Regulation of a Monopoly Generating Externalities*

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Abstract

We consider a monopoly that generates externalities. These depend either on the output level, or on the number of consumers, or both. We characterize optimal prices and construct a global price-cap scheme that decentralizes the second best. The scheme only requires standard accounting data and simple estimates of marginal external costs/benefits. We provide conditions for an iterative implementation process to converge to the second best. We also prove that the mechanism is robust to small errors in external effects parameters. The paper reveals how to correct for external effects without resorting to taxation.

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1 Introduction

Many industries, and in particular the provision of services of general interest, are plagued by two categories of market imperfections: externalities and imperfect competition. Taxation is the usual remedy to externalities, and state ownership and/or administered prices are the traditional responses to imperfect competition. However, the last decades saw a growing reluctance to taxation by governments, a general shift toward private property and the emergence of independent agencies for the enforcement of competition rules. This trend has been accompanied and stimulated by the rise of the so-called "new theory of regulation". Most contributions to the literature treat the two categories of market imperfections separately. On the regulation of a monopoly generating externalities, two "classical" papers are Baron (1985a) and Baron (1985b). Baron (1985a) discusses emission controls vs. pollution taxes under asymmetric information on abatement costs. Baron (1985b) focuses on the coordination of two regulatory agencies which control prices and pollution, respectively. If we restrict ourselves to price regulation as the sole instrument to correct market imperfections, a paper of reference is Oum and Tretheway (1988). It offers extended Ramsey pricing formulae in the externality generating monopoly case. The objective of our paper is to investigate the implementation of the latter proposal. By doing so we contribute to bridge the gap between public economists and the more pragmatic concerns of policy makers.

The setup we construct encompasses a large set of environments. In our model, externalities may depend on either the volume of services delivered by the firm, or on the number of clients, or both.[1] The analysis applies to situations in which access to services and intensity of use can be priced separately.[2] This feature can be observed in most network industries, where externalities are prevalent, and imperfect competition is ordinary.[3] We offer an implementation scheme of the extended price-cap family,[4] which allows the regulator to decen-

[1] An instance in which the greater use of the network by some clients may negatively affect other clients is the telecommunication industry, because of congestion effects. At the same time the telecommunication sector is characterized by the fact a greater number of users enhances the value of each connection to the network, a phenomenon often referred to as "network externality".

[2] For papers with two-part tariffs in models with externalities and imperfect competition, see, among others, Kanemoto (2000), Mitomo (2001) and Blonski (2002).

[3] There are many examples of externality-generating activities which are performed by monopolistic (or quasi-monopolisitc) providers, and are such that access and use can be priced separately. Public utilities which provide energy or water services typically price both access and use, and often generate environmental externalities. Another example are parcel transportation services, where customers are charged according to both the number of consignments (each of which entails displacement of a van for collection, with consequent pollution and road congestion) and the number (or size/weight) of parcels in each consignment.

[4] See Laffont J.J. and J. Tirole (1990a, 1990b)

tralize the second-best allocation. Our analysis shares with other recent papers the objective to decentralize a relatively elaborate policy by means of a simple and "easy-to-use" instrument. Tanaka (2007) considers price regulation and investment efforts. De Fraja and Iozzi (2008) study "price-and-quality" regulation and Bergantino et al. (2010) examine the same problem in an oligopolistic context. Our paper is complementary to these contributions in that, beyond the fact that it addresses a different problem, it also considers the issue of robustness.

The paper is organised as follows. Section 2 introduces the model. In Section 3 we present two benchmarks: the first-best and the profit-maximising price structure. In Section 4 we characterize the socially optimum prices when the producer is required to break-even. In Section 5 we propose a regulatory mechanism which allows the regulator to decentralize the corresponding second-best allocation. Convergence and robustness are discussed and a short concluding section completes the paper.

2 The model

A monopolist delivers a total quantity X of output to N consumers at a cost $C(X,N)$. Each consumer is charged a fee equal to a for having access to the service. The service is sold at a unit price b. A consumer of type $\theta \in [0,+\infty[$ obtains gross surplus $S_\theta(x_\theta, X, N)$ from consuming the quantity x_θ when there are N consumers whose demands add up to X.

Assumption 1 *(individual surplus).* $S_\theta(x_\theta, X, N)$ *is:*
a) quasi-linear;
b) twice continuously differentiable;
c) increasing and concave in x_θ, i.e. $(\partial S_\theta/\partial x_\theta) > 0$ *and* $\left(\partial^2 S_\theta/\partial x_\theta^2\right) < 0$*;*
d) increasing in θ, i.e. $(\partial S_\theta/\partial \theta) > 0$*;*
e) with marginal surplus also increasing in θ*, i.e.* $\left(\partial^2 S_\theta/\partial x_\theta \partial \theta\right) > 0$.

Let the net surplus of a consumer of type θ be

$$V_\theta(a,b) \equiv \sup\{S_\theta[x_\theta(b,X,N), X, N] - [a + bx_\theta(b,X,N)], S_\theta(0,X,N)\}, \tag{1}$$

where $x_\theta(b,X,N)$ denotes individual demand conditional on having access to the service, uniquely defined by the condition[5]

$$\frac{\partial S_\theta(x_\theta, X, N)}{\partial x_\theta} = b. \tag{2}$$

[5] Existence and uniqueness are ensured by assumptions 1b) and 1c). As income effects are ruled out by assumption 1a), individual demand conditional on access does not depend on a.

Let $\theta_m(a,b)$ be the marginal type, *i.e.* the type of consumers indifferent between not dealing with the firm or instead paying for access and purchasing $x_{\theta_m}(b,X,N)$, as defined by the equation

$$S_{\theta_m}\left(x_{\theta_m}(b,X,N),X,N\right)-\left(a+bx_{\theta_m}(b,X,N)\right)=S_{\theta_m}(0,X,N). \qquad (3)$$

Assumptions 1d) ensures that individual net surplus $V_\theta(a,b)$ is increasing in θ.[6] Assumptions 1d) and 1e) guarantee uniqueness of θ_m.[7]

Given the above, consumers who find it beneficial to get access to the service are all and only those with $\theta \geq \theta_m$. Assumption 1e) also ensures[8] that individual demand is also increasing in θ, i.e. $(dx_\theta/d\theta)>0$.

If the population is distributed over types according to the density function $g(\theta)$, the number of consumers and aggregate demand are respectively:

$$N = \int_{\theta_m}^{+\infty} g(\theta)\,d\theta. \qquad (4)$$

$$X = \int_{\theta_m}^{+\infty} x_\theta(b,X,N)g(\theta)\,d\theta. \qquad (5)$$

3 Two benchmarks

3.1 First-best prices

We now briefly characterize the first-best allocation, using prices as decision variables.

Maximization of social welfare, which can be written as

$$W(a,b)=\int_0^{+\infty} V_\theta(a,b)\,g(\theta)\,d\theta+aN(a,b)+bX(a,b)-C\left(X(a,b),N(a,b)\right), \qquad (6)$$

[6] For $\theta<\theta_m$ this is obvious as gross and net surplus coincide. For $\theta \geq \theta_m$, it follows from the envelope theorem :

$$\frac{d}{d\theta}\left\{S_\theta\left[x_\theta(b,X,N),X,N\right]-\left[a+bx_\theta(b,X,N)\right]\right\}=\frac{\partial S_\theta}{\partial\theta}+\left(\frac{\partial S_\theta}{\partial x_\theta}-b\right)\frac{dx_\theta}{d\theta}.$$

[7] Assumption 1d) ensures that $(\partial V_\theta/\partial\theta)=(\partial S_\theta/\partial\theta)$ (see the previous footnote). Assumption 1e) ensures that

$$\frac{\partial S_\theta(x_\theta,X,N)}{\partial\theta}>\frac{\partial S_\theta(0,X,N)}{\partial\theta}.$$

[8] Demand monotonicity follows immediately from differentiation of equation (2) wrt θ:

$$\frac{\partial^2 S_\theta}{\partial\theta\partial x_\theta}+\frac{\partial^2 S_\theta}{\partial x_\theta^2}\frac{dx_\theta}{d\theta}=0.$$

leads to the following conditions (see Appendix):

$$a = \frac{\partial C}{\partial N} - E_N, \tag{7}$$

$$b = \frac{\partial C}{\partial X} - E_X, \tag{8}$$

where E_N and E_X denote the *marginal* external effects, that is the impact on the aggregate surplus of, respectively, an additional connection and an additional unit of service:

$$E_N = \int_0^{+\infty} \frac{\partial S_\theta}{\partial N} g(\theta)\, d\theta, \tag{9}$$

$$E_X = \int_0^{+\infty} \frac{\partial S_\theta}{\partial X} g(\theta)\, d\theta. \tag{10}$$

Note that E_N and E_X can be either positive or negative. It is immediately seen that, in the absence of externalities, conditions (7) and (8) boil down to plain marginal cost pricing. If instead the externality terms E_N and/or E_X are negative, the first-best allocation requires access and/or consumption to be priced above marginal cost. If this secures the provider non-negative profit, the regulator may think of implementing the first-best allocation *via* direct price control. Of course, this requires that the regulator can make use of reliable estimates of both marginal costs C_X and C_N and marginal external costs (or benefits) E_X and E_N. To the contrary, if the externality terms E_N and/or E_X are positive, the first-best requires pricing below marginal cost, which would most likely cause the provider to operate at a loss. Therefore in Section **4** we study a second-best situation where welfare maximization is subject to a break-even constraint. Before doing that, however, we briefly consider the profit-maximizing price structure, that provides us with another useful reference point.

3.2 Profit-maximising price structure

Maximization of profit, given by

$$\Pi = aN + bX - C(X, N), \tag{11}$$

leads to the following couple of Lerner formulae (see Appendix A.2):

$$\frac{a - \widetilde{C}_N}{a} = \frac{1}{\epsilon_N}, \tag{12}$$

$$\frac{b - \widetilde{C}_X}{b} = \frac{1}{\epsilon_X}, \tag{13}$$

where ϵ_N and ϵ_X are respectively the price-elasticity of the demand for access and the standard price-elasticity:

$$\epsilon_N = -\frac{a}{N}\frac{dN}{da}, \tag{14}$$

$$\epsilon_X = -\frac{b}{X}\frac{dX}{db}; \tag{15}$$

while $\widetilde{C}_N$ and $\widetilde{C}_X$ as defined by

$$\widetilde{C}_N = \frac{\partial C}{\partial N} - \left[b - \frac{\partial C}{\partial X}\right]\left(\frac{dX}{da}/\frac{dN}{da}\right), \tag{16}$$

$$\widetilde{C}_X = \frac{\partial C}{\partial X} - \left[a - \frac{\partial C}{\partial N}\right]\left(\frac{dN}{db}/\frac{dX}{db}\right), \tag{17}$$

are respectively the "virtual cost of connection", that is the cost of giving access to an additional consumer, net of the change in profit associated with the change in sales that takes place when the number of accesses vary[9] and the "virtual cost of service" to be interpreted along similar lines.

There is a difference, however. The virtual connection cost (16) can be rewritten so as to only make reference to magnitudes that are (at least in principle) directly computable. More precisely[10],

$$\widetilde{C}_N = \frac{\partial C}{\partial N} - \left(b - \frac{\partial C}{\partial X}\right)\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}, \tag{18}$$

where

$$E_{xN} = \int_{\theta_m}^{+\infty}\left(\frac{-\partial^2 S_\theta}{\partial x_\theta^2}\right)^{-1}\frac{\partial^2 S_\theta}{\partial x_\theta \partial N} g(\theta)\, d\theta,$$

$$E_{xX} = \int_{\theta_m}^{+\infty}\left(\frac{-\partial^2 S_\theta}{\partial x_\theta^2}\right)^{-1}\frac{\partial^2 S_\theta}{\partial x_\theta \partial X} g(\theta)\, d\theta,$$

indicate the change in service demand that is induced by a unit change in, respectively, N and X.[11] Observe that, if the latter magnitude is positive and

[9] The virtual cost of a good extends the notion of cost in that it considers both possibles losses for the provider in addition to production cost (these losses appear with a positive sign) and possible gains different from revenues connected with the sale of the good in question (these gains appear with a negative sign), that ensue from selling an additional unit of the good thanks to a change in its price (this holds, separately, for the good 'access' and the good 'service').

[10] The reformulation of $\frac{dX}{da}/\frac{dN}{da}$ that permits to rewrite (16) as (18) is developed in Appendix A.2.2).

[11] In interpreting E_{xX} observe that $\left(-\partial^2 S_\theta/\partial x_\theta^2\right)$ is nothing but the reduction in marginal surplus, in absolute value, that is caused by a unit increase in x_θ. Thus $\left(-\partial^2 S_\theta/\partial x_\theta^2\right)^{-1}\left(\partial^2 S_\theta/\partial x_\theta \partial X\right)$ is the change in x_θ that is triggered by the effect of an additional unit of X on $\left(\partial S_\theta/\partial x_\theta\right)$.
Interpretation of E_{xN} can be made along similar lines.

sufficiently weak[12], the term $1/(1-E_{xX})$ is a multiplier that amplifies the second term on the right-hand side of (18) - this term represents the correction to be applied to marginal cost in order to obtain virtual marginal cost.

On the contrary, virtual marginal cost (17) cannot be similarly rewritten.[13] However, on the condition that access price a is set to its profit-maximizing level given by (12), we can reformulate the optimal pricing rule (13) as follows

$$\frac{b-\widehat{C}_X}{b} = \frac{1-x_{\theta_m}/\overline{x}}{\widehat{\epsilon_X}}, \tag{19}$$

where $\overline{x} = X/N$ denotes average consumption,[14] $\widehat{C}_X$ and $\widehat{\epsilon_X}$ as defined by

$$\widehat{C}_X = \frac{\partial C}{\partial X} - N\left(\frac{\partial S^+_{\theta_m}}{\partial X} - \frac{\partial S^-_{\theta_m}}{\partial X}\right), \tag{20}$$

$$\widehat{\epsilon_X} = -\frac{b}{X}\frac{\partial\widehat{X}/\partial b}{1-E_{xX}}, \tag{21}$$

(with$\left(\partial\widehat{X}/\partial b\right) = \int_{\theta_m}^{+\infty}(\partial x_\theta/\partial b)\, g(\theta)\, d\theta$) are alternative definitions of virtual cost and price elasticity respectively (see Appendix $A.2.3$). Condition (20) accounts for the fact that, depending on the economic environment, the external effects may impact the sole actual consumers or the whole population.[15] Observe that (21) is the price elasticity that would obtain were the number of consumers N kept constant. In fact, $\left(\partial\widehat{X}/\partial b\right)/(1-E_{xX})$ represents the inframarginal change in demand.

Comparing the profit maximizing price b, as defined by (19), with the price level that would be suggested by a naïve application of the Lerner formula (i.e. one that fails to consider the link with the sale of access), is far from straightforward. If either (i) there are no externalities or (ii) agents are affected by the externalities that derive from X irrespective of whether they are connected and consume or not, the alternative virtual marginal cost $\widehat{C}_X$ equals plain marginal cost $(\partial C/\partial X)$. Absent externalities, it is also immediate that $\widehat{\epsilon_X} \leq \epsilon_X$.

[12] Of course $E_{xX} = 1$ is not admissible. However, $E_{xX} > 1$ is hardly admissible too. It would indeed imply that the direct impact on X of a change in either price would trigger an even greater indirect increase of X. So in the following we limit ourselves to situations in which $E_{xX} < 1$.

[13] The reason is that, differently from $\frac{dN}{da}/\frac{dX}{da}$, we have no closed form for $\frac{dN}{db}/\frac{dX}{db}$.

[14] Observe that demand monotonicity implies $x_{\theta_m}/\overline{x} < 1$.

[15] The expressions $\left(\partial S^+_{\theta_m}/\partial X\right)$ and $\left(\partial S^-_{\theta_m}/\partial X\right)$ denote the marginal effect of total consumption on, respectively, the marginal consumer who chooses to access the services and the marginal consumer who does not. If the externality extends to the whole population, their difference amounts to zero and the virtual cost $\widehat{C}_X$ does not differ from the standard marginal cost $(\partial C/\partial X)$. If, on the contrary, external effects impact the sole consumers, this is to be considered as part of the cost of delivering services.

However, the ranking of $\widehat{\epsilon_X}/\left(1-x_{\theta_m}/\overline{x}\right)$ and ϵ_X remains ambiguous since the multiplier $1/\left(1-x_{\theta_m}/\overline{x}\right)$ is greater than 1, so even in this simple case we cannot reach neat conclusions. This contrasts with the determination of the access price a, as defined by (12): in the absence of externalities, $\widetilde{C}_N$ is always smaller than $\left(\partial C/\partial N\right)$, so the profit maximising access price a is always lower than the level that would result from a *naïve* application of the Lerner formula.

Notice also that in the first-best formulae the externalities enter through the terms E_X and E_N, that express them in terms of impacts on social surplus, while here they operate through the terms E_{xN} and E_{xX}, that measure the corresponding impacts on demand (which is what concerns a profit-maximizing provider).

4 Second-best prices

We now turn to the second-best solution which consists in maximizing W subject to the producer break even constraint $\Pi \geq 0$. Obviously this only makes sense under the following:

Assumption 2 *First-best prices as defined by* (7) *and* (8) *do not allow the firm to break-even.*

Let $\mathcal{L}$ be the Lagrangean expression associated with the second-best problem, while λ is the multiplier of the break-even constraint. From Assumption **2** we know that $\lambda > 0$, *i.e.* the constraint is binding. The following first-order conditions hold:

$$\frac{\partial \mathcal{L}}{\partial a} = \int_0^{+\infty} \left[\frac{\partial S_\theta}{\partial X}\frac{dX}{da} + \frac{\partial S_\theta}{\partial N}\frac{dN}{da} - 1_{\theta \geq \theta_m}\right] g(\theta)\, d\theta$$
$$+ (1+\lambda)\left[N + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{da}\right] = 0, \qquad (22)$$

$$\frac{\partial \mathcal{L}}{\partial b} = \int_0^{+\infty} \left[\frac{\partial S_\theta}{\partial X}\frac{dX}{db} + \frac{\partial S_\theta}{\partial N}\frac{dN}{db} - x_\theta(a,b)\, 1_{\theta \geq \theta_m}\right] g(\theta)\, d\theta$$
$$+ (1+\lambda)\left[X + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{db}\right] = 0. \qquad (23)$$

Condition (22)-(23) can be rewritten as (see Appendix A.3)

$$a - \widetilde{C}_N = \frac{\lambda}{1+\lambda}\frac{a}{\epsilon_N} - \frac{1}{1+\lambda}\left[E_N + E_X\left(\frac{dX}{da}/\frac{dN}{da}\right)\right], \qquad (24)$$

$$b - \widetilde{C}_X = \frac{\lambda}{1+\lambda}\frac{b}{\epsilon_X} - \frac{1}{1+\lambda}\left[E_X + E_N\left(\frac{dN}{db}/\frac{dX}{db}\right)\right]. \qquad (25)$$

As shown in Appendix $A.2.2$,

$$\frac{dX}{da} / \frac{dN}{da} = \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}.$$

Thus equation (24) can be rewritten more explicitly as

$$a - \widetilde{C}_N = \frac{\lambda}{1+\lambda} \frac{a}{\epsilon_N} - \frac{1}{1+\lambda} \left(E_N + E_X \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}} \right). \tag{26}$$

Equation (25) has the drawback that it rests on the ratio $\left(\frac{dN}{db} / \frac{dX}{db}\right)$ which does not admit an explicit expression. However, if a is indeed optimally chosen by the monopolist, the latter condition can be rewritten as (See Appendix $A.3$):

$$b - \widehat{C}_X = \frac{\lambda}{1+\lambda} \left(1 - \frac{x_{\theta_m}}{X/N}\right) \frac{b}{\widehat{\epsilon_X}} - \frac{1}{1+\lambda} \left[E_X - N \left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X} \right) \right]. \tag{27}$$

The second-best allocation is thus defined by means of equations (26) and (27) , that can be interpreted along the lines proposed in the previous sections.[16] Yet, these formulae do not lend themselves to handy application either. Indeed, the regulator should avail itself of reliable estimates of the not so obvious magnitudes there contained in order to compute second-best prices. So in the next section we propose a price-cap formula that makes implementation of the second best easier.

5 Decentralization and global price-cap

The previous section presents the pricing policy (a^*, b^*) that would be chosen by a welfare maximizing (and well-informed) regulator subject to a break-even constraint. We now show that in the case of a profit-maximizing provider, the second-best can be obtained through a global price-cap scheme - *i.e.* a constraint imposing an upper limit on the weighted average of the two prices.

5.1 Ideal price-cap

The provider is allowed to choose prices a and b so as to maximize its profits, defined in (11) , under the following regulatory constraint

$$\alpha a + \beta b \leq \bar{p} + \varphi N + \psi X, \tag{28}$$

[16] These second-best conditions remind us of Oum and Tretheway (1988, p. 312), who write: "the markups are computed on the basis of marginal private costs and a fraction of external costs". This applies especially to formulations (24) and (25) where, differently from $\widetilde{C}_N$ and $\widetilde{C}_X$, the external effects are divided by $1 + \lambda$.

where the weights α, β, φ, ψ and the cap $\bar{p}$ are exogenously given to it. The main idea behind this "extended version" of the usual price-cap approach is that, if N or X generate positive (resp. negative) external effects, the firm should be rewarded (resp. punished) for their provision. This is done by relaxing (resp. strengthening) the price-cap $\bar{p}$.

We denote by $\mathcal{M}$ the Lagrangean of the provider's maximisation problem and by μ the multiplier associated to the constraint (28).
First-order conditions of the maximization problem are:

$$\frac{\partial \mathcal{M}}{\partial a} = N + \left(a - \frac{\partial C}{\partial N} + \mu\varphi\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X} + \mu\psi\right)\frac{dX}{da} - \mu\alpha = 0, \quad (29)$$

$$\frac{\partial \mathcal{M}}{\partial b} = X + \left(a - \frac{\partial C}{\partial N} + \mu\varphi\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X} + \mu\psi\right)\frac{dX}{db} - \mu\beta = 0. \quad (30)$$

Equation (29) rewrites simply as (see appendix $A.4$):

$$a - \widetilde{C}_N = \left(1 - \mu\frac{\alpha}{N}\right)\frac{a}{\epsilon_N} - \mu\left(\varphi + \psi\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right). \quad (31)$$

If a is optimally chosen by the firm according to (31), equation (30) rewrites (See Appendix $A.4$):

$$b - \frac{\partial C}{\partial X} + \mu\psi + N\left(1 - \mu\frac{\alpha}{N}\right)\left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right)$$

$$= \left(1 - \mu\frac{\beta}{X}\right)\left[1 - \left(\frac{1 - \mu\alpha/N}{1 - \mu\beta/X}\right)\frac{x_{\theta_m}}{X/N}\right]\frac{b}{\widehat{\epsilon_X}} \quad (32)$$

The second-best is only obtained if equations (22)–(23) are solved by the couple (a, b) that is defined by (31) and (32). Comparing the latter equations, respectively, with (26) and (27), that determine the second-best solution (a^*, b^*), we see that this is the case when

$$\mu = \frac{1}{1 + \lambda^*}, \quad (33)$$

and

$$\alpha = N(a^*, b^*), \quad \beta = X(a^*, b^*), \quad \varphi = E_N^*, \quad \psi = E_X^*, \quad (34)$$

where the upperscript $*$ stands to indicate that the values are those attached to the second-best optimum.

By definition when $a = a^*$ and $b = b^*$, the provider makes zero-profits. Conversely, if the cap $\bar{p}$ is set in such a way that the provider can just break even and if at the same time the weights are set according to (34), then (33) must also be satisfied. Formally:

Proposition 1 (Ideal price-cap) *The price cap defined by* (28) *and* (34) *,with* $\bar{p}$ *such that the firm breaks even but cannot make strictly positive profits, leads a profit-maximizing monopolist to set prices at their second-best levels defined by* (26) *and* (27) *.*

Using (34), the price cap formula (28) becomes, *at optimum*,

$$(a - E_N^*) N (a^*, b^*) + (b - E_X^*) X (a^*, b^*) \leq \bar{p}. \tag{35}$$

This formulation shows that in the ideal price cap formula the provider's earnings are to be computed using accounting prices that take into consideration marginal external effects. Imagine that, as in the telecom example mentioned at the beginning, increasing the number of accesses causes external benefits (due to the enlargement of the network), while increasing the quantity of service causes external costs (due to congestion). According to (35), the provider's revenue from selling accesses is to be computed at a discounted price, while the sale of services is to be computed at a surcharge: the price cap constraint is so designed as to push the provider to sell more connections and fewer calls in the above mentioned circumstances than in the absence of externalities.

A comment is in order concerning the informational requirements of the ideal price cap. According to (34) the weights associated with the externality-generating variables N and X, that is $\varphi = E_N^*$ and $\psi = E_X^*$, are their external marginal costs/benefits. Clearly, if externalities are to be taken into account by the regulator, E_X and E_N are the most straightforward magnitudes one can think of. The weights attached to prices (α and β), instead, are just market demands for the corresponding goods, a standard result for price-cap schemes. So, differently from the direct computation of the second-best allocation through (26) and (27), here the regulator does *not* need to have any knowledge of properties of demand *functions* such as price elasticities ϵ_N and ϵ_X or *demand multipliers* $1/(1 - E_{xN})$ and $1/(1 - E_{xX})$, but only of demand *levels* N and X at the (constrained) optimum.

Yet, one can object that these are not known to the regulator either. However, beside simplicity the price cap approach offers a further implementation advantage: it lends itself to being applied iteratively, with parameters being adjusted at each step so as to obtain better and better approximations to the ideal formula. Here below we propose an iterative formula for the extended price cap considered above, and prove its monotonic convergence under reasonable assumptions. This allows us to presume that, even if some of these assumptions were violated to a limited extent, convergence to the second best would still occur.

5.2 Implementation

We make the following assumptions. Regarding externalities:

Assumption 3 *External effects are linear in N and X, respectively. Formally*

$$E_N(N,X) = \overline{E}_N \quad \text{and} \quad E_X(N,X) = \overline{E}_X.$$

Regarding the regulator:

Assumption 4
(a) *In each period $t = 1, 2...$ the regulatory agency knows prices, quantities and total costs of the previous period* - i.e. $a_{t-1}, b_{t-1}, X_{t-1} \equiv X(a_{t-1}, b_{t-1})$, $N_{t-1} \equiv N(a_{t-1}, b_{t-1})$, *and* $C_{t-1} \equiv C(X_{t-1}, N_{t-1})$.
(b) *The regulation takes the form of the following constraint on prices a and b*

$$\begin{aligned} aN_{t-1} + bX_{t-1} \leq\ & C_{t-1} + \overline{E}_N [N(a,b) - N_{t-1}] \\ & + \overline{E}_X [X(a,b) - X_{t-1}]. \end{aligned} \tag{36}$$

Regarding the provider:

Assumption 5
(a) *In period 0, the last before constraint (36) applies, profit is non-negative; formally* $\Pi_0 \equiv a_0 N_0 + b_0 X_0 - C_0 \geq 0$.
(b) *In each period $t = 1, 2...$ the provider maximizes current profit.*
(c) *Demand and cost functions, $X(a,b)$, $N(a,b)$ and $C(X,N)$ are known to the provider, and do not change over time.*

Finally, regarding the net benefits derived by the population from the services delivered by the monopolist:

Assumption 6 *Net consumer surplus minus external effects as defined by*

$$\begin{aligned} Z(a,b) =\ & \int_0^{+\infty} V_\theta(a,b)\, g(\theta)\, d\theta \\ & - \overline{E}_N N(a,b) - \overline{E}_X X(a,b) \end{aligned}$$

is a convex function of prices a and b.

According to Assumption **5 (b)**, the provider maximizes current profits myopically, ignoring the influence that its current choices exert on how the parameters of the regulatory constraint are set in the following period. Together with Assumptions **4**, **5 (a)** and **5 (c),** this implies that one should only observe nonnegative profits. In fact, by keeping prices unchanged the provider ensures the same profit as in the previous period, and in general can do better. Notice that in case profit were negative and there were suspects of strategic price manipulation, the regulator could stick to the current parameters of the price cap constraint until profits resume positive, as suggested by Vogelsang and Finsinger (1979).

In conventional demand analysis, surplus convexity is implied by the hypothesis that demand is decreasing in prices. As emphasized by De Fraja and Iozzi (2008), surplus convexity cannot be warranted in more complex environments - like theirs or the one of the present paper, where complexity is due to externalities. However, the way Z is defined makes Assumption **6** close to the usual hypothesis that (absent externalities) consumer surplus is a convex function of prices.

The following proposition holds.

Proposition 2 (Implementation) *Under Assumptions* ***- 4- 5 - 6*** *the regulatory mechanism* (36) *yields a pattern of prices that*
(i) *causes social welfare to increase from period to period, and*
(ii) *converges to the (second-best) social optimum.*

(Proof. See Appendix **A.4**).

Observe that perfect foresight rather than myopia would not alter the results substantially: it would only make convergence slower. See Vogelsang and Finsinger (1979), p.167. In fact, a forward-looking firm can indeed induce the regulator to slack the regulatory constraint in the next period, but this can only be done by deviating from profit maximization in the current period. Moreover, the firm can only prevent the regulator from fastening the constraint in the next period by making zero profits in the current period. Thus, in the end, it is not in the firm's interest to block entirely the operation of the mechanism that drives it to the second best.

5.3 Robustness

A natural question in a world of imperfect information is whether the regulatory mechanism proposed in (36) is robust to small errors in estimates of *marginal* externalities E_N and E_X (remember that the optimal choice of parameters as $\varphi = E_N^*$ and $\psi = E_X^*$). We prove the following:

Proposition 3 (Robustness) *Let* $W^R(\varphi, \psi)$ *denote the level of social welfare obtained by applying the regulatory mechanism proposed above, where* φ *and* ψ

are the regulatory parameters defined in (28). *For the parameter values defined in it is* (34) *it is:*

$$\frac{dW^R}{d\varphi} = 0 \quad \textit{and} \quad \frac{dW^R}{d\psi} = 0.$$

(Proof. See Appendix **A.4**).[17]

This shows that, were the parameters of the price cap incorrect, due to small errors by the regulator in estimating the intensity of externalities, the consequences for the level of social welfare are of the second order.

6 Concluding comments

Since the 70s the literature has been quite pessimistic as to the possibility of remedying externalities in practice.[18] See among others Diamond and Mirrlees (1973), and Littlechild (1975), as to consumption externalities and Green and Sheshinski (1976), and Barnett (1980), as to production externalities. The general picture nowadays does not appear to be much more optimistic. As reminded by Cornes and Sandler (1996), the Arrowian approach to externality (incomplete markets) and the Pigouvian remedies that followed (tax and subsidies) are very much rooted in competitive equilibrium analysis. As a consequence, they display intrinsic limitations in circumstances, like the one we consider, where there is imperfect competition. Furthermore, in these environments informational problems are particularly severe as imperfect markets also result in a loss of information regarding both consumers' preferences and producer technology.

In this paper, building on the "new theory of regulation", we prove that, despite the complexity of optimal allocations, these can be decentralized by means of a simple and informationally parsimonious regulatory scheme, namely, an extended price-cap. This also evidences that taxation is by no means the only way to correct for externalities.

More precisely, we consider a monopolistic service provider, assuming that both access and intensity of use can be priced. Externalities may depend on aggregate consumption, number of users, or both. The setup is general enough to allow non-consumers to be affected by the externality. Firstly, we characterize various allocations of interest, including the second-best, defined as the social optimum when the monopolist is subject to the break-even constraint. Secondly

[17] Note that the linearity of external effects is not needed to obtain this robustness result.

[18] For instance Littlechild (1975) declares: "An attempt has been made, however, to characterize the optimal tariffs in terms of operational parameters, such as demand elasticities with respect to price, income, and the number of other subscribers in the system (...) In practice, things are much more complex than assumed here (...) It seems to me doubtful whether the present methods can be extended to give useful insights."

and more importantly, we propose an original regulatory policy of the price-cap category in order to decentralize the second best solution. While direct computation of the second best is quite complex, the proposed mechanism appears to be quite simple. Moreover, it only relies on standard accounting data and direct estimates of marginal external effects generated by aggregate consumption and the number of users. An additional advantage of this mechanism is its transparency. The estimates of its key parameters, the above mentioned marginal externalities, and the criteria for working them out, lend themselves quite straightforwardly to public debate. We also study an iterative application of the price cap that allows for practical implementation, and provide conditions for the process of parameter adjustment to ensure convergence to the second best. Finally, we show that the mechanism is robust as to the possibility that the regulator may rely on incorrect estimates of externalities. More precisely, we prove that such errors only exert second order effects on social welfare.

A promising extension of the analysis carried out in these pages is inclusion of taxes and subsidies into the regulatory framework considered here. We leave it for future research.

7 References

Barnett A. H., [1980], "The Pigouvian Taxe Rule Under Monopoly", *The American Economic Review*, **70**, 1037-1041.

Baron, D., [1985a], "Regulation of Prices and Pollution Under Incomplete Information", *Journal of Public Economics*, **28**(2), 211-231.

Baron, D., [1985b], "Noncooperative Regulation of a Nonlocalized Externality", *RAND Journal of Economics*, 1985b, 16(4), 553-568.

Bergantino A., E. Billette de Villemeur, and A. Vinella [2010], "Partial Regulation in Vertically Differentiated Industry", *Journal of Public Economic Theory*, forthcoming

Blonski, M., [2002] "Network Externalities and Two-Part Tariffs" in *Telecommunication Markets, Information Economics and Policy.* **14**(1), 95-109

Bontems, P. and J.-M. Bourgeon, [2007], "Environmental Policy and Permitting", *Journal of Public Economic Theory*, **9**(5), 771–792.

Cornes, R. and T. Sandler, [1996], *The Theory of Externalities, Public Goods and Club Goods.*

De Fraja, G. and A. Iozzi [2008], "The quest for quality: a quality adjusted dynamic regulatory mechanism", *Journal of Economics and Management Strategy*, **17**(4), 1011-1040.

Diamond, P. and J. Mirrlees, [1973], "Aggregate Production with Consumption Externalities", *Quarterly Journal of Economics*, **87**(1), 1-24.

Green, J. and E. Sheshinski, [1976], "Direct Versus Indirect Remedies for Externalities", *Journal of Political Economy*, **84**, 797-808.

Kanemoto, Y., [2000], "Price and Quantity Competition among Heterogeneous Suppliers with Two-Part Pricing: Applications to Clubs, Local Public Goods, Networks, and Growth Controls", *Regional Science and Urban Economics,* **30**(6), 587-608.

Laffont J.J. and Tirole J. [1990a], "The regulation of multi-product firms, Part 1: theory", *Journal of Public Economics*, **43**, pp. 1–36.

Laffont J.J. and Tirole J. [1990b], "The regulation of multi-product firms, Part 2: Applications to competitive environments and policy analysis", *Journal of Public Economics*, **43**, pp. 37–66.

Littlechild, S.G., [1975]. "Two-part tariffs and consumption externalities", *Bell Journal of Economics and Management Science* **6**, 661–670.

Martin, R. E., [1986], "Externality Regulation and the Monopoly Firm", *Journal of Public Economics* **29**, 347-362.

Mitomo, H., [2001], "The Political Economy of Pricing: Comparing the Efficiency Impacts of Flat Rate vs. Two-Part Tariffs", *Communications and Strategies*, **0**(44): 55-70.

Oum T. H. and M. W. Tretheway, [1988], "Ramsey pricing in the presence of externality costs", *Journal. of Transport Economics and Policy*, **22**(3), 307-317.

Roques, F.A. and N.S. Savva, [2006], "Price Cap Regulation and Investment Incentives under Demand Uncertainty", *Cambridge WP in Economics* 0636.

Sappington D. [2005], "Regulating Service Quality: A Survey", *Journal of Regulatory Economics*, **27**(2), 123-154.

Tanaka, M. [2007], "Extended Price Cap Mechanism for Efficient Transmission Expansion under Nodal Pricing", *Networks and Spatial Economics* **7**, 257–275.

Vogelsang I. and G. Finsinger, [1979], "Regulatory adjustment process for optimal pricing by multiproduct firms", *Bell Journal of Economics*, **10**, 157-171.

A Appendix

A.1 First-Best Allocation

The impacts of the two prices on individual net surplus (or indirect utility function) are respectively

$$\begin{aligned}\frac{dV_\theta}{da} &= \frac{\partial S_\theta}{\partial X}\frac{dX}{da} + \frac{\partial S_\theta}{\partial N}\frac{dN}{da} - 1_{\theta \geq \theta_m},\\ \frac{dV_\theta}{db} &= \frac{\partial S_\theta}{\partial X}\frac{dX}{db} + \frac{\partial S_\theta}{\partial N}\frac{dN}{db} - 1_{\theta \geq \theta_m} x_\theta\left(a,b\right).\end{aligned}$$

Differentiation of (6) with respect to a and b and simple manipulations lead to the following FOCs:

$$\begin{aligned}\frac{dW}{da} &= \left(a - \frac{\partial C}{\partial N} + E_N\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X} + E_X\right)\frac{dX}{da} = 0\\ \frac{dW}{db} &= \left(a - \frac{\partial C}{\partial N} + E_N\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X} + E_X\right)\frac{dX}{db} = 0.\end{aligned}$$

Conditions (9)and (10) are an obvious solution of the system above.

A.2 Profit maximizing prices

A.2.1 Lerner formulae

Maximization of (11) gives rise to the following system of F.O.Cs:

$$\frac{d\Pi}{da} = N + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{da} = 0, \tag{37}$$

$$\frac{d\Pi}{db} = X + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{db} = 0. \tag{38}$$

Equation (37) can be written as

$$a - \frac{\partial C}{\partial N} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{da}/\frac{dN}{da} = \frac{a}{\epsilon_N}, \tag{39}$$

Thanks to definition (16), condition (39) can be written as (12).
Similarly, equation (38) can be written as

$$b - \frac{\partial C}{\partial X} + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{db}/\frac{dX}{db} = \frac{b}{\epsilon_X}, \tag{40}$$

whence (13).

A.2.2 Computation of $\frac{dX}{da}/\frac{dN}{da}$.

A change in a induces a shift in the marginal type θ_m, hence a change in the number of consumers N. More precisely, we know from equation (4) that

$$\frac{dN}{da} = -g\left(\theta_m\right)\frac{d\theta_m}{da}, \tag{41}$$

where, from the monotonicity of V_θ, the derivative $(d\theta_m/da)$ is certainly positive.

As already pointed out, due to quasi-linearity the access fee a has no direct impact on the individual demand of infra-marginal consumers. However it does impact on $x_\theta\left(b, X, N\right)$ indirectly as a consequence of the externalities. To assess this impact we differentiate (2) wrt a, obtaining:

$$\frac{dx_\theta}{da} = \left(\frac{-\partial^2 S_\theta}{\partial x_\theta^2}\right)^{-1}\left[\frac{\partial^2 S_\theta}{\partial x_\theta \partial X}\frac{dX}{da} + \frac{\partial^2 S_\theta}{\partial x_\theta \partial N}\frac{dN}{da}\right]. \tag{42}$$

We now turn to aggregate demand.
By definition,

$$\frac{dX}{da} = \int_{\theta_m}^{+\infty}\frac{dx_\theta}{da}g\left(\theta\right)d\theta - g\left(\theta_m\right)x_{\theta_m}\frac{d\theta_m}{da},$$

whence, thanks to (42) and (41), we obtain

$$\frac{dX}{da} = \frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\frac{dN}{da}. \tag{43}$$

A.2.3 Computation of the modified Lerner formula for price b

We first show that

$$\frac{dX}{db} = \frac{1}{1 - E_{xX}}\frac{\partial \widehat{X}}{\partial b} + \left(\frac{x_{\theta_m} + E_{xN}}{1 - E_{xX}}\right)\frac{dN}{db}; \tag{44}$$

We then establish that:

$$\frac{dN}{db}/\frac{dN}{da} = x_{\theta_m} - \left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right)\frac{1}{1 - E_{xX}}\frac{\partial \widehat{X}}{\partial b}, \tag{45}$$

before to establish the lerner formula equation (19).

Decomposition of $\frac{dX}{db}$: The effect of a change in the price b on the consumption of X can be decomposed into a marginal effect and an infra-marginal effect:

$$\frac{dX}{db} = -x_{\theta_m}g\left(\theta_m\right)\frac{d\theta_m}{db} + \int_{\theta_m}^{+\infty}\frac{dx_\theta}{db}g\left(\theta\right)d\theta. \tag{46}$$

From equation (4), we know that the first term of the right-hand side of (46) writes

$$-x_{\theta_m} g(\theta_m) \frac{d\theta_m}{db} = x_{\theta_m} \frac{dN}{db}. \tag{47}$$

The second term represents the impact of a change in b on infra-marginal consumption. It can in turn be decomposed into a direct and an indirect effect, the latter resulting from the presence of externalities. Indeed, differentiating with respect to b equation (2) that defines individual consumption yields

$$1 = \frac{\partial^2 S_\theta}{\partial x_\theta^2} \frac{dx_\theta}{db} + \frac{\partial^2 S_\theta}{\partial x_\theta \partial X} \frac{dX}{db} + \frac{\partial^2 S_\theta}{\partial x_\theta \partial N} \frac{dN}{db}.$$

It follows that

$$\frac{dx_\theta}{db} = \left(\frac{-\partial^2 S_\theta}{\partial x_\theta^2} \right)^{-1} \left[-1 + \left(\frac{\partial^2 S_\theta}{\partial x_\theta \partial X} \right) \frac{dX}{db} + \left(\frac{\partial^2 S_\theta}{\partial x_\theta \partial N} \right) \frac{dN}{db} \right]$$

so that, by introducing

$$\frac{\partial \widehat{X}}{\partial b} = \int_{\theta_m}^{+\infty} \frac{\partial x_\theta}{\partial b} g(\theta)\, d\theta = \int_{\theta_m}^{+\infty} \left(\frac{\partial^2 S_\theta}{\partial x_\theta^2} \right)^{-1} g(\theta)\, d\theta,$$

we have

$$\int_{\theta_m}^{+\infty} \frac{dx_\theta}{db} g(\theta)\, d\theta = \frac{\partial \widehat{X}}{\partial b} + E_{xX} \frac{dX}{db} + E_{xN} \frac{dN}{db}. \tag{48}$$

The two remaining terms on the right-hand side of (48) reflect the *indirect* effect of a change in b on the demand of infra-marginal consumers.
Finally, plugging (47) and (48) into (46) , yields equation (44).

Computation of $\frac{dN}{db} / \frac{dN}{da}$: As a first step we differentiate wrt a equation (3) that defines the marginal type θ_m, obtaining

$$\begin{aligned} & \frac{\partial S_\theta^+}{\partial x_\theta} \frac{dx_{\theta_m}}{da} + \frac{\partial S_\theta^+}{\partial X} \frac{dX}{da} + \frac{\partial S_\theta^+}{\partial N} \frac{dN}{da} + \frac{\partial S_\theta^+}{\partial \theta} \frac{d\theta_m}{da} - \left(1 + b \frac{dx_{\theta_m}}{da} \right) \\ = \quad & \frac{\partial S_\theta^-}{\partial X} \frac{dX}{da} + \frac{\partial S_\theta^-}{\partial N} \frac{dN}{da} + \frac{\partial S_\theta^-}{\partial \theta} \frac{d\theta_m}{da} \end{aligned}$$

where S_θ^+ stands for $S_{\theta_m}(x_{\theta_m}, X, N)$, *i.e.* the gross surplus of the marginal consumer who actually gets access to the service, while S_θ^- stands for $S_{\theta_m}(0, X, N)$, *i.e.* the gross (and net at the same time) surplus of the marginal consumer who actually opts for not accessing the service. By the envelope theorem, this boils down to:

$$\left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X} \right) \frac{dX}{da} + \left(\frac{\partial S_\theta^+}{\partial N} - \frac{\partial S_\theta^-}{\partial N} \right) \frac{dN}{da} + \left(\frac{\partial S_\theta^+}{\partial \theta} - \frac{\partial S_\theta^-}{\partial \theta} \right) \frac{d\theta_m}{da} = 1. \tag{49}$$

Similarly, differentiating wrt b equation (3) gives

$$\begin{aligned}
&\frac{\partial S_\theta^+}{\partial x}\frac{dx_{\theta_m}}{db}+\frac{\partial S_\theta^+}{\partial X}\frac{dX}{db}+\frac{\partial S_\theta^+}{\partial N}\frac{dN}{db}+\frac{\partial S_\theta^+}{\partial \theta}\frac{d\theta_m}{db}-\left(x_{\theta_m}+b\frac{dx_{\theta_m}}{db}\right)\\
=\;&\frac{\partial S_\theta^-}{\partial X}\frac{dX}{db}+\frac{\partial S_\theta^-}{\partial N}\frac{dN}{db}+\frac{\partial S_\theta^-}{\partial \theta}\frac{d\theta_m}{db}
\end{aligned}$$

hence

$$\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right)\frac{dX}{db}+\left(\frac{\partial S_\theta^+}{\partial N}-\frac{\partial S_\theta^-}{\partial N}\right)\frac{dN}{db}+\left(\frac{\partial S_\theta^+}{\partial \theta}-\frac{\partial S_\theta^-}{\partial \theta}\right)\frac{d\theta_m}{db}=x_{\theta_m}. \tag{50}$$

From (41) and (43) , equation (49) rewrites:

$$\left[\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right)+\left(\frac{\partial S_\theta^+}{\partial N}-\frac{\partial S_\theta^-}{\partial N}\right)-\frac{1}{g\left(\theta_m\right)}\left(\frac{\partial S_\theta^+}{\partial \theta}-\frac{\partial S_\theta^-}{\partial \theta}\right)\right]\frac{dN}{da}=1.$$

From (47) and (44) ,equation (50) rewrites:

$$\begin{aligned}
x_{\theta_m}\;=\;&\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right)\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\\
&+\left[\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right)+\left(\frac{\partial S_\theta^+}{\partial N}-\frac{\partial S_\theta^-}{\partial N}\right)-\frac{1}{g\left(\theta_m\right)}\left(\frac{\partial S_\theta^+}{\partial \theta}-\frac{\partial S_\theta^-}{\partial \theta}\right)\right]\frac{dN}{db}
\end{aligned}$$

hence equation (45) .

The final step: Thanks to equation (44) we can rewrite the FOC (38) as

$$\begin{aligned}
0\;=\;&X+\left(b-\frac{\partial C}{\partial X}\right)\left(\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right)\\
&+\left(a-\frac{\partial C}{\partial N}+\left(b-\frac{\partial C}{\partial X}\right)\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right)\frac{dN}{db}
\end{aligned} \tag{51}$$

If the price a is set to its profit-maximizing level (12) with the virtual connection cost formulated as in (18), the previous equation boils down to:

$$0=X+\left(b-\frac{\partial C}{\partial X}\right)\left(\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right)-N\left(\frac{dN}{db}/\frac{dN}{da}\right). \tag{52}$$

Plugging in equation (45) we obtain

$$0=X+\left(b-\frac{\partial C}{\partial X}\right)\left(\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right)-N\left[x_{\theta_m}-\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right)\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right],$$

whence

$$b-\left[\frac{\partial C}{\partial X}-N\left(\frac{\partial S_{\theta_m}^+}{\partial X}-\frac{\partial S_{\theta_m}^-}{\partial X}\right)\right]=\frac{\left(1-E_{xX}\right)\left(Nx_{\theta_m}-X\right)}{\partial \widehat{X}/\partial b},$$

that is

$$\frac{b-\left[\frac{\partial C}{\partial X}-N\left(\frac{\partial S_{\theta_m}^{+}}{\partial X}-\frac{\partial S_{\theta_m}^{-}}{\partial X}\right)\right]}{b}=\frac{1-x_{\theta_m}/\left(X/N\right)}{\left[-\frac{b}{X}\frac{\partial \widehat{X}/\partial b}{1-E_{xX}}\right]}.$$

Using the definitions of $\widehat{C}_X$ and $\widehat{\epsilon_X}$ and $\overline{x}$ one immediately obtains equation (19).

A.3 Second_best

A.3.1 Computation of equation (24)

Making use of notations (10) and (9), the FOC condition (22) yields directly

$$-\lambda N=\left[E_N+(1+\lambda)\left(a-\frac{\partial C}{\partial N}\right)\right]\frac{dN}{da}+\left[E_X+(1+\lambda)\left(b-\frac{\partial C}{\partial X}\right)\right]\frac{dX}{da} \tag{53}$$

from which, dividing by (dN/da), rearranging and using the definition (14), we obtain

$$\lambda\frac{a}{\epsilon_N}=\left(E_N+E_X\frac{dX/da}{dN/da}\right)+(1+\lambda)\left[\left(a-\frac{\partial C}{\partial N}\right)+\left(b-\frac{\partial C}{\partial X}\right)\frac{dX/da}{dN/da}\right].$$

Another division by $(1+\lambda)$ and recourse to definition (16) yields equation (24).

A.3.2 Computation of equation (27)

Making use of notations (10) and (9), the FOC condition (23) yields directly

$$-\lambda X=\left[E_N+(1+\lambda)\left(a-\frac{\partial C}{\partial N}\right)\right]\frac{dN}{db}+\left[E_X+(1+\lambda)\left(b-\frac{\partial C}{\partial X}\right)\right]\frac{dX}{db}, \tag{54}$$

whence, using (44) we obtain:

$$\begin{aligned}-\lambda X \;=\;& (1+\lambda)\left[\left(b-\frac{\partial C}{\partial X}\right)+\frac{1}{1+\lambda}E_X\right]\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\\ &+\left(E_N+E_X\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right)\frac{dN}{db}\\ &+(1+\lambda)\left[a-\frac{\partial C}{\partial N}+\left(b-\frac{\partial C}{\partial X}\right)\left(\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right)\right]\frac{dN}{db}.\end{aligned}$$

Thanks to (53), the later equation rewrites first as

$$\begin{aligned}&+\left(E_N+E_X\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right)\\ &+(1+\lambda)\left[a-\frac{\partial C}{\partial N}+\left(b-\frac{\partial C}{\partial X}\right)\left(\frac{x_{\theta_m}+E_{xN}}{1-E_{xX}}\right)\right]\end{aligned}$$

$$-\lambda X = (1+\lambda)\left[\left(b - \frac{\partial C}{\partial X}\right) + \frac{1}{1+\lambda}E_X\right]\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b} - \lambda N\left(\frac{dN}{db}/\frac{dN}{da}\right)$$

and then, using (45), as

$$-\lambda X = (1+\lambda)\left[\left(b - \frac{\partial C}{\partial X}\right) + \frac{1}{1+\lambda}E_X\right]\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b} - \lambda N\left[x_{\theta_m} - \left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right)\frac{1}{1-E_{xX}}\frac{\partial \hat{X}}{\partial b}\right].$$

Using the price elasticity of infra-marginal consumers $\widehat{\epsilon_X}$ defined in (21) and dividing by $(1+\lambda)\frac{1}{1-E_{xX}}\frac{\partial \hat{X}}{\partial b}$, we get:

$$b - \frac{\partial C}{\partial X} + \frac{1}{1+\lambda}E_X + \frac{\lambda}{1+\lambda}N\left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right) = \frac{\lambda}{1+\lambda}\left(1 - \frac{x_{\theta_m}}{X/N}\right)\frac{b}{\widehat{\epsilon_X}}$$

Equation (27) is obtained by plugging in the definition of virtual marginal cost (20).

A.4 Regulation and Global Price Cap

A.4.1 Computation of equation (31)

By using (43), equation (29) rewrites directly as:

$$-N + \mu\left[\alpha - \left(\varphi + \psi\frac{x_{\theta_m} + E_{xN}}{1-E_{xX}}\right)\frac{dN}{da}\right] = \left[a - \frac{\partial C}{\partial N} + \left(b - \frac{\partial C}{\partial X}\right)\frac{x_{\theta_m} + E_{xN}}{1-E_{xX}}\right]\frac{dN}{da}. \tag{55}$$

It follows that

$$a - \frac{\partial C}{\partial N} + \left(b - \frac{\partial C}{\partial X}\right)\frac{x_{\theta_m} + E_{xN}}{1-E_{xX}} = \left(1 - \mu\frac{\alpha}{N}\right)\frac{a}{\epsilon_N} - \mu\left(\varphi + \psi\frac{x_{\theta_m} + E_{xN}}{1-E_{xX}}\right).$$

Using the definition of virtual connection cost $\widetilde{C}_N$ (16), one gets directly (31).

A.4.2 Computation of equation (32)

Plugging equations (44) and (45)into equation (30) gives

$$\begin{aligned}
0 &= X - \mu\beta + \left(b - \frac{\partial C}{\partial X} + \mu\psi\right)\left(\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right) \\
&\quad + \left[a - \frac{\partial C}{\partial N} + \mu\varphi + \left(b - \frac{\partial C}{\partial X} + \mu\psi\right)\frac{x_{\theta_m} + E_{xN}}{1-E_{xX}}\right]\frac{dN}{db}, \text{ or else} \\
0 &= X - \mu\beta + \left(b - \frac{\partial C}{\partial X} + \mu\psi\right)\left(\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right) \\
&\quad + \left[a - \frac{\partial C}{\partial N} + \left(b - \frac{\partial C}{\partial X}\right)\frac{x_{\theta_m} + E_{xN}}{1-E_{xX}} + \left(\mu\varphi + \mu\psi\frac{x_{\theta_m} + E_{xN}}{1-E_{xX}}\right)\right] \\
&\quad \cdot\left(x_{\theta_m} - \left(\frac{\partial S_\theta^+}{\partial X} - \frac{\partial S_\theta^-}{\partial X}\right)\left[\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right]\right)\frac{dN}{da}.
\end{aligned}$$

Using (55), the square parenthesis can be replaced by $N\left(1-\mu\frac{\alpha}{N}\right)/\left(\frac{dN}{da}\right)$, so, after rearrangement, one obtains

$$\begin{aligned} 0 &= X\left[\left(1-\mu\frac{\beta}{X}\right)-\left(1-\mu\frac{\alpha}{N}\right)\frac{x_{\theta_m}}{X/N}\right] \\ &+\left(b-\frac{\partial C}{\partial X}+\mu\psi+N\left(1-\mu\frac{\alpha}{N}\right)\left(\frac{\partial S_\theta^+}{\partial X}-\frac{\partial S_\theta^-}{\partial X}\right)\right)\left(\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}\right). \end{aligned}$$

Dividing both sides by $\frac{1}{1-E_{xX}}\frac{\partial \widehat{X}}{\partial b}$ and using (21) yields (32).

A.4.3 Proof of Proposition 2

We first prove *(i)*. Assumption **4 (c)** ensures us that

$$\frac{\partial Z}{\partial a}=-N \quad \textit{and} \quad \frac{\partial Z}{\partial b}=-X.[19] \tag{55}$$

So, thanks to Assumption **6** it is

$$\begin{aligned} V_t-V_{t-1} \quad &\geq \quad E_N^0\left[N_t-N_{t-1}\right]+E_X^0\left[X_t-X_{t-1}\right] \\ &\quad -N_{t-1}\left(a_t-a_{t-1}\right)-X_{t-1}\left(b_t-b_{t-1}\right), \end{aligned} \tag{56}$$

where $V_t=\int_0^{+\infty}V_\theta\left(a_t,b_t\right)g\left(\theta\right)d\theta$ denotes net consumer surplus at date t.[20] If the regulatory constraint (36) holds true, it must be the case that

$$\begin{aligned} &\left(a_t-a_{t-1}\right)N_{t-1}+\left(b_t-b_{t-1}\right)X_{t-1}\leq \\ &-\Pi_{t-1}+E_N^0\left[N_t-N_{t-1}\right]+E_X^0\left[X_t-X_{t-1}\right], \end{aligned} \tag{57}$$

or

$$E_N^0\left[N_t-N_{t-1}\right]+E_X^0\left[X_t-X_{t-1}\right]\geq-\left(a_t-a_{t-1}\right)N_{t-1}-\left(b_t-b_{t-1}\right)X_{t-1}+\Pi_{t-1} \tag{58}$$

[19] In all cases,

$$\begin{aligned} \frac{\partial Z}{\partial a} &= -N+\left(E_N-E_N^0\right)\frac{dN}{da}+\left(E_X-E_X^0\right)\frac{dX}{da}, \\ \frac{\partial Z}{\partial b} &= -X+\left(E_N-E_N^0\right)\frac{dN}{db}+\left(E_X-E_X^0\right)\frac{dX}{db}. \end{aligned}$$

In the particular case where $E_N=\bar{E}_N=E_N^0$ and $E_X=\bar{E}_X=E_X^0$, it is

$$\frac{\partial Z}{\partial a}=-N \quad \text{and} \quad \frac{\partial Z}{\partial b}=-X.$$

[20] Indeed, convexity of $Z\left(a,b\right)$ implies that

$$Z_t-Z_{t-1}\geq\left.\frac{\partial Z}{\partial a}\right]_{t-1}\left(a_t-a_{t-1}\right)+\left.\frac{\partial Z}{\partial b}\right]_{t-1}\left(b_t-b_{t-1}\right).$$

Substituting (58) into (56) yields

$$V_t - V_{t-1} \geq \Pi_{t-1}, \tag{59}$$

so it is

$$W_t - W_{t-1} \equiv \Pi_t - \Pi_{t-1} + V_t - V_{t-1} \geq \Pi_t \geq 0: \tag{60}$$

social welfare W_t does indeed increase from period to period.

Point *(ii)* results from the fact that increasing series that are bounded from above must converge. From (59) and profit positivity we know that the series V_t is also increasing.We know that both W_t and V_t are bounded from above, as well. So both series converge. The same can be said of their difference $\Pi_t = W_t - V_t$ (which is *not* necessarily monotonic). By (60), it must be the case that lim $\Pi_{t\rightarrow+\infty} = 0$.

Under Assumption **4 - 5,** the provider sets the prices a_t and b_t as to maximize its profits as defined in (11), under the global price-cap constraint (36) that can be rewritten as:

$$\alpha a_t + \beta b_t \leq \bar{p} + \varphi N(a_t, b_t) + \psi X(a_t, b_t), \tag{61}$$

where $\alpha = N_{t-1}$, $\beta = X_{t-1}$, $\varphi = E_N^0$, $\psi = E_X^0$ and $\bar{p} = C_{t-1} - E_N^0 N_{t-1} - E_X^0 X_{t-1}$. This says that the prices (a_t, b_t) satisfy both (31) and (32) together with $\Pi(a_t, b_t) = \Pi_t$. If the policy (a^*, b^*) is well defined, there is a unique solution to the system (31)-(32) that also satisfies $\Pi(a, b) = 0$. As a result, the sequence of prices (a_t, b_t) converges to the socially optimal prices defined in section **4**.

A.4.4 Robustness

Let $\left(a^R, b^R\right)$ define the prices of, respectively, N and X that are set by the profit-maximising monopolist when *(i)* it is subject to the regulatory constraint (28) and *(ii)* it exactly breaks-even. Obviously, both prices (and quantities N and X as well) depend upon the parameters in the cap, in particular φ and ψ. We denote by $W^R(\varphi, \psi)$ social welfare as a function of these two parameters, defined as $W^R(\varphi, \psi) = W\left(a^R, b^R\right)$. Similarly, $\Pi^R(\varphi, \psi)$ denotes firm's profits in such a situation. In what follows, we investigate the impact upon social welfare of small changes in φ and/or ψ.

By definition, if the firm exactly breaks even, $\Pi^R(\varphi, \psi) \equiv 0$. Differentiating, one gets

$$\begin{aligned} \frac{d\Pi^R}{d\varphi} &= \frac{\partial \Pi^R}{\partial a}\frac{da^R}{d\varphi} + \frac{\partial \Pi^R}{\partial b}\frac{db^R}{d\varphi} = 0, \\ \frac{d\Pi^R}{d\psi} &= \frac{\partial \Pi^R}{\partial a}\frac{da^R}{d\psi} + \frac{\partial \Pi^R}{\partial b}\frac{db^R}{d\psi} = 0. \end{aligned}$$

We also know that $(\partial\Pi/\partial a)$ and $(\partial\Pi/\partial b)$ can be decomposed as follows:

$$\begin{aligned}\frac{\partial\Pi}{\partial a} &= N + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{da}, \\ \frac{\partial\Pi}{\partial b} &= X + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{db}.\end{aligned}$$

Therefore it is:

$$\begin{aligned}0 &= \left[N + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{da}\right]\frac{da^R}{d\varphi} \\ &\quad + \left[X + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{db}\right]\frac{db^R}{d\varphi} \qquad (62)\end{aligned}$$

and

$$\begin{aligned}0 &= \left[N + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{da}\right]\frac{da^R}{d\psi} \\ &\quad + \left[X + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{db}\right]\frac{db^R}{d\psi}. \qquad (63)\end{aligned}$$

Similarly, changes in total welfare can be decomposed in two parts:

$$\begin{aligned}\frac{dW^R}{d\varphi} &= \frac{\partial W^R}{\partial a}\frac{da^R}{d\varphi} + \frac{\partial W^R}{\partial b}\frac{db^R}{d\varphi}, \\ \frac{dW^R}{d\psi} &= \frac{\partial W^R}{\partial a}\frac{da^R}{d\psi} + \frac{\partial W^R}{\partial b}\frac{db^R}{d\psi},\end{aligned}$$

where

$$\begin{aligned}\frac{\partial W}{\partial a} &= \left(a - \frac{\partial C}{\partial N} + E_N\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X} + E_X\right)\frac{dX}{da}, \\ \frac{\partial W}{\partial b} &= \left(a - \frac{\partial C}{\partial N} + E_N\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X} + E_X\right)\frac{dX}{db}.\end{aligned}$$

Now, since the firm is subject to regulation, there exists a real parameter μ such that

$$N + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{da} = \mu\left[\alpha - \varphi\frac{dN}{da} - \psi\frac{dX}{da}\right], \quad (64)$$

$$X + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{db} = \mu\left[\beta - \varphi\frac{dN}{db} - \psi\frac{dX}{db}\right]. \quad (65)$$

(A re-statement of the FOCs (29)-(30)). It follows that

$$\begin{aligned}\frac{\partial W^R}{\partial a} &= E_N\frac{dN}{da} + E_X\frac{dX}{da} + \mu\left[\alpha - \varphi\frac{dN}{da} - \psi\frac{dX}{da}\right] - N, \\ \frac{\partial W^R}{\partial b} &= E_N\frac{dN}{db} + E_X\frac{dX}{db} + \mu\left[\beta - \varphi\frac{dN}{db} - \psi\frac{dX}{db}\right] - X,\end{aligned}$$

for some μ.

Consider now the impact on social welfare of small errors in the parameter φ. By definition:

$$\begin{aligned}
\frac{dW^R}{d\varphi} &= \frac{\partial W^R}{\partial a}\frac{da^R}{d\varphi} + \frac{\partial W^R}{\partial b}\frac{db^R}{d\varphi} \\
&= \left\{ E_N \frac{dN}{da} + E_X \frac{dX}{da} + \mu \left[\alpha - \varphi \frac{dN}{da} - \psi \frac{dX}{da} \right] - N \right\} \frac{da^R}{d\varphi} \\
&\quad + \left\{ E_N \frac{dN}{db} + E_X \frac{dX}{db} + \mu \left[\beta - \varphi \frac{dN}{db} - \psi \frac{dX}{db} \right] - X \right\} \frac{db^R}{d\varphi}.
\end{aligned}$$

Now (62) together with (64)-(65) also tells us that

$$\begin{aligned}
0 &= \left[N + \left(a - \frac{\partial C}{\partial N} \right) \frac{dN}{da} + \left(b - \frac{\partial C}{\partial X} \right) \frac{dX}{da} \right] \frac{da^R}{d\varphi} \\
&\quad + \left[X + \left(a - \frac{\partial C}{\partial N} \right) \frac{dN}{db} + \left(b - \frac{\partial C}{\partial X} \right) \frac{dX}{db} \right] \frac{db^R}{d\varphi} \\
&= \mu \left[\alpha - \varphi \frac{dN}{da} - \psi \frac{dX}{da} \right] \frac{da^R}{d\varphi} + \mu \left[\beta - \varphi \frac{dN}{db} - \psi \frac{dX}{db} \right] \frac{db^R}{d\varphi};
\end{aligned}$$

Thus $\left(dW^R / d\varphi \right)$ simplifies to

$$\begin{aligned}
\frac{dW^R}{d\varphi} &= \left\{ E_N \frac{dN}{da} + E_X \frac{dX}{da} - N \right\} \frac{da^R}{d\varphi} \\
&\quad + \left\{ E_N \frac{dN}{db} + E_X \frac{dX}{db} - X \right\} \frac{db^R}{d\varphi}.
\end{aligned}$$

Moreover the identity obtained above

$$0 = \left[\alpha - \varphi \frac{dN}{da} - \psi \frac{dX}{da} \right] \frac{da^R}{d\varphi} + \left[\beta - \varphi \frac{dN}{db} - \psi \frac{dX}{db} \right] \frac{db^R}{d\varphi},$$

rewrites

$$0 = \left[N^* - E_N^* \frac{dN}{da} - E_X^* \frac{dX}{da} \right] \frac{da^R}{d\varphi} + \left[X^* - E_N^* \frac{dN}{db} - E_X^* \frac{dX}{db} \right] \frac{db^R}{d\varphi}.$$

for the parameter values defined in (34). Thus, at the second best, as defined by (22)–(23) and for the parameter values defined in (34) it is

$$\frac{dW^R}{d\varphi} = 0.$$

In words, an error on the parameter $\varphi \equiv E_N^*$ only exerts a second order impact on welfare.

Similarly, by definition:

$$\begin{aligned}\frac{dW^R}{d\psi} &= \frac{\partial W^R}{\partial a}\frac{da^R}{d\psi} + \frac{\partial W^R}{\partial b}\frac{db^R}{d\psi} \\ &= \left\{E_N\frac{dN}{da} + E_X\frac{dX}{da} + \mu\left[\alpha - \varphi\frac{dN}{da} - \psi\frac{dX}{da}\right] - N\right\}\frac{da^R}{d\psi} \\ &+ \left\{E_N\frac{dN}{db} + E_X\frac{dX}{db} + \mu\left[\beta - \varphi\frac{dN}{db} - \psi\frac{dX}{db}\right] - X\right\}\frac{db^R}{d\psi}.\end{aligned}$$

Now (63) together with (64)-(65) also tells us that

$$\begin{aligned}0 &= \left[N + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{da} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{da}\right]\frac{da^R}{d\psi} \\ &+ \left[X + \left(a - \frac{\partial C}{\partial N}\right)\frac{dN}{db} + \left(b - \frac{\partial C}{\partial X}\right)\frac{dX}{db}\right]\frac{db^R}{d\psi} \\ &= \mu\left[\alpha - \varphi\frac{dN}{da} - \psi\frac{dX}{da}\right]\frac{da^R}{d\psi} + \mu\left[\beta - \varphi\frac{dN}{db} - \psi\frac{dX}{db}\right]\frac{db^R}{d\psi}\end{aligned}$$

Thus $\left(dW^R/d\psi\right)$ simplifies to

$$\begin{aligned}\frac{dW^R}{d\varphi} &= \left\{E_N\frac{dN}{da} + E_X\frac{dX}{da} - N\right\}\frac{da^R}{d\psi} \\ &+ \left\{E_N\frac{dN}{db} + E_X\frac{dX}{db} - X\right\}\frac{db^R}{d\psi}.\end{aligned}$$

Moreover the identity

$$0 = \left[\alpha - \varphi\frac{dN}{da} - \psi\frac{dX}{da}\right]\frac{da^R}{d\psi} + \left[\beta - \varphi\frac{dN}{db} - \psi\frac{dX}{db}\right]\frac{db^R}{d\psi},$$

rewrites

$$0 = \left[N^* - E_N^*\frac{dN}{da} - E_X^*\frac{dX}{da}\right]\frac{da^R}{d\psi} + \left[X^* - E_N^*\frac{dN}{db} - E_X^*\frac{dX}{db}\right]\frac{db^R}{d\psi}.$$

for the parameter values defined in (34) .It follows that, at the second best, as defined by (22)–(23) and for the parameter values defined in (34) it is

$$\frac{dW^R}{d\psi} = 0.$$

In words, an error on the parameter $\psi \equiv E_X^*$ has only a second order impact on welfare.