time t is defined by:*

$$\mathbb{P}(X^{sys} \leq CoVaR_{q,t}^{1,...,S}|C(X_t^1),...,C(X_t^S)) = q, \tag{2.42}$$

and the set of intermediations $\{1,2,...,S\}$'s contribution to financial system is denoted by:

$$\Delta CoVaR_{q,t}^{1,...S} = CoVaR_{q,t}^{X^1=VaR_q^1,...,X^S=VaR_q^S} - CoVaR_{q,t}^{X^1=Median^1,...,X^S=Median^S}. \tag{2.43}$$

Different from previous case, the author focuses on the case where the conditioning event is described by $\{X_t^i \leq VaR_{q,t}^i\}$[38] for $i = 1,...,S$. Hence $\Delta CoVaR_{q,t}^{1,...S}$ denotes the difference between the VaR of financial system conditional on a set of financial intermediations $\{1,...,S\}$ being in a tail event and the VaR of financial system conditional on the set of financial intermediations $\{1,...,S\}$ being in a median state at time t.

Note that, this measure captures a spillover effect, which is characterized

[37] SeeArtzner, Delbaen, Eber, and Heath (1997) for details.

[38] Ergun and Girardi (2012) propose a method to capture more severe distress events that are farther in the tail.

by the contribution of systemic risk of a set of FIs is in distress to the financial system. This measure has two advantages. First, it allows to calculate the total contribution of systemic risk in the financial system, with this total contribution, one can apply an allocating rule to attribute the total risk to each intermediation. This total systemic risk is characterized by when all the intermediations are in distress, what is its effect to the system Value at Risk. Suppose there are N intermediations in the system, the total contribution of systemic risk is given by:

$$\mathbb{P}(X^{sys} \leq CoVaR_{q,t}^{1,\dots,N} | C(X_t^1), \dots, C(X_t^N)) = q, \tag{2.44}$$

and

$$\Delta CoVaR_{q,t}^{1,\dots N} = CoVaR_{q,t}^{X^1 = VaR_q^1, \dots, X^N = VaR_q^N} - CoVaR_{q,t}^{X^1 = Median^1, \dots, X^N = Median^N}. \tag{2.45}$$

Secondly, it allows to calculate the marginal contribution of intermediation i for a given set of intermediations S is already in distress. Denote $\Delta_i(S)$ the marginal systemic risk contribution of one specific intermediation i is determined by:

$$\Delta_i(S) = \Delta CoVaR(S \cup \{i\}) - \Delta CoVaR(S), \text{ for } S \subset N, \ i \notin S. \tag{2.46}$$

This measure has another advantage, that is to inform the regulators that which distressed intermediation should be bailed-out during the financial crisis. For example, in September 2008, Lehman Brother and AIG are both in financial distress at the same time and then Federal reserve have decided to bailout AIG. However, based on Multi-$CoVaR$ one can calculate $\Delta(S)$ for both LB and AIG and to bailout the one has a larger marginal contribution of systemic risk. Interestingly, this measure also provide us the systemic importance of different class, which can be seen as different sectors in the financial system, or different regions in the financial system, etc. Such information is also very useful for regulators to maintain the stability of financial system as a whole.

Shapley value. The Shapley value methodology was initially proposed in the circumstance of cooperative games (see Shapley (1953)), in which a group of players generates a share "value" (e.g. wealth, cost) for a group as a whole. The Shapley value of a player in a game turns out to be his expected marginal contribution over the set of all permutations on the set of players. For example, a group of agents would like to connect to a server in order to benefit a high speed functioning of their own PC, however, the maintain of the server is costly, the Shapley value is a fair and efficient allocation rule to share the costs among agents and the Shapley value of an agent is his expected marginal contribution over all possible set of the agents.

In order to apply the Shapley value methodology to a financial system, it is sufficient to define a so-called "characteristic function". This function is unchanged over the set of all permutations on the set of intermediations and map each subsystem into a risk measure. The characteristic function, v, should accept as input anyone of the $2^N - 1$ subsystems[39] of intermediations and should deliver the system-wide risk measures when applied to the entire system.

The derivation of Shapley values involves the following process.

There are N players, which are financial intermediations. Let $v : 2^N \to R^+$ be a function defining the systemic risk for each subset of N, where N denotes the entire financial system. v is assumed to be monotone (i.e. $v(S) \geq v(S')$ for all $S' \subseteq S \subseteq N$), and $v(\phi) = 0$. The objective is to find non-negative systemic risk attribution $\{Sh_i\}_{i \in N}$ such that:

Axiom 1 (Additivity/Efficiency): $\Sigma_{i \in N} Sh_i = v(N)$,

Axiom 2 (Dummy axiom): If i is such that $\Delta_i(S) = v(\{i\})$ for all $i \notin S$, then $Sh_i = v(\{i\})$,

Axiom 3 (Symmetry): If $i \neq j$ such that $\Delta_i(S) = \Delta_j(S)$ for all $i, j \notin S$, then $Sh_i = Sh_j$,

Axiom 4 (Linearity): Suppose $v(S) = v_1(S) + v_2(S)$ where v_1 and v_2 are also non-negative monotone functions with $v_1(\phi) = v_2(\phi) = 0$, and $\{Sh_i^1\}_i$ are systemic risk shares for v_1-risk and $\{Sh_i^2\}_i$ are systemic risk shares for v_2-risk, then $Sh_i = Sh_i^1 + Sh_i^2$, for all i, defines the systemic risk shares for v-risk.

There is a unique way to satisfy axioms 1, 2, 3 and 4, called Shapley value and Shapley value for intermediation i is:

$$Sh_i(v) = \Sigma_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{i\}) - v(S)), \tag{2.47}$$

[39]Since the empty set ϕ does not play a role in contributing the value.

where n is the total number of intermediations and the sum extends over all subsets S of N not containing intermediation i. This formula can be interpreted as the expected marginal contribution of intermediation i over the set of all permutations on the set of intermediations.

Tarashev, Borio, and Tsatsaronis (2010) use individual risk measures, such as Value-at-Risk and Expected Shortfall[40] to be the characteristic function, to capture the intermediation's systemic risk importance by implementing Shapley value methodology as an allocation rule. On top of that, the paper examines different drivers of systemic importance and their interactions and illustrate how the allocation methodology can be used as a basis of policy intervention with macroprudential objectives.

Drehmann and Tarashev (2011) aim at combining the literature on (a) banks' systemic importance and (b) interbank networks.[41] It explores two different approaches to measure systemic importance: a) bank's participation in systemic events which is referred to as the participation approach (PA) and 2) banks' contribution to systemic risk considering interbank network which is referred to as generalized contribution approach (GCA). Here, the participation approach is the same as in Tarashev, Borio, and Tsatsaronis (2010). In this approach, we have $v^P(S) - v^P(S - \{i\}) = E(L_i|\text{systemic event})$, which depends on i but not on S (the second axiom of Shapley value), and L_i is the loss of institution i . Therefore one can obtain:

$$Sh_i(N,\ v^P) - ShV_i(N - \{k\},\ v^P) = Sh_k(N,\ v^P) - Sh_k(N - \{i\},\ v^P) = 0. \quad (2.48)$$

The first equality in this expression (2.48) holds due to the fairness criterion of Shapley value, which means that the increment of the Shapley value on institution i caused by the presence of institution k equals the increment of the Shapley value of institution k caused by the presence of institution i.[42] The second equality in expression (2.48) means that the presence of institution i have no effect on the Shapley value of institution k and vice versa. Then the Shapley value of bank i is simply the loss it is expected to generate conditional on the systemic events: $Sh_i(v^P) = E(L_i|\text{systemic event})$. Note that, this is the *systematic*

[40] This is a modified version of Expected Shortfall, since the conditional part of the measure is the whole system is in distress instead of the individual is in distress.

[41] Here, I refer the bank as intermediation (FI) in the previous cases.

[42] This result is from Mascolell, Whinston, and Green (1995).

risk measure, and *not* systemic risk (In the paper, the authors call it participation approach), and we will see later this measure is in the same spirit as in Huang, Zhou, and Zhu (2010) and **?**.

To closer to the original idea behind Shapley value, which means that the we should measure the risk that a bank generates on its own as well as this bank's contributions to the risk in each subsystem of other banks. Then, GCA suggests that, when we evaluate the risk of a subsystem from which bank i is excluded, we cannot simply consider the risk that this bank generates when entire system is in place. The characteristic function of this approach is: $v^G(S) \equiv E(\sum_{i \in S} L_i^S | \sum_{i=S} L_i^S \geq \text{threshold})$ for any $S \subseteq \Sigma$.

At the entire system level, this two approaches coincide, as $v^G(\Sigma) = v^P(\Sigma) = ES(\Sigma)$. But the allocation of this system-wide risk differs between the two approaches for two important reasons. First, v^G allows the losses incurred by the non-bank creditors of bank i to depend on the subsystem considered: L_i^S. Here, default is distinguished by fundamental default and contagion default as presented in EN01, thus the probability of default of one bank is equal to the sum of fundamental PD and contagion PD. To construct probability distribution of losses, we implement Eisenberg and Noe's clearing algorithm in each approach, only one time in PA (in the case of entire system is in place) and $2^n - 1$ times in GCA (for each subsystem in a system with n banks, empty set is excluded). Second difference between v^P and v^G is due to the fact that, v^G incorporates conditioning events that changes with sub system. Thus, in contrast to v^P, v^G measures risk as the expected shortfall in each subsystem: $v^3(S) = ES(S)$. This leads to the following special case of the general Shapley value formula in (2.47):

$$Sh_i(v) = \Sigma_{S \subseteq N \backslash \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (ES(S \cup \{i\}) - ES(S)). \tag{2.49}$$

In contrast to the PA, the increment of systemic risk that is caused by the joint presence of two banks is split into equal halves, which GCA attributes to each of these banks. Or formally:

$$ShV_i(N,\ v^G) - ShV_i(N - \{k\},\ v^G) = ShV_k(N,\ v^G) - ShV_k(N - \{i\},\ v^G) > 0. \tag{2.50}$$

The strict inequality indicates that, v^G does convey the extent to which one bank affects the riskiness of another. That is v^G captures the contribution of this interbank link to systemic risk and splits it equally between bank i and bank k. By extension, a Shapley value under v^G attributes the risk created by an interbank link equally to the interbank lender and the interbank borrower. A natural question will be raised here. As $ES(S)$ is a coherent measure and satisfies the subadditivity axiom (see Artzner, Delbaen, Eber, and Heath (1997)), the last term in expression (2.49) $(ES(S \cup \{i\}) - ES(S))$ could be negative, which violate the assumption that the characteristic function should be monotone. To overcome this potential drawback, Cao (2012) proposes to use Multi-$CoVaR$ as the characteristic function to allocate the contagion risk of the banks in the system. The intuition is that when there is two banks which are in distress, its systemic risk contribution should be greater than only one of these two banks is in distress and so on. The paper shows that, by the additivity of Shapley value, the estimated aggregate systemic risk is efficiently distributed to each bank; however, the standard $CoVaR$ measure may under-estimate the systemic risk during the boom and over-estimate the systemic risk during the recession.

2.5.1.2 Bilateral contagion risk

Joint probability of default (Jpod). Segoviano and Goodhart (2009) propose a methodology to capture inter-linkages effects between banks, which they call CIMDO. CIMDO is based on a copula formula and it is a non-parametric method, which extracts the link between events rather than predetermining laws of motion. The objective of the methodology is to find the joint distribution, which best fits a prior joint distribution according to the information criteria and is consistent with the probability of distress for each bank. The latter are calculated through a market based approach, which considers that the financial health of an institution is reflected in the CDS spreads. Two concerns with the use of CDS spreads have to do with how liquid these markets are and the amount of trading noise, which can bias the results. It follows 4 steps of modeling banking system's stability and distress dependence. Step1, we conceptualize the banking system as a portfolio of banks. Step2, for each of the banks included in the portfolio, we obtain empirical measurements of probabilities of distress (PoDs). Step3, Making use of the Consistent Information Multivariate Density Optimization (CIMDO) methodology and taking as input variables the individual banks' PoDs estimated in the previous step, we cover the banking system's multivariate density (BSMD). Step4, based on BSMD, we estimate the proposed banking stability measures (BSMs), such as the probability of default of bank A given that the bank B is default.

Granger causality networks. Billion, Getmansky, Lo, and Pelizzon (2010) propose two econometric measures of systemic risk that capture the interconnectedness among the monthly returns of hedge funds, banks, brokers, and insurance companies based on principal components analysis (PCA) and Granger-causality tests. The authors find that all four sectors have become highly interrelated over the past decade, increasing the level of systemic risk in the finance and insurance industries. Their measures can also identify and quantify financial crisis periods, and seem to contain predictive power for the current financial crisis. Their philosophy is that statistical relation-ships between returns can yield valuable indirect information about the build-up of systemic risk.

CoVaR. Note that $CoVaR$ measure can also provide us that bilateral contagion risk between banks. We can slightly change the definition 2 by proposing bank j's VaR conditional on bank i is already in distress, $Pr(X^j \leq CoVaR_q^{j|C(X^i)}|C(X^i)) = q$. In this case, $CoVaR$ measure implies the bank i's risk contribution to the bank j.

2.5.2 Systematic risk measures

MES. Acharya, Pederson, Phillipon, and Richardson (2010) and Brownlees and Engle (2011) propose that each FI's exposure to a systematic event can be captured by Marginal Expected Shortfall. More precisely, let N denote the set of firms in the economy. The return generated in the system at time t can be measured by the value weighted average asset return of all firms, which is market return (The same in AB11). This approach starts from a description of the bivariate process of FIs and market returns:

$$\begin{aligned} X_{mt} &= \sigma_{mt}\epsilon_{mt} \\ X_{it} &= \sigma\rho_{it}\epsilon_{it} + \sqrt{1-\rho_{it}^2}\xi_{it} \\ (\epsilon_{mt}, \xi_{it}) &\sim F. \end{aligned}$$

Note that ρ_{it} is the correlation between FI i's return and market return at time t. σ_{it} and σ_{mt} are respectively FI i's return volatility and market return volatility. The disturbances, ϵ_{mt} and ξ_{it} are independent and identically distributed over time and have zero mean unit variance and zero covariance. However, they are not assumed to be independent.

Straightforward algebra shows us that this measure can be decomposed in a function of 1) Volatility, 2) Correlation and 3) Tails expectations of the standardized innovations distribution:

$$\begin{aligned} MES_{it-1} &= \mathbb{E}_{t-1}(X_{it}|X_{mt} \leq C) \\ &= \underbrace{\sigma_{it}}_{\text{volatility}} \underbrace{\rho_{it}}_{\text{correlation}} \underbrace{\mathbb{E}_{t-1}(\epsilon_{mt}|\epsilon_{mt} \leq C/\sigma_{mt})_{it}}_{\text{tail expectation}} + \\ & \quad \underbrace{\sigma_{it}}_{\text{volatility}} \underbrace{\sqrt{1-\rho_{it}^2}}_{\text{correlation}} \underbrace{\mathbb{E}_{t-1}(\xi_{it}|\epsilon_{mt} \leq C/\sigma_{mt})_{it}}_{\text{tail expectation}}. \end{aligned}$$

And the conditional probability of a systemic event is:

$$\mathbb{P}_{t-1}(r_{mt} \leq C) = \mathbb{P}(\epsilon_{mt} \leq C/\sigma_{mt}).$$

Notice that in this framework, such probability is time varying: the higher the volatility the higher the probability of observing a loss above fixed threshold.

From a theoretical point of view, it is akin to a Beta computed within the Capital Asset Pricing Model (CAPM). The difference rests on the fact that the MES measures a so-called "tail-Beta", which is a projection of FI's return to a space where the market return is lower than a specific threshold. By contrast, in the CAPM, the Beta is simply the projection of a FI's return to the market return, without any threshold. An economic interpretation about this measure is that, what is the loss of one specific FI in case of a large loss happened in the financial system. Now we can see that, this measure is in the same spirit of Tarashev, Borio, and Tsatsaronis (2010).

The estimation of MES is proceed as follow. In wide world of GARCH specifications, TGARCH is picked to model the *volatility* in order to capture a so-called "leverage effect". The evolution of the conditional variance dynamics in this model is given by:

$$\sigma_t^2 = \omega_G + \alpha_G r_{t-1}^2 + \gamma_G r_{t-1}^2 I_{t-1}^- + \beta_G \sigma_{t-1}^2,$$

with $I_{t-1}^- = r_t \leq 0$. The model is estimated by Quasi-MLE which guaranties the consistency of the estimator. We have:

$$\sqrt{T}(\hat{\theta}^{MLE} - \theta^o) \sim N(0, J_0^{-1} I_0 J_0^{-1}),$$

with $\hat{I} = \frac{1}{T}\sum S_t(\hat{\theta})S_t'(\hat{\theta})$ and $\hat{J} = \frac{1}{T}\sum \frac{\partial^2 logL}{\partial\theta\partial'\theta}$ is the Hessian of total log-likelihood function. The time varying *correlation* is modeled by using DCC approach. Correlation matrix is given by:

$$P_t = diag(Q_t)^{-\frac{1}{2}}Q_t diag(Q_t)^{-\frac{1}{2}},$$

and DCC specification is defined as:

$$Q_t = (1-\alpha_C-\beta_C)S + \alpha_C\epsilon^*_{t-1}\epsilon^*_{t-1} + \beta_C Q_{t-1}.$$

We use the estimation method mentioned above for this dynamic conditional correlation. The last part need to be estimated is *Conditional Expected Tail*:

$$E_{t-1}(\epsilon_{mt}|\epsilon_{mt} \leq \kappa) \text{ and } E_{t-1}(\xi_{it}|\epsilon_{mt} \leq \kappa).$$

A non-parametric kernel estimation approach is implemented to improve the efficiency of estimators. Let

$$K_h(t) = \int_{\infty}^{t/h} k(u)du,$$

where $k(u)$ is a kernel function and h is a positive bandwidth. Therefore, we have

$$\hat{E}_h(\epsilon_{mt}|\epsilon_{mt} \leq \kappa) = \frac{\frac{1}{n}\sum_{t=1}^{n}\epsilon_{mt}K_h(\epsilon_{mt}-\kappa)}{\hat{P}_h},$$

and

$$\hat{E}_h(\xi_{it}|\epsilon_{mt} \leq \kappa) = \frac{\frac{1}{n}\sum_{t=1}^{n}\xi_{it}K_h(\epsilon_{mt}-\kappa)}{\hat{P}_h},$$

where $\hat{P} = \frac{1}{n}\sum_{t-1}^{n} K_h(\epsilon_{mt}-\kappa)$. An advantage of non-paramretric estimator defined above is tat they are smooth function of cutoff point κ which, in turn, deliver smooth estimates of MES as a function of C/σ_{mt}.

Distressed Insurance Premium. The Distressed Insurance Premium (DIP) is proposed as an ex ante systemic risk metric by Huang, Zhou, and Zhu (2010) and it represents a hypothetical insurance premium against a systematic financial distress, defined as total losses that exceed a given threshold, say 15%, of total FI liabilities. The methodology is general and can apply to any pre-selected group of FIs with public tradable equity and CDS contracts. Each FI's marginal contribution to systemic risk is a function of size, probability of default (PoD), and asset correlation. The last two components need to be estimated from market data.

Let L_i denote the loss of institution i's liability with $i = 1, ..., N$; and $L = \Sigma_{i=1}^{N} L_i$ is the total loss of portfolio. Then the distress insurance premium (DIP) is given by the risk-neutral expectation of the loss exceeding certain threshold level:

$$\text{DIP} = \mathbb{E}^{Q}[L | L \geq \text{threshold}],$$

and this DIP formula can be implemented with Monte Carlo simulation Huang, Zhou, and Zhu (2009). The marginal contribution of the systematic DIP can be characterized by

$$\frac{\partial \text{DIP}}{\partial L_i} \equiv \mathbb{E}^{Q}[L_i | L \geq \text{threshold}].$$

Again, from the property of the measure, this measure is equivalent to Marginal Expected Shortfall (MES).

SRISK. The SRISK is proposed by Acharya, Engle, and Richardson (2012) and Brownlees and Engle (2011) extends the MES in order to take into account the balance sheet data - the liabilities of the FI. The SRISK corresponds to the expected capital shortfall of a given FI, conditional on the whole financial system is in a systematic event. In this perspective, The FI with the largest capital losses are supposed to be the one which has the largest exposure to this systematic distress. Acharya, Engle, and Richardson (2012) define the SRISK as;

$$\text{SRISK}_{it} = \max[0; kD_{it} - (1 - k)W_{it}(1 - \text{LRMES}_{it})],$$

where k is the prudential capital ratio, D_{it} is the quarterly book value of total liabilities, and W_{it} is the daily market capitalization or market value of equity and LRMES_{it} is the long-run marginal expected shortfall.[43] Note that, the SRISK measure is increasing in liabilities and decreasing in capital, so it can be seen as an implicit increasing function of the quasi-leverage (capital/debt).

CoVaR. Note that $CoVaR$ measure can also provide us that systematic exposure of banks. We can slightly change the definition 2 by proposing bank i's VaR conditional on the whole is already in distress, $Pr(X^i \leq CoVaR_q^{i|C(X^{sys})} | C(X^{sys})) = q$. In this case, $CoVaR$ measure implies the bank i's exposure to the systematic event.

[43] Acharya, Engle, and Richardson (2012) propose to approximate this LRMES using the daily MES as $\text{LRMES} \simeq 1 - \exp(18 \times \text{MES}_{it})$.

2.6 Macroprudential policy and Monetary policy

Farhi and Tirole (2012) study a collective moral hazard problem in the banking system. They show that imperfect targeting of distressed institutions in times of crisis makes private leverage choices among banks strategic complements. If authorities are perceived to be tough during the crisis period, each bank has an incentive to issue less short-term debt or hold more liquidity reserves. On the other hand, if the central bank is perceived to be "kind", banks have an incentive to engage in maturity mismatch, which induces each individual bank to issue more short-term debt and invest in long-run illiquid assets. In addition, they show that under the policy response, banks choose the correlation of their shock with that of other banks (Acharya (2009)). A "kind" central bank is referred to be the one that will lower the interest rate when there is a severe shock in the economy. However, this policy not only raises the issue we have seen above, but also sow the seeds for the next crisis, because more unworthy projects will be financed since the borrowing cost is reduced and it exposes banks to future liquidity problems. Due to the time-inconsistency problem, that a "kind" central bank will be always there at crisis time.

Farhi and Tirole (2012) also propose a transfer policy to boost the banks' wealth when an aggregate shock occurs. The mechanism is similar to Holmstrom and Tirole (1997) and Brunnermeier and Pedersen (2009), when a bank's wealth is high, they have more chance to get funds from outside investors which prevents a "credit rationing"; or is strengthens bank's margin constraint as in equation (2.9), thereby stabilizing asset prices. However, in reality policy makers are less well informed, so they cannot detect which banks are those banks which received a shock. So, the interest rate policy is then always part of the optimal policy mix. Direct transfer policy is only optimal in Farhi and Tirole (2012) if a large fraction of FIs is affected by the crisis.

Stein (2011) also points out that FI may issue too much short-term debt and may generate negative fire-sales externalities. This is because FIs only consider the benefit that they gain from such behavior, however, they do not internalize the social cost that they generated from doing so. In a crisis period, FIs are forced to sell their assets at fire-sale price to honor their short-term debt causing a negative externality on other banks as in Lorenzoni (2008).

In this context, a monetary policy based approach - "cap-and-trade" of money-creation permits can implement the optimal outcome. This approach is characterized by the reserve requirements on FIs, and the price of these permits - the interest rate on reserves[44] - reveals information about banks' investment opportunities to the regulator. For example, when the price of the permits goes up, this suggests that banks in the aggregate have strong investment opportunities, and so the regulator should loosen the cap by putting more permits into the system. The paper states that, if there are only commercial banks in the economy that play a role of lenders, the conventional monetary policy can be used to regulate this externality efficiently, but this view is far away from the modern financial system. So when large shadow banking system is present in the economy, some additional regulatory measures have to be imposed.

Beau, Clerc, and Mojon (2012) use a DSGE model and focus on the price stability to illustrate the interactions between macroprudential policy and monetary policy. Specifically, this paper mainly focuses on the time-dimension of macroprudential policy, and authors consider that this macroprudential policy is designed to lean against the wind of booms and busts financial cycles. If the inflation cycle and booms/busts cycle are positively correlated then standard monetary policy and macroprudential policy are complementary. If these two cycles are negatively correlated then the two types of policy are conflicting. The compatibility of the two policies depends very much on the type of shocks; the demand shocks lead to complementarities and supply shocks lead to conflicts.

2.7 Conclusion

I would like to close this survey by noting that the study of systemic risk should not be treated in isolation. As we have seen in the main body of the article, many mechanisms behind systemic risk are triggered by financial friction in the economy, and these frictions often come with the asymmetric information between the agents in the economy; therefore it is hard for us to achieve the *first-best* allocation. How to design an efficient regime to cope with this agency cost is at the core of the research in this domain.

[44]Can be seen as the outside opportunity of these reserves.

Chapter 3

Multi-CoVaR and Shapley value: A Systemic Risk Measure

3.1 Introduction

The recent financial crisis has demonstrated the adverse effects of a large scale breakdown of financial intermediation both for other banks as well as for the rest of the economy. This is embodied in after the failure of Lehman Brother, the entire financial market has become extremely volatile, at the same time, the manufacture industry was deeply impacted. The concern that a specific bank defaults would trigger a domino effect in the financial sector is often brought forward to justify large scale government intervention and bailouts of failed institutions. But this increases the burden on taxpayers and sows the seeds of the next crisis.[1] The failure of a bank causes these externalities because financial institutions are directly linked through interbank loans and derivatives. However, before the 2007-2009 financial crisis, banking regulation was based on individual risk, that is to say the adverse consequences that a bank brings for other banks as well as the economy as a whole was not considered by the regulator. Moving towards a new regulation framework, economic researchers and regulators propose to implement a *macroprudential* policy, which aims to mitigate the risk of the financial system as a whole (systemic risk) and to stabilize the financial system. To efficiently regulate systemic risk in banking sector need two steps: 1) measure each institution's marginal systemic risk contribution to the whole system; 2) based on these measures, propose an efficient requirement standard to resist the

[1]See Farhi and Tirole (2012).

systemic risk.[2] In this paper, I mainly focus on the first step of macro-prudential regulation and propose a new methodology to measure the institution's marginal systemic risk contribution.

The methodology proposed in this paper is complementary to Adrian and Brunnermeier (2011) CoVaR methodology, which is one of the most popular systemic risk measures in the literature. In Adrian and Brunnermeier (2011), authors use $\Delta CoVaR$, which is the difference between the VaR of the financial system conditional on a given financial institution being in a tail event and the VaR of financial system conditional on this financial institution being in a normal state, to capture the marginal contribution of systemic risk. Note that the VaR of the financial system conditional on a given financial institution being in a tail event is also known as $CoVaR$. However, a specific feature of modern financial crisis is that there may be several institutions in financial distress at the same time, as a result, it is difficult to accurately measure a specific institution's systemic risk contribution taken in isolation while the effect could channel through other financial institutions themselves in distress at the same time. As a consequence, I modify the standard $\Delta CoVaR$ to the multi-$\Delta CoVaR$. Unlike the standard $\Delta CoVaR$, multi-$\Delta CoVaR$ is the difference between the VaR of financial system conditional on a given *set of financial institutions* being in a tail event and the VaR of the financial system conditional on this *set of financial institutions* being in a normal state. With this complementary extension, multi-$\Delta CoVaR$ captures the systemic risk contribution of several distressed financial institutions at the same time; furthermore it can capture the marginal systemic risk contribution of an institution that just joined the "distressed club" by taking the difference of the multi-$\Delta CoVaR$ of the set including the new one to the club and the multi-$\Delta CoVaR$ of the original set. Henceforth, given the specific feature of modern financial crisis, multi-$\Delta CoVaR$ can provide very useful information to build a macroprudential regulation framework.

The property of multi-$\Delta CoVaR$ is very useful for regulators. First, it allows to calculate the total contribution of systemic risk in the financial system, with this total contribution, one can apply an allocating rule to attribute the total risk to each bank. This total systemic risk is measured by the systemic risk contribution when all the institutions in the system are in distress. This total systemic risk measure can be set as a benchmark for regulators, since the sum of each institution's systemic risk can never surpass (less than) this benchmark,

[2]Efficient requirement standard refers to the total systemic requirement can be implemented at individual level without over or under-regulation.

otherwise there is an over-regulation (under-regulation) in the industry which may hurt the real economy. Second, it allows to calculate the marginal contribution of a bank for a given set of institutions is already in distress. This has the advantage to inform the regulators that which distressed institution should be bailed-out more urgently during the financial crisis. For example, in September 2008, Lehman Brother and AIG are both in financial distress at the same time and then Federal reserve has decided to bailout AIG. However, based on multi-$CoVaR$ measure one can calculate the marginal contribution for both LB and AIG and to bailout the one which has a larger marginal contribution of systemic risk. The third, the multi-$CoVaR$ can provide the systemic risk contribution of different groups. Again, this property is very useful for the regulators. Put the set of institutions differently for various sectors such as deposit banking sector, insurance sector, investment banking sector or government-sponsored enterprise (GSE), one can obtain the systemic risk contribution for these different sectors and regulate each sector appropriately.

Another advantage of multi-$\Delta CoVaR$ is that one can apply to Shapley value methodology, which satisfies a set of axioms, to allocate systemic risk to each financial institution. The Shapley value methodology was initially proposed in the circumstance of cooperative games by Shapley (1953), in which a group of players generates a shared "value" (e.g. wealth, cost) for a group as a whole. The Shapley value of a player in a game turns out to be his expected marginal contribution over the set of all permutations on the set of players. This methodology can be applied in my setting, where financial institutions are connected via correlated high risk activities that trigger systemic risk in the system. The "value" mentioned above is the systemic risk generated by financial institutions. The additivity axiom of Shapley value states that the sum of each institution's Shapley value of systemic risk contribution is exactly equal to the multi-$\Delta CoVaR$ of all the financial institutions in the system being in financial distress, which means that the Shapley value methodology allocates overall systemic risk in an efficient way. This coincides with Gourieroux and Monfort (2011), who suggest that systemic risk measures should be additive. While in the contrary, the sum of each standard $\Delta CoVaR$ is larger then the multi-$\Delta CoVaR$ of all the financial institutions in the system being in financial distress; that is to say, if I regulate financial institutions with the standard $\Delta CoVaR$, I implicitly punish the whole financial system, since the regulation is based on some systemic risk which does not even exist. Therefore, it would decrease the credit volume of banks and hurt the real economy.

After introducing the allocation methodology of how to distribute the systemic risk to each financial institution, I now turn to the estimation of the multi-$CoVaR$. There are many possible ways to calculate it. In this paper, I directly use the market returns of the financial system and of individual financial institutions for multi-$CoVaR$ calculation. For simplicity, at the very beginning, I assume the returns of the financial system and financial institutions follow a multi-t distribution to construct the distribution of financial system's return conditional on the returns of the financial institutions.[3] To that purpose, I use GARCH model and Dynamic Conditional correlation (DCC) (Engle (2002)) to construct the variance-covariance matrix of the conditional distribution. I loosen this assumption to calculate individual financial institution's VaR: instead I use a nonparametric bootstrap technology implemented in GARCH model proposed by Pascual, Nieto, and Ruiz (2006) and Christofferson and GonCalves (2005). With all the ingredient I need, then I apply Shapley value methodology to allocate systemic to each financial institution.

I apply the methodology of calculating the systemic importance in two banking panels, French panel and Chinese panel. Given these two banking systems, my measure provides the total systemic risk in the financial system and marginal contribution of systemic risk for each financial institution. I find that for both French and Chinese panels the overall systemic risk during the 2007-2009 financial crisis is larger than total systemic risk during the European sovereign debt crisis at the peak, and Chinese banks have little impact from the European crisis.

The remainder of the paper is organized as follows. I provide a literature review in section 3.2. In section 3.3, I develop the methodology, illustrate the differences with existing measures, show the properties of my measure and provide the calculation method of the measure. In section 3.4, I present the econometric framework and estimation. Section 3.5 gives the results and show the importance of additivity . In section 3.6, I briefly propose a regulation tool based on this measure. Section 3.7 concludes.

[3]The normal distribution has been criticized since its quantile function doesn't capture the fat tail events. I try to make distribution assumption as less as possible. In this case, I have to assume the returns follow a multi-t distribution, however, I loosen this assumption when I calculate the individual VaR.

3.2 Literature review

This paper belongs to a growing empirical literature on systemic risk. The main contribution of the paper is that I focus on the aggregate contagion[4] risk of a set of financial institutions and use an efficient allocation rule to attribute this aggregate contagion risk to each financial institution in order to determine their systemic importance. The efficient allocation means that the overall aggregate contagion risk is all attributed to each financial institution without waste and won't be attributed by any risks that don't exist, hence we have that the sum of attributed risk to each financial institution equals the overall aggregate contagion risk.

The most of empirical literature have particular interest are those that treat explicitly the financial system as a portfolio of institutions. Lehar (2005) uses a sample of international banks from 1988 to 2002, to estimate the dynamics and correlations between bank asset portfolios. After the 2007-2009 financial crisis, two strands of the literature have been emerged. The first strand of the literature captures the bank's sensitivity to a systematic shock in the whole financial system. In Huang, Zhou, and Zhu (2010), they compute first the total market loss when a systematic shock takes place and then attribute this loss to each financial institution to capture the systemic importance. Similarly, **?** proposed a measure called "Marginal Expected Shortfall" (MES); this measure allows to investigate what is the loss of one specific institution in case of a large loss in the financial system. From a theoretical point of view, it is akin to a Beta computed within the Capital Asset Pricing Model (CAPM). The difference rests on the fact that the MES measures a so-called "tail-Beta", which is a projection of institution's return to a space where the market return is lower than a specific threshold. In **?**, the authors construct an econometric framework to compute the MES. In their model, the MES is consist of 3 parts: the volatility of institutions' market capitalization return, their correlation and the conditional tail expectation. Adrian and Brunnermeier (2011) also suggest a measure exposure-$CoVaR$ to capture this effect. Greenwood, Landier, and Thesmard (2011) propose a measure to gauge a bank's exposure to sector-wide deleveraging. This specific measure is focusing on the fire-sale effect in the financial market.

Another strand of the literature gauges systemic risk by the contagion

[4]Some papers are focusing on the exposure of banks when a systematic shock happens, e.g. Huang, Zhou, and Zhu (2010); **?**; others are focusing on the contagion, e.g. Segoviano and Goodhart (2009); Billion, Getmansky, Lo, and Pelizzon (2010); however Greenwood, Landier, and Thesmard (2011) allows to distinguish these two risks.

through the financial market. This contagion is twofold. On the one hand, this contagion could be considered as the bilateral contagion, which means one bank's sensitivity to a shock of another bank and vice verca. Segoviano and Goodhart (2009) propose a methodology to capture such inter-linkages contagion. The objective of the methodology is to find the joint distribution, which best fits a prior joint distribution according to the information criteria and is consistent with the probability of distress for each bank. This gives the probability of default of one bank given another bank is in distress. In Billion, Getmansky, Lo, and Pelizzon (2010), they propose a measure of connectedness based on principal-component analysis and Granger-causality networks. They find that financial institutions are highly interrelated over the past decade. On the other hand, this contagion could be considered as the contagion of a bank to the whole financial system, however, one can argue that this is also the contribution of financial institutions to overall systemic risk. Adrian and Brunnermeier (2011) use the $CoVaR$ measure to characterize one bank's systemic risk contribution to the financial system. In their framework, this measure is the Value at Risk of the whole financial sector conditional on a given institution being in distress. Tarashev, Borio, and Tsatsaronis (2010) and **?** use individual risk measures to capture an institution's systemic importance by implementing Shapley value methodology; they also focus on the contribution of institutions to systemic risk. However, in this paper, I use a systemic risk measure to capture the systemic importance of institutions and then allocate the measure by Shapley value methodology. In Greenwood, Landier, and Thesmard (2011), their method also allows to capture one bank's contribution to the overall deleveraging risk, but, again, their measure is specifically focused on the fire-sale effect.

There are also two very useful survey about the systemic risk measures Benoit, Colletaz, Hurlin, and Perignon (2012) and Bisias, Flood, Lo, and Valavanis (2012).

3.3 Multi-CoVaR and Shapley value methodology

In this section, I present how the Shapley value methodology can be applied to the CoVaR measure and its properties.

3.3.1 Remind CoVaR

Recall that Value-at-Risk (VaR) is defined as the solution to

$$\mathbb{P}(r_t \leq VaR_t^q) = q,$$

VaR_t^q is the q-quantile of the return r_t. Note that, with this definition, VaR_t^q is typically a negative number.

Adrian and Brunnermeier (2011) defined $CoVaR_{q,t}^{sys|i}$ as the VaR of financial system conditional on some event $C(r_t^i)$ of institution i at time t. That is, the $CoVaR_{q,t}^{sys|i}$ for financial system and confidence level q when the institution i is on some event $C(r^i)$ at time t is defined by:

$$\mathbb{P}(r_t^{sys} \leq CoVaR_{q,t}^{sys|C(r_t^i)} | C(r_t^i)) = q, \tag{3.1}$$

and institution i's contribution to system is measured by:

$$\Delta CoVaR_{q,t}^{sys|i} = CoVaR_{q,t}^{sys|r_t^i \in \{\text{adverse case}_t^i\}} - CoVaR_{q,t}^{sys|r_t^i \in \{\text{normal case}_t^i\}}. \tag{3.2}$$

In practice, they focus on $\{r^i = VaR_q^i\}$ as the conditioning event and simplify the notation $CoVaR_q^{sys|r^i = VaR_q^i} = CoVaR_q^i$ and $\Delta CoVaR_q^{sys|i} = \Delta CoVaR_q^i$, meanwhile they focus on $\{r^i = \text{Median}^i\}$ in the normal case as the conditioning event, $CoVaR_{q,t}^{sys|r^i = \text{Median}^i}$. Hence, $\Delta CoVaR_q^i$ denotes the difference between the VaR of financial system conditional on the financial institution i being in a tail event and the VaR of financial system conditional on the financial institution i being in a normal state. Note that, they also define the system returns as the weighted sum of individual returns at each time t.

3.3.2 Multi-CoVaR

After remind the definition of CoVaR of Adrian and Brunnermeier (2011) above. They propose $\Delta CoVaR$ as a systemic measure to quantify the risk spillover effects, what is the impact on the financial system if one specific institution is in financial distress. However, during the financial crisis, I have seen that several financial institutions may have been in financial distress at the same time, therefore an interesting extension of CoVaR can be defined as follow:

Definition 4. I denote by $CoVaR_{q,t}^{1,\dots,S}$ the VaR of the financial system conditional on some event $\{C(r_t^1),\dots,C(r_t^S)\}$ of a set of institutions $\{1,\dots,S\}$ at time t. That is $CoVaR_{q,t}^{1,\dots,S}$ for financial system and confidence level q when the set of institutions $\{1,\dots,S\}$ is on some event $\{C(r_t^1),\dots,C(r_t^S)\}$ at time t is defined by:

$$\mathbb{P}(r_t^{sys} \leq CoVaR_{q,t}^{1,\dots,S}|C(r_t^1),\dots,C(r_t^S)) = q, \tag{3.3}$$

and the set of institutions $\{1,2,...,S\}$'s contribution to financial system is denoted by:

$$\Delta CoVaR_{q,t}^{1,\dots,S} = CoVaR_{q,t}^{r^1 \leq VaR_q^1,\dots,r^S \leq VaR_q^S} - CoVaR_{q,t}^{-\alpha\sigma_t^1 \leq r_t^1 \leq \alpha\sigma_t^1,\dots,-\alpha\sigma_t^S \leq r_t^S \leq \alpha\sigma_t^S}. \tag{3.4}$$

Unlike above, I focus on $\{r_t^i \leq VaR_{q,t}^i\}$, for $i = 1,...,S$, in the adverse case as the conditioning event and simplify the notation, $CoVaR_{q,t}^{sys|r_t^1 \leq VaR_{q,t}^1,\dots,r_t^S \leq VaR_{q,t}^S} = ACoVaR_{q,t}^S$; I focus on $\{-\alpha\sigma_t^i \leq r_t^i \leq \alpha\sigma_t^i\}$, for $i = 1,\dots,S$, in the normal case as the conditioning event and simplify the notation, $CoVaR_{q,t}^{sys|-\alpha\sigma_t^1 \leq r_t^1 \leq \alpha\sigma_t^i,\dots,-\alpha\sigma_t^S \leq r_t^S \leq \alpha\sigma_t^S} = NCoVaR_{q,t}^S$. Note that the normal case is characterized as an α-standard deviation around the mode, where I assume the mode is 0 and σ_t^i is the conditional standard deviation of institution i. It is straightforward to see that, $\Delta CoVaR_q^{sys|1,\dots,S} = ACoVaR_{q,t}^S - NCoVaR_{q,t}^S$, letting $\Delta CoVaR_q^{sys|1,\dots,S} = \Delta CoVaR_q^S$. Hence $\Delta CoVaR_{q,t}^S$ denotes the difference between the VaR of financial system conditional on a set of financial institutions $\{1,\dots,S\}$ being in a tail event and the VaR of financial system conditional on the set of financial institutions $\{1,\dots,S\}$ being in a normal state at time t.[5]

The economics of multi-$CoVaR$ are quite similar to those of standard $CoVaR$. Both of them quantifies the spillover effects by measuring institution(s) add(s) to the global risk of the financial system. The spillover effects are embodied in several ways. First, if several banks are selling off their mark-to-market assets to meet their obligations, it will lower the price of these assets and further decrease the values of the banks who hold these assets, consequently it will hurt market liquidity and harm banks ability of raising new fund and even more likely trigger the insolvency problem of banks. This implies the second spillover effect, when

[5] Note that this is different from the initial definition of Adrian and Brunnermeier (2011), where they let the return exactly equals to its VaR as the adverse case and the return exactly equals to its median as the normal case. See also Ergun and Girardi (2012), they have changed the initial definition of conditional event in the standard CoVaR.

several banks meet the insolvency problem at the same time, it will drag other banks who hold direct debt contract of distressed banks into trouble, and this is so called domino effect. Last, when GSIFIs are insolvent, the government will intervene to bail-out these banks due to the "too-big-to-fail" effect and increase the burden of taxpayers and sow the seeds for the next crisis. Note that, this multi-$CoVaR$ captures spillover effects, and measure the contribution of systemic risk of a set of banks when it is in distress.

3.3.2.1 Calculation of Multi-CoVaR

As to the VaR for individual institutions, I compute $ACoVaR^S_{q,t}$ and $NCoVaR^S_{q,t}$ for a set of institutions in the same spirit in order construct the $\Delta CoVaR^S_{q,t}$ measure. The equation (3.3) can be reformulate as:

$$\frac{\mathbb{P}(r_t^{sys} \leq CoVaR_{q,t}^{1,\ldots,S}, C(r_t^1), \cdots, C(r_t^S))}{\mathbb{P}(C(r_t^1), \cdots, C(r_t^S))} = q. \tag{3.5}$$

To compute $ACoVaR^S_{q,t}$, I replace the conditional event $C(r^i_t)$ by $\{r^i_t \leq VaR^i_{q,t}\}$. Since the individual risk measure $VaR^i_{q,t}$ can be easily obtained,[6] therefore I can calculate the denominator of equation (3.5), which generates a joint probability q_d, (d for denominator),

$$\int_{-\infty}^{VaR^1_{q,t}} \cdots \int_{-\infty}^{VaR^S_{q,t}} D_{S,t}(r^1_t, \ldots, r^S_t) dr^1_t \cdots dr^S_t = q_d, \tag{3.6}$$

where $D_{S,t}(\cdot)$ denotes the probability density function of a S-dimensional random vector $r_{S,t} = (r^1_t, \ldots, r^S_t)'$. Hence, the numerator in the equation (3.5) can be rewritten in the same way,

$$\int_{-\infty}^{ACoVaR^S_{q,t}} \int_{-\infty}^{VaR^1_{q,t}} \cdots \int_{-\infty}^{VaR^S_{q,t}} D_{S+1,t}(r^{sys}_t, r^1_t, \ldots, r^S_t) dr^{sys}_t dr^1_t \cdots dr^S_t = q \times qd, \tag{3.7}$$

where $D_{S+1,t}(\cdot)$ denotes the probability density function of a $S+1$-dimensional random vector $r_{S+1,t} = (r^{sys}_t, r^1_t, \ldots, r^S_t)'$. As the $ACoVaR^S_{q,t}$ is the only unknown in the equation (3.7), it can be solved numerically.

Regarding to the $NCoVaR^S_{q,t}$, the conditional event is $\{-\alpha\sigma^i_t \leq r^i_t \leq \alpha\sigma^i_t\}$ and the conditional standard deviation σ^i_t can be obtained by implementing

[6] I will present how to calculate the individual VaR in the next section.

a GARCH process. The denominator of equation (3.5), in this case, gives,

$$\int_{-\alpha\sigma_t^1}^{\alpha\sigma_t^1} \cdots \int_{-\alpha\sigma_t^S}^{\alpha\sigma_t^S} D_{S,t}(r_t^1, \ldots, r_t^S) dr_t^1 \cdots dr_t^S = p_d, \tag{3.8}$$

and the numerator gives,

$$\int_{-\infty}^{NCoVaR_{q,t}^S} \int_{-\alpha\sigma_t^1}^{\alpha\sigma_t^1} \cdots \int_{-\alpha\sigma_t^S}^{\alpha\sigma_t^S} D_{S+1,t}(r_t^1, \ldots, r_t^S) dr_t^{sys} dr_t^1 \cdots dr_t^S = q \times p_d, \tag{3.9}$$

therefore, I can compute $NCoVaR_{q,t}^S$ with the same procedure as for $ACoVaR_{q,t}^S$, thus the $\Delta CoVaR_{q,t}^S$ is obtained by taking the difference between $ACoVaR_{q,t}^S$ and $NCoVaR_{q,t}^S$.

3.3.2.2 Properties of Multi-CoVaR

This measure has three advantages. First, it allows to calculate the total contribution of systemic risk in the financial system, with this total contribution, one can apply an allocating rule to attribute the total risk to each bank. This total systemic risk is measured by the systemic risk contribution when all the institutions in the system are in distress. Suppose there are N banks in the system, the total contribution of systemic risk is given by:

$$\mathbb{P}(R^{sys} \leq CoVaR_{q,t}^{1,\ldots,N} | C(r_t^1), ..., C(r_t^N)) = q, \tag{3.10}$$

and

$$\Delta CoVaR_{q,t}^N = ACoVaR_{q,t}^N - NCoVaR_{q,t}^N. \tag{3.11}$$

Note that the equation (3.10) characterizes an extreme case in the system, where all the institutions are in financial distress, and the equation (3.11) gives the related systemic risk contribution in this scenario, which is the overall systemic risk contribution in the system. This total systemic risk measure can be set as a benchmark for regulators, since the sum of each institution's systemic risk can never surpass (less than) this benchmark, otherwise there is an over-regulation (under-regulation) in the industry which may hurt the real economy.

Secondly, it allows to calculate the marginal contribution of bank i for a given set of institutions S is already in distress. Denote $\Delta_i(S)$ the marginal systemic risk contribution of one specific institution i is determined by:

$$\Delta_i(S) = \Delta CoVaR(S \cup \{i\}) - \Delta CoVaR(S), \text{ for } S \subset N,\ i \notin S. \tag{3.12}$$

The equation (3.12) has the advantage to inform the regulators that which distressed institution should be bailed-out more urgently during the financial crisis. For example, in September 2008, Lehman Brother and AIG are both in financial distress at the same time and then Federal reserve has decided to bailout AIG. However, based on (3.12) one can calculate $\Delta(S)$ for both LB and AIG and to bailout the one which has a larger marginal contribution of systemic risk.

The last, the multi-$CoVaR$ can provide the systemic risk contribution of different groups. Again, this property is very useful for the regulators. Put the set of institutions S differently for various sectors in the financial system as deposit banking sector, insurance sector, investment banking sector or government-sponsored enterprise (GSE), one can obtain the systemic risk contribution for these different sectors and regulate each sector appropriately. Moreover, we can put the different set of institutions as different regions or different countries to analyze a geographical distinction of systemic risk contribution to better inform the international establishment as OECD, IMF, and BIS etc.

On top of that, this extension about standard $CoVaR$ allows us to implement Shapley value methodology to attribute the systemic risk to each financial institution in the system, which has some related advantages to the design of prudential regulation issue.

3.3.3 Shapley value

In this paper, the Shapley value plays a role as a systemic risk distributor, which means that I use Shapley value methodology as an allocation rule to assign the overall systemic risk contribution, as in (3.11), to each institution in the financial system. An introduction of Shapley value is presented below.

The Shapley value methodology was initially proposed in the circumstance of cooperative games, in which a group of players generates a share “value” (e.g. wealth, cost) for a group as a whole. The Shapley value of a player in a game turns out to be his expected marginal contribution over the set of all permutations on the set of players. For example, a group of agents would like to connect to a server in order to benefit a high speed functioning of their own PC, however, the maintain of the server is costly, the Shapley value is a fair and efficient allocation rule to share the maintaining costs among agents and the Shapley value of an agent is his expected marginal contribution over all possible set of the agents. This methodology can be applied in our case, where financial institutions are connected in financial market with high correlated risk activities that trigger systemic risk in the system. The "value" mentioned above is the systemic risk generated by financial institutions (banks).

In order to apply the Shapley value methodology to a financial system, it is sufficient to define a so-called "characteristic function". As mentioned above, I define this "characteristic function" as Multi-$CoVaR$. This function is the same over the set of all permutations on the set of banks and map each subsystem into a risk measure. The characteristic function, v, should accept as input anyone of the $2^N - 1$ subsystems[7] of banks and should deliver the system-wide risk measures when applied to the entire system.

The derivation of Shapley values involves the following process.

There are N players in a superadditive game, which are financial institutions. Let $v : 2^N \to R^+$ be a function defining the systemic risk for each subset of N, and $v(\phi) = 0$. The objective is to find non-negative systemic risk attribution $\{Sh_i\}_{i \in N}$ such that:

- Axiom 1 (Additivity/Efficiency): $\Sigma_{i \in N} Sh_i = v(N)$,
- Axiom 2 (Dummy axiom): If i is such that $\Delta_i(S) = v(\{i\})$ for all S such that $i \notin S$, then $Sh_i = v(\{i\})$,
- Axiom 3 (Symmetry): If $i \neq j$ such that $\Delta_i(S) = \Delta_j(S)$ for all S such that $i, j \notin S$, then $Sh_i = Sh_j$,
- Axiom 4 (Linearity): Suppose $v(S) = v_1(S) + v_2(S)$ where v_1 and v_2 and assume $v_1(\phi) = v_2(\phi) = 0$, and $\{Sh_i^1\}_i$ are systemic risk shares for v_1-risk and $\{Sh_i^2\}_i$ are systemic risk shares for v_2-risk, then $Sh_i = Sh_i^1 + Sh_i^2$, for all i, defines the systemic risk shares for v-risk.

There is a unique way to satisfy axioms 1, 2, 3 and 4, called Shapley value and Shapley value for bank i is:

[7] Since the empty set ϕ does not play a role in contributing the value.

$$Sh_i(v) = \Sigma_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{i\}) - v(S)), \tag{3.13}$$

where n is the total number of banks and the sum extends over all subsets S of N not containing bank i. This formula can be interpreted as the expected marginal contribution of bank i over the set of all permutations on the set of banks.

Now, I am going to talk about the economic meaning of these axioms. Axiom 1 states that the sum of of Shapley value equals the aggregated systemic risk, this additivity property also called efficiency property, since the total systemic risk can be attributed among banks with no loss and with no gain. This property has a big advantage in macro-prudential policy, if macro-prudential tools are based on some sub(super)-additivity systemic risk measures, given the linear relationship between the macro-prudential regulation and systemic regulation, supervisors will punish (subsidize) the economy for no reason. Axiom 2 says that if the systemic risk of i is orthogonal of any other $j \neq i$'s systemic risks, then the systemic risk share of i should be exactly equals to its systemic risk alone. This is the case where standard CoVaR coincide with Shapley value methodology, whereas the systemic risk of i is not orthogonal of any other institutions' systemic risks, this is the reason why the standard CoVaR is different from Shapley value methodology. Axiom 3 enforces the fairness among players, for any two different banks, if their marginal systemic risk contribution is the same for any subsets $S \subset N$, then their Shapley value should be the same, therefore, they should be charged the same as well. The last axiom suggests that this systemic risk can be decomposed into two independent risks, for example these two risks are from two different services in the bank. Then to obtain the systemic risk shares it is sufficient to calculate the risk of each services and take the sum the the two. If one can decompose this systemic risk, I can apply the Shapley value methodology in a decentralized way.

3.4 Estimation

There are several ways to calculate $CoVaR$. Adrian and Brunnermeier (2011) propose to use quantile regression to estimate time-varying $\Delta CoVaR$ and VaR. They use a set of state variables as regressors to estimate q-quantile parameters to fit estimated $CoVaR$ and VaR, a big advantage of this method is that they do not consider any specific distribution on random variables in order to obtain the time-varying $CoVaR_t$ and VaR_t. In this paper, I use another econometric framework to illustrate the methodology presented above. In order to avoid the process of which state variables should be selected,[8] I work directly with the returns of the individuals. To clarify the idea, I firstly identify which ingredients are indispensable for calculating multi-$CoVaR$: 1) a probability density function of the returns (both system and individuals), 2) the conditional volatility of individual returns to define the so called "normal case", 3) the idiosyncratic risk VaR to characterize the "adverse case" for individuals.

3.4.1 Multi-t distribution

From the equations (3.6)-(3.9), the key ingredient in computing multi-$CoVaR$ is the joint probability density function. An important stylized fact in financial market is that the asset returns often have fatter tails than normal distribution, and I assume the vector $r_{S+1,t}$ follows a multi-t distribution with mean 0 and variance-covariance Σ_t (v for degree of freedom):

$$\begin{pmatrix} r_t^{sys} \\ r_t^1 \\ \vdots \\ r_t^S \end{pmatrix} \sim t_v \left(\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{sys,t}^2 & \rho_{sys1,t}\sigma_{sys,t}\sigma_{1,t} & \cdots & \rho_{sysS,t}\sigma_{sys,t}\sigma_{S,t} \\ \rho_{1sys,t}\sigma_{1,t}\sigma_{sys,t} & \sigma_{1,t}^2 & \cdots & \rho_{1S,t}\sigma_{1,t}\sigma_{S,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{Ssys,t}\sigma_{S,t}\sigma_{sys,t} & \rho_{S1,t}\sigma_{S,t}\sigma_{1,t} & \cdots & \sigma_{S,t}^2 \end{pmatrix} \right),$$

where $\sigma_{i,t}$ denotes the conditional volatility of i and $\rho_{ij,t}$ denotes the conditional correlation between i and j. Therefore, to estimate the multi-t distribution in each time t, it is sufficient to estimate the conditional volatilities and conditional correlations. Note that, this is the only case where I make a distribution assumption to calculate the multi-$CoVaR$.

3.4.2 Volatility

As mentioned before, I need to compute individual's conditional volatility σ_t^i at first place to calculate $NCoVaR_{q,t}^S$. To do so, I use the GARCH process. In wide world of GARCH specifications, TGARCH is picked to model volatility to capture so called "leverage effect", which has the fact that a negative return increases variance by more than a positive return of the same magnitude. I also a diagnosis of this GARCH model to show that the model is well specified in the

[8]For different state variables selected, the results of quantile estimation may be different.

Appendix. The evolution of the conditional variance dynamics in this model is given by:

$$\begin{aligned} r_t &= \epsilon_t \sigma_t, \\ \sigma_t^2 &= \omega_G + \alpha_G r_{t-1}^2 + \gamma_G r_{t-1}^2 I_{t-1}^- + \beta_G \sigma_{t-1}^2, \end{aligned}$$

with $I_{t-1}^- = r_t \leq 0$. The model is estimated by Quasi-MLE which guarantees the consistency of the estimator. I have:

$$\sqrt{T}(\hat{\theta}^{MLE} - \theta^o) \sim N(0, J_0^{-1} I_0 J_0^{-1}),$$

with $\hat{I} = \frac{1}{T}\sum S_t(\hat{\theta})S_t^{'}(\hat{\theta})$ and $\hat{J} = \frac{1}{T}\sum \frac{\partial^2 logL}{\partial\theta\partial'\theta}$ is the Hessian of total log-likelihood function. Where $\theta = (\alpha_G, \gamma_G, \beta_G)$.

3.4.3 Correlations

The time varying correlation is modeled by using DCC approach. Correlation matrix is given by:

$$P_t = diag(Q_t)^{-\frac{1}{2}} Q_t diag(Q_t)^{-\frac{1}{2}},$$

where Q is a so called pseudo-correlation matrix and it is positive defined. The DCC specification is defined as:

$$Q_t = (1 - \alpha_C - \beta_C)\overline{Q} + \alpha_C \epsilon_{t-1}^* \epsilon_{t-1}^{*'} + \beta_C Q_{t-1},$$

where ϵ_t^* is the standardized returns with $\epsilon_t^* = \text{diag}(Q) \times \epsilon_t$, $\overline{Q}$ is an intercept matrix with $\overline{Q} = \mathbb{E}[\epsilon_t^* \epsilon_t^{*'}]$. I use the estimation method mentioned above for this dynamic conditional correlation.

3.4.4 Value-at-Risk

Instead of using a distribution based approach to calculate VaR, here, I loosen the multi-t distribution assumption and use non-parametric bootstrap methodology to determine individuals VaR. The nonparametric bootstrap allows us to estimate the sampling distribution of a statistic empirically without making assumptions about the form of population, and without deriving the sampling distribution explicitly. The key bootstrap concept is that the population is to the sample as the sample is to the bootstrap sample. Then, I proceed the bootstrap technique in the following way.

For a given institution series of returns $\{r_{i,1}, ..., r_{i,T}\}$, consider a TGARCH model as in the previous case, whose parameters have been estimated by Quasi-MLE. Then I can obtain the standardized residuals, $\hat{\epsilon}_{i,t} = \frac{r_{i,t}}{\hat{\sigma}_{i,t}}$, where $\hat{\sigma}^2_{i,t} = \hat{\omega}^i_G + \hat{\alpha}^i_G r^2_{i,t-1} + \hat{\gamma}^i_G r^2_{i,t-1} I^-_{i,t-1} + \hat{\beta}^i_G \sigma^2_{i,t-1}$, and $\hat{\sigma}^2_{i,1}$ is long-run variance of the sample.

To implement the bootstrap methodology, it is necessary to obtain bootstrap replicates $R^*_{i,T} = \{r^*_{i,1}, ..., r^*_{i,T}\}$ that mimic the structure of original series of size T. $R^*_{i,T}$ are obtained from following recursion (Pascual, Nieto, and Ruiz (2006))

$$
\begin{aligned}
\hat{\sigma}^{*2}_{i,t} &= \hat{\omega}^i_G + \hat{\alpha}^i_G r^{*2}_{i,t-1} + \hat{\gamma}^i_G r^{*2}_{i,t-1} I^{*-}_{i,t-1} + \hat{\beta}^i_G \hat{\sigma}^{*2}_{i,t-1}, \\
r^*_{i,t} &= \epsilon^*_{i,t} \hat{\sigma}^*_{i,t}, \text{ for } t = 1, ..., T,
\end{aligned}
$$

where $\hat{\sigma}^{*2}_{i,1} = \hat{\sigma}^2_{i,1}$ and $\epsilon^*_{i,t}$ are random draws with replacement from the empirical distribution of standardized residuals $\hat{\epsilon}_{i,t}$.[9] This bootstrap method incorporate uncertainty in the dynamics of conditional variance in order to make useful to estimate VaR. Given the bootstrap series $R^*_{i,T}$, I can obtain estimated bootstrap parameters, $\{\hat{\omega}^{i,b}_G, \hat{\alpha}^{i,b}_G, \hat{\gamma}^{i,b}_G, \hat{\beta}^{i,b}_G\}$, The bootstrap of historical values are obtained from following recursions

$$
\begin{aligned}
\hat{\sigma}^{b*2}_{i,t} &= \hat{\omega}^{i,b}_G + \hat{\alpha}^{i,b}_G r^2_{i,t-1} + \hat{\gamma}^{i,b}_G r^2_{i,t-1} I^-_{i,t-1} + \hat{\beta}^{i,b}_G \hat{\sigma}^{b*2}_{i,t-1}, \\
\hat{r}^{b*}_{i,t} &= \epsilon^*_{i,t} \hat{\sigma}^{b*}_{i,t} \text{ for } t = 1, ..., T,
\end{aligned}
$$

where $\hat{\sigma}^{b*}_1$ is the long-run variance of the bootstrap sample R^{b*}_T, note that the historical values is based the original series of return and on the bootstrap parameters. I repeat the above procedure B times, and estimated $\widehat{VaR^*}^i_t(q)$ is k^{th}-order of series $\hat{r}^{b*}_t$, for $b = 1, ..., B$, where $k = B \times q$.

3.5 Empirical Analysis

I apply the methodology and econometric framework described in the previous sections and examine the systemic risk in in France banking system and China banking system.

[9]It is necessary to sample with replacement, because one would otherwise simply reproduce the original sample.

French Panel		Chinese Panel	
Name	Ticker	Name	Ticker
BNP Paribas	BNP	Industrial & Commercial Bank of China	ICBC
Société Générale	SoGen	China Construction Bank	CBC
Crédit Agricole	CrdAgl	Bank of China	BC
Natixis	Natixis	Bank of Communications	CCB
Crédit Mutuel	CrdMtl	China Merchants Bank	CMBC

Table 3.1: Banking Panel.

3.5.1 Data

I study two different banking panels in this paper, French panel and Chinese panel. I choose the top five French banks that are in the top 50 world banks which based on the market capitalization as of January 20, 2012, as the French banking system, from April 19, 2002 to June 29, 2012; and I also choose the top five Chinese banks that are in this top 50 world banks panel, as my Chinese banking system, from October 27, 2006 to June 29, 2012. I extract weekly returns and market capitalization from Bloomberg. The goal is to analyze the systemic risk in these two banking panels (table 3.1) in terms of market capitalization returns separately.[10]

Figure 3.1 gives visual insights on the booms and busts of the French and Chinese banking system. The figure shows the cumulative system returns in both banking panels, from April 2002 to June 2012 for French panel (blue line) and from October 2006 to June 2012 for Chinese panel (red line). The blue line experiences a loss from the start to the mid 2003, and then had a steep growth between mid 2003 and June 2007. Starting from July 2007, the fall of financial market has been dramatic, with the large gains transforming into the huge losses. The system hit the bottom in March 2009 and start a slow recovery that is then interrupted by the 1st European crisis of May 2010 and 2nd European crisis during the summer 2011. The red line also had a steep growth between the beginning of the sample and the early 2008, however, they incurred the losses just several months after French panel faced the losses, this can be explained by the geographical transfer of crisis have some lags. The Chinese system hit the bottom in March 2009 and start a slow recovery that is interrupted only during the 2nd European debt crisis.

[10] As There is no public data for BPCE (Group BPCE was founded in 2009) and Agriculture Bank of China (It was listed on the Hong Kong stock exchanges in July 2010) during the financial crisis 2007-2009, these two banks are not considered in this paper.

Figure 3.1: Cumulative system market capitalization returns of France panel and China panel.

Figure 3.2 and figure 3.3 show that QQ-plot of French panel and Chinese Panel. In line with the stylized fact of financial data, there are fat tails on the returns in both panels, therefore it is irrational to assume the returns of banks follow a multi-normal distribution, which justify my previous assumption that the returns follows a multi-t distribution. An interesting result found from these 2 figures is that French banks have much larger fat tails then Chinese banks, this may be explained by the French banks are more affected by the 2007-2009 financial crisis than Chinese banks do.

3.5.2 Full sample estimation result

I used the methodology introduced in Section 3.3 and section 3.4 to analyze the panels. TGARCH and DCC models are fitted on each bank over the whole sample period and in this section I report information on the parameter estimates, fitted series and check on the autocorrelations.

In table 3.2 I show all banks and the systems' parameter estimates of the TGARCH (left side) and the DCC (right side) models for both panel, note that the DCC is each bank's returns dynamic conditional correlation with the system returns. The TGARCH parameters do not fluctuate much, but for Chinese banks, they are less subject to the so called "leverage effect". The point estimates are in line with the typical TGRACH estimates, with slightly higher αs and γs together with lower βs implying a higher level of unconditional kurtosis. Turning to the

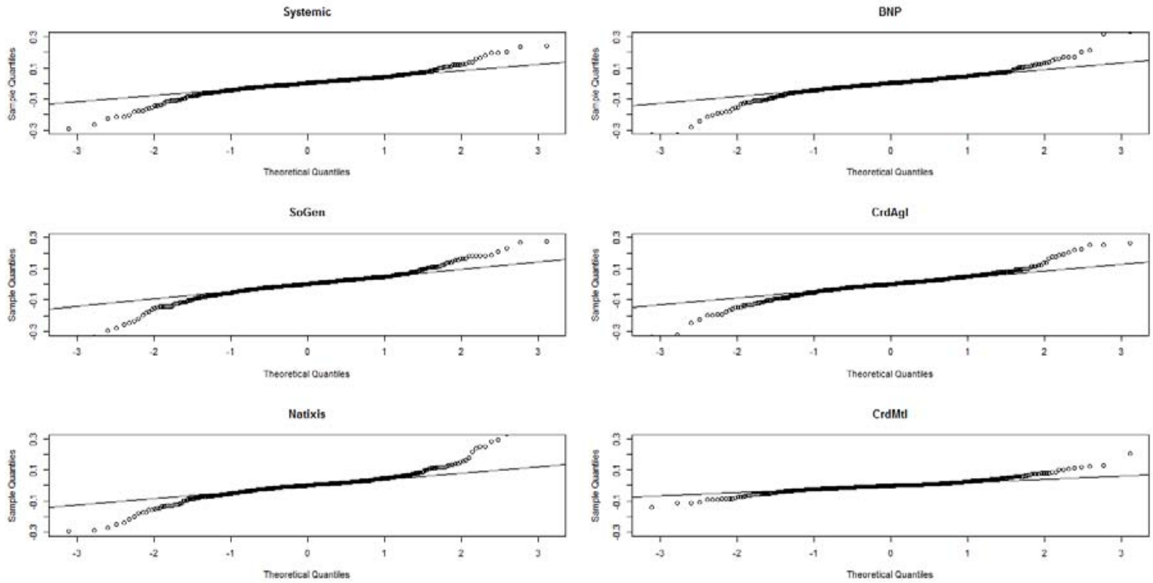


Figure 3.2: QQ-plot of market capitalization returns of French panel.

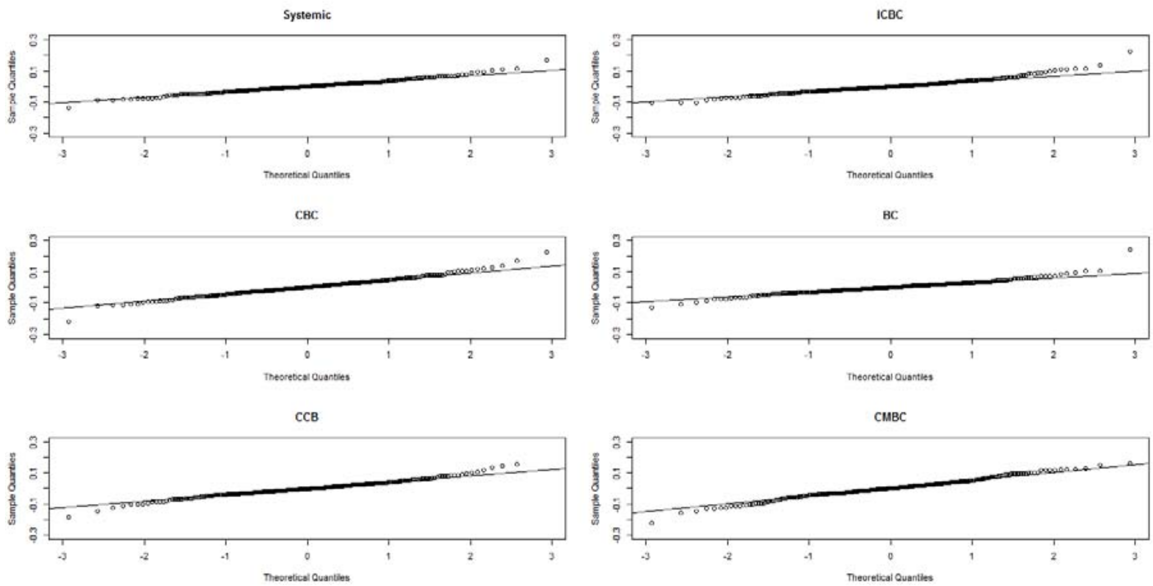


Figure 3.3: QQ-plot of market capitalization returns of Chinese panel.

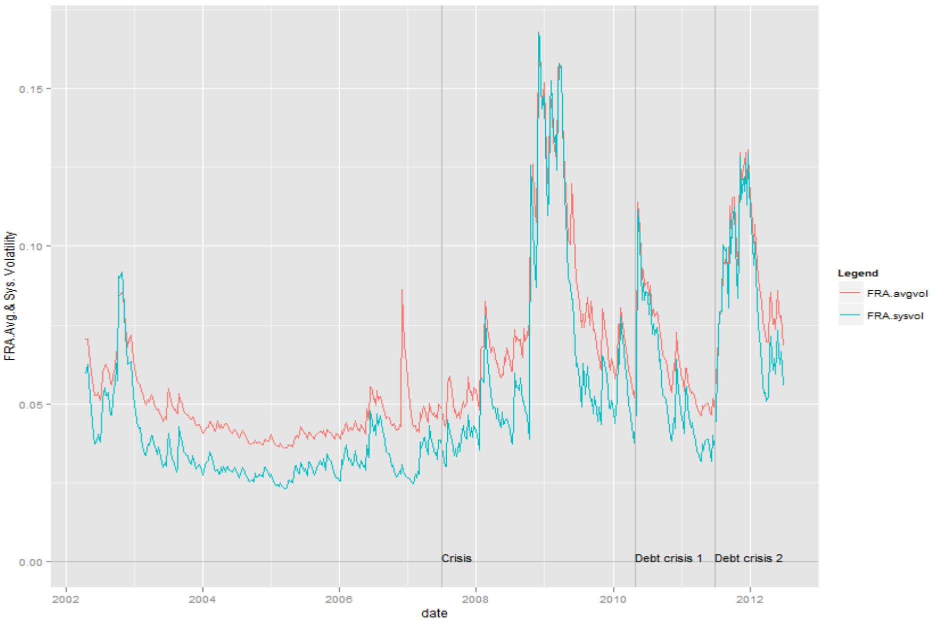
French Panel	α_G	γ_G	β_G	α_C	β_C
FRA. System	0.13	0.07	0.79		
BNP	0.12	0.10	0.77	0.03	0.96
SoGen	0.09	0.04	0.86	0.01	0.98
CrdAgl	0.05	0.04	0.91	0.05	0.94
Natixis	0.06	0.22	0.71	0.13	0.84
CrdMtl	0.04	0.03	0.92	0.01	0.98
Chinese Panel	α_G	γ_G	β_G	α_C	β_C
CHN. System	0.06	0.05	0.85		
ICBC	0.12	0.00	0.86	0.00	0.98
CBC	0.01	0.24	0.74	0.03	0.96
BC	0.11	0.00	0.83	0.00	0.98
CCB	0.02	0.00	0.97	0.30	0.50
CMBC	0.06	0.02	0.92	0.02	0.96

Table 3.2: TGARCH and DCC estimation results.

DCC, parameters are again close to the typical set of estimates. The diagnostic of autocorrelations is provided in Appendix.

I provide the dynamics of volatility in figure 3.4. The blue line represents the time varying system volatility and the red line represents the average time varying of individuals volatilities of all banks in each panels. In Chinese panel, the system volatility is always lower than the average volatility, this is due to the diversification effect among banks. Moreover both average&system volatilities are relative high during the crisis period to the post crisis period, however, the magnitude of the volatilities are much lower than the French banks. Note that, these two panels behave very differently during the European crisis period, this is because the European debt crisis has a huge direct impact on French banks, but has little impact for Chinese banks. In French panel, during the calm period 2003-mid 2007, both system and average volatilities stay at a low level meanwhile there is a big spread between average volatility and system volatility; this is due to diversification effect when market is in a good period. However, this spread narrows down and volatility level increases sharply when financial crisis starts, this is because of the co-movement of individuals returns and systemic return when “things” are going bad. Therefore, it will be useful to investigate the correlations of returns among banks.

Figure 3.5 displays average correlation by banks in each panel. In French panel, there is an obvious trend of increasing in average correlation from calm period to crisis period, which explains why the diversification effect plays little role during the financial crisis. This implies that the sub-additivity property of



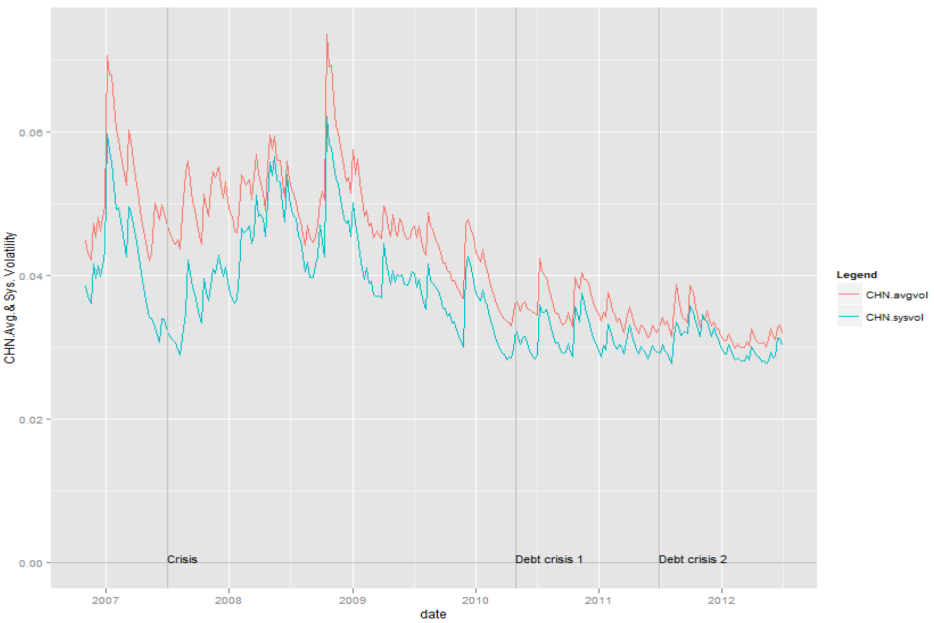


Figure 3.4: Average and system volatility of French system and Chinese system.

risk measures may not hold in some cases, in my view, especially for the systemic risk measures, because a large systemic risk often comes with a high correlation among the agent in the system. It's the same for Chinese panel, the level of correlation increases after the crisis, but the magnitude of correlation in China is larger than in French.

Figure 3.6 provides the average individual Value-at-Risk in French panel and Chinese panel.[11] The average individual VaRs are quite similar with the average volatility in term of evolution, which is in line with the model specification, the return is product of volatility and the innovation. In French panel, during the calm period, the level of individual VaR is low, because the volatility is low. It sharply increased after the crisis, the average VaR almost grew 500% from the bottom during 2004-2005 to the peak in March 2009, and attain a level around −20% of return, which is almost twice larger than the peak in Chinese panel at the same period.

3.5.3 Overall systemic risk and Systemic importance

Figure 3.7 shows that the overall systemic risk[12] (represented in black line and its level is on the right vertical axis), and its level was very low at the beginning of the crisis. Then the total systemic risk moved up significantly after the failure of Lehman Brother. After that, there has been a lot of panic in the market and the total systemic risk reached the peak around 140% of loss in market capitalization return as the overall systemic contribution in March 2009. Since the release of US SCAP around early May 2009, the total systemic risk decrease quickly and returned to the pre-Lehman level. The market has calmed down till the first round of European sovereign debt crisis in May 2010, after the Greece receiving the aid with 14.5 billions euros, the total systemic risk decreased. Almost a year later, June 13 2011, Standard & Poor's has downgraded Greek debt from B to CCC, the total systemic risk raised sharply and reached the peak in summer 2011 around 100% of loss in return after 2007-2009 financial crisis.

[11] Throughout the whole paper, the confidence level is set as 5%.

[12] Delta-Multi CoVaR when all banks in the system are in financial distress, as formulated in equations (3.10) and (3.11).

Figure 3.5: Dynamic Conditional Correlation of French system and Chinese system.

Figure 3.6: Average individual VaR of French panel and Chinese Panel.
Notes: The VaR is calculated at the 5% confidence level and the bootstrap is repeated 1000 times.

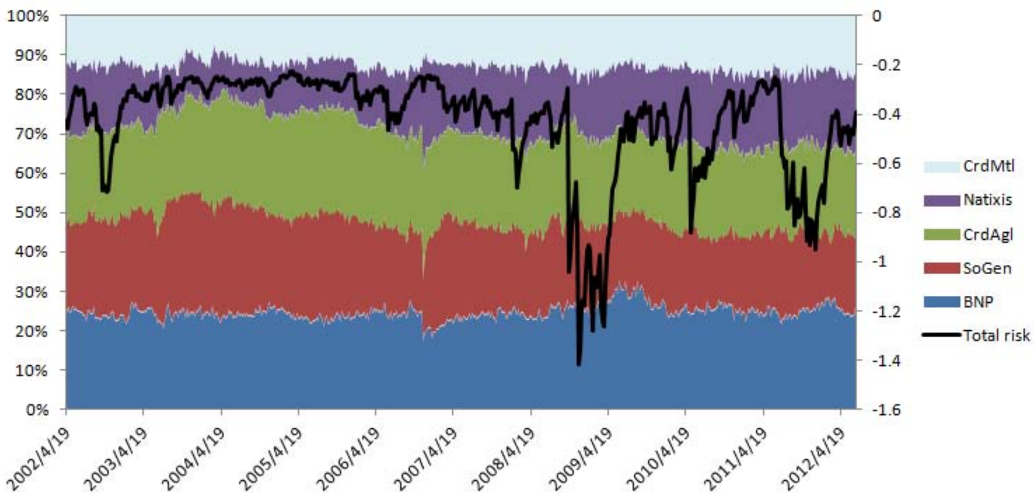


Figure 3.7: Marginal Contribution of Systemic Risk and Total Systemic Risk in French panel.

On the left side of the graph, the axis represents the importance of systemic risk of each financial institution in percentage.[13] The level of systemic importance during the whole sample is quite consistent, there is not much variation of weight of systemic importance among banks. BNP Paribas is the bank that have the most systemic important weight, Crédit Agricole and Société Générale are not too far from BNP Paribas. However, Natixis and Crédit Mutuel have the lowest systemic important weight in the system.

In table 3.3, I provide the summary statistics of the estimated Shapley-CoVaR series and Standard-CoVaR series for all banks in French panel. I first compute the mean of each series in different time horizon to have a general idea about the systemic importance among these banks, then I compute the standard deviation of each series to see the variation of the systemic importance for each bank. Note that the total risk in table 3.3 provides the benchmark for regulators to implement systemic risk measures individually. This benchmark gives the overall systemic risk contribution to the system when all banks are in distress. So the number in "mean column" of table 3.3 can be interpreted as, consider the number for BNP during the whole sample in Shapley-CoVaR case, BNP contributes, in average, -0.106 (with 5% confidence level) of return of market capitalization to the whole system; For standard-CoVaR measure, it contributes more than previous case as -0.285 of return of market capitalization to the whole system. Another interesting result shows in this table is that the more variation (high standard deviation) of the systemic risk series in time horizon comes with

[13]This is achieved by the additivity of Shapley value.

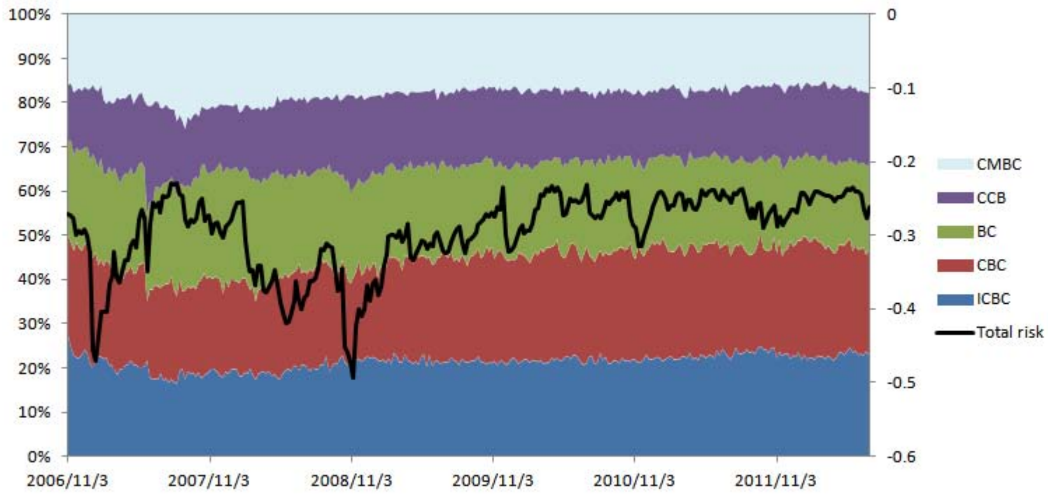


Figure 3.8: Marginal Contribution of Systemic Risk and Total Systemic Risk in Chinese panel.

the more systemic importance of that series.

Figure 3.8 represent the same information as in figure 3.7 for Chinese panel. Since the sample starts from the late 2006, the first peak for the total risk corresponds to the so called "Chinese correction" in early 2007. The second peak comes at the core of 2007-2009 financial crisis, but Chinese banking system doesn't suffer as much as French banking does, this is because the french banks have more connections with U.S. banks. This figure shows us that the most systemically important banks in China are the state banks for Industrial & Commercial Bank of China, China Construction Bank and bank of China. However, two private banks, Bank of Communications and China Merchants Bank have less systemic importance. In general, the magnitude of systemic risk is less in China than in France. Table 3.4 reports the summary statistics of the estimated Shapley-CoVaR series and Standard-CoVaR series for all banks in Chinese panel.

3.5.4 Additivity

Table 3.3 and 3.4 show an interesting result, the sum of individual $\Delta CoVaR$ is larger than the total systemic risk. This difference is quite small during the good period, this is because the correlations between banks are very small at that period. However, during the crisis period the correlations between banks increase sharply, and this implies that sum of individual $\Delta CoVaR$ is greater than the total systemic risk $\Delta CoVaR^{1,\dots,N}$. If the central planner regulate the systemic

Shapley-CoVaR					**Std-CoVaR**			
Whole Sample								
	Mean	Std	Min	Max	Mean	Std	Min	Max
BNP	-0.106	0.054	-0.048	-0.346	-0.285	0.178	-0.106	-0.989
SoGen	-0.096	0.043	-0.045	-0.327	-0.278	0.175	-0.108	-0.990
CrdAgl	-0.098	0.044	-0.054	-0.312	-0.254	0.185	-0.072	-0.990
Natixis	-0.070	0.038	-0.023	-0.223	-0.194	0.157	-0.016	-0.843
CrdMtl	-0.057	0.033	-0.017	-0.247	-0.086	0.078	-0.010	-0.429
Total Risk	-0.427	0.207	-0.227	-1.418	-0.427	0.207	-0.227	-1.418
Financial Crisis								
	Mean	Std	Min	Max	Mean	Std	Min	Max
BNP	-0.143	0.069	-0.071	-0.346	-0.366	0.216	-0.154	-0.989
SoGen	-0.119	0.060	-0.055	-0.327	-0369	0.212	-0.149	-0.990
CrdAgl	-0.124	0.058	-0.066	-0.312	-0.357	0.220	-0.112	-0.990
Natixis	-0.094	0.041	-0.034	-0.223	-0.275	0.178	-0.057	-0.843
CrdMtl	-0.075	0.044	-0.033	-0.247	-0.139	0.090	-0.041	-0.429
Total Risk	-0.556	0.267	-0.294	-1.418	-0.556	0.267	-0.294	-1.418
European Crisis								
	Mean	Std	Min	Max	Mean	Std	Min	Max
BNP	-0.130	0.048	-0.063	-0.237	-0.412	0.166	-0.176	-0.805
SoGen	-0.102	0.042	-0.045	-0.204	-0.397	0.160	-0175	-0.772
CrdAgl	-0.113	0.043	-0.054	-0.216	-0.389	0.163	-0.155	-0.767
Natixis	-0.098	0.033	-0.050	-0.171	-0.319	0.155	-0.106	-0.692
CrdMtl	-0.075	0.029	-0.028	-0.156	-0.138	0.068	-0.047	-0.299
Total Risk	-0.518	0.191	-0.246	-0.947	-0.518	0.191	-0.246	-0.947

Table 3.3: Shapley-CoVaR and Standard CoVaR in French panel.

Notes: The whole sample period is from 4/19/2002 to 6/29/2012, the financial crisis period is from 6/24/2007 to 4/23/2010 and the European crisis period is from 5/1/2010 to 6/29/2012. The α-scale of deviation to define the normal case is set $\alpha = 0.5$. Shapley-CoVaR refers to the methodology proposed in this paper, and Standard-CoVaR is proposes by Adrian and Brunnermeier (2011). Total risk refers to the systemic risk contribution when all these 5 banks are in distress, which is calculated by formulas (3.10) and (3.11). This is also the benchmark for regulators to implement systemic risk measures individually.

Shapley-CoVaR					**Std-CoVaR**			
Whole Sample								
	Mean	Std	Min	Max	Mean	Std	Min	Max
ICBC	-0.063	0.011	-0.038	-0.106	-0.206	0.047	-0.146	-0.358
CBC	-0.067	0.010	-0.043	-0.133	-0.179	0.039	-0.099	-0.350
BC	-0.062	0.014	-0.042	-0.106	-0.174	0.041	-0.124	-0.315
CCB	-0.049	0.012	-0.034	-0.105	-0.158	0.040	-0.090	-0.308
CMBC	-0.053	0.012	-0.037	-0.090	-0.167	0.044	-0.133	-0.330
Total Risk	-0.295	0.053	-0.228	-0.494	-0.295	0.053	-0.228	-0.494

Table 3.4: Shapley-CoVaR and Standard CoVaR in Chinese panel.

Notes: The whole sample period is from 11/3/2002 to 6/29/2012. The α-scale of deviation to define the normal case is set $\alpha = 0.5$. Shapley-CoVaR refers to the methodology proposed in this paper, and Standard-CoVaR is proposes by Adrian and Brunnermeier (2011). Total risk refers to the systemic risk contribution when all these 5 banks are in distress, which is calculated by formulas (3.10) and (3.11). This is also the benchmark for regulators to implement systemic risk measures individually.

risk based on standard $CoVaR$ that will punish the economy by over-regulating, since it will charge more than it should be. Furthermore, it will limit the volume of credit to the real economy, thus amplify the financial crisis and prolong the recovery of the economy. Note that the amount of over-regulation can be very significant, based on French system, the amount of over-regulation during the 2007-09 financial crisis and the European crisis are close to 100% of loss in return which is even larger than the total systemic risk at both periods around 50% of loss in return.

In the Shapley-CoVaR case of table 3.3 and 3.4, the sum of individuals' average measures is exactly equal to the total risk. This is in line with Tarashev, Borio, and Tsatsaronis (2010) and **?**, based on Shapley value methodology, this systemic risk measure has a additive property, which is desirable from an operational perspective, since prudential requirements can be a linear transformation of the marginal contribution if the systemic risk measure is additive. Therefore, it allows the Macro-Prudential tools can be implemented at individual levels. Note that, the "Distressed Insurance Premium" and the "Marginal Expected Shortfall" also have additive property, but these measures are focusing on the exposure of individual institution when a systematic shock take place, they do not measuring the contagion (externality) that an institution contribute to the financial system. However, the systemic risk measure proposed in this paper captures the externality that a specific institutions contribute to the whole system efficiently.

3.6 Regulation

The macroprudential framework deals with systemic risk in both the time dimension (procyclicality) and the cross-sectional dimension (contagion). I have developed a systemic risk measure to deal with the latter, now, let's focus on the former. Adrian and Shin (2011) shows that the procyclical leverage follows directly from the countercyclical nature of risk measure. To weaken the procyclicality effect in the economy, one can change the characteristic of the proportion coefficient[14] of the capital that intermediary holds per unit of risk measure. More specifically, I propose a counter-cyclical coefficient to attain this goal. Based on chapter 13 of Acharya and Richardson (2009) and the measure proposed above, the systemic capital charge (SCC) as a macroprudential tool could be written as:

$$\text{SCC} = \underbrace{\lambda}_{\text{Countercyclical coefficient}} \times Sh(v) \times \text{Value},$$

where $Sh(v)$ is the systemic risk contribution provide by multi-$\Delta CoVaR$ and Shapley value approach introduced in this paper.

This value can be market capitalization if one compute the measure by capitalization and it can be also the asset if the measure is derived by asset. The $Sh(v)$ is the systemic importance of systemic risk expressed in percent of value. However, how to choose the optimal counter-cyclical coefficient λ is still not clear to regulators and economists, this can be put into the agenda of the future research projects.

This macroprudential tool gives the incentives to financial institutions to limit their holding on systemic risk since capital reserves is costly, and counter-cyclical coefficient may weaken the procyclical effect in the economy.

3.7 Conclusion

In this paper I propose a new systemic risk measure to identify financial institutions' systemic importance. I use $Multi-CoVaR$ to calculate the total systemic risk and then use the Shapley value methodology to efficiently allocate total sys-

[14]Under the 1996 Market Risk Amendment of Basel capital accord, this coefficient is equal to three.

temic risk to each financial institution. The additivity property of the Shapley value ensures that the sum of each institution's Shapley value of systemic risk contribution is exactly equal to the $Multi - \Delta CoVaR$ of all the financial institutions in the system being in financial distress, hence the macroprudential policy can potentially be efficiently implemented based on this measure.

However, there are also some future research tasks on the field. First, on the strength of the methodology mentioned above, it is useful to construct a forward looking framework. With TGARCH and DCC models, it is possible to construct a forward looking conditional distribution, a forward looking individual risk is also feasible, since bootstrap technology could deliver this outcome. Second, what is the driving factors of systemic risk is needed, since if these driving factors can be identified, it would be very useful for financial institutions to control their systemic risk and make the financial market more stable. Third, how to design a countercyclical systemic capital charge can be also put into the agenda of the future research projects. Because this countercyclical systemic capital charge may weaken the procyclical effect in the economy.

Chapter 4

Debt maturities, Liquidity risk and Externality[1]

4.1 Introduction

The excessive reliance on short term funding by financial institutions has been considered as the root of the fragility of the financial system as well as the catalyst of the current financial crisis. Indeed many financial institutions were heavily exposed to refinancing risks in wholesale debt markets, and caused externalities on other institutions relying heavily on short term funds. As a result, individual funding decisions have an impact on other institutions, which increases the vulnerability of the latter to liquidity crisis.

We develop a simple 3 periods model where banks are run by homogenous bankers who have the same initial capital. Bankers have access to two types of debt contract, long term and short term. The optimal decision of bank pins down the debt maturity choice. On one hand, at the very beginning when banks choose to issue short term debt to finance their projects, they know that they may have to refinance it at interim stage that their refinancing cost will be high in case of liquidity crisis. Therefore, their expected profits drop when the probability of liquidity crisis increases. On the other hand, when they use long term debt, they do not need refinancing. However, they may misbehave due to the lack of incentives as they do not need to make interim repayment to creditors. But they know that creditors will only accept the contracts ruling out their misbehavior. This reduces the project size they are able to fund and their expected profit. So the basic tradeoff between debt maturities is between liquidity risk of short term

[1]This article is a joint work with Zhao Li.

debt and misbehavior of long term debt.

The project in our model is similar to the one in Holmstrom and Tirole (1997). Banks are run by homogenous bankers who have the same initial capital. Initially, bankers raise funds from the financial market to invest in a constant return to scale risky project. Bankers have a moral hazard problem: they can choose a high success probability project or a low success probability project with some private benefits. To avoid bankers' misbehavior, project sizes and profits are limited. The initial funding market is liquid, there are enough savers willing to supply short term or long term funds. These savers do not have outside investment opportunities other than consumption. As a result, bankers can issue initial debt contracts which just makes savers break even at 0 excess cost.

We introduce an interim aggregate macro shock in the model. This macro shock affects the maturity of bankers' projects. If the shock occurs, all bankers' projects yield nothing at the interim stage, but they can deliver the same expected return at the final stage. If the shock does not occur, all bankers' projects deliver immediately at the interim stage. At the initial stage, we assume all agents know the probability of occurrence of this macro shock. Bankers' maturity choice directly depends on this exogenously given macro shock.

If bankers choose to use short term debt to fund their initial project, they have to refinance their initial debt when a macro shock occurs. Hit by the macro shock, their projects yield nothing at interim stage. They have to wait until the final stage to get the potential return. They can only rely on financial markets to repay the initial debt, otherwise their projects would be wasted. However, they may expose themselves to a liquidity crisis– the savers' cash holding may not be sufficient to meet the total refinancing needs. This means some bankers have to resort to experts for refinancing. Experts are sophisticated investors who have outside investment opportunities. As a result, these bankers are facing an excess refinancing cost. Specifically, the likelihood of a liquidity crisis and the excess refinancing cost during the crisis are both determined by the total amount of short term debt that needs to be repaid.

If bankers choose to use long term debt to fund their initial project, they do not need to make the interim repayment. Therefore, they do not have any refinancing need at all. When hit by the macro shock, they just wait until the final stage. However, consider the case in which the macro shock does not occur. Some bankers may be lucky enough that their projects succeed and deliver a cash flow at the interim stage. Since bankers can freely manage this cash without any interference by creditors, they would steal the whole cash flow generated and run away. This misbehavior could happen between the interim stage and the final stage. Notice that when short term debt is used, bankers have to make interim repayment, thus they cannot misbehave in this way.

To characterize bankers' private optimal choice of maturities, we analyze the market equilibrium with short term debt and with long term debt. At the market equilibrium with short term debt, we found that: when the probability of a macro shock is small, 1. bankers issue a large amount of short term debt at equilibrium, 2. The probability of occurring a liquidity crisis is large, 3. the expected excess refinancing cost is large, 4. the equilibrium profit with short term debt is also large. At the equilibrium with long term debt, we found that the equilibrium profit is insensitive to the macro shock. In sum, when bankers make their private optimal maturities choice, they will choose short term debt when the probability of occurrence of a macro shock is small, and long term debt otherwise.

The comparative statics results with short term debt could offer some insights to understand the $2007-2009$ financial crisis. Before the crisis, banks relied heavily on short term debt because they expected the probability of facing a macro shock to be small, hence expecting small refinancing needs at the interim stage. Their equilibrium profit with short term debt was large. Consequently, they issued a lot of short term debt, knowing that the probability of an interim liquidity crisis and excess refinancing cost during the crisis would be high. However, the macro shock actually hit the economy, the liquidity crisis occurred due to huge amount of short term debt needed to be refinanced, bankers' refinancing cost soared.

Finally, the competitive equilibrium with short term debt may not be socially efficient. Bankers do not internalize the impact on the probability of the liquidity crisis and excess refinancing cost due to their private borrowing decisions. This may generate negative externalities to other banks and creditors. The social planner takes into account the surplus obtained by experts during the crisis refinancing and faces the same incentive compatibility constraint as bankers. When this constraint is binding, the social planner makes the same decision as bankers, there are no externalities. When this constraint is slack, social planner's choice of optimal debt is derived from her unconstrained program. Since she considers individual bankers' impact on probability of the liquidity crisis and excess refinancing cost, the socially optimal debt amount will be lower than the one privately chosen by bankers.

The rest of the paper proceeds as follows. Section 4.2 describes some related literature. Section 4.3 describes the baseline model. Section 4.4 characterizes the optimal debt contract and the market equilibrium when bankers use long-term debt financing and short-term debt financing, then studies bankers' funding choice when s macro shock is exogenously given. Section 4.5 identifies the negative externality caused by over-borrowing of short-term debt in the social case. Section 4.6 justifies some macroprudential regulation related to Basel 3 proposal. Section 4.7 concludes.

4.2 Literature relevance

Our work is related to several strands of literature.

It is first related to the contributions of liquidity risk in the financial system. In Diamond and Dybvig (1983) and Bryant (1980), investors may face a sudden preference shock, then a bank run may destroy the value of banking sector. More precisely, they consider a framework of a fractional reserve system, that banks finance profitable but illiquid projects. Lenders are uncertain about their future liquidity preference. Compared to the market economy, banks can efficiently pool resources together and insure the liquidity risk thus attain the social optimal solution. However, there is also another possibility a lender may have expectation that all the others may withdraw their funds together, thus the rational behavior for her is also to withdraw immediately. If all lenders no matter their actual liquidity preference are doing so, the bank becomes bankrupted. Morris and Shin (2001) have proposed a global game theory which can get rid of the multiplicity of equilibrium in classical model of bank runs. Furthermore, Rochet and Vives (2004) have proved that in a financial system, there does exist solvent but illiquid banks, which cause a coordination failure. More recently, Morris and Shin (2009) have shown the evidence that 2007-2009 financial crisis is a lack of liquidity, not a lack of capital, and showed that the illiquidity risk is the difference between the total credit risk and solvency risk. Both Rochet and Vives (2004) and Morris and Shin (2009) showed that liquidity risk may also comes from a macro shock on fundamental that banks need to raise new fund or fire-sale their asset to continue the project or meet their obligation to short term investors. In Holmstrom and Tirole (1998), they studied a case where whether the private liquidity supply can be efficient for the functioning of productive sector. When there is no aggregate liquidity shock, the efficiency can be attained if firms draw a credit line with intermediaries, which serves as a liquidity insurance to against refinancing risk. However, if an aggregate shock take place, the efficiency can only be achieved by government issuing treasury bond to provide public liquidity supply. **?** identified that the asymmetric information about asset value may lead liquidity freeze in secondary market. In a recent paper by Brunnermeier and Pedersen (2009) have explained how liquidity risk hurts a financial institution even a financial system as a whole. In our model, we first take this liquidity exogenously given, than we formulate this liquidity risk as the cash in the refinancing market is insufficient to roll-over initial debt, therefore banks have to access to a more expansive market to raise funding and lose a part of project's NPV.

Second, our work is related to the strand of literature that stresses disciplinary role of short term debt. Calomiris and Kahn (1991) shows how the issuance of demandable debt would reinforce bankers' monitoring incentives by making it easy for investors to penalize these bankers as soon as depositors began to lose confidence in the bank or its policies. Myers (1977), Flannery (1994) and Diamond and Rajan (2001) also pointed out the disciplinary role of short term debt in financial intermediaries.

The third part of literature to which our paper is related to is the pecuniary externality caused by excessive accessibility in financial markets. Geanakoplos and Polemarchakis (1986) argued that wealth distribution matters when markets are incomplete, and pecuniary externalities generate wealth effect that don't net out. In Caballero and Krishnamurthy (2003) and Lorenzoni (2008), as insurance market is missing, then ex-ante investment generates net pecuniary externalities, which implies competitive equilibrium is constrained inefficient and over-borrowing ex-ante. Gersbach and Rochet (2011) have shown that at market equilibrium, the investment decision of banks is socially inefficient due to banks do not internalize the impact of their decision on asset prices. The consequence of this is that banks may over-invest when they receive a positive macro shock and under-invest when they receive a negative macro shock, hence generating the procyclicality effect. There are some other papers stress on this issue as well, for example Korinek (2011) and Davila (2011) for a survey. Note that we do not have fire-sale externality in our model, however, the pecuniary externality take place since each banker acts atomistically when he raises fund ex-ant, he doesn't take into account his behavior may have some impacts on overall liquidity risk at interim date. Therefore, market equilibrium is not socially efficient.

Our paper is similar to Huberman and Repullo (2010) and Segura and Suarez (2012). Huberman and Repullo (2010) consider a model where banks could directly choose their project's riskiness after the borrowing is done. In their framework, short term debt may always dominate long term debt since it is the only way to secure funding. Our work differs from which that we introduce some uncertainties in economy[2], thus, finance by short term debt or long term debt is depend on the predictions of these uncertainties. Segura and Suarez (2012), the authors consider a dynamic maturity problem that banks could choose the maturity as well as the par value of debt. In their work, liquidity risk is due to the sudden change of preference of the patient creditors, thus results in a higher funding cost to banks. Moreover, market equilibrium is inefficient due to pecuniary externalities in the market for funds during crises and their interaction with banks' refinancing constraints, which is characterized by a too short maturity of debt. However, in our model, this inefficiency is captured by over reliance on short term debt[3] instead of short term debt it-self[4].

4.3 The model

The model has 3 dates $t = 0, 1, 2$. There is a single good in this economy which can be used for investing or consuming. There are two broad classes of agents: bankers and creditors. All agents are risk neutral and they do not discount future consumption.

4.3.1 Agents

Bankers and creditors take decisions in each period. Both classes of agents want to maximize their expected utility. A continuum of measure 1 of homogenous bankers own investment projects. They make the investment decision and they can misbehave. Bankers enter into economy at $t = 0$ and can consume at $t = 1, 2$. Creditors are divided into 3 groups: "patient" savers, "impatient" savers and experts. Each creditor is endowed with 1 unit of good. "patient" savers enter into economy at both $t = 0$ and consume at $t = 2$. "Impatient" savers can enter into economy at $t = 0$ consume at $t = 1$ and enter into economy at $t = 1$ consume at $t = 2$. A mass $\overline{E}$ continuum experts enters into economy at $t = 0$ and consume

[2] These uncertainties are maturity date of project and liquidity risk in interim date.
[3] This is exactly the over borrowing of short term debt.
[4] Note that short term debt have discipline role for bankers.

both at $t = 1, 2$. To summarize, bankers can borrow at $t = 0$ from both types of savers and experts, and can only borrow at $t = 1$ from “impatient”savers and experts.

4.3.1.1 Bankers

At $t = 0$, each banker has the same endowment k and possesses a constant return to scale risky investment project. Banker expands his investment size to i by borrowing $i - k$ from creditors. When the project matures, it delivers payoff:

$$\tilde{y}i = \begin{cases} yi & \text{With probability } p \\ 0 & \text{With probability } 1 - p \end{cases}.$$

Whether the project will succeed or fail is an idiosyncratic event among bankers. There is no liquidation value or bankruptcy cost in the model. There is an interim macro shock that affects the maturity of bankers' projects. If the shock occurs, all bankers projects yield nothing at the interim stage, but they can deliver the same expected return at the final stage. If the shock does not occur, all bankers' projects deliver immediately at the interim stage. At the initial stage, we assume all agents know the probability of occurring this macro shock. Bankers' maturities choice directly depends on this exogenously given macro shock. At $t = 0$, the probability of occurrence of the macro shock is α.

After borrowing the initial funds, bankers can misbehave. We assume there are two kinds of moral hazard problems for 1 unit of investment. First, bankers can choose a lower success probability $p - \triangle p$ in exchange of some private benefits b. Second, bankers can perform asset substitution at $t = 0$. Specifically, substitute the initial project to another one with 0 probability of success, but bankers enjoy a unit private benefit δpy at t = 2. δ measures bankers' “ability to substitute”. Notice that, asset substitution is possible only if bankers choose to borrow long term debt to finance the project, since “patient savers” do not ask bankers for interim repayment.

4.3.1.2 Creditors

We assume at $t = 0$ the economy works well. The measure of entering savers and experts are relatively large compared to the bankers' investment needs. Notice that, at $t = 0$, bankers issue debt only to the savers. Since they do not have outside investment opportunity, borrowing from them is "cheaper" than from experts. Savers decide whether to lend their unit good to bankers or consume it.

At $t = 1$, the measure of savers is an amount $\widetilde{M}$, which is random at $t = 0$. It has a cumulative distribution function H on the interval $[0, \overline{M}]$. We further assume that $\overline{M}$ is large relative to bankers' refinancing needs at $t = 1$.

Experts enter into the economy with superior outside investment opportunities. There are $\overline{E}$ of experts in total and $\overline{E}$ is relatively large compared with bankers' refinancing needs. Each expert can use her unit of good to invest in an outside project with a net return θ. This θ has a support $[0, \overline{\theta}]$ and heterogeneously distributed across experts. We assume that the population of experts with $\theta \leq \hat{\theta}$ has a differentiable and strictly increasing distribution function $F(\hat{\theta})$, with $F(0) = 0$ and $F(\overline{\theta}) = 1$. Define a function: $G(x) = F^{-1}[\frac{1}{\overline{E}}(x)]$ and G is also differentiable and strictly increasing defining on $[0, \overline{E}]$, with $G(0) = 0$ and $G(\overline{E}) = \overline{\theta}$.

4.3.2 Uncertainties and information

As already introduced, bankers' project maturity is affected by an exogenous macro shock. With probability $1 - \alpha$, the shock does not occur and with probability α, the shock occurs. If the shock does not occur, all bankers' projects immediately deliver return at $t = 1$. If it occurs, all bankers' project deliver nothing at $t = 1$ and will deliver the same expected return at $t = 2$ unless initial debt is repaid.

When bankers choose short term debt to fund their projects, they also expose themselves to a liquidity crisis when the macro shock occurs. Since the savers' cash holding is random and distributed on $[0, \overline{M}]$, it is possible that the total amount of short term debt need to be refinanced exceeds the total cash can be offered by the savers. Under this case, some bankers have to resort to experts for refinancing. From $t = 0$'s point of view, bankers can calculate the probability of this liquidity crisis. We will discuss it in details in the following section.

The only asymmetric information is our model is: bankers do not know exactly the return of each expert's outside investment opportunity. So, they can not offer contracts extracting all surplus from experts. Besides this, we can consider we are in a perfect information world.

4.3.3 Interim refinancing

In this subsection, we study a representative banker's interim refinancing decision in detail.

When short term debt is used, the economy may suffer from a liquidity crisis at $t = 1$. If the macro shock occurs, bankers need to refinance a total amount of debt D_1. $t = 1$'s savers could supply a total amount of liquidity M. The refinancing market is perfect liquid if $M \geq D_1$; it is illiquid if $M < D_1$. Under the latter case, bankers have to resort to experts for refinancing the remaining amount of debt $D_1 - M$. The liquidity crisis is more likely to happen either because bankers issue a large amount of short term debt ex ante or there are too few savers ex post.

Given a realization M of $\widetilde{M}$ (in the liquidity crisis case: $M < D_1$), Bankers refinance a proportion $1 - \gamma = \frac{M}{D_1}$ of the total debt D_1 from savers. For the other proportion $\gamma = \frac{D_1 - M}{D_1}$ of the total debt D_1, bankers have to resort to experts for refinancing.

From an ex ante point of view, the probability of liquidity crisis is:

$$\beta = Prob(\widetilde{M} < D_1) = H(D_1).$$

At $t = 0$, for 1 unit of debt that needs to be refinanced, a banker expects the proportion he will refinance from "impatient" experts is:

$$[1 - E(\tilde{\gamma} \mid \widetilde{M} < D_1)],$$

and the proportion he will refinance from experts is:

$$E(\tilde{\gamma} \mid \widetilde{M} < D_1),$$

in the liquidity crisis.

For conciseness, We don't consider the total market freezing case.

4.3.4 Debt Contracts

At $t = 0$, a representative banker can issue 2 kinds of debt contracts to finance his project. First, the banker can issue long term debt contract: borrow $i - k$ at $t = 0$; repay d_l at $t = 2$; creditors cannot claim anything until at the maturity $t = 2$.

Second, the banker can issue short term debt contract: borrow $i - k$ at $t = 0$, repay d_1 at $t = 1$. The cost of refinancing d_1 depends on the aggregate state. If there is no liquidity crisis, banker repays creditors ($t = 1$ savers) d_2^0 at $t = 2$. If liquidity crisis occurs, banker repays creditors ($t = 1$ savers and experts) d_2^1 at $t = 2$.

4.4 Equilibrium debt contract and maturity choice

To analyze bankers' funding choice, we begin by analyzing bankers' optimal debt contract. First, we study the equilibrium with long term debt contract by characterizing the optimal project size and maximum profit. Then, we analyze of equilibrium of using short term debt. To end this section, we present the bankers' equilibrium debt maturity choice. The main result we will present is: bankers make their optimal maturity choice depending on their ex ante perception on the probability of a macro shock. When bankers expect that there is a little chance of macro shock will happen, they will borrow a big amount of short term debt to fund their project in order to maximize their profit. However, once the macro shock takes place, the likelihood of a liquidity crisis is very high and refinancing cost become very high too.

4.4.1 Long term debt

When banker chooses to use long term debt to fund his project, he does not need to make any interim repayment. This leaves the banker a chance to implement asset substitution at $t = 0$,[5] therefore, at $t = 0$, a contract to prevent asset substitution is needed.

We consider an equilibrium "Substitution Prevent" debt contract. At

[5] This illustrates that long term debt lacks of disciplining role.

$t = 2$, the face value of his long term debt is d_l, thus a Substitution Prevent debt contract should satisfies:

$$r_l = yi - d_l \geq \delta yi.$$

In addition, we have to rule out the possibility that the banker "shirks" to choose a low success probability project. banker's incentive compatible constraint is:

$$pr_l \geq (p - \triangle p)r_l + bi,$$

which gives us: $r_l = yi - d_l \geq \frac{bi}{\triangle p}$.

Initially, "patient" savers are willing to lend banker $i - k$ if their lending IR satisfies:

$$pd_l \geq i - k.$$

Since savers behave competitively, their IR constraint will be binding: $d_l = \frac{i-k}{p}$. Denote $\rho_1 = py$, and $\rho_0 = p(y - \frac{b}{\triangle P})$, we state the first proposition to characterize long term debt equilibrium.

Proposition 5. *Under the assumption $A_1 : \delta\rho_1 > \rho_1 - \rho_0$, the profit maximizing size i^* funded by long term debt is:*

$$i^* = \frac{k}{1 - \rho_1 + \delta\rho_1}.$$

Banker's maximum profit is given by:

$$\Pi_l = (py - 1)i^* = \frac{(py - 1)k}{1 - \rho_1 + \delta\rho_1} = \frac{\rho_1 - 1}{1 - \rho_1 + \delta\rho_1}k.$$

.

Proof. Since "patient" savers have no outside investment opportunity and are risk neutral, the banker will get all NPV from the project, which is: $NPV = (py - 1)i$. Moreover, since the project has constant return to scale, banker will always choose a size as large as possible to maximize project's NPV. Since $\delta y > \frac{b}{\triangle p}$, the maximum size is obtained by making the Substitution Prevent constraint binding: $yi - d_l = \delta yi$. Inserting "patient" savers' binding IR $d_l = \frac{i-k}{p}$ into this equation gives us the maximum project size which rule out banker's double moral hazard: $yi^* - \frac{i^*-k}{p} = \delta yi^*$and $i^* = \frac{k}{1-\rho_1+\delta\rho_1}$. Inserting i^* into NPV gives the maximum profit. □

The economic intuitions are, 1.Asset substitution gives the banker higher private benefit than "shirking" ($\delta y > \frac{b}{\triangle p}$). To rule out the banker's ex post substitution, he should be given even more rent than to rule out his "shirking". Anticipate this, the "patient" savers have a even lower incentive to lend, which makes their IR even more tighten,

$$i^* = \frac{k}{1-\rho_1+\delta\rho_1} < \frac{k}{1-\rho_0}.$$

2. Notice that we have: $\frac{\partial i^*}{\partial \delta} < 0$ and $\frac{\partial \Pi_l}{\partial \delta} < 0$. δ measures the banker's "ability to substitute". If δ is high, it means the banker is very good at substituting. He can transfer more money into his own pocket. To prevent substituting from happening, higher δ leads to tighter IR of savers. The reason is that the banker must get a high compensation from substitution prevent contract when his substitution ability is high. This leaves a lower stake to savers. So, they are not willing to lend much. As a result, the debt amount and profit is low when banker's ability to steal is high.

4.4.2 Short term debt

When using short term debt, bankers can never implement any "asset substitution". The reason is that they have to make interim repayment at $t = 1$ when their project matures. The only money they had after making the repayment is their own profit. This shows that short term debt has a disciplinary role vis à vis the bankers. Creditors can prevent asset substitution by asking immediate repayment. However, using short term debt may result in exposing the bankers to an interim liquidity crisis. Remember that with probability β , the bankers have to resort to experts for refinancing their short term debt when macro shock occurs at $t = 1$.

We analyze a representative banker's profit maximizing program at $t = 0$.

We begin by formulating the program in a backward induction manner.

- At t=1

At $t = 1$, if the macro shock does not occur, banker's project succeeds with a probability p. In this case, he can repay d_1 to initial "impatient" savers and get the rest $yi - d_1$. With a probability $1 - p$, banker's project fails, both agents get nothing.

We focus on the case where macro shock occurs and banker needs to refinance his debt d_1, otherwise initial savers take over his project.

At $t = 1$,the randomness of $\widetilde{M}$ resolves, whose realization is M[6]. If $M > D_1$, the banker faces a perfectly liquid refinancing market. We assume that banker has all bargaining power against savers, since he knows that the savers do not have any outside option. Banker can refinance his debt d_1 by promising a repayment d_2^0 to the savers at $t = 2$. The savers' lending IR is given by:

$$pd_2^0 \geq d_1. \tag{4.1}$$

If $M < D_1$, the bankers face a market illiquidity (liquidity crisis). Bankers can only refinance a total amount M from $t = 1$'s savers. For the other part of debt $D_1 - M$, they have to resort to experts. Experts are financiers with outside investment opportunities, bankers must pay excess cost to refinance this proportion of debt. We characterize a market clearing excess cost paid by the bankers. Notice that the aggregate demand for liquidity from experts is $\Phi = \gamma D_1$. To clear the market, we have to set the supply of liquidity $S = \Phi$. Therefore, we have,

$$\gamma D_1 = \Phi = S = F(\hat{\theta})\overline{E}.$$

To understand this condition, first notice that the bankers do not know the net return of each expert's outside investment opportunity. Denote $\hat{\theta}$ as the clearing market "price" for liquidity (excess cost to bankers) offered by the bankers. Only experts with net opportunity return $\theta \leq \hat{\theta}$ are willing to supply their liquidity to bankers. Thus, the total liquidity supply is $S = F(\hat{\theta})\overline{E}$. Market

[6]Notice that, the state of world at $t = 1$ is characterized by this M, and we assume everyone knows it.

clears if $S = \Phi$.From the sector 3.1.2, we have: $\hat{\theta} = G(\gamma D_1)$.

In the case of liquidity crisis $M < D_1$, bankers Bertrand compete with each other for the liquidity from savers. The result of the Bertrand competition is that bankers also have to pay positive a excess refinancing cost to savers, which is exactly $\hat{\theta}$. As a result, bankers pay the same excess refinancing cost to savers and experts. Creditors' lending IR in liquidity crisis is:

$$pd_2^1 \geq (1 + \hat{\theta})d_1. \tag{4.2}$$

At $t = 2$, banker repays γd_2^1 to experts and $(1 - \gamma)d_2^1$ to savers. Their returns after project success in different scenarios are: 1. $r_1 = yi - d_1$ if macro shock does not occur; 2. $r_2^0 = yi - d_2^0$ if no liquidity crisis; 3. $r_2^1 = yi - d_2^1$ in liquidity crisis.

- At t=0

At $t = 0$, the banker borrows $i - k$ from the initial "impatient" savers[7]. The "impatient" savers' $t = 0$ lending IR is:

$$(1 - \alpha)pd_1 + \alpha d_1 \geq i - k. \tag{4.3}$$

Since the banker has no incentive to leave creditors any extra rents, all lending IRs are binding. Moreover, γ is a random variable $\widetilde{\gamma} = \gamma(\widetilde{M}, D_1) = \frac{D_1 - \widetilde{M}}{D_1}$ at $t = 0$.

Although the banker can not substitute asset when short term debt is used, he can still choose to "shirk". To prevent this, the banker's stake in the project should satisfy IC constraint at $t = 0$:

$$\begin{aligned} p[(1-\alpha)r_1 + \alpha(1-\beta)r_2^0 + \alpha\beta E(r_2^1 \mid \widetilde{M} < D_1)] \quad \geq \quad & (p - \triangle p)[(1-\alpha)r_1 + \alpha(1-\beta)r_2^0 \\ & + \alpha\beta E(r_2^1 \mid \widetilde{M} < D_1)] + bi. \end{aligned}$$

[7]This is because we have assumed that at $t = 0$, there is a large amount of saver in the market and bankers will never go to the experts to fund their projects.

The interpretation of the IC is as follow. At $t = 0$, the banker's expected return is: $(1-\alpha)pr_1 + \alpha(1-\beta)pr_2^0 + \alpha\beta pE(r_2^1 \mid \widetilde{M} < D_1)$ if he does not "shirk". The term $E(r_2^1 \mid \widetilde{M} < D_1)$ represents banker's expected return under liquidity crisis scenario. Since savers' $t = 1$ cash holding is a random variable $\widetilde{M}$, the occurrence of liquidity crisis is given by $\beta = prob(\widetilde{M} < D_1)$. The banker expects he will refinance his debt at an excess cost $E(\hat{\theta} \mid \widetilde{M} < D_1)$. As a result, the banker's expected return in liquidity crisis is: $pE(r_2^1 \mid \widetilde{M} < D_1)$. After some modification, IC constraint boils down to:

$$\rho_0 i = p(y - \frac{b}{\triangle p})i \quad \geq \quad (1-\alpha)pd_1 + \alpha d_1 + \alpha\beta E(\hat{\theta} \mid \widetilde{M} < D_1)d_1.$$

For simplification, we denote μ as the unit expected excess refinance cost:

$$\mu = E(\hat{\theta} \mid \widetilde{M} < D_1) = E[G(\tilde{\gamma}D_1) \mid \widetilde{M} < D_1] = \int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c,$$

where H_c is the distribution function of $\widetilde{M}$ conditional on $\widetilde{M} < D_1$. Based on these 2 equations, we can further simplify individual banker's IC:

$$\rho_0 i \geq (1-\alpha)pd_1 + \alpha d_1 + \alpha\beta\mu d_1. \tag{4.4}$$

It is obvious that refinancing during the liquidity crisis has a higher expected cost to the banker. Notice that the expected net interest rate demanded by the savers under no liquidity crisis scenario is: $\lambda_0 = E(\frac{d_2^0}{d_1} - 1) = \frac{1}{p} - 1$ compared to the case in liquidity crisis: $\lambda_1 = E[(\frac{d_2^1}{d_1} - 1)]$, we have:

$$\lambda_1 = E[(\frac{d_2^1}{d_1} - 1) \mid \widetilde{M} < D_1] \quad = \quad (\frac{1}{p} - 1) + \frac{1}{p}E(\hat{\theta} \mid \widetilde{M} < D_1) > \frac{1}{p} - 1 = \lambda_0.$$

From savers' IR constraints, we know that: $d_2^1 > d_2^0 > d_1$. The limited liability constraint for the banker is:

$$r_2^1 = yi - E(d_2^1 \mid \widetilde{M} < D_1) \geq 0. \tag{4.5}$$

At $t = 0$, a representative banker chooses optimal project size i to maximize expected profit:

$$\Pi_s = pyi - i - \alpha\beta\mu d_1 = (\rho_1 - 1)i - \alpha\beta\mu d_1.$$

Note that, the banker cannot obtain the entire NPV generated from the project $(\rho_1 - 1)i$. Under liquidity crisis scenario, banker faces a illiquid refinancing market. He expects that he has to refinance d_1 with an expected excess cost μ. At $t = 0$, this scenario occurs with a probability $\alpha\beta$. A part of the surplus has to be given to creditors. The reason is that the banker cannot offer perfect discrimination contracts to extract all surplus under liquidity crisis scenario.

A representative banker maximizes Π_s subject to constraints (4.1) to (4.5) and taking μ, β as given. To solve banker's program, first note that IR constraints (4.1) to (4.3) are binding, since the banker is not willing to leave creditors any extra rents. The banker's profit maximizing program is:

$$\begin{aligned} max\ \Pi_s &= \rho_1 i - i - \alpha\beta\mu d_1 \\ st. \quad & \rho_0 i \geq (1-\alpha)pd_1 + \alpha d_1 + \alpha\beta\mu d_1 \\ & yi - E(d_2^1 \mid \widetilde{M} < D_1) \geq 0 \\ & d_1 = \frac{i-k}{\alpha + (1-\alpha)p}. \end{aligned}$$

To solve the program, we introduce a lemma and a proposition.

Lemma 6. *Under A_2: $\rho_1 - 1 > max[\underset{\alpha}{max} P_1(\alpha)\beta\mu\ ,\ \rho_0 + \frac{\overline{\theta}}{2} - p]$, A_3: $\rho_1 - \rho_0 < 1$. The representative banker will choose the maximum project size. The size is given by making his individual IC constraint binding. Moreover, the banker's limited liability constraint is strictly satisfied when his IC constraint is satisfied.*

Proof. See Appendix B.1. □

Proposition 7. *Following Lemma 6, given an exogenous value of α, the representative banker's profit maximizing program in short term debt has a unique solution:*

1. $i^{**} = \frac{A(\beta,\mu)}{A(\beta,\mu)-\rho_0}k$,

2. $\Pi_s = \frac{\rho_1-\rho_0}{A(\beta,\mu)-\rho_0}A(\beta,\mu)k - k$, *where* $A(\beta,\mu) = 1 + P_1(\alpha)\beta\mu$ *and* $P_1(\alpha) = \frac{\alpha}{\alpha+p(1-\alpha)}$,

3. *the optimal size and profit are both decreasing with β and μ.*

Proof. See Appendix B.2. □

The intuition is as following. Individual agent acts atomistically. At $t = 0$, a banker makes optimal decision taking all other bankers' choice as given. The optimal decision of the banker depends on three uncertainties: 1) Macro shock; 2) liquidity crisis; 3) expected excess refinancing cost in liquidity crisis. The first uncertainty is determined by exogenous probability of a macro shock, while the two latters are due to the randomness in savers' cash supply at interim date. When making private optimal decision, all these three uncertainties are considered as given for the banker. Furthermore, $A(\beta, \mu) - \rho_0 > 0$ and the multiplier $\frac{A(\beta,\mu)}{A(\beta,\mu)-\rho_0}$ is greater than 1, which shows that the banker can lever her wealth, and $A(\beta, \mu)$ is the marginal cost of the banker.

With the individual banker's optimal solution, we can characterize the competitive equilibrium in short term debt. To begin with, notice that β, μ[8] are:

$$\beta = H(D_1)\ ,\ \mu = E[\hat{\theta} \mid \widetilde{M} < D_1] = \int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c.$$

We then introduce the definition of the market equilibrium in short term debt in our model:

- ## Competitive market equilibrium in short term debt[9]

Definition 8. Given an exogenous parameter α , and the function F and G, an equilibrium in short term debt is a tuple $(I^e, (\beta^e, \mu^e, \overline{\gamma}^e))$ satisfies:

1. Each banker chooses $i^e = \frac{A(\beta^e,\mu^e)}{A(\beta^e,\mu^e)-\rho_0}k$ as the equilibrium project size funded by short term debt. His equilibrium profit in short term debt is: $\Pi_s^e = \frac{\rho_1-\rho_0}{A(\beta^e,\mu^e)-\rho_0}A(\beta^e, \mu^e)k - k$.

2. The aggregate project size at $t = 0$ is:

[8]Under individual banker's private optimal program, the banker takes the total amount of short term debt issued at $t = 0$ as given. Each individual banker choose his best response, namely his optimal investment size, given all other bankers' choice of debt. In our model, each banker behave atomistically, thus his individual choice of investment size or his individual level of short term debt $d_1$will not affect the total amount D_1 prevailed in market.

[9]This equilibrium is a simple Cournot-Nash Equilibrium. At $t = 0$, each bankers calculate a best response function to all other bankers' decisions. Since the atomistic assumption, all other bankers' decisions contribute a total short term debt amount D_1 . The best response function is: $i^{**} = \frac{A(\beta(D_1),\mu(D_1))}{A(\beta(D_1),\mu(D_1))-\rho_0}k$. To solve the equilibrium, we take into account I^{**} and $\beta(D_1)$, $\mu(D_1)$ and the expected clear-market condition $\mu = E[\widetilde{\gamma}G(\widetilde{\gamma}D_1) \mid \widetilde{M} < D_1]$ in $t = 1$.

$$I^e = \frac{A(\beta^e, \mu^e)}{A(\beta^e, \mu^e) - \rho_0} K.$$

The aggregate face value of debt at $t = 1$ is:

$$D_1^e = \frac{P_1(\alpha)}{\alpha}(I^e - K).$$

"Impatient" savers lend bankers $I^e - K$ at $t = 0$ in exchange for repayment $D_1^e = \frac{P_1(\alpha)}{\alpha}(I^e - K)$[10] at $t = 1$. Probability of liquidity crisis is $\beta^e = H(D_1^e)$.

3. At $t = 1$, the refinancing market clears in a way that, without liquidity crisis, savers supply D_1^e in exchange for $t = 2$'s repayment $D_2^{0e} = \frac{D_1^e}{p}$. In liquidity crisis, savers and experts jointly supply an amount of D_1^e liquidity in exchange for $t = 2$'s repayment $D_2^{1e} = \frac{(1+\mu^e)D_1^e}{p}$; experts supply an amount $\overline{\gamma}^e D_1^e$, savers supply an amount $(1 - \overline{\gamma}^e)D_1^e$, with $\overline{\gamma}^e = E(\frac{D_1^e - \widetilde{M}}{D_1^e}|M < D_1^e)$. Excess refinancing cost in liquidity crisis $\mu^e = \int_0^{D_1^e} G(\gamma(\widetilde{M}, D_1^e)D_1^e)dH_c$.

We propose the main result in this section:

Proposition 9. *The market equilibrium in short term debt characterized by tuple $(I^e, (\beta^e, \mu^e, \overline{\gamma}^e))$ exists and is unique.*

Proof. See appendix B.3. □

The comparative static analysis of change in probability of a macro shock on equilibrium outcomes are summarized in the following proposition:

Proposition 10. *Comparative statics: the short term debt equilibrium outcomes: D_1^e, β^e, μ^e response to the change in probability of a macro shock α is as follows:*

1. $\frac{\partial D_1^e}{\partial \alpha} < 0$*: if α decreases, the equilibrium short term debt level at $t = 1$ increases;*

2. $\frac{\partial \beta^e}{\partial \alpha} < 0$ *: if α decreases, the equilibrium probability of liquidity crisis increases;*

3. $\frac{\partial \mu^e}{\partial \alpha} < 0$*: if α deceases, the equilibrium expected excess cost to refinance* 1 *unit of debt in crisis increases.*

[10] Note that, there is mass 1 bankers, thus $K = k$, $i^e = I^e$ and $D_1^e = d_1^e$

Proof. See appendix B.4. □

Note that, $\alpha + (1-\alpha)p$ characterizes the probability of all states at $t = 1$ that a banker has to repay the initial debt. Within these states, α denotes the probability that a banker needs to repay the debt through refinancing, which incurs a expected excess cost if liquidity crisis occurs. Thus, $P_1(\alpha) = \frac{\alpha}{\alpha+(1-\alpha)p}$ denotes the probability that banker has to repay the initial debt with a higher expected cost. As a consequence, the macro shock α has a direct effect on $P_1(\alpha)$, which implies that the bankers will borrow a lot of short term debts when they observe a small macro shock α, since the probability of refinancing their debt at interim date is low. Meanwhile, an indirect effect is characterized by a large D_1^e gives a large probability of liquidity crisis $\beta^e$and a large expected excess refinance cost μ^e. Given the fact that the direct effect surpass the indirect effect, we can obtain the following results.

Corollary 11. *If $\widetilde{M}$ is uniformly distributed on $[0, \overline{M}]$ and F is uniformly distributed on $[0, \overline{E}]$, we have:*

1. $\frac{d^2 D_1^e}{d\alpha^2} > 0$

2. $\frac{\partial \Pi_s^e}{\partial \alpha} < 0$*, If α decreases, the overall effect on individual banker's equilibrium profit in short term debt is Π_s^e increases.*

3. $\frac{\partial i^e}{\partial \alpha} < 0$*, if α decreases, the individual banker will issue more short term debt at $t = 0$, to expand their project size.*

4. $\frac{\partial I^e}{\partial \alpha} < 0$*: since we assume that there is mass 1 of bankers, I^e and i^e coincides.*

Proof. By directly taking derivatives.

The intuition is that a decreasing in α will have direct positive effect on Π_s^e. When α becomes smaller, directly $P_1(\alpha)$ decreases. It means across all $t = 1$ states that the probability of paying a high expected cost state decreases, which implies that the project size and profit increase. The indirect effect of a decreasing in α on i^e and Π_s^e works through β^e and μ^e. From proposition 10, when α decreases, both β^e and μ^e increases. This says that if a macro shock had taken place, they will face a high probability of liquidity crisis as well as a high excess refiance cost. As the indirect effect is dominated by direct effect, these results hold.

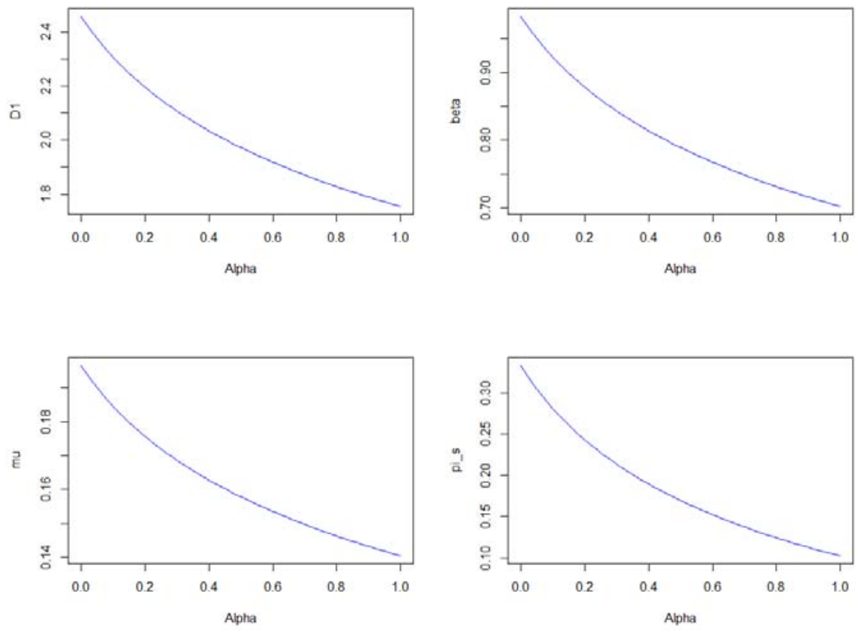


Figure 4.1: Comparative statics in the short term debt equilibrium.
Notes: Here we set $K = k = 1$, $\rho_1 = 1.1, \rho_0 = 0.7$. $p = 0.95$, $\overline{\theta} = 0.4$, $\overline{M} = 2.5$ and $\overline{E} = 2.5$. Note that all these values satisfies the assumption A_2 A_3 and A_4.

4.4.2.1 A numerical example

We now give a numerical example of proposition 10 and corollary 11.

The main theoretical foundation of the numerical result in given by proposition 9. At equilibrium, $\Sigma(D_1^e) = \frac{P_1(\alpha)K}{\alpha}[\frac{1+P_1(\alpha)\frac{\overline{\theta}}{2\overline{ME}}(D_1^e)^2}{1+P_1(\alpha)\frac{\overline{\theta}}{2\overline{ME}}(D_1^e)^2-\rho_0} - 1] - D_1^e = 0$, this gives us when α decreases D_1^e increases, we can see it very clearly in the first graph. Given the parameter values and distribution assumptions, we have: $\beta^e(\alpha) = \frac{D_1^e(\alpha)}{\overline{M}} = \frac{D_1^e(\alpha)}{2.5}$ and $\mu^e(\alpha) = \frac{D_1^e(\alpha)\overline{\theta}}{2\overline{E}} = \frac{0.4}{2*2.5}D_1^e(\alpha)$, after the arrangement and insert into $\Sigma(D_1^e) = 0$, which implies both β^e and μ^e are decreasing in α, as figure 4.1 shows. Further, we have: $A(\beta^e, \mu^e) = 1 + P_1(\alpha)\frac{\overline{\theta}}{2\overline{ME}}D_1^{e2} = 1 + \frac{\alpha}{\alpha+(1-\alpha)*0.95}\frac{0.4}{2*2.5*2.5}D_1^{e2}$. Moreover, we have:

$$
\begin{aligned}
\Pi_s^e &= \frac{\rho_1 - \rho_0}{A(\beta^e, \mu^e) - \rho_0}A(\beta^e, \mu^e)k - k \\
&= \frac{1.4 - 0.7}{1 + \frac{\alpha}{\alpha+(1-\alpha)*0.95}\frac{0.4}{2*2.5*2.5}D_1^{e2} - \rho_0}[1 + \frac{\alpha}{\alpha + (1-\alpha)*0.95}\frac{0.4}{2*2.5*2.5}D_1^{e2} - \rho_0] - 1.
\end{aligned}
$$

By this equation and last graph, at equilibrium, the profit is decreasing in α.

4.4.3 Market equilibrium maturity choice

To end this section, we analysis a representative banker's equilibrium debt maturity choice. We consider a case in which both debts are available, a representative banker makes his private choice between the two maturities of debt.

Proposition 12. *Under all the assumptions we make, the equilibrium maturities choice can be characterized by*

1. *Long term debt can never completely dominate short term debt for all* $\alpha \in (0, 1)$.

2. *If* δ *is large than the threshold* δ_2*,* $\delta_2 < \delta < 1$*: short term debt strictly dominates long term debt, for all* $\alpha \in (0, 1)$.

3. *If* δ *belongs to an interim interval,* $\delta_1 < \delta < \delta_2$*: there is a critical value* $\alpha_c^e \in (0, 1)$ *of the probability of macro shock at* $t = 1$*. A representative banker will choose long term debt if* $\alpha > \alpha_c^e$ *and short term debt if* $\alpha < \alpha_c^e$ *at* $t = 0$*, he knows the exogenous given probability* α*.*

Proof. For δ_1, δ_2 and $\alpha^c$and the proof, see appendix B.6. □

For a representative banker, using long term debt means that the substitution prevent contract is signed ex ante, and his project size and profit are insensitive to α. Using short term debt, he exposes himself to interim liquidity crisis and faces the risk of incurring excess cost of refinancing when there is macro shock. His equilibrium profit in short term debt is decreasing in α, which makes short term debt especially tempting for him when α is small. On the other hand, we have an opposite effect when α is large.

4.4.3.1 A numerical example(continued)

Funding choice:

Equilibrium profit in Short term debt:

$$\Pi_s^e = \frac{1.1-0.7}{1+\frac{\alpha}{\alpha+(1-\alpha)*0.8}\frac{0.4}{2*2.5*2.5}D_1^{e2}-\rho_0}[1 + \frac{\alpha}{\alpha+(1-\alpha)*0.8}\frac{0.4}{2*2.5*2.5}D_1^{e2}] - 1$$

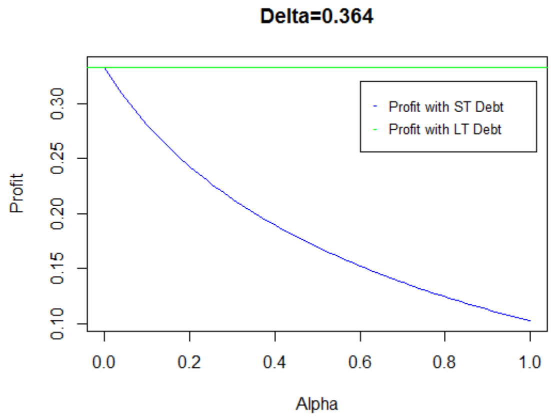


Figure 4.2: Long term debt can never completely dominate short term debt.
Notes: Equilibrium long term debt: $\delta = 0.364$. This graph coincides the first point in proposition 12, the maximum profit of using long term debt is achieved when assumption 1 holds with equality. In this case, long term debt dominates short term debt for all $\alpha \in (0, 1)$.

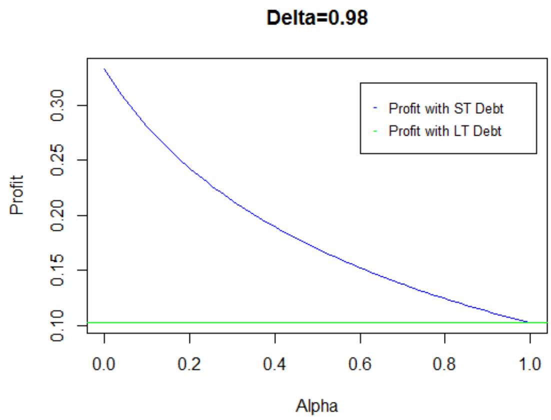


Figure 4.3: short term debt strictly dominates long term debt.
Notes: Equilibrium long term debt: $\delta = 0.98$. This graph illustrates the second point in proposition 12, which tells us that if δ is too high, short term debt will always dominates short term debt.

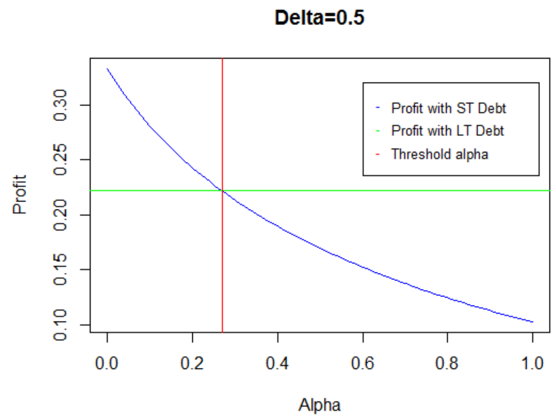


Figure 4.4: Intermediate case.
Notes: Equilibrium long term debt:$\delta = 0.5$. The last graph shows that if macro shock is relatively small (large) short term debt (long term debt) is better than long term debt (short term debt).

4.5 Social Welfare, externalities and funding choice

In this section, we introduce a social planner whose objective is to choose the optimal debt maturity and level to maximize economy's total surplus. At $t = 0$, social planner receives the same information of α as individual bankers do. She then chooses maturities by comparing total expected social surplus under these two kinds of debts. When each program is launched, we assume that social planner faces the same constraints as individual bankers. The result shows that the competitive equilibrium in short term debt may be socially inefficient due to the pecuniary externalities.

4.5.1 Social planner and externality

When using long term debt, there will be no inefficiency. Since the only agents who obtain surplus are bankers, and they do not generate any externalities. Thus, the market equilibrium in long term debt is also socially efficient under the condition that the social planner also completely rules out the possibility of asset substitution.

When using short term debt, different from individual bankers' perspective, social planner does consider that individual bankers' behavior will affect the probability of liquidity crisis as well as the expected excess refinancing cost in crisis. When short term debt is used, both bankers and interim creditors will appropriate some surplus. The social planner's objective function is the sum of bankers' profit and creditors' surplus.

Each Expert provided her 1 unit of money to banks in crisis obtains the difference between the clearing-market excess cost and her individual outside investment opportunity θ. Each savers can get expected surplus $E(\hat{\theta} \mid \widetilde{M} < D_1)$, since they don't have outside investment opportunities. The experts' expected total surplus is:

$$U = \alpha H(D_1) E\{\int_0^{\tilde{\gamma} D_1} [G(\tilde{\gamma} D_1) - G(x)]dx + (1-\tilde{\gamma}) D_1 G(\tilde{\gamma} D_1) \mid \widetilde{M} < D_1\},$$

and bankers' expected total profit is expressed as:

$$\Lambda = (\rho_1 - 1)I - \alpha H(D_1) E[G(\tilde{\gamma} D_1) \mid \widetilde{M} < D_1] D_1.$$

The natural objective function of the social planner is :

$$W = U + \Lambda,$$

and, the social planner has to consider the IC constraint to prevent bankers' misbehavior:

$$\rho_0 I \geq [\alpha + (1-\alpha)p] D_1 + \alpha H(D_1) E[G(\tilde{\gamma} D_1) \mid \widetilde{M} < D_1] D_1.$$

There is an important difference from the individual banker's program: when social planner consider her social welfare maximization program, she internalizes the effect of individual bankers' debt on the crisis probability $\beta = H(D_1)$ and expected excess cost in crisis $\mu = E[G(\tilde{\gamma} D_1) \mid \widetilde{M} < D_1]$.

Therefore the social planner's program can be written as:

$$\begin{aligned}
\max_I W &= \Lambda + U \\
&= (\rho_1 - 1)I - \alpha H(D_1)E[G(\widetilde{\gamma}D_1) \mid \widetilde{M} < D_1]D_1 \\
&\quad +\alpha H(D_1)E\{\int_0^{\widetilde{\gamma}D_1}[G(\widetilde{\gamma}D_1) - G(x)]dx + (1-\widetilde{\gamma})D_1G(\widetilde{\gamma}D_1) \mid \widetilde{M} < D_1\} \\
\text{s.t.} &\quad \rho_0 I \geq [\alpha + (1-\alpha)p]D_1 + \alpha H(D_1)E[G(\widetilde{\gamma}D_1) \mid \widetilde{M} < D_1]D_1,
\end{aligned}$$

where $D_1 = \frac{I-K}{\alpha+(1-\alpha)p}$, $\widetilde{\gamma} = \gamma(\widetilde{M}, D_1)$. We now solve the program and present the following proposition to characterize the social optimal choice of I^o.

Lemma 13. *Following our assumption on distribution function:H and F is uniformly on $[0, \overline{M}]$ and $[0, \overline{\theta}]$, the social planner's objective function is concave in its argument I.*

Proof. See Proof B.6. □

Solve the program, we present the following proposition:

Proposition 14. *For an exogenous macro shock α, compare to the total market equilibrium of size I^e, the social planner will choose a level I^o entails the following properties:*

1. *When IC binds , social planner will choose exactly the same total short term debt volume as in market equilibrium, there are no externalities $I^o = I^e$.*

2. *When IC does not bind, social planner will choose $I^o < I^e$ as the interior solution of her unconstrained maximization program, there are pecuniary externalities.*

3. *There exists a critical value of α^* . When $\alpha < \alpha^*$ $I^o = I^e$ there is no externalities , social planner makes the same choice as individual banker. When $\alpha > \alpha^*$, $I^o < I^e$, due to the pecuniary externalites, social planner chooses a smaller size. There is over-borrowing of short term debt.*

Proof. See Proof B.7 □

The intuition for this result is: depending on the variation of α, when IC binds, the social planner has to chose the project size given by IC. It exactly

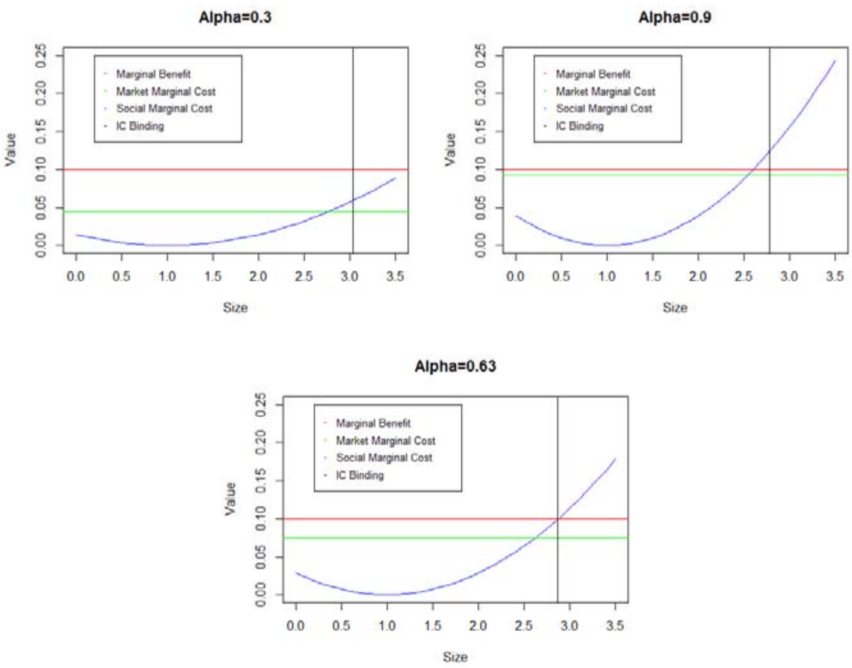


Figure 4.5: Numerical example in social welfare case.

coincides the the market equilibrium. When IC is slack, the social planner will choose the project size as the interior solution of her unconstrained program. And the reason for some values of α, which make the social planner's IC constraints are not binding is that she considers "more" costs than an individual banker does. Her unconstrained program has interior solutions, while the bankers' unconstrained program only gives him the corner solution. The extra costs considered by social planner changes her optimization program to a concave case from a linear program in market equilibrium.

For α small, there will be no externalities. The reason is that even if social planner does consider social marginal costs, which is different from the individual bankers, her project size is always bounded by IC. For some large α, given by the intersection of marginal gain and marginal cost, social planner's solution I^o under unconstrained program and market solution's I^e is different. Therefore, there are externalities, and IC is not binding for these α.

Numerical example continued:

Notice that market equilibrium is not Pareto efficient. The reason is that short term debt does create deviations in the social planner's view, since social planner considers individual banker's short term debt's contribution on liquidity risk and excess refinancing cost. Under market equilibrium, individual bankers do not internalize their individual debt's impact on liquidity risk and excess refinancing cost. As a result, they over use short term debt up till their individual IC constraint is binding. However, social planner takes into account the whole cost of short term debt, higher marginal cost representing marginal increasing in liquidity risk and marginal increasing in excess refinancing cost. Thus, social planner, under some circumstance, will not use short term debt up to the level given by IC binding, which gives a lower level of short term debt.

4.5.2 Social optimal maturity choice

In this section, we consider social planner's maturity choice problem.

Proposition 15. *For given α, we characterize the social optimal maturities choice by*

1.*If $A_1 : \delta < \frac{\rho_1 - \rho_0}{\rho_1} = \delta_1$ holds, long term debt can never dominates short term debt.*

2. *If δ is large than a $\delta_2^{'}$, $\delta_2 < \delta < 1$: for a social planner short term debt strictly dominates long term debt, for all $\alpha \in (0, 1)$, compared to the market equilibrium case, we have: $\delta_2^{'} > \delta_2$.*

3. *If δ belongs to an interim interval, $\delta_1 < \delta < \delta_2^{'}$: there is a critical value $\alpha_c^o \in (0, 1)$ of the probability of macro shock at $t = 1$. Social planner will choose long term debt if $\alpha > \alpha_c^o$ and short term debt if $\alpha < \alpha_c^o$. Moreover, we have $\alpha_c^o > \alpha_c^e$. For $\alpha \in [\alpha_c^e, \alpha_c^o]$, social planner will use short term debt while individual banker uses long term debt. If $\alpha_c^e < \alpha^*$, externalities do not affect maturities choice.*

Proof. Directly follow Proposition 12 and 14. □

4.6 Regulation

After 2007-2009 financial crisis, economists and regulators have realized the importance of macroprudential policy, which aims to address two dimensions of system-wide risk, first, the evolution of system-wide risk over time - the "time dimension" and second, the distribution of risk in the financial system at a given point in time - the “cross-sectional dimension.” This paper gives an insight of regulation on the cross-sectional dimension of macroprudential policy, that banker’s over borrowing of short term debt may cause externalities to other banks. To mitigate this effect, Perotti and Suarez (2011) have proposed a Pigouvian tax on the short term debt to internalize this externality and reduce the total size of the short term debt on the market to attain to the socially optimal level. However, the implementation of this externality correction tool is effective only in the case that the externalities do exist, otherwise, the banking sector is punished for no economic reason and constraint the banks’ credit volume, hence gives a negative impact to the real economy.

4.7 conclusion

We have a model where banks choose their debt maturity structures, weighting short term against long term debt. The basic tradeoff between these two different debts is liquidity risk of short term debt versus misbehavior of long term debt. When using short term debt, at the competitive equilibrium, 1) probability of an interim liquidity crisis, 2) the expected excess refinancing cost during the crisis, 3) bank’s profit, decrease with the probability of the macro shock. In contrast, when using long term debt, banks are insensitive to the macro shock. Then we found, at the competitive equilibrium, bankers prefer short term debt (long term debt) to long term debt (short term debt) when the probability of a macro shock is low (high).

We also show that the competitive equilibrium with short term debt is not socially efficient, bankers borrow too much since they do not internalize their borrowing decision on the probability of having a liquidity crisis and on the refinancing cost during the crisis. So, in order to attain the social optimum, we have to reduce the reliance on short-term debt quantitatively. Therefore, a macro prudential regulation is needed.

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APPENDICES

Appendix A

Appendix of chapter 2

A.1 Diagnostic of TGARCH model

I did a diagnosis of the TGARCH model to see whether these models are well specified or not. The objective of variance modeling is to construct a variance measure, which has the property that standardized squared returns, $r_t^i/\hat{\sigma}_t^i$ have no systematic autocorrelation.

We can see from graph that the standardized squared returns have no autocerrelation as the sticks of different lags are all in their standard error banks. Model is well specified in terms of in sample check.

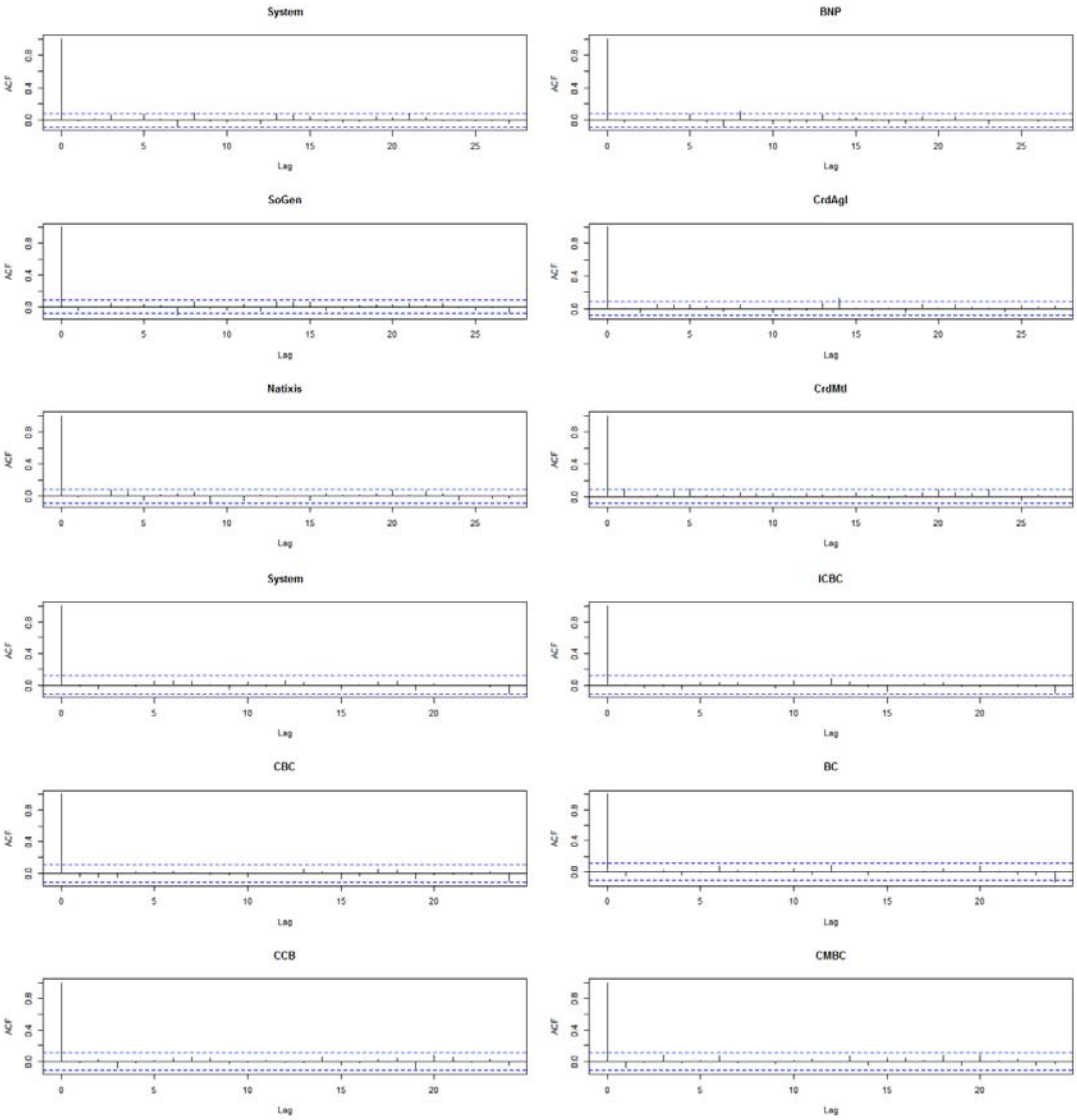

Figure A.1: Autocorrelation function of standardized innovations.

Appendix B

Appendix of chapter 3

B.1 Proof of lemma 6

Proof. From the binding IR constraints (4.3) , we have: $d_1 = \frac{i-k}{\alpha+p(1-\alpha)}$. Inserting it into Π_s, $\Pi_s = (\rho_1 - 1)i - \alpha\beta\mu\frac{i-k}{\alpha+p(1-\alpha)}$.

$$\frac{\partial \Pi_s}{\partial i} = (\rho_1 - 1) - \alpha\beta\mu\frac{1}{\alpha + p(1-\alpha)} = (\rho_1 - 1) - P_1(\alpha)\beta\mu,$$

where $P_1(\alpha) = \frac{\alpha}{\alpha+p(1-\alpha)}$. To guarantee $\frac{\partial \Pi_s}{\partial i} > 0$ always holds we have to assume:

$$\rho_1 - 1 > \underset{\alpha}{max} P_1(\alpha)\beta\mu.$$

As a result of this assumption, the bankers will choose their project size as much as possible to maximize profit. Then, the maximum size is given by his individual IC constraints.

To prove the second result, inserting (4.2) into (4.5) gives us:

$$\rho_1 i \geq (1 + \mu)d_1.$$

From IC constraint (4.4), we have:

$$\rho_1 i \geq (\rho_1 - \rho_0)i + i - k + \alpha\beta\mu d_1.$$

If $(\rho_1 - \rho_0)i + i - k - (1 + \mu - \alpha\beta\mu)d_1 \geq 0$, then $(IC) \Rightarrow (LL)$.

$$(\rho_1 - \rho_0)i + i - k - (1 + \mu - \alpha\beta\mu)\frac{i-k}{\alpha + p(1-\alpha)}$$
$$> \frac{(\rho_1 - \rho_0) + \alpha + p(1-\alpha) - (1 + \mu - \alpha\beta\mu)}{\alpha + p(1-\alpha)}(i-k).$$

To calculate μ, we have by our assumption on distribution function F:

$$\mu = E[G(\widetilde{\gamma}D_1) \mid \widetilde{M} < D_1] = \frac{D_1\overline{\theta}}{2\overline{E}}.$$

$\max\{\mu - \alpha\beta\mu + 1\} = \frac{\overline{\theta}}{2} + 1$ and $\min\{(\rho_1 - \rho_0) + \alpha + p(1-\alpha)\} = \rho_1 - \rho_0 + p$. Thus if assuming $\rho_1 - 1 > \rho_0 + \frac{\overline{\theta}}{2} - p$, we will have $(IC) \Rightarrow (LL)$. Thus in all, if $\rho_1 - 1 > max[\underset{\alpha}{max} P_1(\alpha)\beta\mu\ ,\ \rho_0 + \frac{\overline{\theta}}{2} - p]$, we have the banker will always choose to maximize project size, the size is exactly given by IC constraint and we can always neglect banker's LL constraint since it is implied by IC .

$\square$

B.2 Proof of Proposition 7

Proof. Inserting d_1 into Π_s and IC, and use Lagrange method to solve banker's profit maximizing program, we have:

$$L = \rho_1 i - i - P_1(\alpha)\beta\mu i + P_1(\alpha)\beta\mu k - \nu(i - k + P_1(\alpha)\beta\mu i - P_1(\alpha)\beta\mu k - \rho_0 i),$$

$$\frac{\partial L}{\partial i} = \rho_1 - 1 - P_1(\alpha)\beta\mu - \nu(1 + P_1(\alpha)\beta\mu - \rho_0),$$

$$\frac{\partial L}{\partial \nu} = i - k + P_1(\alpha)\beta\mu i + P(\alpha)\beta\mu k - \rho_0 i.$$

From A_2, we know that $\rho_1 - 1 - P_1(\alpha)\beta\mu > 0$, thus it must be: $\nu > 0$ and $i - k + P_1(\alpha)\beta\mu i - P(\alpha)\beta\mu k - \rho_0 i = 0$. Thus, the optimal size in short term debt is:

$$i^{**} = \frac{A(\beta,\mu)}{A(\beta,\mu) - \rho_0}k,$$

where $A(\beta,\mu) = 1 + P_1(\alpha)\beta\mu$. The maximum profit is given by inserting i^{**} into Π_s:

$$\Pi_s(\beta,\mu) = \frac{\rho_1 - \rho_0}{A(\beta,\mu) - \rho_0}A(\beta,\mu)k - k.$$

It is easily seen that:

$$\frac{\partial i^{**}}{\partial \mu} = \frac{-\rho_0 k}{[A(\beta,\mu)-\rho_0]^2} P_1(\alpha)\beta < 0,$$

$$\frac{\partial \Pi_s}{\partial \mu} = \frac{-\rho_0(\rho_1-\rho_0)k}{[A(\beta,\mu)-\rho_0]^2} P_1(\alpha)\beta < 0.$$

The same we have: $\frac{\partial i^{**}}{\partial \beta} < 0$, $\frac{\partial \Pi_s}{\partial \beta} < 0$.

□

B.3 Proof of Proposition 9

Now, we give a brief prove of proposition 9.

Proof. For a μ, the optimal project size is given by the binding IC constraint :$I^{**} = \frac{A(\beta,\mu)}{A(\beta,\mu)-\rho_0}K$. As a result:

$$D_1^{**} = \frac{P_1(\alpha)}{\alpha}(I^{**}-K) = \frac{P_1(\alpha)}{\alpha}\frac{1+P_1(\alpha)\beta\mu}{1+P_1(\alpha)\beta\mu-\rho_0}K - \frac{P_1(\alpha)}{\alpha}K.$$

Notice that by the clearing condition at $t = 1$ frictional refinancing market: $\mu = \int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c$. Denote H_c as the conditional distribution function of $\widetilde{M}$ conditional on $\widetilde{M} < D_1$. $\beta = H(D_1)$, we have:

$$D_1 = \frac{P_1(\alpha)}{\alpha}\frac{1+P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c}{1+P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c - \rho_0}K - \frac{P_1(\alpha)}{\alpha}K \ (*).$$

The market equilibrium D_1^e will be the solution of the above function $(*)$. First, we prove there exists a solution for $(*)$ and define another function:

$$\Sigma(D_1) = \frac{P_1(\alpha)K}{\alpha}[\frac{1+P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c}{1+P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D)dH_c - \rho_0} - 1] - D_1.$$

If no banker borrows anything, we have $D_1 = 0$. Second, notice that when $\alpha = 0$, we are in the standard Holmstrom and Tirole (1997), the maximum project size for an individual banker is: $\hat{i} = \frac{k}{1-\rho_0}$, thus $\hat{d}_1 = \frac{\hat{i}-k}{p} = \frac{\rho_0 k}{p(1-\rho_0)}$. In aggregation, we have: $\hat{I} = \frac{K}{1-\rho_0}$,and $\hat{D}_1 = \frac{\rho_0 K}{p(1-\rho_0)}$. We consider our function $\Sigma(D_1)$ defined on $[0, \hat{D}_1]$.

Let $D_1 \to 0$.

Since $\beta(0) = H(0) = 0$, $\mu(0) = \int_0^{D_1=0} G(0)dH_c = 0$ and $\rho_0 < 1$, we have:

$$\Sigma(0) = \frac{P_1(\alpha)K}{\alpha}[\frac{1}{1-\rho_0} - 1] > 0.$$

We have: $lim_{D_1 \to 0}\Sigma(D_1) > 0$

Let $D_1 \to \hat{D}_1$.

We can calculate: $lim_{D_1 \to \hat{D}_1}\Sigma(D_1) = \frac{P_1(\alpha)K}{\alpha}[\frac{A(\hat{D}_1)}{A(\hat{D}_1)-\rho_0} - 1] - \frac{1}{p}\frac{\rho_0 K}{1-\rho_0}$, where $A(\hat{D}_1) = 1+P_1(\alpha)H(\hat{D}_1)\int_0^{\hat{D}_1} G(\gamma(\widetilde{M}, \hat{D}_1)\hat{D}_1)dH_c$. Since $\alpha+(1-\alpha)p = p+(1-p)\alpha > p$ for all $\alpha \in (0, 1]$, we have:

$$\frac{P_1(\alpha)}{\alpha} = \frac{1}{\alpha + (1-\alpha)p} < \frac{1}{p}.$$

$$\begin{aligned}
\frac{P_1(\alpha)K}{\alpha}[\frac{A(\hat{D}_1)}{A(\hat{D}_1)-\rho_0} - 1] - \frac{1}{p}\frac{\rho_0 K}{1-\rho_0} &= \frac{P_1(\alpha)K}{\alpha}\frac{A(\hat{D}_1)}{A(\hat{D}_1)-\rho_0} - (\frac{P_1(\alpha)}{\alpha}K + \frac{1}{p}\frac{\rho_0 K}{1-\rho_0}) \\
&< \frac{P_1(\alpha)K}{\alpha}\frac{A(\hat{D}_1)}{A(\hat{D}_1)-\rho_0} - \frac{P_1(\alpha)K}{\alpha}(1 + \frac{\rho_0}{1-\rho_0}) \\
&= \frac{P_1(\alpha)K}{\alpha}[\frac{A(\hat{D}_1)}{A(\hat{D}_1)-\rho_0} - \frac{1}{1-\rho_0}].
\end{aligned}$$

Moreover, for all $\alpha > 0$, we have: $A(\hat{D}_1) > 1$, thus: $[\frac{A(\hat{D}_1)}{A(\hat{D}_1)-\rho_0} - \frac{1}{1-\rho_0}] < 0$. As a result, we know that: $lim_{D_1 \to \hat{D}_1}\Sigma(D_1) < 0$. We could find an equilibrium level $D_1^e \in [0, \hat{D}_1)$ to make $\Sigma(D_1^e) = 0$, existence is proven.

To prove uniqueness solution of $(*)$, $A(D_1) = 1+P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c$, we have:

$$\frac{\partial\Sigma(D_1)}{\partial D_1} = \frac{P_1(\alpha)K}{\alpha}\frac{-\rho_0}{[A(D_1)-\rho_0]^2}\frac{\partial A(D_1)}{\partial D_1} - 1.$$

Notice that $\gamma(D_1, D_1) = \frac{D_1-D_1}{D_1} = 0$, $\gamma'(\widetilde{M}, D_1) = \frac{\widetilde{M}}{D_1^2} > 0$, $G' > 0$ and

$$\frac{\partial \int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c}{\partial D_1} > 0 \ (**).$$

Combing these results, we have:

$$\frac{\partial A(D_1)}{\partial D_1} = \frac{\partial[1 + P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c]}{\partial D_1} > 0.$$

As a result:

$$\frac{\partial \Sigma(D_1)}{\partial D_1} = \frac{P_1(\alpha)K}{\alpha} \frac{-\rho_0}{[A(D_1)-\rho_0]^2} \frac{\partial A(D_1)}{\partial D_1} - 1 < 0.$$

Thus $\Sigma(D_1)$ in monotonic decreasing, we prove the uniqueness of D_1^e.

D_1^e, $\beta^e = H(D_1^e)$, $\mu^e = \int_0^{D_1^e} G(\tilde{\gamma} D_1^e) dH_c = \int_0^{D_1^e} G(\gamma(\widetilde{M}, D_1^e) D_1^e) dH_c$ and $\overline{\gamma}^e = E(\frac{D_1^e - \widetilde{M}}{D_1^e} | M < D_1^e)$ consisting the competitive equilibrium.

□

B.4 Proof of Proposition 10

Proof. First, we are going to calculate:

1. $\frac{\partial D_1^e}{\partial \alpha}$. From the proof of proposition 9, we have D_1^e is given by: $\Sigma(D_1^e) = 0$.

$$\Sigma(D_1^e, \alpha) = \frac{P_1(\alpha)K}{\alpha} [\frac{1 + P_1(\alpha) H(D_1^e) \int_0^{D_1^e} G(\gamma(\widetilde{M}, D_1^e) D_1^e) dH_c}{1 + P_1(\alpha) H(D_1^e) \int_0^{D_1^e} G(\gamma(\widetilde{M}, D_1^e) D_1^e) dH_c - \rho_0} - 1] - D_1^e = 0.$$

Applying implicit function theorem:

$$\frac{dD_1^e}{d\alpha} = -\frac{\frac{\partial \Sigma(D_1^e, \alpha)}{\partial \alpha}}{\frac{\partial \Sigma(D_1^e, \alpha)}{\partial D_1^e}}.$$

To calculate, define again: $A(D_1^e, \alpha) = 1 + P_1(\alpha) H(D_1^e) \int_0^{D_1^e} G(\gamma(\widetilde{M}, D_1^e) D_1^e) dH_c$, we have:

$$\frac{\partial \Sigma(D_1^e, \alpha)}{\partial \alpha} = \frac{-(1-p)K}{[\alpha + (1-\alpha)p]^2} [\frac{A(D_1^e, \alpha)}{A(D_1^e, \alpha) - \rho_0} - 1] + \frac{P_1(\alpha)K}{\alpha} \frac{-\rho_0}{[A(D_1^e, \alpha) - \rho_0]^2} \frac{\partial A(D_1^e, \alpha)}{\partial \alpha} < 0,$$

since $\frac{\partial A(D_1^e, \alpha)}{\partial \alpha} > 0$. As calculated in the last part of proof of proposition 9;

$$\frac{\partial \Sigma(D_1^e, \alpha)}{\partial D_1^e} = \frac{P_1(\alpha)K}{\alpha} \frac{-\rho_0}{[A(D_1^e, \alpha) - \rho_0]^2} \frac{\partial A(D_1^e, \alpha)}{\partial D_1^e} - 1 < 0,$$

since $\frac{\partial A(D_1^e, \alpha)}{\partial D_1^e} > 0$ holds from $(**)$ in proof of proposition 9. As a result, we have:

$$\frac{dD_1^e}{d\alpha} = -\frac{\frac{\partial \Sigma(D_1^e, \alpha)}{\partial \alpha}}{\frac{\partial \Sigma(D_1^e, \alpha)}{\partial D_1^e}} < 0.$$

2. $\frac{\partial \beta^e}{\partial \alpha} = \frac{\partial H(D_1^e)}{\partial D_1^e}\frac{\partial D_1^e}{\partial \alpha} < 0$, since $H^{'} > 0$.

3. $\frac{\partial \mu^e}{\partial \alpha} = \frac{\partial \mu^e}{\partial D_1^e}\frac{\partial D_1^e}{\partial \alpha} = \frac{\partial \int_0^{D_1^e} G(\gamma(\widetilde{M}, D_1^e)D_1^e)dH_c}{\partial D_1^e}\frac{\partial D_1^e}{\partial \alpha} < 0$.

□

B.5 Proof of Proposition 12

Proof. For case 1

A banker's equilibrium profit in long term debt is given by:

$$\Pi_l = \frac{\rho_1 - 1}{1 - \rho_1 + \delta\rho_1}k,$$

with the optimal project size in long term debt funding: $i^* = \frac{k}{1-\rho_1+\delta\rho_1}$. Notice that A_2: $\delta > \frac{\rho_1 - \rho_0}{\rho_1}$ holds and we know that $\frac{\partial \Pi_l}{\partial \delta} < 0$. The maximum profit given by Π_l is just let IC and substitution prevent bind together.[1]

The equilibrium profit when short term debt is used is given by:

$$\Pi_s^e(\alpha) = \frac{\rho_1 - \rho_0}{A^e(\alpha) - \rho_0}A^e(\alpha)k - k,$$

where, equilibrium project size is given by: $i^e = \frac{A^e(\alpha)}{A^e(\alpha)-\rho_0}k$, and $A^e(\alpha) = 1 + P_1(\alpha)\beta^e(D_1^e(\alpha))\mu^e(D_1^e(\alpha))$.

From the comparative static results summarized in Corollary 11: $\frac{\partial \Pi_s^e}{\partial \alpha} < 0$, we have:

$$\underset{\alpha}{max}\Pi_s^e(\alpha) = \Pi_s^e(0) = \frac{\rho_1 - \rho_0}{A^e(0) - \rho_0}A^e(0)k - k = \frac{\rho_1 - \rho_0}{1 - \rho_0}k - k = \frac{\rho_1 - 1}{1 - \rho_0}k.$$

As a result we have:

$$\underset{\alpha}{max}\Pi_s^e(\alpha) = \underset{\delta}{max}\Pi_l.$$

Under A_1, we can not find δ such that $\Pi_l(\delta) > \Pi_s^e$ holds for all $\alpha \in [0, 1]$.

For case 2

We have $\underset{\alpha}{min}\Pi_s^e(\alpha) = \Pi_s^e(1) = \frac{\rho_1 - \rho_0}{A^e(1)-\rho_0}A^e(1)k - k$. If $\Pi_l = \Pi_s^e(1)$, we can find

[1]The bankers' substituting ability is lowest.

a δ_2, which is the solution of $\Pi_l(\delta_2) = \Pi_s^e(1)$:

$$\frac{\rho_1 - 1}{1 - \rho_1 + \delta\rho_1}k = \frac{\rho_1 - \rho_0}{A^e(1) - \rho_0}A^e(1)k - k.$$

We arrange equation and get:

$$\delta_2 = \frac{\rho_1 - 1}{\rho_1}[1 + \frac{A^e(1) - \rho_0}{\rho_1 - 1 - (A(1) - 1)(1 + \rho_0 - \rho_1)}].$$

Whether this $\delta_2 < 1$ or not depends on the distribution assumption of H , F and ρ_1. We simply assume we choose H , F and $\rho_1$carefully enough and $\delta_2 < 1$. Then for any $\delta \in (\delta_2, 1]$, we have

$$\underset{\alpha}{min}\Pi_s^e(\alpha) > \Pi_l(\delta),$$

which means, short term debt will strictly dominates long term debt for $\delta > \delta_2$. If bankers' ability lies in this interval, they will use short term debt for no matter α.

For case 3

If $\delta \in (\delta_1, \delta_2)$. For this case, short term debt can not strictly dominates long term debt for all α, for sure there will be cross for a $\alpha \in (0, 1)$. We denotes this αas α^c. Then we are going to find it. To make the short and long term debt's profit equal,we have:

$$\frac{\rho_1 - \rho_0}{A^e(\alpha) - \rho_0}A^e(\alpha) = 1 + \frac{\rho_1 - 1}{1 - (1 - \delta)\rho_1}.$$

From this equation, we can get:

$$A^e(\alpha) = \frac{\delta\rho_0\rho_1}{\delta\rho_1 - (\rho_1 - \rho_0)(1 - \rho_1 + \delta\rho_1)}.$$

Under A_1: $\delta\rho_1 > \rho_1 - \rho_0$, we have: $\delta\rho_1 > \rho_1 - \rho_0 \Rightarrow \rho_1 - 1 > \frac{(\rho_1 - \rho_0)}{\delta\rho_1}(\rho_1 - 1) \Rightarrow \rho_0 > 1 + \rho_0 - \rho_1 + \frac{(\rho_1 - \rho_0)}{\delta\rho_1}(\rho_1 - 1) \Rightarrow \delta\rho_0\rho_1 > \delta\rho_1(1 + \rho_0 - \rho_1) + (\rho_1 - \rho_0)(\rho_1 - 1)$. Under A_3: $\rho_1 < \rho_0 + 1$, we have: $\delta\rho_1(1 + \rho_0 - \rho_1) + (\rho_1 - \rho_0)(\rho_1 - 1) > 0$, thus $\frac{\delta\rho_0\rho_1}{\delta\rho_1(1+\rho_0-\rho_1)+(\rho_1-\rho_0)(\rho_1-1)} > 1$, and $\frac{\delta\rho_0\rho_1}{\delta\rho_1(1+\rho_0-\rho_1)+(\rho_1-\rho_0)(\rho_1-1)} = \frac{\delta\rho_0\rho_1}{\delta\rho_1-(\rho_1-\rho_0)(1-\rho_1+\delta\rho_1)}$, which gives us:

$$A^e(\alpha) = \frac{\delta\rho_0\rho_1}{\delta\rho_1 - (\rho_1 - \rho_0)(1 - \rho_1 + \delta\rho_1)} > 1.$$

Because $A^e(\alpha) = 1 + P_1(\alpha)\beta^e(\alpha)\mu^e(\alpha)$, which means:

$$P_1(\alpha)\beta^e(\alpha)\mu^e(\alpha) = \frac{(\rho_1 - 1)(\delta\rho_1 + \rho_0 - \rho_1)}{\delta\rho_1 - (\rho_1 - \rho_0)(1 - \rho_1 + \delta\rho_1)} > 0.$$

After some rearrangements:

$$\alpha = \frac{p\eta}{p\eta + \beta^e(\alpha)\mu^e(\alpha) - \eta},$$

where $\eta(\delta) = \frac{(\rho_1-1)(\delta\rho_1+\rho_0-\rho_1)}{\delta\rho_1-(\rho_1-\rho_0)(1-\rho_1+\delta\rho_1)}$. α_c^e is the solution of this function

$$\alpha_c^e = \frac{p\eta}{p\eta + \beta^e(\alpha^c)\mu^e(\alpha^c) - \eta}.$$

From the above derivation, when $\delta \in (\delta_1, \delta_2)$, we must have: $\alpha^c \in (0,1)$.

Last, by corollary 11: $\frac{\partial \Pi_s^e}{\partial \alpha} < 0$, this α_c^e is unique. Π_l is insensitive to α, thus when $\alpha > \alpha_c^e$, we have $\Pi_s^e < \Pi_l$; when $\alpha < \alpha_c^e$, we have: $\Pi_s^e > \Pi_l$.

□

B.6 Proof of Lemma 13

Proof. First, notice that we can simplify the social planner's objective function:

$$W = (\rho_1 - 1)I - \alpha H(D_1)\int_0^{D_1}\int_0^{\gamma(\widetilde{M},D_1)D_1} G(x)dxdH_c.$$

To check the social planner's program is a well defined one, we direct check the case that the distribution function H and F is uniformly on $[0,\overline{M}]$ and $[0,\overline{\theta}]$. Thus we have: $H' > 0$, $F' > 0$ and $H'' = 0$, $F'' = 0$, $\gamma(\widetilde{M}, D_1) = \frac{D_1-\widetilde{M}}{D_1}$ and $D_1 = \frac{P_1(\alpha)}{\alpha}(I - K)$. $\frac{\partial D_1}{\partial I} = \frac{P_1(\alpha)}{\alpha} > 0$ and $\frac{\partial^2 D_1}{\partial I^2} = 0$. Moreover, we have: $G(\gamma(\widetilde{M}, D_1)D_1) > 0$ and $G'(\gamma(\widetilde{M}, D_1)D_1) > 0$[2].

$$\begin{aligned}\frac{\partial W}{\partial I} &= (\rho_1 - 1) - \alpha H'(D_1)\int_0^{D_1}\int_0^{\gamma(\widetilde{M},D_1)D_1} G(x)dxdH_c\frac{\partial D_1}{\partial I} \\ &\quad -\alpha H(D_1)\int_0^{D_1}[\gamma'(\widetilde{M},D_1)D_1 + \gamma(\widetilde{M},D_1)]G(\gamma(\widetilde{M},D_1)D_1)dH_c\frac{\partial D_1}{\partial I},\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2 W}{\partial I^2} &= -\alpha H'(D_1)\int_0^{D_1}[\gamma'(\widetilde{M},D_1)D_1 + \gamma(\widetilde{M},D_1)]G(\gamma(\widetilde{M},D_1)D_1)dH_c(\frac{\partial D_1}{\partial I})^2 \\ &\quad -\alpha H'(D_1)\int_0^{D_1}[\gamma'(\widetilde{M},D_1)D_1 + \gamma(\widetilde{M},D_1)]G(\gamma(\widetilde{M},D_1)D_1)dH_c(\frac{\partial D_1}{\partial I})^2 \\ &\quad -\alpha H(D_1)\int_0^{D_1}[\gamma''(\widetilde{M},D_1)D_1 + 2\gamma'(\widetilde{M},D_1)]G(\gamma(\widetilde{M},D_1)D_1)dH_c(\frac{\partial D_1}{\partial I})^2 \\ &\quad -\alpha H(D_1)\int_0^{D_1}[\gamma'(\widetilde{M},D_1)D_1 + \gamma(\widetilde{M},D_1)]^2G'(\gamma(\widetilde{M},D_1)D_1)dH_c(\frac{\partial D_1}{\partial I})^2.\end{aligned}$$

Something needs to be mentioned: $\gamma'(\widetilde{M}, D_1) = \frac{\widetilde{M}}{D_1^2} > 0$ and $\gamma''(\widetilde{M}, D_1) = -\frac{2\widetilde{M}}{D_1^3}$. But

[2] Since F is uniform on $[0,\overline{\theta}]$, $G = \frac{\overline{\theta}}{E}x$. actually $G' = \frac{\overline{\theta}}{E} > 0$

we have: $\gamma^{''}(\widetilde{M}, D_1)D_1 + 2\gamma^{'}(\widetilde{M}, D_1) = -\frac{2\widetilde{M}}{D_1^3}D_1 + 2\frac{\widetilde{M}}{D_1^2} = 0$. As a result of this, we can claim that:

$$\begin{aligned}\frac{\partial^2 W}{\partial I^2} &= -2\alpha H^{'}(D_1)\int_0^{D_1}[\gamma^{'}(\widetilde{M}, D_1)D_1 + \gamma(\widetilde{M}, D_1)]G(\gamma(\widetilde{M}, D_1)D_1)dH_c(\frac{\partial D_1}{\partial I})^2 \\ &\quad -\alpha H(D_1)\int_0^{D_1}[\gamma^{'}(\widetilde{M}, D_1)D_1 + \gamma(\widetilde{M}, D_1)]^2 G^{'}(\gamma(\widetilde{M}, D_1)D_1)dH_c(\frac{\partial D_1}{\partial I})^2.\end{aligned}$$

$$\frac{\partial^2 W}{\partial I^2} < 0.$$

□

B.7 Proof of Proposition 14

Proof. Notice first the social planner's well defined objective function is:

$$max_I\ W = (\rho_1 - 1)I - \alpha H(D_1)\int_0^{D_1}\int_0^{\gamma(\widetilde{M}, D_1)D_1} G(x)dxdH_c.$$

Notice that as before, this H_c is actually a conditional distribution function (conditional on $\widetilde{M} < D_1$). And IC becomes:

$$\rho_0 I \geq (1 + P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c)(I - K).$$

Notice that : $D_1 = \frac{I-K}{\alpha+(1-\alpha)p}$ by IR. We solve the program by Lagrangian method. Denoting L the associated Lagrangian, and v by the multiplier.

$$\begin{aligned}L &= (\rho_1 - 1)I - \alpha H(D_1)\int_0^{D_1}\int_0^{\gamma(\widetilde{M}, D_1)D_1} G(x)dxdH_c \\ &- \nu\{[1 + P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c](I - K) - \rho_0 I\}.\end{aligned}$$

By Kuhn-Tucker theorem:

$$\begin{aligned}
\frac{\partial L}{\partial I} &= (\rho_1 - 1) - \alpha H^{'}(D_1)\int_0^{D_1}\int_0^{\gamma(\widetilde{M},D_1)D_1} G(x)dxdH_c\frac{\partial D_1}{\partial I} \\
&- \alpha H(D_1)\int_0^{D_1}[\frac{\partial\gamma(\widetilde{M},D_1)}{\partial I}D_1 + \gamma(\widetilde{M},D_1)]G(\gamma(\widetilde{M},D_1)D_1)dH_c\frac{\partial D_1}{\partial I} \\
&- \nu\{[P_1(\alpha)H^{'}(D_1)\int_0^{D_1} G(\gamma(\widetilde{M},D_1)D_1)dH_c \\
&\quad +P_1(\alpha)H(D_1)\int_0^{D_1}[G^{'}(\gamma(\widetilde{M},D_1)D_1)(\frac{\partial\gamma(\widetilde{M},D_1)}{\partial I}D_1 + \gamma(\widetilde{M},D_1))dH_c](I-K)\frac{\partial D_1}{\partial I} \\
&\quad +[1 + P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M},D_1)D_1)dH_c] - \rho_0\} = 0,
\end{aligned}$$

$$\frac{\partial L}{\partial \nu} = 1 + P_1(\alpha)H(D_1)\int_0^{D_1}\gamma(\widetilde{M},D_1)G(\gamma(\widetilde{M},D_1)D_1)dH_c](I-K) - \rho_0 I,$$

and $\nu\{[1 + P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M},D_1)D_1)dH_c](I-K) - \rho_0 I\} = 0$. Notice that: $\gamma(D_1, D_1) = 0$.

To study social planner's program: Given a value of α, we have the social planner's marginal benefit from issuing 1 unit of debt:$\rho_1 - 1$, and her marginal cost from issuing 1 unit of debt:

$$\begin{aligned}
\text{Marginal Cost} &= P_1(\alpha)H(D_1)\int_0^{D_1}\gamma(\widetilde{M},D_1)G(\gamma(\widetilde{M},D_1)D_1)dH_c \\
&+ P_1(\alpha)H^{'}(D_1)\int_0^{D_1}\int_0^{\gamma(\widetilde{M},D_1)D_1} G(x)dxdH_c \\
&+ P_1(\alpha)H(D_1)\int_0^{D_1}[\gamma^{'}(\widetilde{M},D_1)D_1]G(\gamma(\widetilde{M},D_1)D_1)dH_c.
\end{aligned}$$

Compared to the individual banker's program, the marginal cost of issuing 1 unit of debt is:

$$P_1(\alpha)H^{'}(D_1)\int_0^{D_1}\int_0^{\gamma(\widetilde{M},D_1)D_1} G(x)dxdH_c + P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M},D_1)D_1)dH_c.$$

We study 2 cases: Case 1: If $\nu > 0$, for all value of α we have IC constraints are binding:

$$[1 + P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M},D_1)D_1)dH_c](I-K) = \rho_0 I.$$

We denote the social planner's solution if all IC constraints are binding as $\overline{D_1^o}$. For all α:

$$\overline{I^o}(\alpha) = \frac{1 + P_1(\alpha)H(\overline{D_1^o})\int_0^{\overline{D_1^o}} G(\gamma(\widetilde{M},\overline{D_1^o})\overline{D_1^o})dH_c}{1 + P_1(\alpha)H(\overline{D_1^o})\int_0^{\overline{D_1^o}} G(\gamma(\widetilde{M},\overline{D_1^o})\overline{D_1^o})dH_c - \rho_0}K.$$

we have:

$$\overline{D_1^o} = \frac{P_1(\alpha)}{\alpha}(\overline{I^o} - K) = \frac{P_1(\alpha)K}{\alpha}\frac{1 + P_1(\alpha)H(\overline{D_1^o})\int_0^{\overline{D_1^o}} G(\gamma(\widetilde{M}, \overline{D_1^o})\overline{D_1^o})dH_c}{1 + P_1(\alpha)H(\overline{D_1^o})\int_0^{\overline{D_1^o}} G(\gamma(\widetilde{M}, \overline{D_1^o})\overline{D_1^o})dH_c - \rho_0} - \frac{P_1(\alpha)K}{\alpha}.$$

As before, a simplified equation:

$$\frac{P_1(\alpha)K}{\alpha}[\frac{1 + P_1(\alpha)\frac{\bar{\theta}}{2\overline{ME}}(\overline{D_1^o})^2}{1 + P_1(\alpha)\frac{\bar{\theta}}{2\overline{ME}}(\overline{D_1^o})^2 - \rho_0} - 1] = \overline{D_1^o},$$

exactly, coincides the market equilibrium program for all value of α. We can get $\overline{I^o}(\alpha) = [\alpha + (1-\alpha)p]\overline{D_1^o} + K$.

Case 2: If $\nu = 0$ for all value of α. We have IC constraints are all slack for all state of α. Then for all α:

$$\begin{aligned}(\rho_1 - 1) &= \alpha H'(D_1)\int_0^{D_1}\int_0^{\gamma(\widetilde{M}, D_1)D_1} G(x)dxdH_c\frac{\partial D_1}{\partial I} \\ &+\alpha H(D_1)\int_0^{D_1}[\gamma'(\widetilde{M}, D_1)D_1 + \gamma(\widetilde{M}, D_1)]G(\gamma(\widetilde{M}, D_1)D_1)dH_c\frac{\partial D_1}{\partial I}.\end{aligned}$$

and IC constraint : $[1 + P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)dH_c](I - K) - \rho_0 I > 0$. Notice that $D_1 = \frac{P_1(\alpha)}{\alpha}(I - K)$, thus $\frac{\partial D_1}{\partial I} = \frac{P_1(\alpha)}{\alpha}$.

$$\begin{aligned}\rho_1 - 1 &= P_1(\alpha)H(D_1)\int_0^{D_1}\gamma(\widetilde{M}, D_1)G(\gamma(\widetilde{M}, D_1)D_1)dH_c \\ &+P_1(\alpha)H(D_1)\int_0^{D_1}\gamma'(\widetilde{M}, D_1)D_1G(\gamma(\widetilde{M}, D_1)D_1)dH_c \\ &+P_1(\alpha)H'(D_1)\int_0^{D_1}\int_0^{\gamma(\widetilde{M}, D_1)D_1} G(x)dxdH_c.\end{aligned}$$

Notice that the first 2 terms of the above equation can be written as: $P_1(\alpha)H(D_1)\int_0^{D_1} G(\gamma(\widetilde{M}, D_1)D_1)d$
We denote the social planner's solution if all IC constraints are slack as $\underline{D_1^o}$. Then for all α:

$$\rho_1 - 1 = P_1(\alpha)H(\underline{D_1^o})\int_0^{\underline{D_1^o}} G(\gamma(\widetilde{M}, \underline{D_1^o})\underline{D_1^o})dH_c + P_1(\alpha)H'(\underline{D_1^o})\int_0^{\underline{D_1^o}}\int_0^{\gamma(\widetilde{M}, \underline{D_1^o})\underline{D_1^o}} G(x)dxdH_c.$$

By the assumption on distribution we can simplify it to:

$$\underline{D_1^o} = \sqrt{\frac{3\overline{ME}(\rho_1 - 1)}{2P_1(\alpha)\bar{\theta}}}.$$

Solving this equation gives us: $\underline{D_1^o}$ as a function of α. Again, we can get: $\underline{I^o} = [\alpha + (1-\alpha)p]\overline{D_1^o} + K$. We claim that social planner's optimal choice of short term level:

$$I_1^o(\alpha) = min\{\overline{I^o}(\alpha), \underline{I^o}(\alpha)\}.$$

We denote the α make $\overline{I^o}(\alpha) = \underline{I^o}(\alpha)$ happen as α^*. To make sure of this, we must keep $\overline{I^o}(\alpha)$ and $\underline{I^o}(\alpha)$ cross and cross only once for $\alpha \in [0,1]$, thus $\alpha^* \in [0,1]$ and it is unique. Since we have already known that $\frac{\partial I^e}{\partial \alpha} < 0$ and from the social planner's optimal program, we have $\frac{\partial I^o}{\partial \alpha} < 1$. And when $\alpha \to 0$, D_1^e binds by $\hat{D}_1$, thus $\overline{I^o} = I^e$ binds by $\overline{I} = \frac{K}{1-\rho_0}$. However, when $\alpha \to 0$, $\underline{D_1^o} \to \infty$. Thus $\underline{I^o} \to \infty$. Thus, to make them cross, we need to restrict $\underline{I^o}(0) < \overline{I^o}(0) = I^e(0)$ when $\alpha = 1$. Moreover, when $\alpha = 1$, $D_1 = I - K$. We can directly restrict parameter values to satisfy: $\underline{D^o}(0) < \overline{D^o}(0)$. From the above, we have: $\underline{D_1^o}(1) = \sqrt{\frac{3\overline{ME}(\rho_1-1)}{2\overline{\theta}}}$ and $\overline{D_1^o}(1)$ as the solution of $K[\frac{1+\frac{\overline{\theta}}{2\overline{ME}}(\overline{D_1^o})^2}{1+\frac{\overline{\theta}}{2\overline{ME}}(\overline{D_1^o})^2-\rho_0} - 1] = \overline{D_1^o}$, thus the final restriction on parameters $(\rho_1, \rho_0, \overline{\theta}, p, \overline{M}, \overline{E})$ will be:

$$\sqrt{\frac{3\overline{ME}(\rho_1-1)}{2\overline{\theta}}} < \overline{D_1^o}(1),$$

and $\overline{D_1^o}(1)$ as the solution of $K[\frac{1+\frac{\overline{\theta}}{2\overline{ME}}(\overline{D_1^o})^2}{1+\frac{\overline{\theta}}{2\overline{ME}}(\overline{D_1^o})^2-\rho_0} - 1] = \overline{D_1^o}$. Since the social planner consider extra cost than individual banker, which mean her unconstrained program's solution: $\underline{D_1^o}$ has steep slope than the constrained one.(given by market equilibrium), thus $\overline{D_1^o}(\alpha)$ and $\underline{D^o}(\alpha)$ will cross only once.

$\square$

Abstract: This thesis analysis the inefficiencies which may trigger the systemic risks in the financial system and studies the related measures to quantify such risks. The first article surveys the systemic risk in the financial system and the related macro-prudential policy: 1) the pro-cyclicality effect is harmful to the whole financial system as well as to the real economy; 2) the contagion risk among financial institutions. The second article of thesis proposes a new systemic risk measure to efficiently capture the systemic importance of each financial institution within a given system. The term systemic risk refers to the contagion risk to which each bank contributes to the financial system. The third article of thesis analysis the debt structure in the banking sector. Banks choose their debt maturity structure by weighting short term against long term debt. The externalities caused by over borrowing in short term debt exist only when the probability of macro shock is large.

Keywords: Systemic Risk, Systemic Risk Measures, Macroprudential Policy, Banking Model.

Résumé: Cette thèse analyse les sources d'inefficacité qui peuvent générer un risque systémique au sein du system financiers et étudie les différentes mesures associées. Le premier article présente une revue de la littérature sur le risque systémique et la politique macroprudentielle : 1) les effets négatifs de la procyclicité pour le système financier dans son ensemble ainsi que pour l'économie réelle ; 2) le risque de contagion entre institutions financières. Le second article de la thèse propose une nouvelle mesure du risque systémique visant à capturer efficacement l'importance systémique de chaque institution financière au sein d'un système donné. Le troisième article de la thèse analyse la structure de la dette des banques. Les banques choisissent la maturité de leur dette à court et/ou long terme. Les externalités négatives générées par l'excès de financement de court terme n'apparaissent que lorsque la probabilité d'un choc macroéconomique est suffisamment large.

Keywords: Risque systémique, Mesures de Risk Systémique, Politique Macroprudentielle, Modèle Bancaire.