From Pigou to Extended Liability:  
On the Optimal Taxation of Externalities under Imperfect Financial Markets.*

Jean Tirole †

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Abstract

Pigovian taxation of externalities has limited appeal if the tortfeasor has insufficient resources to pay the damage when it occurs. To defend Pigovian taxation in the presence of judgment-proof agents, its proponents point at the many institutions extending liability to third parties. Yet little is known about the validity of Pigou’s analysis in this context.

The paper analyzes the costs and benefits of extended liability and investigates whether full internalization is called for in the presence of agency costs between potential tortfeasors and providers of guarantees. Its contribution is two-fold. It first shows that the better the firms’ corporate governance and the stronger their balance sheet, the more closely taxes should track the corresponding externality. It then develops the first analysis of extended liability when guarantors themselves may be judgment-proof and the extension of liability may give rise to further externalities. Relatedly, it derives the curvature of the optimal taxation of externalities in a multi-plant firm.

Keywords: Pigovian taxation, externalities, financial constraints, extended liability, snowball effects.

JEL numbers: D62, D82, H23, J65, Q58

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†Toulouse School of Economics.
1 Introduction

An important contribution of economics to public policy rests on the precept that price signals should force producers of externalities to internalize the welfare of other economic agents. Pigou (1920)’s celebrated insight on the taxation of externalities provides an intellectual foundation for a variety of policies from pollution taxes/permits to experience rating; it also underlies the expectation damages remedy for breach in commercial law.

From a policy perspective, though, Pigou’s internalization principle — the requirement that the polluter pay the damage inflicted on the pollutee — raises the concern that the polluter have insufficient resources to pay the damages when it occurs. In legal terms, the polluter may be “judgment-proof”. Shallow pockets may substantially reduce the policy appeal of Pigovian taxation. And indeed, many environmental, personal or financial obligations are evaded through bankruptcy and abandonment of operations.  

Proponents of Pigovian taxation retort that, in the presence of solvency concerns, potential polluters should be forced to contract with other corporate entities or financial intermediaries who will step in and guarantee payments in case of damage. Such extended liability prevents polluters from escaping their duties.

In support of this view, many vehicles for extending liabilities to a third party have developed, either spontaneously or through regulation, over the years. Mandatory car insurance covers damages inflicted on third parties. Banks and governments guarantee firms’ payments to suppliers (through letters of credit) or workers (through unemployment insurance funds). Deposit insurance funds and capital adequacy requirements protect depositors against poor risk management by financial institutions. In some payment systems, banks can increase their overdraft facility by securing some guarantee from other banks. Manufacturing units often benefit from an explicit or implicit backing of external claims by a parent company. Extended liability is also ubiquitous for environmental damages.

1Boyd (2002) provides extensive evidence on unmet environmental liabilities. Ringleb and Wiggins (1990) demonstrate that, following the introduction of liability laws, the number of small corporations increases substantially. The interpretation is that firms try to evade liabilities by creating judgment-proof entities in risky activities.

2In the US, CERCLA (1990) requires insurance or financial coverage for firms dealing with hazardous waste. The European directive 2004/35/CE similarly calls for Member States to take measures to encourage the use of insurance.

3As Boyd (2002) notes.

“Financial assurance is demanded of a wide variety of U.S. commercial operations, including municipal landfills, ships carrying oil or hazardous cargo, hazardous waste treatment facilities, offshore oil and gas installations, underground gas tanks, wells, nuclear power stations, and mines. Firms needing assurance can purchase it in the form of insurance, surety obligations, bank letters of credit, and deposit certificates. Alternatively, firms can establish trust funds or escrow accounts dedicated to future obligations. Most
This paper revisits Pigovian thinking by considering financially constrained agents monitored by financial intermediaries, business partners, or parent companies. The framework is otherwise a standard one in which the agent, a firm, may impose a negative externality on a third party, a worker, the environment or the taxpayer. The firm, run by an entrepreneur, has limited financial resources. The state can demand extended liability: that is ask the intermediary to pay part of or the full penalty when the firm is judgment-proof. Extending liability however can improve welfare only if the intermediary has more information than the state. In the tradition of corporate finance, the paper thus assumes that the intermediary acts as a delegated monitor. As usual in that field, the income that is pledgeable to financial intermediaries is

not the firm’s total surplus, as the entrepreneur enjoys a rent.

Several questions can be raised in this context. First, what are the costs of extended liability? Should firms and their guarantors be held accountable for the full externality, as Pigou suggested, or, to the contrary, should some of the social cost be socialized? Second, can the extension of liability to other industrial companies jeopardize otherwise healthy activities? 

The paper obtains two sets of insights:

1. **Well-diversified guarantors.** We first assume that the guarantor is never judgment proof himself. One may have in mind a well-diversified financial intermediary. A natural benchmark principle is then the “delegation principle”: Because the guarantor is better informed about the potential tortfeasor than the state, the cost of the externality should be passed through to the guarantor, who then writes an optimal second-best contract with the firm. This delegation principle extends the validity of Pigovian reasoning to firms with limited funds but potential guarantors.\(^4\)

We derive conditions under which this delegation principle holds, and unveil two exceptions. The first departure from the delegation principle occurs when the contract between the firm and the intermediary is not second best due to ex post renegotiation. We find that a partial extension of liability, by strengthening the intermediary’s bargaining position, helps the firm obtain financing and raises social welfare. We relate the extension of liability to the firm’s strength of balance sheet and to the quality of monitoring. In particular, liability is extended more, the stronger the firm’s balance sheet.

The second exception occurs when firms are heterogenous. A partial extension of

\(^4\)This delegation principle is broader than that offered by Segerson and Tietenberg (1992) in that it allows intermediaries to be imperfectly informed about potential tortfeasors.
liability then favors financially weak firms, who are more likely to cause the damage, to the detriment of stronger ones. Limited extended liability allows the state to keep the financially weak firms “on board” while keeping externalities at a reasonable level.

2) **Snowball effects and judgment-proof guarantors.** Parent companies, customers, suppliers, or other industrial companies may be best placed to monitor the potential tortfeasor. An extension of liability to them may jeopardize their own activities. That is, judgment proofness may propagate through the guaranteeing chain. Similarly, banks may face hardship when the borrowers they guarantee fail, and re-insurers may not be able to cover the insurance companies’ losses. Worse still, the extension of liability may lead to the extension of externalities. For example, a monitor who is asked to cover the cost of layoffs by the monitoree may be forced to lay workers off himself.

Such snowball effects are investigated in a tractable “nested information structure” model of independent interest. To reflect the possibility that guarantors themselves be judgment-proof, we consider a model with $n$ ex ante symmetric firms with limited funds and mutually monitoring each other. Each firm faces uncertainty; for example, in the layoff interpretation, each firm may face a productivity shock that makes continued employment less attractive. Using the employment interpretation as an illustration, the realization of uncertainty unfolds in the following way: First, a random number $m \in \{0, \cdots, n\}$ of firms are unviable, leading to as many closures. Second, the common productivity $y$ of the $n-m$ remaining and potentially viable firms is drawn from a continuous distribution $G_{n-m}(y)$. The number $m$ is called the “breadth” of the shock, while the distributions $G_{n-m}(\cdot)$ are ordered through the standard hazard-rate comparison, giving rise to a notion of “depth” of the shock. The state knows neither the breadth nor the depth of the shock.

It is shown that it is optimal to (partially) extend liability if breadth and depth are negatively correlated. Intuitively, if the shock on potentially viable firms is likely to be small (in a stochastic sense) when other firms shut down, then recovering the cost of the externality through extended liability creates only limited snowball effects.

By contrast, liability is not extended (despite mutual monitoring) when shocks are independent. And liability is “negatively extended” (virtuous firms receive support when others create externalities) if breadth and depth are positively correlated. The overarching principle is thus the use of the information conveyed by the existence of externalities about the ability of others to bear extended liability. The other key lesson is that snowball effects substantially mitigate the benefits of extended liability.

This $n$-firm snowball effect admits a re-interpretation in terms of a single, financially constrained firm with $n$ different activities. The results then imply that the per-layoff tax schedule is decreasing (respectively constant, increasing) with the number of layoffs
if the breadth and the depth of the shock are negatively (respectively not, positively) correlated.

The paper is organized as follows: Section 2 sets up the basic model, and shows that it applies to a variety of policy environments, including experience rating, environmental taxation and prudential regulation. Section 3 derives the optimal level of extended liability when firms are ex ante homogenous and shows how policy should account for the firms’ balance sheet strength and for capital market imperfections. It then extends the analysis to ex ante firm heterogeneity. Section 4 investigates the extension of liability to other industrial companies and analyzes snowball effects. Finally, section 5 concludes with potential alleys for research.

*Relationship to the literature:* There is by now a large literature on the effects of extended liability on the level of environmental care, which I will not try to review exhaustively (a useful collection of state-of-the-art contributions can be found in Boyer et al 2007). This paper differs from this literature in several important respects. First, unlike a number of papers it allows a full range of instruments for the government. Namely, the latter can set up a fund (for example, a deposit or unemployment insurance fund) collecting money from firms regardless of whether they exert externalities. The absence of a fund creates a role for third parties as conduits for the state; with full instrument range by contrast, the financial intermediary or business partner is not needed for levying the contribution to the fund and extended liability makes sense only if the financial intermediary or business partner monitors the tortfeasor. We accordingly cast the argument in a (rather general) corporate finance framework.

Second, it focuses on different effects; in particular, starting from the delegation principle as a Pigovian benchmark, it unveils three factors, the renegotiation of financial contracts, the heterogeneity of firms, and snowball effects that call for limits to extended liability. It also contains the first analysis of extended liability with financially constrained guarantors.

Pitchford (1995) makes the important points that the tortfeasor’s incentives to exert care need to be preserved, and that an increase in lender liability, because it must be offset by a payment to the lender in the absence of accident, reduces the tortfeasor’s incentives. Thus in such an “efficiency wage framework” in which managerial incentives are traded off against rents, full extended liability is in general dominated by partial lender liability. The lender in the model is a conduit for government policy, collecting income from the firm on the government’s behalf as the government cannot levy contributions to a fund in the absence of accident; the (partial) extension of liability is one way of implementing
the government’s optimal policy and is not needed if the government can impose direct liability and collect taxes for a fund, as is assumed in this paper. Balkenborg (2001) extends Pitchford’s analysis by assuming that the lender has market power instead of being competitive. Che and Spier (2008) investigate the potential gaming of liability through the choice of financial structure. They show that an optimal regulation of the financial structure involves a limited amount of senior debt together with a more junior tort claim, further borrowing being treating as even more junior.

Several papers (e.g., Boyer and Laffont 1997, Balkenborg 2004) have analyzed extended liability in an adverse selection context. Boyer and Laffont consider a Bolton-Scharfstein (1990) model of refinancing. They first show that the firm optimally avoids the common agency externality that arises when one entity finances the investment and another guarantees the liability. The government chooses a level of liability and trades off the tortfeasor’s informational rent and the shadow cost of public funds. When the latter is low, the bank does not renew its financing often enough from a social efficiency perspective; less-than-full extended liability is then one way of subsidizing the firm and thereby reinforcing its balance sheet. Subsidization of credit constrained firms is always suboptimal in our model since the government puts no (or less than full) weight on their welfare, and so the Boyer-Laffont effect is absent.

Hiriart and Martimort (2006, 2007), like this paper, allow for a full range of instruments and posit that the entrepreneurs and principals (the analog of investors in this paper’s model) form a coalition. Their work is in the tradition of the common agency literature, with investors not internalizing the impact of their incentive contract with the entrepreneur on social welfare. This common agency problem is costly for society if and only if the principal has substantial bargaining power; extended liability then is a way to extract some of the principal’s rent. By contrast, in this paper the principal (competitive investors) has no rent, and so the Hiriart-Martimort effect is absent.

The paper ignores monitoring incentives by the state. This is of course a gross oversimplification. In practice, the state defines standards, checks guarantors’ solvency, and monitors firm behavior both ex ante and ex post (for example through court investigations, as in Boyer and Porrini 2008). The effects studied in this paper would however still be present with a more pro-active state.

Finally, the snowball section is related to the literature on financial contagion. Allen and Gale (2000) formalizes contagion as resulting from an incompleteness in the structure of claims. Closer in spirit to this paper, Rochet and Tirole (1996) studies “optimal

contagion” in a bilateral banking relationship as part of an incentive scheme meant to induce monitoring. Section 4 abstracts from the monitoring incentive by assuming that mutual monitoring within a group of \( n \) firms is cheap, and studies whether joint liability can help control externalities onto third parties; furthermore, as already stated, it contains a first analysis of the taxation of multiple externalities by a financially constrained firm.

2 Model

2.1 Set up

Because experience rating is one of the most important policy applications of Pigovian taxation we will couch our model in a labor context; as we will later show, simple relabellings transform the model into ones of environmental taxation or prudential regulation. The model is similar to that in Blanchard and Tirole (2008), except for the assumption that firms have shallow pockets and may resort to financial intermediaries.

Figure 1 summarizes the model’s timing and notation.

- **Government sets**
  - unemployment benefit \( \mu \)
  - layoff tax \( f \) and payroll tax \( \tau \).

- **Firms**
  - invest \( I \)
  - hire worker.

- **Worker’s productivity** \( y \)
  - realized (drawn from distribution \( G \)).

- **Firm**
  - keeps worker if \( y \geq \bar{y} \)
  - fires worker if \( y < \bar{y} \),
  - for some endogenous threshold \( \bar{y} \).

The economy is composed of a continuum of mass 1 of workers, a continuum of mass (at least) 1 of entrepreneurs, and the state. Tastes and technology are as follows:

- **Entrepreneurs** are risk neutral. Each entrepreneur can start and run a firm (free entry). There is a fixed cost of creating a firm, \( I \), which is the same for all entrepreneurs.

If a firm is created, a worker is hired, and the productivity of the match is then revealed. Productivity is given by \( y \) and is drawn from cdf \( G(y) \), with density \( g(y) \) on \((-\infty, +\infty)\). The firm can either keep the worker and produce, or lay the worker off, who then becomes unemployed. Realizations are iid across firms; that is, there is no aggregate risk.\(^6\)

\(^6\)Clearly this assumption is least unrealistic when the unemployment insurance scheme is economy-wide. Sectoral insurance schemes are much more prone to aggregate shocks, as demonstrated by the West Virginia Coal Workers’ mutual compensation insurance scheme.
The entrepreneur, but neither the worker nor the state (also called the “unemployment insurance fund” or “government”), observes $y$. The worker and the state observe only whether the worker remains employed or not.

The representative entrepreneur has initial assets $A \geq 0$ and is protected by limited liability. $A$ can exceed or be smaller than $I$. Firms have the same net worth and so are ex ante identical. Section 3.3 extends the analysis to allow for ex ante firm heterogeneity, but for the moment we keep the model as simple as possible in order to highlight its key features.

- **Workers** are risk averse, with utility function $U(.)$. When remaining employed, a worker receives wage $w$ and utility $U(w)$. Absent unemployment benefits, utility if unemployed is given by $U(b)$ (so $b$ is the wage equivalent of being unemployed). When receiving unemployment benefit $\mu$, the unemployed worker has utility $U(b+\mu)$. Workers are not subject to moral hazard, either on the job or when unemployed.\(^8\) They just need to be insured.

The *first-best policy* consists in providing the worker with full insurance ($w = b + \mu$) and in keeping the worker whenever the realized income $y$ exceeds the worker’s opportunity cost $b$. Compensating the worker when unemployed (or cleaning up a bankrupt firm’s site in the environmental context) however may be made difficult by the fact that firms are likely to have no income when shutting down. Money must somehow be levied on healthy ones in order to cover the unemployment benefit. This can be accomplished in two ways: a payroll tax (a mandatory contribution to a clean-up fund in the environmental context), at the cost of encouraging more shutdown; and extended liability, at the cost of making it more difficult for firms to raise funds (or find industrial partners). We accordingly turn to the model’s two other actors: the state and the investors.

- **The state** collects payroll tax $\tau$ and layoff tax $f$ and distributes unemployment benefit $\mu$. We could allow the state to set a job creation subsidy $\sigma$ ($\sigma$ negative in case of a tax). but this instrument is redundant if the firm is financed by a deep-pocket intermediary.

\(^7\)In particular, and as we will posit, the state can levy a layoff tax, but cannot engage in ex post monitoring to try to assess whether the layoff was “justified”. In contrast, Hiriart et al (2007) allow ex post monitoring as well as ex ante monitoring in a model in which the firm can collude with its monitor(s). They analyze whether the ex ante and ex post monitoring functions should be merged or separated.

\(^8\) See Blanchard and Tirole (2008) for an analysis of the impact of worker incentive considerations on optimal employment protection in an environment in which firms do not face financial constraints. Incomplete worker insurance (due to moral hazard on the job or aimed at providing job search incentives) implies that a dismissal exerts a negative externality both on the unemployment insurance fund and on the dismissed worker, calling for more-than-full experience rating (a related result is in Mongrain-Roberts 2005, who show that the absence of unemployment insurance calls for excessive retention).
as we will shortly assume.⁹ “Full experience rating” corresponds to \( f = \mu \), i.e. to a contribution rate \( c \equiv f/\mu \) equal to 1. The state must break even.

Given free entry of firms, it makes sense to focus on the maximization of worker welfare. Note, though, that our results would carry through even if the firms could make a profit and the state put some weight on that profit, as long as the state puts a higher weight on consumer welfare. Given policy \( \{\tau, f, \mu\} \) and the entrepreneurs’ free-entry condition and/or the financiers’ break-even condition, the wage \( w \) will be market determined. Thus even though the government does not directly set \( w \), its policy indirectly sets it, and so in the optimizations below we will just consider that the government sets \( w \) through proper choices of instruments.

We posit the existence of an (endogenous) cutoff productivity \( \bar{y} \) above which the firm continues and retains the worker (as will be the case in the examples below).⁹ The state’s goal is to maximize the workers’ expected utility over instruments \( \{\tau, f, \mu\} \):

\[
\max_{\{\tau, f, \mu\}} \{G(\bar{y})U(b + \mu) + (1 - G(\bar{y}))U(w)\}. \tag{1}
\]

Its budget constraint writes:

\[
-G(\bar{y})(\mu - f) + (1 - G(\bar{y}))\tau = 0. \tag{2}
\]

We will later add two constraints to this maximization problem: the entrepreneurs’ participation and financing constraints.

- **Investors.** There are deep-pocket, risk-neutral (or well-diversified) investors. If these investors are uninformed (i.e., if, like the state, they can only observe whether workers are employed or unemployed), their presence does not alter the set of feasible utilities. Whatever the investors do, the state could already do by itself. Accordingly, and in line with the policy-oriented literature,¹¹ we assume that investors act as delegated monitors. As in Hiriart and Martimort (2006), we view the firm and its investors as forming a coalition that acts optimally to the government’s policy. In particular, the coalition sets the threshold \( \bar{y} \) under which the firm lays the worker off. The simplest interpretation is that the entrepreneur and investors contract and then jointly observe the realization of \( y \); they then implement the continuation decision specified in the contract (continue if

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⁹It is also redundant if there is no intermediary and the firm has enough funds to finance the initial investment \((A \geq I)\). So its only purpose is to make up for a stand-alone firm’s missing equity, \( I - A \), if any.

¹⁰The private determination of a cut-off \( \bar{y} \) is the analog of the choice of prevention care in environmental economics models. Both are moral hazard variables that impact the level of externalities. See Section 2.4.

¹¹E.g., Boyd (2002).
and only if $y \geq \bar{y}$). But it need not be the case that investors observe $y$ already at the
date of the continuation decision. More generally, the key modeling feature is that the
coalition’s contract impacts the probability of layoff and therefore the level of externality.

We assume that the presence of a monitor is optimal. Like in the corporate finance
literature, though, monitoring involves a cost $m$.

We assume that for the first-best continuation rule, the investment is socially valuable:

**Assumption 1 (positive net present value)**

$$\int_b^{+\infty} (y - b)dG(y) > I + m.$$  

Monitoring is imperfect and the entrepreneur enjoys a rent in case of continuation.
Investors cannot receive ex post more than the pledgeable portion $y - R(y, \bar{y})$ of the firm’s
income. In general, managerial rents can take a variety of forms.

**Assumption 2 (managerial rent)**

The entrepreneur’s realized rent is a non-decreasing function of realized productivity $y$ and
a non-increasing function of the cut-off $\bar{y}$ above which production occurs and the worker
is retained:

$$R = R(y, \bar{y}).$$

The entrepreneur’s rent is equal to 0 in case of shutdown.

The no-rent-if-shutdown property will result from optimal contracting between entre-
preneurs and investors in the examples below. It should be noted, though, that our results
carry over to the more general case in which the entrepreneur receives rent $R_0$ in case of
shutdown, as long as it is lower than the rent that he would receive under continuation.
To keep expressions simple, and because it is often satisfied by corporate finance models,
we assume that $R_0 = 0$.

Investors, who are competitive and have deep pockets, lend $I - A$ to entrepreneurs
(if $I > A$; otherwise, we can adopt the accounting convention that they receive $A - I$).
We ignore for the moment potential incentive issues with regard to monitoring. Section

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12Suppose that $y = \pi - \rho$, where $\pi$ is a known payoff in case of continuation and $\rho$ is a liquidity
shock (with ex ante cumulative distribution $1 - G(\pi - \rho)$) that must be withstood in order to continue.
Suppose that only the entrepreneur observes $\rho$. Still the investors can control the continuation decision
by providing the entrepreneur with liquid assets (or a credit line) $\bar{\rho} = \pi - \bar{y}$.

13Section 3.4 will allow for endogenous monitoring (which includes the possibility of no monitoring).

3.4 generalizes the analysis to an endogenous choice of monitoring intensity by the intermediary. One may for the moment have in mind that the investors spend a screening cost $m$ (otherwise, they are exposed to liability on other types of “bad firms”), that, as a byproduct, makes them knowledgeable about the entrepreneur’s activity and write the financial contracts as posited above.\footnote{Relaxing this assumption would lead us to add the constraint that the entrepreneur not sign contracts with non-monitoring investors. It is immediate to check that the extra constraint is non-binding in the “ex ante contracting” case below. With ex post contracting, one can find conditions under which non-monitoring contracts are not viable.}

- **Value and rents.**

As usual, it is useful to start with the real allocation (characterized by the cutoff $\bar{y}$ and the utilities), and then proceed to examine whether the optimal allocation can be implemented by available instruments (here $\mu, f, \tau$). In this spirit, let

$$N(\bar{y}) \equiv \int_{\bar{y}}^{+\infty} (y - b) dG(y)$$

and

$$R(\bar{y}) \equiv \int_{\bar{y}}^{+\infty} R(y, \bar{y}) dG(y)$$

denote the net social productivity and the entrepreneur’s expected rent for cut-off $\bar{y}$.

Assumption 2 implies that the entrepreneur ex post benefits from a more lenient continuation policy:

**Corollary 1** (entrepreneur stake in a lenient continuation rule)

$$R'(\bar{y}) = \frac{d}{d\bar{y}} \left[ \int_{\bar{y}}^{+\infty} R(y, \bar{y}) dG(y) \right] < 0.$$ 

The expected net productivity $N(\bar{y})$ is single-peaked and is maximized at $\bar{y} = b$.

**Assumption 3** (single peakedness of investor revenue)

The function $N(\bar{y}) - R(\bar{y})$ is single-peaked in the cut-off $\bar{y}$. From Corollary 1, its maximum is reached at some $\bar{y}^* > b$.

For example, when the rent depends only on the realized income ($R(y, \bar{y}) = r(y)$), then $\bar{y}^* - r(\bar{y}^*) = b$.

- **Investors-entrepreneur coalition.**
Given public policy \( \{\tau, f, \mu\} \), the net value for the coalition is given by:  

\[
V_E - (I + m) = -G(\bar{y})f + \int_{\bar{y}}^{+\infty} (y - w - \tau) dG(y) - (I + m).
\]  

(3)

Because the investors break even, the entire surplus goes to the entrepreneur, who therefore receives net utility:  

\[
V_E - (I + m).
\]

To produce, the entrepreneur must secure financing from the investors. Given the presence of non-pledgeable income, the maximum amount that the entrepreneur can pledge is given by:  

\[
V_I \equiv -G(\bar{y})f + \int_{\bar{y}}^{+\infty} (y - w - \tau - R(y, \bar{y})) dG(y)
\]  

(4)

Let \( \bar{Y}^+ \) denote the value that maximizes pledgeable income.  

For example, when \( R(y, \bar{y}) = r(y) \), then \( \bar{Y}^+ - r(\bar{Y}^+) = w + \tau - f \).

For investors to be willing to lend funds in amount \( I - A \), the following condition must hold:

\[
V_I \geq (I + m) - A.
\]  

(5)

The entrepreneur will not receive more than \( R(y, \bar{y}) \) whenever the financing constraint \( (5) \) is binding.

The optimal contract between the entrepreneur and the investors is illustrated graphically in Figure 2:

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\[16\] Positing, without loss of generality, that the entrepreneur brings his entire net worth \( A \) to the venture and that the investors pay the full layoff tax, the net utilities are

\[
\int_{\bar{y}}^{+\infty} R(y, \bar{y}) dG(y) - A + t
\]

for the entrepreneur, and

\[
-G(\bar{y})f + \int_{\bar{y}}^{+\infty} [y - (w + \tau - R(y, \bar{y}))] dG(y) - (I + m - A) - t
\]

for the investors, where \( t \geq 0 \) denote the expected extra rent granted to the entrepreneur (equivalently, he brings only \( A - t \) upfront and keeps \( t \)). The investors must break even (condition (4) below), which, substituting for \( t \), yields (3). \( t > 0 \) at the optimal policy corresponds to a non-binding financing constraint.

\[17\] The entrepreneur’s gross utility is then \( A + (V_E - I - m) \).

\[18\] Note that \( \bar{Y}^+ \) is defined in a way analogous to \( \bar{y}^* \), but for an arbitrary institutional environment \( \{w, \tau, f\} \), while \( \bar{y}^* \) is relative to the more specific situation in which the “net wage bill” is equal to the worker’s opportunity cost \( w + \tau - f = 0 \).
For $A \geq A_1$, the bilaterally efficient cut-off $\bar{y} = w + \tau - f$ prevails, as it allows investors to break even. For $A_1 > A \geq A_2$, the entrepreneur, whose preferred cut-off is $w + \tau - f$, must make a concession to investors and accept less continuation (increase $\bar{y}$). When $A$ falls to $A_2$ ($\bar{y}$ goes to $\bar{y}^+$), though, no more concessions can raise the pledgeable income and the investment can no longer be financed.

When $A < A_1$, there is less continuation than under the ex-post efficient rule ($\bar{y} > w + \tau - f$). Yet, ex-post renegotiation is not an issue, as long as $\bar{y} \in [w + \tau - f, \bar{y}^+]$, as value cannot be increased (thereby generating potential gains from renegotiation) without reducing the pledgeable income. The existence of an incompressible entrepreneurial rent makes the utility in part non-transferable, and hampers renegotiation in this region. This remark, together with Figure 2, implies that we can ignore the possibility of renegotiation of $\bar{y}$.

**Discussion: nature of externality and role of monitoring.**

In this model, the externality exerted by the investors-entrepreneur coalition relates to the choice of the layoff threshold $\bar{y}$. When contemplating whether to preserve the job, the coalition does not internalize the wage loss incurred by the worker (or, equivalently, the cost for the unemployment insurance fund if the worker is insured against unemployment).

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$^{15}$The externality is here linked to the choice of continuation, e.g., of how much liquidity the entrepre-
It is also worth commenting on the role of monitoring. In this basic model, monitoring by an intermediary allows investors to recoup income in good states of nature. It thereby harnesses the firm’s contributive capability and allows the firm to (indirectly, through investors) pay a layoff tax in bad states of nature. This is obviously an important aspect of monitoring. Another important aspect consists for the monitor in exerting her control rights so as to change the distribution of income; alternatively the monitor may affect what happens to the worker in case of layoff: By providing on-the-job generalist training or by making its employees more connected and visible to the outside, the firm can reduce the length of unemployment (which is equivalent to increasing b in this model).20 Monitors thus can exercise their control rights to either reduce the probability of bankruptcy/layoff or to improve the sake of laid-off workers. As Section 3.4 however will show, our analysis fully carries over to this other dimension of monitoring, showing that there is no substantive difference between the “contributive-capability-enhancement” and the “ex-ante intervention” views of monitoring.

2.2 Examples

Example 1 (stealing or incentive payment): Suppose that the entrepreneur can steal a fraction $\beta$ of realized income. Equivalently, $\beta y$ could be associated with an incentive payment aimed at preventing the entrepreneur from taking (output-contingent) private benefits. Then

$$R(y, \bar{y}) = \beta y 1_{\{y \geq \bar{y}\}}.$$

The monitoring cost can in this example be viewed as a cost required to prevent the entrepreneur from stealing the entire output.21

Example 2 (perks and prestige from office): Suppose that the entrepreneur enjoys non-monetary private benefit $B$ from being in office.22 Then

$$R(y, \bar{y}) = B 1_{\{y \geq \bar{y}\}}.$$ 

In a milder version, the entrepreneur’s private benefit would depend on the firm’s scale or labor force, bringing Example 2 closer to Example 1. Examples 1 and 2 can more

20Interestingly, experience rating in the US is assessed ex post on the realized length of unemployment of laid-off workers.

21Recent work in corporate finance that emphasizes diversion or tunneling includes Ellingsen and Kristiansen (2008).

22In the presence of a private benefit, we adopt the convention that total productivity $y$ includes the private benefit $B$, so the monetary revenue is only $y - B$. 

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generally be combined as
\[ R(y, \bar{y}) = r(y) \mathbb{1}_{\{y \geq \bar{y}\}}, \]
with \( 0 \leq r(y_2) - r(y_1) < y_2 - y_1 \) for \( y_2 > y_1 \) (less-than-unitary increments).

In Examples 1 and 2, \( N - R \) is trivially single-peaked, with
\[ \bar{y}^* - r(\bar{y}^*) = b, \]
that is:
\[ \bar{y}^* = \frac{b}{1 - \beta} \quad \text{(Example 1)} \quad \text{and} \quad \bar{y}^* = b + B \quad \text{(Example 2)}. \]

Given \( w, \tau \) and \( f, \)
\[ \frac{\partial V_E}{\partial y} = -(\bar{y} - w - \tau + f)g(\bar{y}) \quad \text{and} \quad \frac{\partial V_I}{\partial y} = -(\bar{y} - w - \tau + f - r(\bar{y}))g(\bar{y}). \]
\( \bar{y}^+ \) is defined (uniquely since increments in \( r \) are less than unitary) by:
\[ \bar{y}^+ - r(\bar{y}^+) = w + \tau - f. \]

**Example 3 (ex post bargaining):** In this example, there is no incompressible rent per se. implying that the gains from trade between entrepreneur and investors can be shared arbitrarily between them. However, the entrepreneur can renegotiate ex post and possibly get more of the gains from trade than was specified ex ante when contracting with investors. Suppose that the investors observe realized productivity \( y \), but that the entrepreneur is indispensable for production and can capture a fraction \( \beta \in (0, 1) \) of the ex post rent of the entrepreneur-financiers coalition through ex post bargaining about managerial compensation.

Given policy \( \{\mu, \tau, f\} \), the coalition wants to continue whenever this raises the joint payoff of the entrepreneur and the investors:
\[ y - (w + \tau) \geq -f \Leftrightarrow y \geq \bar{y} = w + \tau - f. \]
The rent from continuation to be shared is then
\[ y - (w + \tau) + f = y - \bar{y}. \]

Ex post bargaining about managerial compensation\(^2\) allocates a fraction \( \beta \) (the entre-
\[ \text{A rather different analysis of ex post bargaining is conducted in Blanchard and Tirole (2008). There, under ex post wage bargaining (firms and workers bargain over wages after investments have been sunk), an increase in the layoff tax makes it easier for workers to capture some of the upside profit. Given the firms' breakeven condition, these upside gains are offset by lower wages or lower unemployment benefits when the firm is less productive, destroying insurance overall. Ex post wage bargaining thus calls for less-than-full experience rating. Note that bargaining occurs between the firm and the workers and that the latter are risk averse. Risk aversion is key to the extent that ex post wage bargaining destroys insurance. Here the firm and its financiers are both risk neutral.} \]

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preneur’s bargaining power) to the entrepreneur:

\[ R(y, \overline{y}) = \beta(y - \overline{y}) \mathbb{1}_{\{y \geq \overline{y}\}}. \]

The parameter \( \beta \) can be viewed as a measure of “bad governance.” In more general models of corporate finance, the index of governance quality is taken to be the fraction of income (here, \( 1 - \beta \)) that investors can put their hands on. So, by a slight abuse of terminology, we will say that corporate governance improves when \( \beta \) decreases.

Under ex post bargaining

\[ N' - \mathcal{R}' \equiv (b - \overline{y})g(\overline{y}) + \beta(1 - G(\overline{y})). \]

The cut-off rule \( \overline{y}' \) that maximizes pledgeable income is given by:

\[ \overline{y}' = b + \beta \frac{1 - G(\overline{y}')} {g(\overline{y}')}. \]

and is unique provided that the cumulative distribution \( G \) satisfies the standard \textit{monotone hazard rate assumption} that \( g(y)/[1 - G(y)] \) is nondecreasing.

### 2.3 Optimal government policy

In the tradition of mechanism design, our strategy will consist in obtaining an upper bound on what the state can achieve, and then turning to the implementation issue. Using the government budget constraint

\[ -G(\overline{y})(\mu - f) + [1 - G(\overline{y})] \tau \geq 0 \]

to eliminate taxes from the constraints, the optimization problem can be stated as:

\[
\max_{\{w, \mu, \overline{y}\}} \{ G(\overline{y})U(b + \mu) + (1 - G(\overline{y}))U(w) \}
\]

subject to:

\[
-G(\overline{y})\mu + \int_{\overline{y}}^{+\infty} (y - w) dG(y) \geq I + m
\]

\[
-G(\overline{y})\mu + \int_{\overline{y}}^{+\infty} [y - w - R(y, \overline{y})] dG(y) \geq I + m - A.
\]

At the optimum, the workers are fully insured:

\[ w = b + \mu. \]
And so we can rewrite the optimal government policy program as:

$$\max_{\{\mu, \bar{y}\}} \{\mu\}$$  \hspace{1cm} (I)

subject to the entrepreneur’s participation and investors’ break-even constraints:

$$\mu \leq \int_{\bar{y}}^{+\infty} (y - b)dG(y) - I - m$$
$$\mu \leq \int_{\bar{y}}^{+\infty} [y - b - R(y, \bar{y})]dG(y) - I - m + A.$$

Program 1 can thus be rewritten as:

$$\max \{\mu\}$$
$$\{\mu, \bar{y}\}$$

s.t.

$$\mu \leq N(\bar{y}) - (I + m)$$ \hspace{1cm} (6)

$$\mu \leq N(\bar{y}) - R(\bar{y}) - (I + m) + A.$$ \hspace{1cm} (7)

### 2.4 Reinterpretations

While we couched our model in terms of labor regulation, the same program (I), with slight changes in notation, captures several other important applications of the Pigovian principle.

#### 2.4.1 Environmental taxation

In the environmental reinterpretation, the state aims at minimizing pollution. Minimizing pollution, just like maximizing worker income or welfare in the labor context, is constrained by the creation margin, that the entrepreneurs be enticed to create firms and that they be able to attract financing. A firm has uncertain productivity $y$ with distribution $G$ on $(-\infty, +\infty)$. It further uses a facility that will require a clean-up cost $\gamma$ whether it stops operating.\(^{24}\) The pollution damage in the absence of depollution is $D > \gamma$. Letting $X \in [0, 1]$ denote the fraction of shut-down facilities that are cleaned up, the first-best allocation solves:

$$\max_{\{\bar{y}, X\}} \left\{ \int_{\bar{y}}^{+\infty} ydG(y) - [\gamma X + D(1 - X)]G(\bar{y}) \right\}$$

\(^{24}\)One may have in mind that maintenance during operations make this clean-up cost irrelevant (disused industrial buildings, gas pumps,...). But the results are unchanged if active firms also pay a cost for keeping the site clean.
This yields a full clean-up, \( X = 1 \), and a continuation threshold \( \bar{y} = -\gamma \).

In a second-best environment, the state does not directly control the continuation threshold \( \bar{y} \) and is subject to a budget constraint. Each active firm contributes a tax \( \tau \) to a government-run environmental fund, that covers some of the clean-up costs associated with firms that have ceased operations. Let \( f \) denote the firm (entrepreneur plus investors)’s liability when closing a site. The fund’s budget constraint is:

\[
1 - G(\bar{y}) \tau = G(\bar{y}) (\gamma X - f).
\]

We assume that the state wants firms to exist (they deliver some sufficiently large benefit, which we omit). Letting, as earlier, \( R(y, \bar{y}) \) denote the entrepreneurial rent in case of continuation, the state’s objective is to maximize consumer surplus, i.e., to minimize pollution:

\[
\max_{\{X, \bar{y}\}} \left\{ - (1 - X) G(\bar{y}) D \right\}
\]

subject to the participation conditions for entrepreneur and investors:

\[-G(\bar{y}) f + \int^{+\infty}_{\bar{y}} (y - \tau) dG(y) \geq I + m \]

and

\[-G(\bar{y}) f + \int^{+\infty}_{\bar{y}} \left[ y - \tau - R(y, \bar{y}) \right] dG(y) \geq I + m - A.\]

Using the government’s budget constraint and letting \( \mu \equiv G(\bar{y}) \gamma (X - 1) \) denote (minus) the social cost of uncleared facilities, this program can be rewritten as:

\[
\max_{\{\mu, \bar{y}\}} \{ \mu \} \\
\text{st} \\
\mu \leq \int^{+\infty}_{\bar{y}} (y + \gamma) dG(y) - (I + m + \gamma) ;
\]

and

\[
\mu \leq \int^{+\infty}_{\bar{y}} \left[ y + \gamma - R(y, \bar{y}) \right] dG(y) - (I + m + \gamma) + A.
\]

which is nothing but (I) (with \( b = -\gamma \) and \( m \) replaced by \( m + \gamma \)). The social productivity of continuing, which was equal to \( y - b \) in the labor market paradigm, is here equal to \( y + \gamma \).

Because the environmental context figures prominently in the debate on extended liability, it is worth commenting further on the role of intermediaries. As was discussed
in Section 2.1, the foundations of extended liability stem from two equally important effects of monitoring: contributive-capability enhancement and ex-ante intervention. The former, modeled here, builds on the fact that when the firm exerts externalities in bad states of nature, the government can ask for a contribution by intermediaries and trading partners, who share some of the firm’s rent in good states of nature. According to the ex-ante intervention view by contrast, the intermediary for example intervenes in the firm’s management so as to force the firm to provide generalist training and thereby reduce the unemployment length in case of layoff (labor paradigm) or (its counterpart in the environmental paradigm) to reduce the probability of having to clean up the site when it is shut down.\textsuperscript{25} Section 3.4 will show that the contributive capability-enhancement and ex-ante-intervention views lead to identical results.

2.4.2 Prudential regulation

Depositors deposit savings equal to 1 in a financial institution which invests the money into an asset with random return \( y \) with distribution \( G \) on \((-\infty, +\infty)\). Let \( \mu \) denote the gross return for the depositors. Healthy financial institutions pay \( \tau \) to the deposit insurance fund, which guarantees the full return \( \mu \) to depositors even if the financial institution fails. Each financial institution must secure collateral \( f \) from other financial institutions (which together must pay cost \( m \) to monitor); this collateral goes to the deposit insurance fund in case of failure. Let, as before, \( y \) and \( \bar{y} \) denote the financial institution’s realized income and threshold. Then, the deposit insurance fund’s budget constraint writes:

\[
G(\bar{y})(\mu - f) = |1 - G(\bar{y})|\tau.
\]

The state’s objective is to maximize depositors’ welfare. Its program is therefore:

\[
\max_{\{\mu, \bar{y}\}} \{\mu\}
\]

s.t.

\[
u \leq \int_{\bar{y}}^{+\infty} ydG(y) - (I + m)
\]

and

\[
u \leq \int_{\bar{y}}^{+\infty} [y - R(y, \bar{y})]dG(y) - (I + m) + A.
\]

and is a special case of Program 1.

\textsuperscript{25}This can be formalized by letting \( X_v(m) \), an increasing function of \( m \), denote the fraction of sites that do not need to be cleaned up when closed. The clean-up cost is then equal to \( G(\bar{y})\gamma[X - X_v(m)] \). The other expressions are unchanged.
3 The extension of liability to financial intermediaries

3.1 The social optimum

Returning to Program I, we are led to consider four regions:

(a) **Deep-pocket firms** ($A \geq A^*$):

In this region, the production-efficient policy ($\bar{y} = b$) is optimal and is implemented through “Pigovian” full experience rating ($f = \mu$). The financing constraint (7) is not binding and $\mu$ is determined by (6):

$$\mu = N(b) - (I + m).$$

The entrepreneur’s minimum net worth for this regime to prevail is given by:

$$A^* = \int_{b}^{+\infty} R(y, b) dG(y).$$

That is, the entrepreneur must have enough cash up front to pay for its future rent under production-efficient continuation.

The deep-pocket case always prevails in an Arrow-Debreu/no-agency-cost world ($\beta = 0$ in the ex post bargaining and the stealing examples, $B \equiv 0$ in the private benefit one). Then only (6) is binding, and so at the optimum of the program, $\bar{y} = b$, which, together with $\bar{y} = w + \tau - f$ (which always holds in the absence of agency cost) and the government’s budget constraint yields full experience rating: $\mu = f$.

More generally, full experience rating prevails when the agency cost is small and $A$ is large. In the following, we will progressively reduce $A$, but a similar exercise can be performed with respect to an increase in the agency cost.

(b) **Average balance sheet** ($A^{**} \leq A < A^*$):

In this region, efficient production $\bar{y} = b$ does not generate enough pledgeable income. The firm must shut down over a wider range of realizations of $y$ in order to boost pledgeable income.

Pledgeable income is raised by increasing $\bar{y}$ toward $\bar{y}^*$ (where $\bar{y}^*$ was defined in Assumption 3 as the maximizer of $N - R$). In this region, both the entrepreneur’s participation constraint (6) and the financing constraint (7) are binding and

$$R(\bar{y}) = A.$$  \hspace{1cm} (8)
Then
\[ \mu = N(\bar{y}) - (I + m). \]

\( A^{**} \) is given by:
\[ \mathcal{R}(\bar{y}^*) = A^{**}. \]

(c) Weak balance sheet \((A^{***} \leq A < A^{**})\)

Pledgeable income can no longer be increased once the cut-off \(\bar{y}^*\) has been reached. In this third region, only the financing constraint \((7)\) is binding (the firm enjoys an ex ante rent) and
\[
\mu = \int_{\bar{y}^*}^{+\infty} [y - b - R(y, \bar{y}^*)] dG(y) - I - m + A \\
= N(\bar{y}^*) - I - m - [\mathcal{R}(\bar{y}^*) - A].
\]

This region holds as long as investment benefits workers, i.e. \( \mu \geq 0 \), or \( A \geq (I + m) + \mathcal{R}(\bar{y}^*) - N(\bar{y}^*) \). In this region, in which entrepreneurs enjoy a rent, the optimal solution maximizes pledgeable income so as to transfer as much as possible to the worker.

(d) Destitute firms \((A < A^{***})\)

Finally, if the value of \( \mu \) given by \((9)\) is negative, investment is not consistent with the workers’ interests.

Figure 3 depicts the socially optimal policy as a function of the representative firm’s wealth \( A \). For \( A \geq A^* \), firms have deep pockets and so production is efficient. As \( A \) decreases, firms must make concessions to investors by hoarding less liquidity and therefore liquidate more often. When the net worth falls below \( A^{**} \), further increasing the probability of liquidation (\( \bar{y} \) above \( \bar{y}^* \)) hurts even investors and therefore no longer attracts them. Wage (and unemployment benefit) then make up one-for-one for the lack of funds.
Figure 3

The resulting cut-off rule, expressed as a function of the firm’s balance sheet strength (measured by $A$), is depicted in Figure 4.

Figure 4

**Proposition 1.** There exist thresholds $A^* > A^{**} > A^{***}$ for the firm’s strength of balance sheet, such that:
\[ b < \bar{y} < \bar{y}^* \quad \text{for} \quad A^* > A > A^{**} \]

and

\[ \bar{y} = \bar{y}^* \quad \text{for} \quad A^{**} \geq A \geq A^{***}, \]

where \( \bar{y}^* \) is the cut-off that maximizes pledgeable income;

- for \( A^{***} > A \), there is no investment.

### 3.2 Implementation: modified Pigovian taxation

As we discussed, the externality relates to the fact that the entrepreneur (and investors) does not internalize the loss \( w - b = \mu \) incurred by the worker when laid off. Because the worker is optimally insured, this externality is de facto transferred to the unemployment insurance fund. A key focus of the implementation of the optimal allocation is therefore the extent to which the firm internalizes the cost \( \mu \) that a layoff imposes on the unemployment insurance fund. This (gross) contribution rate \( c \) is defined by

\[ c = \frac{\hat{f}}{\mu} \]

and is unitary under Pigovian taxation since Pigovian taxation consists in setting a tax equal to the marginal externality at the optimal solution.

Note that an a priori better definition (which we will use in Sections 3.3 and 4) is in terms of the net contribution

\[ \hat{c} = \frac{\hat{f}}{\mu} = \frac{f - \tau}{\mu} \]

since the firms’ incentive to lay workers off is determined by the net layoff tax, \( f - \tau \). However, as long as firms are homogenous (unlike in Section 3.3) and there is a single activity (unlike in Section 4),

\[ 1 - \hat{c} = \frac{1 - c}{1 - G(\bar{y})} \]

from the government’s budget constraint, and so the concepts of full or partial Pigovian taxation are identical under the two definitions.

**Deep pocket firms.**

If \( A > A^* \), firms have deep pockets. A unit contribution rate \( c = f/\mu = 1 \), i.e., \( f = \mu \) and \( \tau = 1 \) (Pigovian taxation), leads firms to choose the production-efficient productivity threshold.
Shallow pocket firms.

The rest of the section assumes that firms have shallow pockets but sufficient net worth for the investment to be undertaken \( A^{**} \leq A < A^* \). Here we distinguish between two situations:

### 3.2.1 Commitment

Suppose that the entrepreneur and his financiers can commit to a second-best contract specifying cutoff \( \overline{y} \). Let us show that Pigovian taxation \( \{ f = \mu, \tau = 0 \} \) implements the social optimum. To see this, note that in the special case of Pigovian taxation,

\[
V_E = N(\overline{y}) \quad \text{and} \quad V_I = N(\overline{y}) - R(\overline{y}),
\]

and so the maximization with respect to \( \overline{y} \) of \( V_E \) subject to \( V_I \geq (I + m) - A \) yields the same solution as Program I.

**Proposition 2.** Under commitment, Pigovian taxation \( \{ f = \mu \} \) yields the social optimum. In other words, the government extends a liability equal to the full externality to the intermediary.

*Remark:* Interestingly, Pigovian taxation, while a natural focal point, is not the only way to implement the optimum. Suppose that the rent can be written as \( r(y)1_{[y \geq \overline{y}]} \) (where \( r(y) \) is weakly increasing, with less-than-unitary increments). Then any layoff tax \( f \) in an interval around \( \mu \) implements the second best, namely any \( f \) satisfying\(^{20}\)

\[
\frac{\mu - f}{1 - G(\overline{y})} + r(\overline{y}) \geq \overline{y} - b > \frac{\mu - f}{1 - G(\overline{y})}.
\]

\(^{20}\)Let \( \hat{N}(\overline{y}) \) and \( \hat{R}(\overline{y}) \) denote the net productivity and rent for a given government policy \( \{ f, \mu, \tau \} \):

\[
V_E = \hat{N}(\overline{y}) \equiv \int_{\overline{y}}^{+\infty} [y - (b + \mu) - \tau] dG(y) - fG(\overline{y}),
\]

\[
V_I = \hat{N}(\overline{y}) - \hat{R}(\overline{y}) \equiv \int_{\overline{y}}^{+\infty} [y - (b + \mu) - \tau - r(y)] dG(y) - fG(\overline{y}).
\]

To implement the optimal allocation and when \( A^* > A \geq A^{**} \), \( \{ f, \mu, \tau \} \) must satisfy the government's budget constraint as well as:

\[
\hat{N}(\overline{y}) = I + m
\]

\[
\hat{N}(\overline{y}) - \hat{R}(\overline{y}) = I + m - A
\]

\[
\hat{N}'(\overline{y}) < 0
\]

\[
\hat{N}'(\overline{y}) - \hat{R}'(\overline{y}) \geq 0.
\]

Note, first, that the second condition is always satisfied, given that \( \hat{N} - \hat{R} = N - R \) and \( \overline{y} \) satisfies \( ^{8} \). Similarly, the first condition is always satisfied given that \( N(\overline{y}) = I + m \) and given the government’s budget constraint. So we are left with the government’s budget constraint and the last two conditions. Using the former and substituting into the latter, and using \( r(\overline{y}) \geq \overline{y} - b > 0 \) (since the optimum is at or to the left of \( \overline{y}^* \)), we obtain (10).
3.2.2 Ex post bargaining

Under ex post bargaining (Example 3), the firm and the investors share the gains from continuation and the cut-off $\bar{y}$ is given by the ex-post bilaterally-efficient continuation policy, given the government’s policy:

$$\bar{y} = [b + \mu] + \tau - f.$$  \hspace{1cm} (11)

- If $A^* > A > A^{**}$, firms choose the productivity threshold so as to satisfy the financing constraint and the free-entry condition with equality. Using the government budget constraint, $(\mu - f)G(\bar{y}) = \tau[1 - G(\bar{y})]$ and the bilaterally-efficient continuation policy rule:

$$f = \mu - [1 - G(\bar{y})](\bar{y} - b) < \mu.$$  \hspace{1cm} (12)

The contribution rate is lower than 1.

- If $A < A^{**}$, then only the financing constraint is binding. In this case, the optimal cut-off is given by $\bar{y}^*$, and $f = \mu - [1 - G(\bar{y}^*)](\bar{y}^* - b) < \mu$. The wage and the unemployment benefit must adjust one-for-one as $A$ decreases ($dw/\partial A = 1$ where $w = b + \mu$).

The key result is that when firms have shallow pockets the optimal contribution rate is below 1. Ex post bargaining creates an inefficiency in the ex ante bilateral contract between the entrepreneur and the investors.\footnote{In particular if the net value is positive ($V_E > I + m$) but pledgeable income under ex post bargaining is insufficient to support investment ($V_I < I + m - A$), the two parties would like ex ante to contract on a smaller share of the surplus for the entrepreneur in order to let the investors break even; such a commitment however is not credible as by assumption the inalienability of the entrepreneur’s human capital allows him to ex post blackmail the investors and to extract a fraction $\beta$ of the surplus under continuation.}

Taxation is less than the full Pigovian prescription ($f = \mu$) in order to put the investors in a stronger bargaining position and thereby increase pledgeable income, ultimately benefitting the workers.

Let us perform some comparative statics on the gross contribution rate (the same results hold for the net contribution rate).

For $A \in [A^{**}, A^*]$, 

$$c = 1 - \frac{[1 - G(\bar{y})](\bar{y} - b)}{\int_{\bar{y}}^{+\infty} (y - b) dG(y) - (I + m)}.$$ 

And so $\partial c / \partial \bar{y} < 0$ given that $\bar{y} \in [b, \bar{y}^*]$ implies that the numerator of the fraction is increasing and that the denominator is decreasing in $\bar{y}$. 

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Because $\bar{y}$ is given by $A = \int_{\bar{y}}^{+\infty} \beta(y - \bar{y})dG(y),$

\[
\frac{\partial c}{\partial \beta} < 0 \quad \text{and} \quad \frac{\partial c}{\partial A} > 0.
\]

The optimal scheme makes firms more accountable when governance improves and when firms become wealthier (have stronger balance sheets).

Finally, for $A < A^{**}$

\[
c = 1 - \frac{[1 - G(\bar{y}^{*})](\bar{y}^{*} - b)}{\int_{\bar{y}^{*}}^{+\infty} [y - b - \beta(y - \bar{y}^{*})]dG(y) - (I + m) + A}.
\]

It can be checked that the same comparative statics with respect to $\beta$ and $A$ hold.

**Proposition 3.** Under ex post bargaining, shallow-pocket firms must shut down more often ($\bar{y} \geq b$) than if they had substantial net worth (were deep-pocket firms). Layoffs are excessive from a first-best perspective, but are a necessary concession to investors. The optimal policy involves less internalization than that prescribed by Pigou’s rule: The gross and net contribution rates, $c \equiv f/\mu$ and $\tilde{c} \equiv (f - \tau)/\mu$, are lower than 1. These contribution rates increase with the strength of the firm’s balance sheet ($A$) and with the quality of their governance ($1 - \beta$).

**Remark:** Propositions 2 and 3 show that the key difference between Example 3 and Examples 1 and 2 is that the contract between entrepreneurs and financiers in Example 3 is a third-best contract, whose distance to the second-best one is influenced by public policy, and not the fact that the rent function is a function of $y$ only in Examples 1 and 2.\(^{28}\)

\(^{28}\)To illustrate this point, consider the situation in which the entrepreneur invests in creating private benefits for himself. The timing goes as follows: the financial contract specifies cut-off $\bar{y}$; then the entrepreneur incurs private cost $c$ in order to create private benefit $B(c)$ if he is retained; finally $y$ is realized and the contract is implemented (commitment case). The entrepreneur chooses $c$ so as to maximize $-c + [1 - G(\bar{y})]B(c)$ and so the gross rent can be written as

\[r(\bar{y})\mathbb{I}_{\{y > \bar{y}\}}\]

where $r(\bar{y}) \equiv B(c(\bar{y}))$ is a decreasing function of $\bar{y}$: an entrenched entrepreneur (i.e., with a contract specifying a low $\bar{y}$) has an incentive to create large private benefits. Despite the fact that $r$ depends on $\bar{y}$, Proposition 2 applies.
3.3 Heterogeneity and cross-subsidies

Suppose now that there are “strong” and “weak” firms, in proportions \((\rho, 1 - \rho)\). The productivity distributions for these two types are \(G_H(y)\) and \(G_L(y)\), respectively, where \(G_H\) dominates \(G_L\) in the sense of first-order stochastic dominance (the firms differ only in this distribution). While financial intermediaries can tell the two types apart, the government cannot; the best the government can do is to offer an incentive compatible menu \(\{f_H, \tau_H\}, \{f_L, \tau_L\}\) in which firms self select. By contrast, it is still socially optimal to offer undifferentiated unemployment insurance \(\mu\) and thereby induce a uniform wage \(w\) in the economy. With obvious notation, the policy must satisfy the government’s budget constraint:

\[
\rho \left[ G_H(\bar{y}_H)(f_H - \mu) + [1 - G_H(\bar{y}_H)]\tau_H \right] + (1 - \rho) \left[ G_L(\bar{y}_L)(f_L - \mu) + [1 - G_L(\bar{y}_L)]\tau_L \right] \\
= \rho \Delta_H + (1 - \rho) \Delta_L \geq 0.
\]

As usual, welfare (\(\mu\)) is equal to social surplus,

\[
\rho \int_{\bar{y}_H}^{+\infty} (y - b) dG_H(y) + (1 - \rho) \int_{\bar{y}_L}^{+\infty} (y - b) dG_L(y) - (I + m),
\]

minus the rents enjoyed by the firms compared to the absence of investment.

To compute the strong firms’ rent, which is determined by the payoff they get by mimicking the weak types, let us assume for simplicity that their productivity is sufficiently high that they face no credit constraint when mimicking the weak type; and so their cutoff is \(w + \tau_L - f_L\). [The reasoning generalizes to the case in which mimicking the low type makes the high type credit-constrained.] The high types’ rent is then

\[
\rho \left[ \int_{w+\tau_L-f_L}^{+\infty} [y - (w + \tau_L)] dG_H(y) - G_H(w + \tau_L - f_L) f_L \right] \\
= \rho \left[ \int_{w+f_L}^{+\infty} [y - w + f_L] dG_H(y) - [1 - G_L(\bar{y}_L)] f_L - G_L(\bar{y}_L) \mu - \Delta_L \right],
\]

where

\[
\tilde{f}_L = f_L - \tau_L
\]

is the net layoff tax for the weak types. Furthermore,

\[
\bar{y}_L \geq w - \tilde{f}_L
\]

(with strict inequality if the weak types are credit constrained).

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Note that, for a given $\bar{y}_L$, the derivative of the strong types’ rent with respect to $\hat{f}_L$:

$$\rho \left[ [1 - G_H(w - \hat{f}_L)] - [1 - G_L(\bar{y}_L)] \right]$$

is strictly positive, and so it is optimal to choose $\hat{f}_L$ as small as possible, that is such that\(^{29}\)

$$\bar{y}_L = w - \hat{f}_L.$$  

Maximizing the difference between social surplus and rent with respect to $\bar{y}_L = w - \hat{f}_L$ and using first-order stochastic dominance ($G_H(\bar{y}_L) < G_L(\bar{y}_L)$) yields:

$$\bar{y}_L > b,$$

or equivalently

$$\hat{f}_L < 0.$$  

Thus the net contribution rate $\tilde{c}_L \equiv (f_L - \tau_L)/\mu$ is smaller than 1. By contrast, the high type’s net contribution rate $\tilde{c}_H \equiv (f_H - \tau_H)/\mu$ is unitary.

**Proposition 4.** Firm heterogeneity leads to less than full Pigovian taxation, and this even under commitment.

Proposition 4 extends a similar result obtained by Blanchard and Tirole (2008) in the absence of agency cost. Cross-subsidization through less-than-full-Pigovian taxation thus also holds in the presence of extended liability. Intuitively, a high contribution rate/internalization generates high profits for those firms that are unlikely to lay off workers. Incomplete Pigovian taxation helps redistribute income from firms to workers.

### 3.4 Extension to other forms of monitoring, and incentives to monitor

As we discussed, the monitoring activity can accomplish two purposes: contributive-capability enhancement and ex-ante intervention. The first purpose (that stems from the intermediary’s prevention of tunneling and monitoring of the repayment) can be modeled by a rent function $R(y, \bar{y}, m)$ that depends on the intensity of monitoring $m$, which we

\(^{29}\)The rent is also reduced by decreasing $\Delta_L$, i.e., by increasing $\Delta_H = -(1 - \rho)/\rho \Delta_L$. Note though that the strong type’s rent must be equal to

$$\int_b^{+\infty} (y - b)dG_H(y) - (I + m + \Delta_H),$$

so $\Delta_L$ and $\Delta_H$, for given $\bar{y}_L$, $\hat{f}_L$ and $\mu$ are given by this equality
now take to be variable instead of fixed.$^{30}$ Ex-ante intervention consists in monitoring in order to exercise control rights. This exercise shifts the distribution of income $G(y, m)$ (advising), possibly by constraining the entrepreneur’s rent $R(y, \bar{y}, m)$ (disciplining). It can also affect the expected opportunity value $b(m)$; as we discussed, the firm can in the labor context provide general training that enables the worker to find a job again more quickly when unemployed, and, in the environmental context, reduce the probability that the site needs to be cleaned up when closed. Crucially, we assume that $b(m)$ is observable by the state, a reasonable assumption in these two contexts.$^{31}$

We only sketch the analysis, which follows the steps of our previous treatment. Let

$$N(\bar{y}, m) \equiv \int_{\bar{y}}^{+\infty} [y - b(m)]dG(y, m) + b(m)$$

and

$$R(\bar{y}, m) \equiv \int_{\bar{y}}^{+\infty} R(y, \bar{y}, m)dG(y, m).$$

As in Section 3.1, one can obtain an upper bound for social (worker) welfare by noting that full insurance ($w = b(m) + \mu$) is still optimal and by solving Program I:$^{32}$

$$\max \{w\}$$

$$\begin{align*}
\text{s.t.} \quad & w \leq N(\bar{y}, m) - (I + m) \\
& w \leq N(\bar{y}, m) - R(\bar{y}, m) - (I + m) + A.
\end{align*}$$

(13)

As in Section 3.2.1, let us implement the corresponding upper bound in the commitment case through Pigovian taxation. From now on, we denote by $\{\bar{y}, m\}$ the cutoff and monitoring intensity given by Program I’, and by $\{\tilde{y}, \tilde{m}\}$ an arbitrary choice by the coalition. Pigovian taxation requires an ex-post assessment of the externality:$^{33}$

$$f(\tilde{m}) \equiv \mu + [b(m) - b(\tilde{m})].$$

(15)

The socially optimal value, $m$, and the chosen level, $\tilde{m}$, will be equal under the implementing scheme, and so $\tau = 0$.

---

$^{30}$m can be single- or multi-dimensional for the purpose of our analysis.
$^{31}$The opportunity value may be a random function of $m$. Because of risk neutrality, we care only about the expressed value $b(m)$.
$^{32}$Because $b$ is endogenous, we must work with $w$ as the objective function rather than $\mu$.
$^{33}$As is the case in the environment context, and also the case in the labor context under US-style experience rating.
Let us assume for example that constraints (13) and (14) are both binding and let \( \theta \) denote the ratio of the two multipliers. From Program I, the socially optimal values \( \{\tilde{y}, \tilde{m}\} \) maximize
\[
N(\tilde{y}, \tilde{m}) - \tilde{m} - \frac{\theta}{1 + \theta} R(\tilde{y}, \tilde{m}).
\]
And because Pigovian taxation eliminates externalities on third parties (here the workers), \( (\tilde{y}, \tilde{m}) \) also maximizes\(^{34}\)
\[
\frac{|V_E - (I + m)| + \theta |V_I - (I + m - A)|}{1 + \theta} = N(\tilde{y}, \tilde{m}) - \tilde{m} - \frac{\theta}{1 + \theta} R(\tilde{y}, \tilde{m}) - \left[I + [\mu + [b(m)] - \frac{\theta}{1 + \theta} A\right].
\]
(16)

Thus, if the entrepreneur and the investors can contract on \( \{\tilde{y}, \tilde{m}\} \), they choose \( \tilde{y} = \tilde{y} \) and \( \tilde{m} = m \).

When the entrepreneur and the investors can contract on the threshold (pick \( \tilde{y} \)) but not on the monitoring intensity, the monitor must be incentivized to choose \( \tilde{m} = m \) if entrepreneurial welfare is to be maximized given the state’s policy. The monitor properly internalizes the tax \( f \) as long as she is made accountable for it. But the monitor’s incentive to monitor must further be aligned with that of the coalition (given by (16)) with respect to the impact of \( \tilde{m} \) on \( G \) and \( R \). Indeed, Pagano and Roell (1998) show that disciplining monitors have excessive incentives to monitor if they receive the entire pledgeable income, as they do not internalize the entrepreneurial rent.\(^{35}\) Conversely, one may intuit that the pledgeable income may, for the same reason, provide an advisor with insufficient incentives. This suggests that the coalition will set some incentive scheme \( z(y) \) for the monitor, that differs from pledgeable income. Indeed, letting the monitor pay the layoff tax, the monitor solves:
\[
\int_{\tilde{y}}^{+\infty} z(y) dG(y, \tilde{m}) - G(\tilde{y}, \tilde{m}) f(\tilde{m}) - \tilde{m}.
\]
(17)

Suppose, first, that monitoring may change the distribution of income (advising) and the worker’s opportunity cost, but not the entrepreneur’s rent. Then, for (16) and (17) to deliver the same solution and therefore for \( \tilde{m} \) to be equal to \( m \), the coalition can choose:
\[
z(y) = y - [b(m) + \mu] - \frac{\theta}{1 + \theta} R(y, \tilde{y}).
\]

\(^{34}\)Note that the wage \( w = \mu + b(m) \) is a market wage and is accordingly independent of the actual choice \( \tilde{m} \) by the firm. So for example \( V_E \equiv -G(\tilde{y}) [\mu + b(m) - b(\tilde{m})] + \int_{\tilde{y}}^{+\infty} [y - [\mu + b(m)] dG(y, \tilde{m}).
\]

\(^{35}\)The monitor’s role is then not to capture the rent, but to shift \( G \) or monitor compliance of \( b.\)
In some other models of corporate finance by contrast, an increase in monitoring intensity reduces the expected managerial rent (and induces a first-order-stochastic dominance improvement in the distribution of $G$ or an increase in $b$). Still, it is in general feasible to construct a function $z(\cdot)$ providing the monitor with the proper incentives.\textsuperscript{36}

Finally, we consider implementation in the ex-post bargaining case. Again, we let $\{\mu, \bar{y}, m\}$ denote the socially optimal levels. If $b$ is a monotonic function of $\tilde{m}$, then the government can prevent deviations away from $m$ and the optimal allocation can be implemented. Suppose, next, that $b$ is independent of $\tilde{m}$. The state chooses $\mu$ and sets layoff tax $f$ such that

$$f = \mu - [1 - G(\bar{y}, m)][\bar{y} - b],$$

as in Section 3.2.1, the payroll tax $\tau$ being chosen to satisfy the government’s budget constraint,

$$[1 - G(\bar{y}, m)]\tau = G(\bar{y}, m) [\mu - f].$$

Note that if the optimal solution $(\tilde{y}, \tilde{m}) = (\bar{y}, m)$ is to be implemented, there is now an externality on the government’s budget constraint, unlike under Pigovian taxation: If $\tilde{m}$ shifts $G$, then a lower intensity of monitoring makes layoffs more likely and creates a government deficit. It would therefore seem (but I have not performed the analysis) that the state is vulnerable to a choice by the coalition that increases the probability of layoff; in that case, the state would have too few instruments and the socially optimal policy might further reflect the necessity to induce $m$ from the coalition.

**Proposition 5.** The analysis broadly carries over to an endogenous monitoring effort, meant to enhance the firm’s contributive capability and/or to exercise control rights so as to boost income and/or to reduce the externality: Under weak conditions, Proposition 1 (socially optimal allocation) and 2 (Pigovian implementation under commitment) carry through; so does Proposition 3 (implementation under ex post bargaining) if monitoring affects the externality.

\textsuperscript{36}For any increasing $z(y)$, $K(\tilde{m}) \equiv \int_{\bar{y}}^{+\infty} z(y) dG(y, \tilde{m})$ is increasing in $\tilde{m}$ if $\tilde{m}$ shifts $G$ in the FOSD sense. To implement $\tilde{m} = m$, any increasing function $z(\cdot)$ can be scaled up or down $(\lambda z(\cdot))$ to achieve the desired $K'(m)$. The function $z$ must further be selected so as to create enough curvature. We leave the technical details to the reader.
4 The extension of liability to other industrial companies: snowball effects

Liability is often extended to industrial partners as well as financial intermediaries. This section focuses on a specific issue: the possibility of snowball effects. While our treatment of financial intermediaries has so far assumed perfect diversification, this section allows the guarantor’s activity to be affected by the failure of the tortfeasor. In the context of unemployment insurance this lack of diversification raises the concern that even if the guarantor can pay the layoff taxes, it will do so at the expense of its own activities. For example, in order to pay the layoff tax associated with the closing down of firm 2, firm 1 may choose to close its otherwise healthy activity.

4.1 n-activity set-up

There are \( n \) industrial firms, \( i = 1, \ldots, n \), each with one activity and one job. Each firm has asset \( \bar{f} \equiv A - I \geq 0 \), invests \( I \) and hires one worker.

We will assume that firms perfectly monitor each other and so there is no agency cost among them. They form a coalition and thus can be described as a single firm (the “coalition”). Thus, an alternative interpretation of this model is one of a single firm with multiple activities or plants.

Stochastic structure. We make the following assumption on the joint distribution of productivities:

**Nested uncertainty structure.** With probability \( p_k (k \in \{0, \ldots, n\}, \sum_{k=0}^{n} p_k = 1, p_0 > 0) \), exactly \( k \) jobs are potentially viable; their common productivity \( y \) is drawn from distribution \( G_k(y) \) with density \( g_k(y) \). The \( (n-k) \) other jobs are very costly socially and must be suppressed.

The lack of diversification is consistent with the idea that firms with related activities are best placed to monitor each other. The nested uncertainty structure corresponds to a two-dimensional adverse selection problem: The government observes neither the number of viable activities \( k \) nor the productivity \( y \) in those activities.

The number of non-viable activities, \( n-k \), can be called the *breadth* of the productivity shock. The *depth* of the productivity shock for a given \( k \) will be measured by (minus) the hazard rate \( [1 - G_k(y)]/g_k(y) \). The breadth and the depth of the productivity shock

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37 This is an important consideration in practice. A case in point is extended liability regulation for environmental matters in the US.

38 Following Section 3.4, though, the information structure could be endogenized.
are positively (negatively) correlated if a smaller (larger) number of viable activities corresponds to a worse distribution of the productivity parameter $y$ in those viable activities: that is (for positive correlation) when $\ell < k$:

$$\frac{1 - G_\ell(y)}{g_\ell(y)} \leq \frac{1 - G_k(y)}{g_k(y)} \quad \text{for all } y.$$ 

This condition can be rewritten in terms of elasticities of (the supply of) layoffs:

when $\ell < k$: $\frac{yg_\ell(y)}{1 - G_\ell(y)} \geq \frac{yg_k(y)}{1 - G_k(y)}$ for all $y$.

A positive correlation then refers to a higher elasticity of (per-activity) layoffs when fewer activities are viable.

Public policies. On the government side, the state pays unemployment benefits $\mu$. The state observes only the number of layoffs. When the coalition retains $\ell$ employees and dismisses the $(n - \ell)$ others, the state levies payroll tax $\ell \tau_\ell$ (that is, $\tau_\ell$ per employee) and layoff tax $(n - \ell) f_{n-\ell}$ (that is, $f_{n-\ell}$ per layoff). Naturally, what matters is the total payment

$$T_\ell \equiv \ell \tau_\ell + (n - \ell) f_{n-\ell}$$
made by the coalition. Indeed, and assuming as earlier the absence of macroeconomic uncertainty (there are many such coalitions), the government’s budget constraint writes:

$$\sum_{k=1}^{n} p_k \left[ (1 - G_k(\bar{y}_k)) \left[ k\tau_k + (n - k)(f_{n-k} - \mu) \right] + G_k(\bar{y}_k)n(f_n - \mu) \right] \geq 0. \quad (18)$$

where $\bar{y}_k$ is the cut-off for the $k$ viable activities. In the economy, a fraction $p_k$ of coalitions have $k$ viable jobs. A fraction $G_k(\bar{y}_k)$ of these shut down, creating deficit $n(\mu - f_n)$ for the unemployment insurance fund. A fraction $1 - G_k(\bar{y}_k)$ keep their $k$ viable jobs, generating $k\tau_k$ in payroll taxes and net cost $(n - k)(\mu - f_{n-k})$ in unemployment benefits. Let $\ell(k, y) \leq k$ denote the number of workers kept in state $(k, y)$. Similarly, let $U(k, y)$ denote the ex post rent of the coalition when it has $k$ viable activities with productivity $y$ each:

$$U(k, y) = \ell(k, y)y - T_\ell(k, y) = \max_{\{0 \leq \ell \leq k\}} \{\ell y - T_\ell\}.$$ 

From the envelope theorem,

$$U(k, y) \geq \int_{-\infty}^{y} \ell(k, z)dz.$$  

By the same argument as before, the state is in a position to fully insure workers, and so it does. This implies that unemployment benefits are the same for all unemployed, irrespective of the state of the coalition that laid them off, and given by $\mu = w - b$. 

32
4.2 The snowball effect: heuristics

To illustrate the snowball effect and the possibility that layoff taxes encourage layoffs, consider the case of two firms/jobs (n = 2). For convenience, refer to coalitions for which both activities are viable (k = 2) as “strong coalitions” (although, of course, the productivity y of the two jobs may be low and so a strong coalition may still fire its workers), and to coalitions for which a single activity is viable (k = 1) as “frail coalitions.”

Consider the ex-post strong coalitions’ choice of laying off zero or two workers. If they keep both workers, their profit is given by 2(y - w - τ₂). If they lay both workers off, their profit is given by -2f₂. Thus, their threshold productivity is given by:

$$\bar{y}_2 = (w + \tau_2) - f_2.$$  

Ex-post frail coalitions face the choice of laying off one or two workers. If they lay one worker off, their profit is given by y - (w + τ₁) - f₁. If they lay both workers off, their profit is given by -2f₂. Thus, their threshold productivity is given by:

$$\bar{y}_1 = (w + \tau_1) - (2f_2 - f_1).$$  \hspace{1cm} (19)

Equation (19) shows that the frail coalition is more likely to shut down altogether if the layoff-tax schedule is more concave, i.e., if f₁ grows keeping 2f₂ constant (which it will be, as we will show that 2f₂ = 2f for f small enough when n = 2).

This expression embodies the concern described earlier: An increase in the layoff tax f₁ encourages layoffs by frail coalitions. It creates a “spillover” or “snowball effect” on the otherwise healthy activity. Suppose for example that μ ≤ 2f < 2μ. Then full experience rating is feasible for one layoff (f₁ = μ), but not for two (f₂ ≤ f < μ). Applying full experience rating to a single layoff implies a “layoff tax discount” and makes the second layoff rather cheap; for example if μ = 2f, then the second layoff is free.

Note also that if the state levies the maximum fine (f₁ = 2f) for a single layoff, then the coalition’s continuation policy is limited by its ability to pay the worker: \( \bar{y}_1 = w + \tau_1 \). With a lower layoff tax f₁, the state could have made it possible (as well as privately desirable) for the coalition to cover the wage bill when the productivity is just below this wage bill. In this sense, the snowball argument captures the idea that layoff taxes take away what would have been paid to remaining workers.

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\(^{39}\)We here assume that strong coalitions keep both workers or none. See section 4.3 for the justification of this assumption.

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33
4.3 The optimal allocation

Using the result that workers are fully insured, the state designs a mechanism so as to maximize $U(b + \mu)$. The workers’ benefit ($n\mu$) over and above their reservation wage ($nb$) is equal to the expected ex post value minus the coalition’s utility:

$$\sum_{k} p_k \int_{-\infty}^{+\infty} \ell(k, y)(y - b)dG_k(y) + n\bar{f} - U,$$

where $U$ is the coalition’s ex ante utility.\footnote{One can envision the coalition as handing over its free cash flow $n\bar{f}$ to the government and deriving an ex post rent $U$. We will emphasize a different interpretation in the implementation, though.}

An upper bound on the workers’ utility is therefore obtained by solving:

$$\max \quad \{\ell(\cdot, \cdot) \mid \ell(k, y) \leq k \text{ for all } y\} \quad \{\sum_{k} p_k \int_{-\infty}^{+\infty} \ell(k, y)(y - b)dG_k(y) + n\bar{f} - U\}$$

s.t.

$$U \geq n(I + \bar{f}) \quad (20)$$

$$U \geq \sum_{k} p_k \int_{-\infty}^{+\infty} \ell(k, y)[1 - G_k(y)]dy. \quad (21)$$

Let $\theta$ and $\lambda$ denote the (non-negative) shadow prices of constraints (20) (coalition’s participation constraint) and (21) (coalition’s incentive constraint), respectively.

The first-order conditions are:

$$\theta + \lambda = 1 \quad (22)$$

and

for all $k$, \( \ell(k, y) \equiv \begin{cases} k \text{ if } y \geq \bar{y}_k \\ 0 \text{ if } y < \bar{y}_k \end{cases} \)

where

$$\frac{\bar{y}_k - b}{\bar{y}_k} \equiv \lambda \frac{[1 - G_k(\bar{y}_k)]}{\bar{y}_kg_k(\bar{y}_k)} \quad (23)$$

Condition (23) can be given a Ramsey pricing interpretation: The solution involves a trade-off between efficiency (which would call for $\bar{y}_k = b$) and rent extraction (related to the elasticity of layoffs, $g_k(\bar{y}_k)/[1 - G_k(\bar{y}_k)]$).

Thus, $\bar{y}_k \geq b$ and the coalition maintains $k$ jobs or none.

If $G_k(y) = G(y)$ for all $k$ (symmetric distributions), condition (23) shows that the thresholds $\bar{y}_k$ must all be equal:

$$\bar{y}_k = \bar{y} \text{ for all } k.$$
Finally, if breadth and depth are positively (negatively) correlated, $\bar{y}_k$ is increasing (decreasing) in $k$. Intuitively, if the layoff behavior is very elastic when few activities are viable, it is important to prevent layoffs, that is to have a relatively low cutoff, $\bar{y}_k$, when $k$ is small.

**Full internalization.** Full internalization occurs when $\bar{y}_k = b$, or equivalently $\lambda = 0$. Only the firm’s participation constraint (20) is then binding, and so

$$\bar{f} \geq \bar{f}^*, \quad (24)$$

where

$$n(I + \bar{f}^*) = \Sigma_k p_k \int_{b}^{+\infty} (y - b) dG_k(y). \quad (25)$$

Deep-pocket coalitions (as defined by (24)) fully internalize the cost of their layoffs.

**Less-than-full internalization.** For shallow-pocket coalitions ($\bar{f} < \bar{f}^*$), $\lambda > 0$ and so $\bar{y}_k > b$. For $\bar{f}_* \leq \bar{f} < \bar{f}^*$ (where $\bar{f}_*$ is defined below), both constraints are then binding ($\theta$ and $\lambda$ are both strictly positive). Furthermore,

$$\frac{d\mu}{df} = 1 - \theta = \lambda > 0,$$

and so, like in the one-activity case, workers are better off when firms have stronger balance sheets.

When $\bar{f} < \bar{f}_*$, then $\lambda = 1$ and $\theta = 0$. The thresholds $\bar{y}_k$ are given by the “monopoly pricing equations”:

$$\frac{b - \bar{y}_m}{\bar{y}_k} = \frac{1 - G_k(\bar{y}_m)}{\bar{y}_k g_k(\bar{y}_m)}.$$  

The state in a sense maximizes the tax revenue, so as to redistribute it to workers. $\bar{f}_*$ is given by

$$n(I + \bar{f}_*) = \Sigma_k p_k \int_{\bar{y}_m}^{+\infty} (y - \bar{y}_m) dG_k(y).$$

While it delivers a number of new insights, the multi-activity analysis also generalizes the single-activity one: Full internalization obtains for firms with strong balance sheets: layoffs become more likely as the balance sheet deteriorates, until the pledgeable income is maximized.

**Proposition 6.** Under the nested uncertainty structure, the optimal cutoffs satisfy:

$$\frac{b - \bar{y}_k}{\bar{y}_k} = \lambda \left[ \frac{1 - G_k(\bar{y}_k)}{\bar{y}_k g_k(\bar{y}_k)} \right].$$
(i) The cutoffs are increasing (\(s_1 \leq s_2 \leq \cdots \leq s_n\)) if the breadth and depth of the productivity shock are positively correlated, and decreasing if they are negatively correlated. 

(ii) Full internalization (\(s_k = b\) for all \(k\) as \(\lambda = 0\)) occurs when balance sheets are strong (\(f \geq f^*\)), while \(0 < \lambda < 1\) for \(f^* < f < f^*\) and \(\lambda = 1\) for \(f \leq f^*\).

### 4.4 The shape of the layoff tax schedule: how much extended liability?

We now turn to the implementation of the allocation derived in Section 4.3 through a “layoff tax schedule”. The implementation of the optimal policy in the case of deep-pocket coalitions is straightforward: It suffices to levy layoff tax \(f = \mu\) per layoff. The government’s budget is then balanced and the firms select the socially optimal destruction margin (\(s_k = b\)). Let us therefore turn to the more interesting case of shallow-pocket firms.

As we noted earlier, firms care only about the total payment \(T_k\) when \(\ell\) activities remain in operation. Thus, and quite generally, we cannot identify a layoff tax schedule without making an assumption on the payroll tax schedule. We accordingly (and reasonably) assume that the payroll tax schedule is linear:

\[
\tau_\ell = \tau \quad \text{for all } \ell.
\]

Let

\[
\mathcal{F}_m \equiv mf_m
\]

denote the total layoff tax for \(m\) layoffs.

We begin with the case of independent distributions, and then generalize the analysis.

**Independent distributions.**

Suppose that \(G_k(y) = G(y)\) for all \(k\). The optimal policy can then be implemented by *linear* payroll and layoff taxes:

\[
\tau_k = \tau \text{ and } f_k = f = f^* \quad \text{for all } k. \tag{26}
\]

For any \(k\), the choice of a cut-off is the same as in a single-activity firm:

\[
\bar{y} = (w + \tau) - f. \tag{27}
\]

---

\(^{41}\)To compute \(\tau\) and \(f\), note that, using (18) and (26), the government’s budget constraint becomes in the independent case:

\[
|\Sigma_k p_k |1 - G(\bar{y})| \|	au = |\Sigma_k p_k |n - |1 - G(\bar{y})||\mu - f\|.
\]

For \(f \in [f^*, f^*]\), in equilibrium, \(\bar{y}\) is given by (20) and (21) combined:

\[
[\Sigma_k p_k] \left[ \int_\bar{y}^{+\infty} (y - \bar{y}) dG(y) \right] = n(1 + f).
\]
Negative correlation between depth and breadth.

Let us choose the layoff tax schedule \( \{ \mathcal{F}_n \} \) and the per-worker payroll tax \( \tau \) so that with \( k \) viable activities, the cutoff \( \bar{y}_k \) makes keeping the \( k \) activities and shutting down a matter of indifference:

\[
k(\bar{y}_k - w - \tau) - \mathcal{F}_{n-k} = -\mathcal{F}_n. \tag{28}
\]

To check for incentive compatibility, we must verify that all “downward incentive constraints” (DIC) are satisfied:

\[
k(\bar{y}_k - w - \tau) - \mathcal{F}_{n-k} \geq \ell(\bar{y}_k - w - \tau) - \mathcal{F}_{n-\ell} \quad \text{for all } \ell < k. \tag{DIC}
\]

This amounts to

\[-\mathcal{F}_n \geq \ell(\bar{y}_k - \bar{y}_\ell) - \mathcal{F}_n,
\]

which is indeed satisfied if \( \bar{y}_k \) is decreasing in \( k \) (negative correlation).

Rewriting (28):

\[
\frac{\mathcal{F}_n - \mathcal{F}_{n-k}}{k} = -(\bar{y}_k - w - \tau),
\]

and so the average cost of incremental layoffs (full closure) is increasing in \( k \), i.e., decreasing with the number of layoffs. Put differently, the per-layoff tax decreases with the number of layoffs.

Positive correlation between depth and breadth.

Let us now choose the layoff tax schedule \( \{ \mathcal{F}_k \} \) and the per-worker payroll tax \( \tau \) so that the “downward adjacent incentive compatibility constraints” (DAIC) are binding.

For all \( k \):

\[
k(\bar{y}_k - w - \tau) - \mathcal{F}_{n-k} = (k - 1)(\bar{y}_k - w - \tau) - \mathcal{F}_{n-k+1}
\]

or

\[
\bar{y}_k - (w + \tau) = -(\mathcal{F}_{n-k+1} - \mathcal{F}_{n-k}). \tag{DAIC}
\]

\( \mu \) (and therefore \( w = b + \mu \)) is given by

\[
\mu = [\Sigma_k p_k k] \left[ \int_{\bar{y}}^{+\infty} (y - b) dG(y) \right] - nI.
\]

When \( \mathcal{T} < \mathcal{T}_* \), then \( \bar{y} = \bar{y}^m \) and

\[
\mu \equiv \Sigma_k p_k k |1 - G(\bar{y}^m)| |\bar{y}^m - b| + n f.
\]

Finally, condition (26) together with the expression of \( \mu \) just derived yields \( f = \mathcal{T} \) and \( \tau \).

\(^{42}\)Note that if (DIC) is satisfied for \( \bar{y}_k \), then it is a fortiori satisfied for all \( y > \bar{y}_k \).
Let us show that if the cutoffs are increasing in \( k \) (positive correlation), then (DAIC) implies more generally that the (DIC) constraints are satisfied:

\[
k(\bar{y}_k - w - \tau) - \mathcal{F}_{n-k} = (k-1)(\bar{y}_{k-1} - w - \tau) - \mathcal{F}_{n-k+1} + (k-1)(\bar{y}_k - \bar{y}_{k-1})
\]

\[
= (k-2)(\bar{y}_{k-1} - w - \tau) - \mathcal{F}_{n-k+2} + (k-1)(\bar{y}_k - \bar{y}_{k-1})
\]

\[
= (k-2)(\bar{y}_k - w - \tau) - \mathcal{F}_{n-k+2} + (\bar{y}_k - \bar{y}_{k-1})
\]

\[
\geq (k-2)(\bar{y}_k - w - \tau) - \mathcal{F}_{n-k+2}
\]

if \( \bar{y}_k \geq \bar{y}_{k-1} \). And so forth. Thus if the cutoff \( \bar{y}_k \) is increasing in \( k \) (positive correlation), the allocation is incentive compatible.

Furthermore, the marginal layoff tax, \( \mathcal{F}_{n-k+1} - \mathcal{F}_{n-k} \), is decreasing in \( k \), that is increasing in the number of layoffs: the layoff tax schedule is convex.

Extended liability. Recall that the model admits two interpretations: one with a single, multi-activity firm, and another with \( n \) single-activity firms. The latter allows us to discuss extended liability. Note first that in the case of independent distributions, it is optimal to treat the firms separately and levy per-layoff tax \( f \).\(^{43}\) There is no extended liability.

More generally, let us posit that each firm that shuts down pays its maximal layoff tax \( f \) and let us normalize the layoff tax schedule in a natural way:\(^{44}\)

\[
\mathcal{F}_n = n \bar{f} \quad \text{and} \quad \mathcal{F}_0 = 0.
\]

When the per-layoff tax decreases with the number of layoffs, the layoff tax paid by a firm that retains its workers while \( m \) firms shut down,

\[
\frac{\mathcal{F}_m - m \bar{f}}{n-m},
\]

is positive: liability is extended to other firms. It however is extended less and less as the number of layoffs increases.

\(^{43}\)We assume that \( \bar{f} \leq \bar{f}^* \) in the discussion of extended liability. Otherwise, full internalization at the firm level obtains and there is no extended liability.

\(^{44}\)When the (DAIC) constraints are binding (convex tax schedule), \( \mathcal{F}_n - \mathcal{F}_0 = - \sum_{a=1}^{n} (\bar{y}_a - w - \tau) \); choose the payroll tax \( \tau \) so as to obtain \( \mathcal{F}_0 = 0 \). In the case of a concave tax schedule, \( \mathcal{F}_n - \mathcal{F}_0 = n(\bar{y}_n - w - \tau) \), so set \( \tau \) such that

\[
\bar{f} = \bar{y}_n - (w + \tau).
\]

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When the layoff tax schedule is convex by contrast, the firms that retain their workers receive a break. This can be interpreted as a reduction in the payroll tax when other firms lay their workers off.

**Proposition 7.** Extending liability to other industrial companies raises the possibility of “snowball effects”. Under the nested uncertainty structure, when $\bar{f} < \bar{f}$, the state may not apply full experience rating even when feasible (i.e., in case of partial layoffs).

(i) With independent distributions, the optimal allocation can be implemented through a linear layoff-tax-cum-payroll-tax scheme.

(ii) The per-layoff tax decreases with the number of layoffs (the layoff tax schedule is convex) if the depth and breadth of the productivity shock are negatively (positively) correlated.

(iii) In the multi-firm interpretation of the model, liability is extended if and only if the depth and breadth of the productivity shock are negatively correlated.

## 5 Summary and conclusion

Should firms and their guarantors be made accountable for the full externality, as Pigou suggested? The argument in favor of this “generalized Pigou principle” is that the government should delegate the monitoring to a private party who, if made fully accountable, will write efficient second-best contracts with the firm. We validated this “delegation principle”, but unveiled two limitations: First, it no longer holds if contracts between financial intermediaries and firms are not second-best efficient, as we illustrated through the case of ex post renegotiation. The public policy then plays a dual role of promoting Pigovian internalization and of facilitating the firm’s access to a monitor. The contribution rate should then be less than unitary. Second, when firms are heterogenous, a limited taxation of externalities (again, a contribution rate below one) cross-subsidizes the weaker firms and limits the rents of stronger ones.

Second, tapping profits in another (monitoring) industrial company has its limits, as extending liability may extend externalities. Snowball effects were studied in a tractable nested-information-structure model of independent interest, showing that the liability should not be extended when the partners’ failure conveys bad or no news about the guarantors’ own activities. Overall, the case for extended liability is much weaker when the guarantor himself has a weak balance sheet and the structure of shocks makes propagation likely.

It was further shown that for a single, multi-activity firm, the per-layoff tax should decrease (be constant, increase) with the number of layoffs if the breadth and depth of productivity shocks are negatively (not, positively) correlated.
 Needless to say, this paper does not exhaust the topic. Another reason why the private sector's exposure might optimally be limited is that the government, through its policies, affects the probability of creation of externalities: in the unemployment application, firm productivity and unemployment length depend on public policy; similarly, authorities impact the environmental and industrial accident externalities, or the level of a bankrupt bank's liabilities. Then, the state may have to be held accountable. Finally, the stochastic environment in our analysis of snowball effects (within a firm or a coalition of firms under extended liability) generated a two-dimensional adverse selection problem. More generally, the Pigovian analysis should develop multidimensional adverse selection modeling in order to analyze the optimal taxation of externalities in multi-activity environments. These models as our two-dimensional one, will also prove most useful to deepen our rather imperfect understanding of the role of governments in alleviating the risk and impact of financial contagion.

Limited access to capital markets is an ubiquitous feature of developed and less-developed economies. The widespread policy concerns about the impact of layoff or environmental taxes on financially fragile companies or of stringent prudential regulation on financially fragile financial institutions deserve more attention from economic theorists and empiricists. We hope that this paper will encourage further work in this direction.
References


